Towards thermalization of Quark-Gluon Plasma

――――A quantum simulation on Schwinger model

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w/ Shuzhe Shi & Li Yan

Ref: arxiv: 2412.00662

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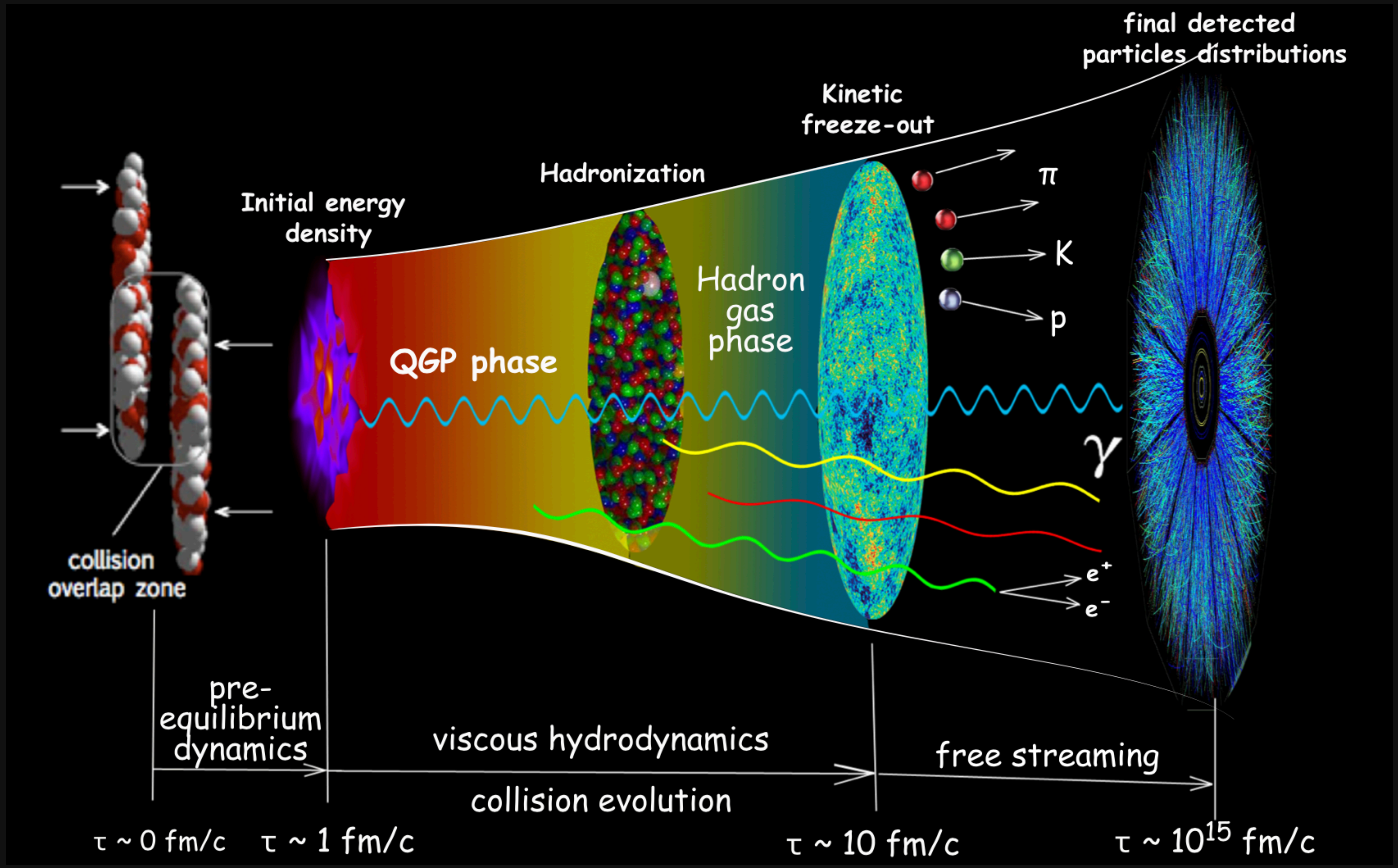
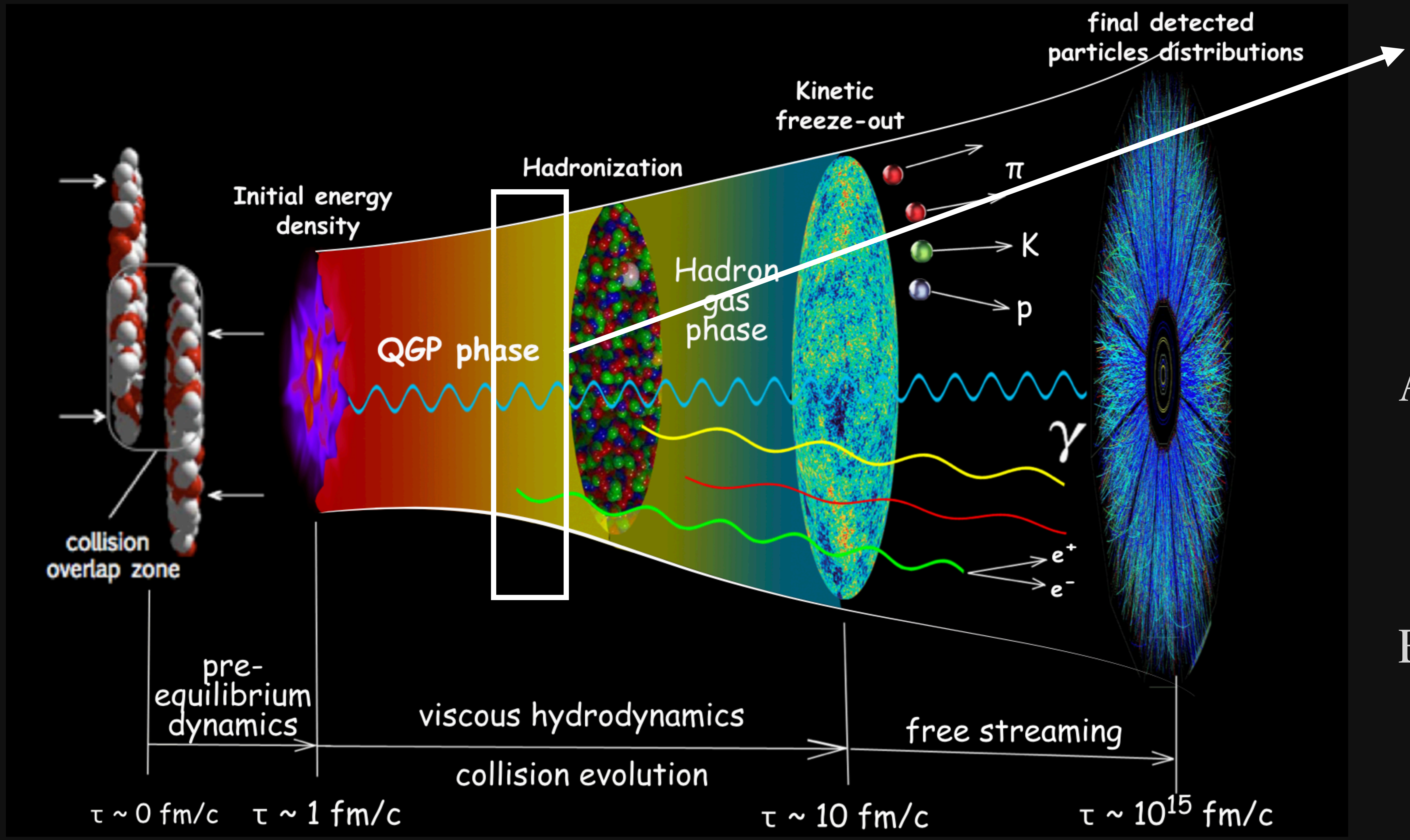
Introduction



Introduction



Hydrodynamically evolved system
in local equilibrium?

Rapid decrease of degrees of freedom

Attractor behavior?

M. P. Heller and M. Spalinski, Phys. Rev. Lett. 115, no. 7, 072501(2015)

J. P. Blaizot and L. Yan, Phys. Lett. B 780, 283-286 (2018)

Shile Chen and Shuzhe Shi, *Phys.Rev.C* 111 (2025) 2, L021902

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Emergence of effective theory?

Motivation

For a strongly coupled many body system, (e.g. quark-gluon plasma), how can we describe its thermalization within an ab-initio, real-time simulation?

A toy model

Schwinger model (1+1-D QED)

$$\mathscr{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

QCD-like properties

Chiral Condensate **Confinement** **CP violation**

Topological θ angle **String breaking mechanism**

Time-independent Hamiltonian

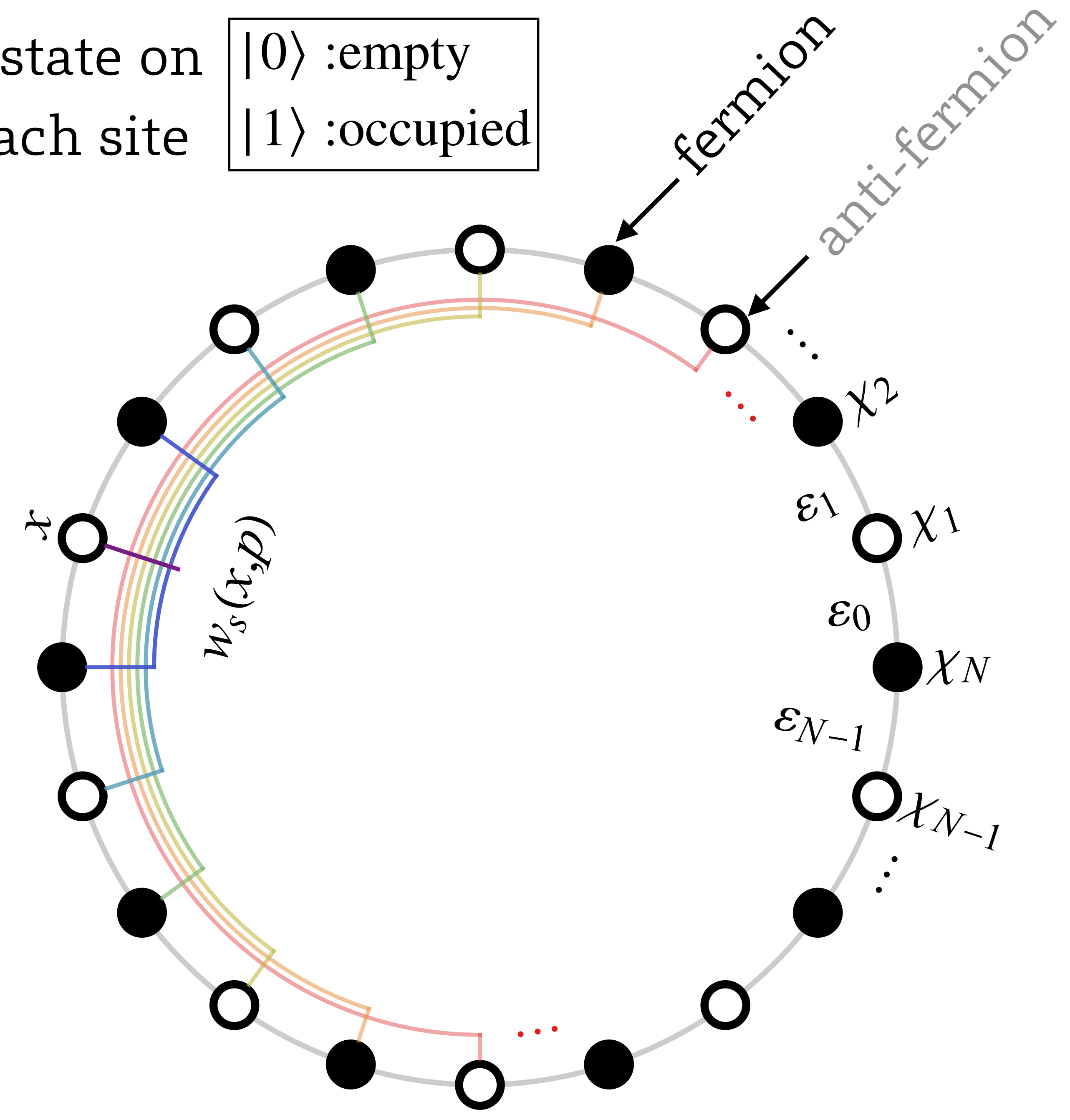
$$H = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1\partial_x - g\gamma^1 A - m)\psi \right) \mathrm{d}x\,.$$

On a lattice

Hilbert space

Staggered fermion

2-state on each site | $|0\rangle$:empty | $|1\rangle$:occupied



Hamiltonian operator

$$H = \int \left[-\bar{\psi}(i\gamma^1\partial_z + m)\psi + \frac{1}{2}\left(g \int_0^x \bar{\psi}\gamma^0\psi \right)^2 \right] \mathrm{d}z$$

$\frac{1}{a}$ ← Energy scales

Staggered fermion component representation in periodic boundary condition

$$H_{PBC} = \sum_{n=1}^{N} \left(-\frac{i}{2}\frac{1}{a}(\chi_n^\dagger e^{i\phi_n}\chi_{n+1} - \chi_{n+1}^\dagger e^{-i\phi_n}\chi_n) + (-1)^n m_0 \chi_n^\dagger \chi_n + \frac{ag^2}{2}\varepsilon_n^2 \right)$$

Jordan-Wigner

$$\chi_n = \frac{X_n - iY_n}{2} \prod_{m=1}^{n-1} (-iZ_m)$$

$$X_n \equiv I \otimes \cdots \otimes I \otimes \sigma_x \otimes I \otimes \cdots \otimes I$$

n^{th}

Quantum thermalization in an isolated system

Quantum distribution function: *Wigner function*

$$W_{ab}(t,z,p) = \int \langle \Psi_t | \bar{\psi}_a(z+\frac{y}{2})\psi_b(z-\frac{y}{2}) | \Psi_t \rangle e^{ipy} dy$$

Decomposition

Scalar   Axial vector charge

$$W = W_s + W_0\,\gamma^0 + W_1\,\gamma^1 - iW_p\,\gamma^5$$

Vector charge   Pseudo-scalar

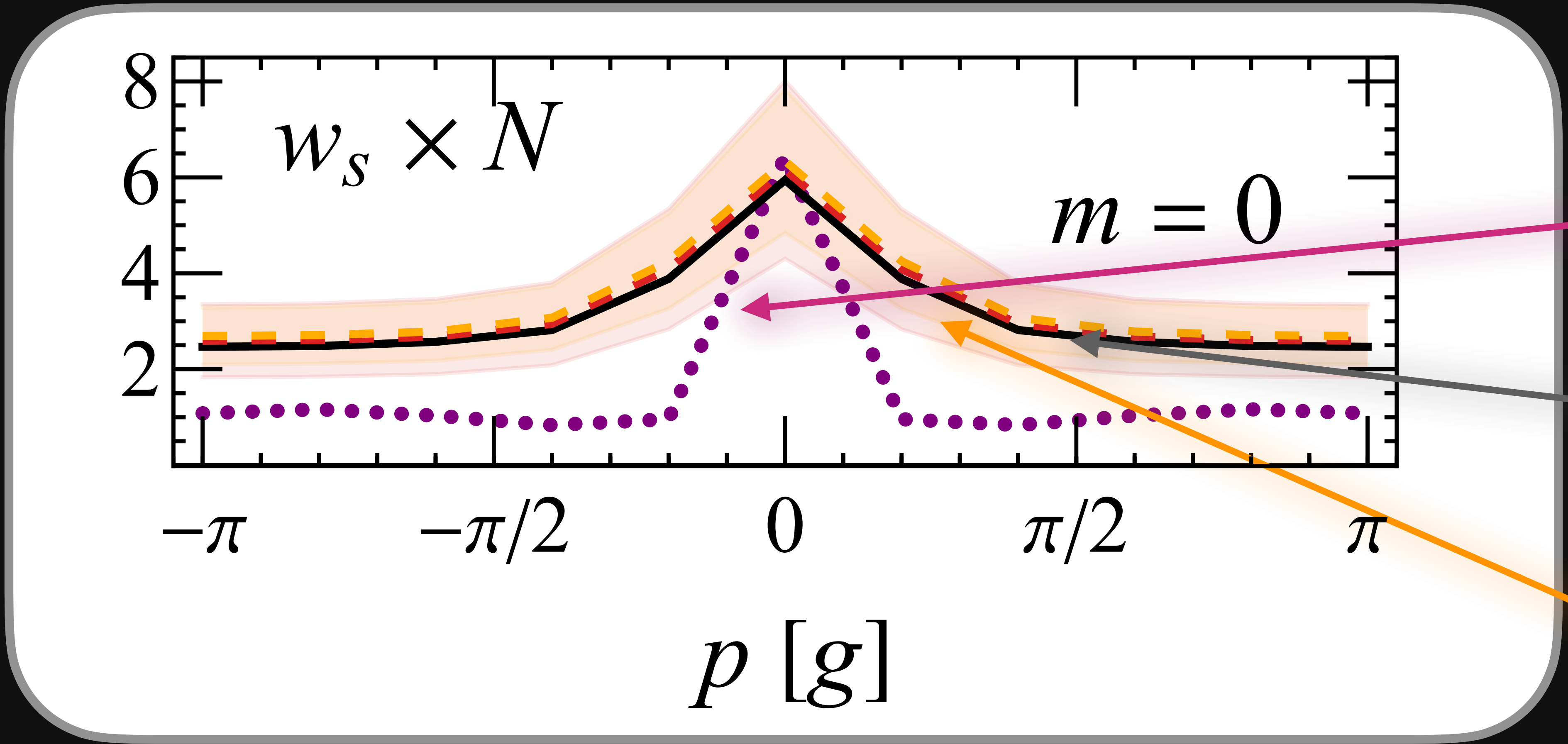
Time evolution

$$H = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1\partial_x - g\gamma^1 A - m)\psi \right) \mathrm{d}x$$

$$|\psi(t)\rangle = \exp(-i\,H\,t)\,|\psi(0)\rangle$$

$$W(t) = \langle \psi(t) | \hat{W} | \psi(t) \rangle$$

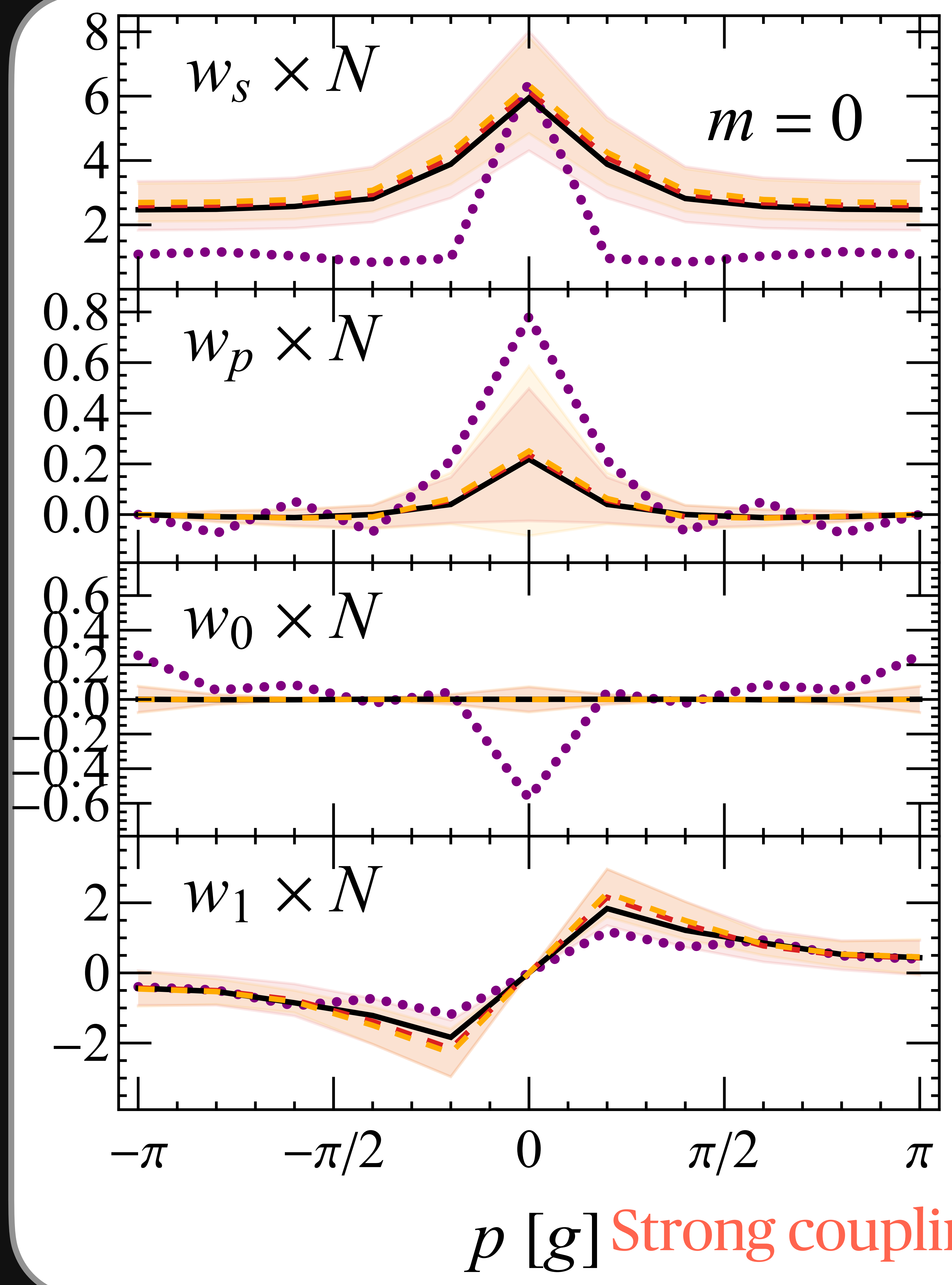
Quantum thermalization in an isolated system



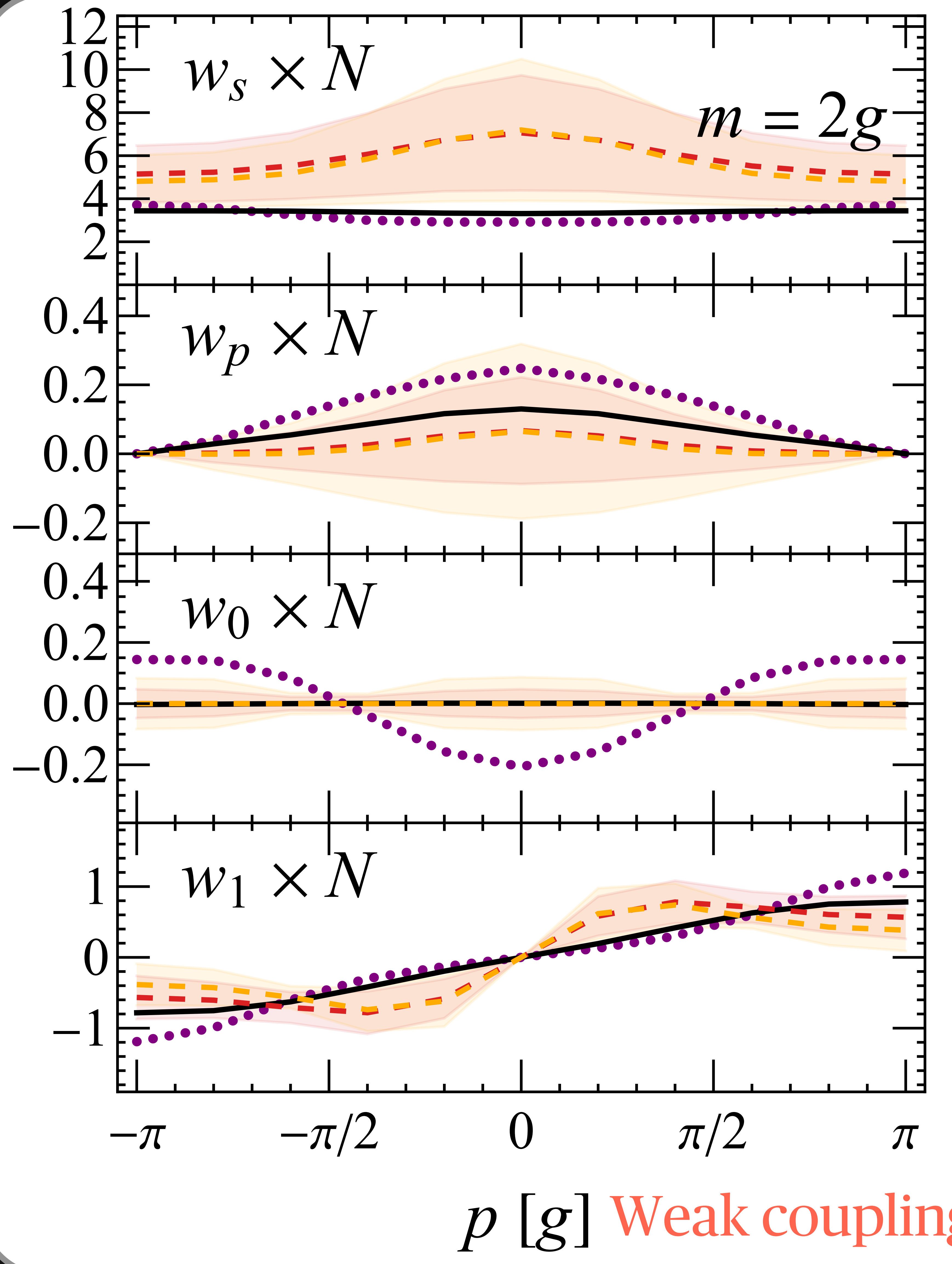
initial condition

long-time average

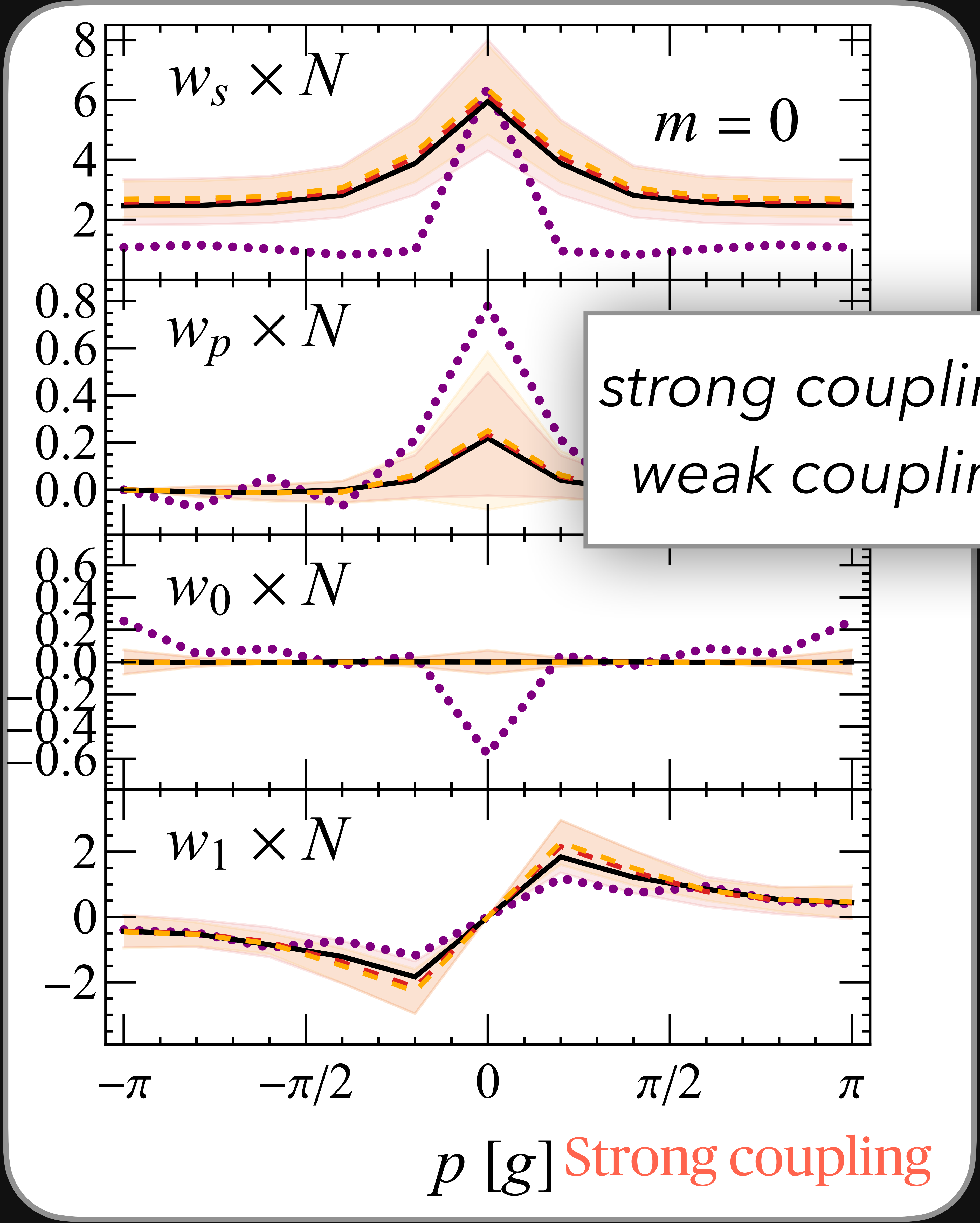
MCE/CE average

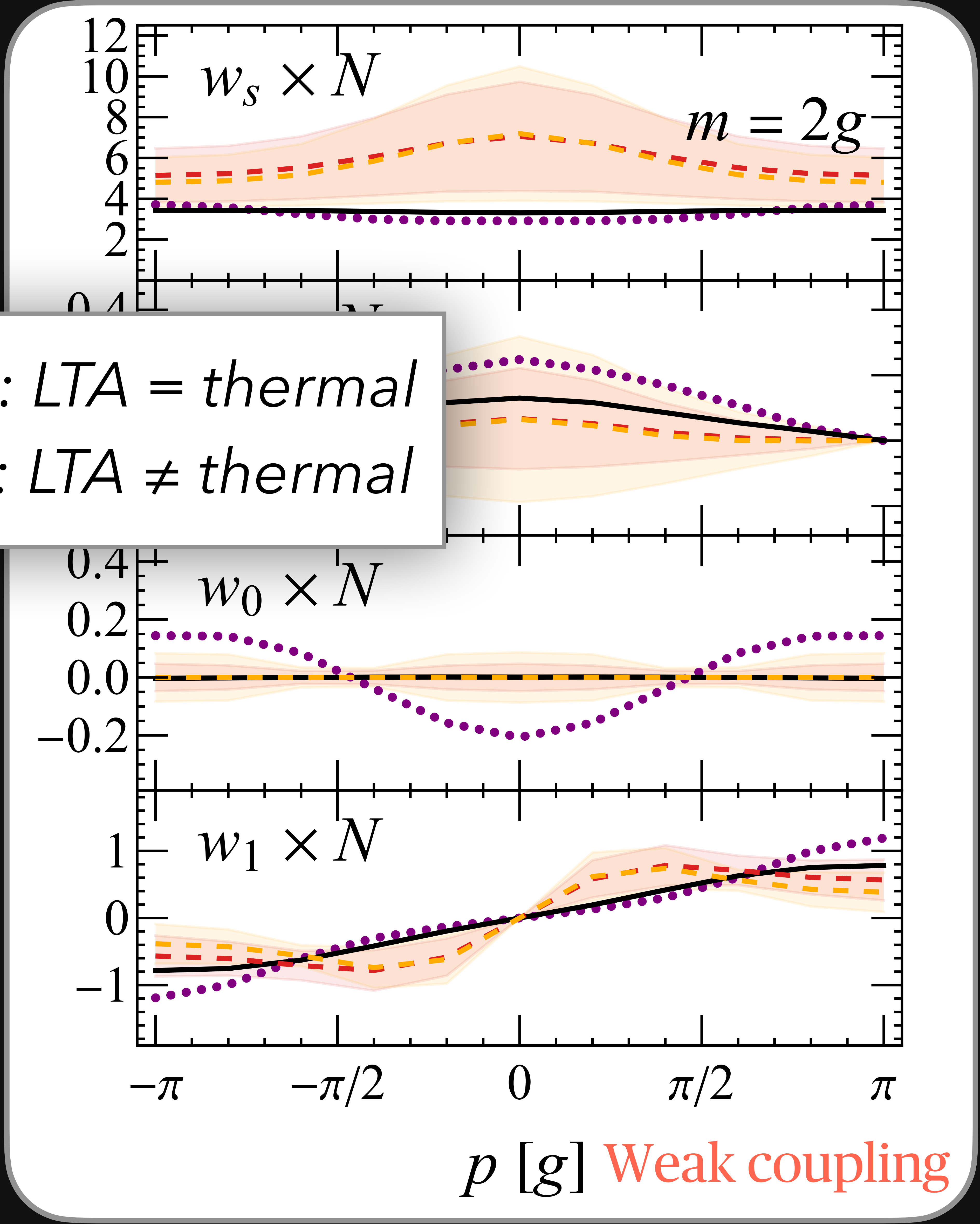


Strong coupling



Weak coupling





strong coupling: LTA = thermal

weak coupling: LTA ≠ thermal

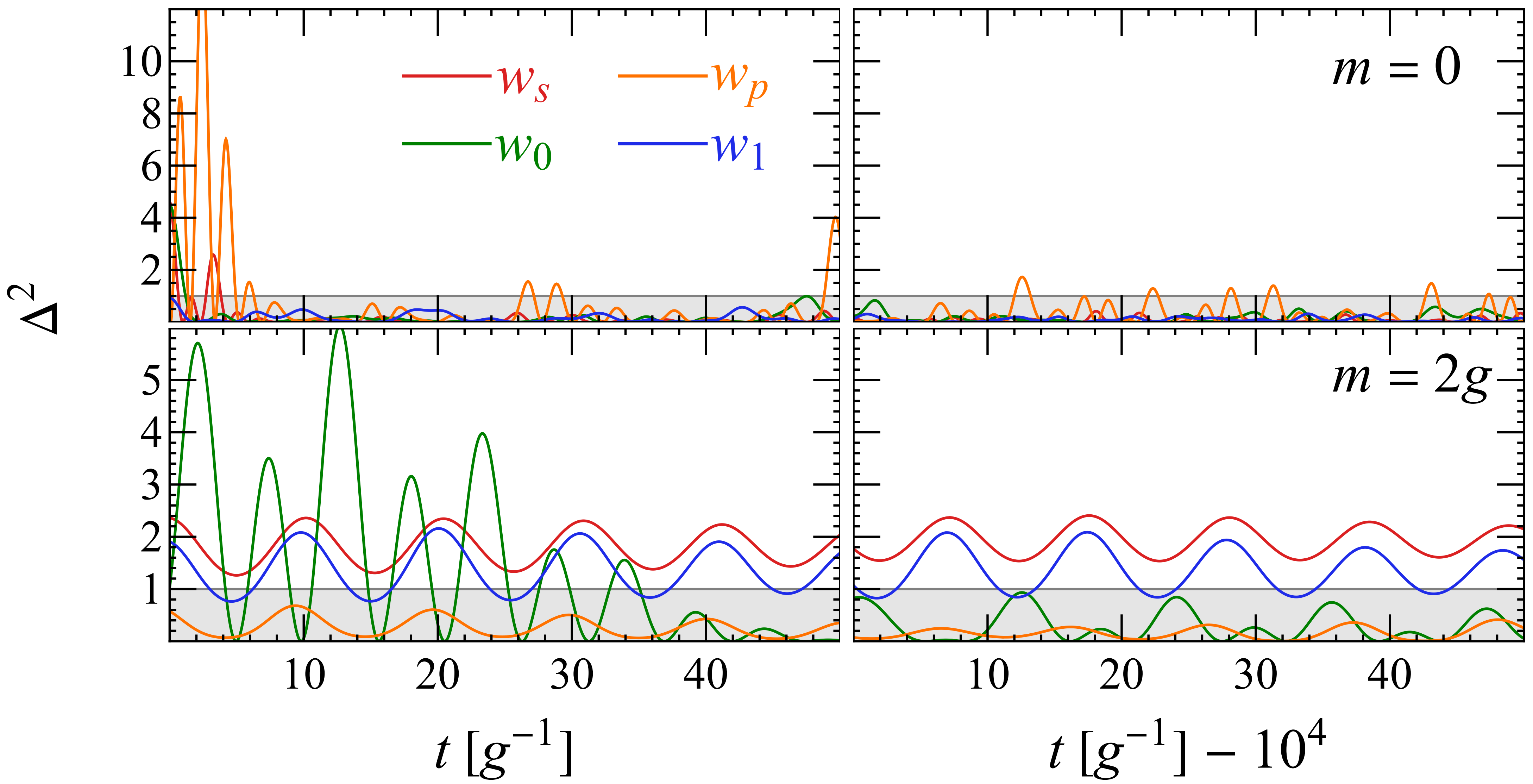
Quantum thermalization in an isolated system

$$\Delta_i^2(p,t) \equiv \left(\frac{\langle \hat{w}_i(p) \rangle_t - \langle \hat{w}_i(p) \rangle_{\mathrm{MCE}}}{\delta_i^{\mathrm{MCE}}(p)} \right)^2$$

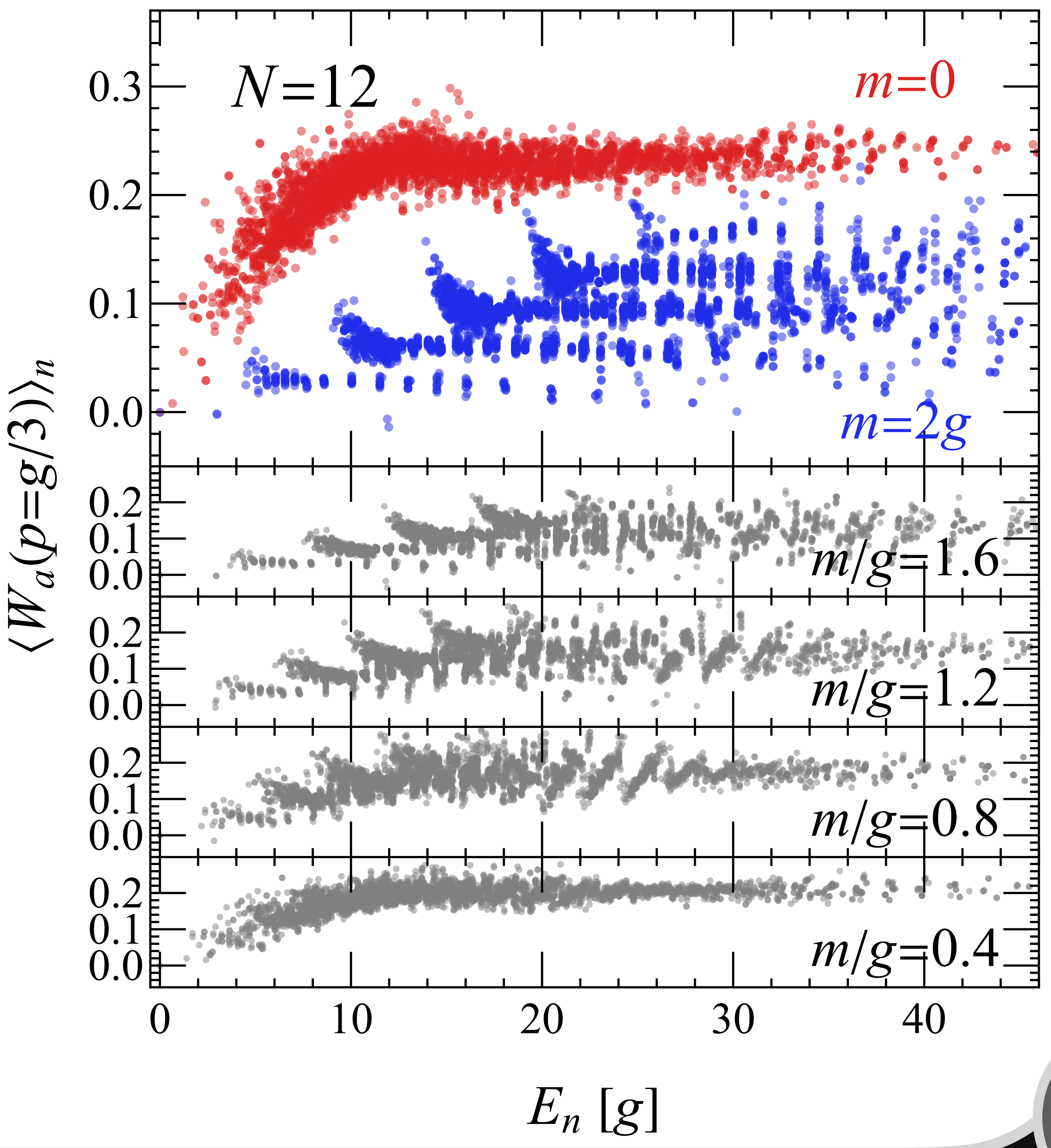
$$O_{nm} = O(\bar{E})\delta_{nm} + \Omega(\bar{E}, E_n - E_m) r_{nm}$$

$$O_{nm} = O(\bar{E})\delta_{nm} + \Omega(\bar{E}, E_n - E_m) r_{nm} \,.$$

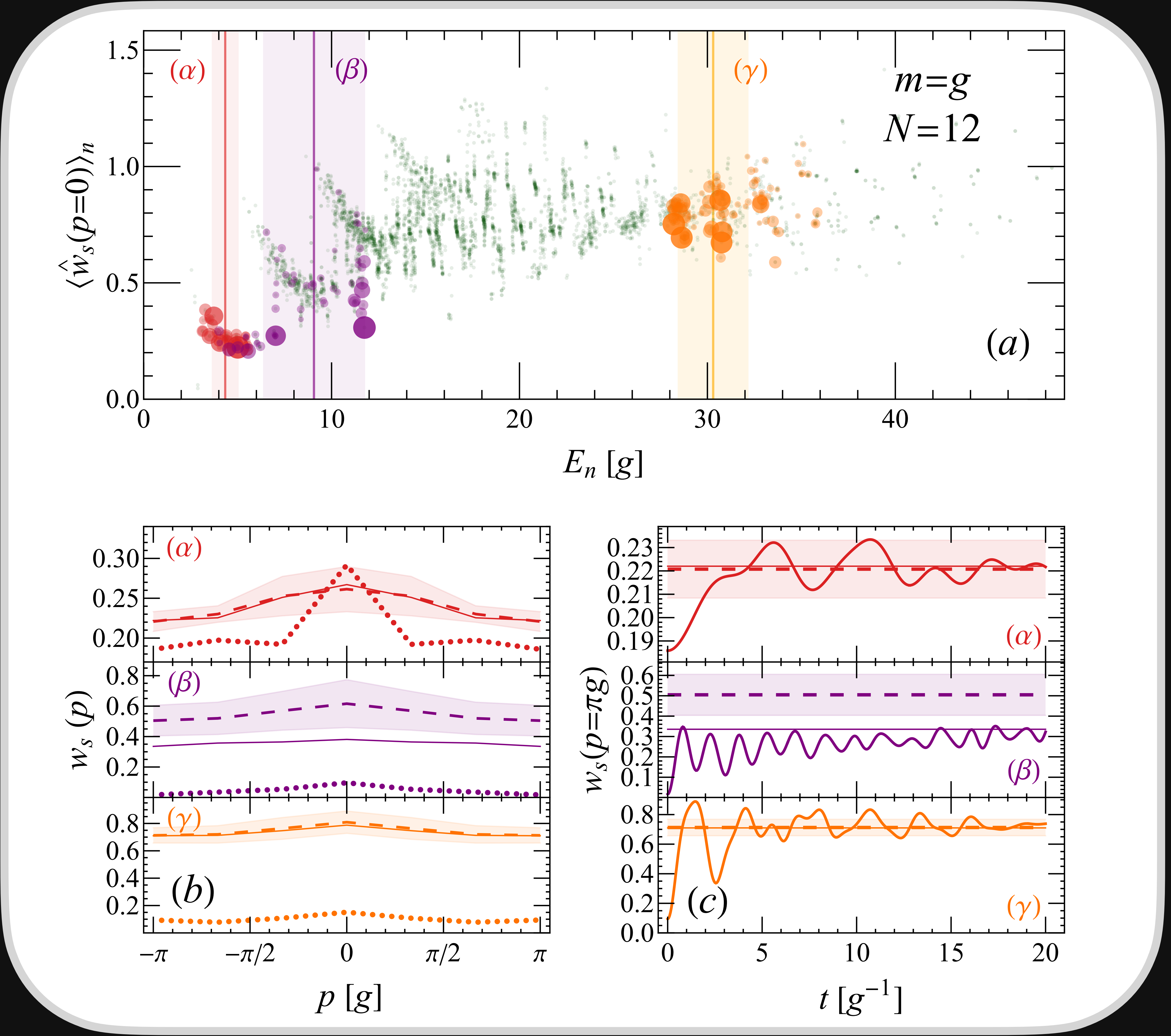
Real-time evolution



Eigenstate thermalization hypothesis



Quantum thermalization in an isolated system



$$O_{nm} = O(\bar{E})\delta_{nm} + \Omega(\bar{E}, E_n - E_m) r_{nm}$$

Weak eigenstate thermalization hypothesis

Initial state-dependent thermalization behavior

(a). Low energy area

Cover a single branch

Quantum thermalized

(b). Mid energy area

Cover different branches

Not thermalized

(c). High energy area

Cover a single branches

Quantum thermalized

Quantum thermalization in an isolated system

$$O_{nm} = O(\bar{E})\delta_{nm} + \Omega(\bar{E}, E_n - E_m) r_{nm}$$

Violation of ETH & Discrete symmetry dependence

In weak coupling limit $\hat{H} \to \hat{H}_{fermion}$

Mass creation operator $$\hat{Q}^\dagger_\alpha \equiv \frac{1}{N_{\text{perm.}}} \sum_{\substack{\text{perm.} \\ |\mathscr{A}| + |\mathscr{B}| = \alpha}} \left(\prod_{j \in \mathscr{A}} \chi^\dagger_j \otimes \prod_{k \in \mathscr{B}} \chi_k \otimes \prod_{i \in \overline{\mathscr{A} \cup \mathscr{B}}} I_i \right)$$

$$[\hat{H}_{\text{fermion}}, \hat{Q}^\dagger_\alpha] = \alpha\, m\, \hat{Q}^\dagger_\alpha \qquad |Q_\alpha\rangle \equiv \hat{Q}^\dagger_\alpha |1010\ldots\rangle$$

$$\hat{H}_{\text{fermion}} |Q_\alpha\rangle = (E_0 + \alpha\, m) |Q_\alpha\rangle \qquad \hat{H} |Q_\alpha\rangle = (E_0 + \alpha\, m)(1 + O(g/m)) |Q_\alpha\rangle$$

Almost energy eigenstate

$\boxed{\text{Parity odd}} \longleftarrow w_p$ $\qquad$ $w_s \longrightarrow \boxed{\text{Parity even}}$

$$\langle Q_\alpha | \hat{w}_p | Q'_\alpha \rangle = 0$$

Quantum thermalization in an isolated system

Reversal of parity

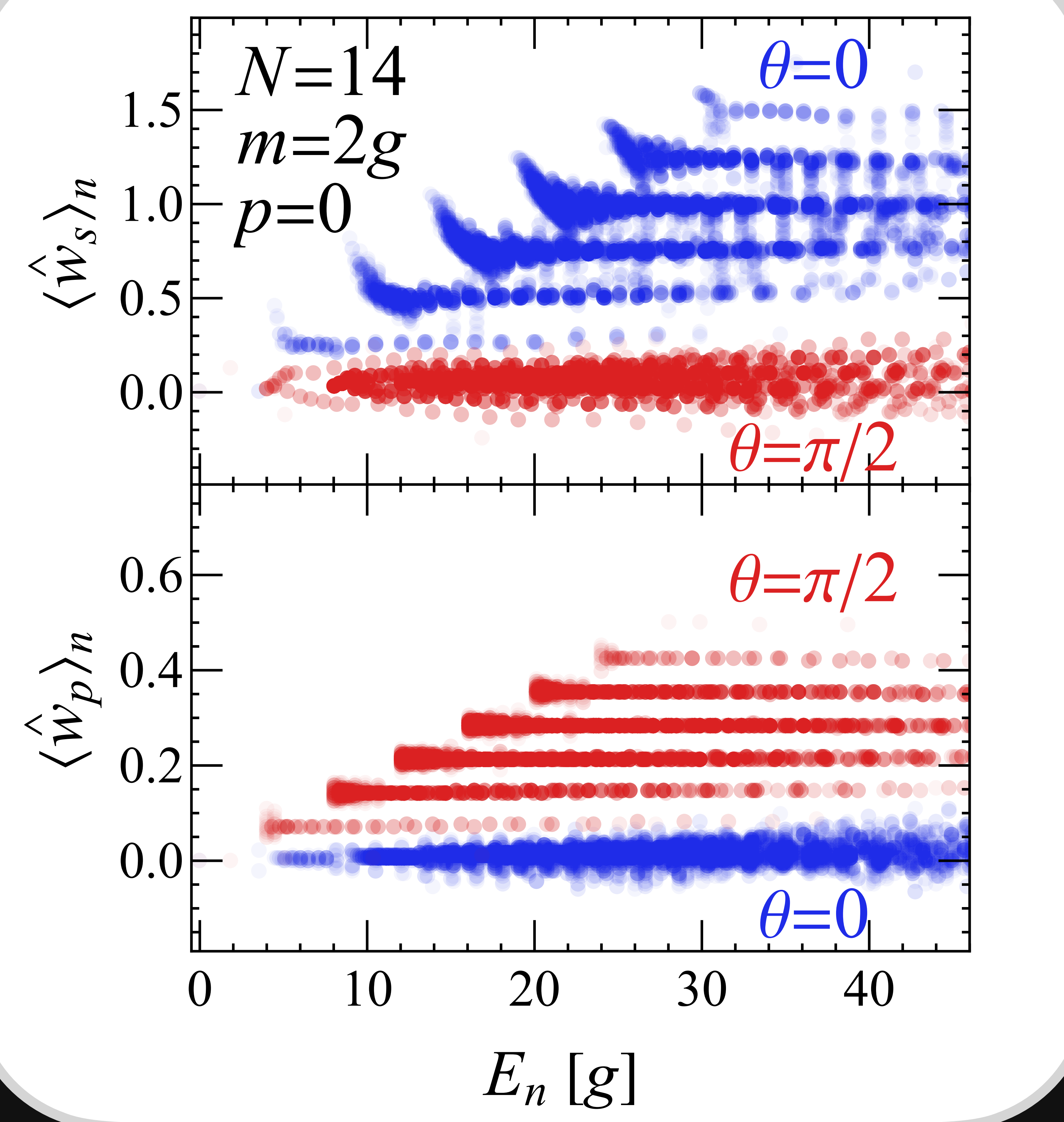
$$H = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1\partial_x - g\gamma^1 A - m e^{i\theta\gamma^5})\psi \right) \mathrm{d}x$$

$\theta = \frac{\pi}{2}$ $\qquad$ $\theta = 0$

$m\,\bar{\psi}\gamma^5\psi$ $\qquad$ $m\,\bar{\psi}\psi$

$$w_s(x,p) \sim \int_{x-y} e^{ip(x-y)}\bar{\psi}(x)\psi(y)$$

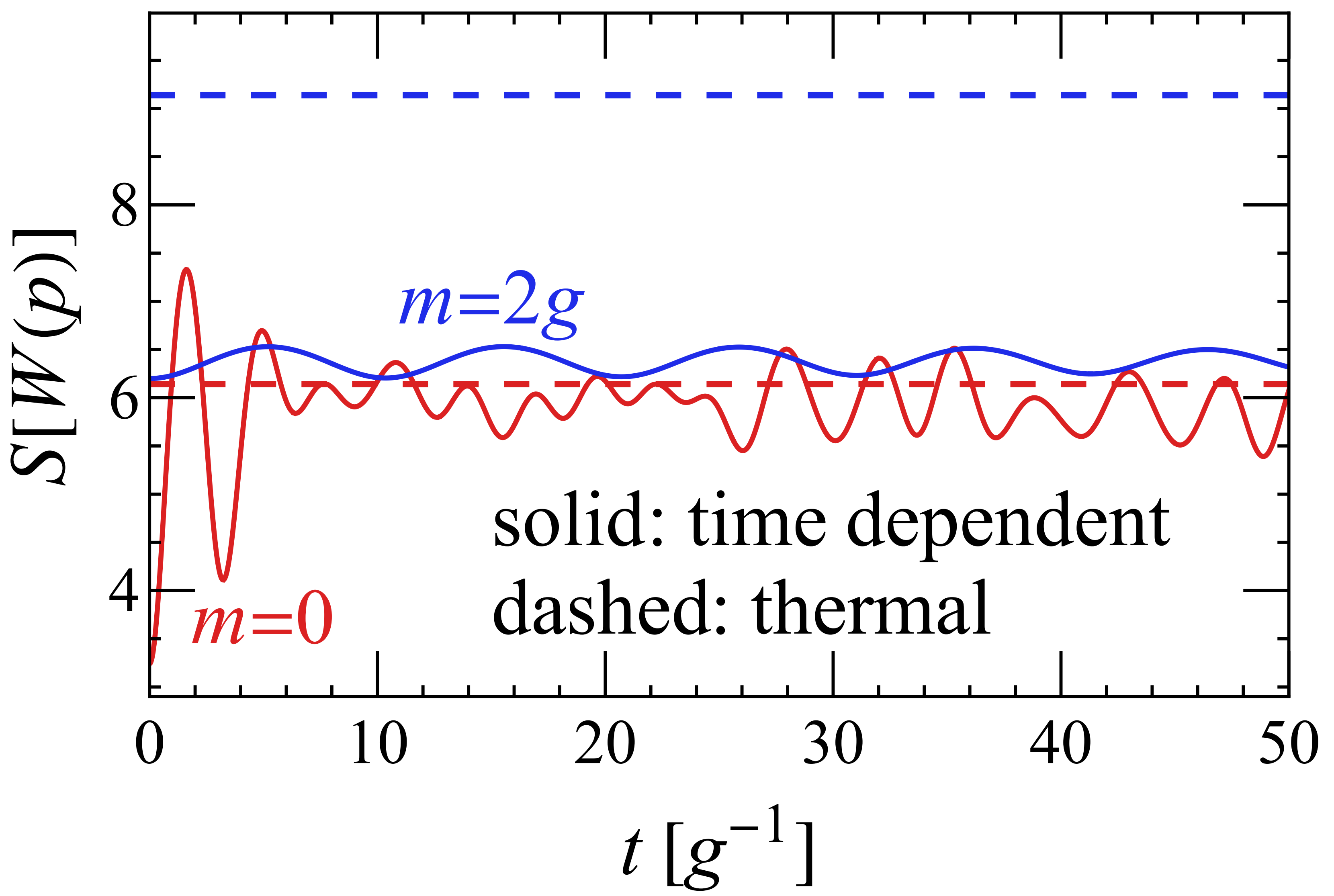
$$w_p(x,p) \sim \int_{x-y} e^{ip(x-y)}\bar{\psi}(x)\gamma^5\psi(y)$$



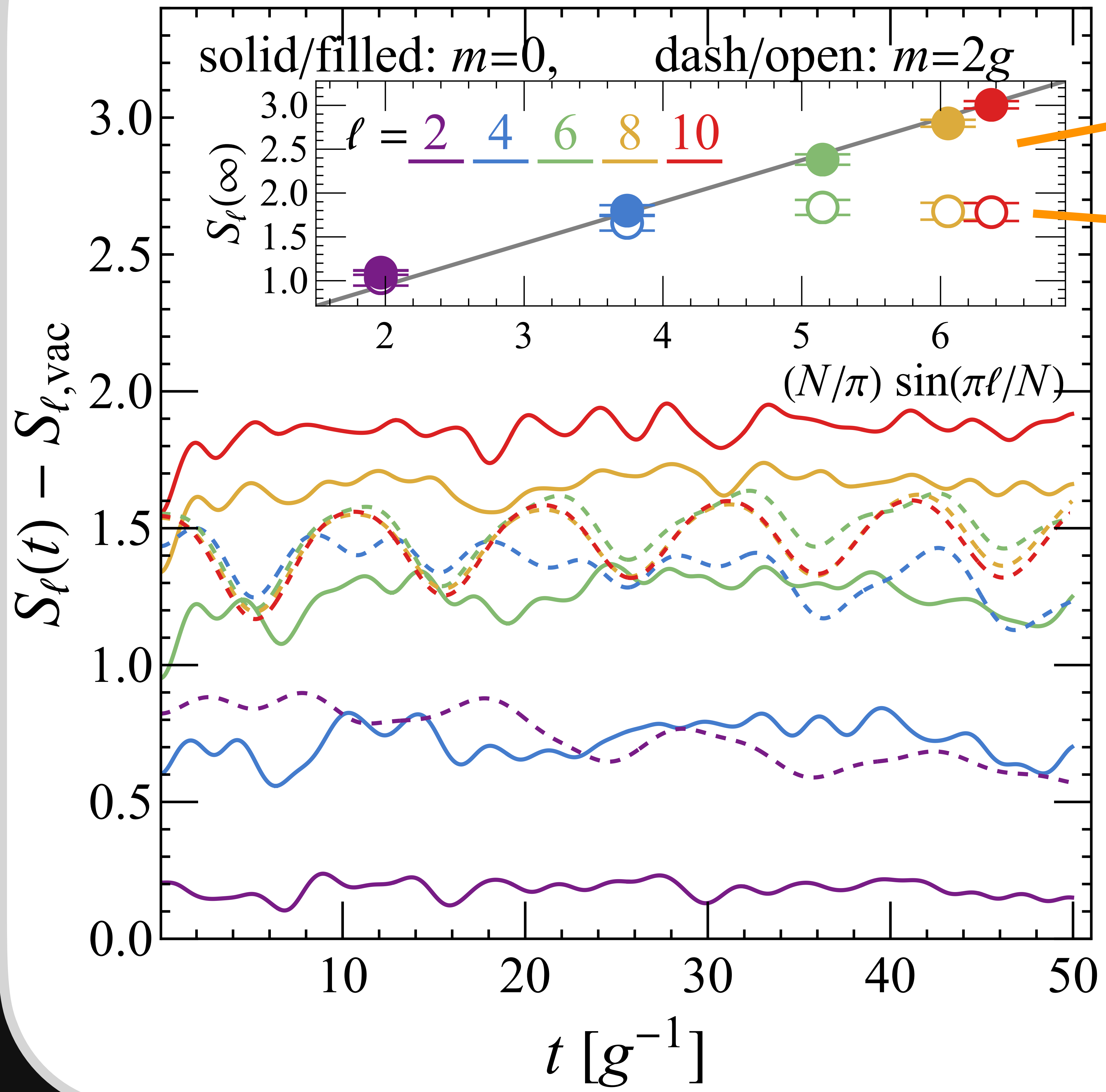
Quantum thermalization in an isolated system

Entropy production

Boltzmann entropy



Entanglement entropy



Volume law

Area law

Summary & outlook

In this work we study the thermalization behavior of quantum distribution function and its link with the quantum thermalization hypothesis. And find the relation between different component with certain global symmetry.

- Subsystem thermalization and extraction of diffusion mode

- Extract hydrodynamic mode in the quantum system evolution

Onset of hydrodynamics in a strongly coupled system based on quantum many-body calculation

May 29, 2025, 12:15 PM
15m
Room 2: 3F Conference Hall #301 (DCC)

Contributed Oral Prese...
Hot and Dense Nucl...
Parallel Session

Talk by Shuzhe Shi, Thur. 12:15

- Thank you!