# Bayesian Inference of fine-features of dense matter EOS from future high-precision data of neutron star radii


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- Examples of expected high-precision R measurements
- Bayesian analyses with mocked high-precision R data
- What we have learned

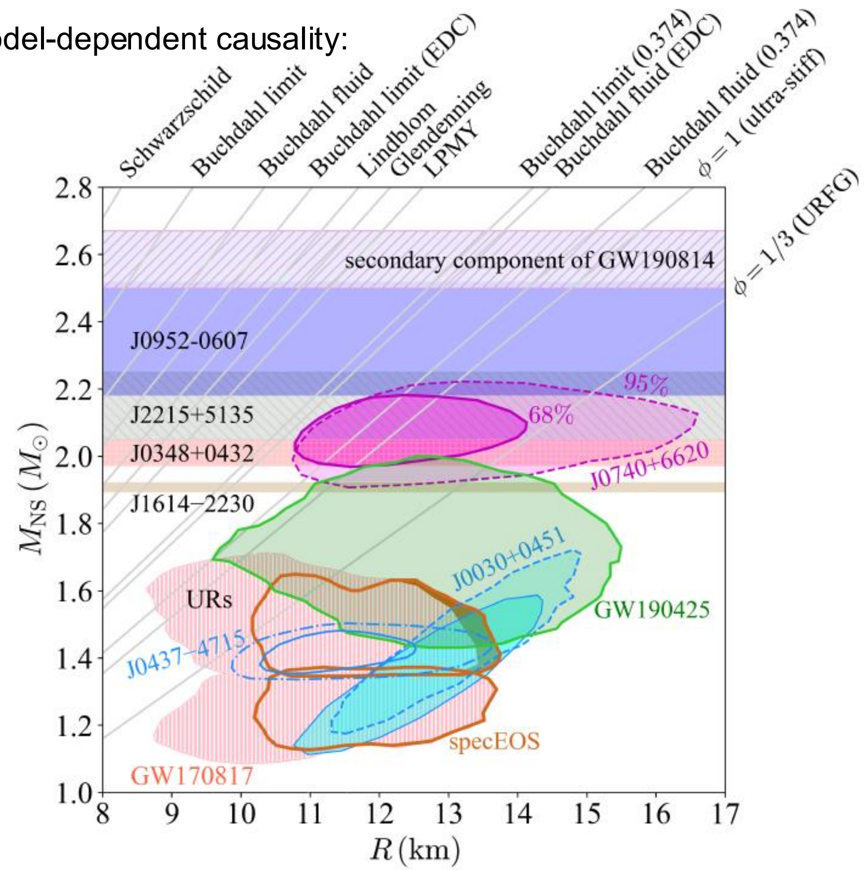

Office of Science
U.S. DEPARTMENT OF ENERGY

## Best estimate of current precision $\sigma_R$>1.0 km

GW170817, $R_{1.4}$=11.9 ± 1.4 km

### Currently available mass-radius observations

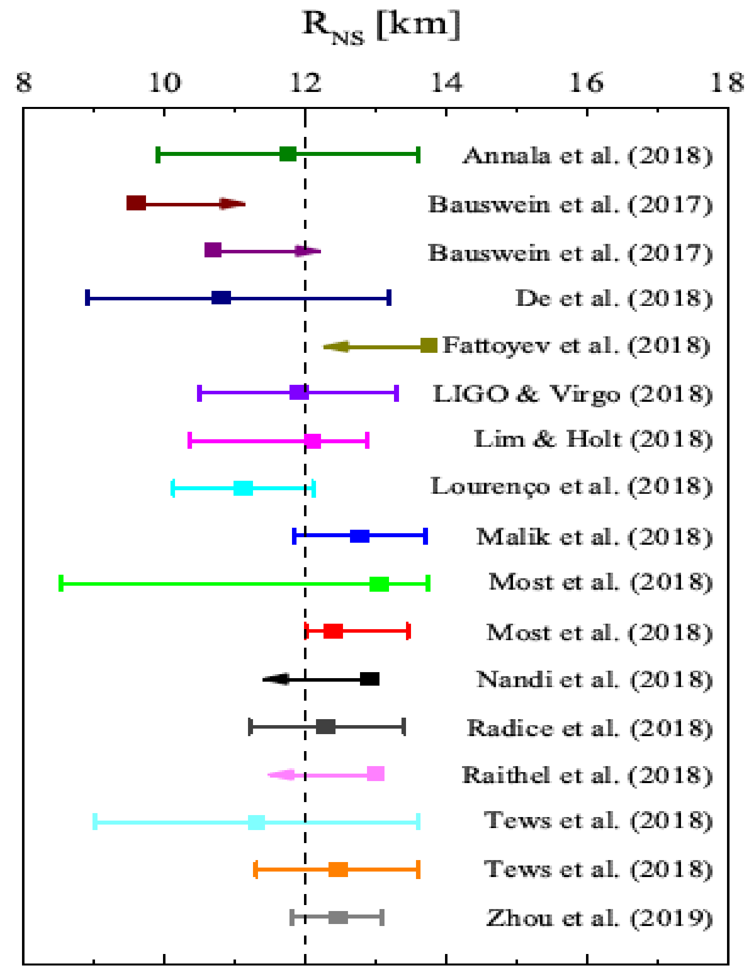
Model-dependent causality:



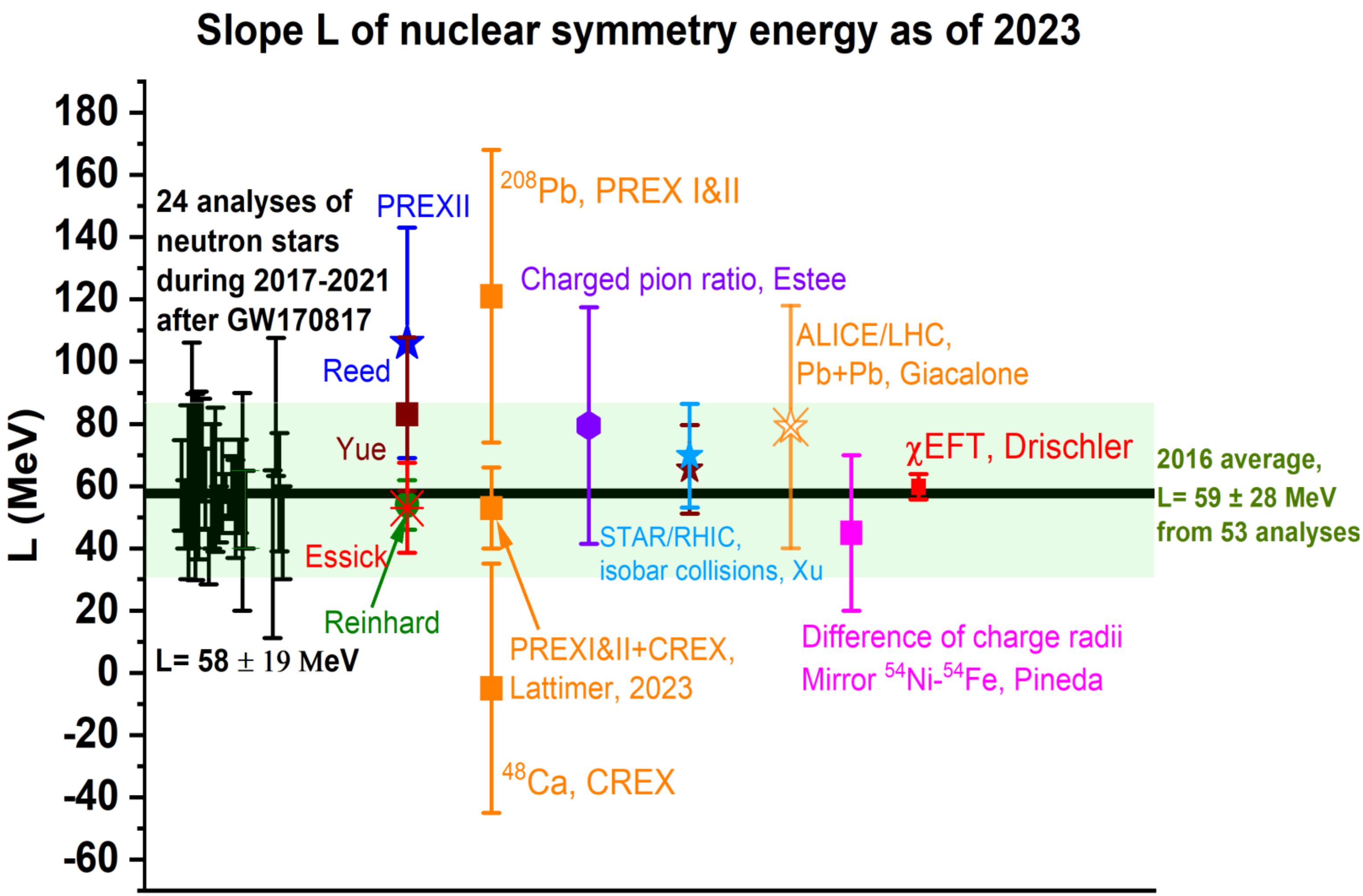

B.J. Cai and B.A. Li, EPJA 61,55 (2025)

Various analyses during 2-years after GW170817

$$R_{1.4} = 12.0 \pm 1.13$$

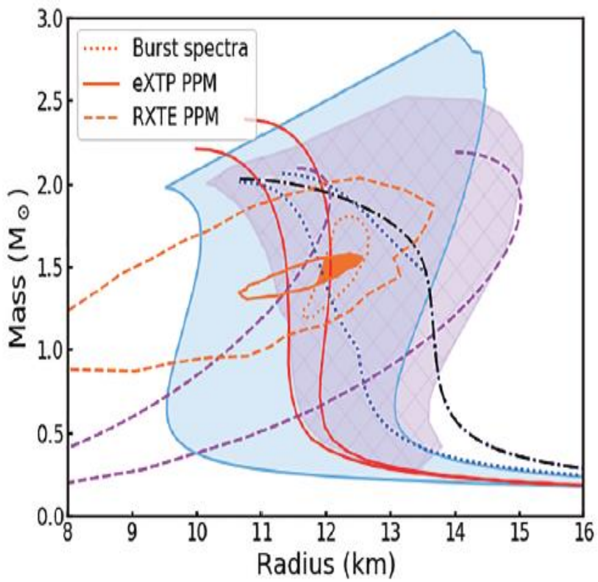



B.A. Li et al., EPJA 55, 117 (2019)




**GW170817 did NOT tell us anything new about nuclear EOS compared to what we knew before, while they are consistent.**
**Despite of its historical importance in many other aspects.**

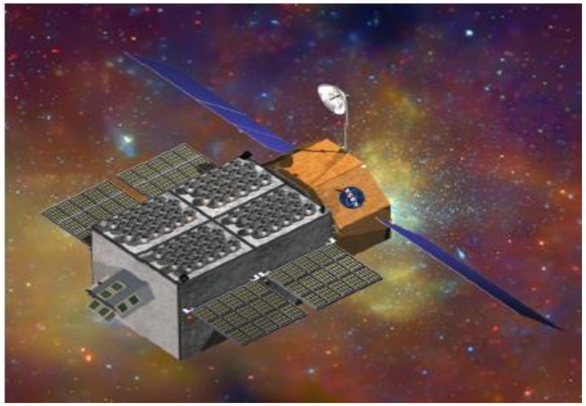
Nai-Bo Zhang and Bao-An Li, EPJA 59, 86 (2023) *eXTP: The enhanced X-ray Timing and Polarimetry mission*

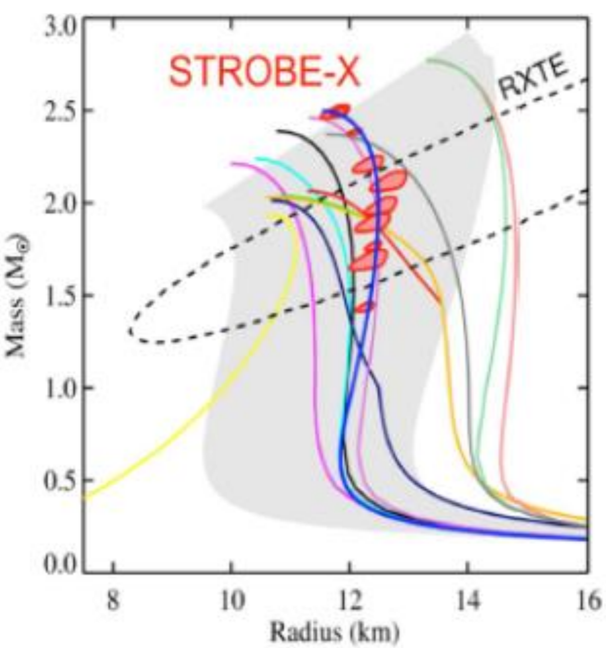



S. N. Zhang et al., arXiv:1812.04020v1

STROBE-X

*STROBE-X*: X-ray Timing and Spectroscopy on Dynamical Timescales from Microseconds to Years






STROBE-X Science Working Group

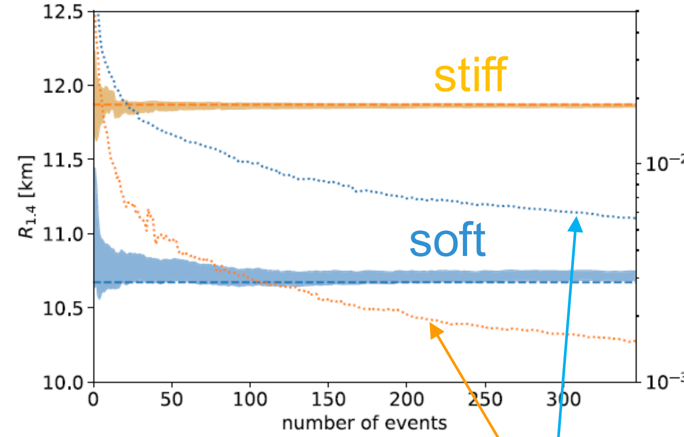
Paul S. Ray et al. 1903.03035

The Astrophysical Journal, 955:45 (8pp), 2023 September 20 https://doi.org/10.3847/1538-4357/acf12f


## Prospects for a Precise Equation of State Measurement from Advanced LIGO and Cosmic Explorer

Daniel Finstad[1,2,3], Laurel V. White[4], and Duncan A. Brown[4]

determine $R_{1.4}$, the radius of a 1.4 $M_\odot$ neutron star. We find that, with 321 signals, the LIGO–Virgo network is able to measure $R_{1.4}$ to better than 2% precision for all equations of state we consider; however, we also find that achieving this precision could take decades of observation, depending on the equation of state and the merger rate. On the other hand, we find that with one year of observation, Cosmic Explorer will measure $R_{1.4}$ to better than 0.6% precision. In both cases, we find that systematic biases, such as from an incorrect mass prior, can significantly impact measurement accuracy, and efforts will be required to mitigate these effects.



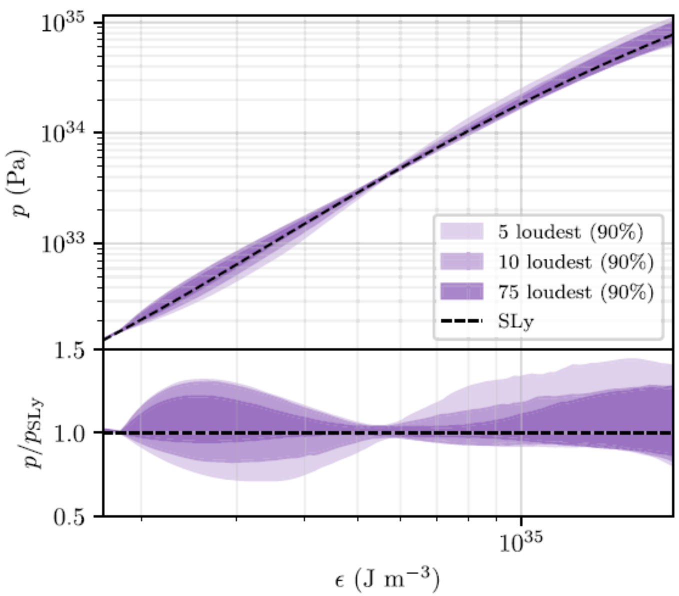

**1-year Cosmic Explorer operation**

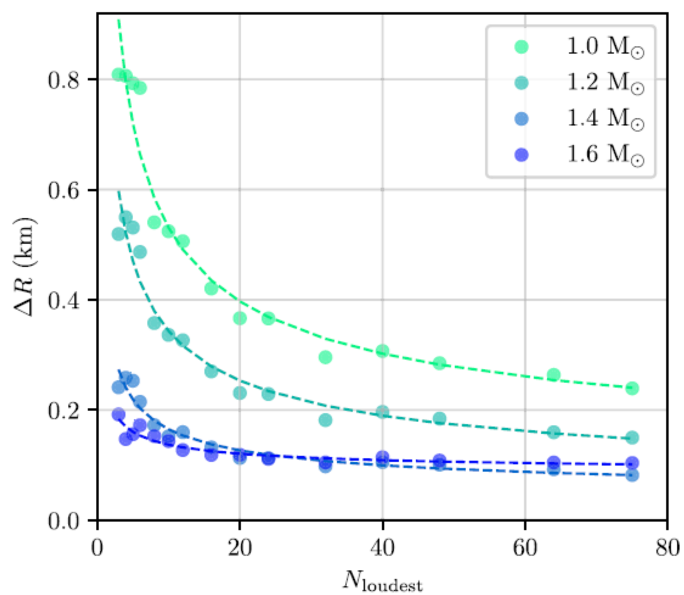
Relative error

## Precision constraints on the neutron star equation of state with third-generation gravitational-wave observatories

PHYSICAL REVIEW D **110**, 043013 (2024)

Kris Walker[1,2,*] Rory Smith[3,†] Eric Thrane[2,3] and Daniel J. Reardon[4,5]



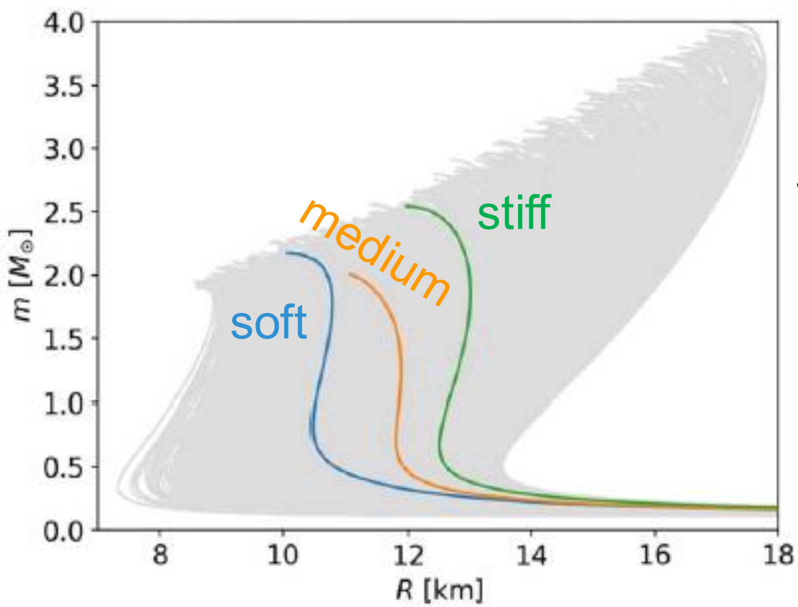

**1-year operation**

**Cosmic Explorer**
**+**
**Einstein Telescope**

**The proposed high-precision measurements of neutron star radii are SUPER-expensive in terms of money, man-power and time, and no guarantee….**

**Probably many high-value scientific returns**

What can we really learn from high-precision measurements of neutron star radii?



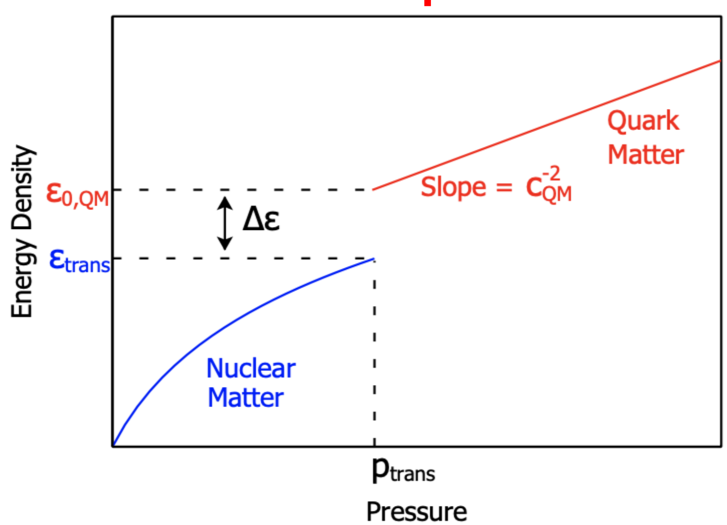

D. Finstad, L.V. White and D.A. Brown, APJ 955, 45 (2023)

**Basic issues regarding the EOS:**

Stiffness of neutron star matter has many components. Which part is actually most important for determining R?

Can the precise R tell anything about hadron-quark phase transition or quark matter in neutron star cores?

**1st order Hadron-quark transition**



# Bayesian inferences from mocked 3 sets of high-precision R data

$R_{1.4}$=11.9 ±σ, $R_{1.4}=R_{1.6}=R_{1.8}=R_{2.0}$=11.9 ±σ, $R_{2.0}$=11.9 ±σ

with σ varying from 1.0 km to 0.1 km

9-parameters meta-model EOS with a first-order hadron-quark phase transition satisfying all existing constraints from nuclear and astrophysics

$$\varepsilon(p) = \begin{cases} \varepsilon_{\rm HM}(p) & \rho < \rho_t \\ \varepsilon_{\rm HM}(p_t) + \Delta\varepsilon + C_{\rm qm}^{-2}(p - p_t) & \rho > \rho_t \end{cases}$$

Constant speed of sound (css) model of quark matter
M. Alford, S. Han and M. Prakash, PRD 88, 083013 (2013)

$\varepsilon_{\rm HM}(p)$ Charge-neutral npeμ matter at ß-equilibrium

$$E(\rho,\delta) = E_0(\rho) + E_{\rm sym}(\rho)\cdot\delta^2 + \mathcal{O}(\delta^4)$$

$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2}(\frac{\rho-\rho_0}{3\rho_0})^2 + \frac{J_0}{6}(\frac{\rho-\rho_0}{3\rho_0})^3$$

$$E_{\rm sym}(\rho) = E_{\rm sym}(\rho_0) + \frac{L}{3}\left(\frac{\rho}{\rho_0}-1\right) + \frac{K_{\rm sym}}{18}\left(\frac{\rho}{\rho_0}-1\right)^2 + \frac{J_{\rm sym}}{162}\left(\frac{\rho}{\rho_0}-1\right)^3 + \mathcal{O}\left[\left(\frac{\rho}{\rho_0}-1\right)^4\right]$$

Crust: Negele-Vautherin (NV) EOS for the inner and Baym-Pethick-Sutherland (BPS) for the outer crust.

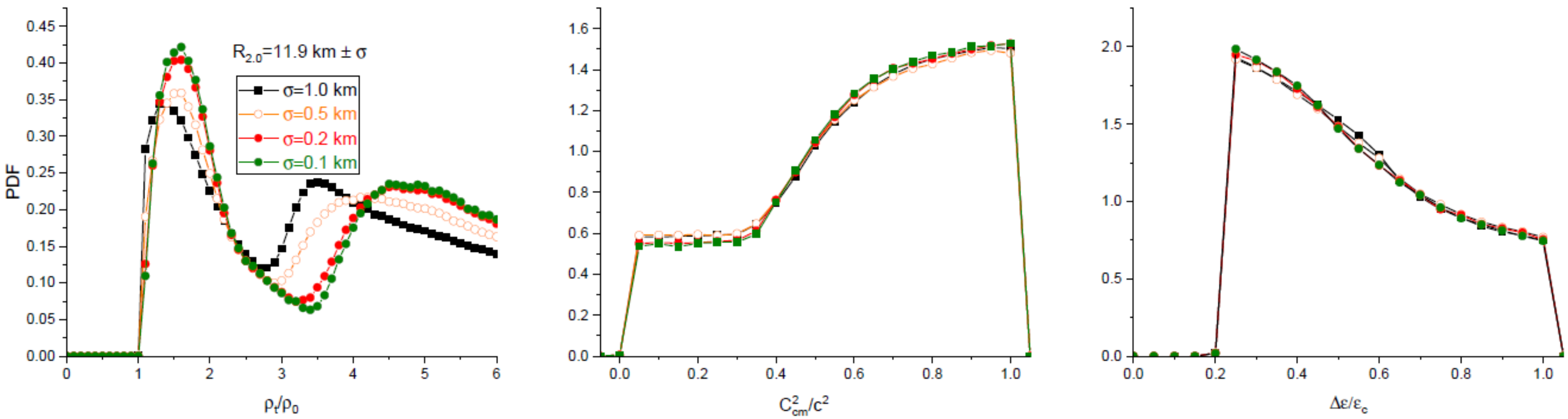
TABLE I. Prior ranges of the nine EOS parameters (MeV).

| Parameters | Lower limit | Upper limit |
|---|---|---|
| $K_0$ | 220 | 260 |
| $J_0$ | -400 | 400 |
| $K_{\rm sym}$ | -400 | 100 |
| $J_{\rm sym}$ | -200 | 800 |
| $L$ | 30 | 90 |
| $E_{\rm sym}(\rho_0)$ | 28.5 | 34.9 |
| $\Delta\epsilon/\epsilon_t$ | 0.2 | 1.0 |
| $C_{\rm qm}^2/c^2$ | 0.0 | 1.0 |
| $\rho_t/\rho_0$ (case-A) | 1.0 | 6.0 |
| $\rho_t/\rho_0$ (case-B) | 3.0 | 6.0 |

Hadron-quark transition density

**Wen-Jie Xie and Bao-An Li**
**APJ 883, 174 (2019), APJ 899, 4 (2020)**
**Bao-An Li et al., PRD 110, 103040 (2024);**
arXiv:2505.00194

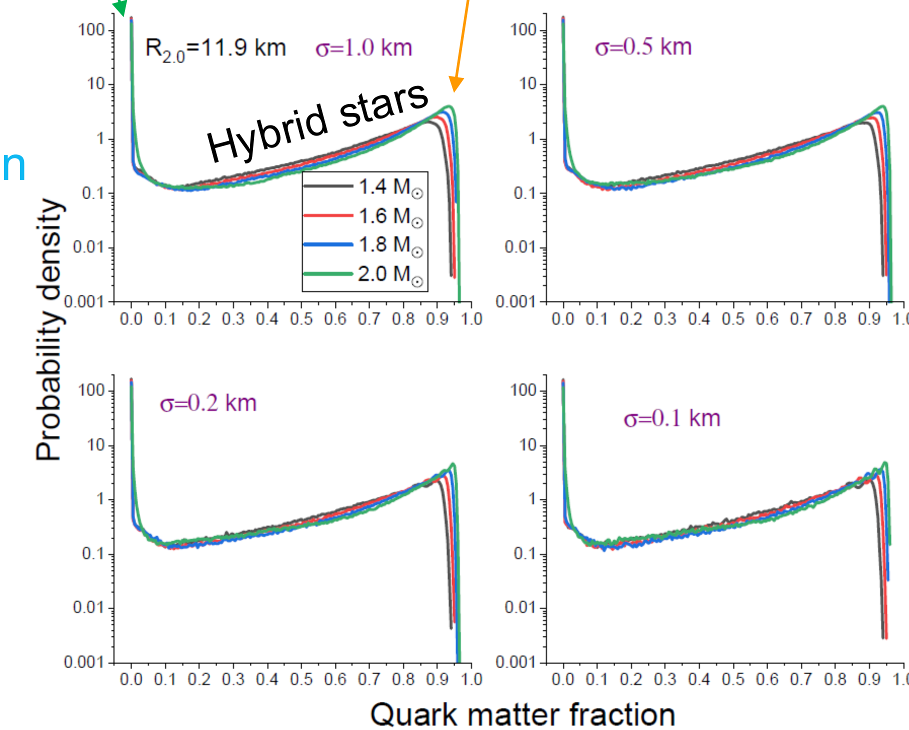
# Bayesian posterior probability distribution functions of hadron-quark transition properties




Prior range: (1-6)$\rho_0$

Normal neutron stars

From the 1st peak of PDF($\rho_t$)




The 1st peak in PDF($\rho_t$) around (1.7-2.0)$\rho_0$ is sufficient, but unnecessary, --> a fake transition

Bayesian statistics can NOT replace physics

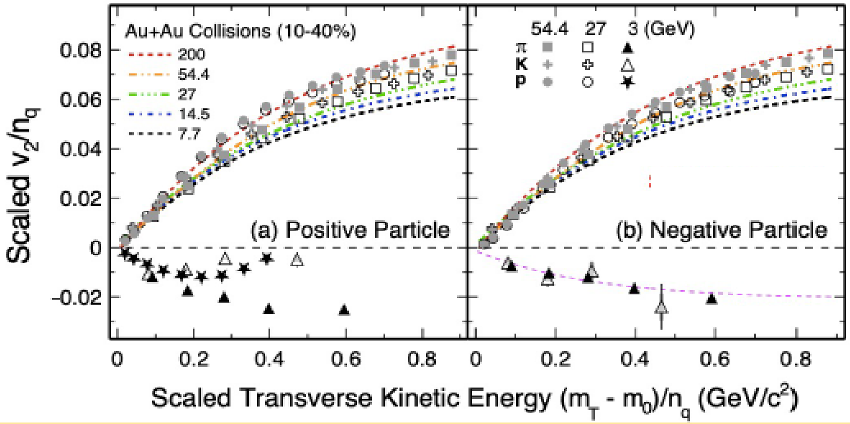
The second peak in PDF($\rho_t$) is sufficient, physical and consistent with indications of recent BES/RHIC experiments


Bao-An Li, Xavier Grundler, Wen-Jie Xie, Nai-Bo Zhang, **arXiv:2505.00194**

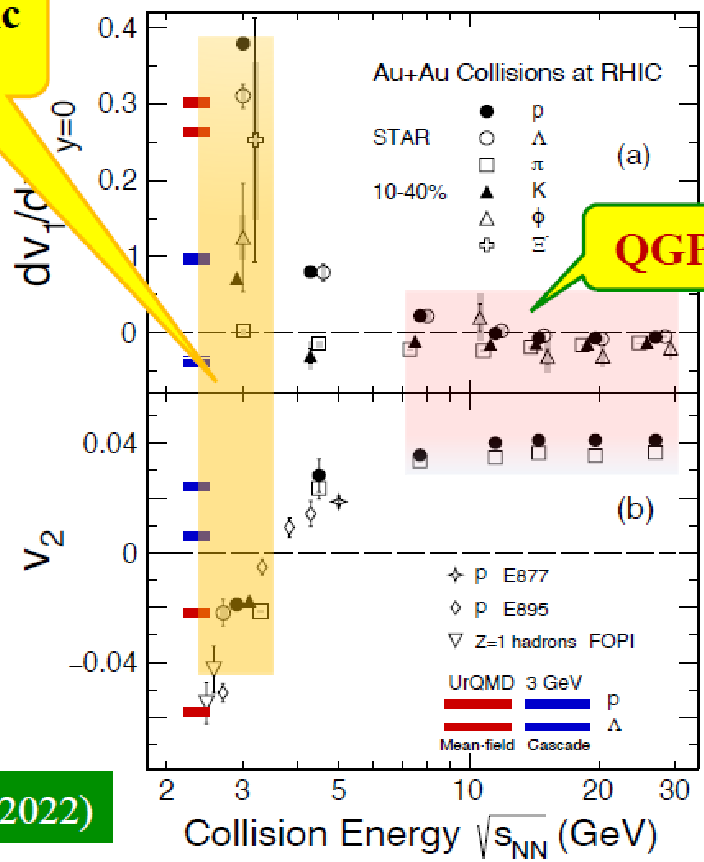
**Nu Xu**,

**indications on hadron-quark transition density in hot dense matter from BES/RHIC**

# Disappearance of Partonic Collectivity

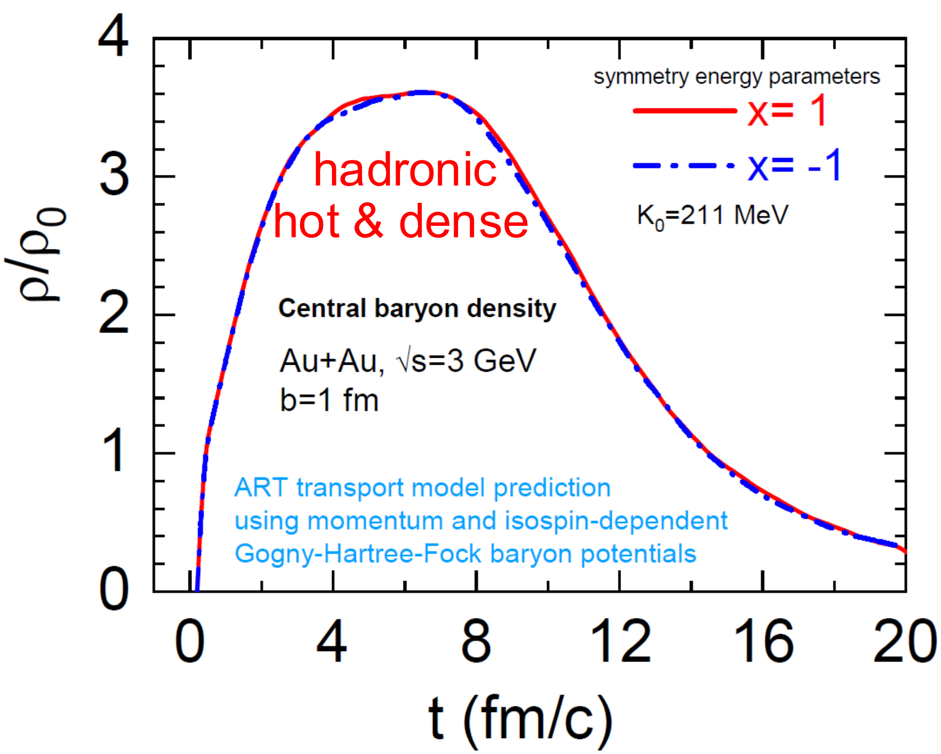



Hadronic Matter

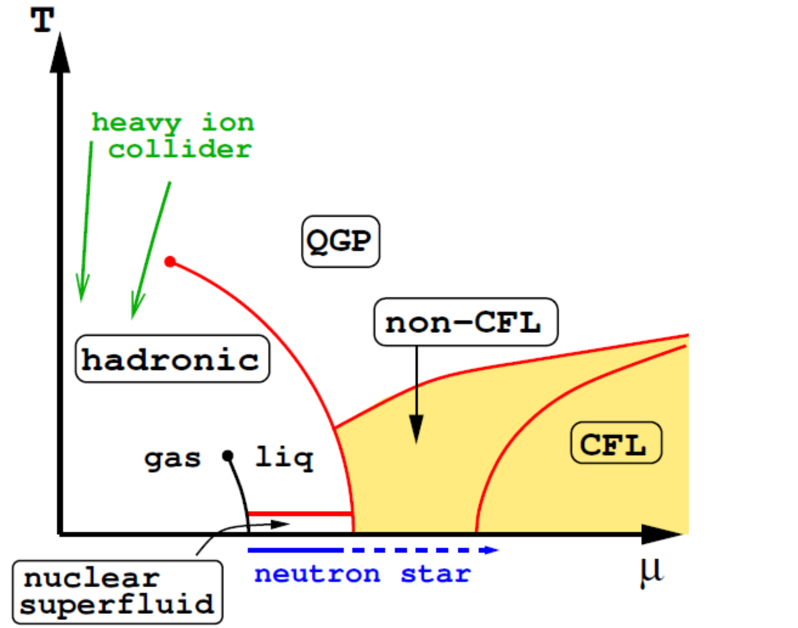



- At **3 GeV**, NCQ scaling is absent；
- Transport model calculations, with baryonic mean field, reproduce both $v_1$ and $v_2$ results；
- **hadronic interactions dominant!**

STAR: PLB**827**, 137003(2022)




G.C. Yong, B.A. Li, Z.G. Xiao and Z.W. Lin
Phys. Rev. C **106**, 024902 (2022)



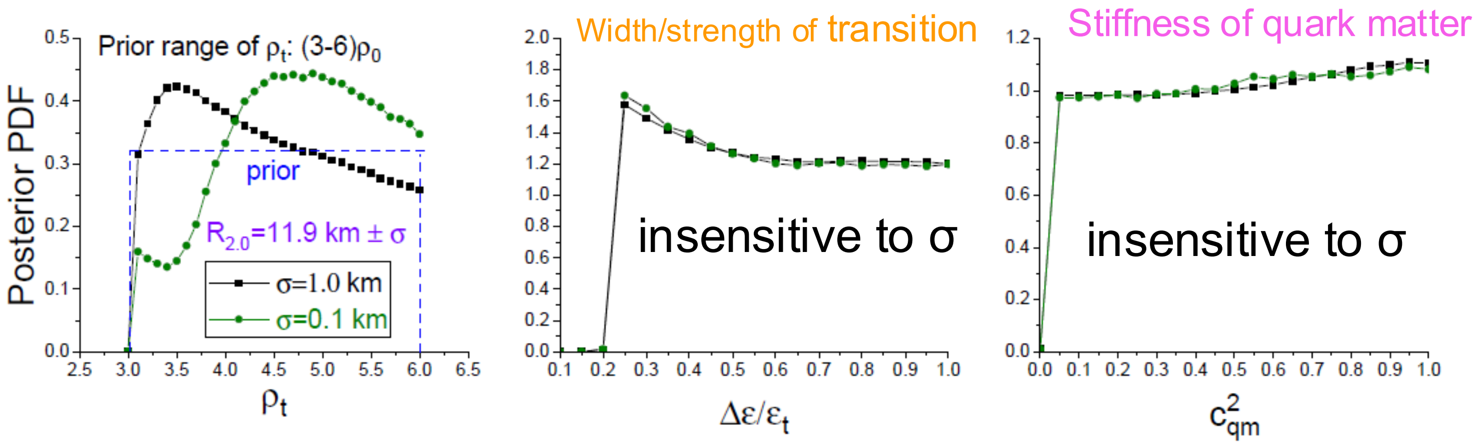

Mark G. Alford, Andreas Schmitt,
Krishna Rajagopal and Thomas Schafer,
Review of Modern Physics, 80, 1455 (2008)

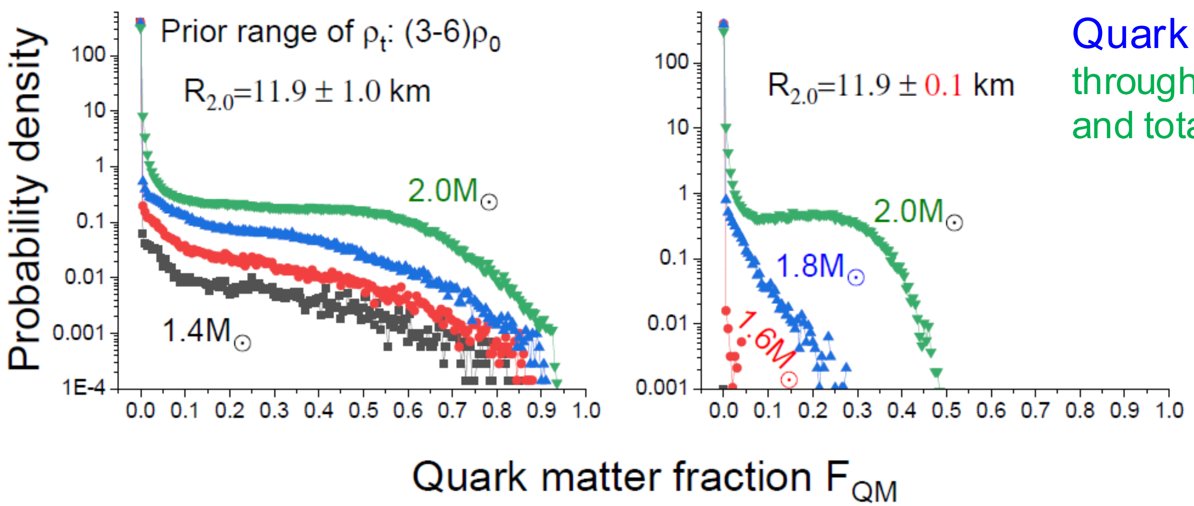
**Indication (BES/RHIC data and transport model simulation):**
**The 1st order hadron-quark phase transition can NOT happen below about 3.6$\rho_0$ in cold (T=0) neutron stars**

⇨ **Modify the prior range of hadron-quark transition density in Bayesian analyses**

Precision radius measurement is a sensitive probe of the hadron-quark transition density but not the strength of the transition nor the stiffness of quark matter

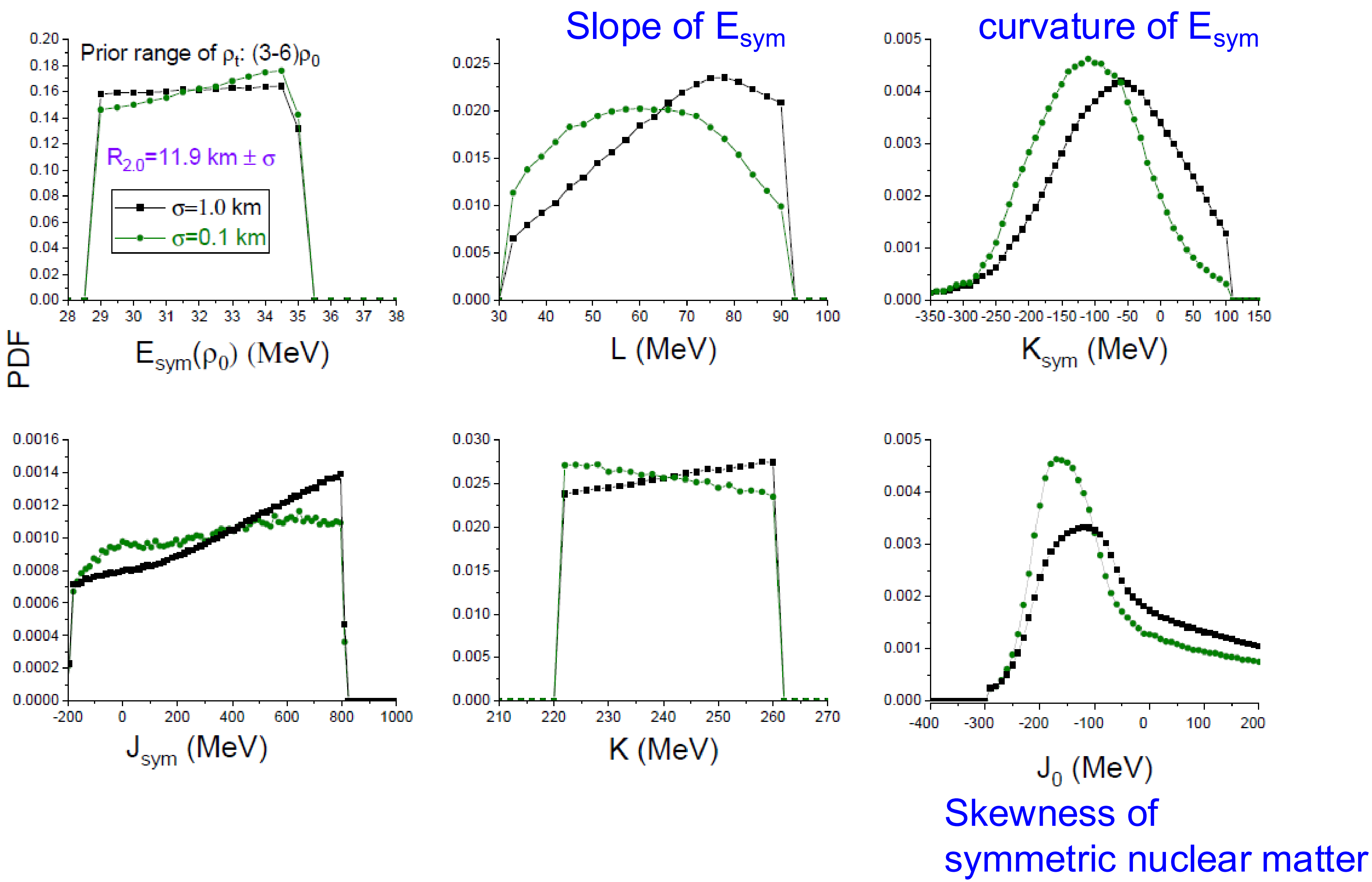
$\rho \propto M/R^3$: small change in R has a big impact on the average value and profile of density



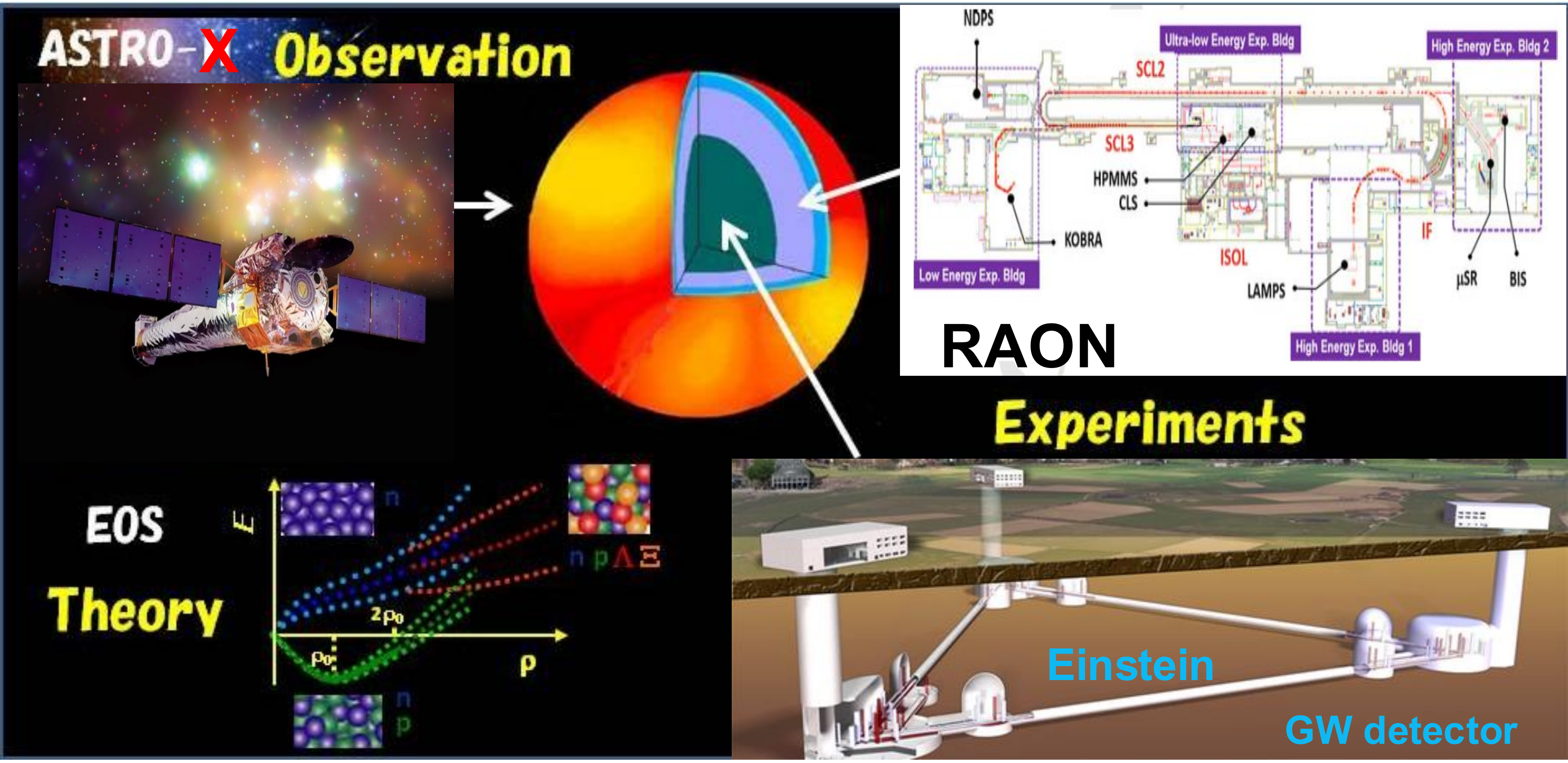

Quark matter fraction is sensitive to σ through the transition density $\rho_t$ and total mass conservation

# High-precision radius measurements are sensitive to high-density hadronic matter EOS



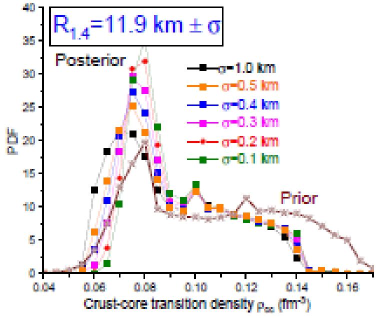
# High-precision NS radius measurements to σ=0.1 km

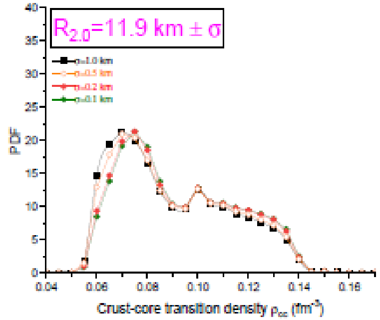
1. CAN probe sensitively the hadron-quark transition density and high-density hadronic matter EOS, P®=0, the radius is determined mostly by hadronic EOS
2. Can NOT probe sensitively properties of quark matter
3. Statistics can NOT replace physics, inputs from nuclear theory and experiments are important to avoid misleading but best combinations of EOS parameters from Bayesian analyses of neutron star observational data

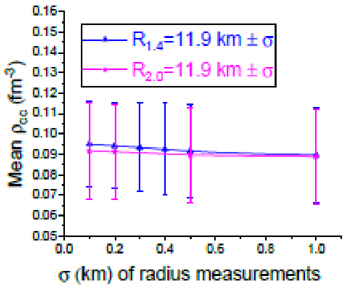


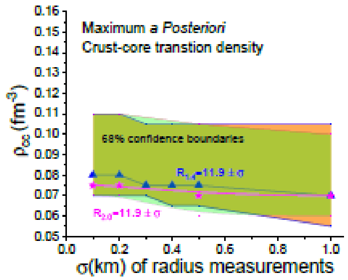
# Backup slides

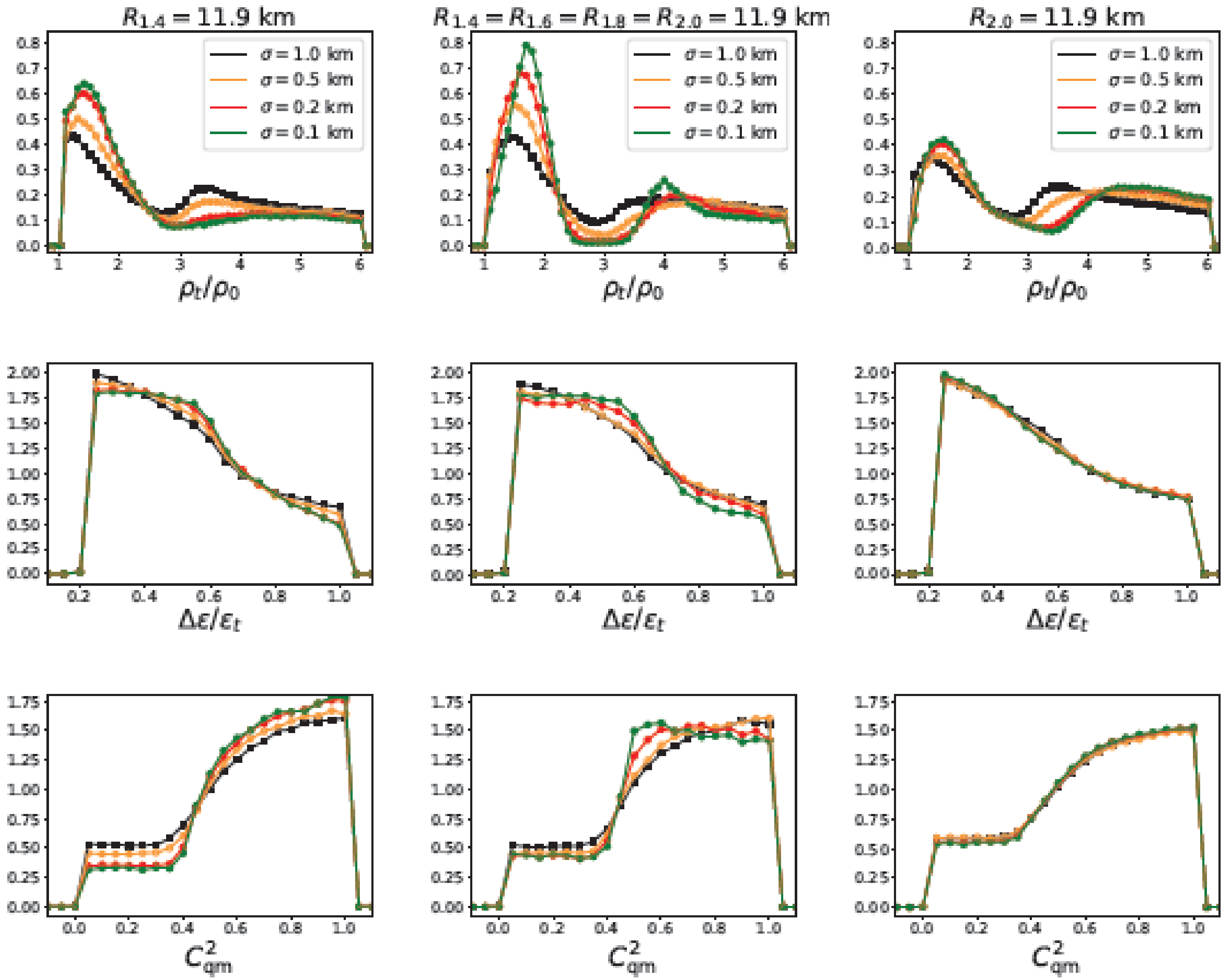
## PDF of crust-core transition density

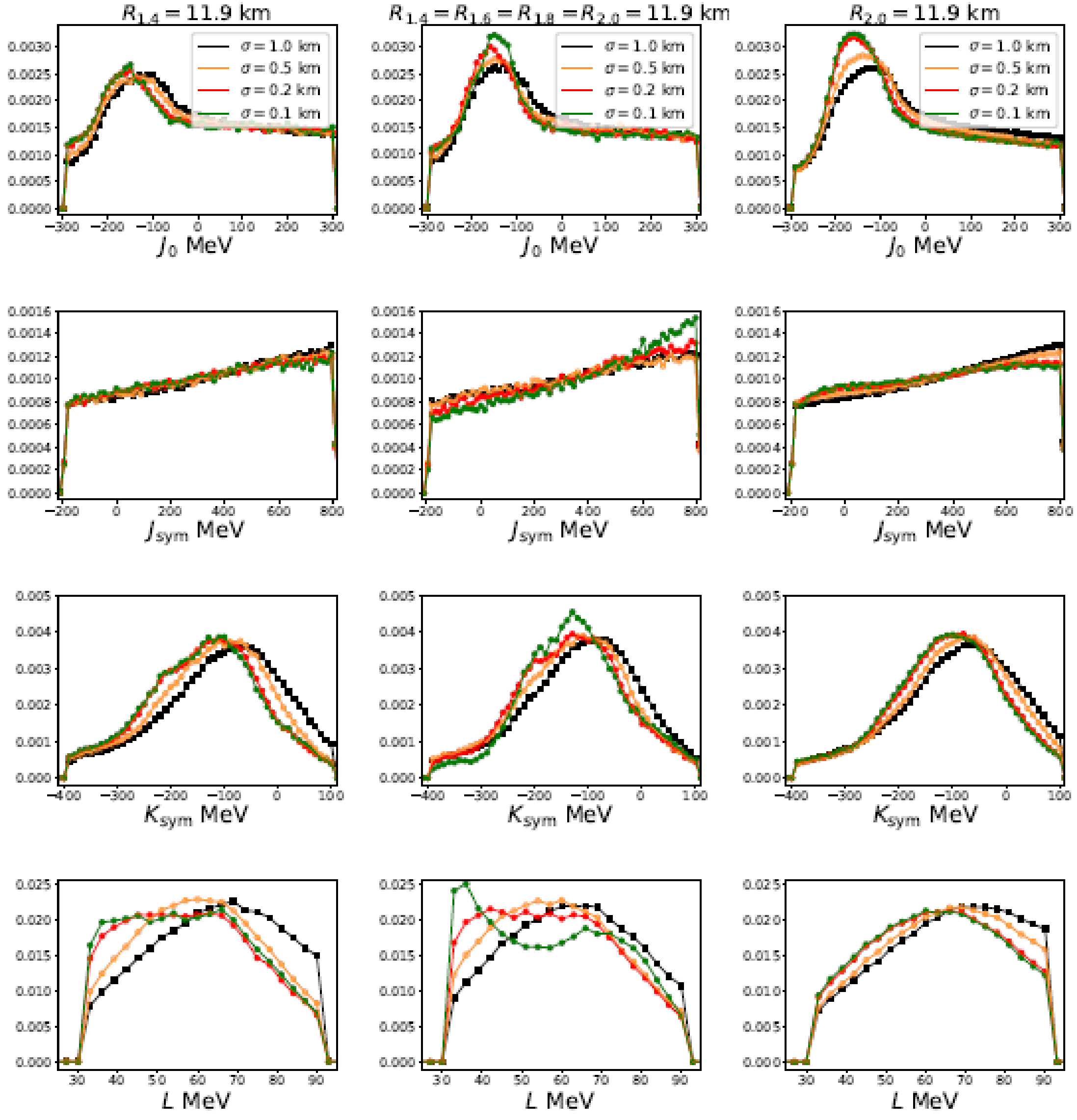


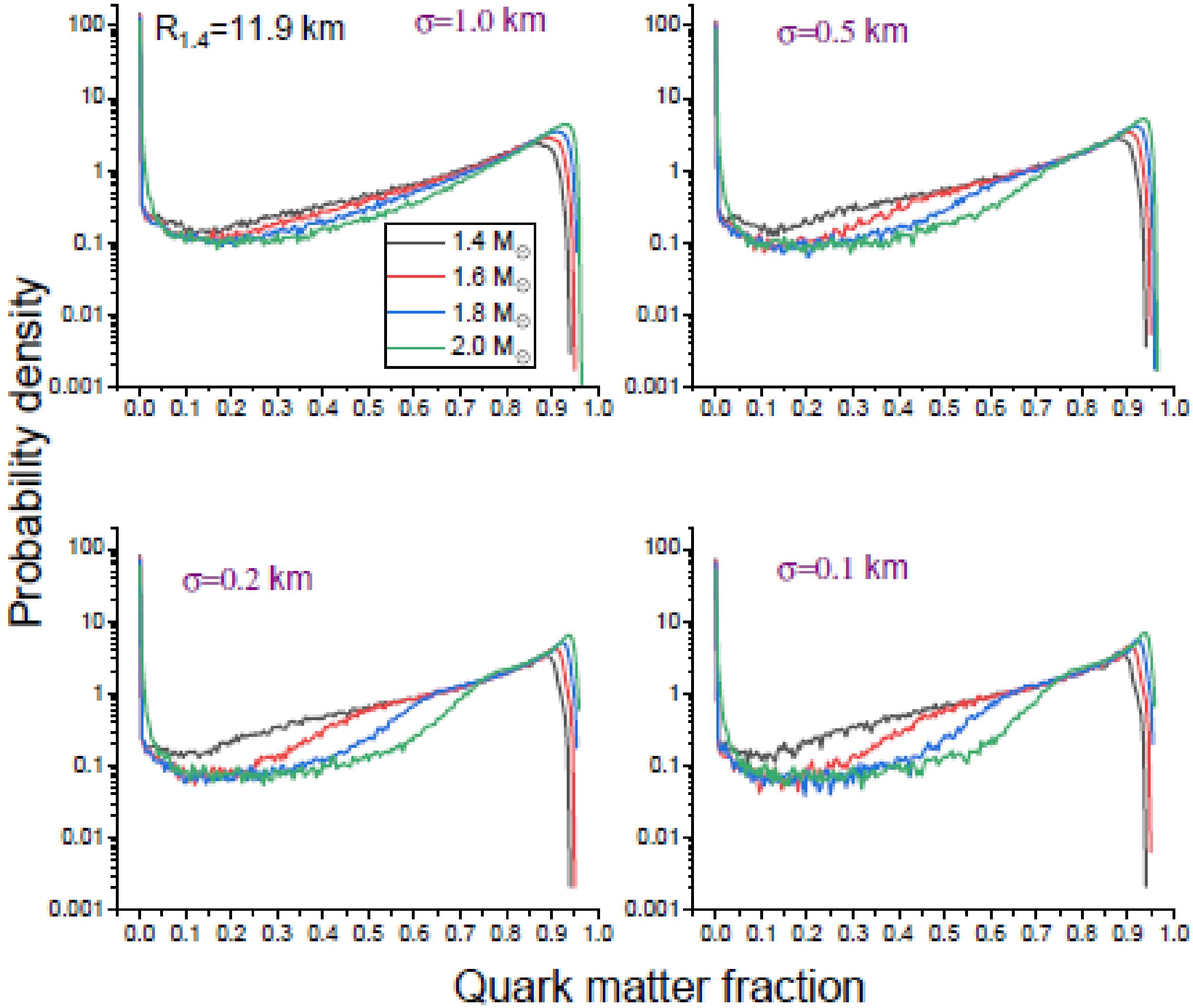



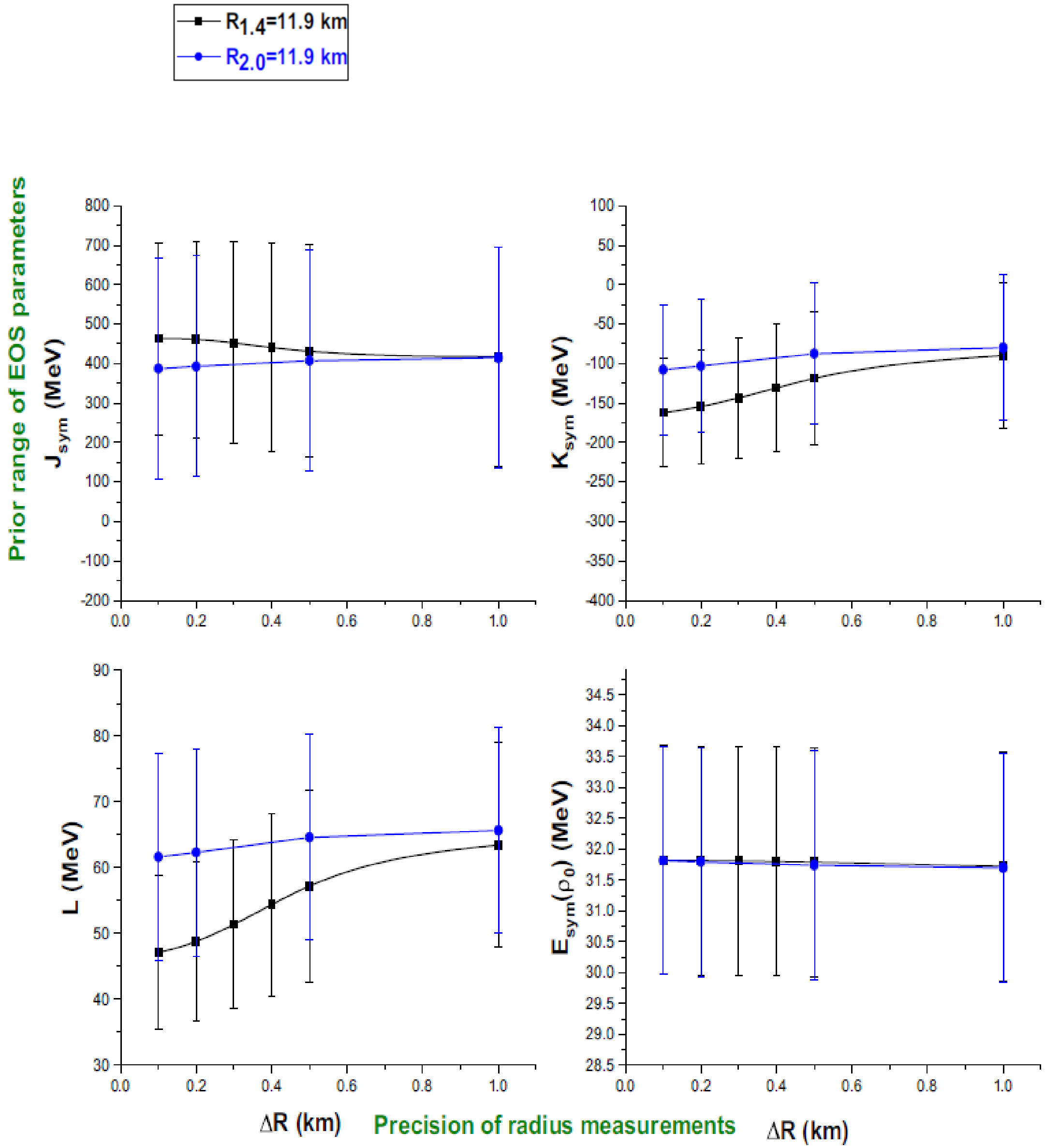




$R_{1.4} = 11.9$ km
$R_{1.4} = R_{1.6} = R_{1.8} = R_{2.0} = 11.9$ km
$R_{2.0} = 11.9$ km
$\sigma = 1.0$ km
$\sigma = 0.5$ km
$\sigma = 0.2$ km
$\sigma = 0.1$ km
$\rho_t/\rho_0$
$\Delta\epsilon/\epsilon_t$
$C^2_{\mathrm{qm}}$

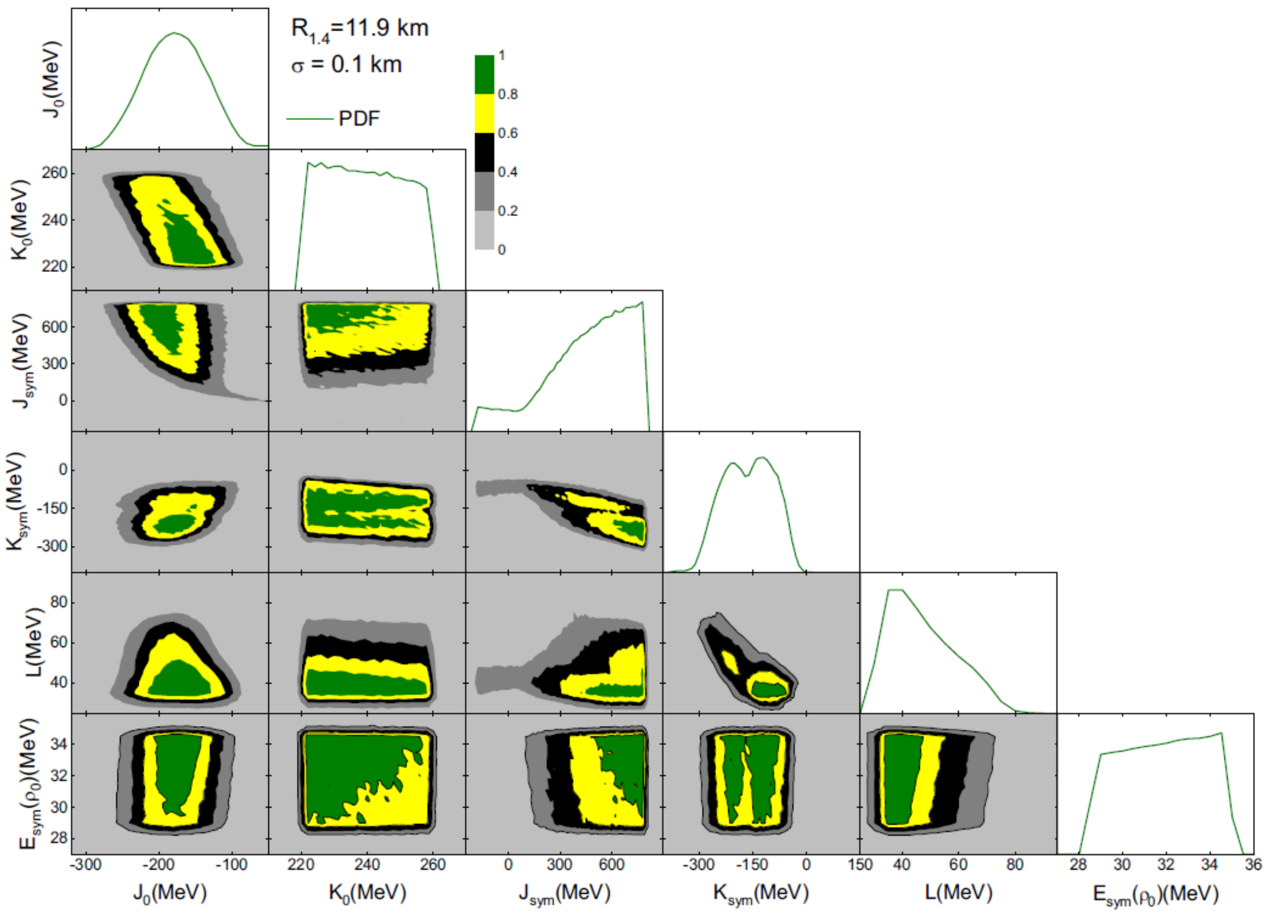

$R_{1.4} = 11.9$ km
$R_{1.4} = R_{1.6} = R_{1.8} = R_{2.0} = 11.9$ km
$R_{2.0} = 11.9$ km
$\sigma = 1.0$ km
$\sigma = 0.5$ km
$\sigma = 0.2$ km
$\sigma = 0.1$ km
$J_0$ MeV
$J_{sym}$ MeV
$K_{sym}$ MeV
$L$ MeV



R1.4=11.9 km
σ=1.0 km
σ=0.5 km
σ=0.2 km
σ=0.1 km
1.4 M⊙
1.6 M⊙
1.8 M⊙
2.0 M⊙
Probability density
Quark matter fraction



$R_{1.4}$=11.9 km
$R_{2.0}$=11.9 km
Prior range of EOS parameters
$J_{sym}$ (MeV)
$K_{sym}$ (MeV)
L (MeV)
$E_{sym}(\rho_0)$ (MeV)
ΔR (km)
Precision of radius measurements
ΔR (km)

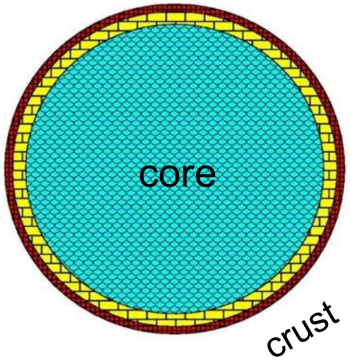


$R_{1.4}$=11.9 km
σ = 0.1 km
PDF
1
0.8
0.6
0.4
0.2
0
$J_0$(MeV)
$K_0$(MeV)
$J_{sym}$(MeV)
$K_{sym}$(MeV)
L(MeV)
$E_{sym}(\rho_0)$(MeV)

# How does the symmetry energy affect neutron star properties?

**(1) The proton fraction x is determined by the $E_{sym}(\rho)$ through charge neutrality and beta-equilibrium conditions:**

$$x = 0.048[E_{sym}(\rho)/E_{sym}(\rho_0)]^3(\rho/\rho_0)(1-2x)^3$$




Critical for the cooling mechanism of protoneutron stars and associated neutrino emissions, appearance of hyperons, kaon condensation, baryon resonances…..

**(2) The pressure in the npe matter at beta equilibrium:**

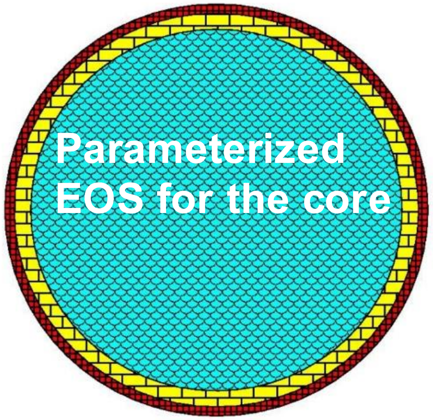
$$P(\rho,\delta) = \rho^2\left[\frac{dE_0(\rho)}{d\rho} + \frac{dE_{\rm sym}(\rho)}{d\rho}\delta^2\right] + \frac{1}{2}\delta(1-\delta)\rho E_{\rm sym}(\rho)$$

**(3) The crust-core transition density and pressure is determined by setting the incompressibility of neutron star matter =0 (speed of sound becomes imaginary):**

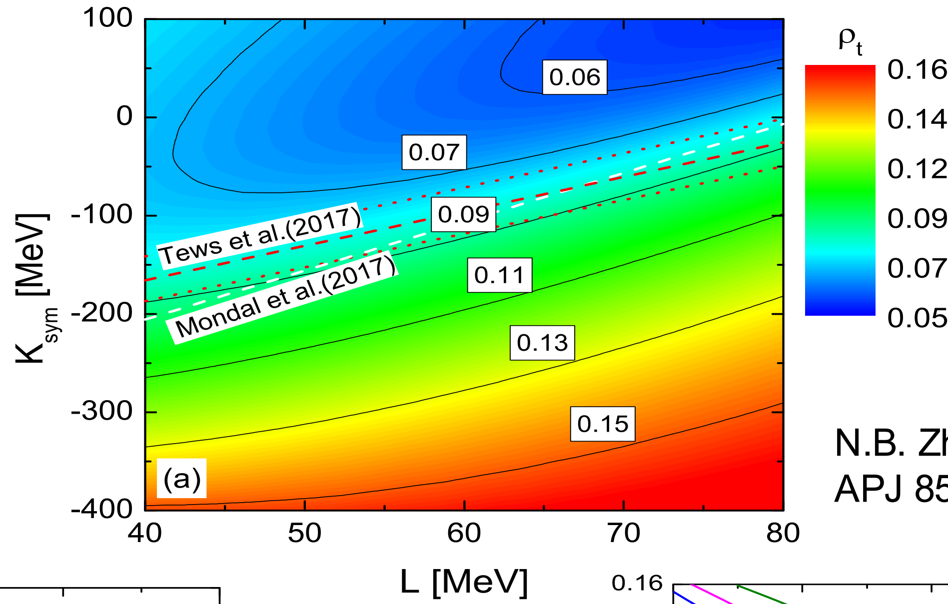
$$K_\mu = \rho^2\frac{d^2E_0}{d\rho^2} + 2\rho\frac{dE_0}{d\rho} + \delta^2\left[\rho^2\frac{d^2E_{sym}}{d\rho^2} + 2\rho\frac{dE_{sym}}{d\rho} - 2E_{sym}^{-1}\left(\rho\frac{dE_{sym}}{d\rho}\right)^2\right] = 0$$

**Lattimer & Prakash, Phys. Rep., 442, 109 (2007)**

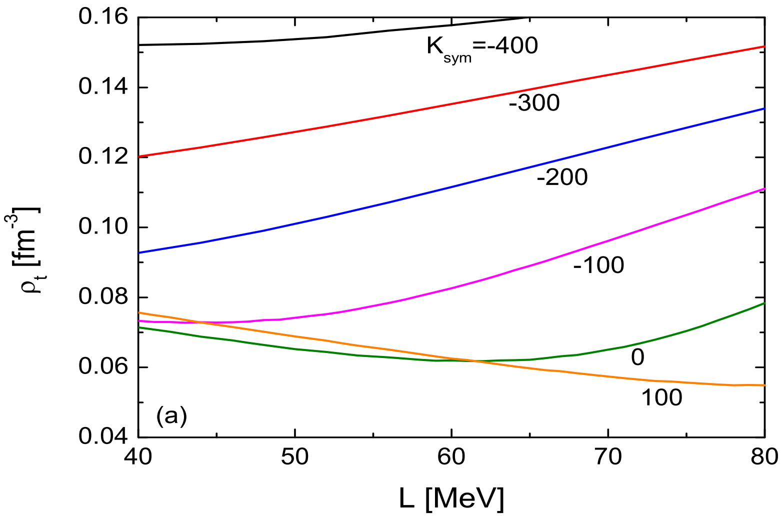
# Effects of symmetry energy on the crust-core transition density

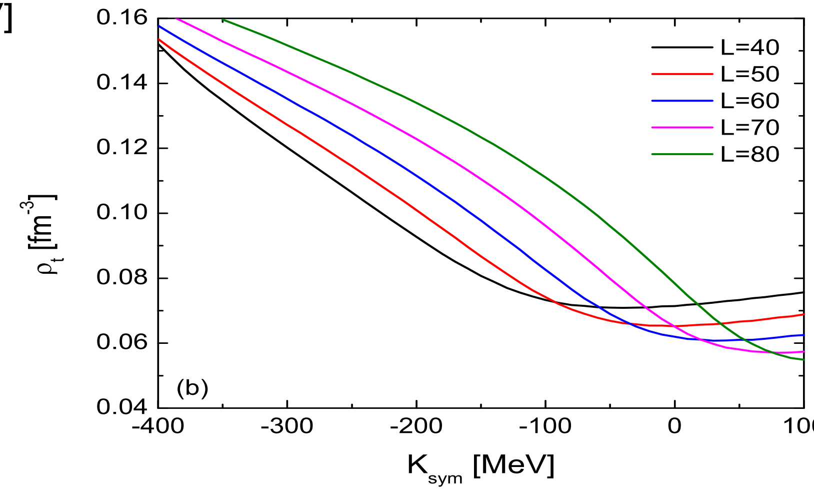



NV+BPS EOS
for the crust




N.B. Zhang, B.A. Li and J. Xu, APJ 859, 90 (2018)




At the crust-core transition: Incompressibility in neutron stars at $\beta$ equilibrium = 0

$$K_\mu = \rho^2 \frac{d^2 E_0}{d\rho^2} + 2\rho \frac{dE_0}{d\rho} + \delta^2 \left[ \rho^2 \frac{d^2 E_{sym}}{d\rho^2} + 2\rho \frac{dE_{sym}}{d\rho} - 2E_{sym}^{-1} (\rho \frac{dE_{sym}}{d\rho})^2 \right]$$

Lattimer & Prakash, Phys. Rep., 442, 109 (2007)

Table 1. The radius $R_{1.4}$ data used in this work.

| Radius $R_{1.4}$ (km) (90% confidence level) | Source | Reference |
|---|---|---|
| $11.9^{+1.4}_{-1.4}$ | GW170817 | (Abbott et al. 2018) |
| $10.8^{+2.1}_{-1.6}$ | GW170817 | (De et al. 2018) |
| $11.7^{+1.1}_{-1.1}$ | QLMXBs | (Lattimer & Steiner 2014) |
| $11.9\pm0.8$, $10.8\pm0.8$, $11.7\pm0.8$ | Imagined case-1 | this work |
| $11.9\pm0.8$ | Imagined case-2 | this work |

Wen-Jie Xie and Bao-An Li
ApJ 883, 174 (2019)
ApJ 899, 4 (2020)

**Posterior probability distribution** $$P(\mathcal{M}|D) = \frac{P(D|\mathcal{M})P(\mathcal{M})}{\int P(D|\mathcal{M})P(\mathcal{M})d\mathcal{M}},$$ **(Bayes' theorem)**

**Likelihood:** $$P[D(R_{1,2,3})|\mathcal{M}(p_{1,2,\cdots 6})] = \prod_{j=1}^{3} \frac{1}{\sqrt{2\pi}\sigma_{\mathrm{obs},j}} \exp[-\frac{(R_{\mathrm{th},j} - R_{\mathrm{obs},j})^2}{2\sigma^2_{\mathrm{obs},j}}],$$

**Meta-modeling of nuclear EOS**

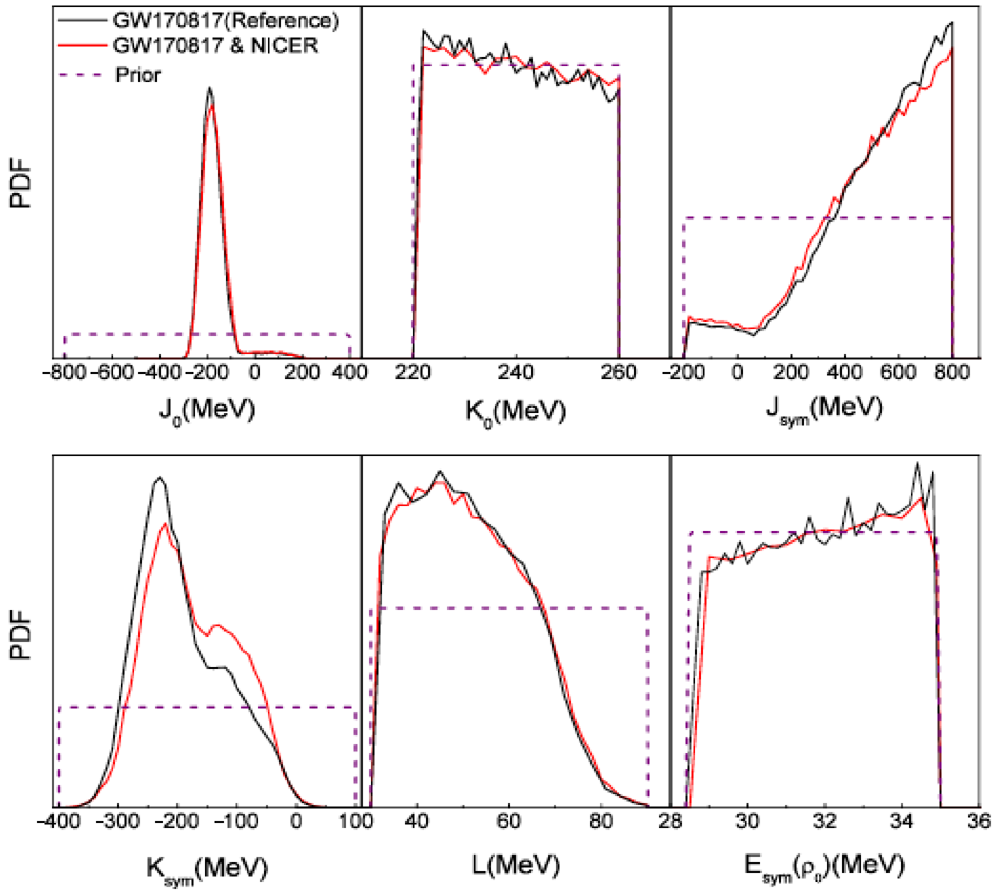
**Uniform prior distribution P(M) in the ranges of**

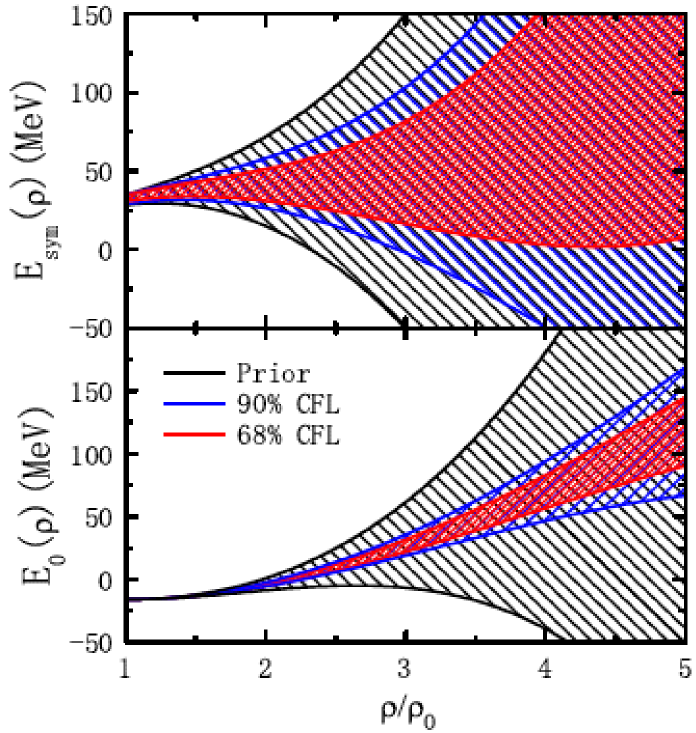
Table 2. Prior ranges of the six EOS parameters used

| Parameters | Lower limit | Upper limit (MeV) |
|---|---|---|
| $K_0$ | 220 | 260 |
| $J_0$ | -800 | 400 |
| $K_{\mathrm{sym}}$ | -400 | 100 |
| $J_{\mathrm{sym}}$ | -200 | 800 |
| $L$ | 30 | 90 |
| $E_{\mathrm{sym}}(\rho_0)$ | 28.5 | 34.9 |

# Bayesian inference of high-density $E_{sym}$ from the radii $R_{1.4}$ of canonical neutron stars in 6D EOS parameter space

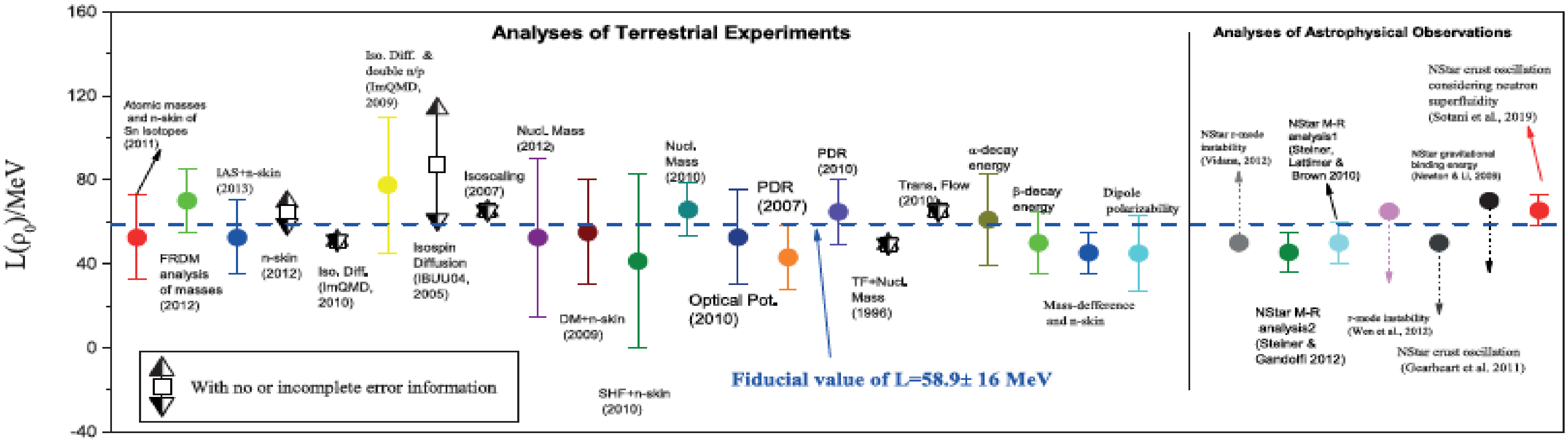
# Posterior probability distribution function (PDF) of 6 EOS parameters from Bayesian analyses of GW170817 & NICER data for the canonical PSR J0030+0451 of masses around 1.4 solar mass




Wen-Jie Xie and Bao-An Li
APJ 883, 174 (2019)
APJ 899, 4 (2020)

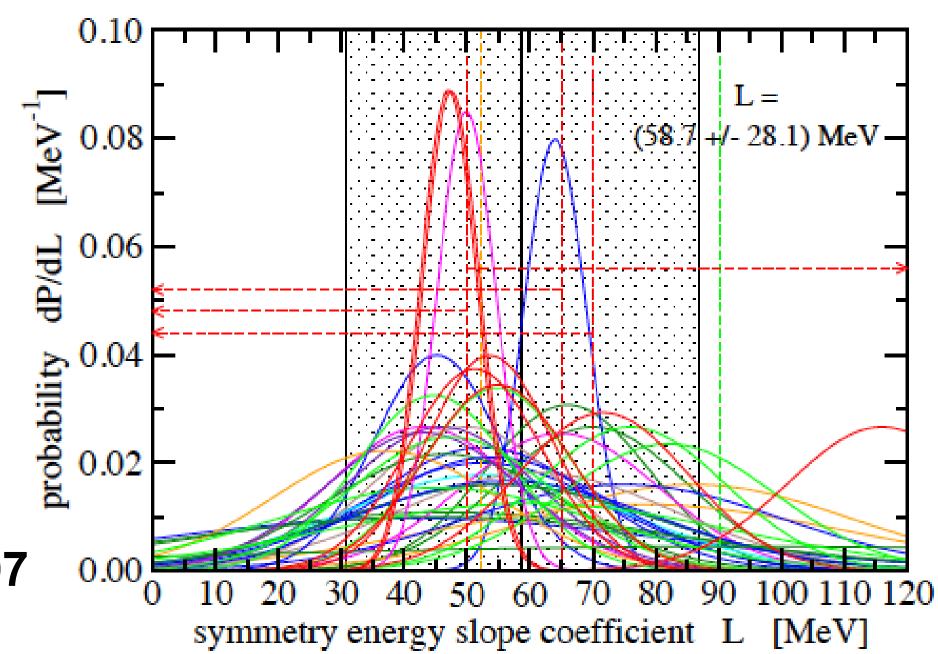
# Constraints on L as of 2013 based on 29 analyses of data




Bao-An Li and Xiao Han, **Phys. Lett. B727 (2013) 276**

## L=58.7±28.1 MeV

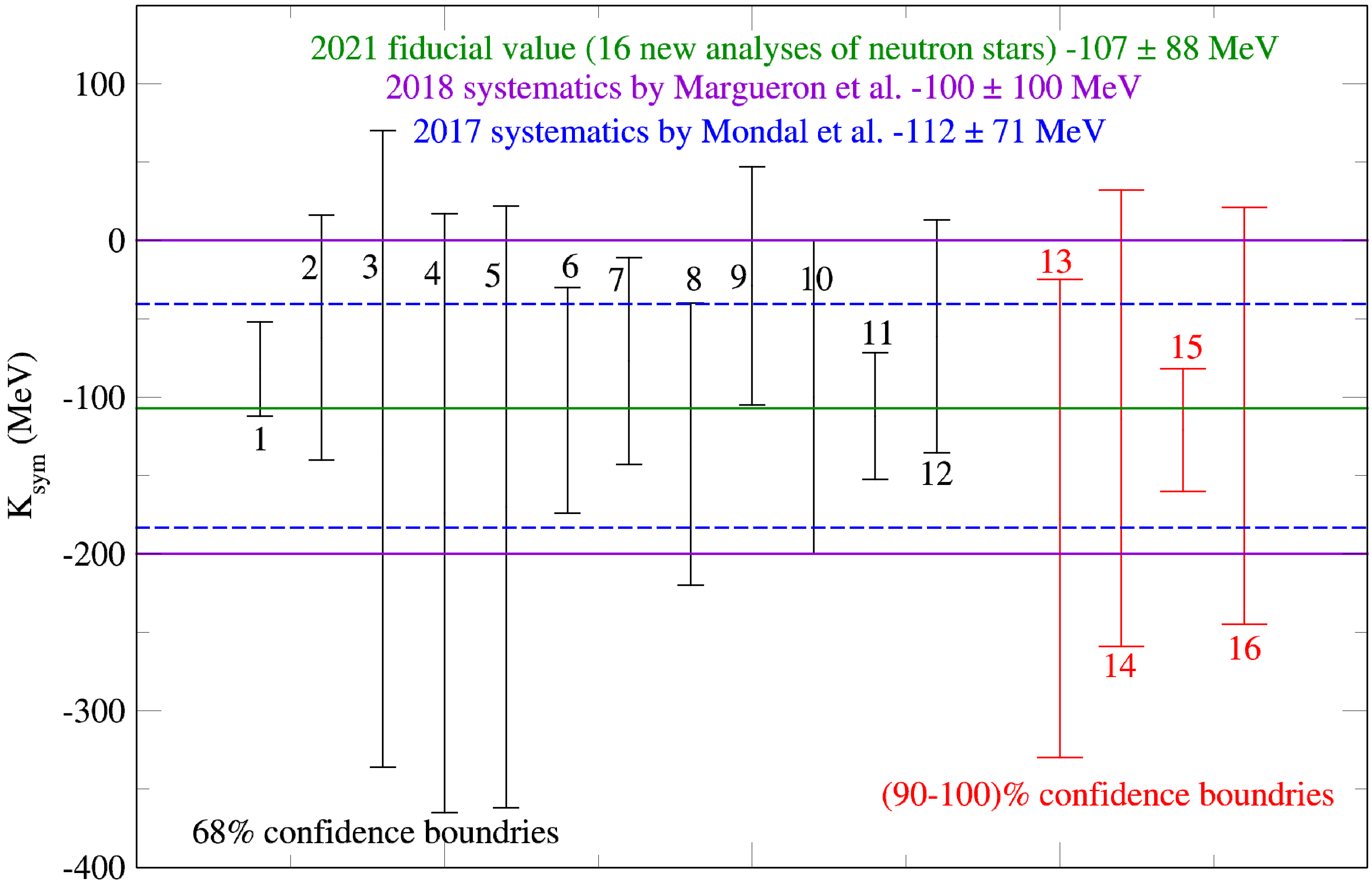
## Fiducial value as of 2016 from surveying 53 analyses



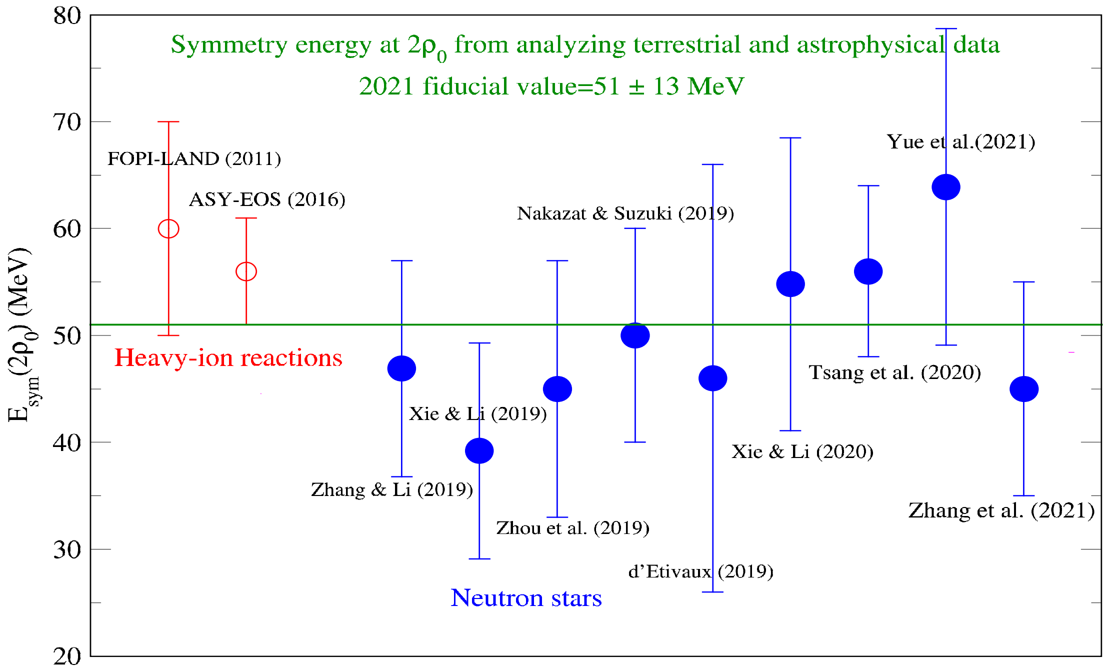

**M. Oertel, M. Hempel, T. Klähn, S. Typel**

**Review of Modern Physics 89 (2017) 015007**

## Curvature of the symmetry energy at saturation density




**Progress in Constraining Nuclear Symmetry Energy Using Neutron Star Observables Since GW170817,**
**B.A. Li, B.J. Cai, W.J. Xie and N.B. Zhang,** *Universe* **2021**, *7*(6), 182; https://doi.org/10.3390/universe7060182




Bao-An Li, Bao-Jun Cai, Wen-Jie Xie, Nai-Bo Zhang, **Universe 7, 182 (2021)**

**Examples of theoretical predictions for $E_{sym}(2\rho_0)$:**

**(1) Chiral EFT,** $E_{\mathrm{sym}}(2\rho_0) \approx 45 \pm 3\ \mathrm{MeV}$

C. Drischler, R. J. Furnstahl, J. A. Melendez, and D. R. Phillips, PRL**125**, 202702 (2020)

**(2) Quantum Monte Carlo,** $E_{\mathrm{sym}}(2\rho_0) \approx 46 \pm 4$

D. Lonardoni, I. Tews, S. Gandolfi, and J. Carlson, Phys. Rev. Research **2**, 022033(R) (2020)

**(3) Relativistic BHF in full Dirac space: 51.6 MeV**

Sibo Wang, Hui Tong, Qiang Zhao, Chencan Wang, Peter Ring, Jie Meng, PRC 106 (2022) 2, L021305

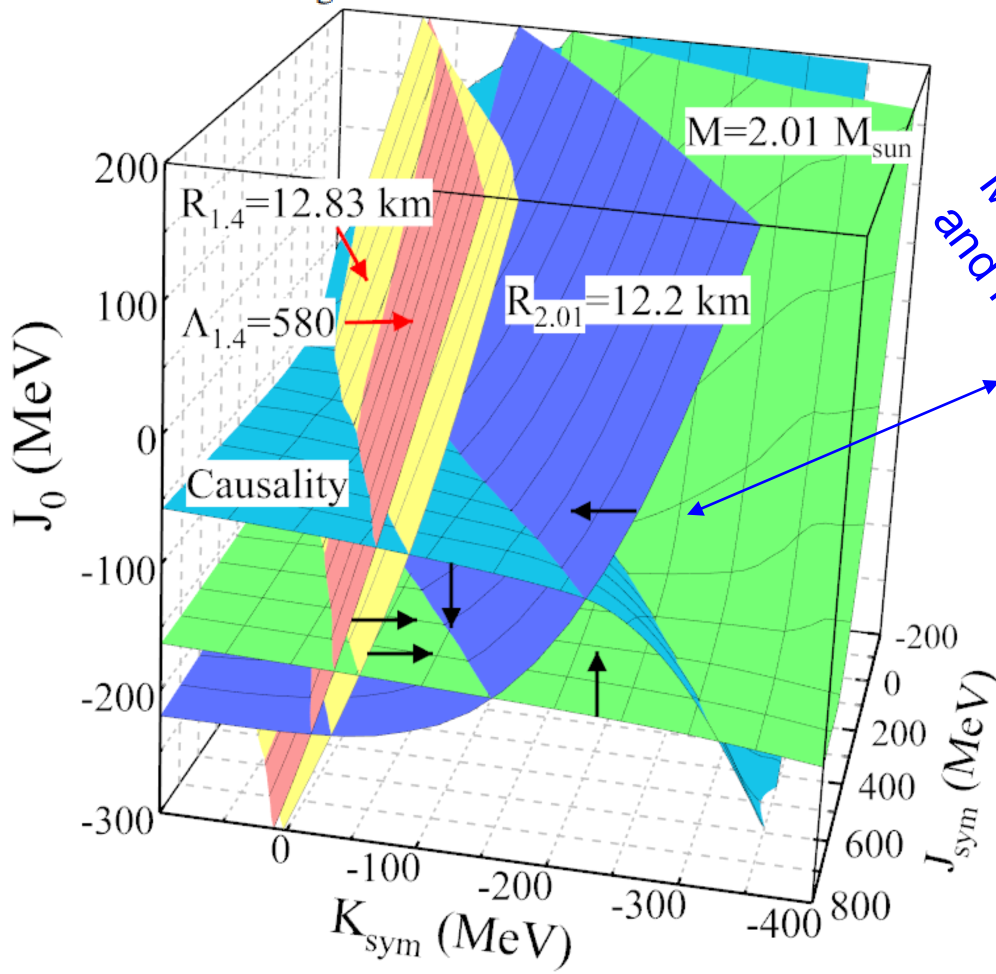
**(4) Relativistic BHF: ~ 53 MeV**

Chencan Wang, Jinniu Hu, Ying Zhang, Hong Shen, *Chin. Phys. C* 46 (2022) 6, 064108

THE ASTROPHYSICAL JOURNAL, 921:111 (11pp), 2021 November 10 https://doi.org/10.3847/1538-4357/ac1e8c


# Impact of NICER's Radius Measurement of PSR J0740+6620 on Nuclear Symmetry Energy at Suprasaturation Densities

Nai-Bo Zhang[1] and Bao-An Li[2]




Power of simultaneous Measurements of mass and radius

**NICER results :**

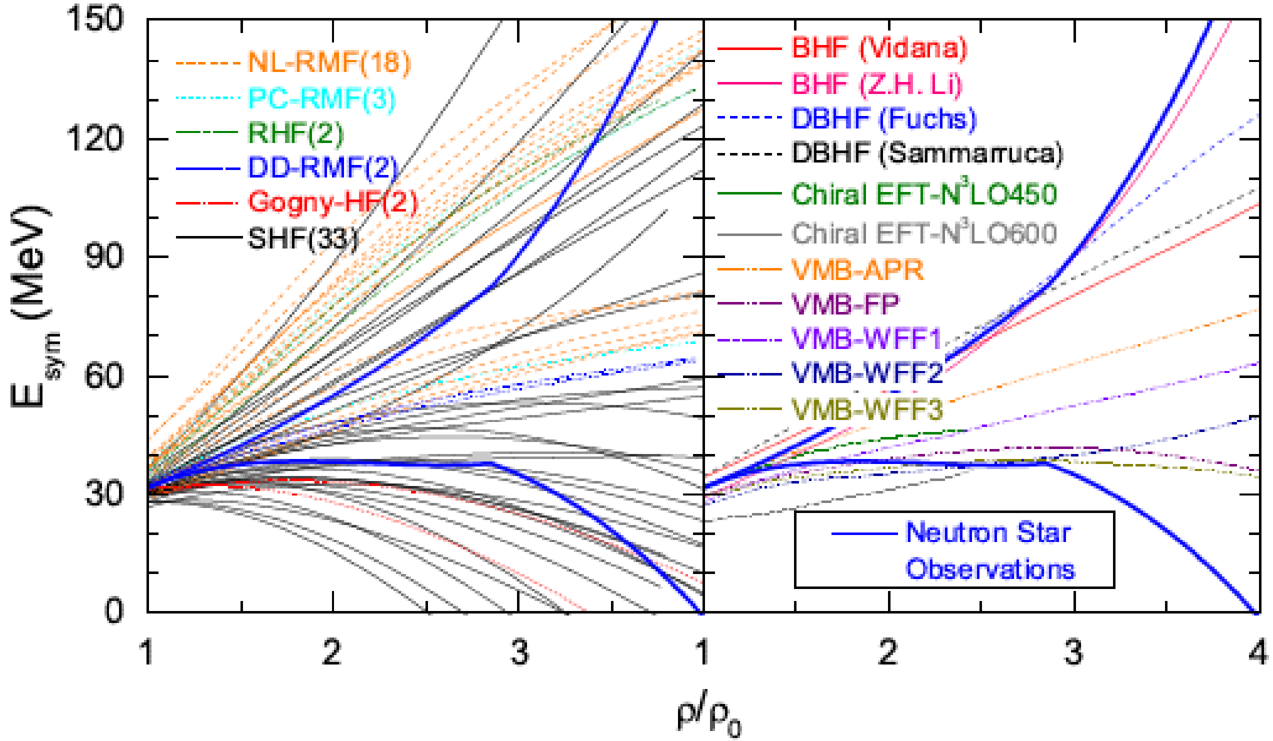
Mass: $2.08 \pm 0.07\, M_\odot$

Radius: $13.7^{+2.6}_{-1.5}$ km (68%) (Miller et al. 2021) or $12.39^{+1.30}_{-0.98}$ km (Riley

**Why is the symmetry energy still so uncertain especially at high densities?**

**Phenomenological Models**
**60 examples**

**Microscopic & *ab initio* Theories**
**11 examples**

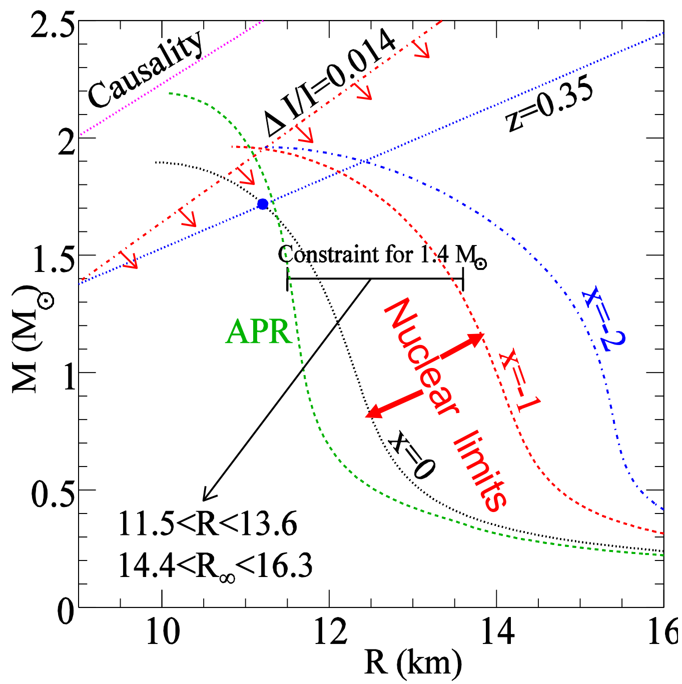


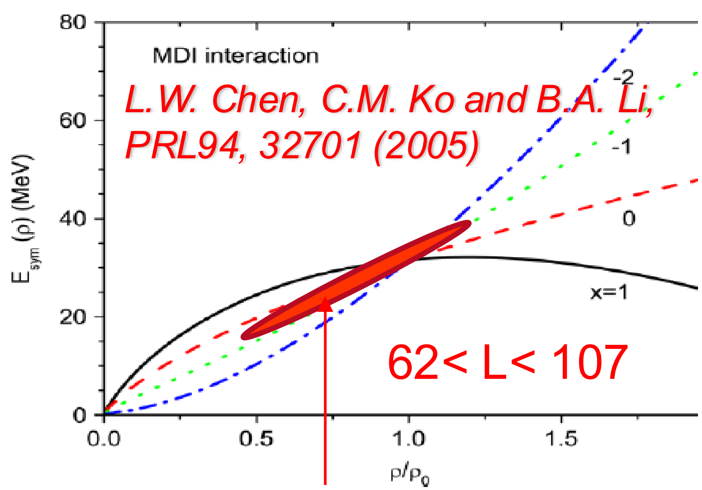

L.W. Chen, Nucl. Phys. Rev. 34, 20 (2017).
N.B. Zhang, B.A. Li, Eur. Phys. J. A 55, 39 (2019).

➢ $E_{sym}$ around (1-2)$\rho_0$ is most relevant for determining the radii of canonical neutron stars, existing 1.4$M_{sun}$NS observations do NOT constrain much $E_{sym}$ above 2$\rho_0$ where SRC effects are important.

# Constraining the radii of neutron stars with terrestrial experiments

**Bao-An Li and Andrew W. Steiner, Phys. Lett. B 642, 436 (2006)**




Fitting isospin diffusion data from MSU