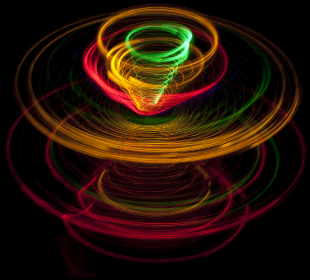
Axial quadrupole and octupole dynamics in even-even and odd mass nuclei

Radu Budaca

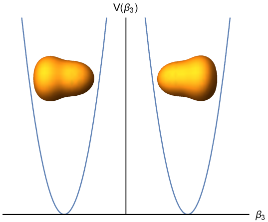
IFIN-HH, Măgurele, România



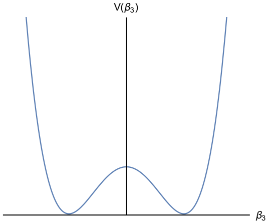
P. Buganu
A. I. Budaca

29th International Nuclear Physics Conference (INPC 2025), May 2025, Daejeon, Republic of Korea.

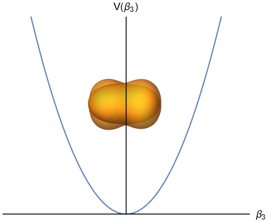
Dynamics of octupole deformation and its spectral signatures



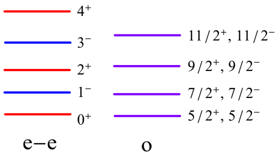
Stable (static, rigid) β_3

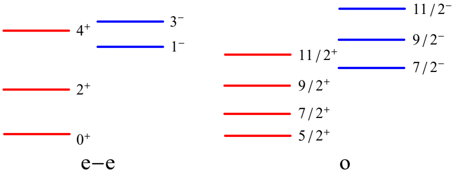


Intermediary dynamical regime (tunneling)



Dynamic β_3 (oscillations)





- Enhanced $E1$ and $E3$ transitions.

Quadrupole-octupole collective Hamiltonian

Quadrupole ($\beta_2 = \alpha_{20}$) and octupole ($\beta_3 = \alpha_{30}$) deformation variables limited to axial symmetry

$\Longrightarrow$

$$H = -\sum_{\lambda=2,3} \frac{\hbar^2}{2B_\lambda} \frac{1}{\beta_\lambda^3} \frac{\partial}{\partial \beta_\lambda} \beta_\lambda^3 \frac{\partial}{\partial \beta_\lambda} + \frac{\hbar^2 \hat{L}^2}{6(B_2\beta_2^2 + 2B_3\beta_3^2)} + U(\beta_2, \beta_3)$$

Solutions $\Phi^{\pm}_{LMK}(\beta_2, \beta_3, \theta) = (\beta_2\beta_3)^{-3/2}\Psi^{\pm}_L(\beta_2, \beta_3)|LMK, \pm\rangle$.

$$\text{Notations:} \quad B = \frac{B_2 + B_3}{2}, \quad \epsilon = \frac{2B}{\hbar^2}E, \quad u = \frac{2B}{\hbar^2}U$$

$$\text{Change of variables:} \quad \tilde{\beta} = \sqrt{\frac{B_2\beta_2^2 + B_3\beta_3^2}{B}}, \quad \tan\phi = \frac{\beta_3}{\beta_2}\sqrt{\frac{B_2}{B_3}}$$

$$\phi = \begin{cases} 0, & \text{Pure quadrupole deformation } (\beta_3 = 0) \\ \pm\pi/2, & \text{Pure octupole deformation } (\beta_2 = 0) \end{cases}$$

$\Downarrow$ Integration over Euler angles θ for $K = 0$

$$\left[-\frac{\partial^2}{\partial\tilde{\beta}^2} - \frac{1}{\tilde{\beta}}\frac{\partial}{\partial\tilde{\beta}} + \frac{L(L+1)}{3\tilde{\beta}^2(1+\sin^2\phi)} - \frac{1}{\tilde{\beta}^2}\frac{\partial^2}{\partial\phi^2} + u(\tilde{\beta}, \phi) + \frac{3}{\tilde{\beta}^2\sin^2 2\phi} - \epsilon_L\right]\Psi^{\pm}_L(\tilde{\beta}, \phi) = 0$$

[V. Yu. Denisov, A. Ya. Dzyublik, Nucl. Phys. A **589** (1995) 17]

Separation of variables

$\Downarrow$

- Separable and ϕ-independent potential $u(\tilde{\beta},\phi)=u(\tilde{\beta})$.
- Factorized wave-function $\Psi_L^{\pm}(\tilde{\beta},\phi)=\psi_L^{\pm}(\tilde{\beta})\chi_L^{\pm}(\phi)$.
- Separation constant $W_L^{\pm}$.

Radial-like equation: $$\left[-\frac{\partial^2}{\partial\tilde{\beta}^2}-\frac{1}{\tilde{\beta}}\frac{\partial}{\partial\tilde{\beta}}+\frac{W_L^{\pm}}{\tilde{\beta}^2}+u(\tilde{\beta})\right]\psi_L^{\pm}(\tilde{\beta})=\epsilon_L\psi_L^{\pm}(\tilde{\beta})$$

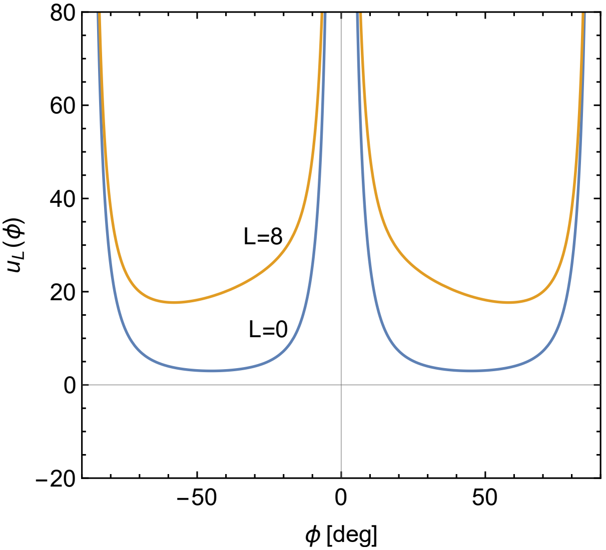
$$u(\tilde{\beta})=\frac{w_0}{\tilde{\beta}^2}+u_0(\tilde{\beta})\begin{cases}\text{Harmonic oscillator potential (HO):} & u_0(\tilde{\beta})=\tilde{\beta}^2\\ \text{Infinite square well potential (ISW):} & u_0(\tilde{\beta})=\begin{cases}0, & \tilde{\beta}\leq 1\\ \infty, & \tilde{\beta}>1\end{cases}\end{cases}$$

Angular equation: $$\left[-\frac{\partial^2}{\partial\phi^2}+u_L(\phi)\right]\chi_L^{\pm}(\phi)=W_L^{\pm}\chi_L^{\pm}(\phi)$$

Intrinsic potential: $$u_L(\phi)=\frac{3}{\sin^2 2\phi}+\frac{L(L+1)}{3(1+\sin^2\phi)}$$

Intrinsic potential

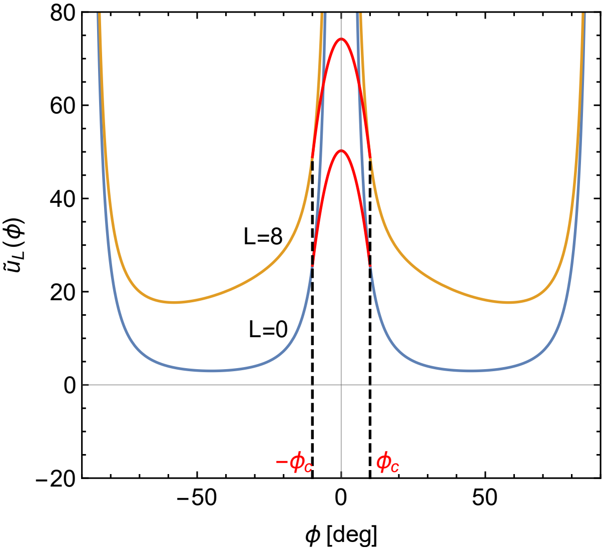
$$u_L(\phi) = \frac{3}{\sin^2 2\phi} + \frac{L(L+1)}{3(1+\sin^2\phi)}$$



Intrinsic potential

$$u_L(\phi) = \frac{3}{\sin^2 2\phi} + \frac{L(L+1)}{3(1+\sin^2\phi)}$$

$$v(\phi) = -a\phi^2 + b$$



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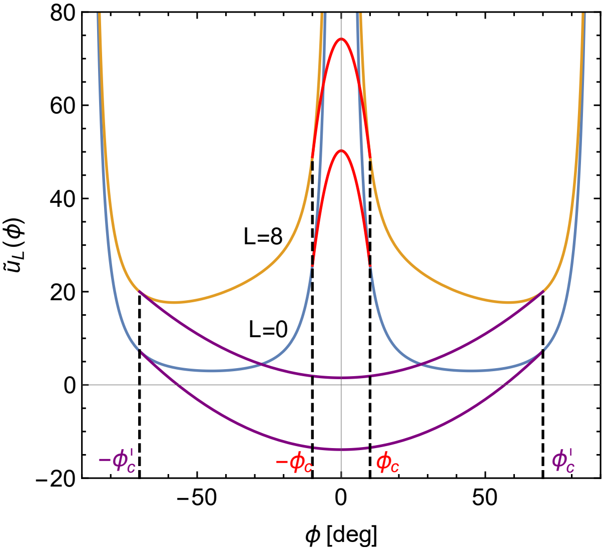
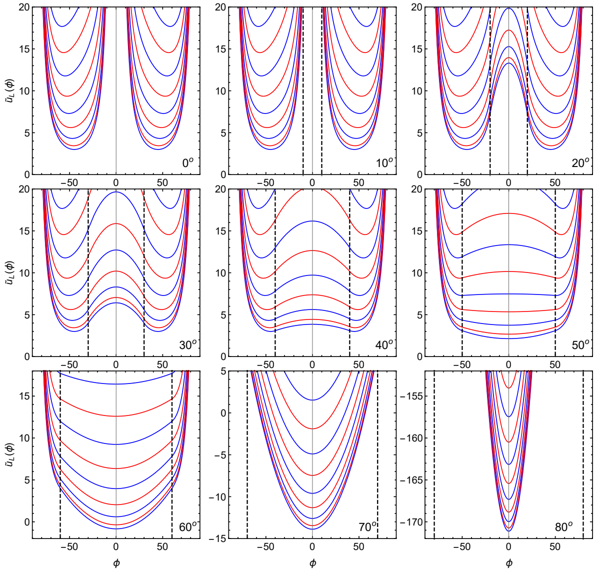


Modified ϕ potential

The modified potential:

$$\tilde{u}_L(\phi) = \begin{cases} v_L(\phi), & |\phi| < |\phi_c| \\ u_L(\phi), & |\phi| \geq |\phi_c| \end{cases}$$



Odd-A systems

- Total angular momentum $\vec{J} = \vec{L} + \vec{j}$
- Strong coupling hypothesis $\Rightarrow$ collective degrees of freedom are dominant
- Intrinsic angular momentum projection $K = \Omega$
- Coriolis interaction

$$L(L+1) \;\rightarrow\; J(J+1) - K^2 + \pi a \delta_{K\frac{1}{2}} (-)^{J+\frac{1}{2}} \left(J + \frac{1}{2}\right)$$

a - decoupling parameter (adjustable).

$\pi = \pi_p \pi_c$ - total parity defined by the parity of the odd nucleon and of the deformed core.

Numerical applications

Independent parameters $|\phi_c|$, w_0 and a (for $K = 1/2$) are determined by fitting the energy ratios $\frac{E(J_i^\pi)}{E(J_{ref}^\pi)}$ using HO or ISW $\tilde{\beta}$ potential.

- Even-even nuclei ($K = 0$ yrast states, HO&ISW)

19 Heavy nuclei $^{222-228}$Ra, $^{224-234}$Th, $^{230-240}$U, $^{236-240}$Pu

[RB, P. Buganu, A.I. Budaca, Phys. Rev. C **106** (2022) 014311]

[RB, P. Buganu, A.I. Budaca, Eur. Phys. J. A **59** (2023) 242]

19 Medium nuclei $^{142-148}$Ba, $^{146-148}$Ce, $^{148-152}$Nd, $^{150-154}$Sm, $^{156-160}$Gd, $^{164-170}$Er

[RB, A.I. Budaca, P. Buganu, Phys. Scr. **99** (2024) 035309]

- 18 Odd-A heavy nuclei (HO) [RB, At. Data Nucl. Data Tables, **161** (2025) 101692]

$K = 1/2$ ^{219,221}Fr, ^{225}Ra, ^{227}Th, ^{237}U, ^{239}Pu, $^{229-233}$Pa

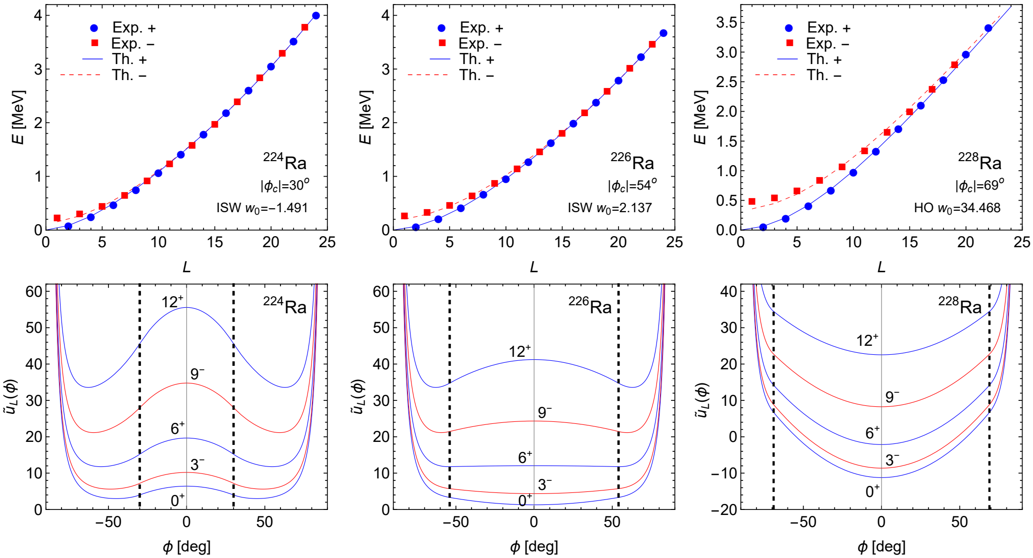
$K = 3/2$ ^{223}Ra, ^{225,227}Ac

$K = 5/2$ ^{221}Ra, ^{231}Th, ^{233}U, $^{241-243}$Am

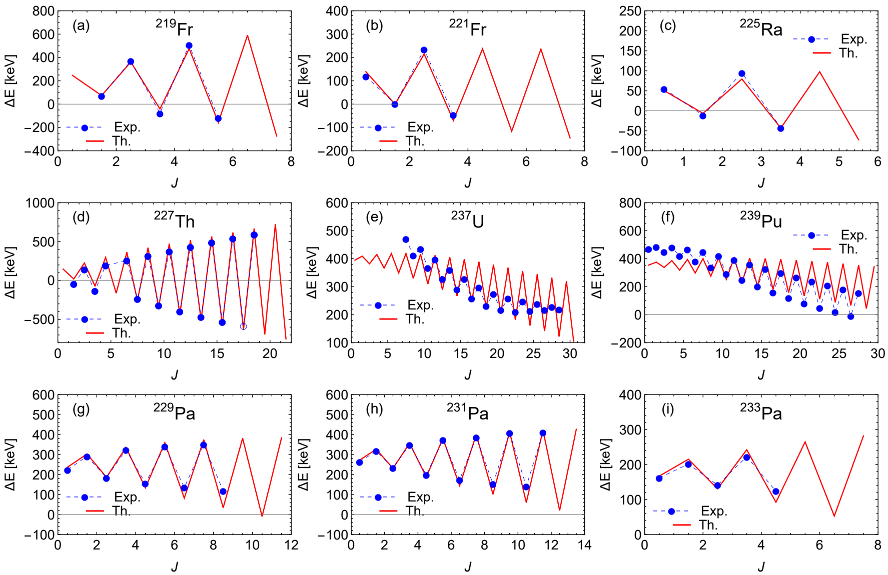
- 3 Odd-A medium nuclei (HO) [RB, At. Data Nucl. Data Tables, **161** (2025) 101692]

$K = 5/2$ $^{155-159}$Eu

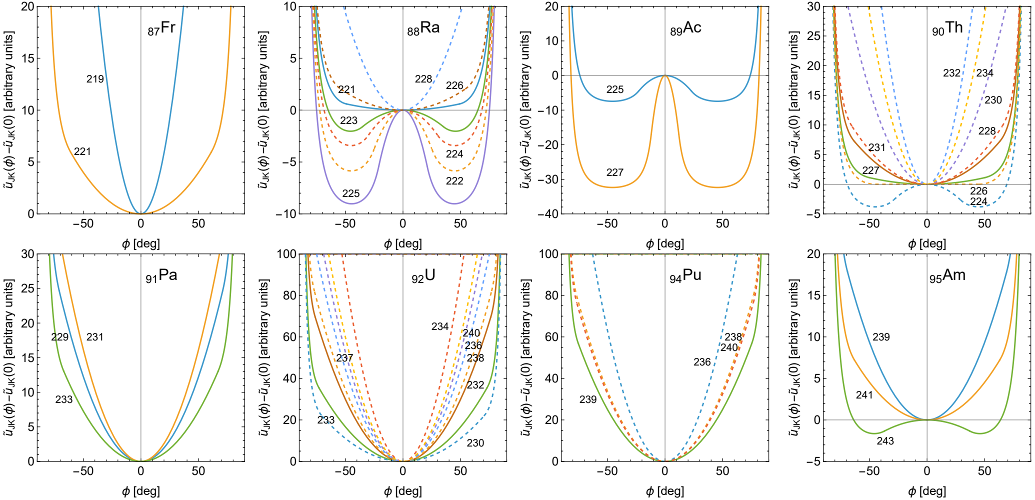
Numerical applications - energy levels (even-even Ra nuclei)



Numerical applications - Energy difference between $K = 1/2$ bands in odd mass nuclei

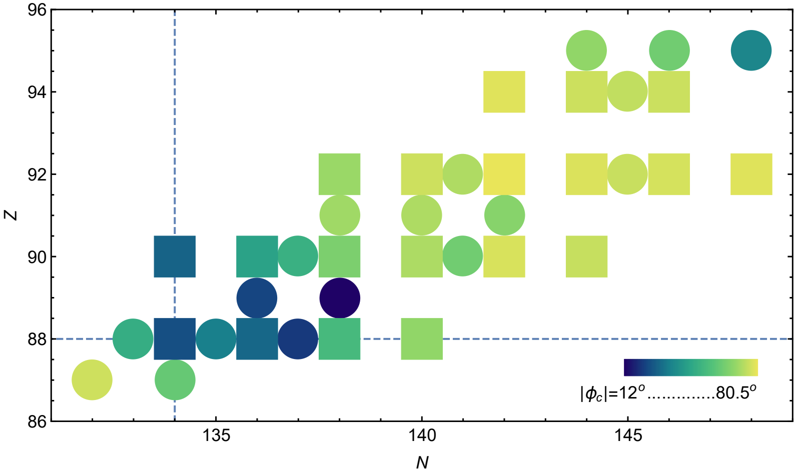


Numerical applications - ϕ ground state potential (heavy nuclei)



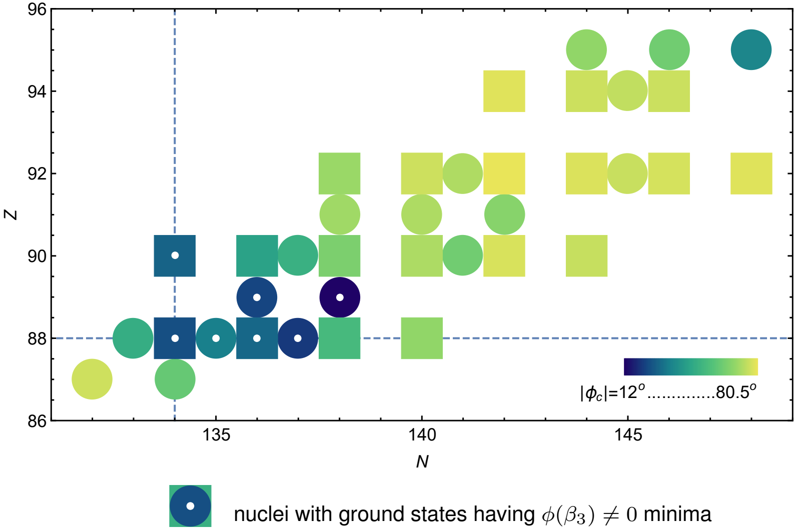
Numerical applications - parameters evolution (heavy nuclei)

[RB, At. Data Nucl. Data Tables, **161** (2025) 101692]



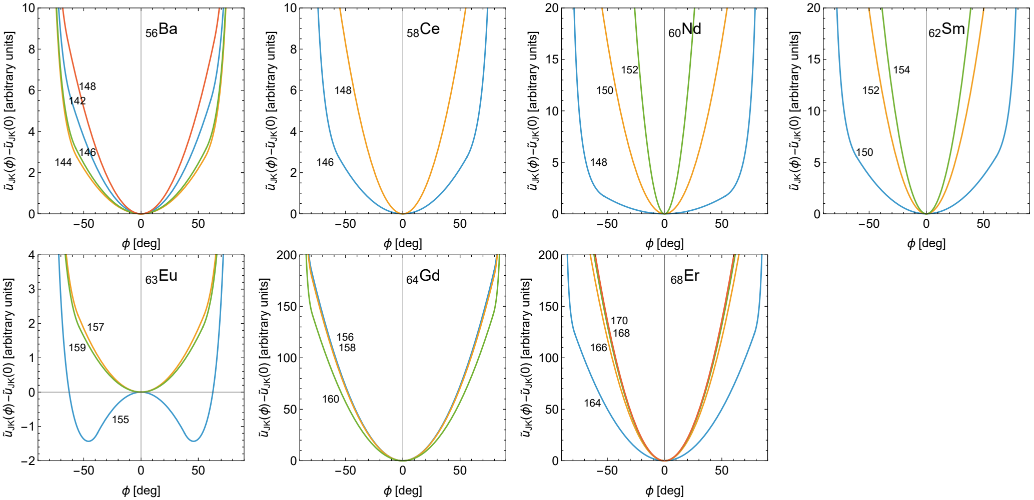
Numerical applications - parameters evolution (heavy nuclei)

[RB, At. Data Nucl. Data Tables, **161** (2025) 101692]

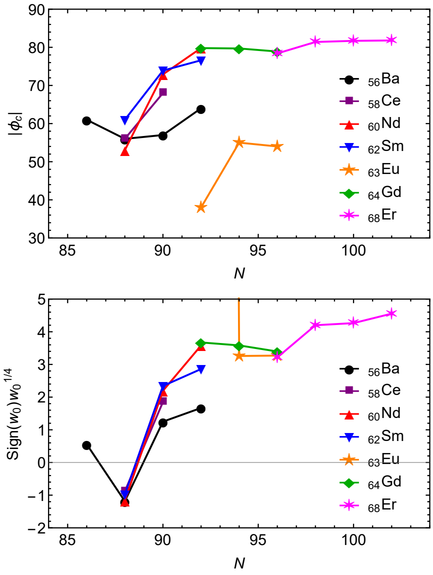


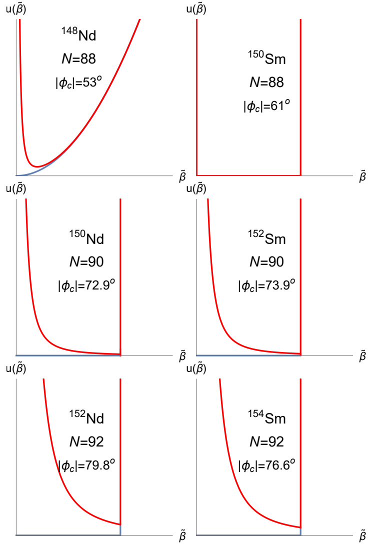
nuclei with ground states having $\phi(\beta_3) \neq 0$ minima

Numerical applications - ϕ ground state potential (medium mass nuclei)

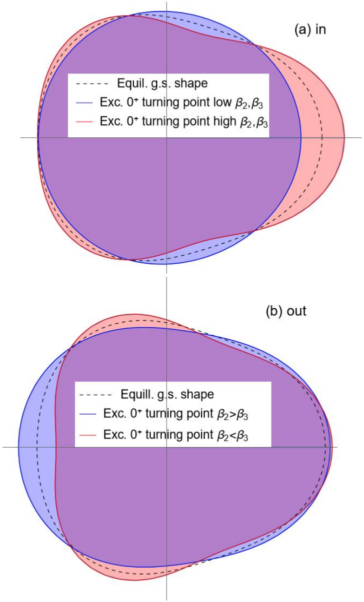


Numerical applications - parameters evolution (medium mass nuclei)

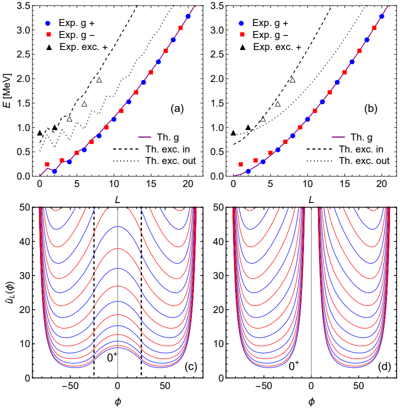




Excited states



^{222}Ra

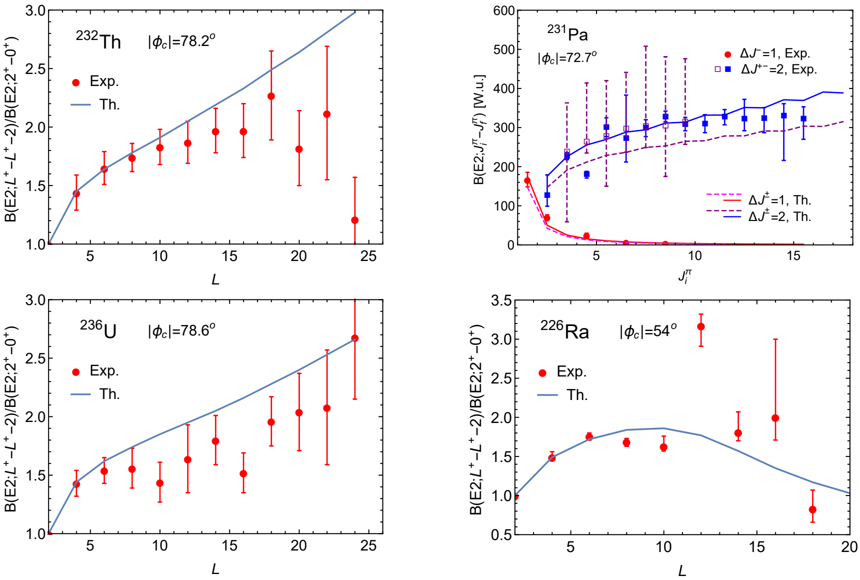


$\tilde{\beta}$ excitation in-phase $E1(\Delta\tilde{\beta}) > 0$

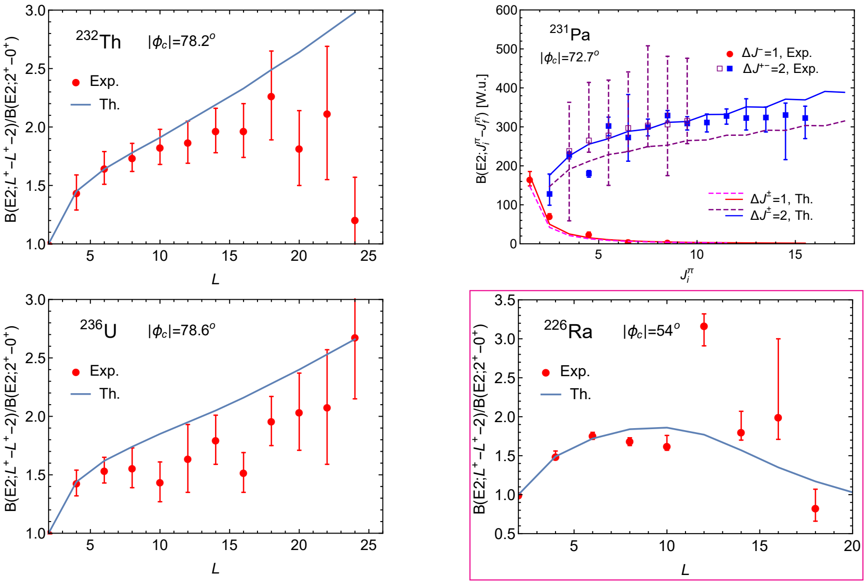
ϕ excitation out-of-phase $E1(\Delta\phi) \approx 0$

[RB, P. Buganu, A.I. Budaca, Eur. Phys. J. A **59** (2023) 242]

Numerical applications - $E2$ transitions



Numerical applications - $E2$ transitions



- $B(E1)$ transition probabilities $$\frac{B(E1; L^- \to (L+1)^+)}{B(E1; L^- \to (L-1)^+)} > 1$$
- $B(E3)$ transition probabilities

^{146}Ba

$$\frac{B(E3; 5^- \to 2^+)}{B(E3; 3^- \to 0^+)} = \begin{cases} 1.52^{+601}_{-85} \text{ Exp.} \\ 1.67 \text{ Th.} \end{cases}$$

$$\frac{B(E3; 7^- \to 4^+)}{B(E3; 3^- \to 0^+)} = \begin{cases} 1.70^{+765}_{-109} \text{ Exp.} \\ 2.31 \text{ Th.} \end{cases}$$

^{148}Nd

$$\frac{B(E3; 5^- \to 2^+)}{B(E3; 3^- \to 0^+)} = \begin{cases} 1.88^{+40}_{-31} \text{ Exp.} \\ 1.91 \text{ Th.} \end{cases}$$

$$\frac{B(E3; 7^- \to 4^+)}{B(E3; 3^- \to 0^+)} = \begin{cases} 2.12^{+51}_{-36} \text{ Exp.} \\ 3.03 \text{ Th.} \end{cases}$$

^{224}Ra

$$\frac{B(E3; 1^- \to 2^+)}{B(E3; 3^- \to 0^+)} = \begin{cases} 5(1) \text{ Exp.} \\ 3.21 \text{ Th.} \end{cases}$$

$$\frac{B(E3; 5^- \to 2^+)}{B(E3; 3^- \to 0^+)} = \begin{cases} 1.45(42) \text{ Exp.} \\ 1.68 \text{ Th.} \end{cases}$$

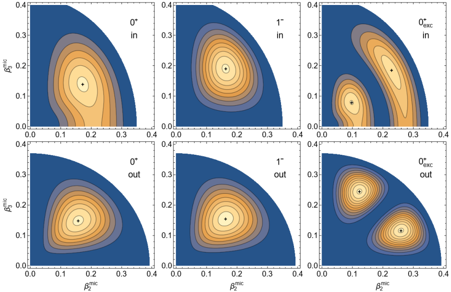
^{236}U

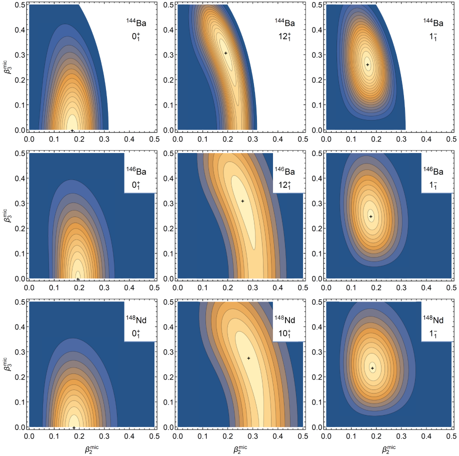
$$\frac{B(E3; 0^+ \to 3^-)}{B(E3; 1^- \to 4^+)} = \begin{cases} 2.58(51) \text{ Exp.} \\ 1.71 \text{ Th.} \end{cases}$$

Transformation to the (β_2, β_3) deformation space

$B(E2), B(E3)$ data $\longrightarrow$ B_2/B_3 and $s(\beta_\lambda^{mic} = s\beta_\lambda)$

^{222}Ra

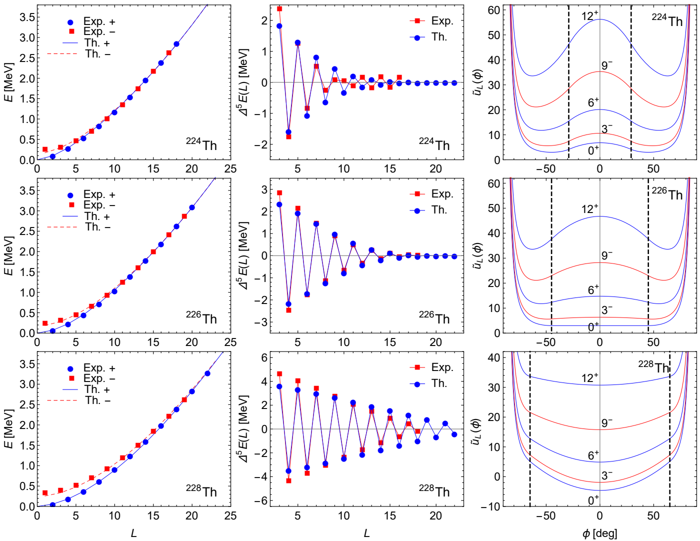




Conclusions

- An axially symmetric geometric model is developed for the quadrupole-octupole interaction, which is managed by an angular variable and whose spin dependence is naturally extracted from the geometry of the nuclear shape.
- Numerical applications on alternate parity and parity doublet bands of heavy and medium mass nuclei, demonstrated the model's ability to describe the evolution of the octupole deformation along an isotopic chain as well as a function of spin within a rotational band.
- Ra and Th nuclei with $A = 224 - 228$ were identified as being part of a critical region for the transition between stable and dynamical (vibration) octupole deformation which commences at different angular momentum.
- The quadrupole shape phase transition at $N = 90$ is accompanied by an increase in the vibrational character of the octupole deformation.
- The distinctive trend predicted for the $E2$ transition rates in critical nuclei needs a firmer experimental confirmation.

Numerical applications - energy levels, $\Delta L = 1$ staggering, and potential evolution of even-even heavy nuclei



$$\Delta^5 E(L) = 6\Delta E(L) - 4\Delta E(L-1) - 4\Delta E(L+1) + \Delta E(L+2) + \Delta E(L-2), \qquad \Delta E(L) = E(L) - E(L-1)$$

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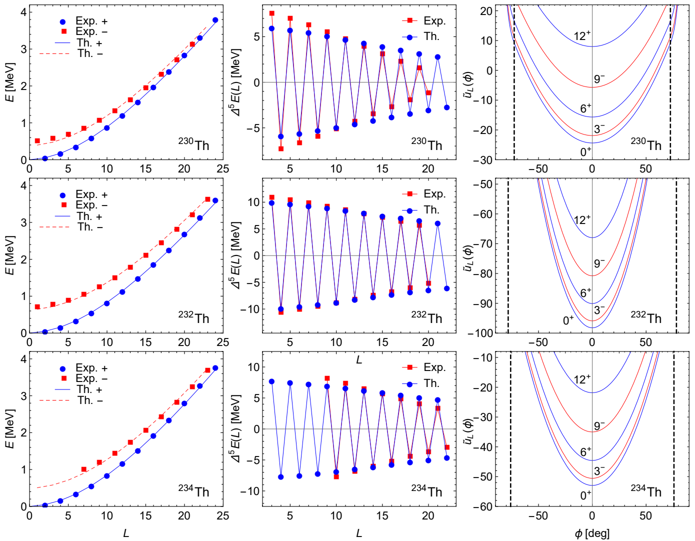

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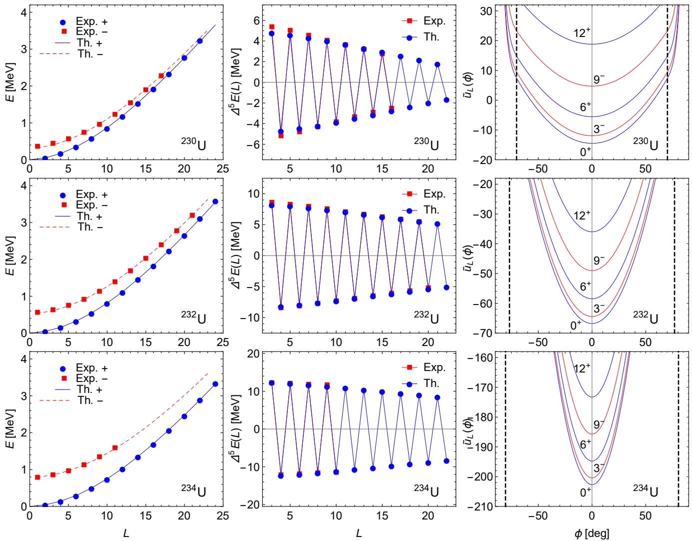

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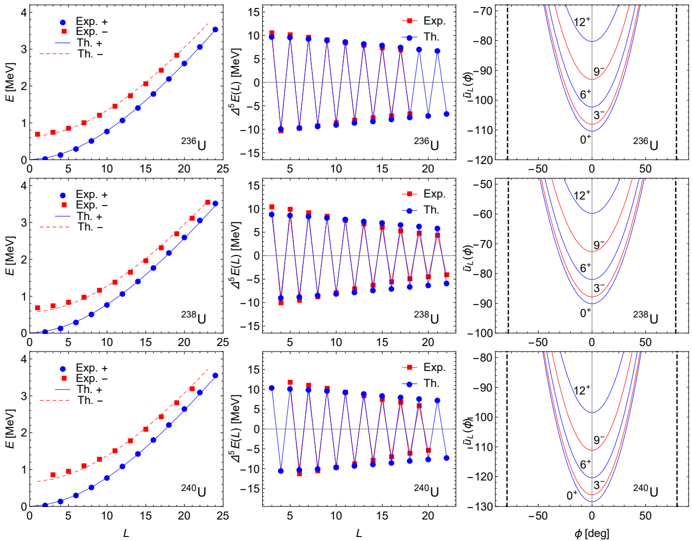

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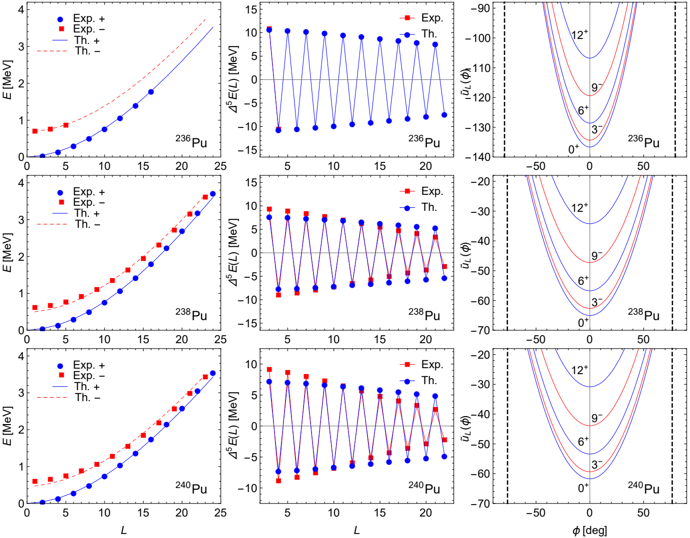
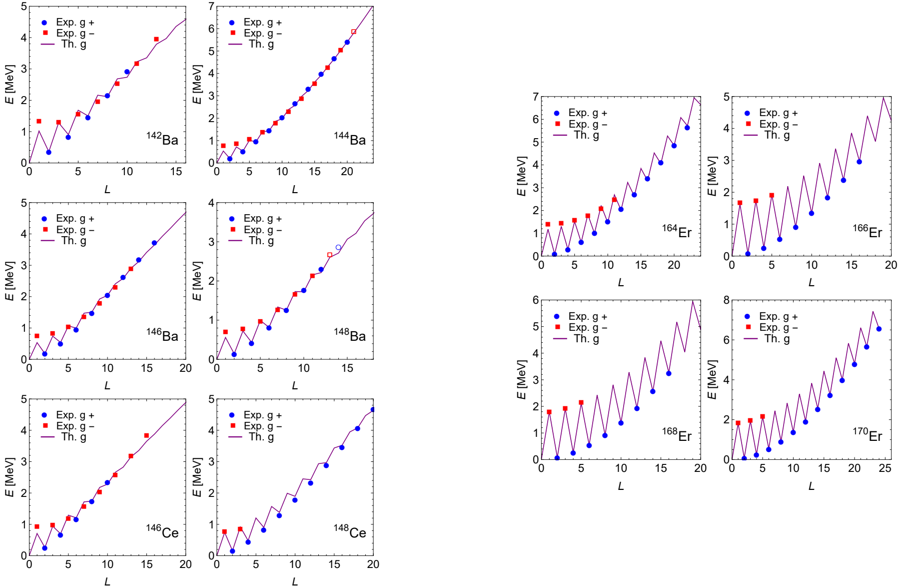
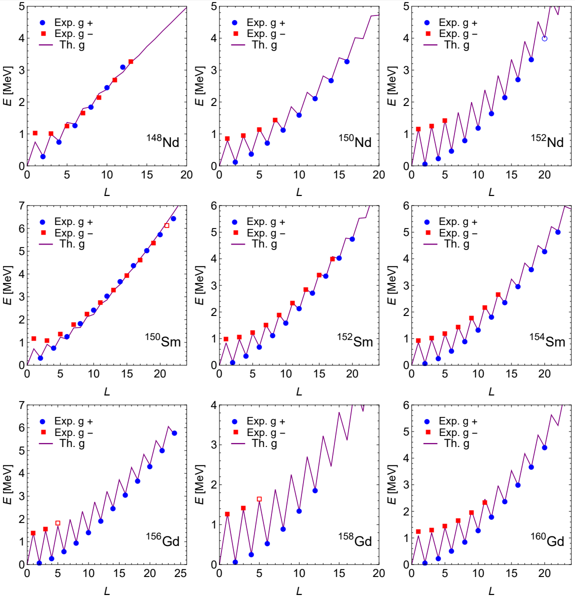


$$\Delta^5 E(L) = 6\Delta E(L) - 4\Delta E(L-1) - 4\Delta E(L+1) + \Delta E(L+2) + \Delta E(L-2), \qquad \Delta E(L) = E(L) - E(L-1)$$

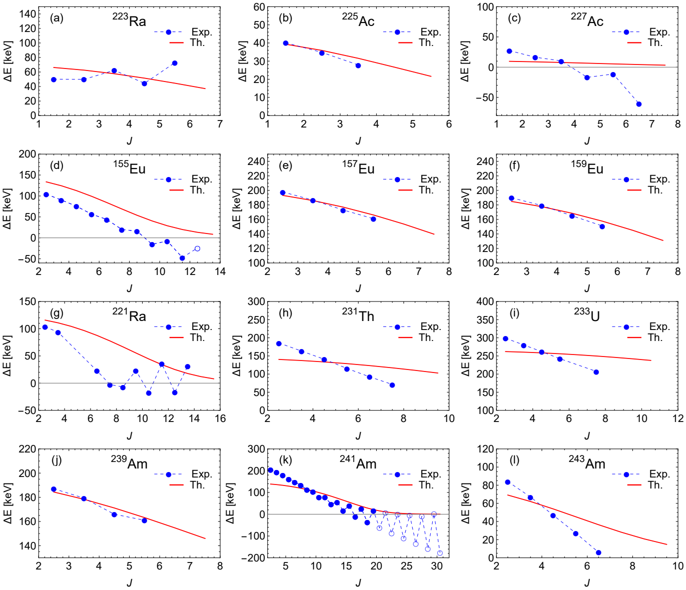
Numerical applications - energy levels of even-even medium nuclei



Numerical applications - energy levels of even-even medium nuclei



Numerical applications - Energy difference for $K \neq 1/2$ bands of odd mass nuclei



Transition probabilities

$$B(E\lambda; Lnp \to L'n'p') = t_\lambda \left(C^{L\,\lambda\,L'}_{0\,0\,0}\right)^2 \left(\tilde{B}^{\lambda}_{Lnp;L'n'p'} I^{\lambda}_{Lp;L'p'}\right)^2$$

$\lambda = 1, 2, 3$ - multipolarity.

t_λ - gathers physical units and normalization factors.

$C^{L\,\lambda\,L'}_{0\,0\,0}$ - Clebsch-Gordan coefficients.

$E1\,(\lambda = 1)$

$$\tilde{B}^{\lambda} = \int_0^{\infty} \tilde{\beta}^3 \psi^p_{Ln}(\tilde{\beta}) \psi^{p'}_{L'n'}(\tilde{\beta}) d\tilde{\beta}$$

$$I^{\lambda} = \int_{-\pi/2}^{\pi/2} \sin 2\phi \chi^p_L(\phi) \chi^{p'}_{L'}(\phi) d\phi$$

$E2\,(\lambda = 2)$

$$\int_0^{\infty} \tilde{\beta}^2 \psi^p_{Ln}(\tilde{\beta}) \psi^{p'}_{L'n'}(\tilde{\beta}) d\tilde{\beta}$$

$$\int_{-\pi/2}^{\pi/2} \cos \phi \chi^p_L(\phi) \chi^{p'}_{L'}(\phi) d\phi$$

$E3\,(\lambda = 3)$

$$\int_0^{\infty} \tilde{\beta}^2 \psi^p_{Ln}(\tilde{\beta}) \psi^{p'}_{L'n'}(\tilde{\beta}) d\tilde{\beta}$$

$$\int_{-\pi/2}^{\pi/2} \sin \phi \chi^p_L(\phi) \chi^{p'}_{L'}(\phi) d\phi$$