Toroidal dipole mode in nuclei: recent results

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Content

- Exotic E1 excitations: pygmy, toroidal, compression

J. Kvasil et al,
PRC 84, 034303 (2011)

- Toroidal dipole resonance (TDR)

V.O. Nesterenko et al,
Phys. Atom. Nucl. 79, 842 (2016)
A. Repko et al PRC, 87, 024305 (2013),
EPJA, 55, 242 (2019)

- Individual toroidal states

V.O. Nesterenko et al, PRL 120, 182501 (2018)
Y. Kanada-En'yo et al, PRC 95, 064319 (2017)

- Search of vortical states in experiment:

V.O. Nesterenko et al, PRC 100, 064302 (2019).

(α,α')

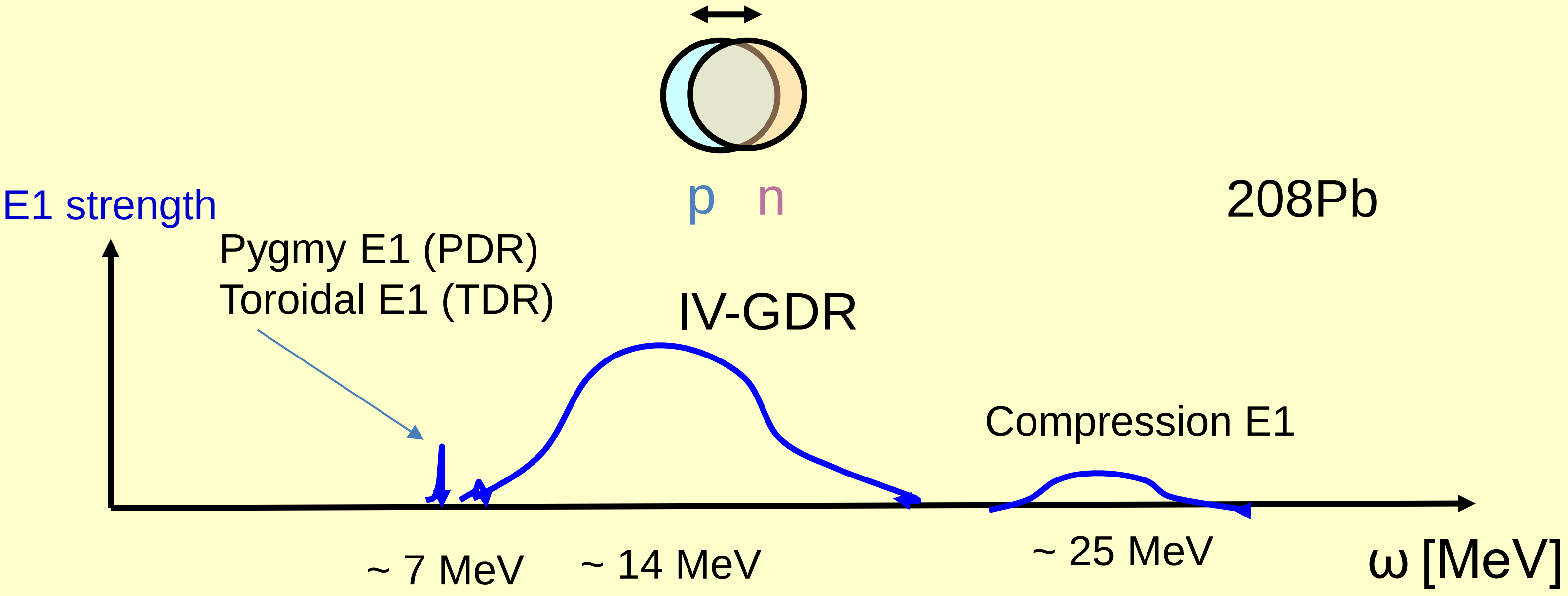
(e,e')

$(e,e'\gamma)$

P. Von Neumann-Cosel, V.O. Nesterenko,
P.I. Vishnevskiy et al,
arXiv 2310.04736 [nucl-exp]

- Comparison of PDR and TDR: example of 40Ca vs 48Ca

Diversity of E1 excitations in nuclei



p n

208Pb

E1 strength

Pygmy E1 (PDR)
Toroidal E1 (TDR)

IV-GDR

Compression E1

~ 7 MeV ~ 14 MeV ~ 25 MeV

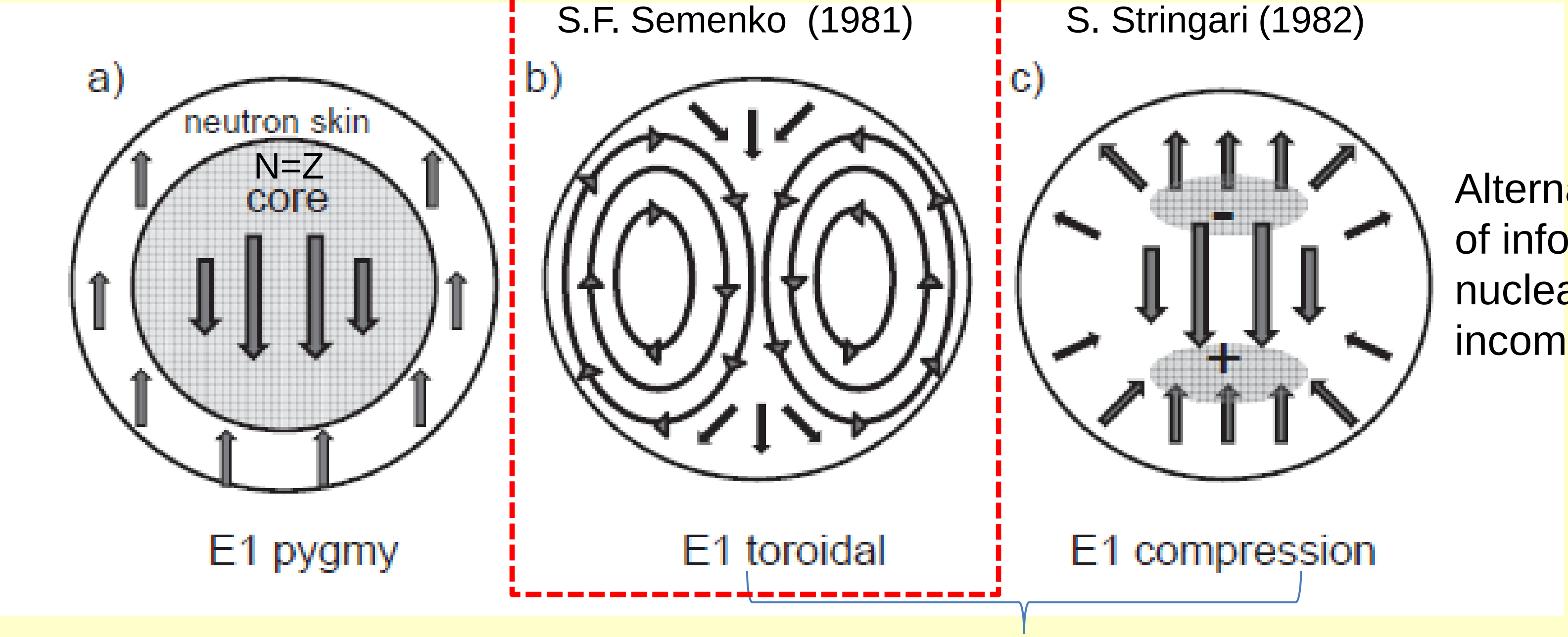
ω [MeV]

Exotic dipole resonances

R. Mohan et al (1971),

V.M. Dubovik (1975)
S.F. Semenko (1981)

M.N. Harakeh (1977)
S. Stringari (1982)



Alternative source of information on nuclear incompressibility

Dominate in E1(T=0) excitation channel
(due to suppression of dominant E1(T=1) motion)

irrotational vortical irrotational

$E = 50 \div 60\ A^{-1/3} MeV$ $E = 50 \div 70\ A^{-1/3} MeV$ $E = 132\ A^{-1/3} MeV$

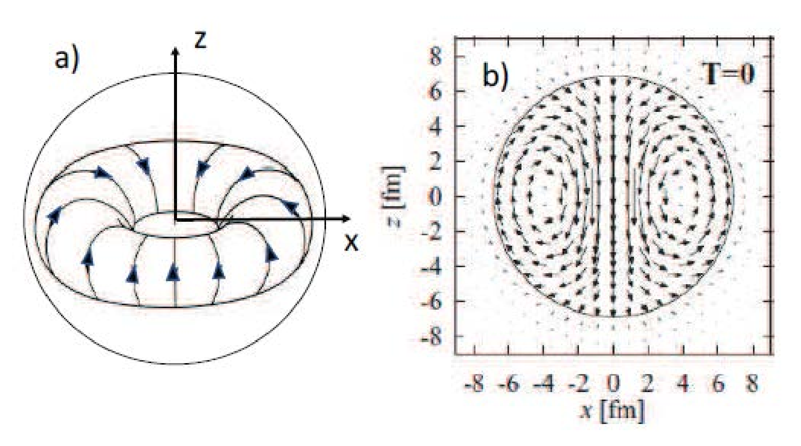
Reviews:
N. Paar et al, Rep. Prog. Phys. 70 691 (2007);
D. Savran et al, Prog. Part. Nucl. Phys. 70, 210 (2013)
VON, J. Kvasil, A. Repko,W. Kleinig, and P.-G. Reinhard, Phys. Atom. Nucl. 79, 842 (2016).

- Different kinds of dipole oscillations with fixed c.m.

Hill’s vortex ring (HIR)

- was suggested by Hill in 1894 as one of the exact solutions of Euler equations
- is one of the most simple cases of a vortical flow caused by nuclear turbulence
- this is a vortex ring
- is very common in liquids and gases but rarely noticed unless the motion of the fluid is revealed by suspended particles (smoke rings)

Current distribution in 208Pb



colored by tobacco particles and so are visible

Each listener of my talk permanently produces (invisible) Hill’s vortex rings in the surrounding air when breathing out!

It would be strange if this elementary vortical mode would by absent in nuclear dynamics. In my talk, I will show that it must exist in nuclei as E1 toroidal resonance.

Justification of the toroidal vortical mode

★ nuclear current density

Following theorem of Chandrasekhar/Moffat, the current distribution can be decomposed as

$$\vec{j}(\vec{r}) = \vec{\nabla}\phi(\vec{r}) + \underbrace{\vec{\nabla}\times[\vec{r}\psi(\vec{r})] + \vec{\nabla}\times\vec{\nabla}\times[\vec{r}\chi(\vec{r})]}_{\text{transversal}}$$

electric moments
E1 GDR, compression

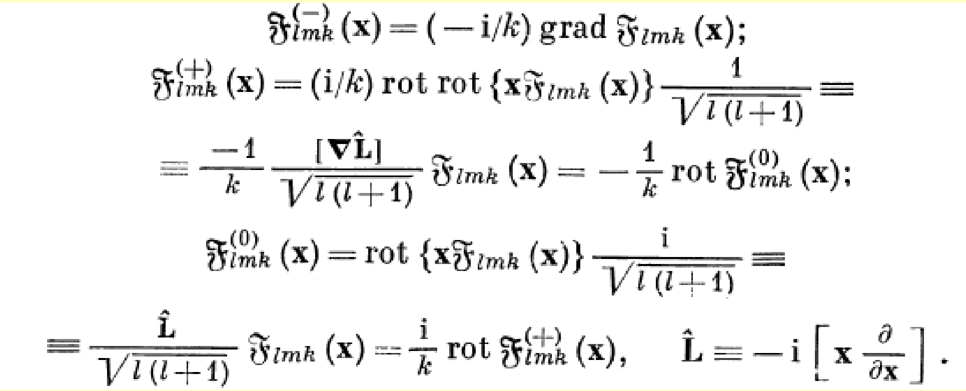
magnetic moments

electric **toroidal** moments

transversal

Unlike Helmholtz expansion $\vec{j}(\vec{r}) = \vec{\nabla}\phi(\vec{r}) + \vec{\nabla}\times[\vec{r}\psi(\vec{r})]$, this decomposition allows to separate magnetic and electric vortical parts

Using eigen-functions $\vec{\mathrm{J}}_{lmk}(\vec{r})$ of vector Helmholtz equation $(\Delta + k^2)\vec{\mathrm{J}}_{lmk}(\vec{r}) = 0$,
one can construct the complete ortho-normalized basis



$$\mathfrak{F}^{(-)}_{lmk}(\mathbf{x}) = (-\mathrm{i}/k)\,\mathrm{grad}\,\mathfrak{F}_{lmk}(\mathbf{x});$$

$$\mathfrak{F}^{(+)}_{lmk}(\mathbf{x}) = (\mathrm{i}/k)\,\mathrm{rot}\,\mathrm{rot}\,\{\mathbf{x}\mathfrak{F}_{lmk}(\mathbf{x})\}\frac{1}{\sqrt{l(l+1)}} \equiv$$

$$\equiv \frac{-1}{k}\frac{[\boldsymbol{\nabla}\hat{\mathbf{L}}]}{\sqrt{l(l+1)}}\mathfrak{F}_{lmk}(\mathbf{x}) = -\frac{1}{k}\,\mathrm{rot}\,\mathfrak{F}^{(0)}_{lmk}(\mathbf{x});$$

$$\mathfrak{F}^{(0)}_{lmk}(\mathbf{x}) = \mathrm{rot}\,\{\mathbf{x}\mathfrak{F}_{lmk}(\mathbf{x})\}\frac{\mathrm{i}}{\sqrt{l(l+1)}} \equiv$$

$$\equiv \frac{\hat{\mathbf{L}}}{\sqrt{l(l+1)}}\mathfrak{F}_{lmk}(\mathbf{x}) = \frac{\mathrm{i}}{k}\,\mathrm{rot}\,\mathfrak{F}^{(+)}_{lmk}(\mathbf{x}), \qquad \hat{\mathbf{L}} \equiv -\mathrm{i}\left[\mathbf{x}\frac{\partial}{\partial\mathbf{x}}\right].$$

V.M. Dubovik and A.A. Cheshkov,
Sov. J. Part. Nucl. v.5, 318 (1975).

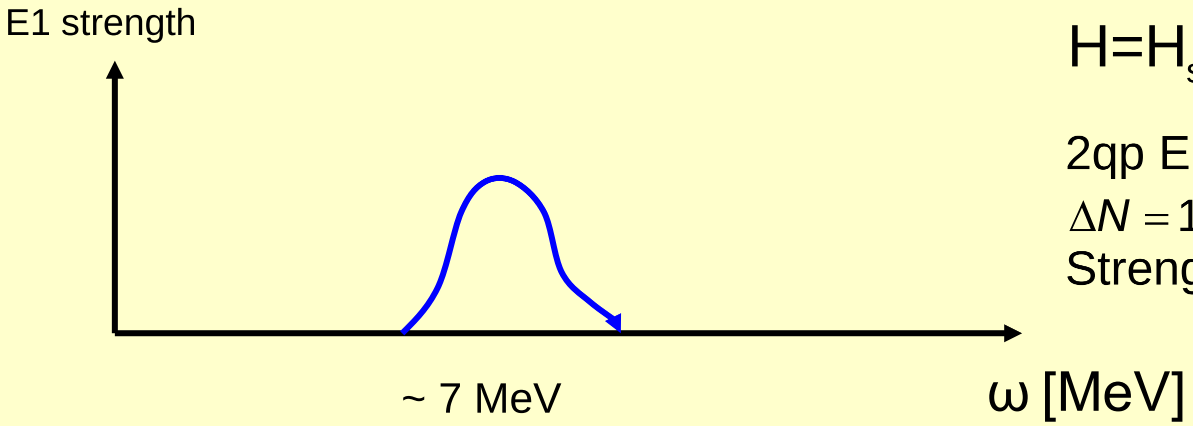
Stefan Nanz, "Toroidal multipole moments in Classical Electrodynamics: An Analysis of their Emergence sand Physical Significance", BestMasters, Springer Spectrum, Karlsruhe, Germany, 2016

Origin of E1 vortial toroidal strength

A. Repko, VON, J. Kvasil and P.-G. Reinhard,
EPJA, 55, 242 (2019)

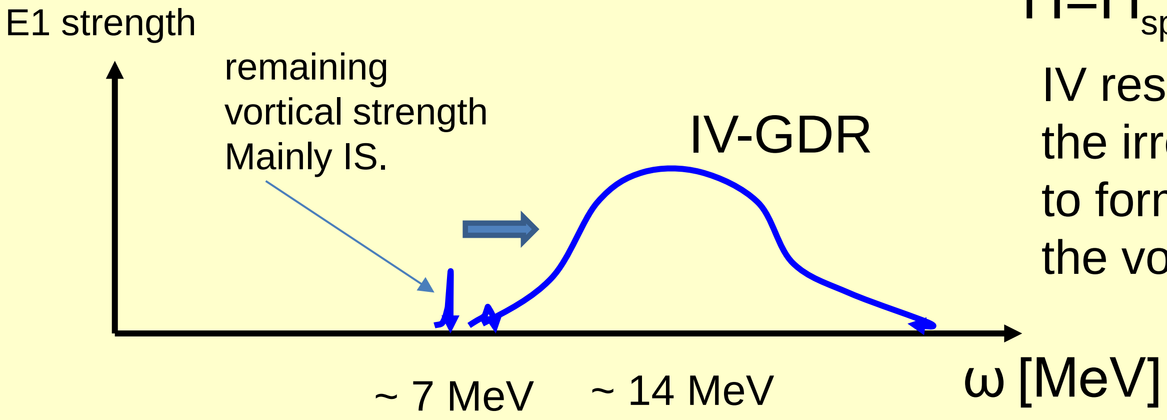
.

E1 toroidal strength must exist in all nuclei at the energy ~ E($\Delta N = 1$).



$H=H_{sp}$

2qp E1 strength from
$\Delta N = 1$ dipole transitions
Strength is mainly irrotational



$H=H_{sp}+H_{res}$

IV residual interaction upshifts
the irrotational E1 strength
to form GDR but leaves at 7 MeV
the vortical (toroidal) strength.

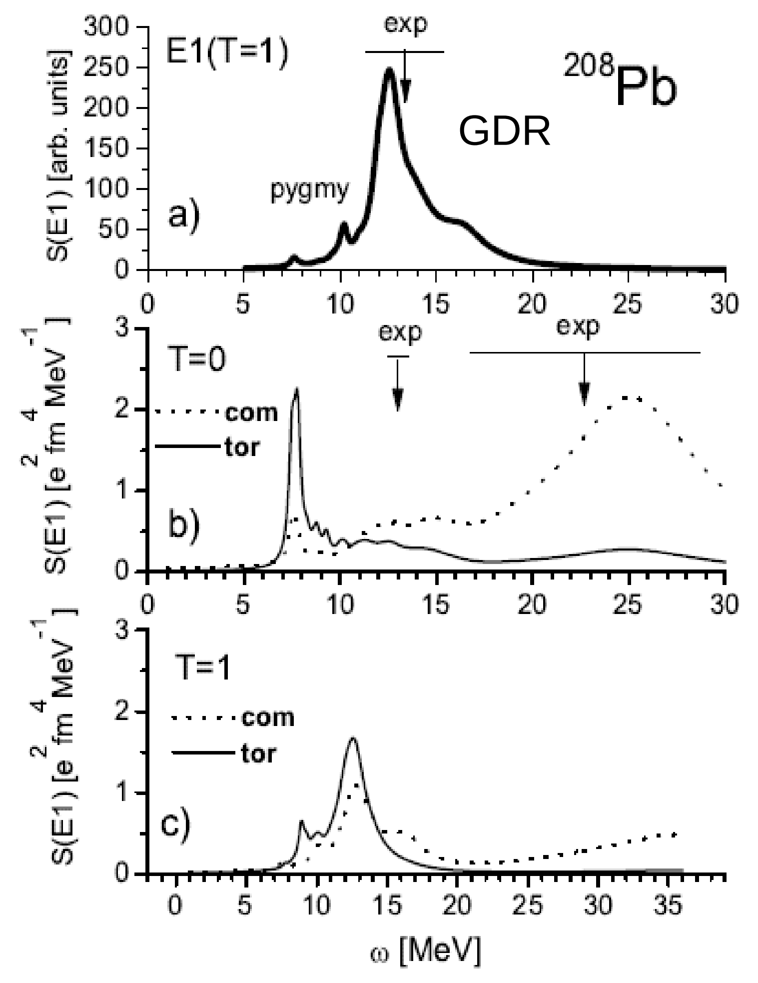
So TDR is indeed a general feature of atomic nuclei

Skyrme QRPA calculations

A. Repko, P.G. Reinhard, VON, J. Kvasil, PRC, 87, 024305 (2013)

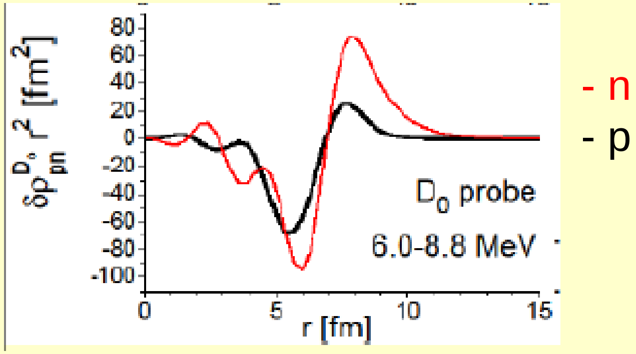
Strength functions

SLy6

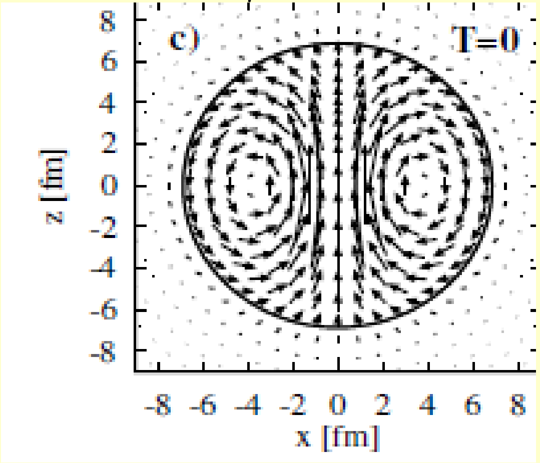


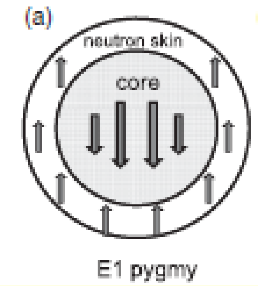
PDR region hosts TDR and CR!

Typical PDR transition density:



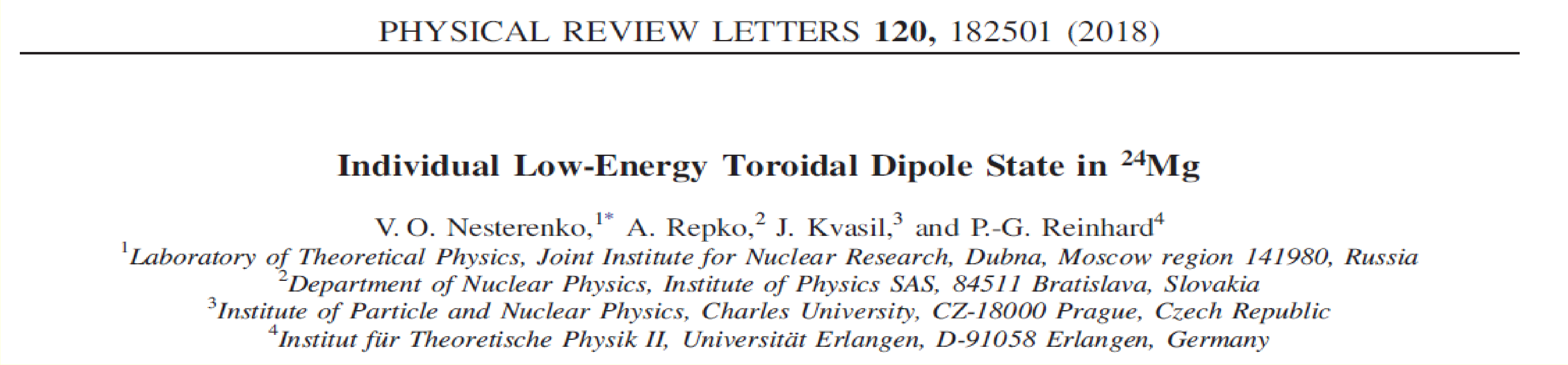
$$\langle \nu \,|\, \hat{j}_{\text{conv}} \,|\, 0 \rangle$$





Nucleon current in the PDR region is mainly toroidal!

Individual toroidal states in light nuclei



PHYSICAL REVIEW LETTERS **120,** 182501 (2018)

Individual Low-Energy Toroidal Dipole State in ^{24}Mg

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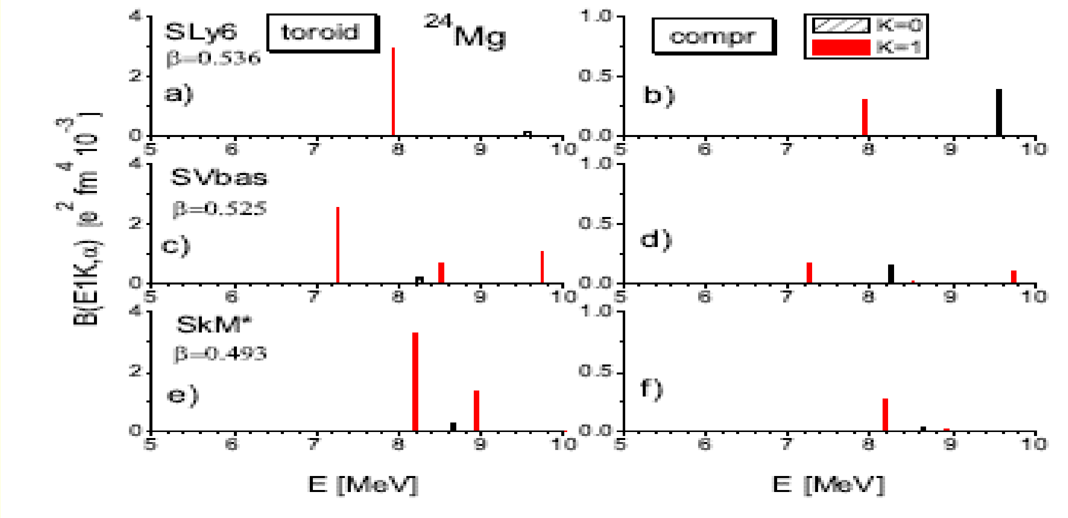
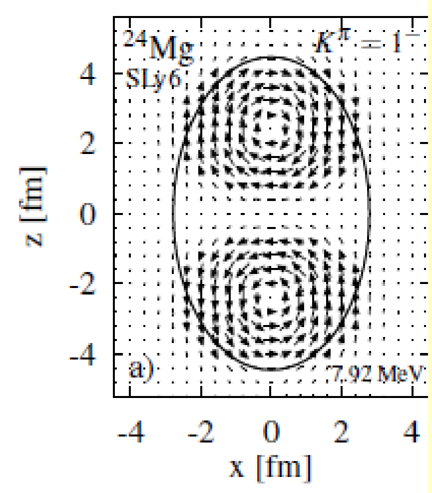
[3]*Institute of Particle and Nuclear Physics, Charles University, CZ-18000 Prague, Czech Republic*
[4]*Institut für Theoretische Physik II, Universität Erlangen, D-91058 Erlangen, Germany*





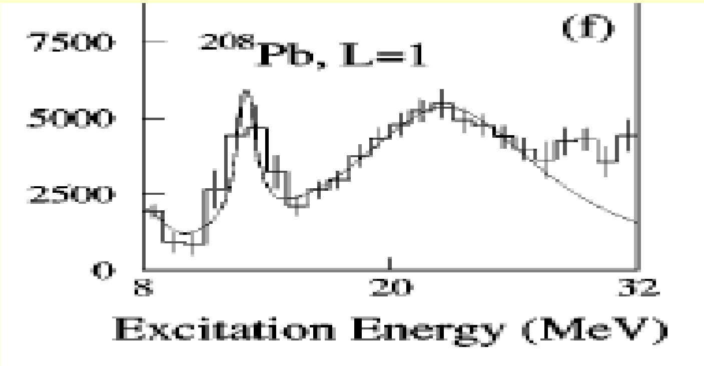
The **lowest** E1(K=1) state in 24Mg should be **toroidal**.

Isoscalar reactions where the dominant contribution of IV-GDR is suppressed

Experiment: (α,α')

Familiar treatment → LE (toroidal) HE (compression)

^{208}Pb
D.Y. Youngblood et al, 1977
H.P. Morsch et al, 1980
G.S. Adams et al, 1986
B.A. Devis et al, 1997
H.L. Clark et al, 2001
D.Y. Youngblood et al, 2004
M.Uchida et al, PRC 69, 051301(R) (2004)



TDR

CDR

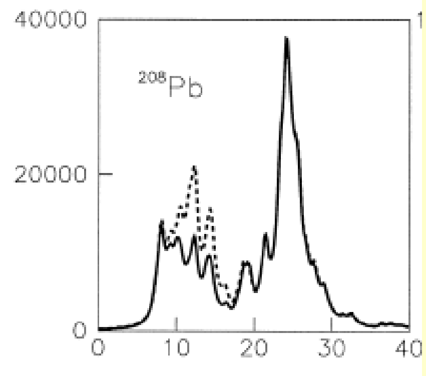
A.Repko, P.-G. Reinhard, V.O.N. and J. Kvasil, PRC 87, 024305 (2013).

There are also exp ISGDR data in

^{56}Fe, ^{58,60}Ni, ^{90}Zr, ^{116}Sn, ^{144}Sm, ...

Theory:

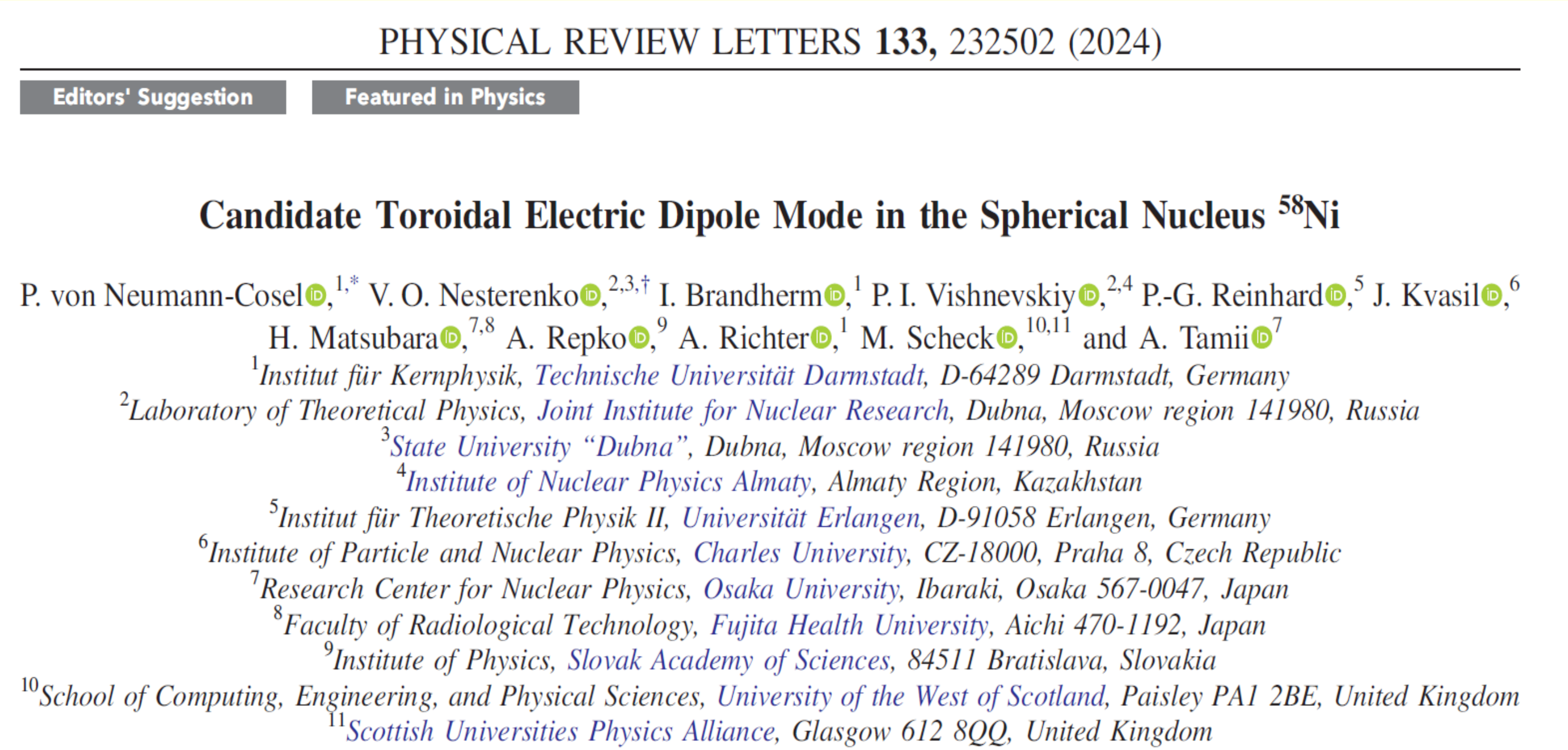
G. Colo et al, PLB 485, 362 (2000)
D. Vretenar et al, PRC, 65, 021301(R) (2002)
N. Paar et al, Rep. Prog. Phys. 70 691 (2007);



Perhaps Uchida observed at 10-17 MeV not TDR but CDR fraction coupled to TDR. Main TDR peak should lie lower at ~ 7-9 MeV.

Anyway (α,α') allows to suppress the dominant IV E1 strength and reveal promising IS toroidal candidates

NEED IN NEW EXPERIMENTS!



PHYSICAL REVIEW LETTERS **133,** 232502 (2024)

Editors' Suggestion **Featured in Physics**

Candidate Toroidal Electric Dipole Mode in the Spherical Nucleus ^{58}Ni

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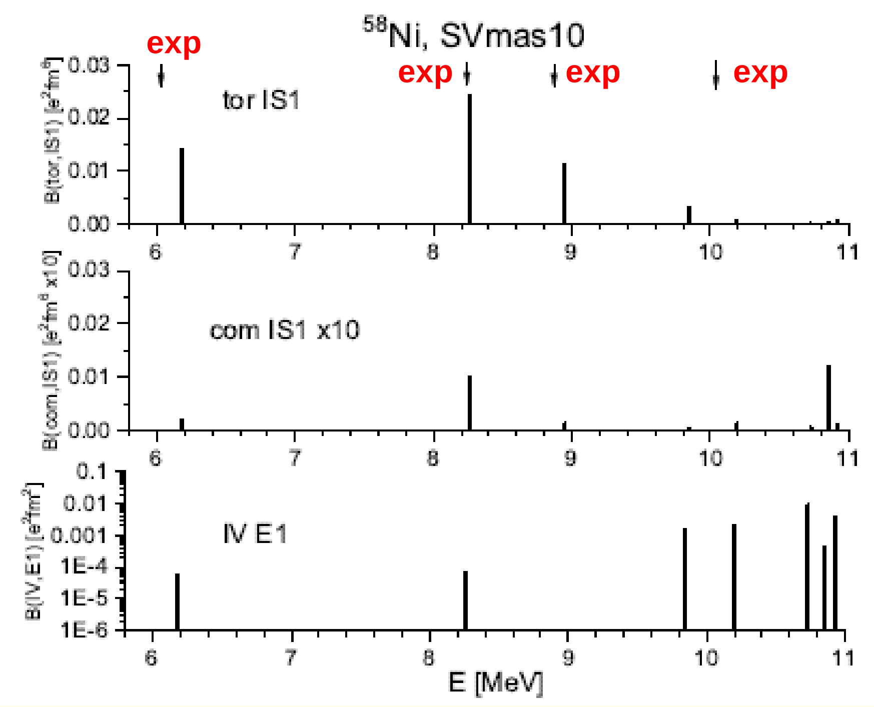
[7] *Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan*
[8] *Faculty of Radiological Technology, Fujita Health University, Aichi 470-1192, Japan*
[9] *Institute of Physics, Slovak Academy of Sciences, 84511 Bratislava, Slovakia*
[10] *School of Computing, Engineering, and Physical Sciences, University of the West of Scotland, Paisley PA1 2BE, United Kingdom*
[11] *Scottish Universities Physics Alliance, Glasgow 612 8QQ, United Kingdom*

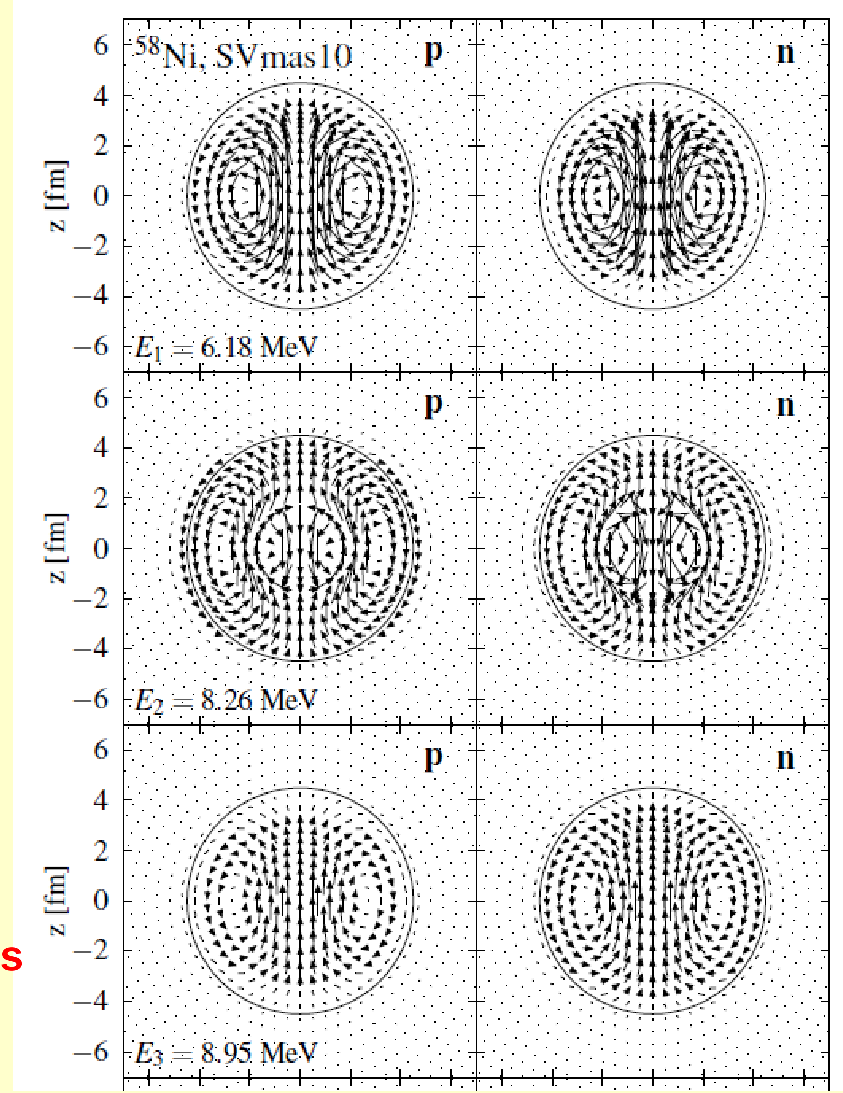
Theory + experiment (TU Darmstadt, 1987) :
the first evidence of PDR in (e,e') reaction.

Individual low-energy toroidal E1 states in spherical 58Ni

Old (e,e') experiments: W. Mettner, A. Richter et al, Nucl. Phys. A473, 160 (1987), Reitz
P. von Neumann Cosel (TU, Darmstadt) :

- large transversal form factors for some low-energy E1 states.
- exp. data from (g,g'), (p,p'), (α,α')
- to check if these states are toroidal?

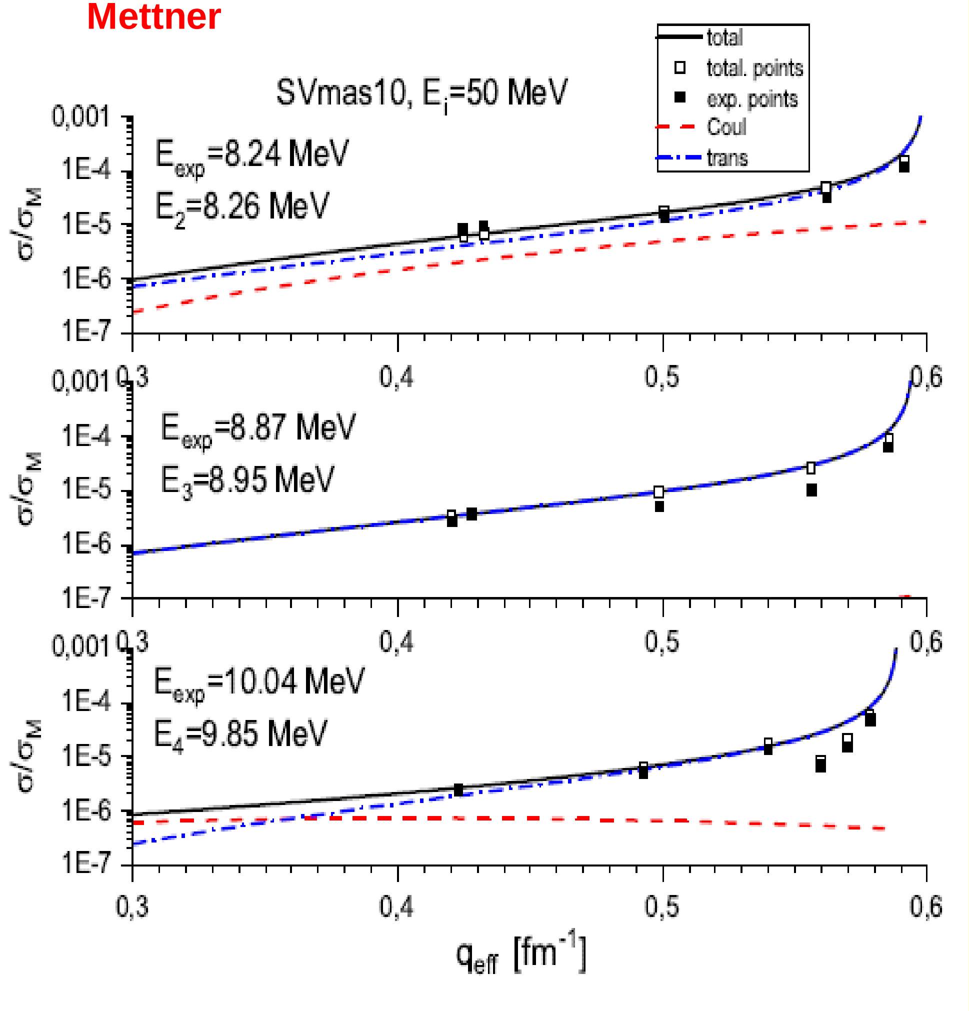




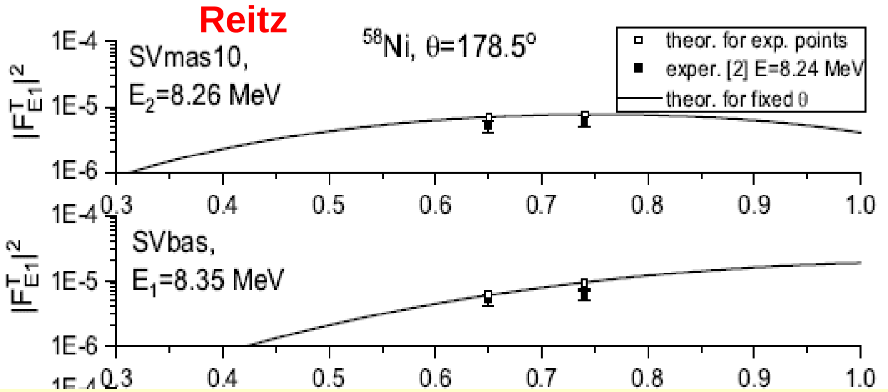
- **large E1 toroidal strength**
- **typical toroidal proton and neutron currents**

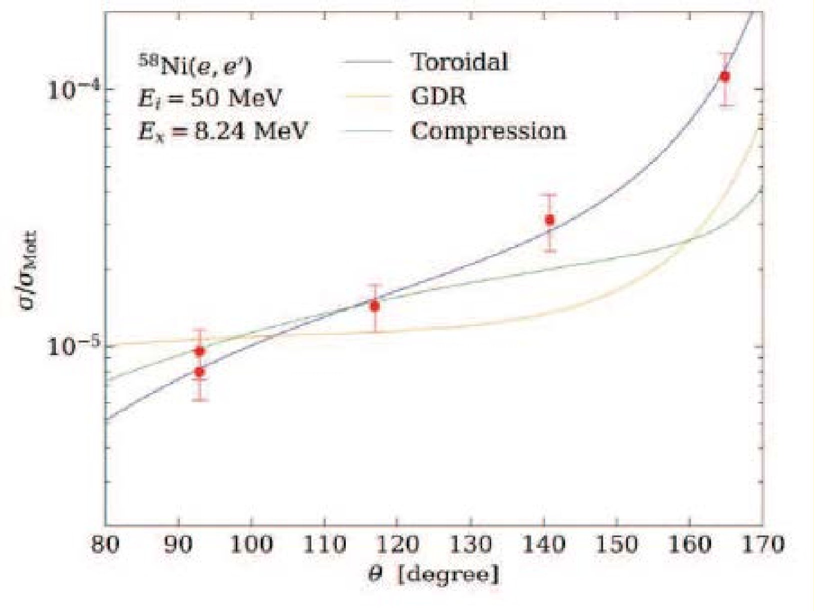
P. Von Neumann-Cosel, V.O. Nesterenko, P.I. Vishnevskiy et al, arXiv 2310.04736 [nucl-exp]

Mettner



Reitz





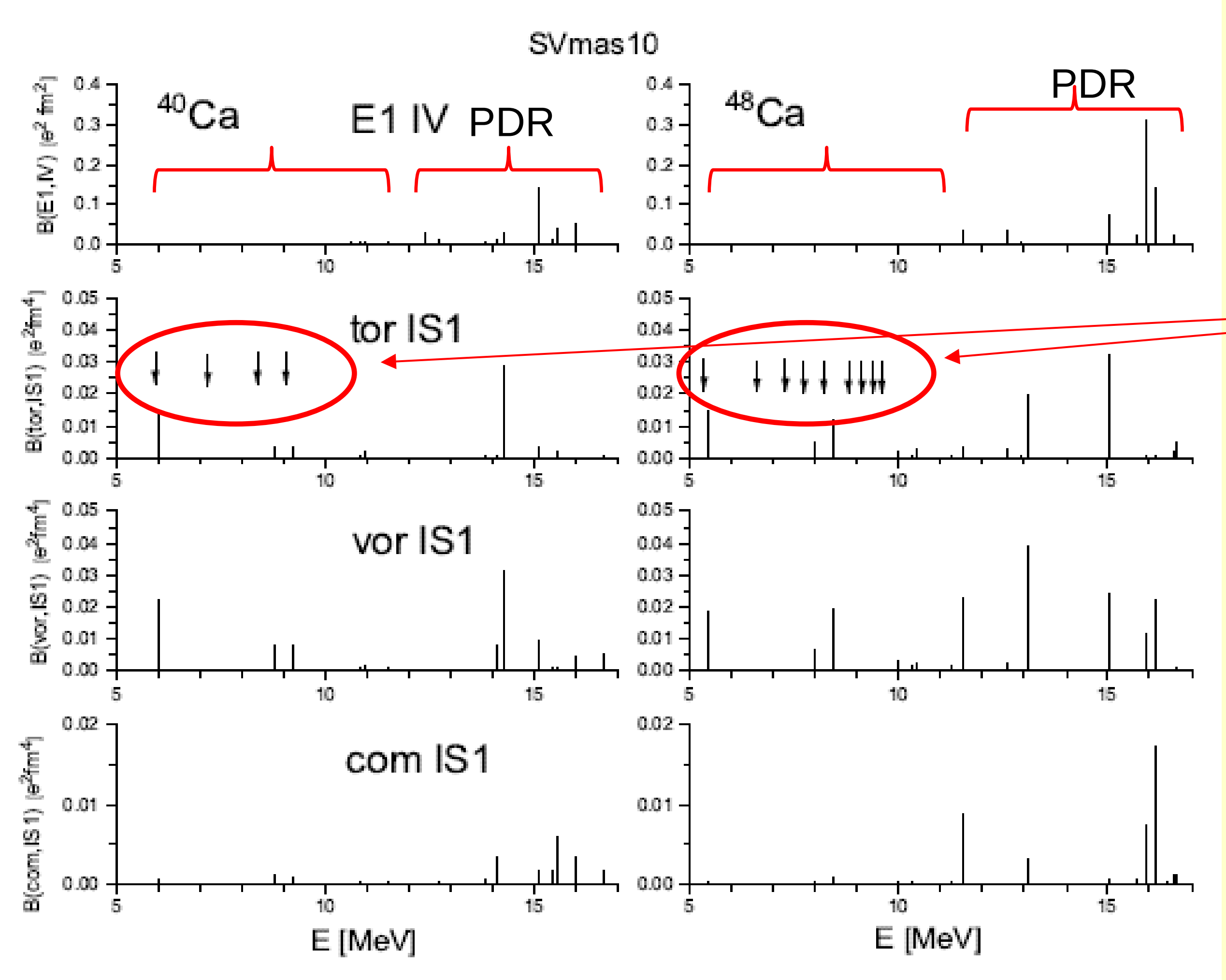
The calculations well describe Mettner's and Reitz's (e,e') data

Just toroidal states (not GDR or compression) describe the slope in Mettner's data

The first (e,e')-based confirmation of existence of E1 toroidal states in nuclei.

TDR vs PDR: example of 40Ca and 48Ca

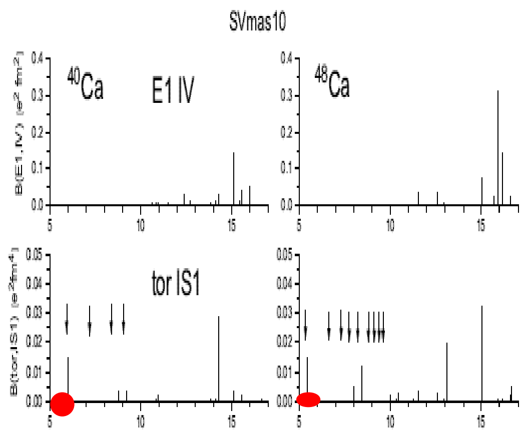
Low-energy QRPA states



observed E1 states

5-12 MeV: dominated by toroidal strength
12-17 MeV: PDR energy region

Currents in TDR energy region ~ 5-6 MeV



States with significant toroidal response:

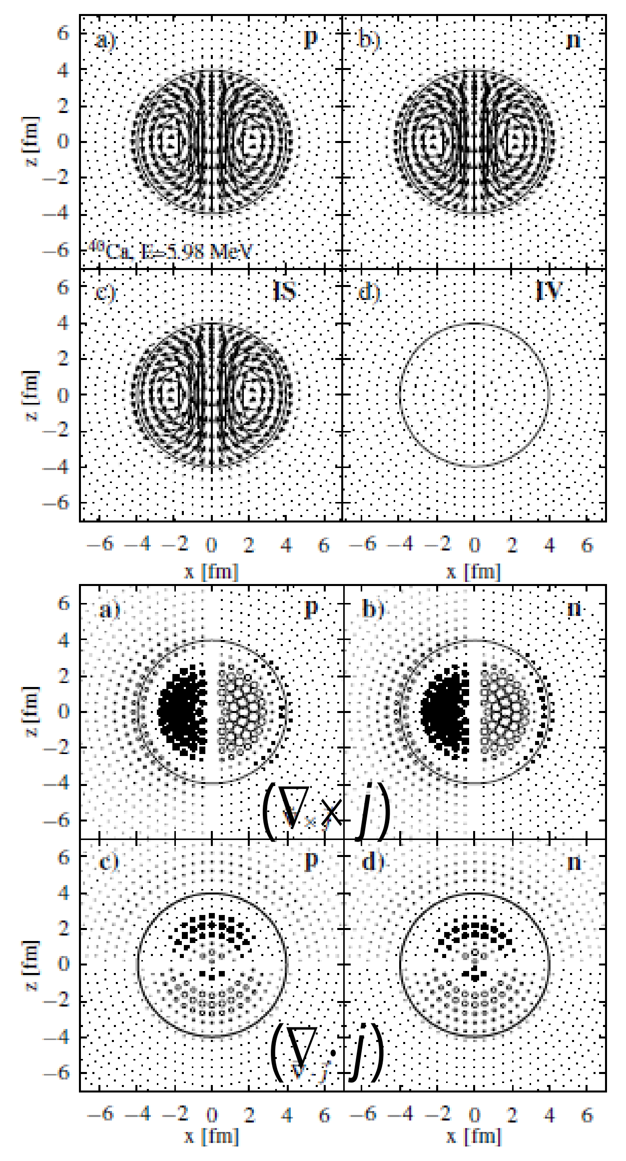
-toroidal isoscalar

- basically vortical

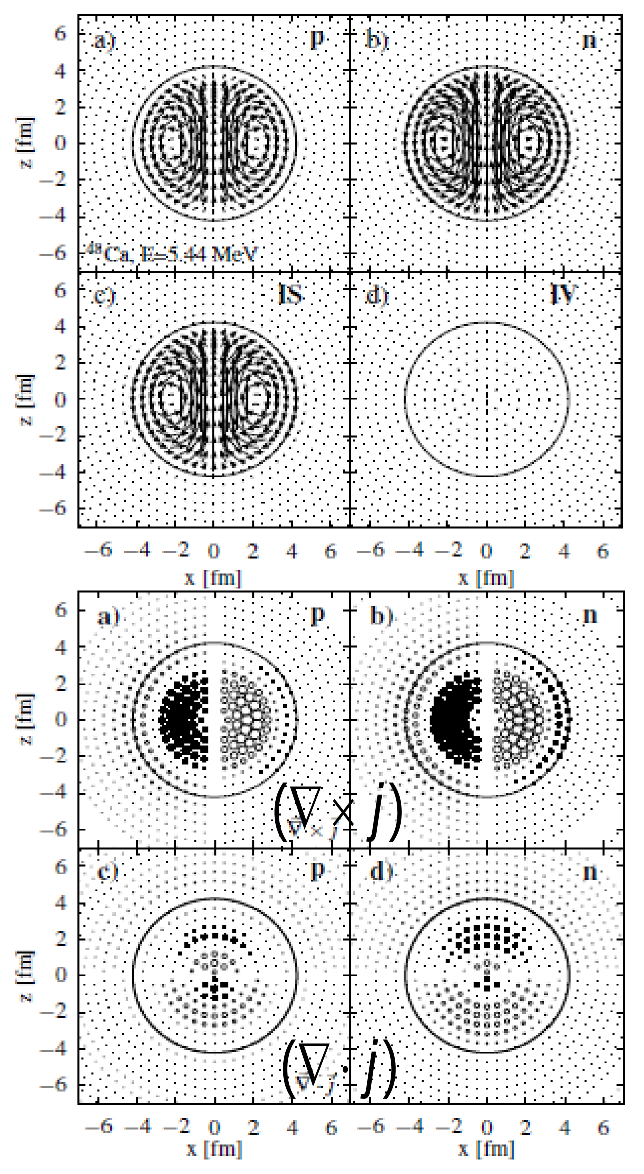
- weak irrotational fraction

- similar for 40Ca and 48Ca

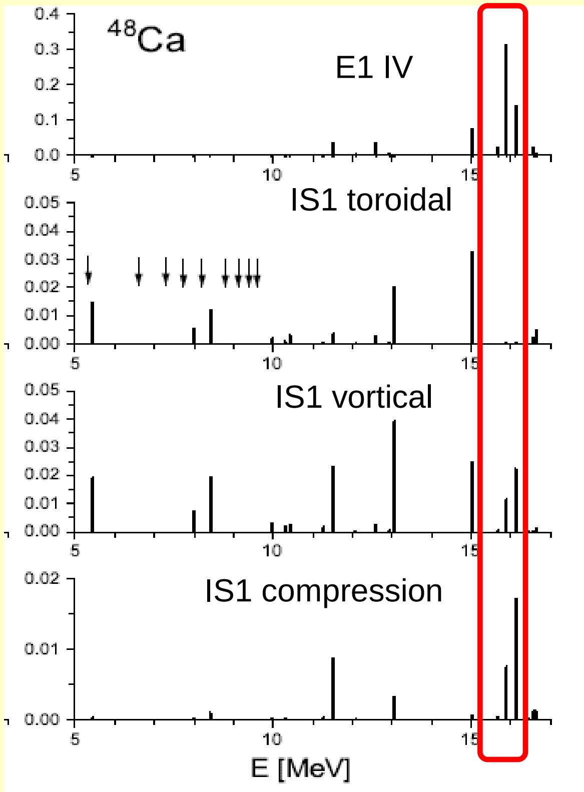
40Ca, E=5.98 MeV

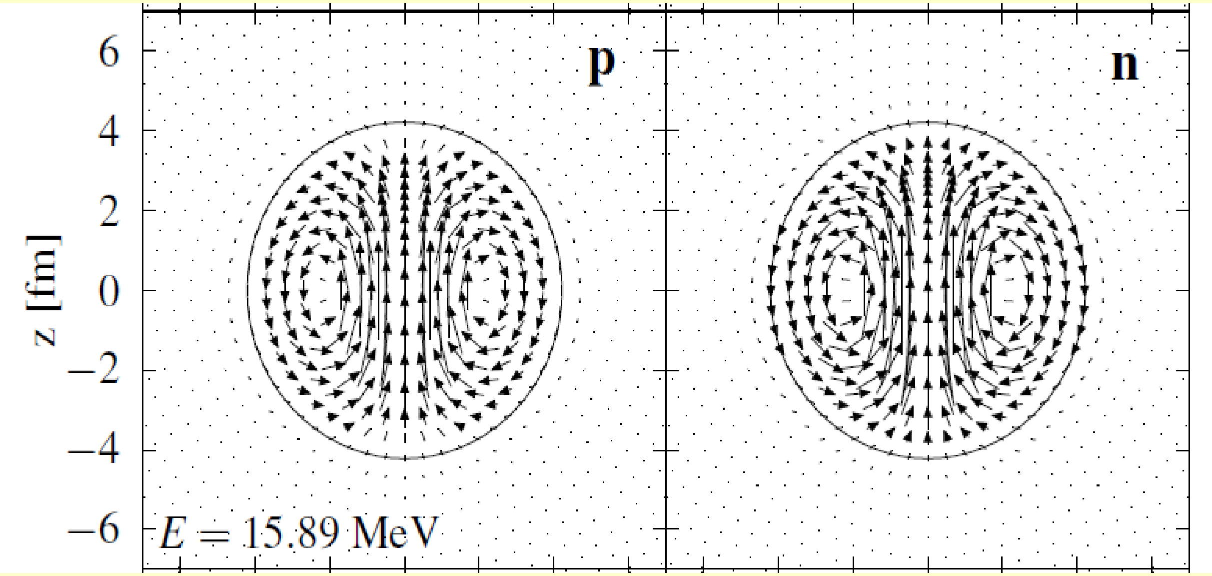


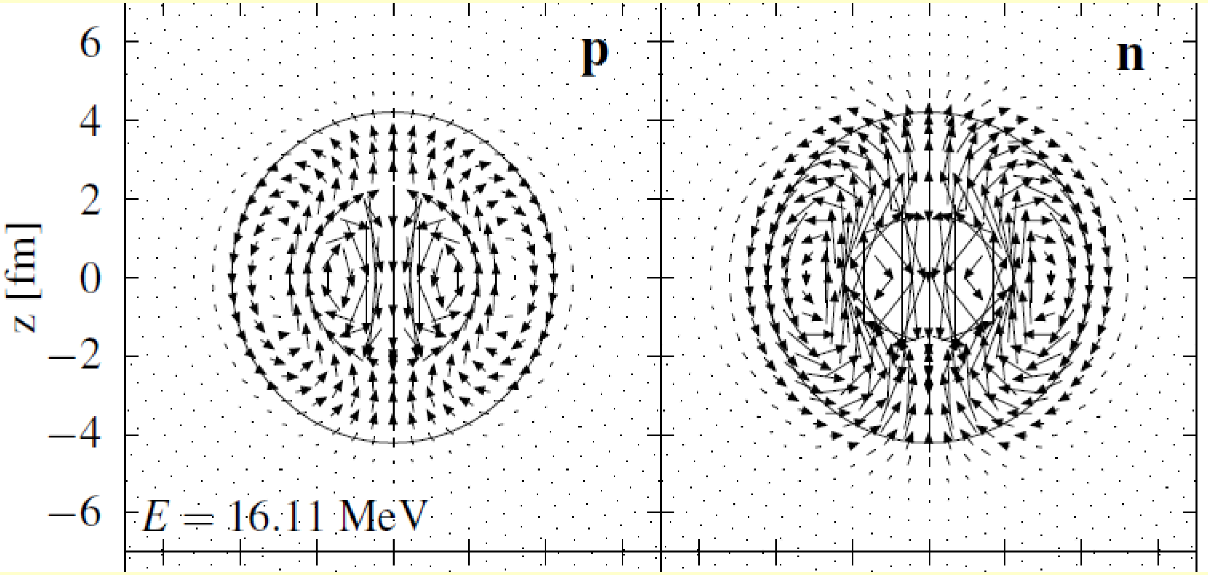
48Ca, E=5.44 MeV



PDR states with maximal B(E1) and small toroidal response in 48Ca





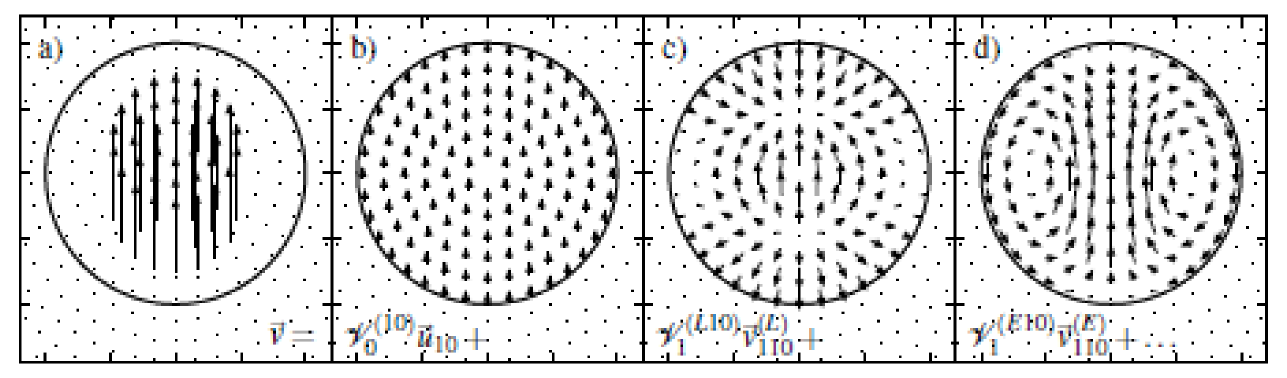


- Vortical + compression flows
- Isoscalar flow inside
- Dominance of neutron flow
  on the nuclear surface
  Surface nuclear flow is part
  of the toroidal one

PDR-like flow can be always decomposed into

- trivial dipole translation
- compression flow
- toroidal flow

Courtesy: A. Repko



PDR-like

common dipole

compression

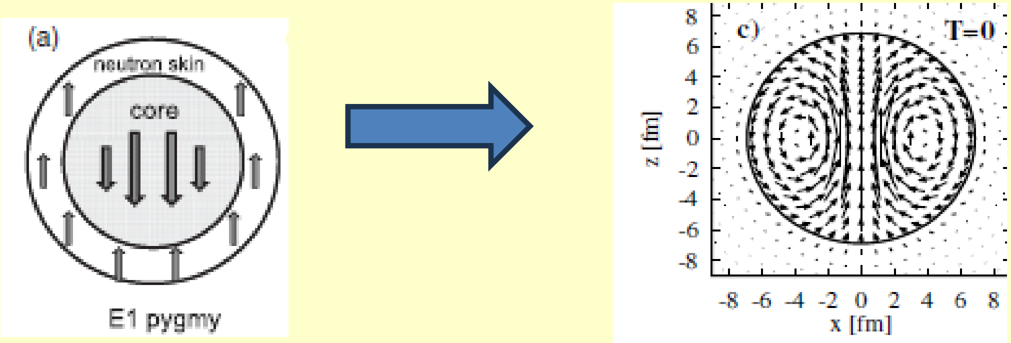
toroidal

So it is natural that PDR-like flow is reduced to combination of toroidal and compression motions.

One may conclude that:

1) The toroidal mode a general feature of atomic nuclei.

2) The familiar PDR is an oversimplified scheme of the actual motion of the nucleons. The dominant flow is toroidal.

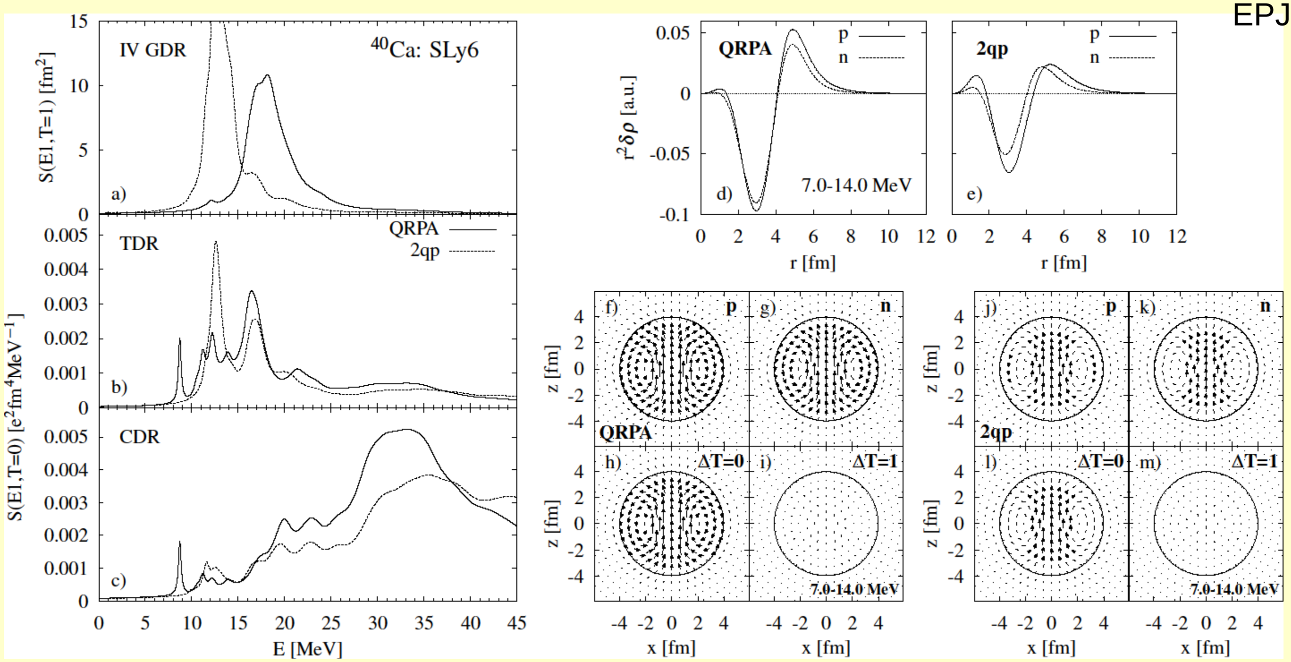


3) We should not revisit previous numerical results on PDR since these results were obtained without using the simplified PDR scheme.

4) The exploration of TDR is important since this resonance:
- is a new kind of nuclear dynamics,
- allows to understand the actual nature of E1 states in PDR energy range,
- competition between vortical TDR and irrotational PDR
- crossover with anapole of Zeldovich, toroid in metamaterials etc

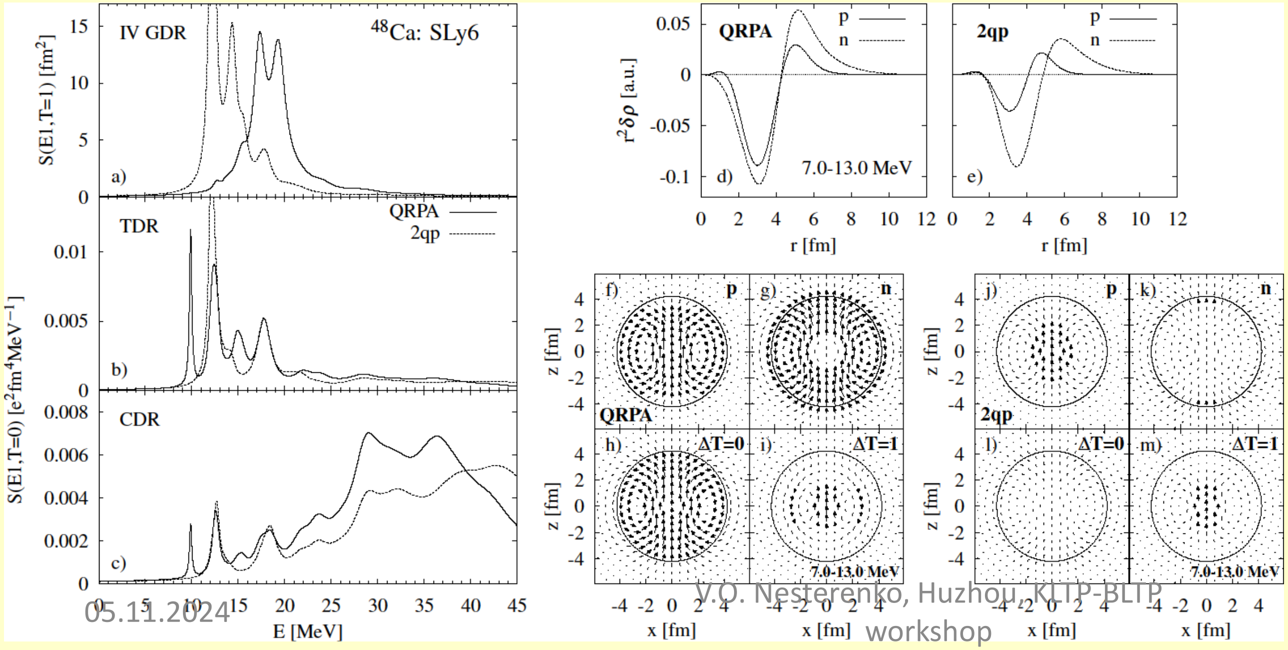
Thank you for attention!

A. Repko, VON, J. Kvasil and P.-G. Reinhard, EPJA, 55, 242 (2019)



40Ca, SLy6

- no PDR but clear TDR
- residual interaction enforces toroidal flow
- protons and neutrons equally contribute
- squeezed toroidal flow



48Ca, SLy6

Basically the same results as for 120Sn, SLy6.

Toroidal mode persists In nuclei independently on the neutron excess

The familiar PDR treatment is a naive and wrong view of the actual flow of the nucleons.

This naive view is based on:

- the existence of the soft dipole mode in nuclei with the neutron excess,
- experimental data for the transition density (TD)

However, the toroidal mode give the same TD!

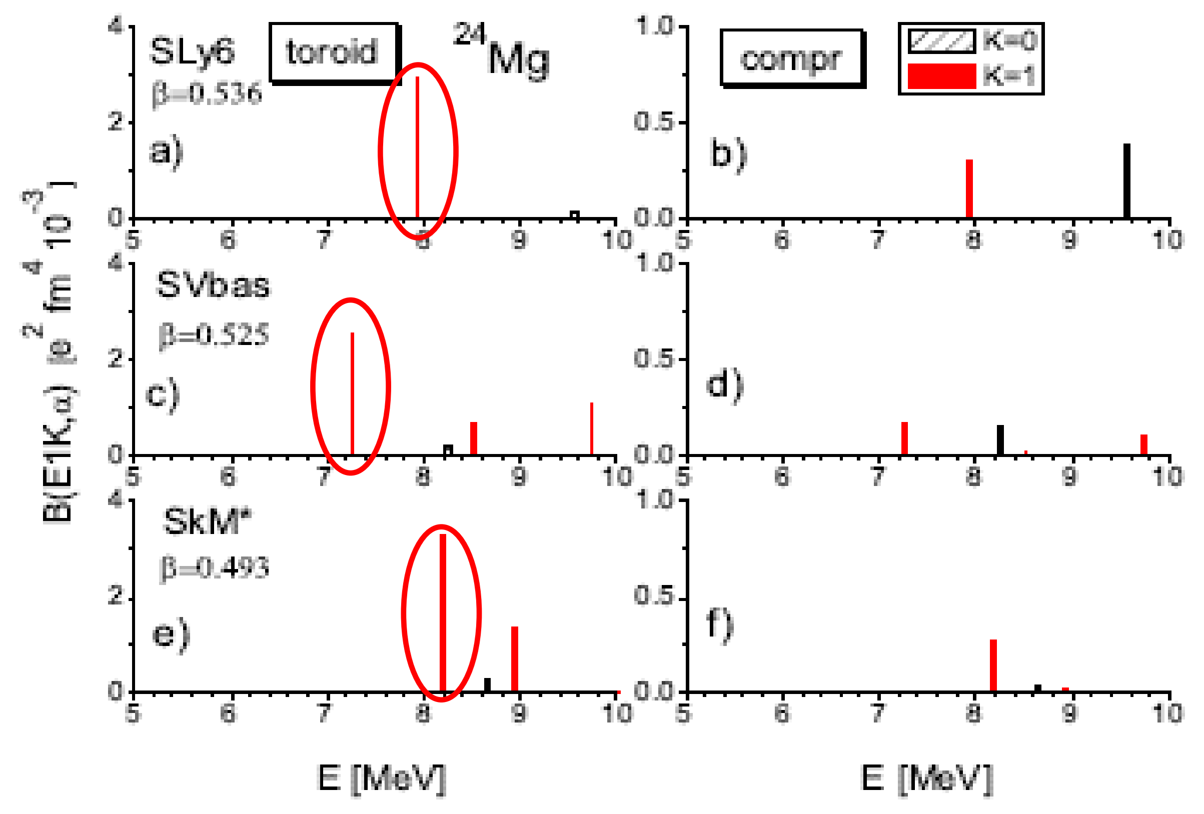
Actually:

- Dipole states at PDR energy region are **mixtures** of toroidal (**dominant !)**, compression and iv dipole flows.
- PDR E1 strength arises in collective states with a large i) compression fraction and ii) irrotational IV part of the toroidal flow.

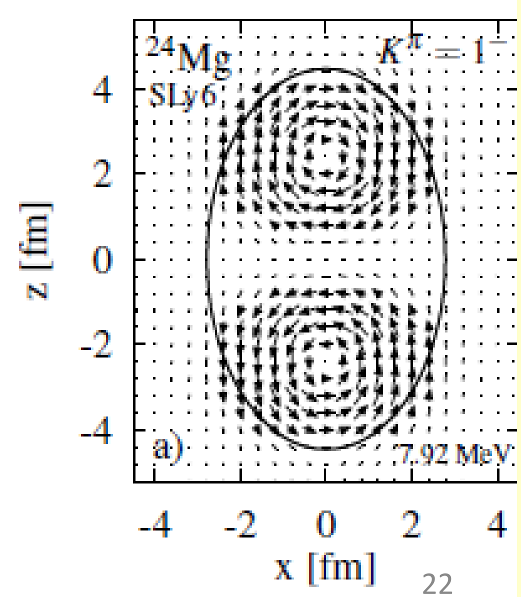
This conclusion does not rule out the previous numerical results for PDR. But allows to understand better its origin.

^{24}Mg $\beta_2^{exp} = 0.605$

VON, A. Repko, J. Kvasil, P.-G. Reinhard,
PRL 120, 182501 (2018)



QRPA results for
SLy6,
SVbas,
SkM*



Persistence of the main result:
the **lowest** toroidal K=1 peak

The remarkable example of individual toroidal state!

Next our step: $(e,e'\gamma)$

C.N. Papanikolas et al, PRL 54, 26 (1985):
first measurement of the relative FL/FT sign in 12C

$$\frac{d^4\sigma}{d\Omega_\gamma d\Omega_e\, d\omega\, dE_\gamma} = \sigma_{\mathrm{Mott}}\left(\frac{\Gamma_{\gamma f}}{\Gamma}\right)\Big\{V_L U_L|F_L(q)|^2 + V_T U_T|F_T(q)|^2 \underbrace{+ V_I U_I \cos\phi_\gamma F_L(q)F_T(q) + V_S U_S \cos 2\phi_\gamma F_T(q)F_T(q)\Big\}},$$

longitudinal-transversal interference terms allow to measure the relative sign of FL and FT.

In the long-wave approximation, Siegert's theorem gives the negative sign:

$$F_T(q) = -\frac{\omega}{q}\left[\frac{\lambda+1}{\lambda}\right]^{1/2} F_L(q) \quad \text{for} \quad q \to \omega.$$

$$j_\lambda(kr) = \frac{(kr)^\lambda}{(2\lambda+1)!!}\left[1 - \frac{(kr)^2}{2(2\lambda+3)} + \ldots\right]$$

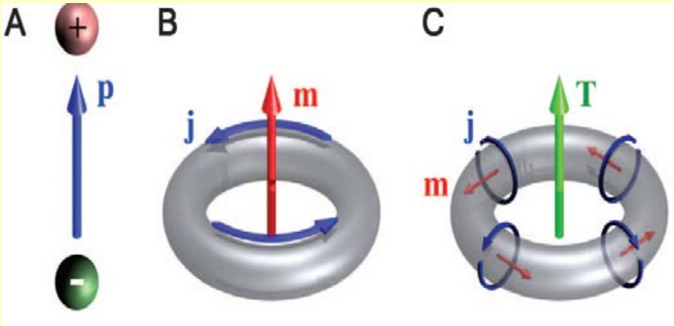
For transversal toroidal E1 states, we expect the opposite sign. This could be a signature of the toroidal mode.
Besides, Hill's vortex can polarize the outcoming photons.

P. Von Neumann-Cosel, V.O. Nesterenko,
P.I. Vishnevskiy et al,
arXiv 2310.04736 [nucl-exp]

$(e,e'\gamma)$ **facilities in TU Darmstadt:**
G. Steinhilber, PhD thesis, TU Darmstadt, 2022.

First experimental discovery of toroidal dipole response in metamaterial

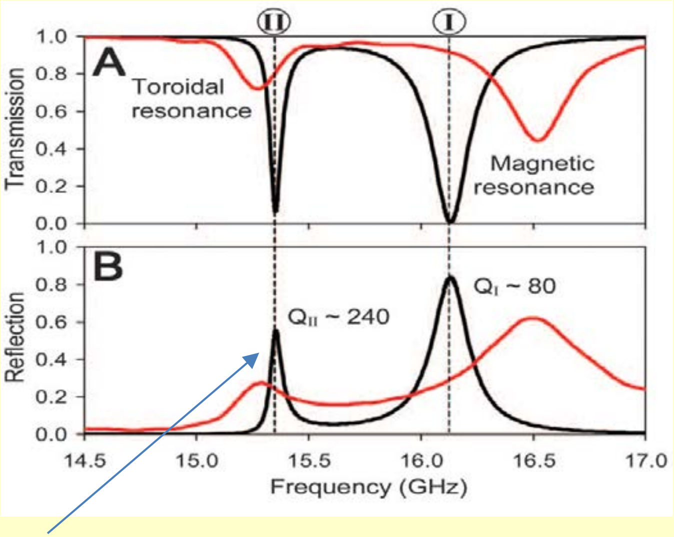
T. Kaelberer, V.A. Fedotov, Papasimakis, D.P. Tsai, N.I. Zheludev, Science 330, 1510 (2010).



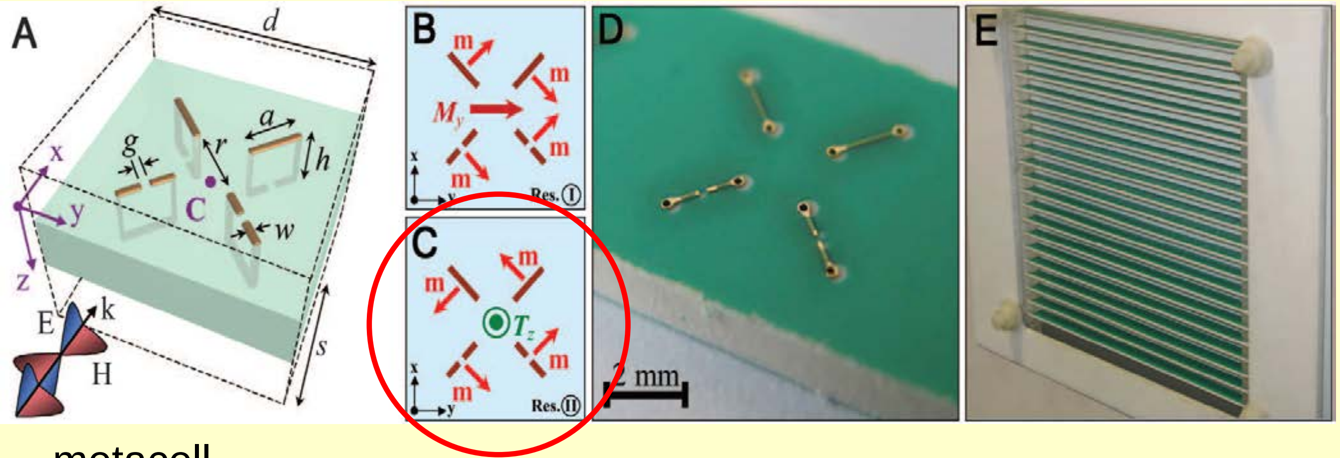
Electric dipole

Magnetic dipole

Toroidal dipole



Toroidal dipole resonance at 15.4 GHz



Metamaterial is irradiated by polarized microwave field at in x-direction

metacell

Toroidal case

N. Papasimakis, V. A. Fedotov, V. Savinov, T. A. Raybould and N. I. Zheludev, **“Electromagnetic toroidal excitations in matter and free space”** Nature Nanomaterials, 2016; http://dx.doi.org/10.1038/nmat4563.

What we get new from the knowledge of the toroidal flow in PDR dipole states?

nuclear current → $\vec{j} = \vec{j}_{vort} + \vec{j}_{irrot}$

** PDR E1 strength is determined by the irrotational fraction of the states

with $(\vec{\nabla} \cdot \vec{j}_{irrot}) \neq 0$, $(\vec{\nabla} \times \vec{j}_{irrot}) = 0$

**Vortical toroidal fraction:

- originates from 2qp vortical components
- competes with irrotational fraction
- in this sense, the vortical fraction affects:
    - PDR E1 strength,
    - dipole polarizability → slope of the symmetry energy, EoS
- allows to understand the diversity of PDR E1 strength in different nuclei

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard,
P. Vesely, PRC, 84, 034303 (2011)

Toroidal E1 operator:

$$\hat{M}_{tor}(E1\mu) = \frac{1}{10\sqrt{2c}}\int d\vec{r}\;[r^3 + \underbrace{\frac{5}{3} r < r^2 >_0}_{\text{cmc}}]\;\vec{Y}_{11\mu}(\hat{\vec{r}})\cdot[\underbrace{\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})}_{\textbf{mainly vortical flow}}]$$

Compression E1 operator:

$$\hat{M}_{com}(E1\mu) = -\frac{i}{10c}\int d\vec{r}\;[r^3 - \frac{5}{3} r < r^2 >_0]Y_{1\mu}\;[\underbrace{\vec{\nabla}\cdot\hat{\vec{j}}_{nuc}(\vec{r})}_{\textbf{irrotational flow}}]$$

$$\dot{\rho} + \vec{\nabla}\cdot\vec{j}_{nuc} = 0$$

$$\hat{M}'_{com}(E1\mu) = \int d\vec{r}\,\hat{\rho}(\vec{r})\;[r^3 - \frac{5}{3} r < r^2 >_0]Y_{1\mu}$$

Toroidal and compression operators are coupled:

$$\hat{M}_{tor}(E1\mu) = -\frac{i}{2\sqrt{3c}}\int d\vec{r}\;\hat{\vec{j}}_{nuc}(\vec{r})\cdot\vec{\nabla}\times(\vec{r}\times\vec{\nabla})[r^3 - \frac{5}{3} r < r^2 >_0]\,Y_{1\mu}(\hat{\vec{r}})$$