The β-decay properties of $N = Z$ nuclei: Role of neutron-proton pairing and the shell model interpretation

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Table of contents

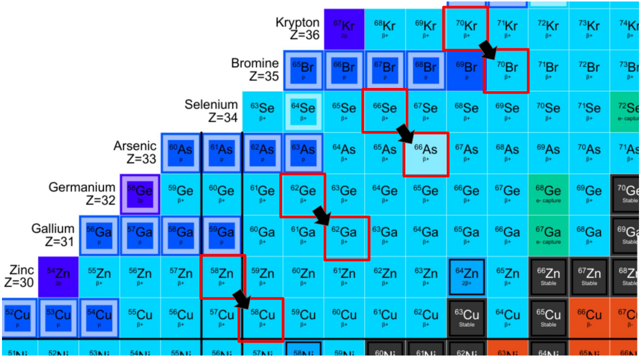
Motivation

- Recent measurement of the beta-decay of the even-even nucleus ^{70}Kr into ^{70}Br. [Ref.: A. Vitéz-Sveiczer *et al.*, Phys. Lett. B **830**, 137123 (2022).]
- An increase in the β-decay strength to the 1_1^+ state in comparison to the β-decay of $Z = N + 2$ system, ^{62}Ge is observed that may indicate increased np correlations in the $T = 0$ channel.

Transitions of interest:

- ^{58}Zn $(0^+_{\text{g.s.}}) \rightarrow {}^{58}$Cu
- ^{62}Ge $(0^+_{\text{g.s.}}) \rightarrow {}^{62}$Ga
- ^{66}Se $(0^+_{\text{g.s.}}) \rightarrow {}^{66}$As
- ^{70}Kr $(0^+_{\text{g.s.}}) \rightarrow {}^{70}$Br

$\rightarrow$ Relavent to the rp-process



[Ref.: https://people.physics.anu.edu.au/ ecs103/chart/]

Gamow-Teller transition strength

- For Gamow-Teller transition: ΔJ=$\pm$1, parity change=No
- The reduced transition probability is expressed as

$$B(\mathrm{GT}) = \frac{g_{\mathrm{A}}^2}{2J_i + 1} |\mathcal{M}_{\mathrm{GT}}|^2$$

g_{A} $\rightarrow$ axial vector coupling constant

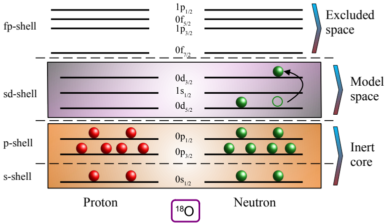
$\mathcal{M}_{\mathrm{GT}}$ $\rightarrow$ Gamow-Teller nuclear matrix element (NME)

- The NMEs for a transition between an initial (i) and final (f) states is given by

$$\mathcal{M}_{\mathrm{GT}} = \underbrace{\sum_{ab} \mathcal{M}_{\mathrm{GT}}(ab)}_{\text{Single-particle matrix elements}} \times \underbrace{(\Psi_f || [c_p^\dagger \tilde{c}_n]_1 || \Psi_i)}_{\text{One-body transition density}}$$

The nuclear shell model

- The nuclear shell-model is the primary tool for understanding the structure of atomic nucleus.
- In this model, interaction takes place among valence nucleons on top of an inert core via residual nuclear forces.

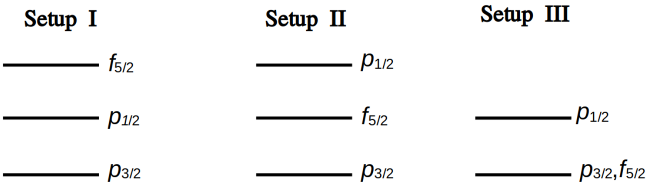


Hamiltonian for a system of A nucleons:

$$H_A = T + V = \underbrace{\sum_{\alpha} \epsilon_\alpha c_\alpha^\dagger c_\alpha}_{\text{single-particle energy}} + \underbrace{\frac{1}{4} \sum_{\alpha\beta\gamma\delta} v_{\alpha\beta\gamma\delta} c_\alpha^\dagger c_\beta^\dagger c_\delta c_\gamma}_{\text{Nucleon-nucleon interaction}}$$

Results

- A schematic calculation in the model space $p_{3/2}p_{1/2}f_{5/2}$ above $N = Z = 28$ shell closure considering pairing matrix elements.
- Only $J = 0$, $T = 1$ and $J = 1$, $T = 0$ two-body matrix elements are taken into account.
- We consider three extreme setups for the single-particle level scheme:



→ Setup III favors pseudo-SU(4) symmetry.

Surface delta interaction

- The matrix elements of the surface delta effective interaction are evaluated as

$$\left\langle j_{\mathrm{a}} j_{\mathrm{b}} \left| V^{\mathrm{SDI}}(1,2) \right| j_{\mathrm{c}} j_{\mathrm{d}} \right\rangle_{JT} = \frac{1}{\sqrt{(1+\delta_{\mathrm{ab}})(1+\delta_{\mathrm{cd}})}} \frac{1}{2} A_T (-1)^{n_{\mathrm{a}}+n_{\mathrm{b}}+n_{\mathrm{c}}+n_{\mathrm{d}}}$$
$$\times \sqrt{(2j_{\mathrm{a}}+1)(2j_{\mathrm{b}}+1)(2j_{\mathrm{c}}+1)(2j_{\mathrm{d}}+1)}$$
$$\times (-1)^{j_{\mathrm{b}}+j_{\mathrm{d}}+l_{\mathrm{d}}+l_{\mathrm{b}}} [1-(-1)^{lc+ld+J+T}] \begin{pmatrix} j_{\mathrm{a}} & j_{\mathrm{b}} & J \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} j_{\mathrm{c}} & j_{\mathrm{d}} & J \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix} -$$
$$[1+(-1)^T] \begin{pmatrix} j_{\mathrm{a}} & j_{\mathrm{b}} & J \\ \frac{1}{2} & \frac{1}{2} & -1 \end{pmatrix} \begin{pmatrix} j_{\mathrm{c}} & j_{\mathrm{d}} & J \\ \frac{1}{2} & \frac{1}{2} & -1 \end{pmatrix}.$$

  - We assume the same coupling strength A_T for both pairing channels for simplicity.

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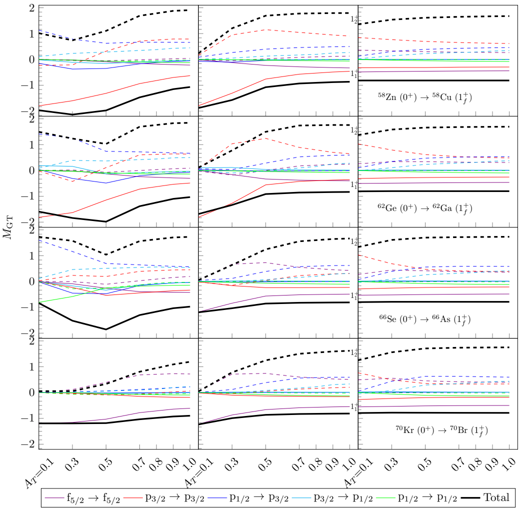

Figure: The solid lines indicate the transition from g.s. to 1_1^+ state, while the dashed lines are for 1_2^+ state.

- Contribution from different single-particle transitions as well as the total M_{GT} for decay to the lowest two 1^+ states.
- The effect of isoscalar and isovector strength on the individual GT matrix elements.
- It is not generally true that the pairing correlation can enhance the GT decay strength.
- Major contributions of different single-particle transitions have the same signs and are constructive.

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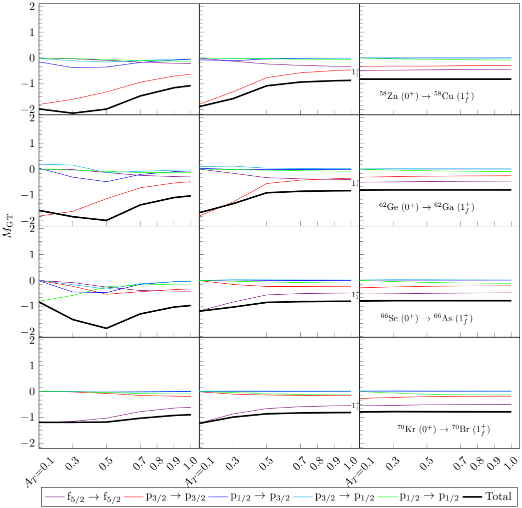

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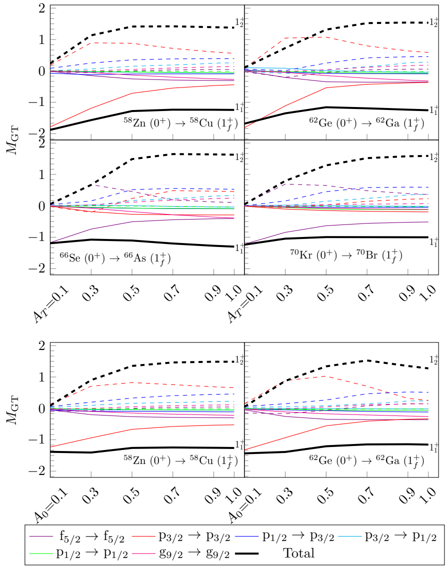

- We have extended our model space by including the $g_{9/2}$ orbital and performed the schematic calculations.
- For the 1_1^+ state, the B(GT) value first decreases with increasing np pairing strength. After a certain strength, it starts increasing.

Figure: The calculated reduced GT transition matrix elements as a function of A_T (the same coupling strength for both pairing channels) in the top panel and as a function of A_0 for the fixed $A_1 = 0.6$ in the bottom panel in the $f_{5/2}pg_{9/2}$ model space.

The realistic shell model calculations

- The focus of the calculation is to determine the sign of the different components of the wave functions and the separate contributions from different one-body transitions.

- We have performed shell-model calculations for all the above transitions with realistic shell-model Hamiltonians.

- Shell model interactions:
  - JUN45 interaction → Core: ^{56}Ni
    Valence space: $f_{5/2}, p_{1/2}, p_{3/2}, g_{9/2}$
  - GXPF1J interaction → Core: ^{40}Ca
    Valence space: $f_{5/2}, f_{7/2}, p_{1/2}, p_{3/2}$

β^+ decay of ^{58}Zn ($0^+_{g.s.}$) $\rightarrow$ ^{58}Cu

A. JUN45 interaction

	SPME	1^+_1 SPME*OBTD	1^+_2 SPME*OBTD
$f_{5/2} \rightarrow f_{5/2}$	-1.195	-0.0223	0.1120
$p_{3/2} \rightarrow p_{3/2}$	1.491	-1.1343	0.4487
$p_{1/2} \rightarrow p_{3/2}$	-1.333	-0.5930	-0.0692
$p_{3/2} \rightarrow p_{1/2}$	1.333	-0.2478	-0.0289
$p_{1/2} \rightarrow p_{1/2}$	-0.471	-0.0057	0.0371
$g_{9/2} \rightarrow g_{9/2}$	2.018	-0.0360	0.0329
$\sum M_{GT}$		-2.0390	0.5325
B_{GT}		4.1577	0.2826
B_{GT}(Expt)		$\leq$0.31	0.54(26)

B. GXPF1J interaction

		Truncated		Full	
	SPME	1^+_1	1^+_2	1^+_1	1^+_2
$f_{7/2} \rightarrow f_{7/2}$	1.8516	0	0	-0.0493	0.0956
$f_{5/2} \rightarrow f_{7/2}$	-2.1381	0	0	0.4919	-0.1726
$f_{7/2} \rightarrow f_{5/2}$	2.1381	0	0	0.4146	-0.3950
$f_{5/2} \rightarrow f_{5/2}$	-1.1952	0.0347	0.0760	0.0246	0.2281
$p_{3/2} \rightarrow p_{3/2}$	1.4907	-0.7936	1.0528	-0.6322	0.5526
$p_{1/2} \rightarrow p_{3/2}$	-1.3333	-0.5838	0.2799	-0.2462	0.0523
$p_{3/2} \rightarrow p_{1/2}$	1.3333	-0.1922	0.0922	-0.5557	0.1139
$p_{1/2} \rightarrow p_{1/2}$	-0.4714	0.0171	0.0290	0.0256	0.0572
$\sum M_{GT}$		-1.5178	1.5299	-0.5266	0.5322
B_{GT}		2.3037	2.3407	0.2773	0.2832
B_{GT}(Expt)		$\leq$0.31	0.54(26)	$\leq$0.31	0.54(26)

β^+ decay of ^{62}Ge $(0^+) \rightarrow {}^{62}$Ga

- The contribution from $g_{9/2}$ is found to be minimal.
- With the GXPF1J interaction, the contributions from $f_{5/2} \rightarrow f_{7/2}$ and $f_{7/2} \rightarrow f_{5/2}$ are in opposite phases which results in a reduction of the total B_{GT} value to 0.1936.
- The interaction with a reduced gap between $p_{3/2}$ and $f_{5/2}$ orbitals is labelled as 'GXPF1J$^-$', whereas that with an increased gap is referred to as 'GXPF1J$^+$'.

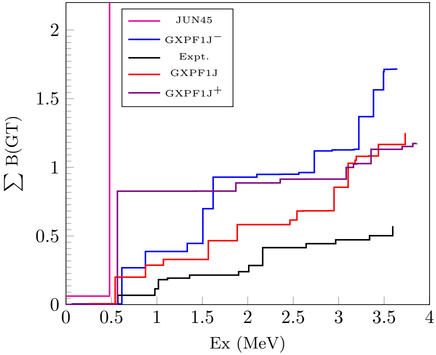

Figure: Comparison of cummulative GT-strength between the experimental data and shell model results.

[Ref. [23]: E. Grodner *et al.*, Phys. Rev. Lett. **113**, 092501 (2014).]
[Ref. [24]: S. E. A. Orrigo *et al.*, Phys. Rev. C **103**, 014324 (2021).]

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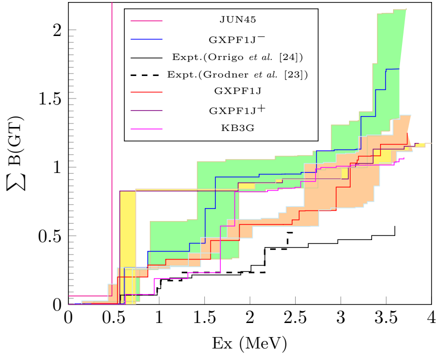

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β^+ decay of ^{70}Kr $(0^+) \to {}^{70}$Br

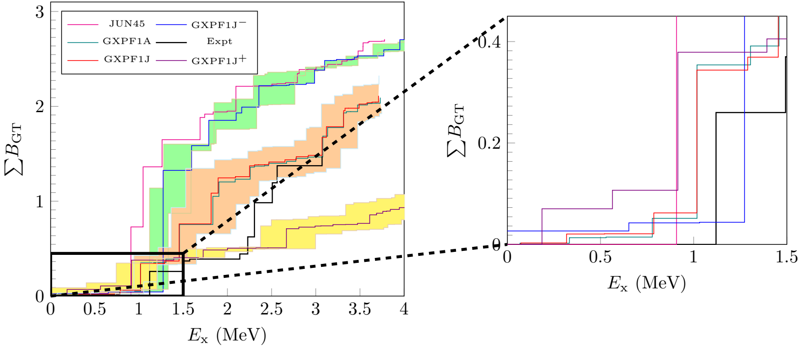

Figure: Distribution of GT strengths in the ^{70}Kr $\to$ ^{70}Br decay corresponding to different shell-model interactions in comparison with the experimental data.

[Ref.: A. Vitéz-Sveiczer *et al.*, Phys. Lett. B **830**, 137123 (2022).]

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- The calculations with JUN45 interaction tend to overestimate the B_{GT} values, due to dominant contribution from the transition $g_{9/2} \to g_{9/2}$.

- The experimental B_{GT} data lies between the results of GXPF1J$^+$ and GXPF1J$^-$.

- Increasing trend with mass number for the first 1^+ in calculations with JUN45. This is due to the enhanced contribution from the $g_{9/2}$ orbital.

	Expt	JUN45	GXPF1J
^{62}Ge$(0^+) \to {}^{62}$Ga(1_1^+)	0.068(6)	0.0621	0.0065
^{70}Kr$(0^+) \to {}^{70}$Br(1_1^+)	0.26(3)	0.7288	0.0028

- We observe that the calculated B_{GT} for 1_1^+ decreases with increasing mass number using GXPF1J interaction.

Conclusion

- The B_{GT} between yrast 0^+ and 1^+ states doesn't necessarily increase with increasing np pairing strength with the same coupling strength.

- By including the $g_{9/2}$ orbital, it can be enhanced due to increased contribution of $g_{9/2}$. The same conclusion is obtained for the different coupling strengths for both pairing channels.

- We have identified the role played by different orbitals on the GT-strength.

- The total accumulated B_{GT} strength increases for GXPF1J interaction.

- Calculation using the interaction with extended model space, containing both $f_{7/2}$ and $g_{9/2}$ orbitals, will be required to provide a better picture.

Thank you.