ID #219

# Dbar-N interaction from Lattice QCD at physical point

INPC 2025

May 27, 2025 @ Daejeon Convention Center


iTHEMS
RIKEN interdisciplinary Theoretical & Mathematical Sciences

Wren Yamada

RIKEN iTHEMS

For HAL QCD collaboration

# Content

This Talk

Study the $\bar{D}N$ interaction with Lattice QCD using configurations generated at "physical point" (pion mass $m_\pi = 137$ MeV) by the HAL QCD collaboration

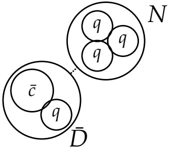
- $\bar{D}N$ potential (LO derivative expansion)
- Scattering quantities (e.g. s-wave phase shift, scattering length, effective range)

Collaborated work by the HAL QCD group.
Especially, Yan Lyu, Kotaro Murakami and Takumi Doi. paper in preparation

- Introduction
- Method: HAL QCD, Lattice Setup
- Results

$\boxed{\bar{D}N \text{ system}}$

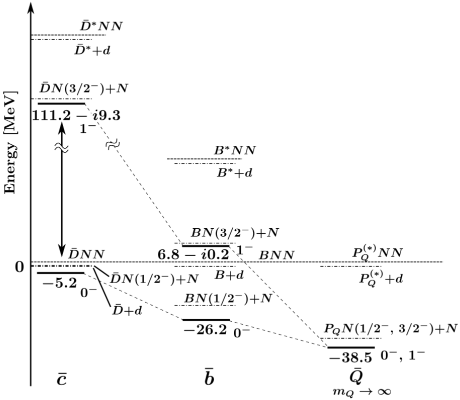
- No $q\bar{q}$ annihilation:
  Bound state → Pentaquark $\Theta_c$



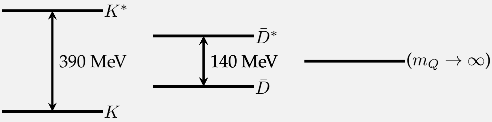

- Charmed nuclei, $\bar{D}NN$...
  Approximate degenarate states
  from HQSS ($j_l \geq 1$)
- in-medium effects to the $\bar{D}$-meson
  e.g. mass & width modification, impurity effects

Hosaka, Hyodo, Sudoh, Yamaguchi, Yasui, PPNP 96, 88 (2017)

*A. Hosaka et al. / Progress in Particle and Nuclear Physics 96 (2017) 88–153*



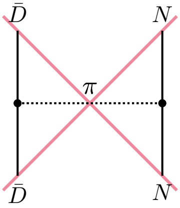
Coupling to the $\bar{D}^*N$ channel (is thought to) enhance attraction ($\leftrightarrow$ $KN$ system)

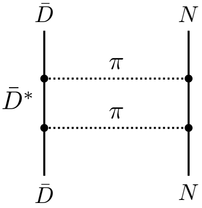



- Contact SU(4)$_\mathrm{F}$ "Tomozawa-Weinberg"

$$C^{(I=0)}_{\bar{D}N\bar{D}N} = 0 \quad \to \quad C^{(I=0)}_{ij} = \begin{bmatrix} 0 & \sqrt{12} \\ \sqrt{12} & -4 \end{bmatrix} \simeq \begin{bmatrix} -6 & 0 \\ 0 & 2 \end{bmatrix}$$

- $\pi$-exchange

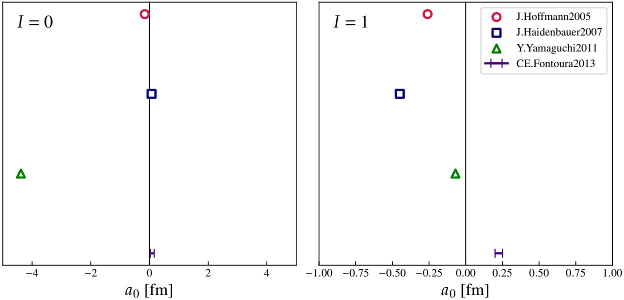


# Current Status: Models

- No consistency of existance/absence of bound state, attractive/repulsive phase shift behavior between models

| Model | Bound State ($I = 0$) [MeV] | Bound State ($I = 1$)[MeV] |
|---|---|---|
| Hoffmann 2005 | × | × |
| Haidenbauer 2007 | × | × |
| Gamermann 2010 | 2805 | × |
| Yamaguchi 2011 | 2804 | × |
| Yamaguchi 2022 | 2804 | 2800 |





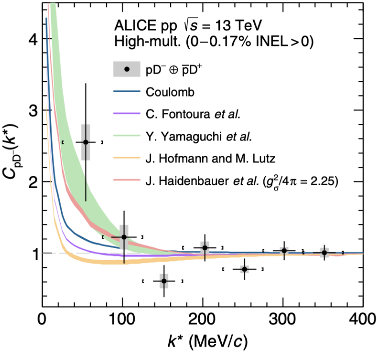
J.Hofmann, M.F.M.Lutz, Nucl.Phys. A763 (2005) 90-139
Y. Yamaguchi, S. Ohkoda, S. Yasui, A. Hosaka, Phys.Rev. D 84 (2011) 014032
D.Gamermann, C.Garcia-Recio, J.Nieves, L.L.Salcedo, L.Tolos, Phys.Rev. D81 (2010) 094016
J.Haidenbauer, G.Krein, U.-G.Meissner, A.Sibirtsev, Eur.Phys.J. A33 (2007) 107-117
C.E. Fontoura, G. Krein, V.E. Vizcarra, Phys.Rev. C 87 (2) (2013) 025206
Y. Yamaguchi, S. Ohkoda, S. Yasui, A. Hosaka, Phys.Rev. D 106, 094001 (2022)

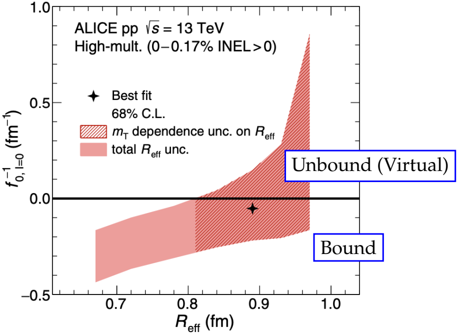
# Current Status: Experiment

$D^- p$ correlation from $pp$ collision

S. Acharya et al. ALICE collab. Phys.Rev.D 106 (2022) 5, 052010

- First experimental study of the two-body scattering of $\bar{D}N$
- Assuming negligible interaction in the $I = 1$ channel

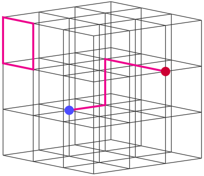



- inverse scattering length $a_0^{-1}$: -0.4 ~ +0.9 fm → Bound state or virtual state

(2-body) scattering on the Lattice

$$\mathcal{C}(t,\vec{r}) = \sum_{\vec{x}} \langle \mathcal{O}_1(t,\vec{x}+\vec{r})\mathcal{O}_2(t,\vec{x})\bar{\mathcal{J}}(0)\rangle$$
$$= \sum_{n} A_n \psi(\vec{r};E_n) e^{-E_n t}$$



Obtain scattering info (Phase shift) from Euclidean-time correlation function:

$$\mathcal{C}(t,\vec{r}) \longrightarrow \text{Phase shift } \delta$$

- Lüscher's finite volume method

  M. Lüscher, Nucl.Phys.B 354 531-578 (1991)

  Energy spectra in finite volume $\{E_n\}$ $\rightarrow$ quatization condition (e.g. Lüscher's formula) $\rightarrow$ Phase shift

- HALQCD method

  N. Ishii, S. Aoki, and T. Hatsuda PRL 99, 022001 (2007)

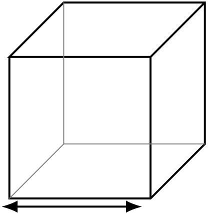
  Temporal + spacial info. $\psi(\vec{r};E_n)$ $\rightarrow$ Potential $\rightarrow$ Phase shift

# Setup

Configuration (F-conf)

Phys. Rev. D 110, 094502 (HAL QCD collab.)

- Volume: $96 \times 96^3$
- Iwasaki gauge action ($\beta = 1.82$)
- 2+1 flavor, $\mathcal{O}$(a)-improved Wilson quark action
- Relativistic heavy-quark action for $c$ quark
  slightly heavy / light $c$ quark (set1 / set2)
- Statistics: 360 configurations $\times$ 96 source $\times$ 4

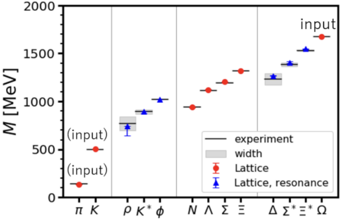


$L \simeq 8$ fm
$m_\pi \simeq 137$ MeV

$a \simeq 0.084$ fm

| | Lattice [MeV] | PDG [MeV] |
|---|---|---|
| $m_\pi$ | 137.1 | 138 |
| $m_N$ | 939.7 | 938 |

| | set-1 [MeV] | set-2 [MeV] | PDG [MeV |
|---|---|---|---|
| $m_D$ | 1880.1 | 1854.3 | 1865 |
| $m_{D^*}$ | 2017.8 | 1994.1 | 2007 |




Phys. Rev. D 110, 094502 (HALQCD)

# Setup

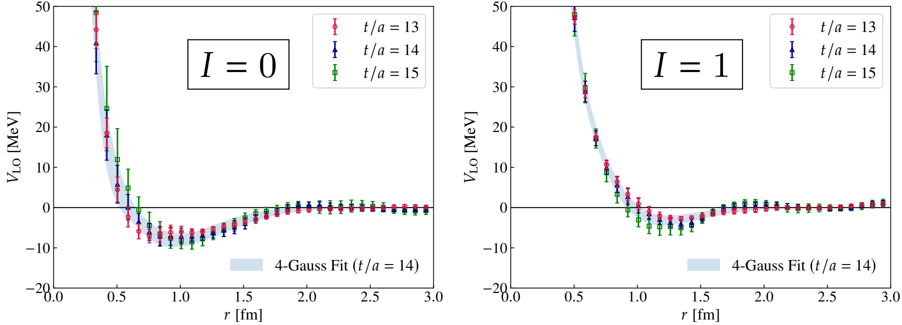
Other remarks:

- Only consider $\bar{D}N$ channel (valid upto $\bar{D}^*N$ threshold)
- Jackknife sampling methods for statistical uncertainties from MC
  sbin = 30 (binsize dependence negligible)
- $A_1^+$ projection + Misner method (T. Miyamoto et.al. Phys. Rev. D 101, 074514 (2020))
  Suppress $l = 4,\ 6,\ \cdots$ contributions
- Linear interpolation of set-1 and set-2 correlators to physical $D^0$ mass (PDG)

# s-wave $\bar{D}N$ potential

LO of the derivative expansion of $V(r, r')$

$$V(\vec{r}, \vec{r}') = \boxed{V_{\text{LO}}(r)}\,\delta(\vec{r} - \vec{r}') + \sum V_k \nabla^k \delta(\vec{r} - \vec{r}')$$




- Repulsive core + Attractive pocket
- $I = 0$ potential is more attractive than $I = 1$ potential in all regions

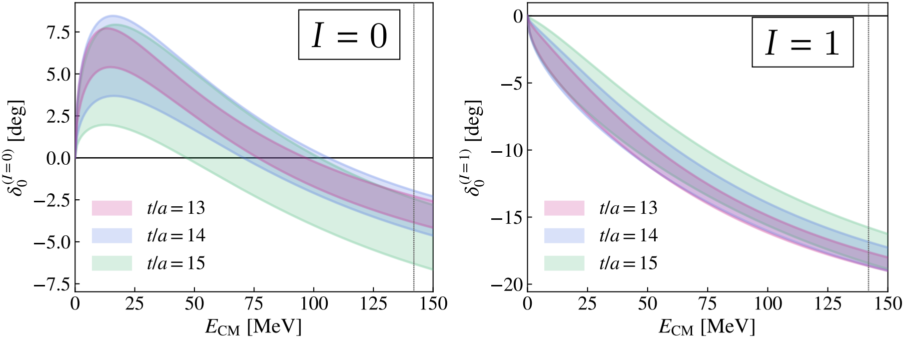
4 Gaussian Fit

$$V_{LO}^{\text{fit}}(r) = \sum_{k=1}^{4} a_i \ \exp[-r^2/b_i^2]$$

| | $\chi^2$/dof ($I = 0$) | $\chi^2$/dof ($I = 1$) |
|---|---|---|
| $t/a = 13$ | 0.31 | 1.36 |
| $t/a = 14$ | 0.46 | 0.82 |
| $t/a = 15$ | 0.24 | 0.66 |

# s-wave Phase Shifts

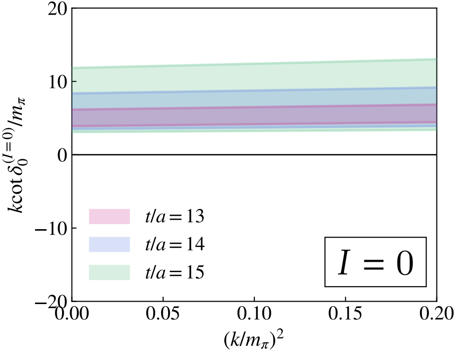
$$\left[H_0 + V_{LO}^{\text{fit}}\right]\psi(r) = E\psi(r) \qquad \psi(r) \sim e^{i\delta_0}\left[\cos\delta_0 j_0(kr) + \sin\delta_0 n_0(kr)\right] \quad (r \gg a)$$

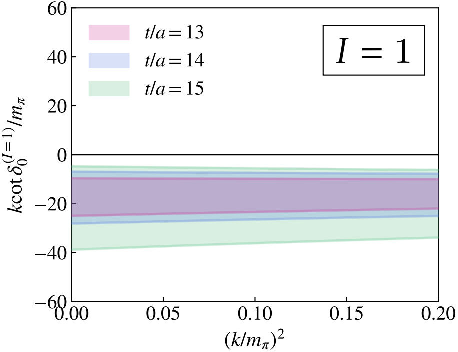



- $I = 0$: Phase shift shows weak attractive behavior in the low-energy region
- $I = 1$: Phase shift shows repulsive behavior in all energy regions
- <u>No bound state in $I = 0$ and $I = 1$ channel</u>

## Effective-Range Expansion

$$k \cot \delta(k) = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \mathcal{O}(k^4)$$




$a_0^{(I=0)} = 0.243(0.105)^{+0.040}_{-0.048}$ fm

$a_0^{(I=1)} = -0.085(0.050)^{+0.015}_{-0.002}$ fm

$r_0^{(I=0)} = 8.42(2.78)^{+1.60}_{-0.00}$ fm

$r_0^{(I=1)} = 14.54(32.10)^{+5.66}_{-0.00}$ fm

- Systematic uncertainties: difference from varying time slices $t/a = 13$-$15$

## Summary

- $\bar{D}N$ system

  Possibility of Pentaquark, Charmed nuclei...
- Limited experimental data input for theoretical studies

  $\rightarrow$ Lattice simulations (expect good signal because no lower open channels, no $q\bar{q}$ annihilation)
- Our Results:
  - (Small) Attractive behavior in the low energy region of $I = 0$ channel
  - Repulsive behavior in the $I = 1$ channel
  - No bound states for both $I = 0$ and $I = 1$
  - Results differ from models (& ALICE results)

## Outlook

- Coupled-channel analysis of $\bar{D}N$-$\bar{D}^*N$

  Coupling of $\bar{D}N$-$\bar{D}^*N$ is important to explain the attraction
- Multi-body systems (e.g. $\bar{D}NN$...) or Femtoscopy analysis using the $\bar{D}N$ HALQCD potential

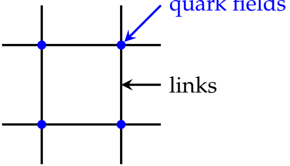
# Supplementary Materials

Numerically evaluating the path integral of QCD

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A\mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-S_{QCD}} \mathcal{O}$$

$$\Big\downarrow \text{Decretize spacetime}$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \prod dU d\bar{\psi} d\psi e^{-S_{LQCD}} \mathcal{O}$$




- Evaluate the decritized path integral with Monte-Carlo methods $\rightarrow \langle \mathcal{O} \rangle$

$\bar{D}N$ system is suited for lattice simulations:

- No open channels ($\leftrightarrow$ $DN$ system has lower open channels e.g. $\pi\Lambda_c$, $\pi\Sigma_c^{(*)}$)
- No $q\bar{q}$ annihilation suited for Lattice simulations
  ($q\bar{q}$ annihilation $\rightarrow$ large computational cost)

# HALQCD method

HALQCD

$$\mathcal{C}(t,\vec{r}) = \sum_{\vec{x}} \langle \mathcal{O}_1(t,\vec{x}+\vec{r})\mathcal{O}_2(t,\vec{x})\bar{\mathcal{J}}(0)\rangle = \sum_n A_n \psi(\vec{r};E_n)e^{-E_n t}$$

$$\psi(\vec{r}) \to \frac{\sin(kr - l\pi/2 + \delta(k))}{kr} e^{i\delta(k)} \quad (r \gg R)$$

$$(\nabla^2 + k^2)\psi(\vec{r};E_n) = \int d\vec{r}' V(\vec{r},\vec{r}')\psi(\vec{r}';E_n)$$

- Energy-independent interaction kernel $V(\vec{r},\vec{r}')$ produces the correct phase shift of the QCD S-matrix in the elastic region

Time-dependent method

Ishii et al. (HAL QCD), PLB712, 437(2012)

$$R(t,\vec{r}) = \frac{\mathcal{C}(t,\vec{r})}{\langle \mathcal{O}_1(t)\rangle \langle \mathcal{O}_2(t)\rangle}$$

$$\left[\frac{1+3\delta^2}{8\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu}\right]R(t,\vec{r}) = \int d\vec{r}' V(\vec{r},\vec{r}')R(t,\vec{r}')$$

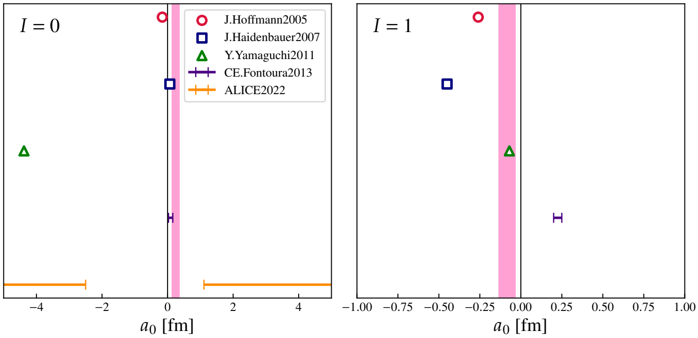
- Excited-state contamination surpressed

# Comparison: Models, Femtoscopy

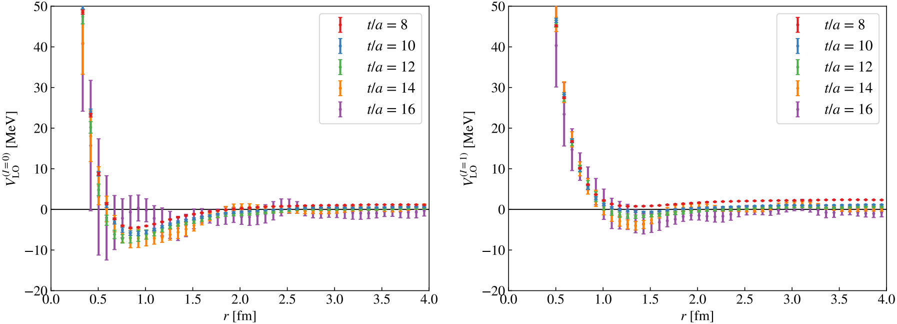
- No bound states in $I = 0$ and $I = 1$

| Study | Bound State ($I = 0$) [MeV] | Bound State ($I = 1$)[MeV] |
|---|---|---|
| Hoffmann 2005 | × | × |
| Haidenbauer 2007 | × | × |
| Gamermann 2010 | 2805 | × |
| Yamaguchi 2022 | 2804 | 2800 |
| ALICE 2022 | △ | — |
| HAL 2025 | × | × |

- Scattering Length $a_0$ (Pink band: Our Result)



# LO potential: $t$-dependence

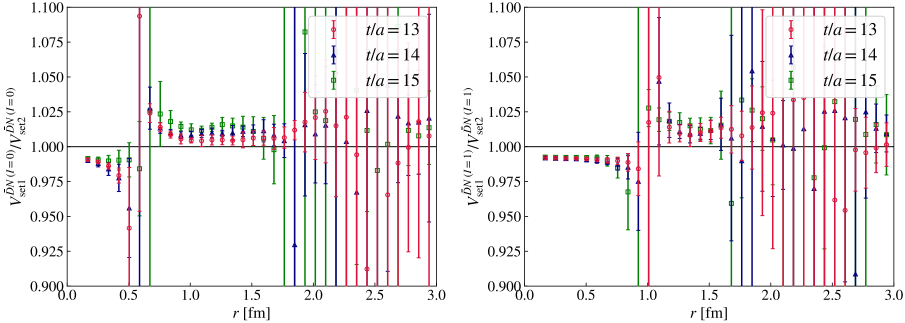



- contributions from excited states at small times → non-zero values at large $r$
- noise too large for time slices larger than $t/a = 16$ (N/S ratio grows exponentially)
- potential stable (independent of time) in window $t/a = 13$ to $t/a = 15$

# LO potential: $m_c$ dependence

Ratio between set-1 LO potential and set-2 LO potential

$$\frac{V^{I}_{LO}}{V^{II}_{LO}}$$




- Heavier $c$ quark $\rightarrow$ stronger attraction ($\sim$ 1-2 %)
  c.f. mass difference of $\bar{D}$ meson between set-1 & set-2 $\approx$ 1.5%

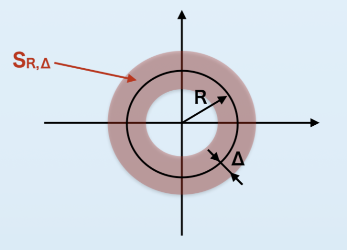
# $\bar{D}N$ potential: Misner Method

Higher partial-wave contamination in the $A_1^+$ R-correlator

- Misner method

  C. W. Misner, Class. Quantum Grav. 21 (2004) S243-S247

  T. Miyamoto et. al. Phys. Rev. D 101, 074514 (2020)




From T.Miyamoto slides 2019

$$R(\vec{r}) \approx \sum_{nlm} c_{nlm} G_n^{R,\Delta}(R) Y_{lm}(\hat{r})$$

$$G_n^{R,\Delta}(R) = \frac{1}{r}\sqrt{\frac{2n+1}{2\Delta}} P_n\left(\frac{r-R}{\Delta}\right)$$

$$c_{nlm} = \langle \tilde{\mathcal{Y}}_{nlm} | \mathcal{Y}_{nlm} \rangle$$

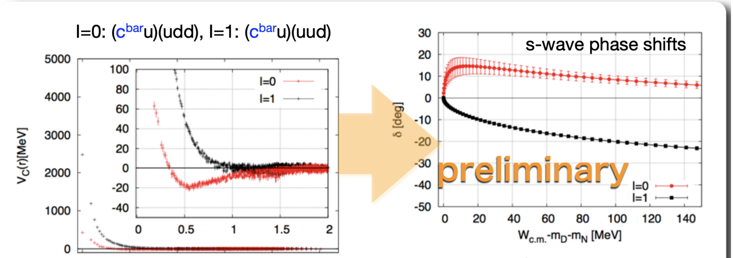
In the following the expansion of the R-correlator is trancated at

- nmax=2, lmax=4
- $\Delta$=1.00 lattice unit

# $m_\pi$ dependence

Comparison to results with heavier pion mass $m_\pi = 410$ MeV (PACS-CS config.)

Y. Ikeda slides 10th APCTP-BLTP/JINR-RCNP-RIKEN Joint Workshop Aug.2016




$$a_{\bar{D}N} = \begin{cases} 0.79(42) & (I=0) \\ -0.20(1) & (I=1) \end{cases}$$

- $I = 0$: smaller pion mass, shallower attractive pocket
  ($m_\pi \simeq 137$ MeV case) has smaller scattering length but is still positive
- $I = 1$: ($m_\pi \simeq 137$ MeV case) has slightly smaller repulsion
  scattering length closer to zero