Empirical Formula for the Maximum mass of neutron stars with Relativistic mean-field theory

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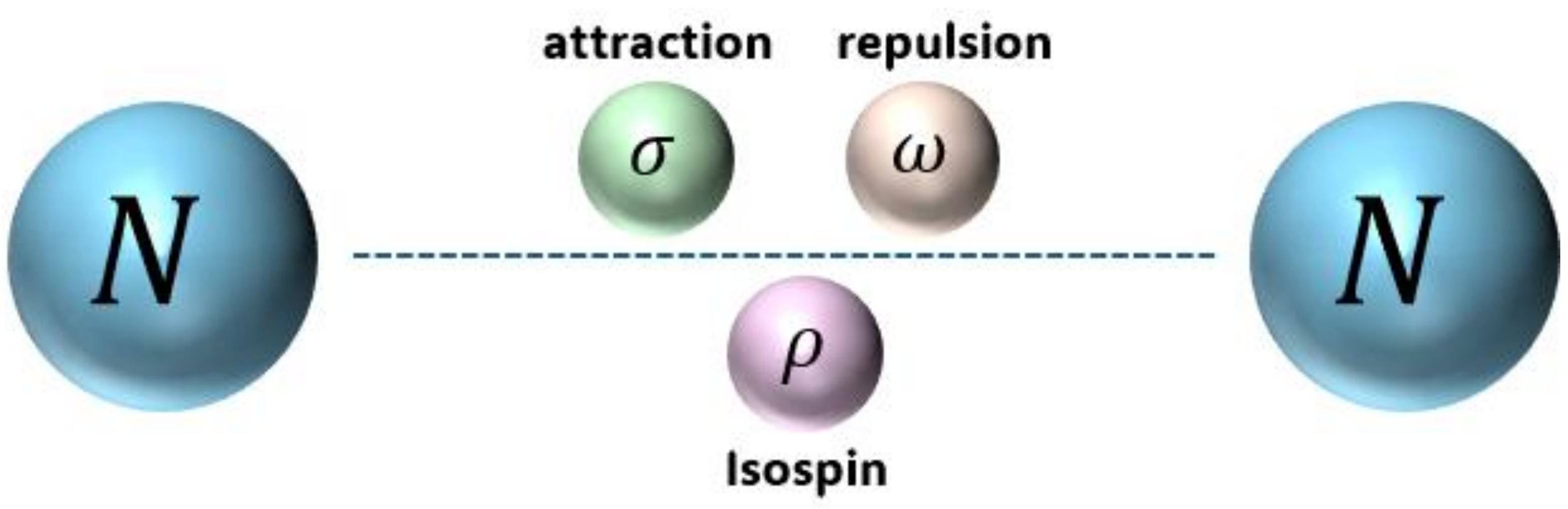
Abstract

The empirical formula for the maximum mass of neutron stars is derived within the framework of the relativistic mean-field model. Observations of massive neutron stars heavier than $\sim 2\,M_\odot$ have ruled out soft equation of state and constrain nuclear interactions, which are valid in dense nuclear matter. Upon the request, numerous attempts have been made to refine the relativistic mean-field model, such as by including the delta meson. However, we find that the maximum mass of a neutron star predicted by the relativistic mean-field model can be primarily determined by the combination of the saturation density, the effective mass at saturation, and the vector meson self-coupling constant. While constraining the pure neutron matter equation of state using Chiral Effective Field Theory (ChEFT) at low densities, 250 parameter sets were generated to derive an empirical formula for the maximum mass of neutron stars and apply the formula with the present relativistic mean field models.

Relativistic Mean Field Theory



$$\begin{aligned}\mathcal{L} = {} & \bar\psi\left(i\gamma^\mu\partial_\mu - M_N\right)\psi + g_\sigma\,\sigma\,\bar\psi\psi - g_\omega\,\omega_\mu\,\bar\psi\gamma^\mu\psi \\ & - g_\rho\,\rho_\mu\,\bar\psi\gamma^\mu\frac{\tau}{2}\psi + \tfrac{1}{2}\,\partial_\mu\sigma\,\partial^\mu\sigma - \tfrac{1}{2}\,m_\sigma^2\,\sigma^2 - \tfrac{1}{3}\,g_2\,\sigma^3 \\ & - \tfrac{1}{4}\,g_3\,\sigma^4 - \tfrac{1}{4}\,\omega_{\mu\nu}\,\omega^{\mu\nu} + \tfrac{1}{2}\,m_\omega^2\,\omega_\mu\,\omega^\mu \\ & + \tfrac{1}{4}\,c_\omega\,(g_\omega^2\,\omega_\mu\omega^\mu)^2 - \tfrac{1}{4}\,\rho_{\mu\nu}\,\rho^{\mu\nu} + \tfrac{1}{2}\,m_\rho^2\,\rho_\mu\,\rho^\mu \qquad (1) \\ & + g_\sigma\,\sigma\,g_\omega^2\,\omega_\mu\omega^\mu\left(c'_{\sigma\omega} + c_{\sigma\omega}\,g_\sigma\,\sigma\right) \\ & + g_\sigma\,\sigma\,g_\rho^2\,\rho_\mu\rho^\mu\left(c'_{\sigma\rho} + c_{\sigma\rho}\,g_\sigma\,\sigma\right) \\ & + c_{\omega\rho}\,g_\omega^2\,\omega_\mu\omega^\mu\;g_\rho^2\,\rho_\mu\rho^\mu\,,\end{aligned}$$

$$\begin{aligned}\varepsilon = {} & \frac{1}{\pi^2}\int_0^{k_p} k^2\sqrt{k^2+M^{*2}}\,dk + \frac{1}{\pi^2}\int_0^{k_n} k^2\sqrt{k^2+M^{*2}}\,dk \\ & + \tfrac{1}{2}\,m_\sigma^2\,\sigma^2 + \tfrac{1}{3}\,g_2\,\sigma^3 + \tfrac{1}{4}\,g_3\,\sigma^4 \\ & - \tfrac{1}{2}\,m_\omega^2\,\omega_0^2 - \tfrac{1}{4}\,c_\omega\,g_\omega^4\,\omega_0^4 + g_\omega\,\omega_0\,\rho \\ & - \tfrac{1}{2}\,m_\rho^2\,\rho_{03}^2 + \tfrac{1}{2}\,g_\rho\,\rho_{03}\,\rho_3 - g_\sigma\,\sigma\,g_\omega^2\,\omega_0^2\left(c'_{\sigma\omega} + c_{\sigma\omega}\,g_\sigma\,\sigma\right) \\ & - g_\sigma\,\sigma\,g_\rho^2\,\rho_{03}^2\left(c'_{\sigma\rho} + c_{\sigma\rho}\,g_\sigma\,\sigma\right) - c_{\omega\rho}\,g_\omega^2\,\omega_0^2\,g_\rho^2\,\rho_{03}^2, \qquad (7)\end{aligned}$$

$$\begin{aligned}P = {} & \frac{1}{3\pi^2}\int_0^{k_p}\frac{k^4}{\sqrt{k^2+M^{*2}}}\,dk + \frac{1}{3\pi^2}\int_0^{k_n}\frac{k^4}{\sqrt{k^2+M^{*2}}}\,dk \\ & - \tfrac{1}{2}\,m_\sigma^2\,\sigma^2 - \tfrac{1}{3}\,g_2\,\sigma^3 - \tfrac{1}{4}\,g_3\,\sigma^4 + \tfrac{1}{2}\,m_\omega^2\,\omega_0^2 + \tfrac{1}{4}\,c_\omega\,g_\omega^4\,\omega_0^4 \\ & + \tfrac{1}{2}\,m_\rho^2\,\rho_{03}^2 + g_\omega\,\omega_0\,\rho + g_\sigma\,\sigma\,g_\omega^2\,\omega_0^2\left(c'_{\sigma\omega} + c_{\sigma\omega}\,g_\sigma\,\sigma\right) \\ & + g_\sigma\,\sigma\,g_\rho^2\,\rho_{03}^2\left(c'_{\sigma\rho} + c_{\sigma\rho}\,g_\sigma\,\sigma\right) + c_{\omega\rho}\,g_\omega^2\,\omega_0^2\,g_\rho^2\,\rho_{03}^2 \qquad (8)\end{aligned}$$

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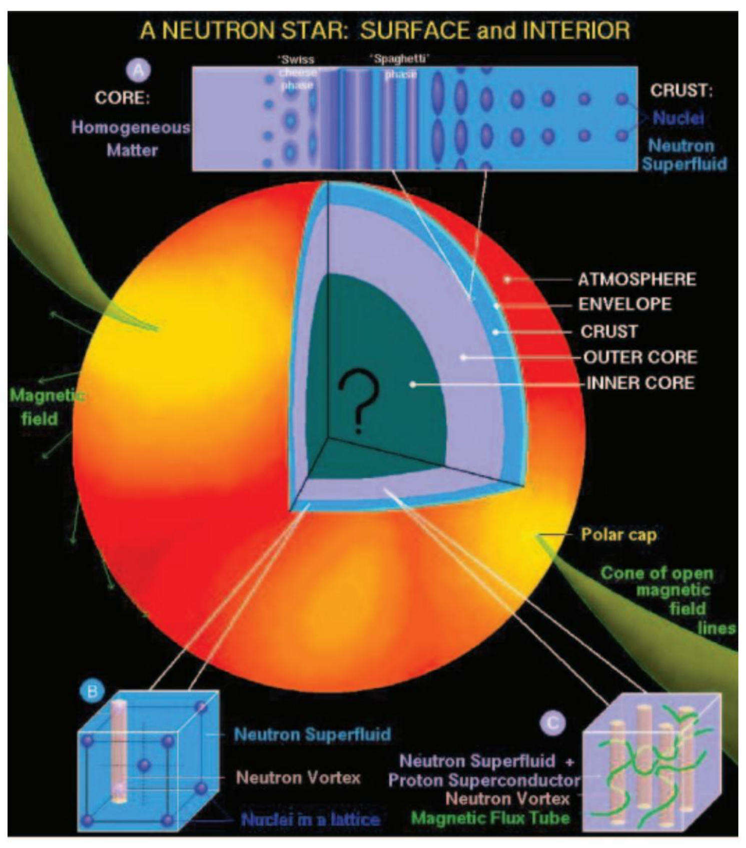
Neutron star



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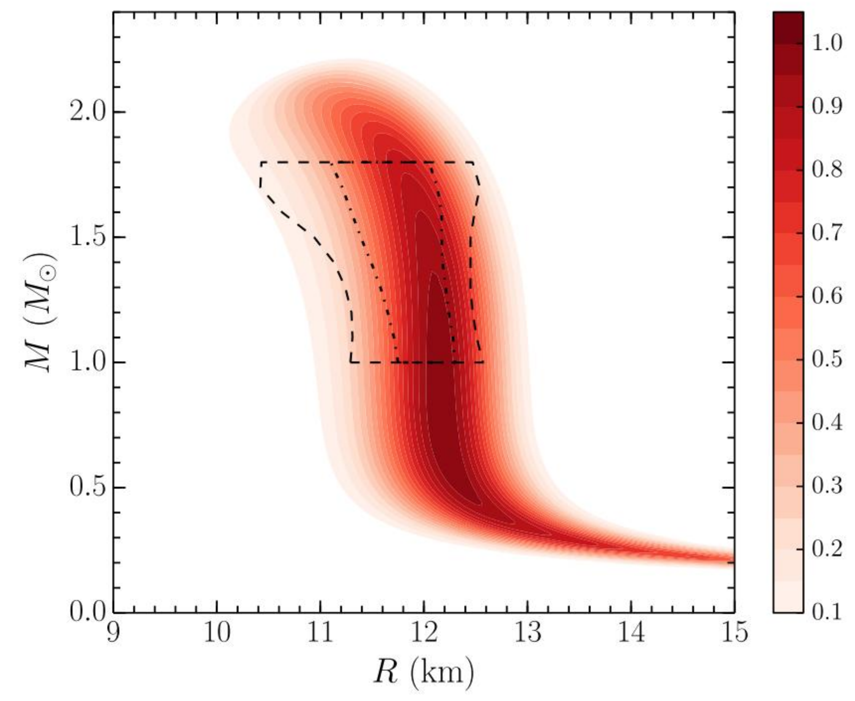

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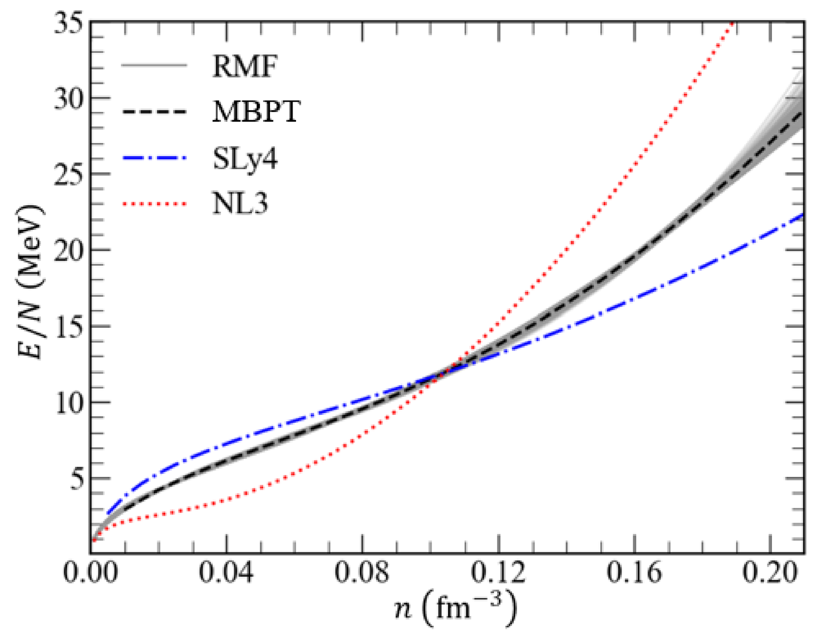
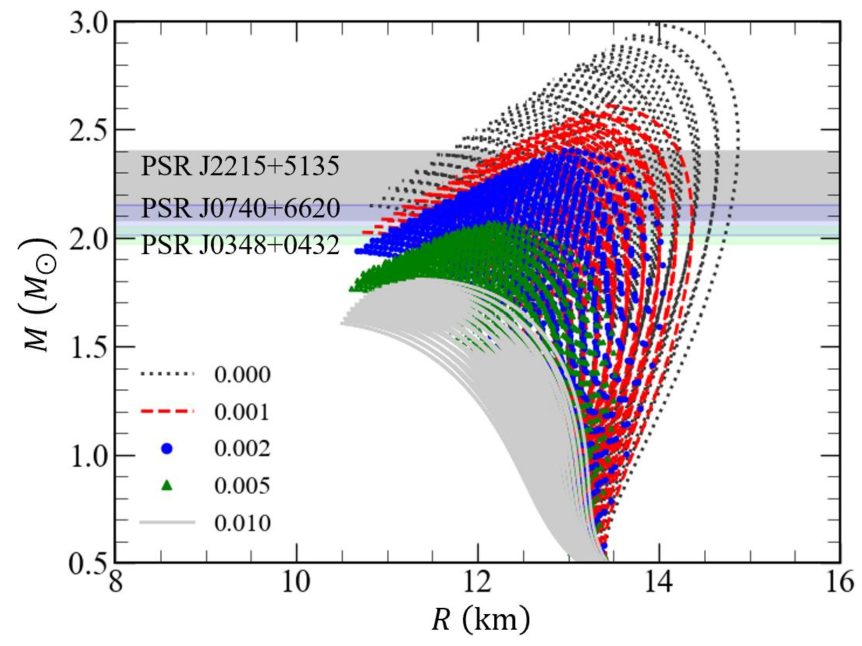
Results

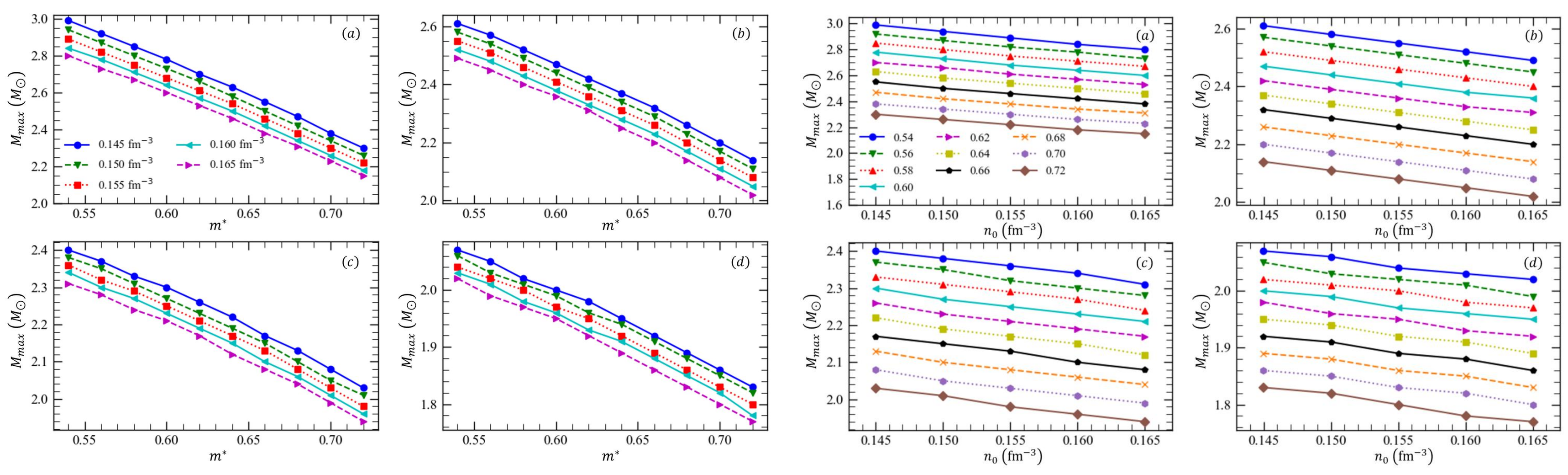
▪TOV neutron stars

$$\frac{dP}{dr} = -\frac{[P(r)+\epsilon(r)][M(r)+4\pi r^3P(r)]}{r[r-2M(r)]} \qquad \frac{dM}{dr} = 4\pi r^2\epsilon(r)$$





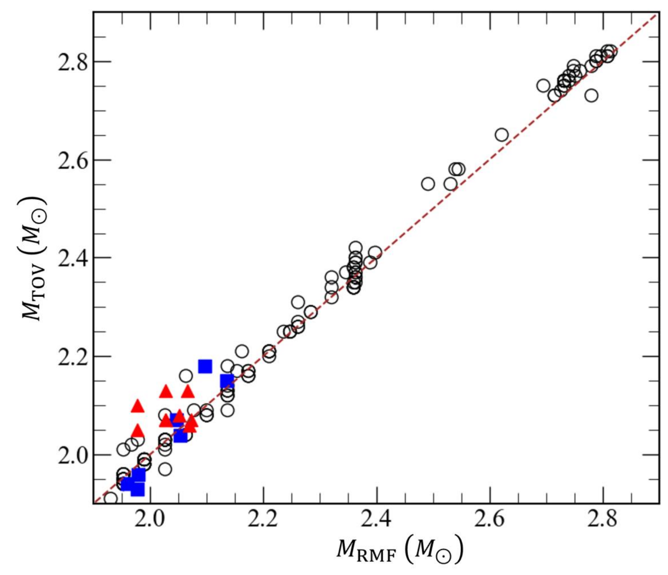
▪Empirical Formula



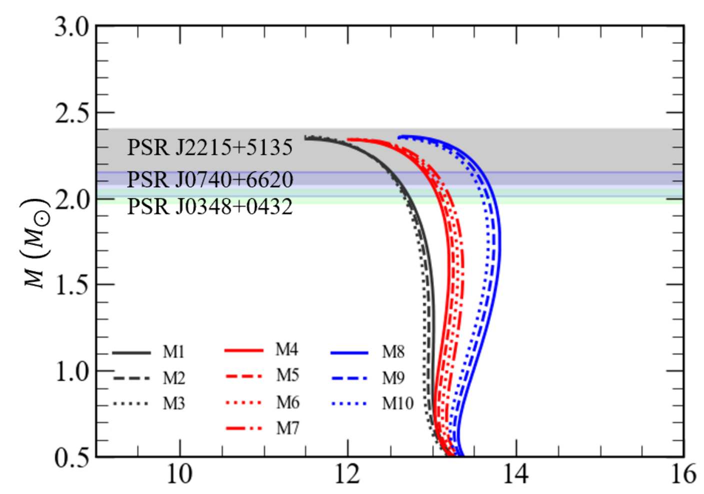
$$M_{max}(n_0, m^*, c_\omega) = f(c_\omega)n_0 + g(c_\omega)m^* + h(c_\omega)$$

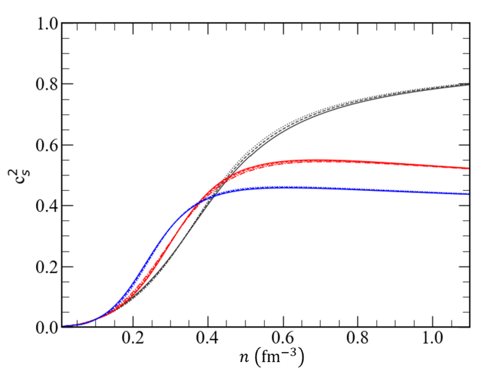
Table 2: Formula Error

Force	J	L	K	Q	B/A	n_0	m^*	c_ω	M_{RMF}	M_{TOV}	Error (%)
IOPB-I	33.36	63.7	222.65	-116.44	-16.11	0.149	0.595	0.00291	2.13579	2.15	0.66
FSUGarnet	30.92	50.96	229.63	8.47	-16.23	0.153	0.579	0.00392	2.04749	2.07	1.09
IUFSU	31.30	47.21	231.33	-290.15	-16.40	0.1546	0.609	0.00500	1.96006	1.94	1.03
IUFSU*	29.85	50.3	235.69	-259.40	-16.02	0.1502	0.605	0.00500	1.97841	1.96	0.94
NL3	37.40	118.5	271.53	202.91	-16.24	0.1482	0.595	0.00000	2.74821	2.78	1.14
FSU-J0	33.45	68.14	229.20	-322.00	-16.31	0.148	0.61	0.00528	1.97745	1.93	2.46
SRV00	33.49	65.23	223.94	-224.08	-16.11	0.149	0.606	0.00353	2.05314	2.04	0.64
NL1	43.46	140.1	211.09	-32.69	-16.42	0.152	0.57	0.00000	2.80829	2.81	0.06
NL2	43.86	129.7	399.37	68.42	-17.03	0.146	0.67	0.00000	2.49107	2.55	2.31
S271	36.46	106.2	271.00	-295.54	-16.24	0.1484	0.7	0.00000	2.36027	2.38	0.83
S271v1	35.74	95.93	271.00	-295.54	-16.24	0.1484	0.7	0.00000	2.36027	2.34	0.87
S271v2	35.06	86.87	271.00	-295.54	-16.24	0.1484	0.7	0.00000	2.36027	2.34	0.87
S271v3	34.42	78.86	271.00	-295.54	-16.24	0.1484	0.7	0.00000	2.36027	2.34	0.87
S271v4	33.83	71.76	271.00	-295.54	-16.24	0.1484	0.7	0.00000	2.36027	2.34	0.87
S271v5	33.27	65.44	271.00	-295.54	-16.24	0.1484	0.7	0.00000	2.36027	2.34	0.87
S271v6	32.74	59.81	271.00	-295.54	-16.24	0.1484	0.7	0.00000	2.36027	2.35	0.44
OMEG1	35.06	70.00	256.00	-300.62	-16.38	0.1484	0.62	0.00362	2.02771	2.13	4.80
OMEG2	33.00	45.00	256.00	-300.56	-16.38	0.1484	0.62	0.00362	2.02772	2.07	2.04
OMEG3	30.00	20.00	256.00	-300.28	-16.38	0.1484	0.62	0.00362	2.02773	2.07	2.04
SRV01	33.75	63.82	221.78	-192.94	-16.11	0.149	0.602	0.00342	2.06893	2.06	0.43
SRV02	33.31	61.49	222.05	-197.82	-16.09	0.149	0.603	0.00336	2.07284	2.07	0.14
SRV03	33.54	58.06	221.72	-188.44	-16.12	0.149	0.601	0.00363	2.05127	2.08	1.38
SRV04	33.34	55.31	221.11	-178.12	-16.11	0.149	0.600	0.00348	2.06564	2.13	3.02
FSU-d6.7	32.75	53.50	229.20	-322.00	-16.31	0.148	0.610	0.00528	1.97745	2.05	3.54
FSU-d6.2	32.53	48.21	229.20	-322.00	-16.31	0.148	0.610	0.00528	1.97745	2.10	5.84



▪M~2.35 $M_\odot$





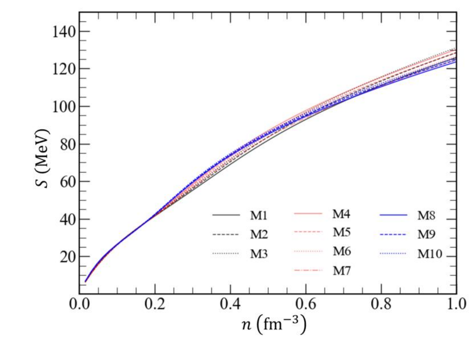


TABLE I. Model properties. $M_{\rm max}$ is the maximum mass (in $M_\odot$), $R_{\rm max}$ and $R_{1.4}$ are radii (in km).

Model	c_ω	n_0	m^*	J (MeV)	L (MeV)	$M_{\rm max}$	$R_{\rm max}$ (km)	$R_{1.4}$ (km)
M1	0	0.14961	0.70	33.4	63.05	2.343	11.51	13.01
M2	0	0.15395	0.69	34.02	66.24	2.350	11.50	12.96
M3	0	0.15829	0.68	34.64	69.51	2.357	11.49	12.91
M4	0.001	0.15904	0.62	34.53	69.94	2.340	12.01	13.18
M5	0.001	0.15459	0.63	33.877	66.46	2.339	12.04	13.23
M6	0.001	0.15014	0.64	33.231	63.05	2.339	12.07	13.29
M7	0.001	0.14569	0.65	32.598	59.69	2.339	12.10	13.35
M8	0.002	0.14760	0.56	32.412	62.71	2.358	12.65	13.73
M9	0.002	0.15220	0.55	33.106	66.35	2.354	12.60	13.66
M10	0.002	0.15679	0.54	33.808	69.99	2.350	12.55	13.59