Generalized parton distributions of the kaon and pion within the nonlocal chiral quark model

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Chiral symmetry breaking and the Nambu-Goldstone bosons

Hadron mass spectra: maximally broken chiral symmetry, eg. N(1/2+, 940) vs N(1/2-,1535).

Spontaneously broken chiral symmetry, $\langle\bar{\psi}\psi\rangle \neq 0 \rightarrow$ massless Nambu-Goldstone boson

Explicit chiral symmetry breaking by current quark masses $m \rightarrow$ Nambu-Goldstone bosons acquire mass M

Gell-Mann - Oakes - Renner relation

$$M^2F^2 = -\,m\langle\bar{\psi}\psi\rangle + \mathcal{O}(m^2)$$

Including strangeness ($m_s \ll \Lambda$), SU(3)$_f$: π, K, η

Breaking SU(3)$_f$ with $m_s \approx 100$ MeV may require significant correction in $\mathcal{O}(m^2)$

Quark structure of the kaon should be different from the pion

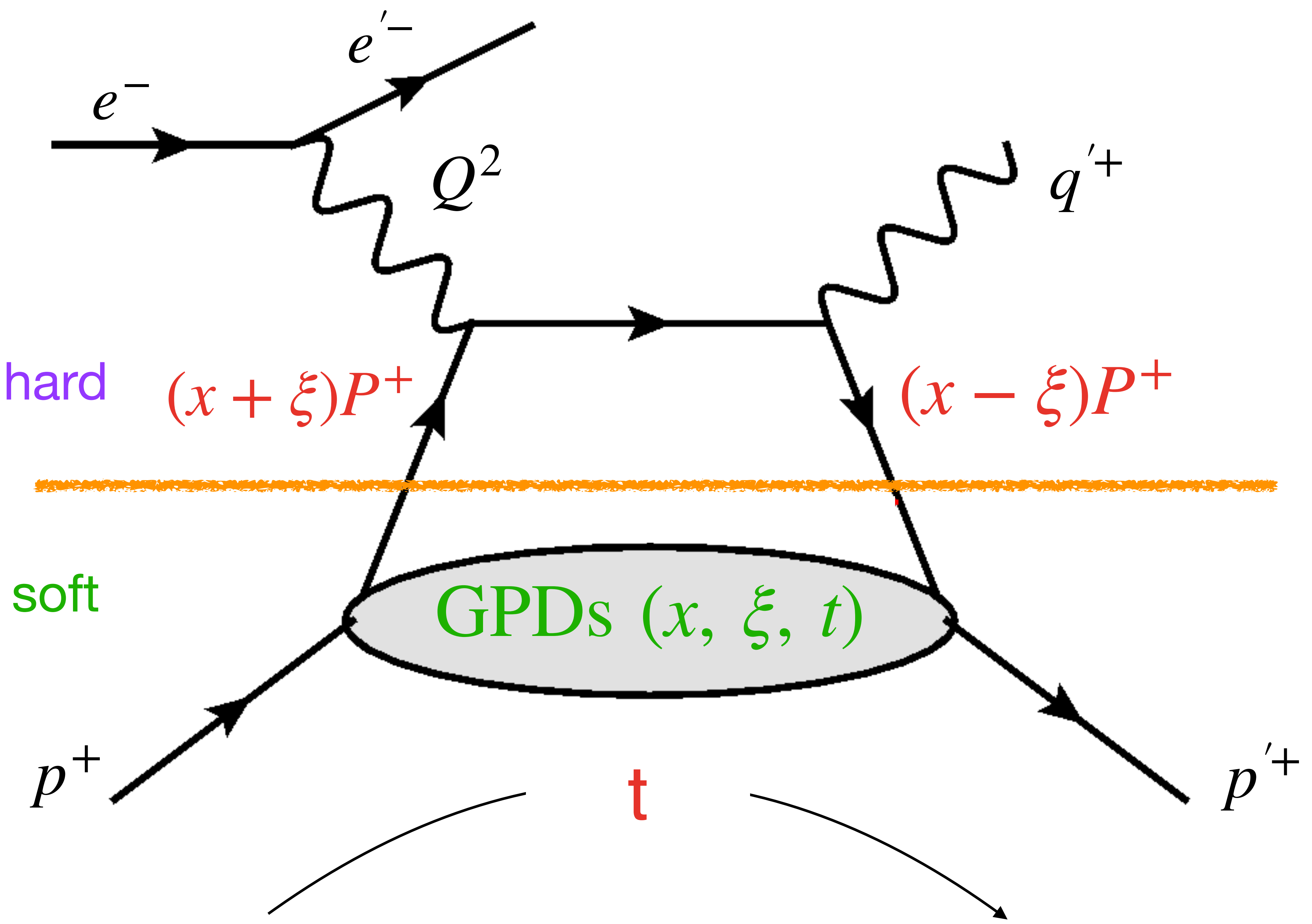
$\rightarrow$ role of m_s in partonic picture (PDFs) and mechanical properties (GFFs) of hadrons ?

$\boxed{\rightarrow \textit{Generalized Parton distributions (GPDs)}}$

Hard exclusive reactions

$$e + p \to e' + p' + \gamma/M$$

Deeply Virtual Compton Scattering / Meson Production (DVCS/DVMP)



Scattering cross-section factorizes as:

hard part (pQCD) $\otimes$ soft part

Q^2 : Virtuality $\to$ hard scattering limit $Q^2 \gg |t|,\ M_t^2,\ M_s^2,\ \cdots,$

p^+ : Light-front (LF) longitudinal momentum of incoming target,

p'^+ : Light-front (LF) longitudinal momentum of incoming target,

P^+ $= (p^+ + p^+)/2$, average hadron momentum,

ξ $= (p^+ - p'^+)/(p^+ + p'^+)$,

asymmetry of longitudinal momentum of target,

$x \pm \xi$: Parton longitudinal momentum fraction,

t : Squared momentum transfer, $\Delta^2 = (q' - q)^2 = (p - p')^2$,

"kick" transverse momentum depending on scattering angle.

Generalized parton distributions (spin=0 hadrons)

Definition (quark)

$$\int \frac{dy^-}{4\pi}\, e^{ixP^+y^-}\langle P'|\, \bar{\psi}_q(0)\, \gamma^+\, \psi_q(y)\, |P\rangle\Big|_{y^+=\mathbf{y}_\perp=0} = \boxed{H^q(x,\xi,t;\mu)}$$

(Chirality even)

$$\int \frac{dy^-}{4\pi}\, e^{ixP^+y^-}\langle P'|\, \bar{\psi}_q(0)\, i\sigma^{+j}\, \psi_q(y)\, |P\rangle\Big|_{y^+=\mathbf{y}_\perp=0} = \frac{i\epsilon_\perp^{ij}q_\perp^i}{2M}E_T^q(x,\xi,t;\mu)$$

(Chirality odd)

Zero momentum transfer of the target hadron → Parton distribution functions

$$H(x,\,0,\,0) = f_1(x)$$

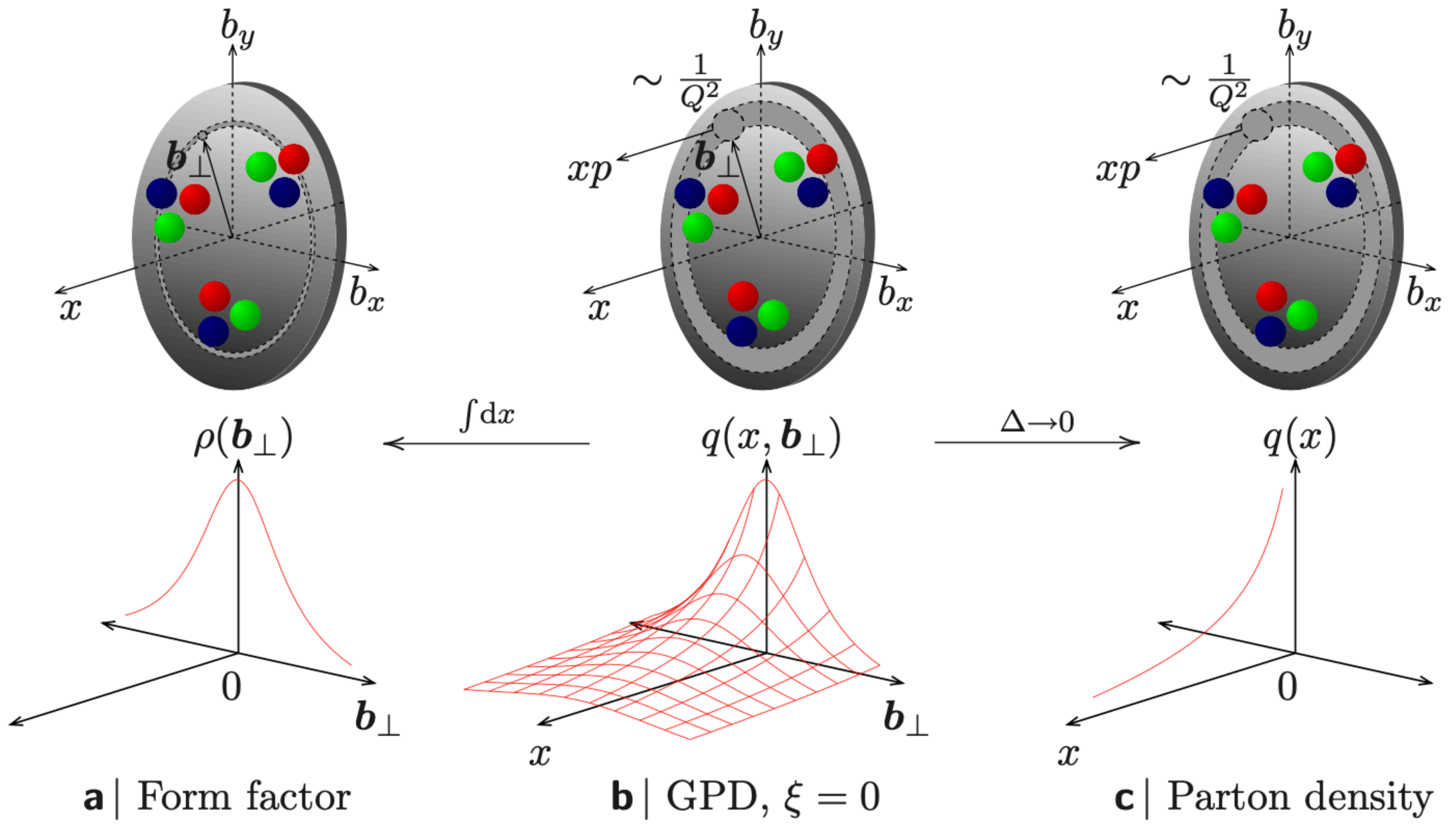
Unpolarized quark distribution

Mellin moments (Polynomiality) → Generalized Form factors

$$2\int_{-1}^{1} dx\, x^{n-1}\, H^q(x,\xi,t) = \sum_{m=0,\text{even}}^{n} \xi^m A^q_{nm}(t)$$

n=1: Electromagnetic form factors

n=2: Gravitational form factors (twist-2)

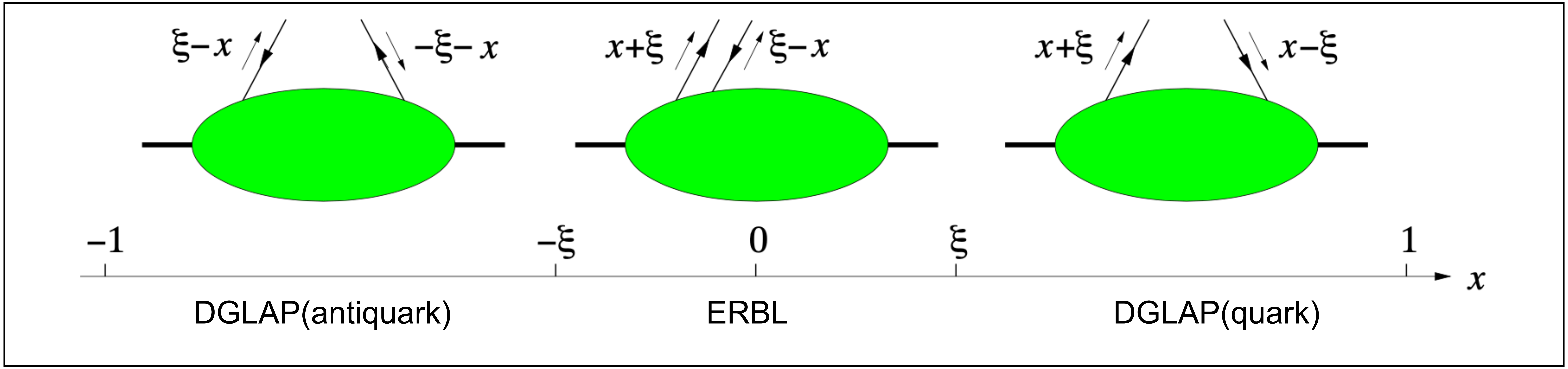


[D. Mueller et al. 1994]

Parton interpretation of GPDs

Parton momentum fraction: combination of x and ξ

[M. Diehl, Phys. Rept. 388 (2003) 41]



1. $x \in [\xi, 1]$: Emission of a quark $x + \xi \geq 0$ and reabsorption $x - \xi \geq 0$.
2. $x \in [-\xi, \xi]$: Emission of a quark $x+\xi \geq 0$ and an antiquark $\xi - x \geq 0$ from the initial proton.
3. $x \in [-1, -\xi]$: Emission of an antiquark $\xi - x \geq 0$ and absorption of an antiquark $-\xi - x \geq 0$.

Theoretical studies on the pion and kaon structures

Gravitational form factors

χPT to $O(p^2)$ for $SU(3)_f$ NGBs [Donoghue and Leutwyler, ZPC52 (1991)]

Crossing and GDAs (Belle data $\gamma\gamma^* \to \pi^0\pi^0$) [Kumano, Song, Teryaev ,PRD 97 (2018)] [Masuda et al, PRD 93 (2016)]

Chiral quark model (non-trivial cancellation of internal pressure) [HDS and H.-Ch. Kim, PRD 90 (2014)]

(… many other studies)

Kaon GPDs within LFWF, DSE (DGLAP region)

[Zhang et al., Phys. Lett. B 815 (2021) 136158.
Raya et al., Chin. Phys. C 46 (1) (2022) 013105
Adhikari et al. , Phys. Rev. D 104 (11) (2021) 114019]

In this work,

we explore the kaon and pion GPDs and GFFs within a nonlocal chiral quark model (ERBL & DGLAP)

by extending the work by Praszalowicz and Rostworowski (pion GPDs in chiral limit)

[Acta Phys. Polon. B 34 (2003) 2699–2730]

Quark one-loop effective action in the large Nc limit

$$\mathcal{S}_{\rm eff} = \int \frac{d^4k}{(2\pi)^4}\bar{\psi}(k)(\not{k} - \hat{m})\psi(k) - \int \frac{d^4k}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\bar{\psi}_f(p)\sqrt{M_f(p)}\,U^{\gamma_5}_{fg}(p-k)\sqrt{M_g(k)}\psi_g(k)$$

$$M(k) = MF^2(k), \qquad U^{\gamma_5}(x) = \exp\left[\frac{i}{F_{\mathcal{M}}}\gamma^5\lambda^a\mathcal{M}^a\right], \qquad \hat{m} = \mathrm{diag}(m_u, m_d, m_s).$$

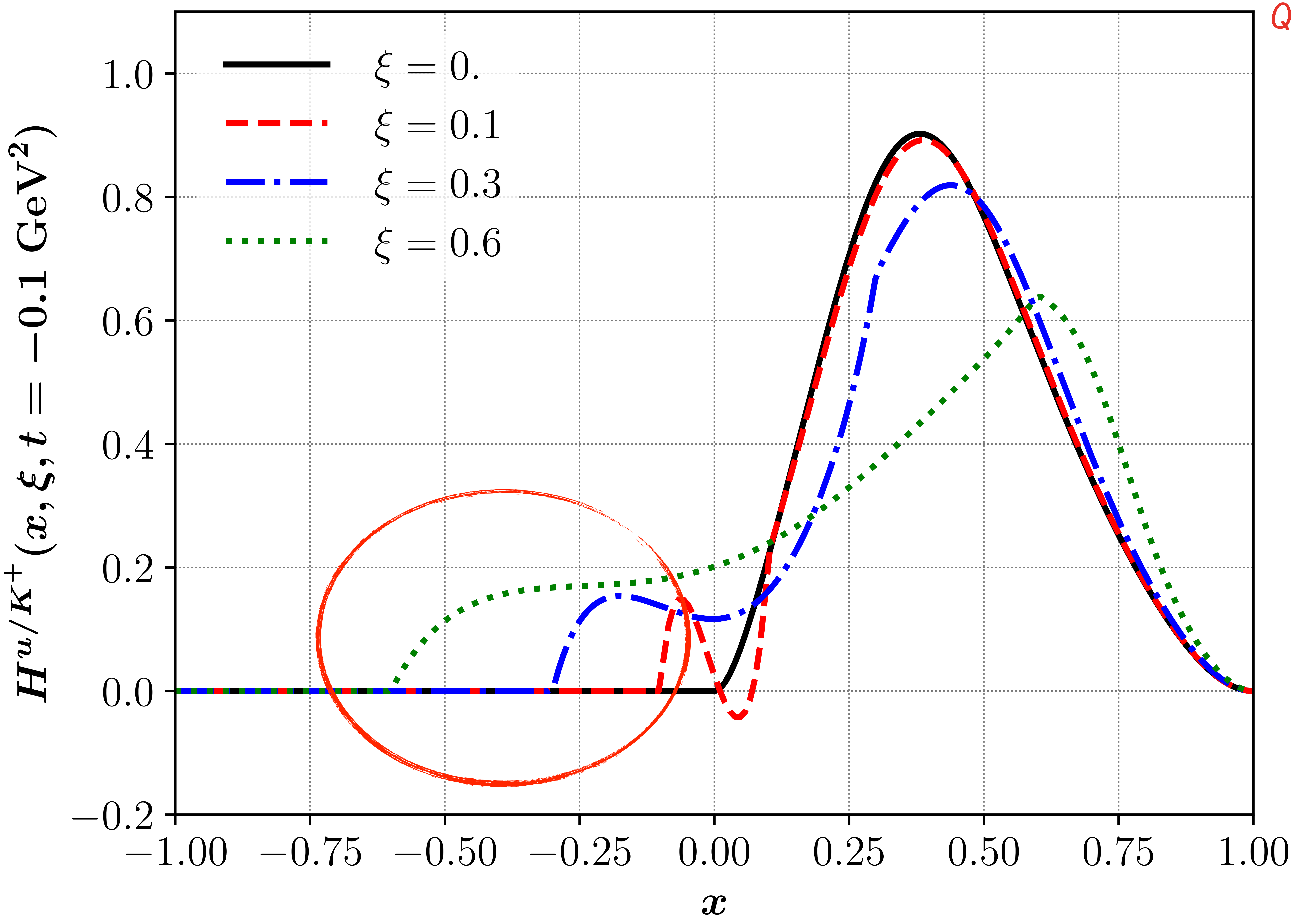
Inspired by the liquid instanton model at low-renormalization point $\mu \sim 1/\bar{\rho}$

$M(0) = 350$ MeV, computed by the gap equation from the instanton vacuum

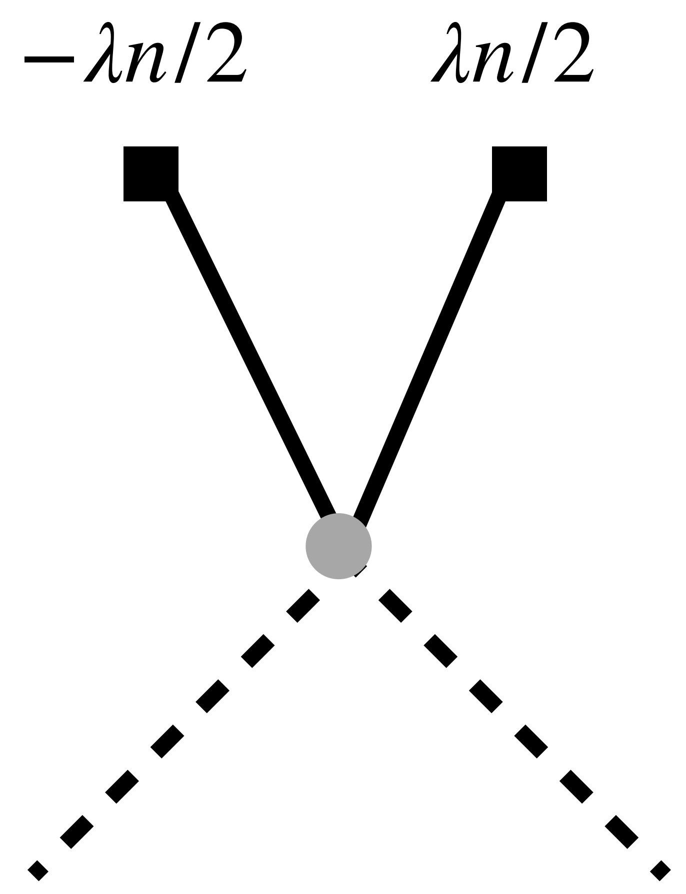
Nonlinear chiral field for $SU(3)_f$ is introduced as well as the current quark masses ($m_u = m_d = 5$ MeV, ms = 100 MeV)

Analytic continuation to Minkowski space is assumed,

with n-pole type quark form factor: $F(k) = \left(\frac{1}{1 - k^2/\Lambda^2 - i\epsilon}\right)^n$, (instanton form factor n = 3/2)

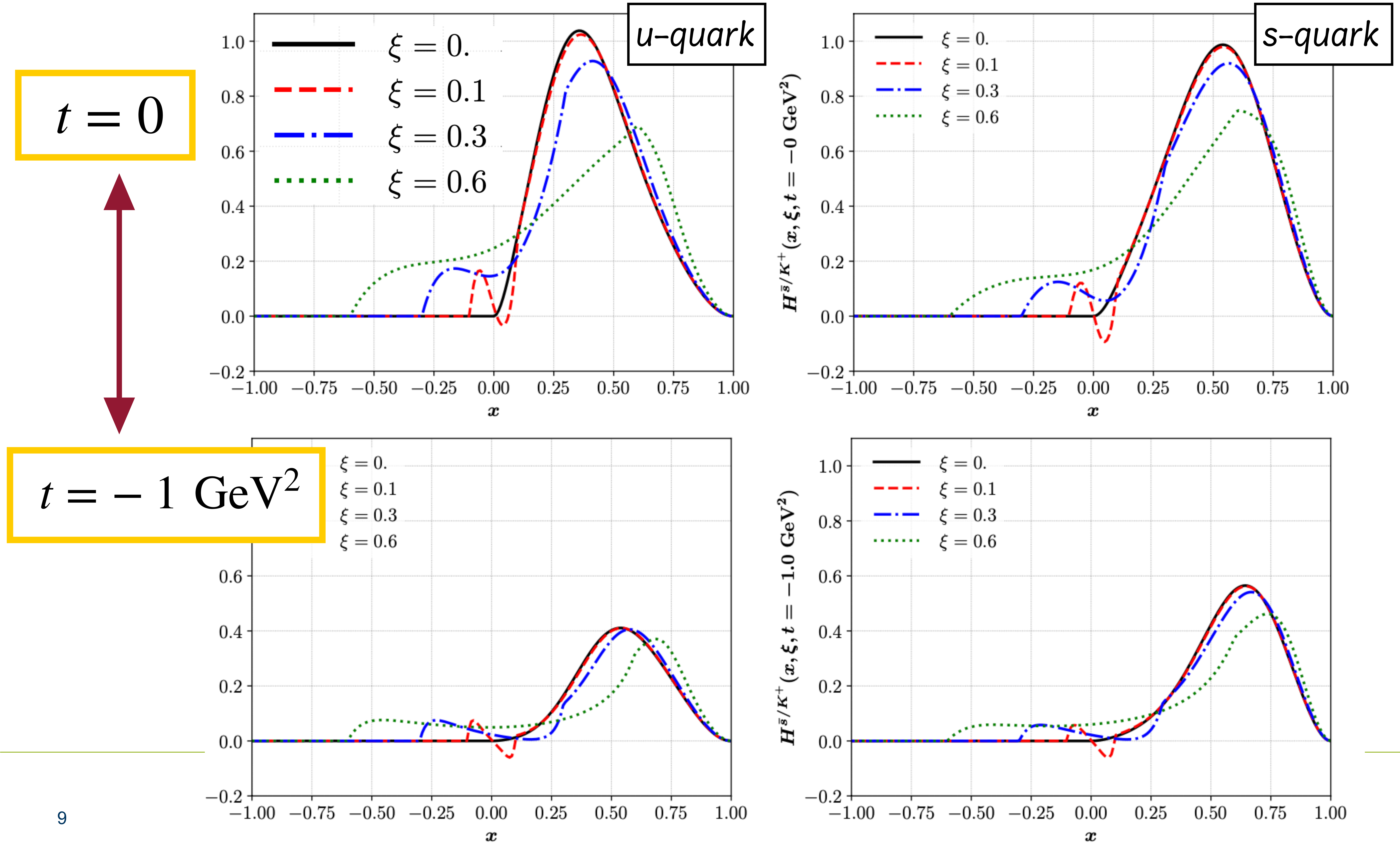


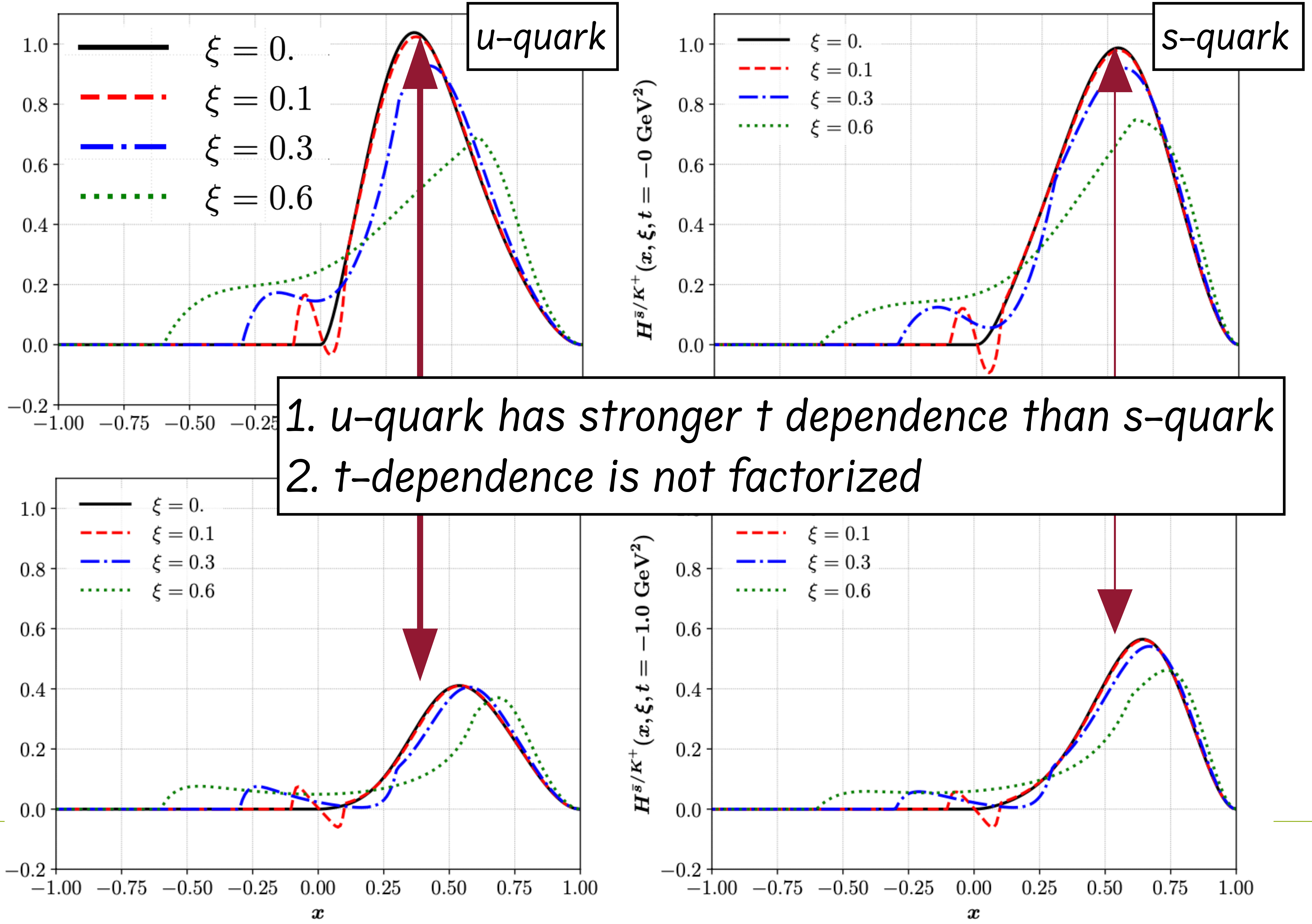
Quark distributions vanish @ $x = -\xi$: indication of no sea-quark



$$M\bar{\psi}_i(k)F(k)K^2_{ij}(k-k')F(k')\psi_j(k')$$

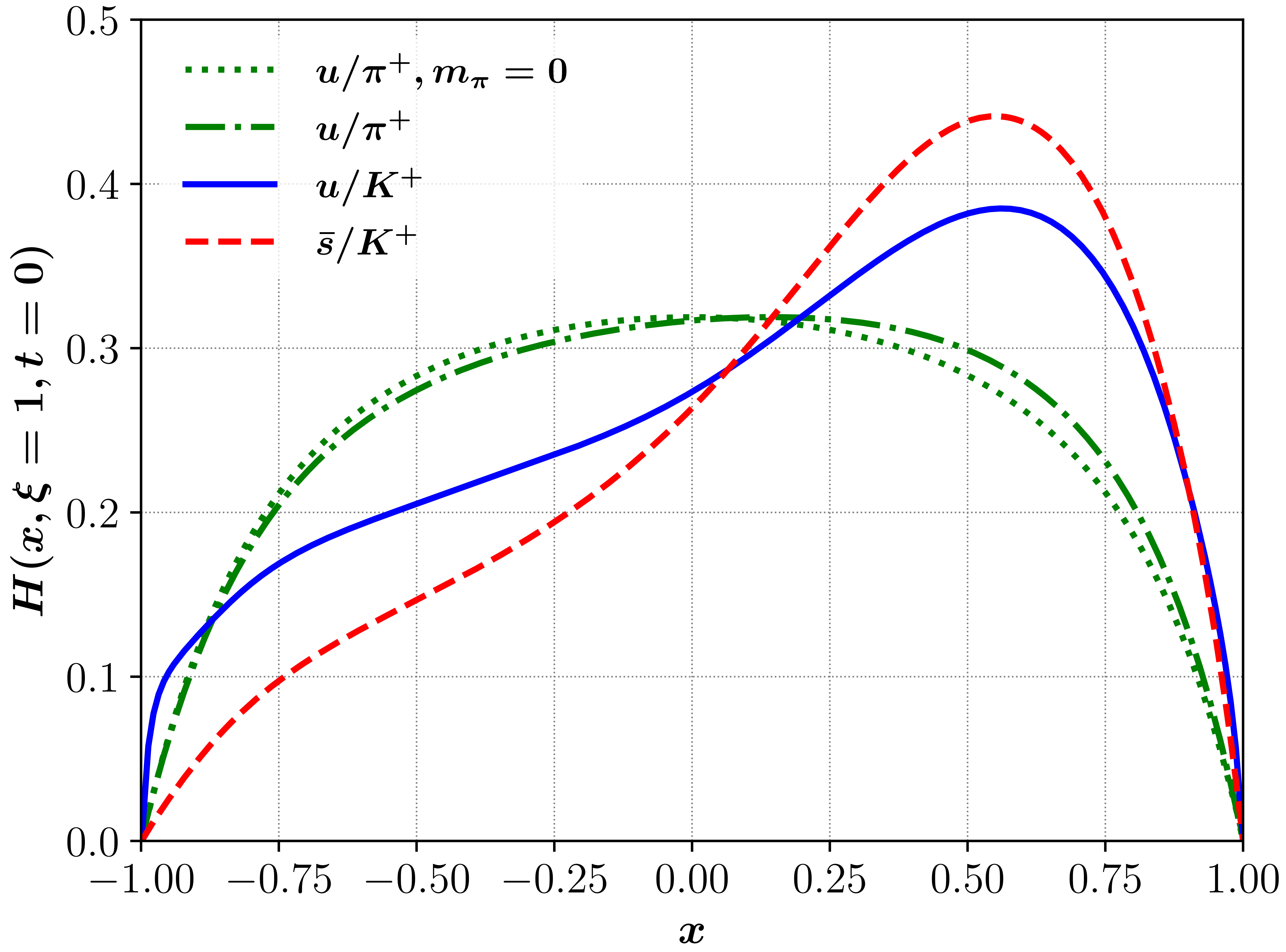
Shoulder-like structure @ERBL mainly due to the qqmm vertices!


$t = 0$
$t = -1\ \mathrm{GeV}^2$
u-quark
s-quark
$\xi = 0.$
$\xi = 0.1$
$\xi = 0.3$
$\xi = 0.6$
$H^{\bar{s}/K^+}(x, \xi, t = -0\ \mathrm{GeV}^2)$
$H^{\bar{s}/K^+}(x, \xi, t = -1.0\ \mathrm{GeV}^2)$
x



1. u-quark has stronger t dependence than s-quark
2. t-dependence is not factorized

GPDs at $\xi = 1$ and explicit chiral symmetry breaking



Chiral symmetry requires H(x,ξ=1,t=0) to be even in x

[M. Polyakov and C. Weiss, Phys.Rev.D 60 (1999) 114017]

Pion ($m_\pi = 0$): exactly symmetric

Pion ($m_\pi = 140$ MeV): slightly distorted

$\bar{s}$ in K^+ u in K^+

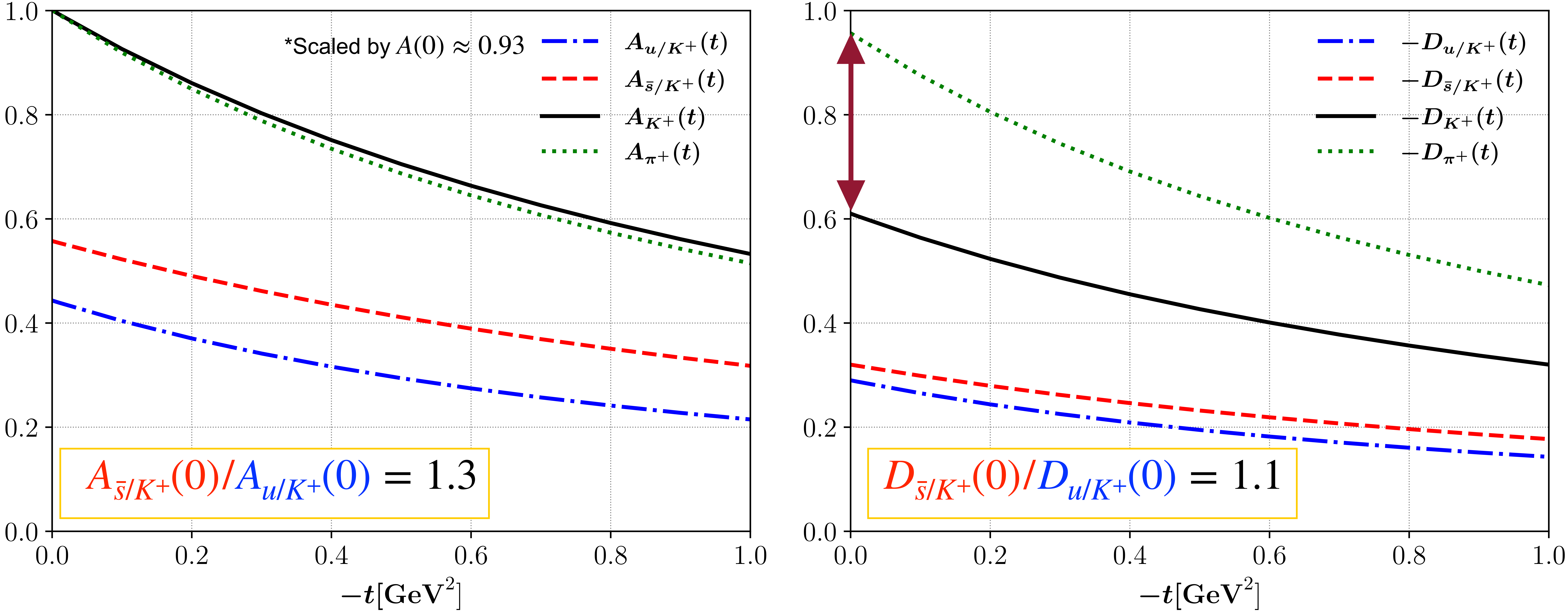
Second Mellin moment: Gravitational form factors

$$\int_{-1}^{1} dx\ x\ 2H(x, \xi = 1, t = 0) = A(0) + D(0) = 0 + \mathcal{O}(m_{\pi,K}^2)$$

Correction: $m_K^2/m_\pi^2 \approx 12$

Gravitational form factors from n=2 Mellin moments

$A(t)$: Distribution of mass
$D(t)$: Pressure and Shear



$$A_K(0) + D_K(0) = 0.36 \text{ vs. } A_\pi(0) + D_\pi(0) = 0.04$$

D-term receives a meson mass correction of order $m^2_{K,\pi}$

Comparison with other predictions

	$A_\pi(0) + D_\pi(0)$	$A_K(0) + D_K(0)$	$A_{\bar{s}/K^+}(0)/A_{u/K^+}(0)$	$D_{\bar{s}/K^+}(0)/D_{u/K^+}(0)$
Current work	0.04	0.36	1.26	1.10
ChPT [Donoghue & Leutwyler 1991]	0.03	0.23	-	-
LFWFs [Raya et al, 2021]	-	-	1.1	1.25
BSE-NJL [Adhikari et al., 2021]	-	-	1.32	-
DSE [Y. Xu et al. 2023]	0.03	0.23	1.56	1.25
BSE-NJL [P. Hutauruk et al. 2016]	-	-	1.38	-
MIT-Lattice [Hackett et al. 2023]	~0.10	-	-	-
ETMC-Lattice [Delmar et al. 2024]	-	-	~1.3	-

Observations

Kaon valence-quark GPDs studied within the nonlocal chiral quark model

u-quark distribution presents stronger t-dependence than the strange quark

Explicit chiral symmetry breaking affects the ERBL region of the kaon GPD significantly

Quark D-terms in Kaon $D^s/D^u \sim 1.1$, $D^{u+s} \sim -0.64$ vs. the ChPT prediction ~0.77

Tasks

Chirality odd GPDs and generalized transverse quark-spin densities

Kaon Sullivan-DVCS process in EIC. Pion case: [Amrath, Diehl, Lansberg, EPJ.C58,179-192]
[Chavez et al, PRL 128 (2022)]

Gravitational form factors of the kaon, $\bar{c}^{u,s}_{K^+}(t)$?

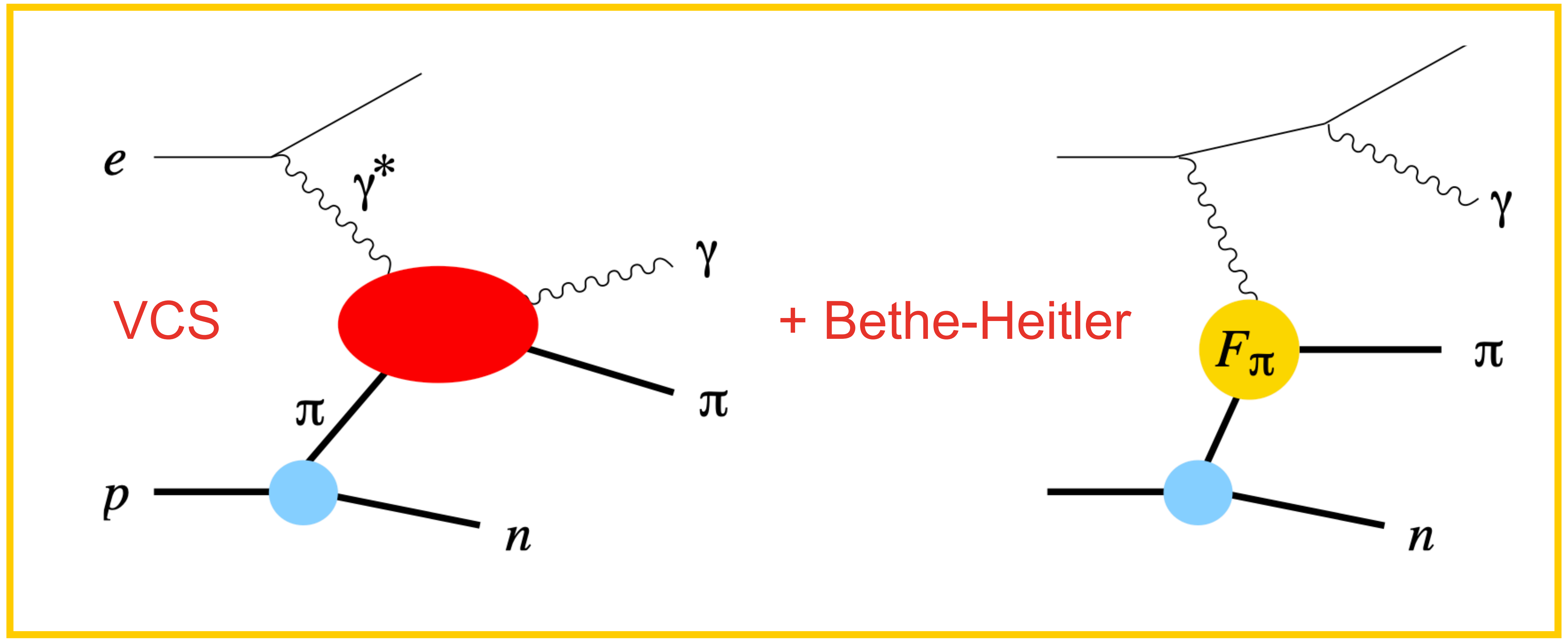
Thank you very much!

Pion and Kaon GPDs from Sullivan-DVCS process

[Amrath, Diehl, Lansberg, EPJ.C58,179-192]

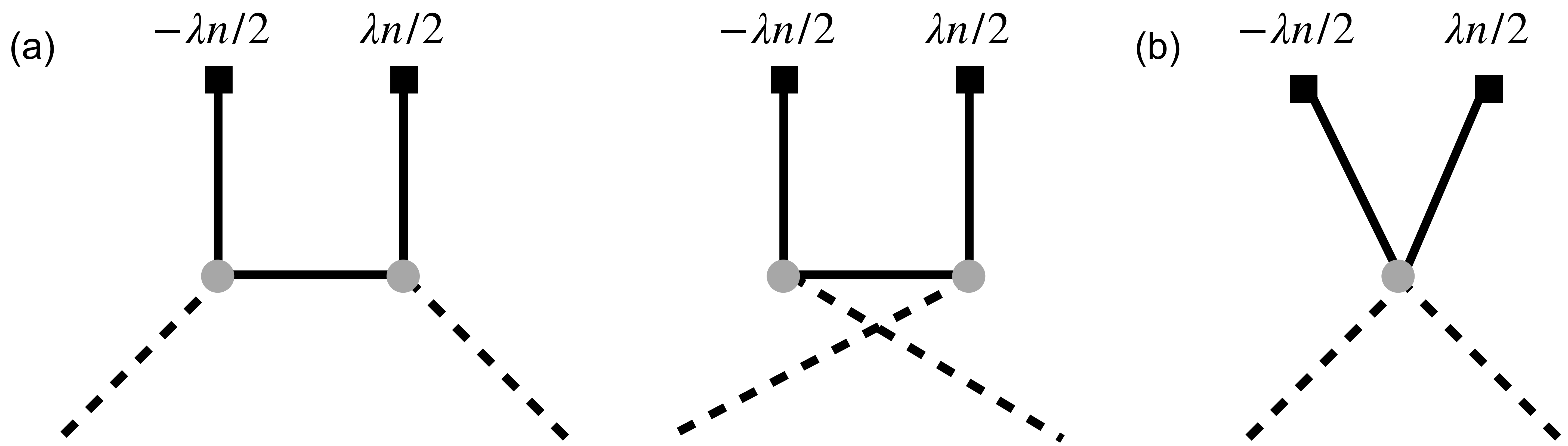
Sullivan-DVCS process

$$ep \rightarrow e'\gamma\pi^+n$$



- Total signal (cross section) is dominated by Bethe-Heitler process [Amrath, Diehl, Lansberg, EPJ.C58,179-192]
- Cross-section (~fb) too small for JLAB 11GeV
- **Feasibility study for EIC : NLO processes and role of gluon** [Chavez et al, PRL 128 (2022)]
- Estimation for the process involving the Kaon GPDs can be studied, eg. $ep \rightarrow e'\gamma K^+\Lambda$

Leading Nc quark-loop diagrams for the kaon valence-quark GPDs



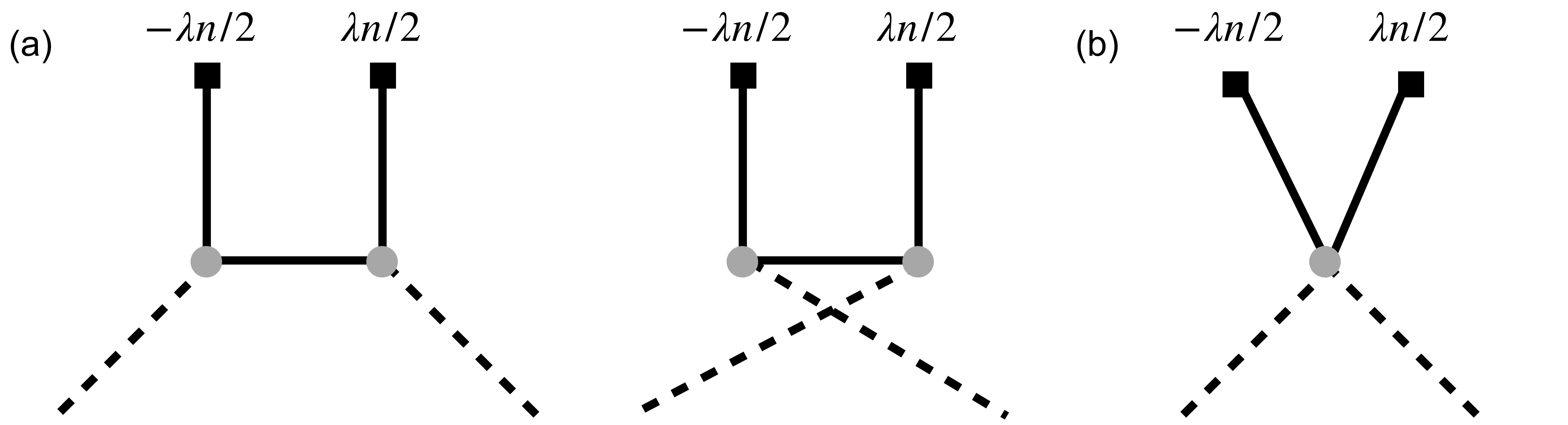
$$H^{u/K^+}(x,\xi,t) = \frac{iN_c}{8(2\pi)^4F_K^2}\int d^2k_\perp \int_{-\infty}^{\infty} dk^- \left[\mathcal{T}^{u\bar{s}}_{(a)}(k-\Delta/2, k-P, k+\Delta/2) + \mathcal{T}^{u}_{(b)}(k-\Delta/2, k+\Delta/2)\right]\Bigg|_{k^+=xP^+}, \tag{33}$$

$$-H^{\bar{s}/K^+}(-x,\xi,t) = \frac{iN_c}{8(2\pi)^4F_K^2}\int d^2k_\perp \int_{-\infty}^{\infty} dk^- \left[\mathcal{T}^{\bar{s}u}_{(a)}(k-\Delta/2, k+P, k+\Delta/2) + \mathcal{T}^{\bar{s}}_{(b)}(k-\Delta/2, k+\Delta/2)\right]\Bigg|_{k^+=xP^+}, \tag{34}$$

$$\mathcal{T}^{fg}_{(a)}(k_1,k_2,k_3) = \mathrm{Tr}_D\left[\gamma^+\frac{\sqrt{M_f(k_1)}}{\not{k}_1 - m_f - M_f(k_1) + i\epsilon}\gamma^5\frac{\sqrt{M_g(k_2)}}{\not{k}_2 - m_g - M_g(k_2) + i\epsilon}\gamma^5\frac{\sqrt{M_f(k_3)}}{\not{k}_3 - m_f - M_f(k_3) + i\epsilon}\right], \tag{35}$$

$$\mathcal{T}^{f}_{(b)}(k_1,k_2) = \mathrm{Tr}_D\left[\gamma^+\frac{\sqrt{M_f(k_1)}}{\not{k}_1 - -m_f - M_f(k_1) + i\epsilon}\frac{\sqrt{M_f(k_2)}}{\not{k}_2 - m_f - M_f(k_2) + i\epsilon}\right]. \tag{36}$$

Leading Nc quark-loop diagrams for the kaon valence-quark GPDs



$$H^{u/K^+}(x,\xi,t) = I_1^{u/K^+}(x,\xi,t)\Theta(x-\xi) + I_2^{u/K^+}(x,\xi,t)\Theta(\xi-|x|) + I_3^{u/K^+}(x,\xi,t)\Theta(\xi-|x|)$$

$$-H^{\bar{s}/K^+}(-x,\xi,t) = I_1^{\bar{s}/K^+}(x,\xi,t)\Theta(-x-\xi) + I_2^{\bar{s}/K^+}(x,\xi,t)\Theta(\xi-|x|) + I_3^{\bar{s}/K^+}(x,\xi,t)\Theta(\xi-|x|)$$

Diagram (a) : I_1(DGLAP) and I_2 (ERBL)

Diagram (b) : I_3 (ERBL)

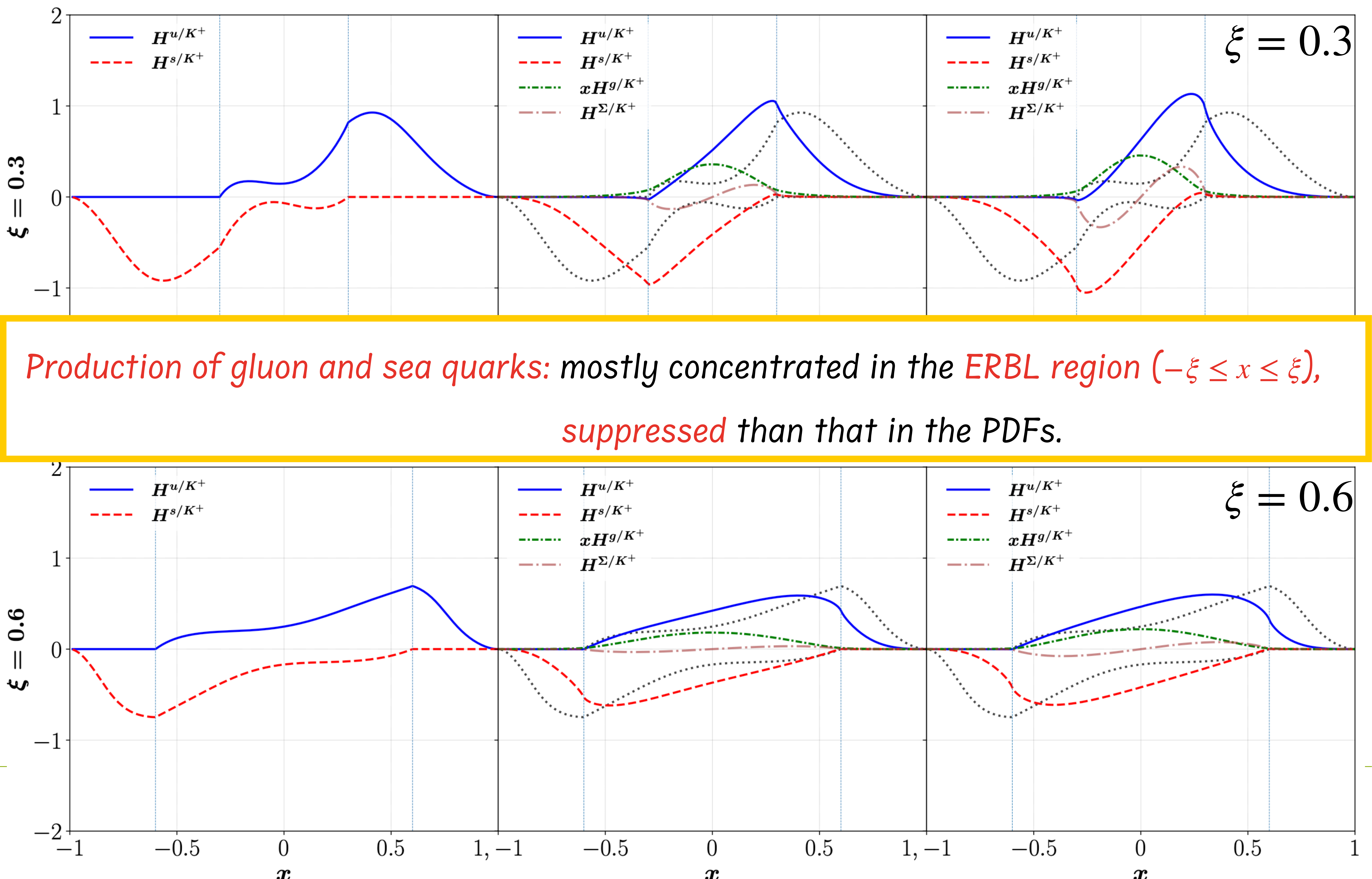
QCD scale evolution of GPDs

- LO (one-loop) splitting functions are computed at early stage
- Studied up to NLO evolution and partially NNLO
- Presents limits to DGLAP ($\xi \to 0$) and ERBL ($\xi \to 1$) evolution equations
- Polynomiality and sum rules are satisfied
- Numerical package **`APFEL++`** developed by V. Bertone (up to LO evolution)
- Model initial scale $\mu_0 = 0.33$ GeV $\to \langle x \rangle_{val}(\mu^2 = 27\ \mathrm{GeV}^2) \approx 0.5$

caveat! Initial scale too small, need to analyze the higher loop corrections

QCD scale evolution of kaon GPDs

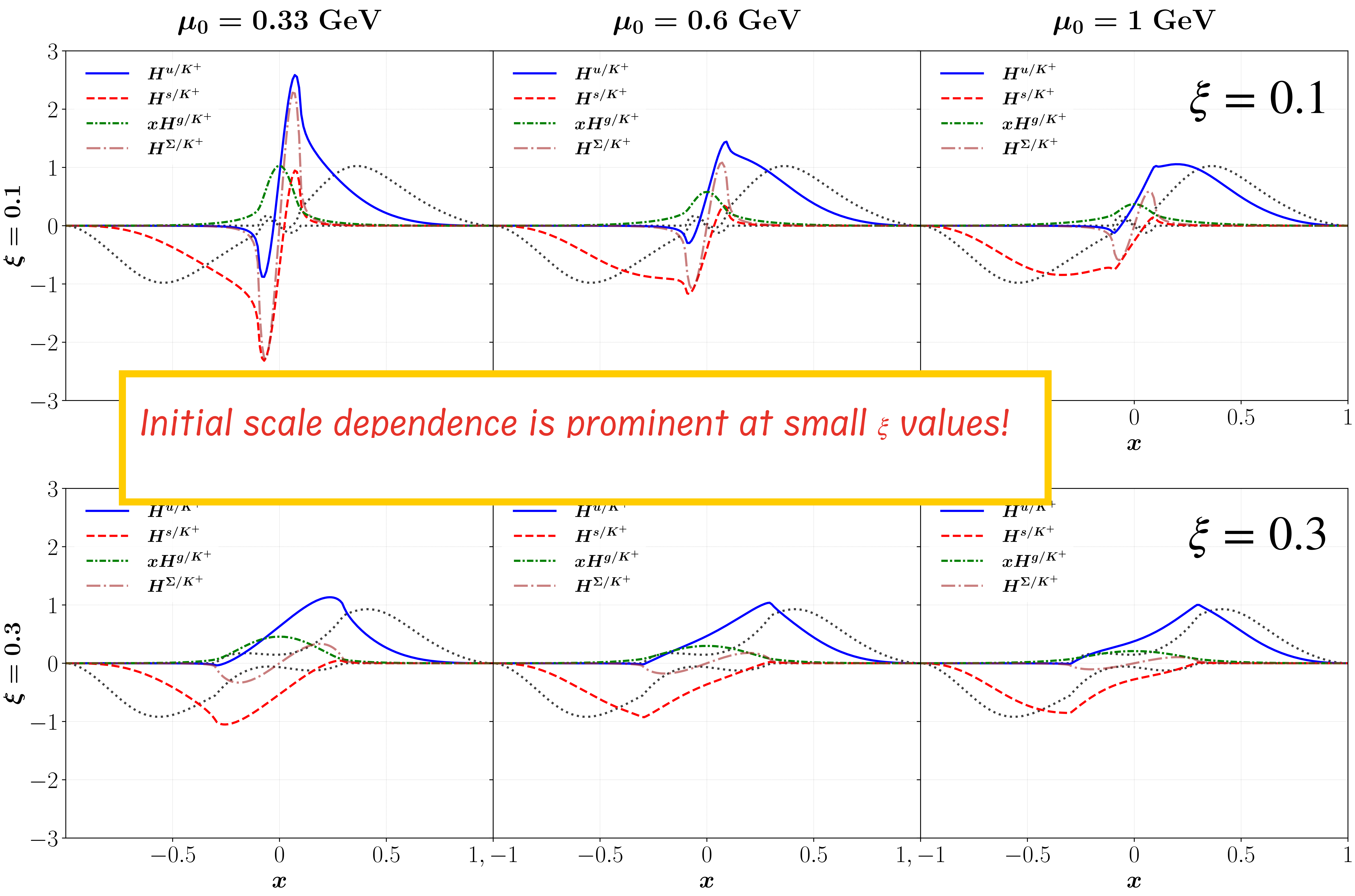
LO evolution by **APFEL++**, $\mu_0 \approx 0.33$ GeV



Production of gluon and sea quarks: mostly concentrated in the *ERBL region* ($-\xi \le x \le \xi$), *suppressed* than that in the PDFs.

QCD scale evolution of kaon GPDs

LO evolution by **APFEL++**



Initial scale dependence is prominent at small ξ values!

Gravitational form factors of the Kaon

$$\langle K^+(p')\,|\,\hat{T}_{\mu\nu}(0)\,|\,K^+(p)\rangle = \left[4P_\mu P_\nu\ A(t) + (q^\mu q^\nu - g^{\mu\nu}q^2)\ D(t)\right]$$

χPT result to O(p²) [Donoghue and Leutwyler, ZPC52 (1991)]

$$A(t) = 1 - 2\,L_{12}^r\frac{t}{F^2}$$

GFF LECs: L_{11}, L_{12}, L_{13}

$$-D(t) = 1 + 2\frac{t}{F^2}(4L_{11}^r + L_{12}^r)$$

$$-16\frac{m_K^2}{F^2}(L_{11}^r - L_{13}^r) + \frac{3t}{4F^2}I_\pi(t) + \frac{3t}{2F^2}I_K(t) + \frac{9t - 8m_K^2}{12F^2}I_\eta(t)$$

$$I(q^2) = \frac{1}{48\pi^2}\left[\ln\frac{\mu^2}{m^2} - 1 + \frac{q^2}{5m^2}\right] + \mathcal{O}(q^4)$$

At -t=0, D=-1 + meson mass correction as follows [Donoghue and Leutwyler, ZPC52 (1991)]

[Hudson and Schweitzer, Phys. Rev. D 96, 114013 (2017)]

$$-D(0) = 1 + \boxed{\frac{16m_K^2}{F^2}(L_{11}^r - L_{13}^r)} - \frac{m_K^2}{64\pi^2F^2}\left[\ln\frac{\mu^2}{m_\eta^2} - 1\right] + \ldots \approx 0.77 \pm 0.15\ \ (\mu = m_\eta)$$

Leading Nc result in the quark model, magnitude is amplified by larger kaon mass (vs. A+D=0.03 for the pion)

Gravitational form factors of the Kaon

$$\langle K^+(p')\,|\,\hat{T}^a_{\mu\nu}(0)\,|\,K^+(p)\rangle = \left[\boxed{4P_\mu P_\nu\, A^a(t)} + \boxed{(q^\mu q^\nu - g^{\mu\nu}q^2)\, D^a(t)} + \boxed{g^{\mu\nu}4\Lambda^2\bar{c}^a(t)}\right]$$

$$\boxed{\begin{aligned} \int_{-1}^{1} dx\, x\, H^{u/K^+}(x,\xi,t) &= A^{u/K^+}(t) + \xi^2 D^{u/K^+}(t) \\ \int_{-1}^{1} dx\, x\, H^{\bar{s}/K^+}(x,\xi,t) &= A^{\bar{s}/K^+}(t) + \xi^2 D^{\bar{s}/K^+}(t) \end{aligned}}$$

$A^a(t)$

Mass distribution of the quarks and gluons inside the kaon

At t=0, second Mellin moment of the unpolarized PDF

Normalization $A^q(0) + A^g(0) = 1$

$D^a(t)$ (D-term)

Dispersion relation of the DVCS (and DVMP) amplitudes

Fundamental, but not related to an obvious symmetry

Internal pressure and shear distributions

Negative for hadrons to satisfy the stability conditions

[Polyakov, Shuvaev hep-ph/0207153]

[Polyakov PLB555 (2003)]

[Polyakov, Schweitzer IJMPA33 (2018)]

$\bar{c}^a(t)$

[M. Polyakov, HDS, JHEP 156 (2018)]

Non-conservation of quark and gluon parts of EMT ~ $g_{\mu\nu}$

Contributes to the mass(00) and the pressure(ii) (quark and gluon portions)

$\sum_q \bar{c}^q + \bar{c}^g = 0$, Smallness of $\sum_q \bar{c}^q(0)$ at low scale, suppressed by instanton packing fraction

Gravitational form factors of the Kaon

[Donoghue and Leutwyler, ZPC52 (1991)]

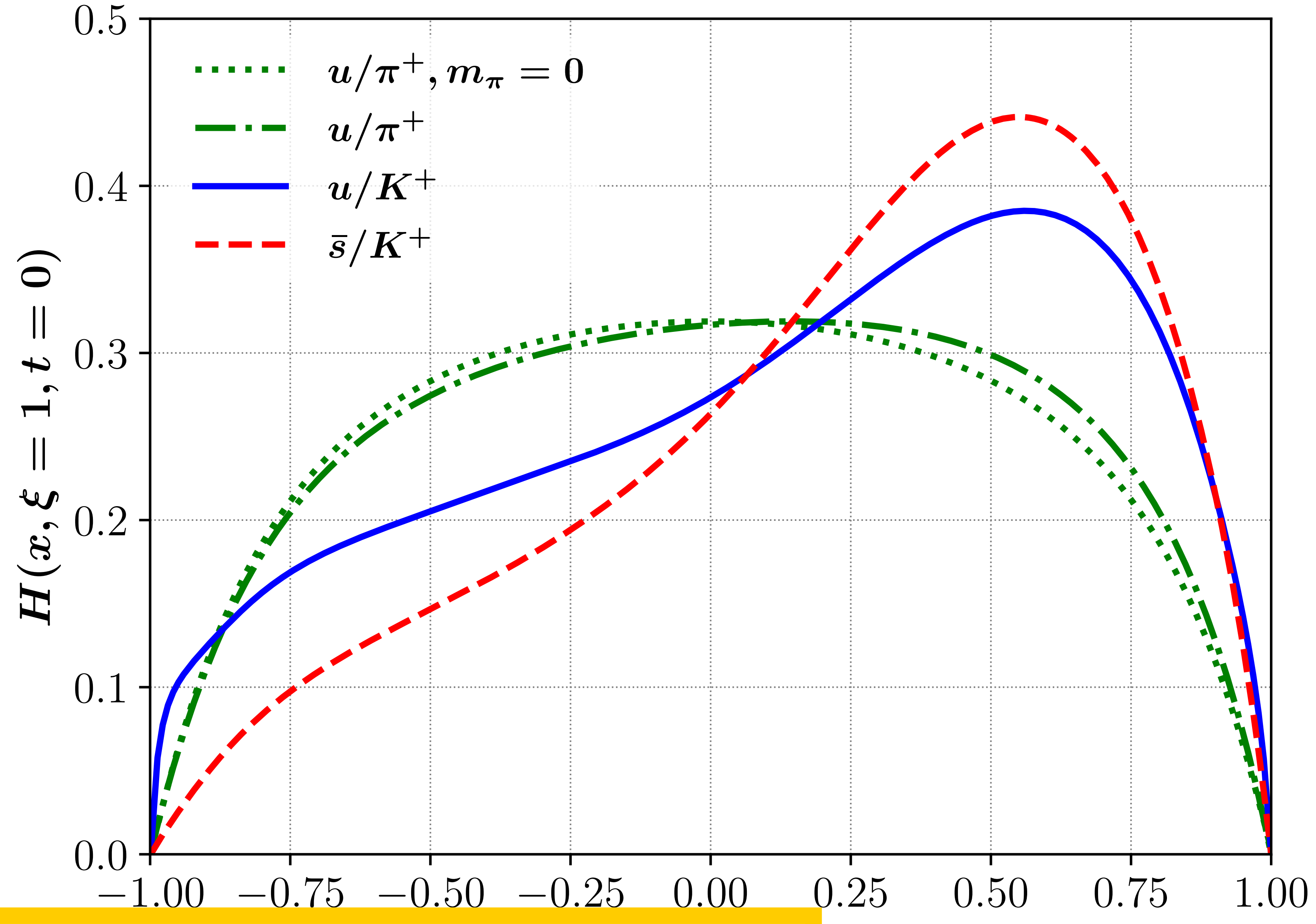
$$A(0) + D(0) = \boxed{\frac{16m_K^2}{F^2}(L_{11}^{\mu} - L_{13}^{\mu})} + \frac{m_K^2}{72\pi^2F^2}\left[\ln\frac{\mu^2}{m_\eta^2} - 1\right] + \ldots \approx 0.33 \pm 0.15$$

[Hudson and Schweitzer, Phys. Rev. D 96, 114013 (2017)]

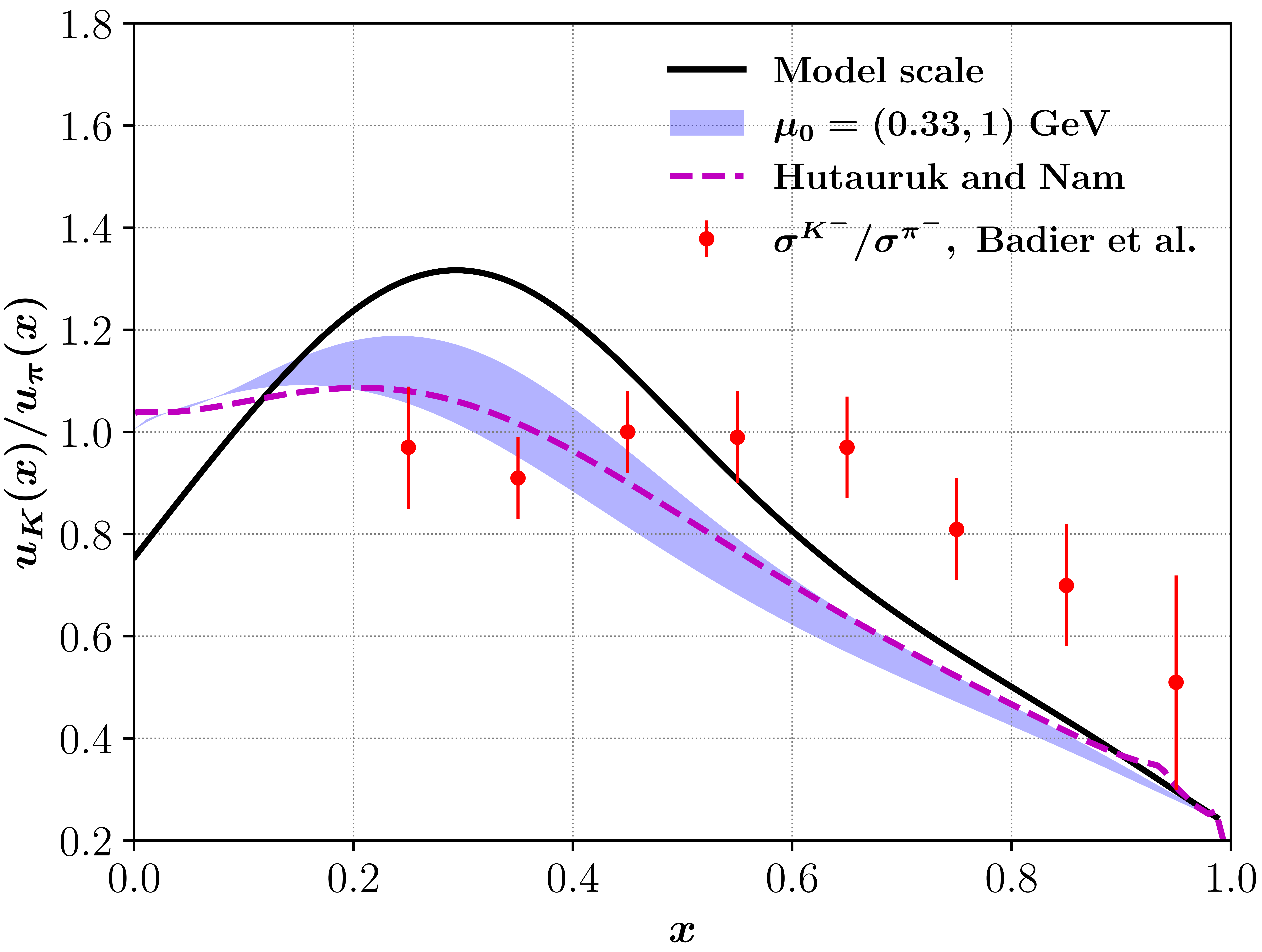
Leading Nc result in the quark model, magnitude is amplified by larger kaon mass (vs. A+D=0.03 for the pion)

$$\boxed{\begin{aligned} \int_{-1}^{1} dx\ x\, H^{u/K^+}(x,\xi,t) &= A^{u/K^+}(t) + \xi^2 D^{u/K^+}(t) \\ \int_{-1}^{1} dx\ x\, H^{\bar{s}/K^+}(x,\xi,t) &= A^{\bar{s}/K^+}(t) + \xi^2 D^{\bar{s}/K^+}(t) \end{aligned}}$$

$$\rightarrow \int_{-1}^{1} dx\ x\, H(x,\xi=1,t=0) = A(0) + D(0)$$



Size of A(0)+D(0) ~ size of asymmetry in xH(x,χ=1,t=0) ~ O(m^2)


Model scale
$\mu_0 = (0.33, 1)$ GeV
Hutauruk and Nam
$\sigma^{K^-}/\sigma^{\pi^-}$, Badier et al.
$u_K(x)/u_\pi(x)$
x
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8