$\Delta I = 2$ Bifurcation as a Characteristic Feature of Scissors Rotational Bands

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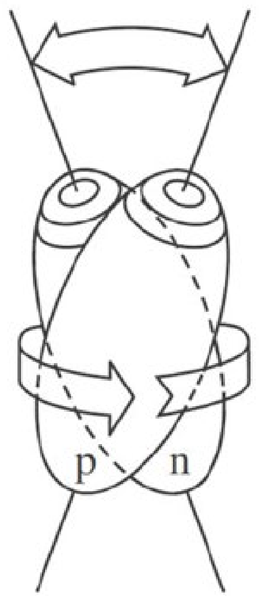
Nuclear Scissors Mode

- In 1978, Iudice and Palumbo predicted the third type of collective vibrational mode in low-lying excitations of atomic nuclei, by using the semi-classical two-rotor model.

- The residual interaction is employed to couple the separated neutron and proton rotors.
- It is assumed that the moving proton and neutron systems are represented by the two blades of a pair of scissors, respectively. Hence the name "Scissors Mode".

N. Lo Iudice and F. Palumbo, Phys. Rev. Lett., 1978, 41: 1532



Scissors mode vibration differs from β and γ vibration

- In β and γ vibration, all nucleons (protons, neutrons) move in phase.
- In scissors mode vibration, protons and neutrons move out of phase.

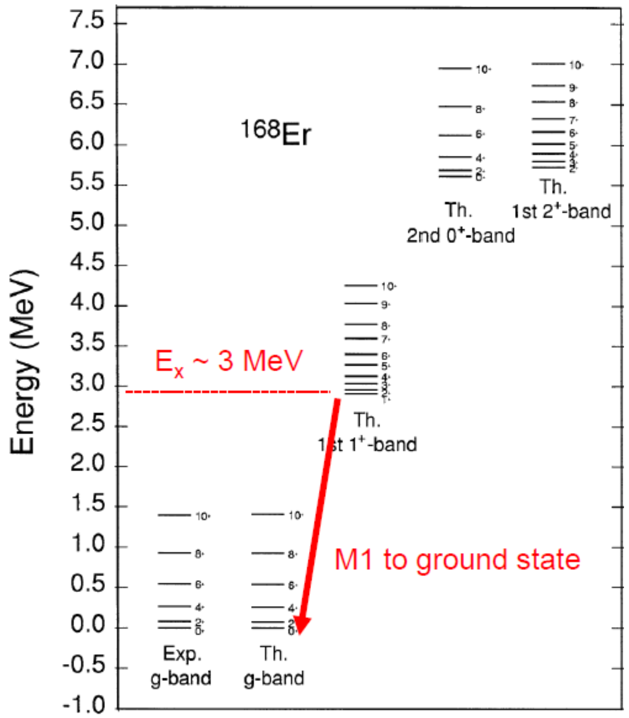
Properties of Scissors Mode

Research on Scissors Mode Vibration

- Theoretical prediction in 1978
- First experimental verification in 1984

Research on the scissors mode is concentrated on the 1^+ state and M1 transition to the nuclear ground state.

- In the study of the scissors mode in rare-earth region nuclei, including near-spherical nuclei and strongly deformed nuclei, there have been two remarkable discoveries.



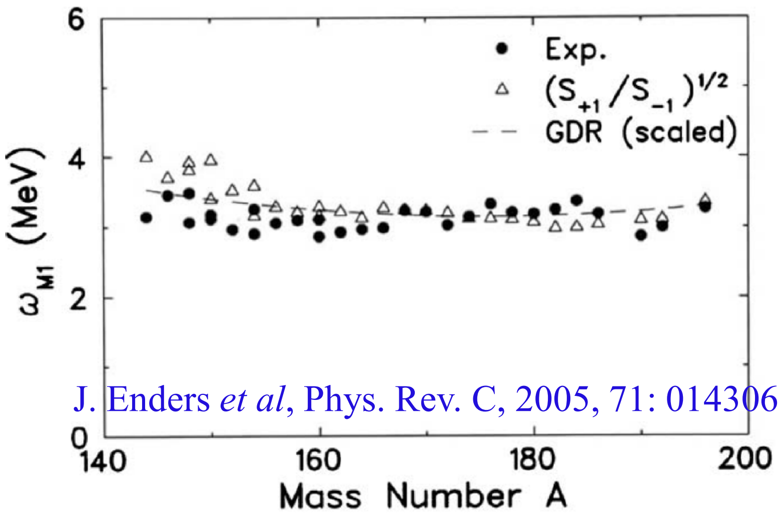
Y. Sun *et al*, Phys. Rev. Lett.,1998, 80: 672; NPA, 2002, 703: 130-151

N. L. Iudice *et al*, Phys. Rev. Lett., 1978, 41: 1532
D. Bohle *et al*, Phys. Lett., 1984, B137: 27
K. Heyde *et al*, Rev. Mod. Phys., 2010, 82: 2365

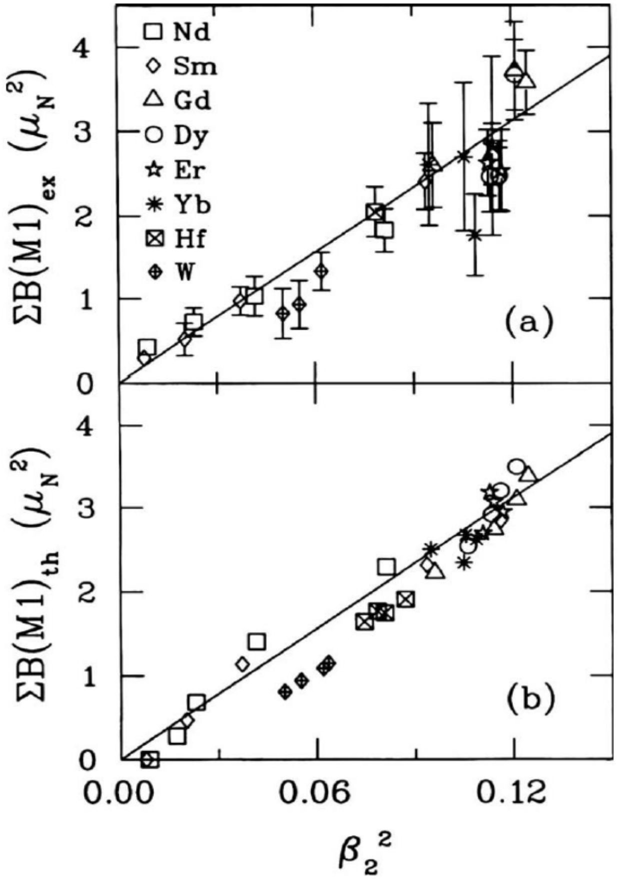
Properties of Scissors Mode

- The excitation energies of the 1^+ states are all around 3-3.5 MeV.

- The sum of the M1 transition strengths B(M1) is linearly proportional to the square of the nuclear deformation parameter.



J. Enders *et al*, Phys. Rev. C, 2005, 71: 014306

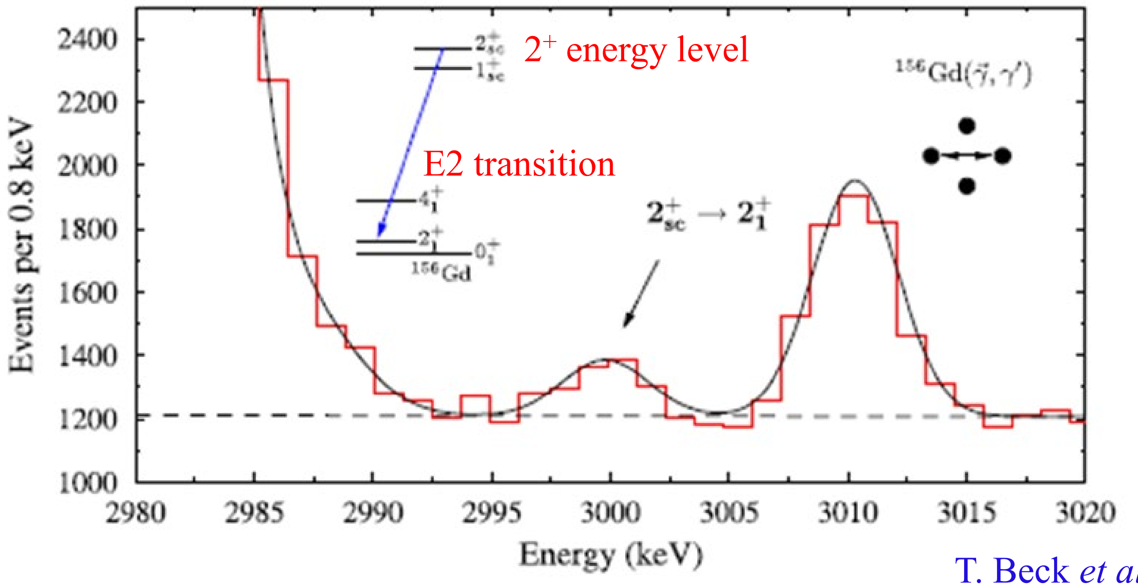


P. Von Neumann-Cosel *et al*, Phys. Rev. Lett.,1995, 75: 4178

- What about the 2^+, 3^+, ... excitation energies?

Discovery of 2^+ State in Scissors Band

- In 2017, the 2^+ energy level above the scissors 1^+ bandhead was measured for the first time.



T. Beck *et al*, Phys. Rev. Lett., 2017, 118: 212502

- Based on the ground band energy: $E(0^+) = 0$, $E(2^+) = 89$ KeV, $E(4^+) = 288$ KeV

- Rotor Formula: $E(I) = AI(I+1)$ ⟶ A~14.8

- Assuming the ground and the scissors band have the same A, then $E_{sc}(2^+) = E_{sc}(1^+) + 59$ KeV

- Experimental measured value： $E_{sc}(2^+) = E_{sc}(1^+) + 19$ KeV

- Study of the scissors mode rotational band in ^{156}Gd using the projected shell model

The Projected Shell Model

- The Hamiltonian employed in the conventional projected shell model typically comprises separable forces.

$$\hat{H} = \hat{H}^0 - \frac{1}{2}\chi \sum_\mu \hat{Q}^{\dagger\mu}\hat{Q}^\mu - G_M \hat{P}^\dagger \hat{P} - G_Q \sum_\mu \hat{P}^{\dagger\mu}\hat{P}^\mu,$$

Spherical Monomer Item — Quadrupole-quadrupole force term — Monopole Pairing Force Term — Quadrupole pairing force term

- The distinctive feature of the extended projected shell model (which we use in this calculate work): performing angular momentum projection separately for protons and neutrons, then coupling them to form the total angular momentum.

- The Hamiltonian can be written in isospin form as three parts: $\hat{H} = \hat{H}_\nu + \hat{H}_\pi + \hat{H}_{\nu\pi}$

$$\hat{H}_\tau = \hat{H}^0_\tau - \frac{1}{2}\chi_{\tau\tau} \sum_\mu \hat{Q}^{\dagger\mu}_\tau \hat{Q}^\mu_\tau - G^\tau_M \hat{P}^\dagger_\tau \hat{P}_\tau - G^\tau_Q \sum_\mu \hat{P}^{\dagger\mu}_\tau \hat{P}^\mu_\tau,$$

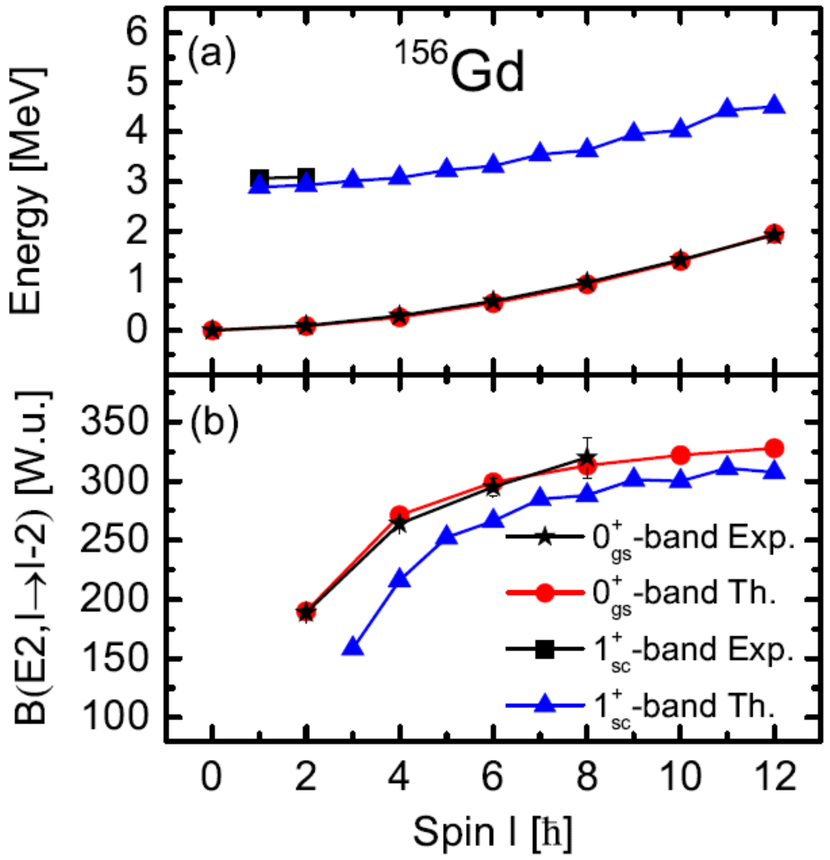
In the equation: $\tau = \nu, \pi$

- $\hat{H}_{\nu\pi}$ is the neutron-proton quadrupole-quadrupole residual interaction, which has the form of a quadrupole-quadrupole interaction:

$$\hat{H}_{\nu\pi} = -\chi_{\nu\pi} \sum_\mu \hat{Q}^{\dagger\mu}_\nu \hat{Q}^\mu_\pi.$$

Y. Sun *et al*, Phys. Rev. Lett.,1998, 80: 672

Rotational Feature of Scissors Band



In Fig.(a), the 1^+_{sc} band has a comparable moment of inertia as the 0^+_{gs} band.

In Fig.(b), the scissors rotational band has a similar but slightly weaker E2 collectivity

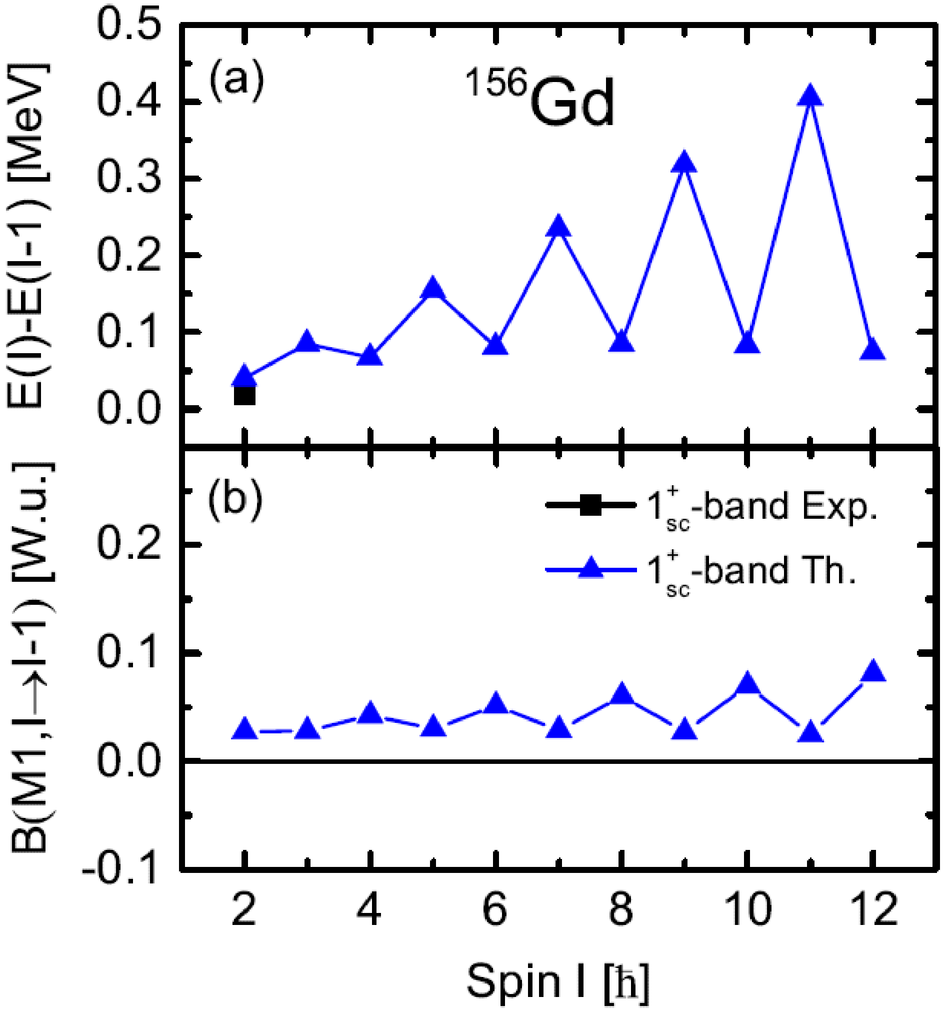
- The rotational bands of the scissors mode and the B(E2) values do not form smooth curves; instead, they exhibit a zigzag pattern between odd and even spins.

Cui-Juan Lv, Fang-Qi Chen, Yang Sun and Mike Guidry, Phys. Rev. Lett.,2022, 129: 042502

Energy Staggering in Scissors Band

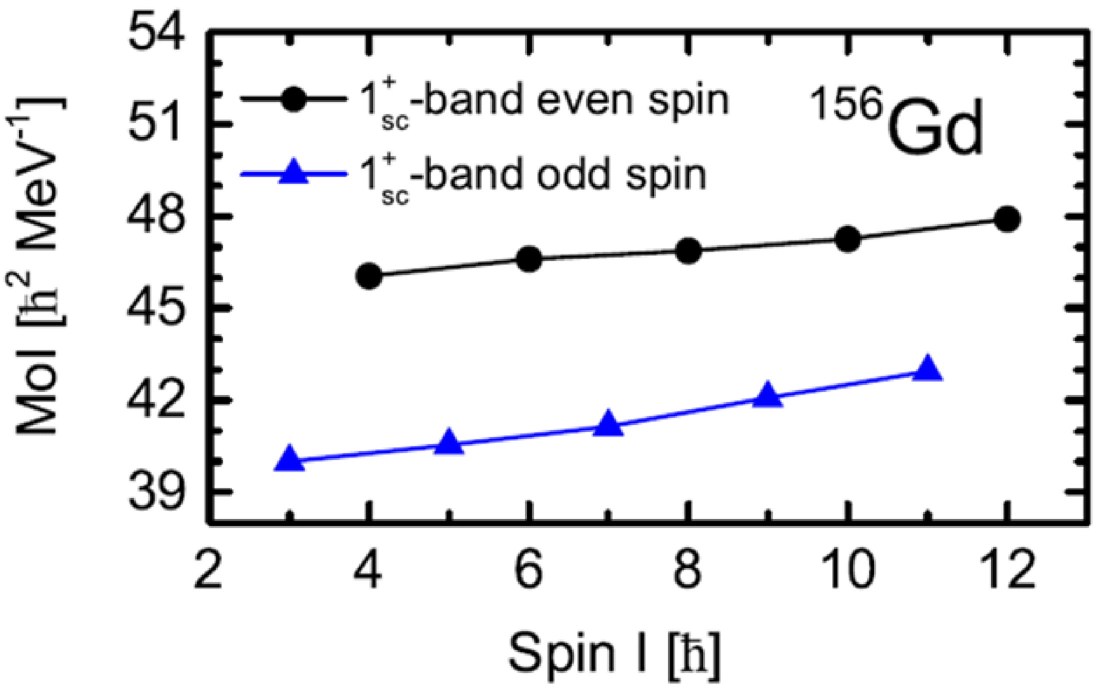
- In Fig.(a), reproduced the only-known first data point ΔE(2), all ΔE(I)s for I = even are lower in energy.

- In Fig.(b), the B(M1;I → I − 1) values are very small in magnitude, but also exhibit clear zigzags.



Cui-Juan Lv, Fang-Qi Chen, Yang Sun and Mike Guidry, Phys. Rev. Lett.,2022, 129: 042502

Moments of Inertia in Odd and Even Spins



- Commonly used expressions for the moment of inertia（MoI）$\mathcal{J}(I)$：

$$\mathcal{J}(I) = \frac{2I - 1}{E(I) - E(I-2)},$$

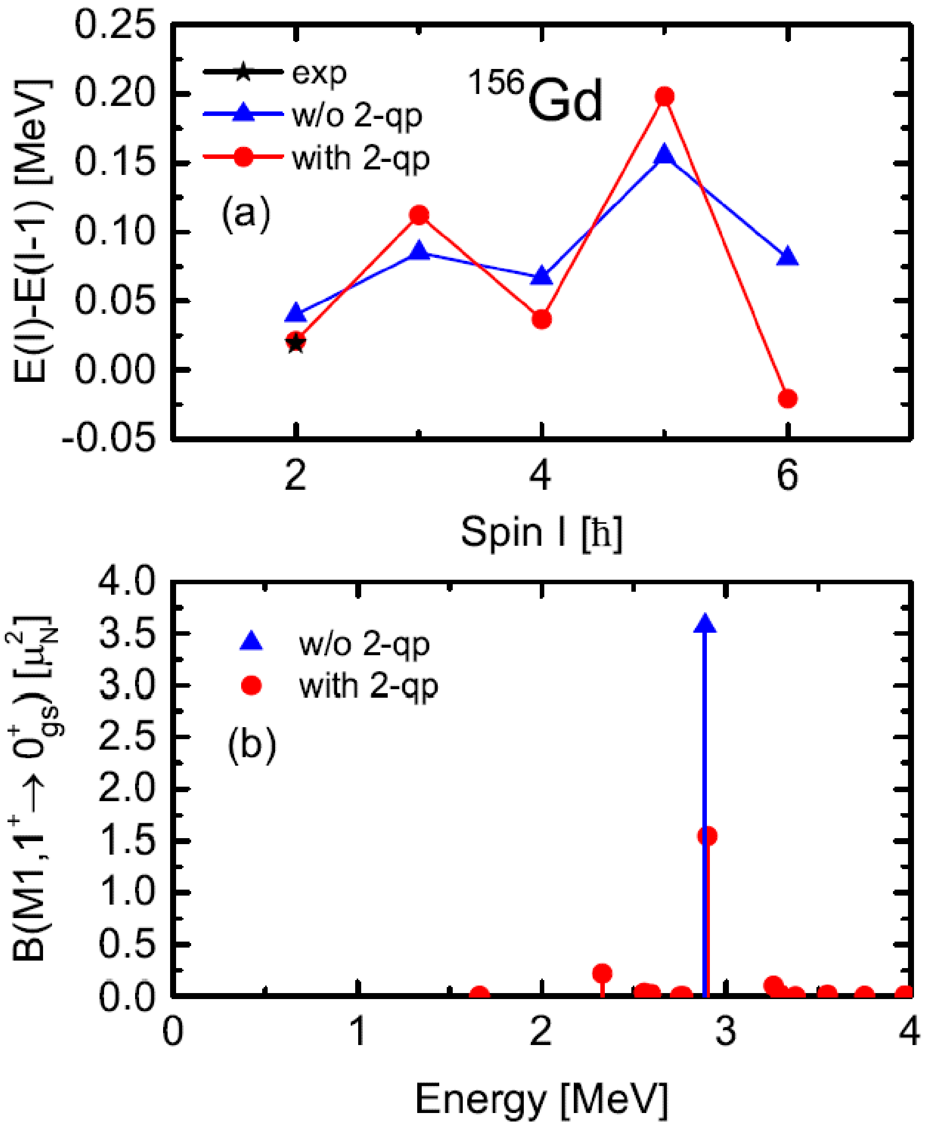
- MoI-even is about 10% larger than MoI-odd, suggesting that the MoI alternates in magnitude between odd- and even- spin states.

Cui-Juan Lv, Fang-Qi Chen, Yang Sun and Mike Guidry, Phys. Rev. Lett.,2022, 129: 042502

Quasi-Particle States Enhance B(M1) Fragmentations

$$|0_\nu\rangle \rightarrow \{|0_\nu\rangle,\ a^\dagger_{\nu,i}a^\dagger_{\nu,j}|0_\nu\rangle\},$$
$$|0_\pi\rangle \rightarrow \{|0_\pi\rangle,\ a^\dagger_{\pi,k}a^\dagger_{\pi,l}|0_\pi\rangle\},$$

- **Added calculations including two-quasi-particle states**
  - In Fig.(a), the coupling of 2-qp states amplifies the staggering and pushes the 2^+_{SC} state down to the exact experimental value.
  - In Fig.(b), mixture of the 2-qp states significantly reduces B(M1, $1^+_{SC} \rightarrow 0^+_{gs}$) from ~$3.5\mu_N^2$ to ~$1.5\mu_N^2$, and causing a fragmented distribution around the scissors M1.

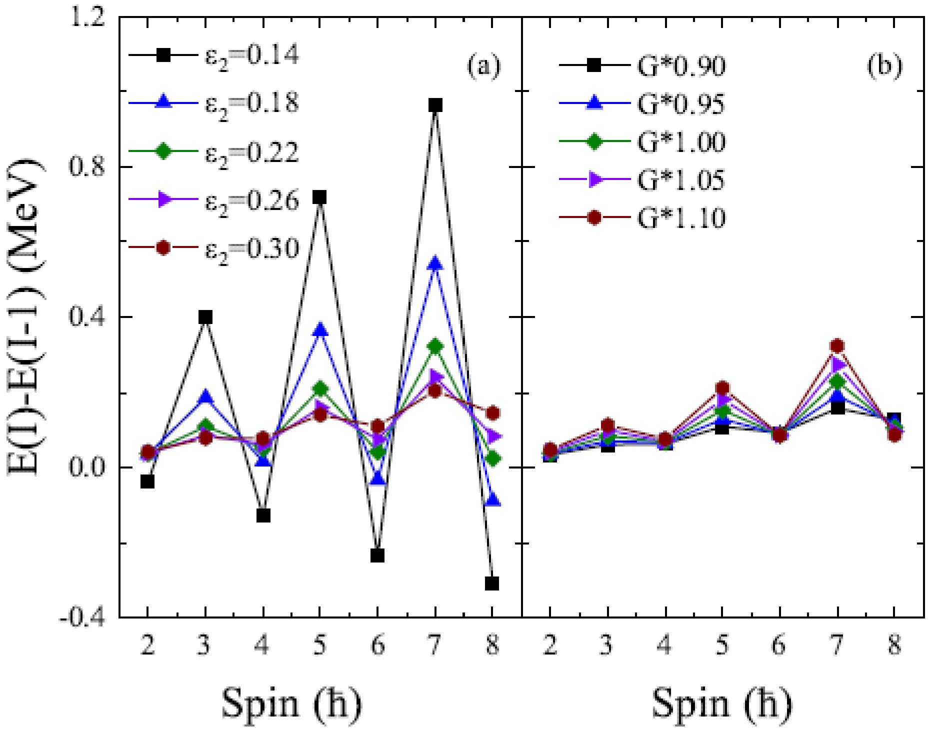


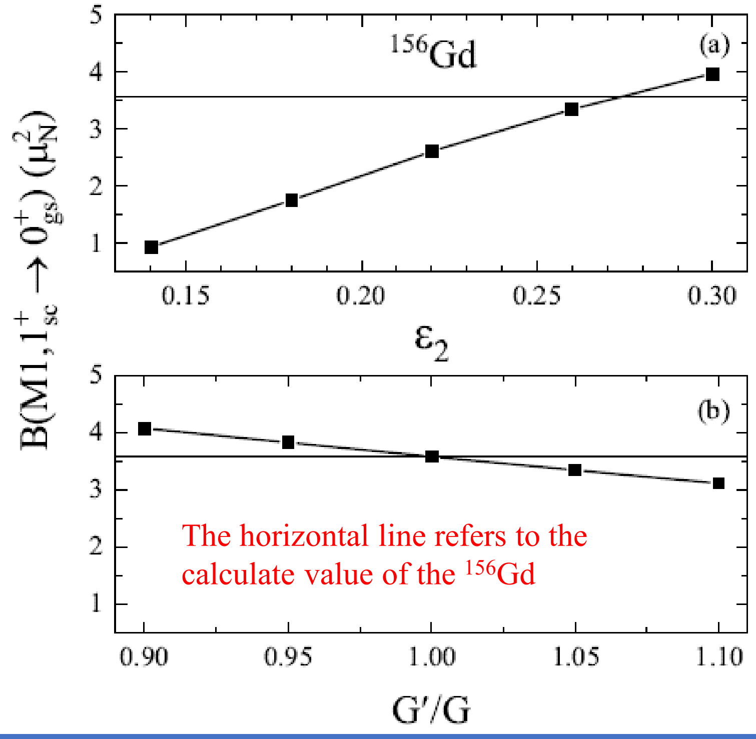
Cui-Juan Lv, Fang-Qi Chen, Yang Sun and Mike Guidry, Phys. Rev. Lett.,2022, 129: 042502

Parameter Influence in Quadrupole and Pairing

- Sensitivity Study of Parameters
  - ΔE(I) and B(M1, $1^+_{sc} \rightarrow 0^+_{gs}$) values of the scissors band, calculated with variation in input deformation parameter ε_2 and monopole-pairing strength G.
  - Between these two factors, the influence of the deformation parameter is substantially greater than that of the monopole-pairing force.

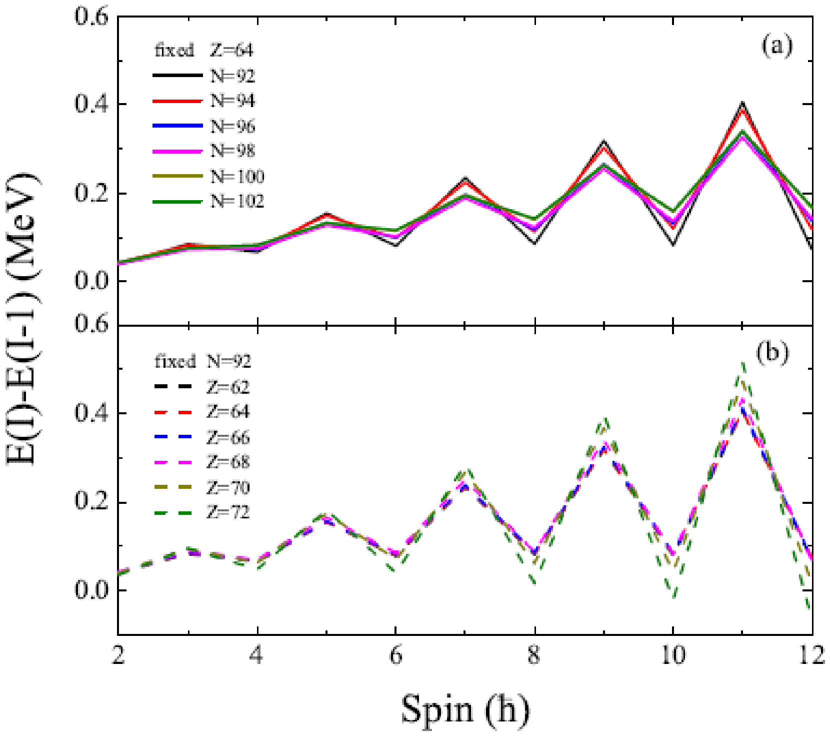
Cui-Juan Lv, Chinese Phys. C, https://doi.org/10.1088/1674-1137/add9f9

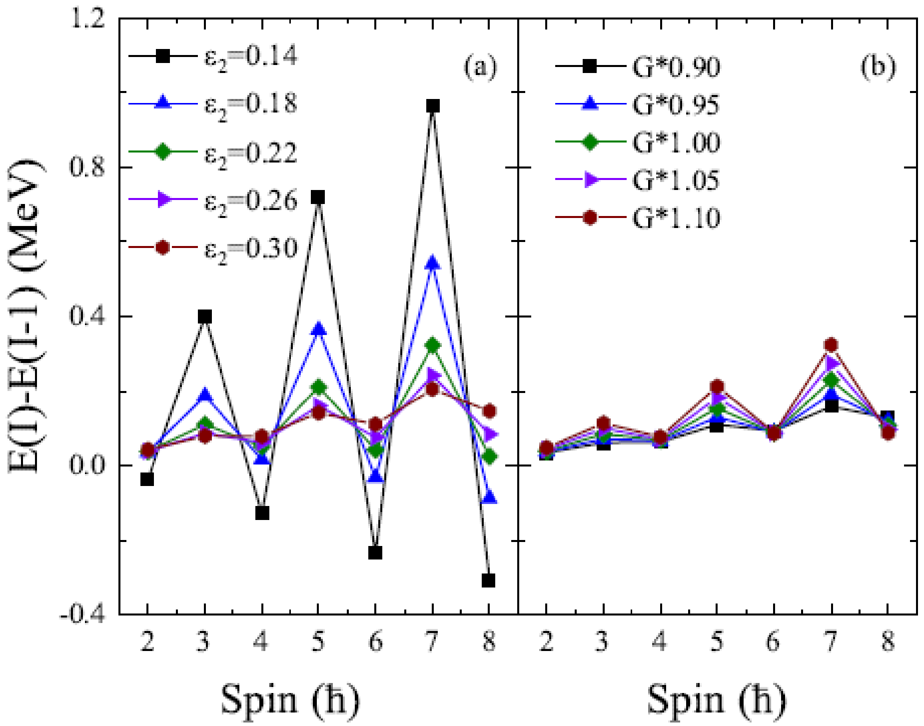




Robustness in Staggering

- The robust occurrence of $\Delta I = 2$ bifurcation in scissors rotation bands
  - employ the same parameters, with only the neutron or proton number altered
  - altering input parameters (quadrupole deformation and pairing force strength)
  - the oscillatory characteristics persist.





Cui-Juan Lv, Chinese Phys. C, https://doi.org/10.1088/1674-1137/add9f9

Summary

- A characteristic feature of the scissors mode that has not been noticed before: A scissors-mode rotational band staggers between odd and even spin states in even–even nuclei, resulting in a $\Delta I = 2$ bifurcation within the band.
- In scissors states such staggering change their moments of inertia back and forth between odd and even spins, in response to angular momentum conservation.
- The 2-qp configurations enhanced the staggering, and causing fragmentation of M1 distribution.
- Between the input parameters, the effect of the deformation parameter on amplitude is substantially greater than that of the monopole-pairing force.
- Demonstrated the robust occurrence of $\Delta I = 2$ bifurcation in scissors rotation bands.

Thank you !