Quark confinement in tetraquark systems

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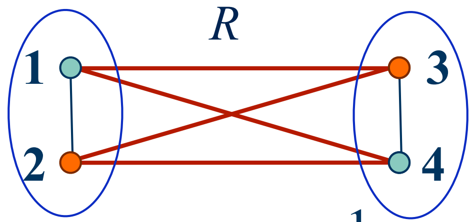
Quark confinement for tetraquarks

- **Conventional quark model: sum of two-body confinement**

$$V=\sum_{i<j}(\lambda_i\cdot\lambda_j)(-ar_{ij}) \qquad (\lambda_i\cdot\lambda_j)=\sum_{\alpha}\lambda_i^{\alpha}\lambda_j^{\alpha}$$



- satisfy color saturation

- induce long-range color van der Waals force

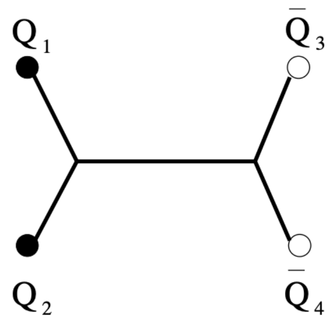
$$\text{cvdW} \propto -\frac{1}{R^3}$$

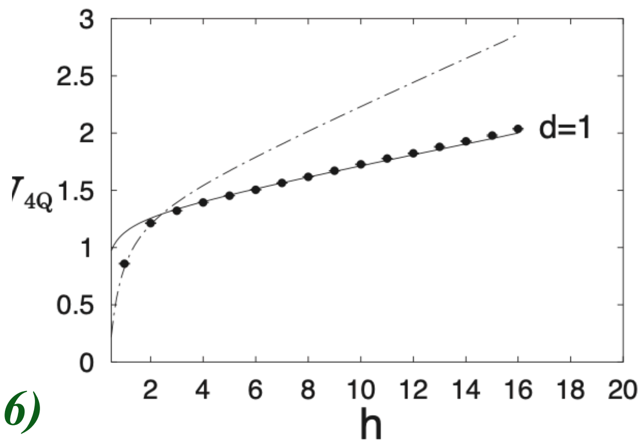
T. Appelquist, W. Fischler, Phys. Lett. B77, 405 (1978)
R.S. Willey, Phys. Rev. D18, 270 (1978)
S. Matsuyama, H. Miyazawa, Prog. Theor. Phys. 61, 942 (1979)

- **Confinement on the lattice**

connect quarks by strings with the minimal lengths

- need string rearrangements to allow color singlet mesons split away





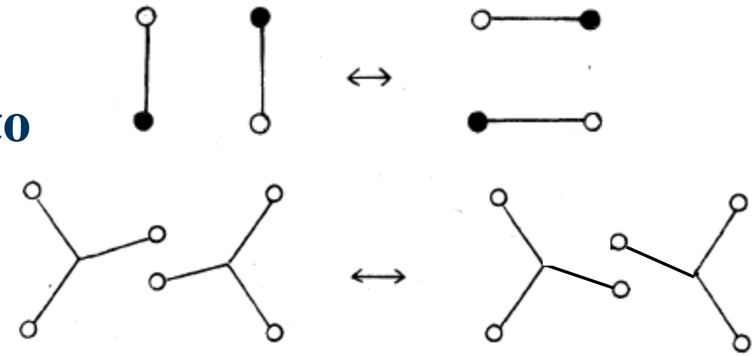
Lattice QCD Wilson loop for tetra quarks
F. Okiharu, et al., J. Mod. Phys. 7 ,774–789 (2016)

String confinement model

- “Reconnection of strings and quark matter”,
  H. Miyazawa, Phys. Rev. D20, 2953 (1979)

  “String Flip-Flop” model
  with reconnections of strings according to
  the spatial configuration of the quarks

$$V_{\text{string}} = \sigma \times \underset{\text{links}}{\text{Min}} \sum r_{\text{link}}$$

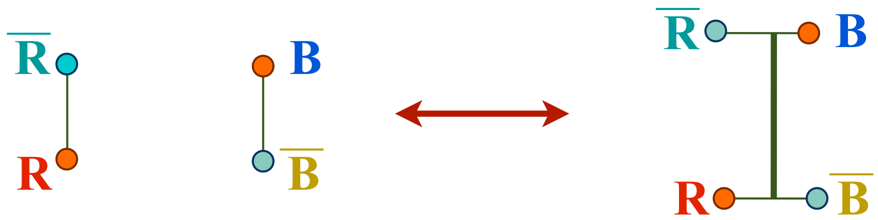


- Similar “string-type” confinement potential models for multi-quark systems were discussed by

  O.W. Greenberg, J. Hietarinta, Phys. Lett. B 86, 309 (1979)
  N. Isgur, J. E. Paton, Phys. Lett. B 124, 247 (1983)
  M. Oka, Phys. Rev. D 31, 2274 (1985).
  F. Lenz, et al., Annals Phys. 170, 65 (1986).
  Y. Koike et al., Nucl. Phys. A449, 635 (1986), PTP S137, 21 (2000).
  G.A. Miller, Phys. Rev D37, 2431 (1998).
  J. Vijande, A. Valcarce, J.M. Richard, Phys. Rev. D 85, 014019 (2012).

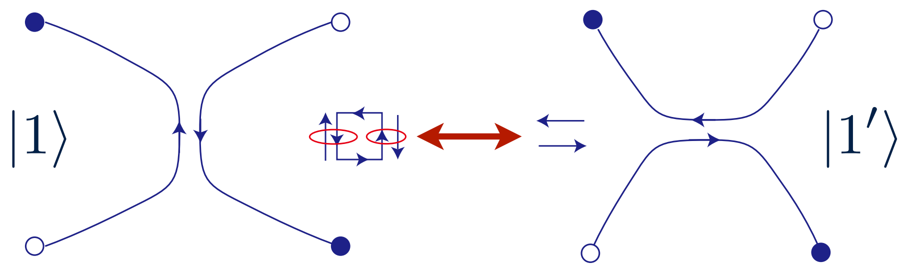
String flip-flop model

- **The string FF model works well for the U(1) charge, but for the color SU(3) theory, color recombination becomes nontrivial. Coupling of hidden color channels is unavoidable.**



Hidden Color channel

- **QCD predicts flip-flop in the *strong coupling expansion***



- **Two states are independent and transferred dynamically. This "string dynamics" can *not* be implemented in the conventional quark model.**

Quark confinement for tetraquarks

- **Reconsider the quark confinement potential for the quark model from the QCD viewpoints.**
  Sum of two-body confinements v.s. String-like confinement
- **We propose to extend the color configuration space of the conventional quark model with a hidden-color $|\mathrm{hc}\rangle\rangle$ state.**
  Its mixing induces extra short-range attraction among the multi-quark systems.
- **Test the new model by applying it to the fully charmed tetraquarks.**

PHYSICAL REVIEW D **108,** L071501 (2023)

Letter

Quark confinement for multiquark systems:
Application to $\boxed{\text{fully charmed tetraquarks}}$

Guang-Juan Wang,[1,2,*] Makoto Oka,[3,2,†] and Daisuke Jido[4,‡]

Quark model - color configurations

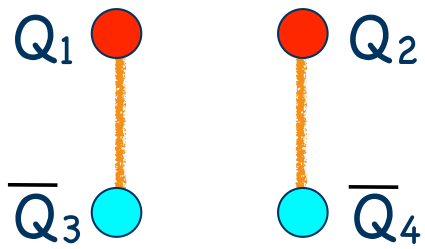
- **Only two independent color states are allowed in Quark Model.**

$$\mathbf{3}\otimes\mathbf{3}\otimes\bar{\mathbf{3}}\otimes\bar{\mathbf{3}}=\boxed{2\times\mathbf{1}}\oplus 4\times\mathbf{8}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$$

- **Color singlet Q_1 Q_2 Q_3^{bar} Q_4^{bar} system is described by**

Two singlets (mesons) states:

$$|1\rangle=|(Q_1\bar{Q}_3)_{\mathbf{1}}(Q_2\bar{Q}_4)_{\mathbf{1}}\rangle \quad |1'\rangle=|(Q_1\bar{Q}_4)_{\mathbf{1}}(Q_2\bar{Q}_3)_{\mathbf{1}}\rangle$$

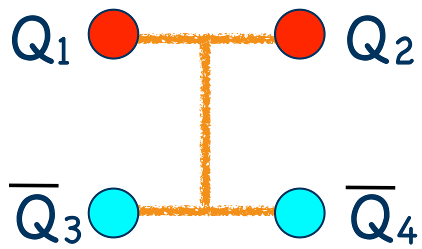


Singlet + hidden color states:

$$|1\rangle=|(Q_1\bar{Q}_3)_{\mathbf{1}}(Q_2\bar{Q}_4)_{\mathbf{1}}\rangle \quad |8\rangle=|(Q_1\bar{Q}_3)_{\mathbf{8}}(Q_2\bar{Q}_3)_{\mathbf{8}}\rangle$$

Diquarks with color 3^{bar} and 6:

$$|3\rangle=|(Q_1Q_2)_{\bar{\mathbf{3}}}(\bar{Q}_3\bar{Q}_4)_{\mathbf{3}}\rangle \quad |6\rangle=|(Q_1Q_2)_{\mathbf{6}}(\bar{Q}_3\bar{Q}_4)_{\bar{\mathbf{6}}}\rangle$$



- ***These bases are all equivalent.***

$$|1\rangle=\sqrt{\frac{1}{3}}|3\rangle+\sqrt{\frac{2}{3}}|6\rangle \qquad |8\rangle=-\sqrt{\frac{2}{3}}|3\rangle+\sqrt{\frac{1}{3}}|6\rangle$$

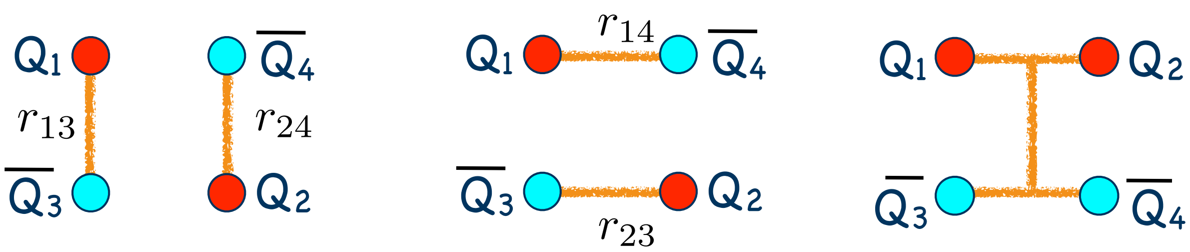
- **Two-meson states are not orthogonal.** $\langle 1|1'\rangle=\frac{1}{3}$

Novel string-like potential

- **If the quarks are the only carriers of color charges, the quark model does not have enough *freedom for color configurations*.**
- **We propose to extend the color Hilbert space of the quark model that can incorporate the color dynamics for multiquark systems.**

G.J. Wang, MO, D. Jido, Phys. Rev. D 108, L071501 (2023)

- **For a tetra-quark systems, we choose 3 color basis states:**



$$|\mathbf{1}\rangle\rangle \equiv |(Q_1 \to \bar{Q}_3)_1 (Q_2 \to \bar{Q}_4)_1\rangle$$

$$|\mathbf{1}'\rangle\rangle \equiv |(Q_1 \to \bar{Q}_4)_1 (Q_2 \to \bar{Q}_3)_1\rangle$$

orthogonal bases

$$|\mathbf{hc}\rangle\rangle \equiv |(Q_1 \leftrightarrow Q_2)_{\bar{3}} \leftarrow (\bar{Q}_3 \leftrightarrow \bar{Q}_4)_3\rangle$$

Novel string-like potential

- **The string confinement potential**

$$\langle\langle \mathbf{1}|V_{\rm ST}|\mathbf{1}\rangle\rangle = \sigma(r_{13}+r_{24}),$$
$$\langle\langle \mathbf{1}'|V_{\rm ST}|\mathbf{1}'\rangle\rangle = \sigma(r_{14}+r_{23}).$$

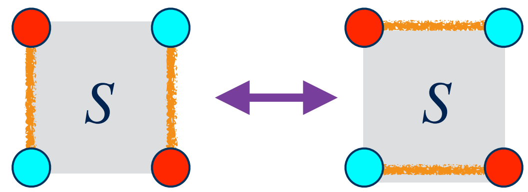
σ: string tension

- **Transitions by quantum tunneling filled the area by gauge field**

$$\langle\langle \mathbf{1}|V_{\rm ST}|\mathbf{1}'\rangle\rangle = \kappa e^{-\sigma S}$$

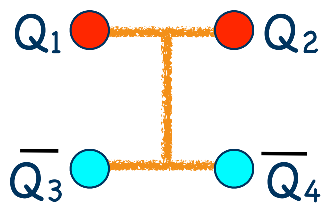
S: Minimal surface area

Y. Koike, O. Morimatsu, K. Yazaki, PTP S137, 21 (2000)



- **Confinement in $|\mathbf{hc}\rangle\rangle$ channel**

$$\langle\langle \mathbf{hc}|V_{\rm ST}|\mathbf{hc}\rangle\rangle = \sigma\left[\frac{r_{13}+r_{24}+r_{14}+r_{23}}{4}+\frac{r_{12}+r_{34}}{2}\right]$$
$$\langle\langle \mathbf{1}|V_{\rm ST}|\mathbf{hc}\rangle\rangle = \langle\langle \mathbf{1}'|V_{\rm ST}|\mathbf{hc}\rangle\rangle = \pm\kappa' \exp\left(-\sigma S\right) \qquad \kappa' = \sqrt{8}\kappa$$

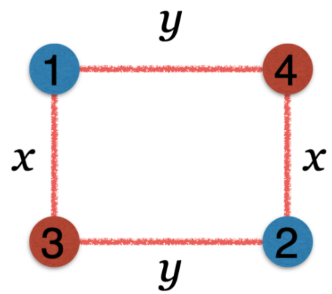


- **3-channel confinement potential (full 4-body potential)**

$$V_{\rm ST} = \begin{array}{cr} \begin{pmatrix} \sigma(r_{13}+r_{24}) & \kappa e^{-\sigma S} & \kappa' e^{-\sigma S} \\ \kappa e^{-\sigma S} & \sigma(r_{14}+r_{23}) & -\kappa' e^{-\sigma S} \\ \kappa' e^{-\sigma S} & -\kappa' e^{-\sigma S} & \frac{\sigma}{4}[r_{13}+r_{24}+r_{14}+r_{23}+2(r_{12}+r_{34})] \end{pmatrix} & \begin{matrix} |\mathbf{1}\rangle\rangle \\ |\mathbf{1}'\rangle\rangle \\ |\mathbf{hc}\rangle\rangle \end{matrix} \end{array}$$

Born-Oppenheimer effective potential

- A rectangle configuration of $QQ\bar{Q}\bar{Q}$



- QM confinement for the non-orthogonal $|\mathbf{1}\rangle, |\mathbf{1}'\rangle$ bases

$$V = \begin{pmatrix} 2\sigma x & (2/3)\sigma(x+y-\sqrt{x^2+y^2}) \\ (2/3)\sigma(x+y-\sqrt{x^2+y^2}) & 2\sigma y \end{pmatrix}$$

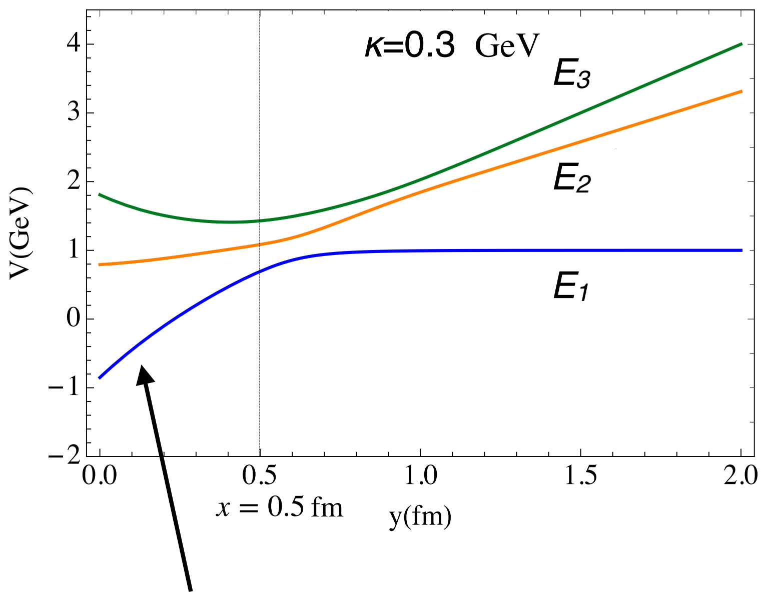
- String confinement for the orthogonal $|\mathbf{1}\rangle\rangle, |\mathbf{1}'\rangle\rangle, |\mathbf{hc}\rangle\rangle$ bases

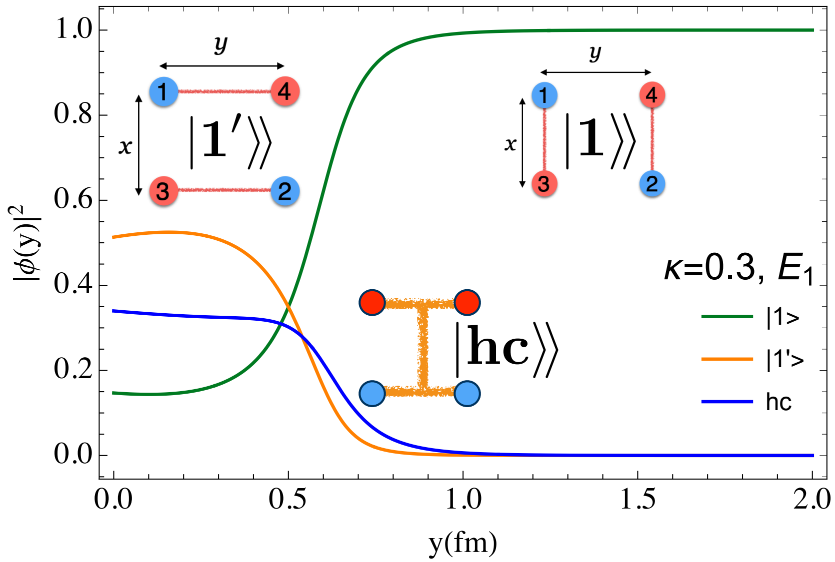
$$V_{\rm ST}(x,y) = \begin{pmatrix} 2\sigma x & \kappa e^{-\sigma xy} & \kappa' e^{-\sigma xy} \\ \kappa e^{-\sigma xy} & 2\sigma y & -\kappa' e^{-\sigma xy} \\ \kappa' e^{-\sigma xy} & -\kappa' e^{-\sigma xy} & \sigma(\frac{x+y}{2}+\sqrt{x^2+y^2}) \end{pmatrix}$$

One free parameter κ ($\kappa' = \sqrt{8}\kappa$), which is not determined from the meson spectrum.

Born-Oppenheimer effective potential

- **Novel String-like confinement: adiabatic potential at fixed *x***





- **A short-range attraction due to the large mixing of the hidden color (hc) state. Compact tetra-quark configurations become important at short distances.**
- **The only free parameter κ will take a value in**

$$0 \le \kappa' = \sqrt{8}\kappa \le 2\sigma a \sim 2\sqrt{\sigma} \sim 1\mathrm{GeV} \longrightarrow \kappa \le 0.3\mathrm{GeV}$$

Confinement + short-range potential

- **Short-range Coulomb + spin-spin forces are superimposed.**

$$V_{\rm cen}(r_{ij}) = V_{\rm Coul} + V_{\rm hyp}$$
$$= \frac{\alpha_s}{r_{ij}} - \frac{8\pi\alpha_s}{3m_i m_j}\left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r_{ij}^2}\mathbf{s}_i\cdot\mathbf{s}_j$$

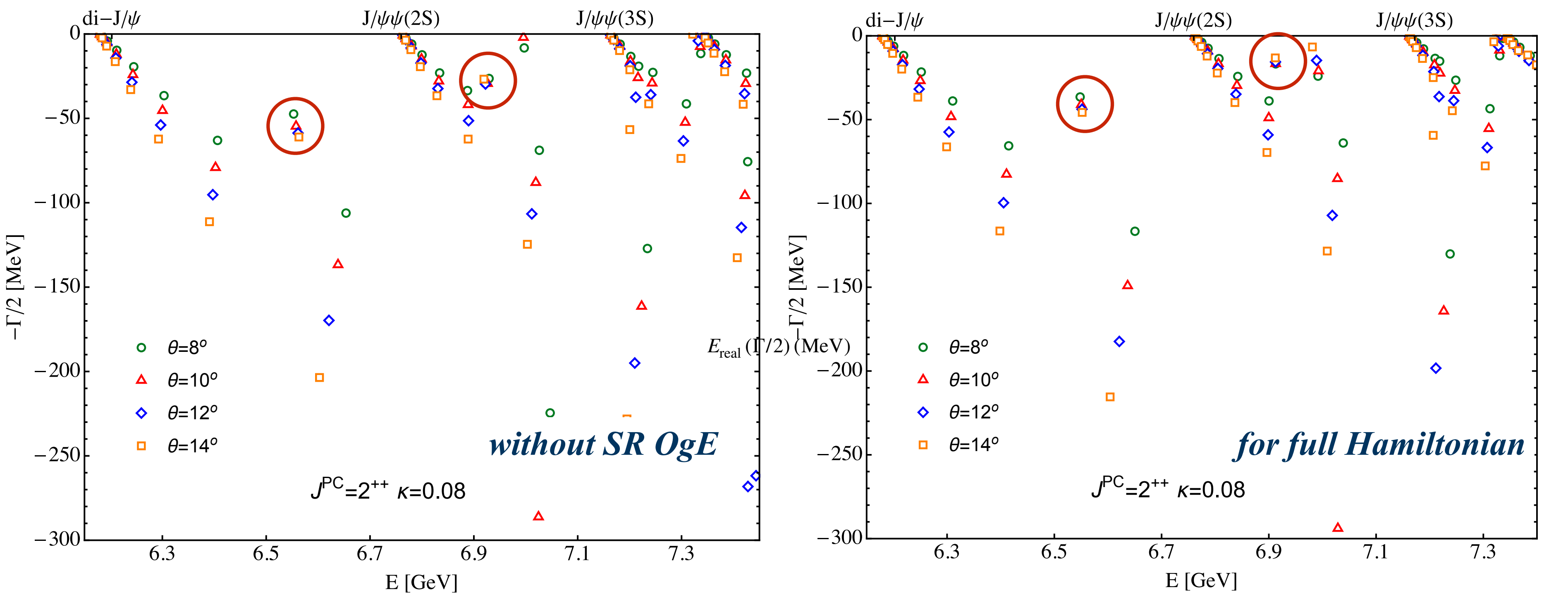
- **Parameters are determined to fit to the charmonium masses and to reproduce the two meson thresholds**

		Mass spectrum (MeV)			
Parameter		$^{2S+1}L_J$	Meson	EXP	THE
α_s	0.5461	$^1S_0$	η_c	2983.9	2984
b [GeV^2]	0.1425	$^3S_1$	J/ψ	3096.9	3092
m_c [GeV]	1.4794	$^3P_0$	χ_{c0}	3414.7	3426
σ [GeV]	1.0946	$^3P_1$	χ_{c1}	3510.7	3506
		$^1P_1$	$h_c(1P)$	3525.4	3516
		$^3P_2$	χ_{c2}	3556.2	3556
		$^1S_0$	$\eta_c(2S)$	3637.5	3634
		$^3S_1$	$\psi(2S)$	3686.1	3675
		$^3S_1$	$\psi(3S)$	4039.0	4076
		$^3S_1$	$\psi(4S)$	4421.0	4412

Tetra-charms with novel confinement

- **Complex scaling spectra of the spin-aligned 2^{++} states**

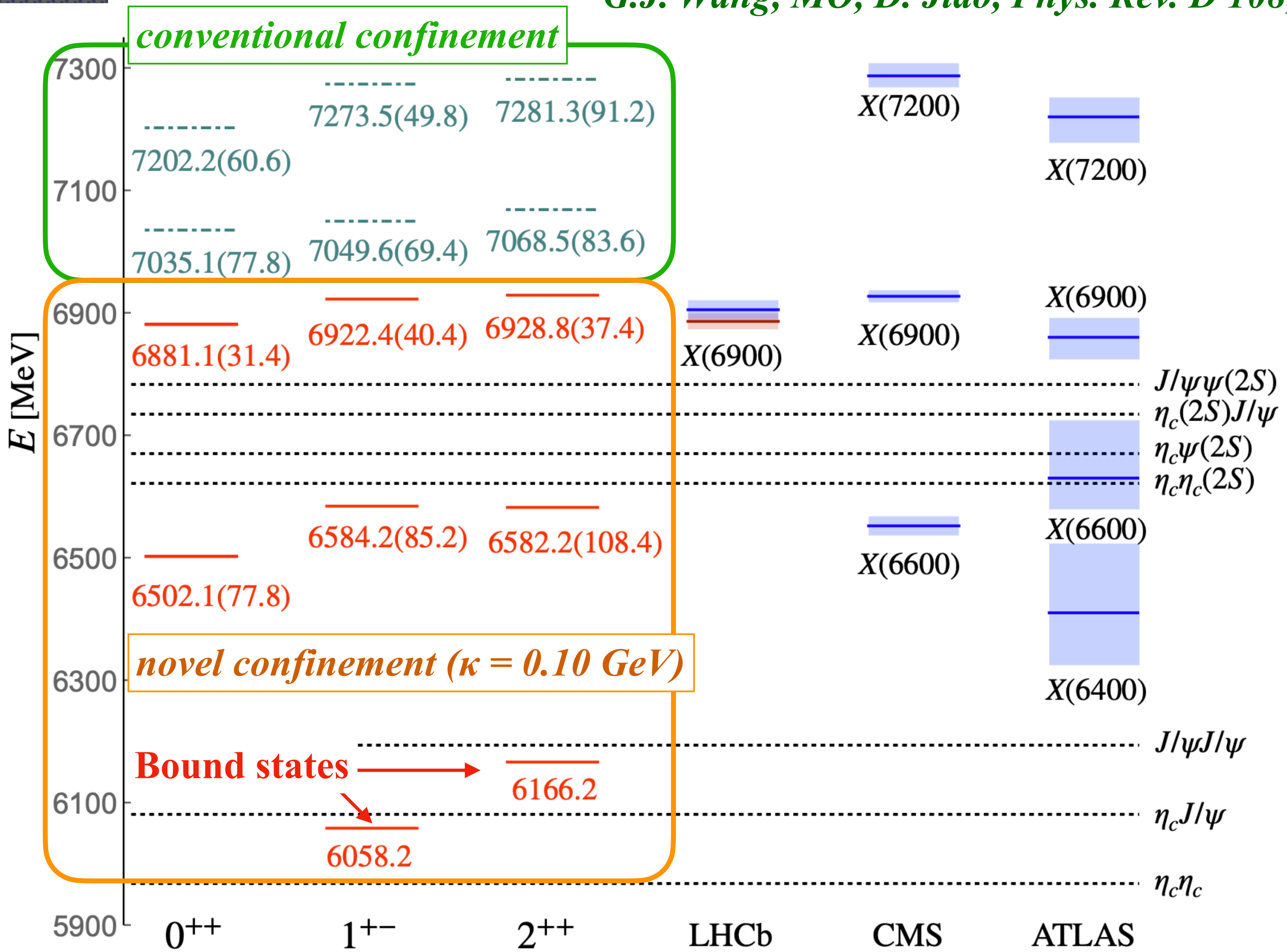
G.J. Wang, MO, D. Jido, Phys. Rev. D 108, L071501 (2023)



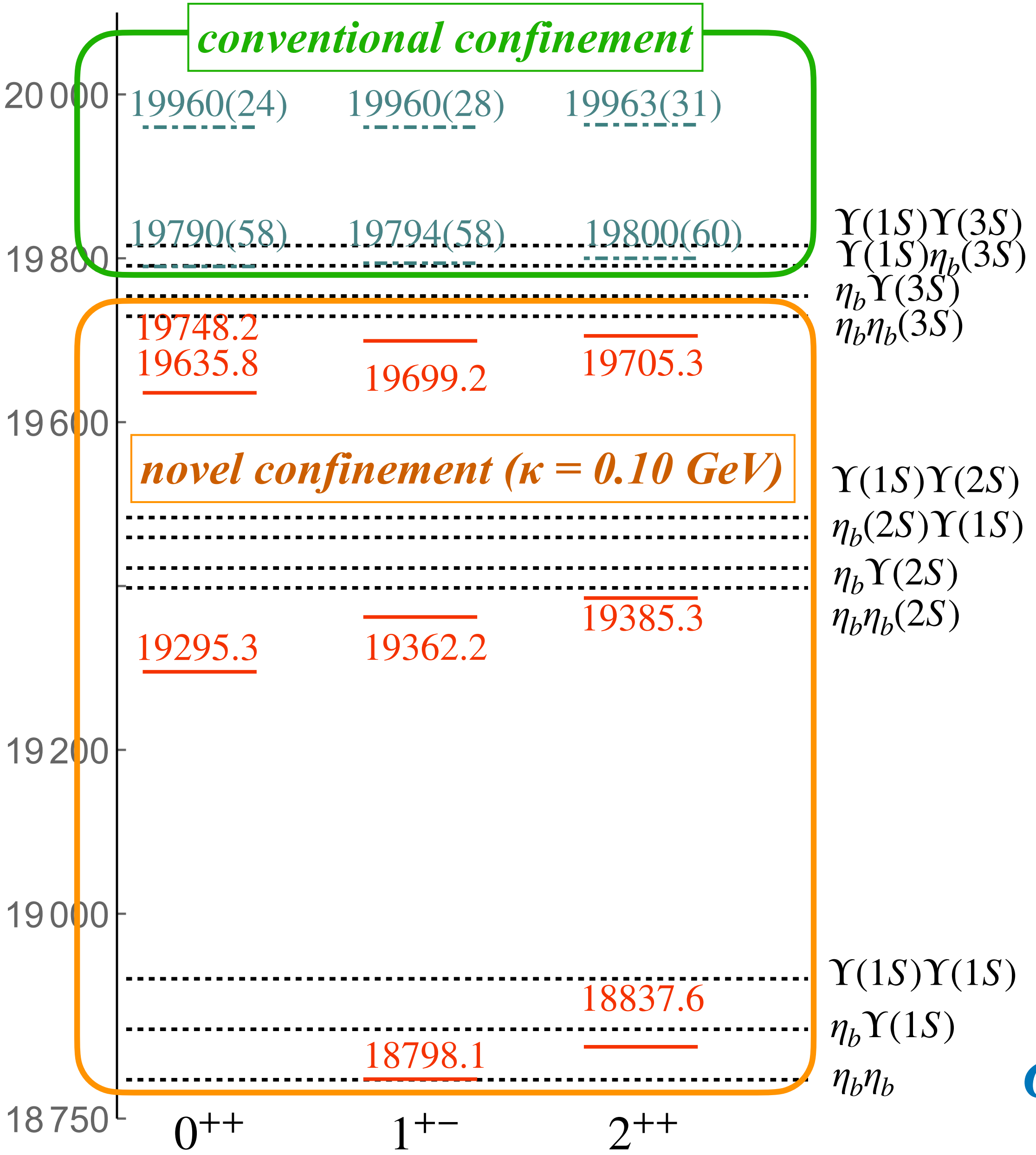
- **The SR OgE interactions play minor role here.**

$cc\bar{c}\bar{c}$ spectrum with novel confinement

G.J. Wang, MO, D. Jido, Phys. Rev. D 108, L071501 (2023)



$bb\bar{b}\bar{b}$ spectrum with novel confinement

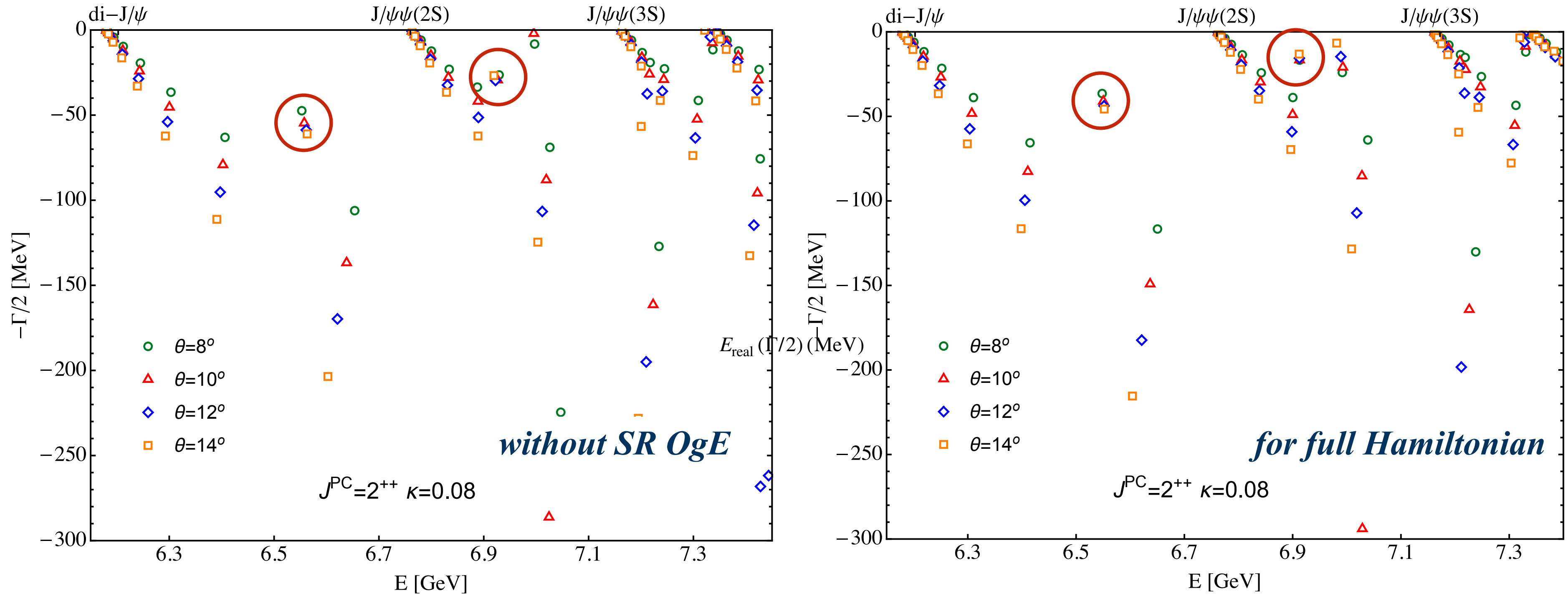


G.J. Wang

$cc\bar{c}\bar{c}$ spectrum with novel confinement

G.J. Wang, MO, D. Jido, Phys. Rev. D 108, L071501 (2023)

Complex scaling spectra of the spin-aligned 2^{++} states

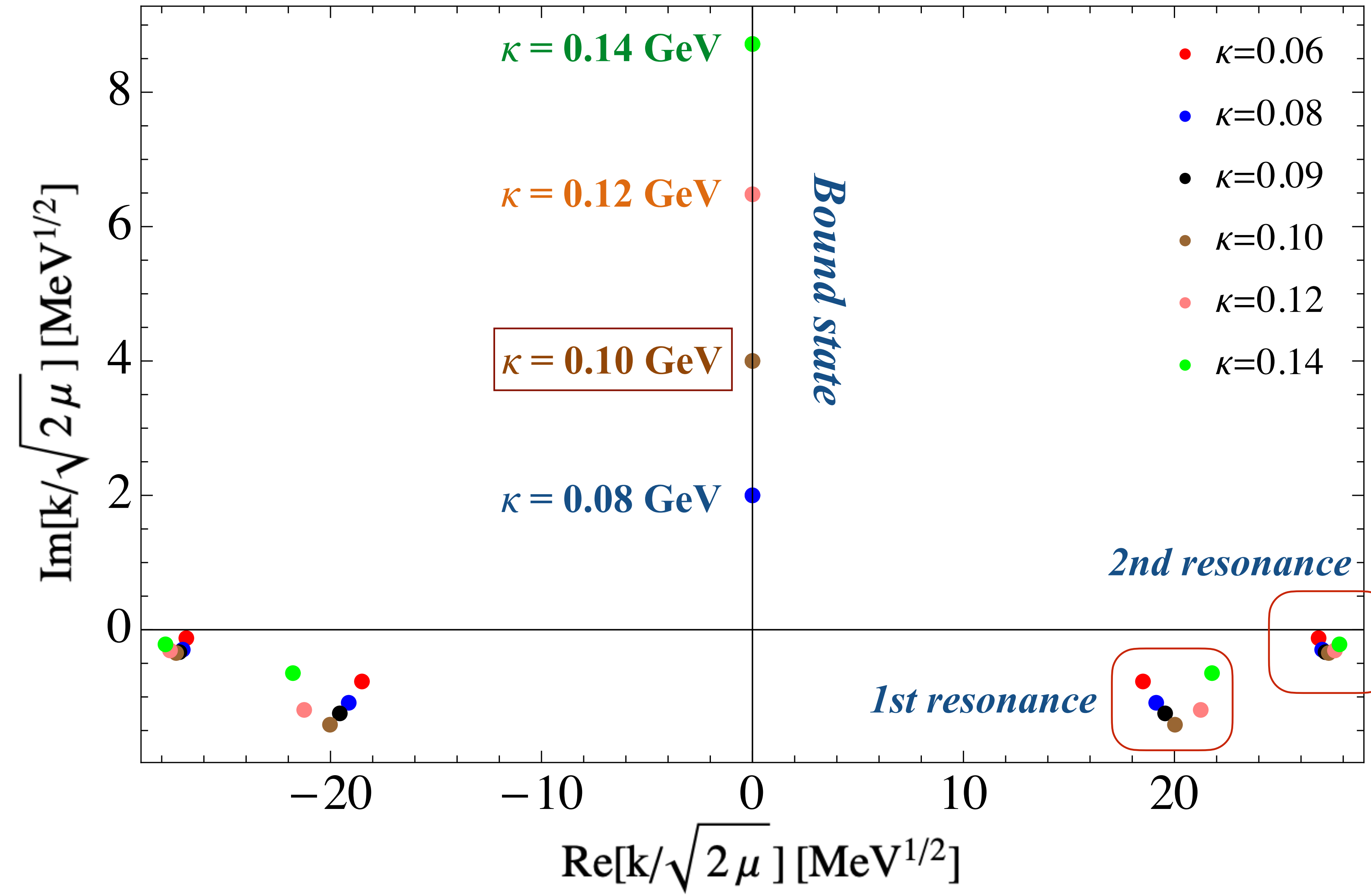


2^{++} bound and resonance states

κ (GeV)	Bound state (MeV)	Binding Ene. (MeV)	1st Res E (Γ/2) (MeV)	2nd Res E (Γ/2) (MeV)
0.08	6180	4	6550 (41)	6913 (17)
0.10	6166	16	6582 (54)	6929 (19)
0.12	6140	42	6631 (51)	6944 (16)

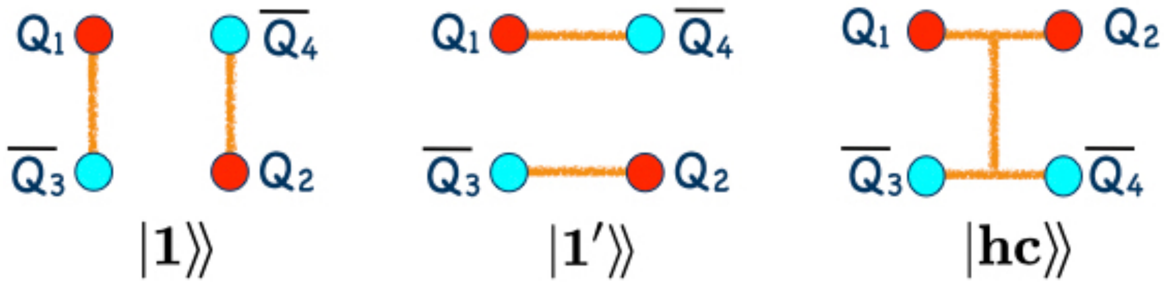
$cc\bar{c}\bar{c}$ spectrum with novel confinement

- **Pole positions of the 2^{++} charmed tetraquark in complex k plane**



Summary

- Confinement in the tetra(multi)-quark system is not trivial nor well established from the quark model viewpoints.
- String-like confinement potential is proposed by extending the color configuration space of the conventional quark model with a compact hidden-color state.



- Mixing of the $|\mathrm{hc}\rangle\rangle$ state induces strong attraction among the multi-quark systems.
- The model is applied to the fully-charmed tetra-quark system. A bound state appears due to the attraction of HC. Complex scaling method provides us with two or more resonances that seem consistent with experiment.