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Nucleon axial charge and form factor from lattice QCD

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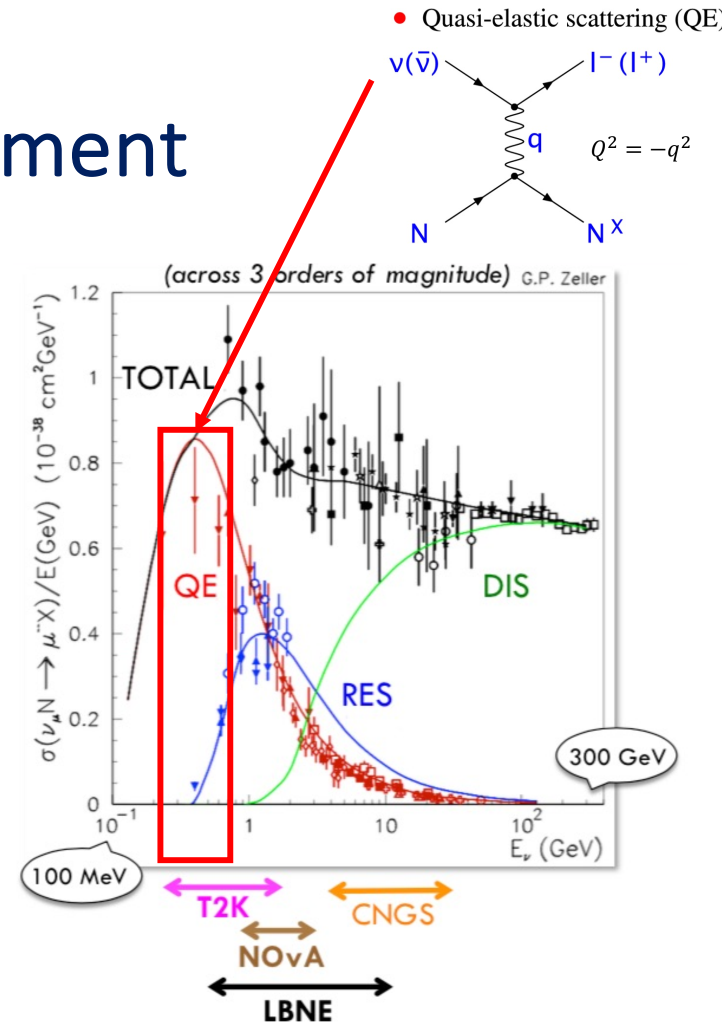
Neutrino Oscillation Experiment

- Upcoming flagship neutrino oscillation experiment such as DUNE (US) and HyperK (Japan), Quasi-elastic (QE) neutrino-nucleon scattering is the dominant interaction process

$$\frac{d\sigma}{dQ^2} \propto f(Q^2, [\mathrm{VFF}], [\mathrm{AFF}], \cdots)$$

- Weak interaction (V-A): Low statistics
  - Vector form factor (VFF)
    → high-statistics electron scattering experiments
  - Axial vector form factor (AFF)
    → Lattice QCD calculation is simple at QE processes and can help constraining experimental cross-section
    → Must provide a complete parametrization function (e.g. z-expansion) of $G_A(Q^2)$ including a covariance matrix of parameters.



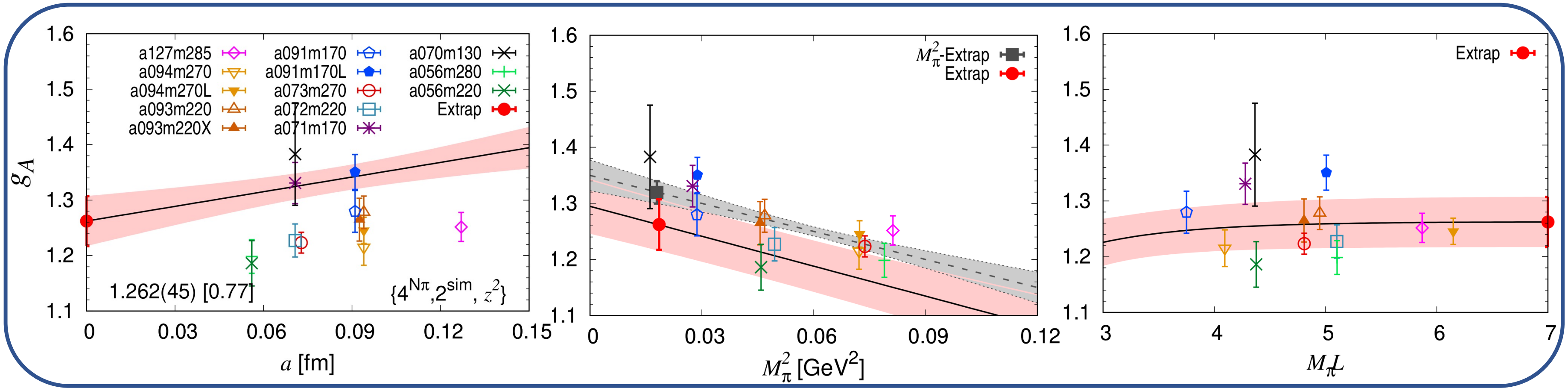
Major Uncertainties in Lattice QCD calculations

- Finite lattice spacing a (UV cut-off effect)
- Chiral fit to get value at physical pion mass
- Finite Volume

- Statistical errors
- Excited state contaminations
- Renormalization

$$g(a, M_\pi, M_\pi L) = c_1 + c_2 a + c_3 M_\pi^2 + c_4 \frac{M_\pi^2 e^{-M_\pi L}}{\sqrt{M_\pi L}}$$

Chiral-Continuum Finite volume extrapolation
of nucleon axial charge g_A [NME preliminary]



This work: two new MILC M_π^{Phys} ensembles

- $N_f = 2 + 1 + 1$ dynamical fermion flavors (isospin symmetric u,d quark masses)
- Gauge ensemble generated by MILC collaboration using HISQ (Highly Improved Staggered Quark) seq quark action

ID	β	a (fm)	M_π (MeV)	M_N (MeV)	L	T	$M_\pi L$	Lattices	N_{HP}	N_{LP}	τ
$a09m135$	6.3	0.087(1)	134(1)	947.1(7.6)	64	96	3.80	5,497	16,491	527,712	{10,12,14,16,18}
$a06m135$	6.7	0.057(0)	136(0)	932.2(8.1)	96	192	3.78	3,990	15,960	430,920	{16,18,20,22,24}

~4x improved statistics

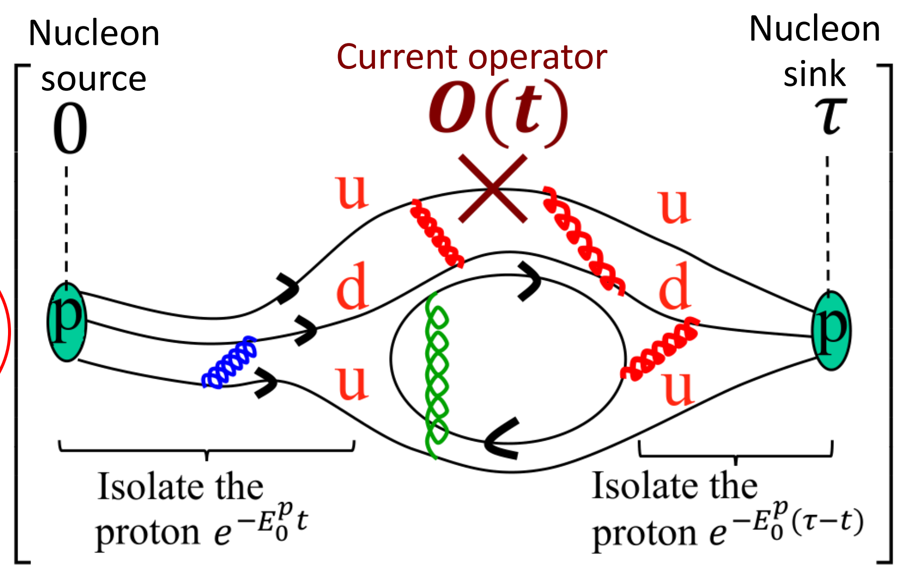
- Improved gauge link smearing, mass parameter tuning
- Larger source-sink time separation
- The largest momentum transfer $\vec{q} = 2\pi\vec{n}/L$ is doubled
  - spacelike 4-momentum transfer $Q^2 = -q^2$ max has increased from $0.45\ \mathrm{GeV}^2$ to $0.82\ \mathrm{GeV}^2$

Calculation of Nucleon Matrix Element

- Nucleon matrix element ($\langle p'|A|p\rangle$, to be decomposed to Form Factors) are extracted from the **<u>3-point correlation function</u>** $\boldsymbol{C(t,\tau) \equiv \langle N^p(\tau) O(t) \bar{N}^p(0)\rangle}$:

Average over the "Gauge ensemble" generated using Markov Chain Monte-Carlo

$$C(t,\tau) = \frac{1}{N_{\text{samp}}} \sum_{\{U\}}$$

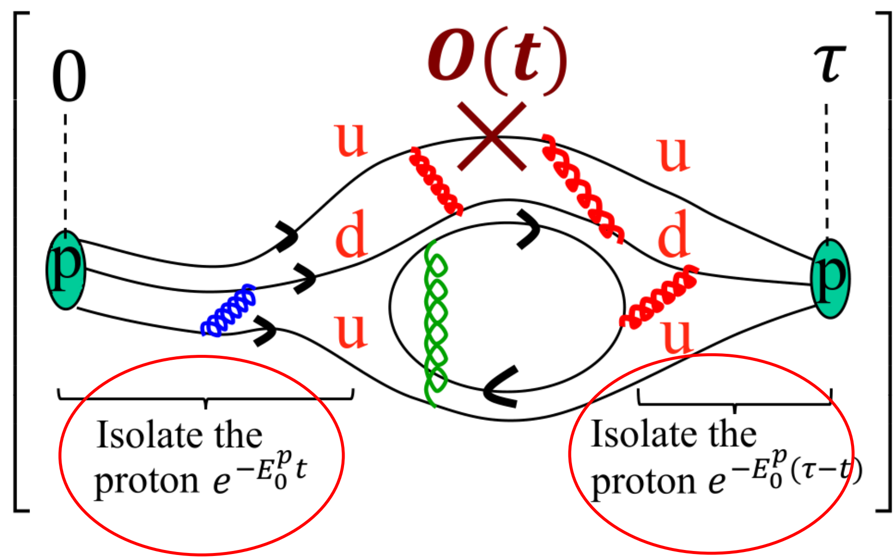


$$\langle N(\boldsymbol{p}_f, s_f)|A_\mu(\boldsymbol{q})|N(\boldsymbol{p}_i, s_i)\rangle$$
$$= \bar{u}_N(\boldsymbol{p}_f, s_f)\left(G_A(q^2)\gamma_\mu + q_\mu \frac{\tilde{G}_P(q^2)}{2M_N}\right)\gamma_5 u_N(\boldsymbol{p}_i, s_i),$$

Excited State (ES) effect is more severe at $\boldsymbol{M_\pi^{\mathrm{Phys}}}$

- Nucleon signal/noise decays $\propto e^{-(E-1.5M_\pi)\tau}$ with Euclidean time τ.
- Nucleon operator creates ground state nucleons (N) plus all excited states with the same quantum number, including $N\pi$, $N\pi\pi$, $N\rho$, $N^*(1440)$, $N^*(1710)$, $\cdots$.

$$C(t,\tau) = \frac{1}{N_{\text{samp}}} \sum_{\{U\}}$$



Jang et al. PRL 124 (2020) 072002
Park et al. PRD 105 (2022) 054505
Jang et al. PRD 109 (2024) 014503

1st excited state masses from two different analysis

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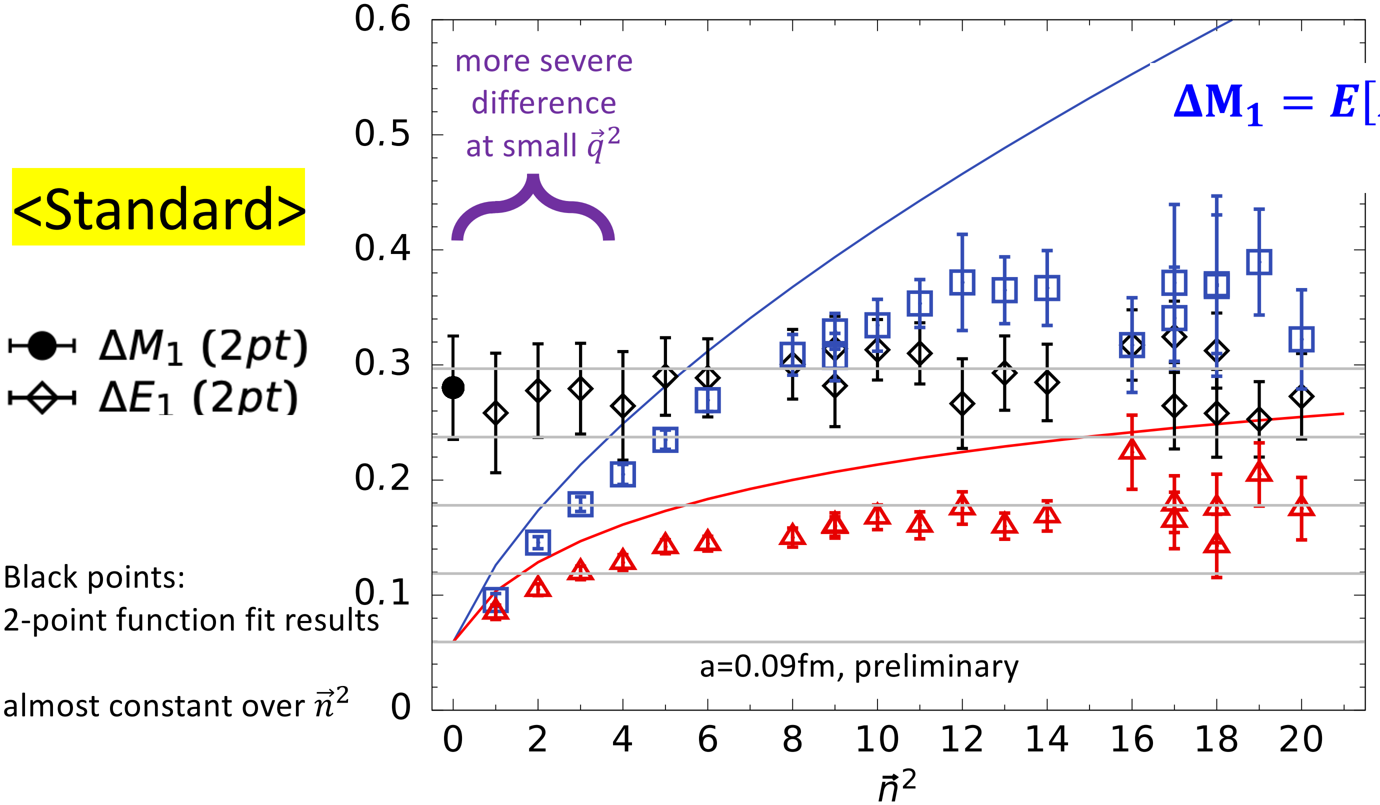


Black points:
2-point function fit results

almost constant over $\vec{n}^2$

<$N\pi$>

$$\mathbf{\Delta M_1} = E[N(\vec{p}) + \pi(-\vec{p})] - M$$

$$\mathbf{\Delta E_1} = E[N(0) + \pi(\vec{p})] - E_N(\vec{p})$$

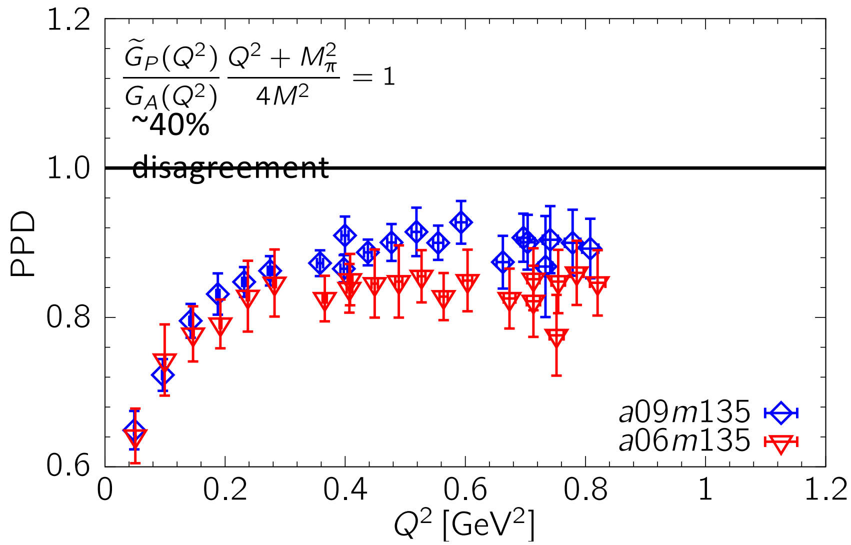
blue / red points
3-point function fit results

similar $\vec{n}^2$ dependence shape as the noninteracting $N\pi$ states where π carries $\vec{p}$

Checking Pion Pole Dominance (PPD) hypothesis

- Relates Induced pseudoscalar and the axial form factors

$$(\widetilde{G_P}(Q^2) = \frac{4M^2}{Q^2+M_\pi^2} G_A(Q^2))$$



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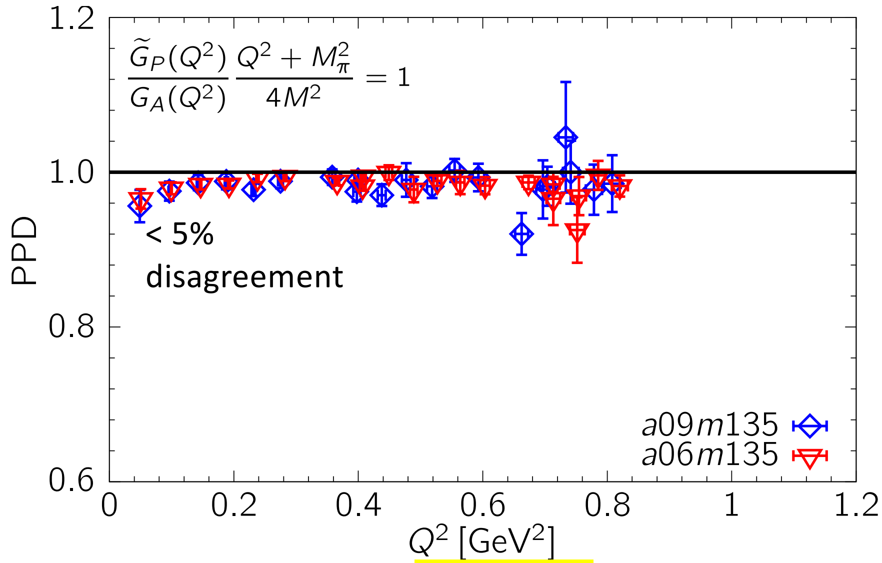


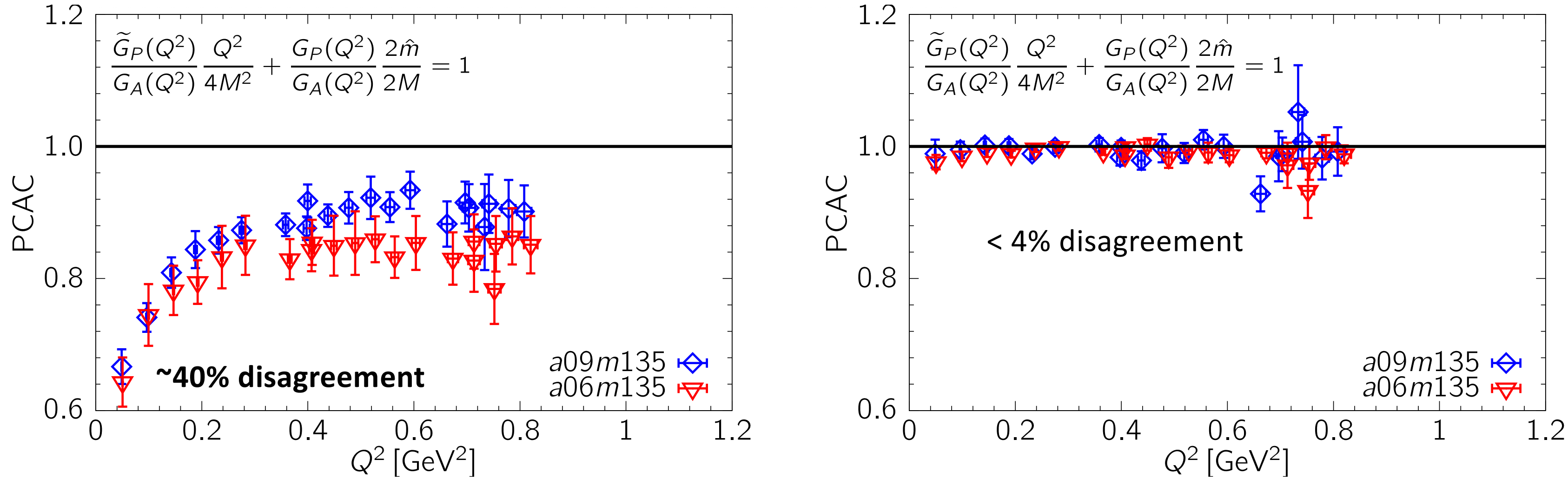
<$N\pi$>

PCAC (Partially Conserved Axial Current) relation

- $\partial_\mu A_\mu(x) = 2\hat{m}P(x)$ where $\hat{m} = Z_m m_{ud} Z_P Z_A^{-1}$
- Applied to nucleon ground state, it relates the 3 nucleon form factors

$$2M_N G_A(Q^2) - \frac{Q^2}{2M_N}\tilde{G}_P(Q^2) = 2\hat{m}G_P(Q^2)$$

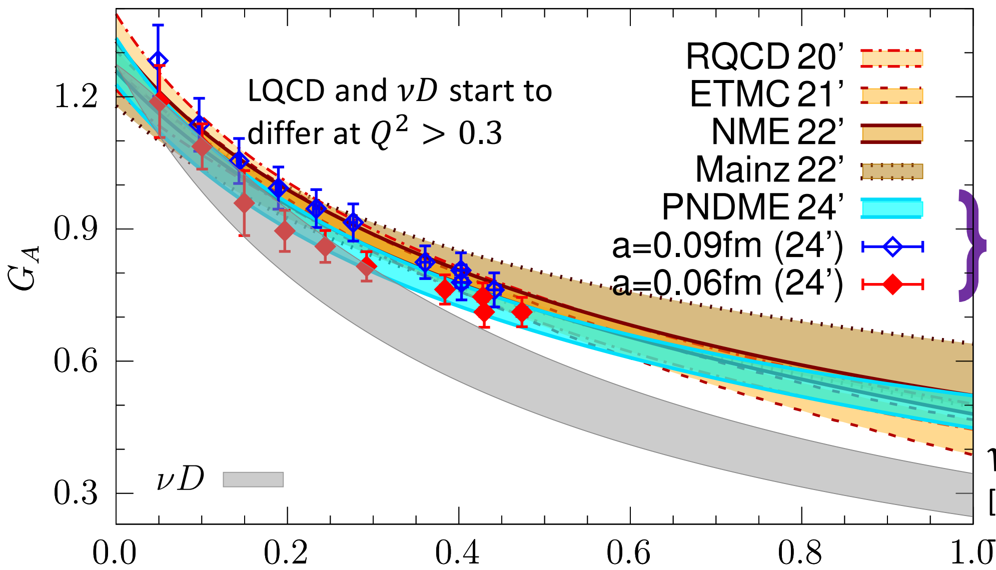
Generalized Goldberger-Treiman Relation



<Standard>   <$N\pi$>

2024 Comparison of lattice results of G_A

RQCD 20’ JHEP 05 126
ETMC 21’ PRD 103, 034509
NME 22’ PRD 105, 054505
Mainz 22’ PRD 106, 074503
PNDME 24’ PRD 109, 014503



Two physical pion mass data in PNDME 24’

νD from z-expansion fit

[A.Meyer et al., PRD 93,113015(2016)]

- The deuterium bubble chamber experiment data (70-80s) have smal statistics (10^3),
- Unknown correction added. Cannot access original data.
- Unresolved nuclear (deuterium) correction
- Dipole fit underestimated G_A uncertainty by x10

Turquoise Band
PNDME 24’

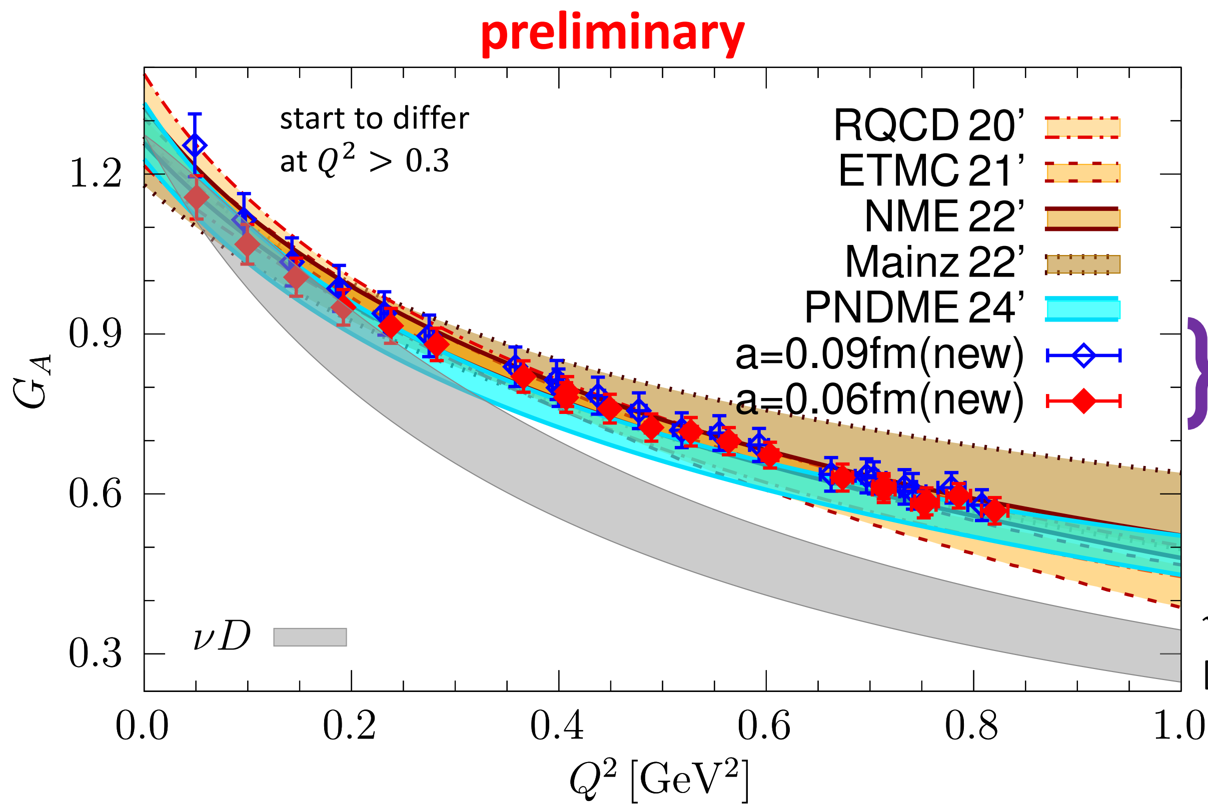
$$G_A(Q^2) = a_0 + a_1 z + a_2 z^2 = 0.876(28) - 1.669(99)z + 0.483(498)z^2,$$

with the correlation matrix:

$$\begin{array}{cc} & \begin{array}{ccc} a_0 & a_1 & a_2 \end{array} \\ \begin{array}{c} a_0 \\ a_1 \\ a_2 \end{array} & \begin{pmatrix} 1.0 & -0.45170 & -0.02966 \\ -0.45170 & 1.0 & -0.24394 \\ -0.02966 & -0.24394 & 1.0 \end{pmatrix} \end{array}$$

Our new data for $\boldsymbol{G_A}$

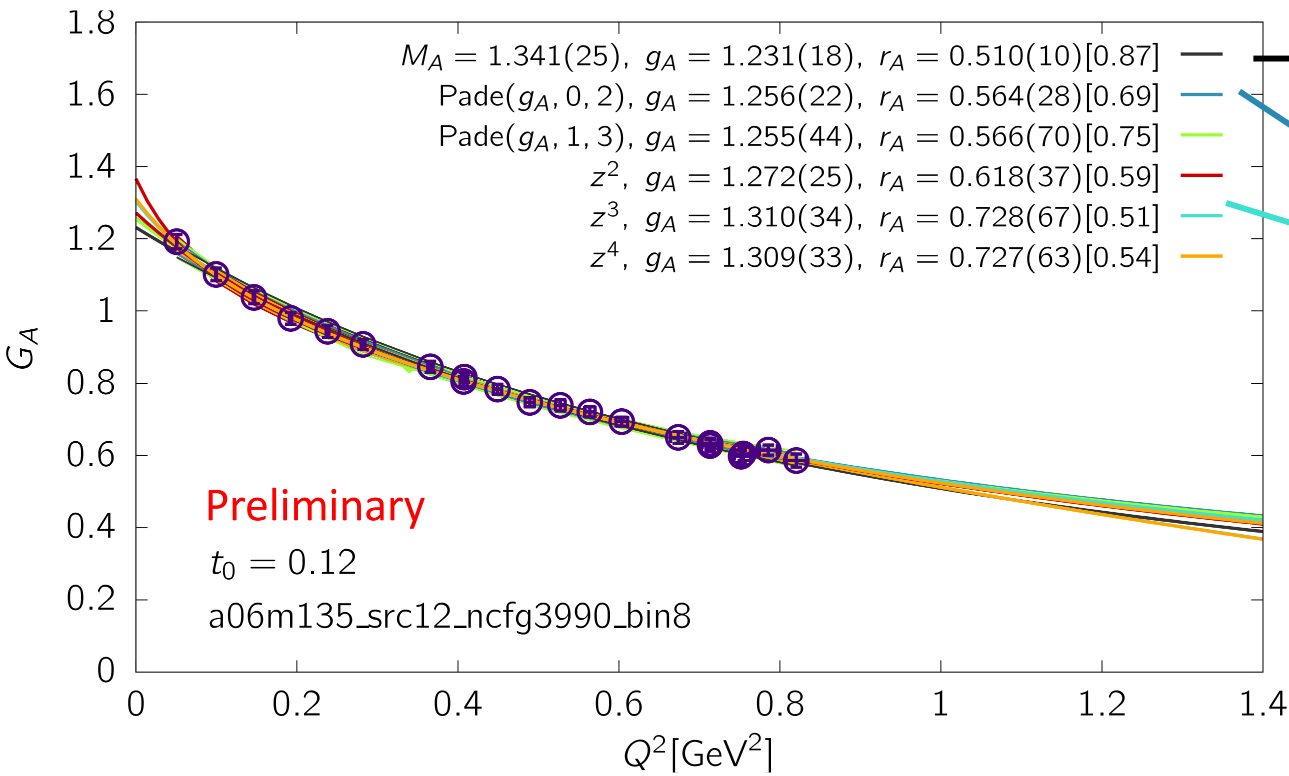
RQCD 20’ JHEP 05 126
ETMC 21’ PRD 103, 034509
NME 22’ PRD 105, 054505
Mainz 22’ PRD 106, 074503
PNDME 24’ PRD 109, 014503



NEW data on the two physical pion mass ensembles

νD from z-expansion fit
[A.Meyer et al., PRD 93,113015(2016)]

G_A^{u-d} : Examined Dipole, Pade and z-expansion fits



- Dipole: $g_A/\left(1+\frac{Q^2}{M_A^2}\right)^2$
  - uncertainty underestimated
- Pade: $\frac{g_A}{1+b_1Q^2+b_2Q^4}$
- z^n-expansion: $\sum_{k=0}^{n} a_k z(Q^2)^k$

z^3 result (renormalized)
gA=1.271(51)
GA= 1.038(34) - 1.54(12) z
+0.89(83) z^2 – 3.2(1.9) z^3

Summary

- Calculating the isovector axial form factor $\boldsymbol{G_A^{u-d}(Q^2)}$ as part of a comprehensive analysis of nucleon structure.
- New data for $\boldsymbol{G_A^{u-d}}$ on 2 physical pion mass ensembles over $0.04 < Q^2 \lesssim 1\ \mathrm{GeV}^2$ with significantly improved statistics and systematics control. This enables a more reliable chiral-continuum extrapolation.
- Result: a z-expansion parametrization with the correlation matrix of the fit parameters.
- $\boldsymbol{G_A^{u-d}(Q^2)}$ is important input in the calculation of the quasi-elastic neutrino-nucleon scattering cross-section. (Needed for DUNE to meet precision goal)

PNDME and NME members

- Tanmoy Bhattacharya (LANL)
- Vincenzo Cirigliano (INT)
- Rajan Gupta (LANL)
- Emanuele Mereghetti (LANL)
- Boram Yoon (NVIDIA)
- Junsik Yoo (LANL)
- Yong-Chull Jang (BNL)
- **Sungwoo Park (LLNL)**
- Santanu Mondal (MSU)
- Huey-Wen Lin (MSU)
- Balint Joo (ORNL)
- Frank Winter (Jlab)

References

PNDME (clover-on-HISQ formulation)

- Charges: Gupta et al, PRD.98 (2018) 034503
- AFF: Gupta et al, PRD 96 (2017) 114503
  Jang et al, PRL 124 (2020) 072002
  Jang et al, PRD 109 (2024) 014503
- VFF: Jang et al, PRD 100 (2020) 014507
- $\sigma_{\pi N}$ Gupta et al, PRL 127 (2021) 242002
- d_n from Θ-term Bhattacharya et al, PRD 103 (2021) 114507
- d_n from qEDM Gupta et al, PRD 98 (2018) 091501
- Moments of PDFs Mondal et al, PRD 102 (2020) 054512
- Proton spin: Lin et al, PRD 98 (2018) 094512
- Flavor diag. charges: Park et al, arXiv:2503.07100

NME (clover-on-clover formulation)

- Charges, VFF, AFF: Park et al, PRD 105 (2022) 054505
- Moments of PDFs Mondal et al, JHEP 04 (2021) 044

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BACKUP

Lepton-nucleon scattering

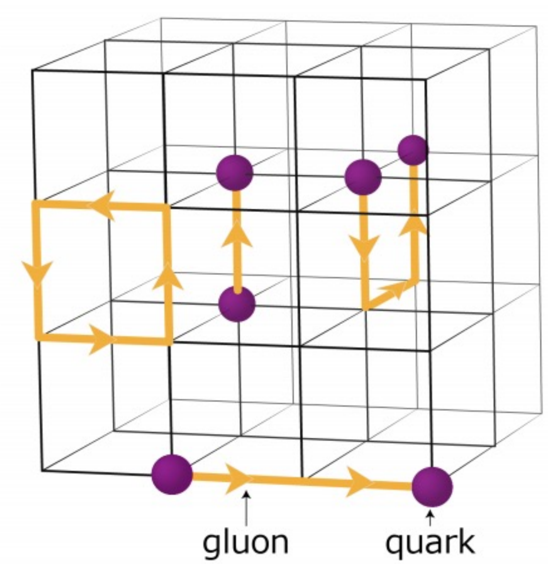
$$\frac{d\sigma}{dQ^2}\begin{pmatrix} \nu_l + n \to l^- + p \\ \bar{\nu}_l + p \to l^+ + n \end{pmatrix}$$
$$= \frac{M^2 {G_F}^2 cos^2\theta_c}{8\pi {E_\nu}^2}\left\{A(Q^2) \pm B(Q^2)\frac{(s-u)}{M^2} + C(Q^2)\frac{(s-u)^2}{M^4}\right\},$$

$$\begin{aligned} A(Q^2) &= \frac{(m^2+Q^2)}{M^2}\left[(1+\tau)F_A^2 - (1-\tau)F_1^2 + \tau(1-\tau)F_2^2 + 4\tau F_1 F_2 \right. \\ & \left. -\frac{m^2}{4M^2}\left((F_1+F_2)^2 + (F_A+2F_P)^2 - 4\left(1+\frac{Q^2}{4M^2}\right)F_P^2\right)\right], \\ B(Q^2) &= \frac{Q^2}{M^2}F_A(F_1+F_2), \\ C(Q^2) &= \frac{1}{4}(F_A^2 + F_1^2 + \tau F_2^2). \end{aligned}$$

F_A = axial form factor
$\tilde{F}_P$ = induced pseudoscalar
$G_E = F_1 - \tau F_2$ Electric
$G_M = F_1 + F_2$ Magnetic
$\tau = Q^2/4M^2$
M=M_n= $M_p \approx$ 939 MeV
$m = M_\pi$

Lattice QCD

[Formulated by K. Wilson (1974). Numerical computation field opened by M. Creutz (1979)]



Lattice QCD is QCD defined on a 4-dimensional Euclidean space-time lattice

- Finite lattice spacing: (a)
- Quark fields $(q, \bar{q})$, Gauge fields (gluons): (U_μ)
- Perturbative & **Numerical (nonperturbative) calculations**

The simulation allows **ab initio** calculations of nonperturbative QCD interactions of quarks and gluons using the **Feynman path integral** formulation of QFT.

Major systematic errors coming from:

- Finite lattice spacing a (UV cut-off effect)
- Chiral fit to get value at physical pion mass
- Finite Volume
- Statistical errors
- **Excited state contaminations**
- Renormalization