

QCD-based approach on isospin symmetry breaking energy density functional

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Isospin symmetry breaking of nuclear interaction

- Nuclear interaction: *almost* isospin symmetric

$$v_{pp}^{T=1} \simeq v_{pn}^{T=1} \simeq v_{nn}^{T=1}$$

Miller, Oppen, and Stephenson. *Annu. Rev. Nucl. Part. Sci.* **56**, 253 (2006)

Isospin symmetry breaking of nuclear interaction

- Nuclear interaction: *almost* isospin symmetric
- **Charge symmetry breaking (CSB)**
 - Difference between p - p int. and n - n int.

$$v_{pp}^{T=1} \simeq v_{pn}^{T=1} \simeq v_{nn}^{T=1}$$

$$v_{\text{CSB}} \equiv v_{nn}^{T=1} - v_{pp}^{T=1} \sim \tau_{zi} + \tau_{zj}$$

- Originates from mass difference of nucleons ($m_p \neq m_n$) and π^0 - η & ρ^0 - ω mixings in meson-exchange process
- Contribute to β term (β^{2n+1} terms) in nuclear EoS
- **Charge independence breaking (CIB)**
 - Difference between like-particle int. and diff.-particle int.

$$v_{\text{CIB}} \equiv \frac{v_{nn}^{T=1} + v_{pp}^{T=1}}{2} - v_{np}^{T=1} \sim \tau_{zi}\tau_{zj}$$

- Originates from mass difference of pions ($m_{\pi^0} \neq m_{\pi^\pm}$)
- Contribute to SNM and β^2 term (β^{2n} terms) in nuclear EoS

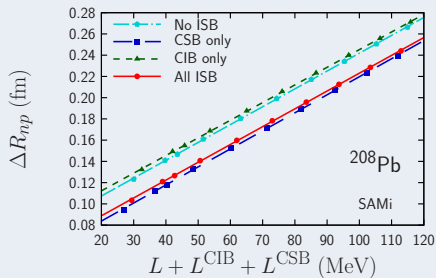
Miller, Opper, and Stephenson. *Annu. Rev. Nucl. Part. Sci.* **56**, 253 (2006)

Isospin symmetry breaking of atomic nuclei

- Different properties of mirror nuclei
 - Mass (Okamoto-Nolen-Schiffer anomaly)
 - Ground-state spin
($^{73}_{38}\text{Sr}$ and $^{73}_{35}\text{Br}$ at NSCL, $^{71}_{36}\text{Kr}$ and $^{71}_{35}\text{Br}$ at RIBF)
 - Shape ($^{70}_{36}\text{Kr}$ and $^{70}_{34}\text{Se}$ at RIBF)
- Finite (negative) neutron-skin thickness $\Delta R_{np} = R_n - R_p$ of $N = Z$ nuclei
 - Okamoto. *Phys. Lett.* **11**, 150 (1964)
 - Nolen and Schiffer. *Annu. Rev. Nucl. Sci.* **19**, 471 (1969)
 - Hoff *et al.* *Nature* **580**, 52 (2020)
 - Wimmer *et al.* *Phys. Rev. Lett.* **126**, 072501 (2021)
 - Algora *et al.* arXiv:2411.00509 [nucl-ex]

Isospin Symmetry Breaking in Nuclear DFT

Neutron-skin thickness of ^{208}Pb



- On top of SAMi-J family, ISB terms are considered

- If we assume the same ΔR_{np} , difference between estimated L_{full} without & that with ISB is 11.1 MeV
CSB contribution 13.9 MeV
CIB contribution -2.7 MeV
- $L_{\text{CIB}} = 2.3 \text{ MeV}$ and $L_{\text{CSB}} = -3.2 \text{ MeV} \rightarrow$ Change of L is 12 MeV

Naito, Colò, Liang, Roca-Maza, and Sagawa. *Phys. Rev. C* **107**, 064302 (2023)

Mysterious of CSB strength (Skyrme t_0 -like interaction)

- **Phenomenological determination**—Referring experimental data
- ***Ab initio* determination**— s_0 extracted from *ab initio* calc.

Method: Naito, Colò, Liang, Roca-Maza, and Sagawa. *Phys. Rev. C* **105**, L021304 (2022)

s_0 -value: Roca-Maza, Colò, and Sagawa. *Phys. Rev. Lett.* **120**, 202501 (2018)

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VMC & AV18: Wiringa. Private communication

SCGF: Rios. Private communication

Summary: Naito, Colò, Liang, Roca-Maza, and Sagawa. *Nuovo Cim. C* **47**, 52 (2024)

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 - $s_0 = -26.3 \text{ MeV fm}^3$ (IAE of ^{208}Pb)
 - $s_0 \simeq -10 \text{ MeV fm}^3$ (MDE and TDE)
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$O(10) \text{ MeV fm}^3$

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- CSB effect in *ab initio* is $\times 0.1$ of that in DFT?!?! **Open problem**

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- **We need to determine ISB strength microscopically!**

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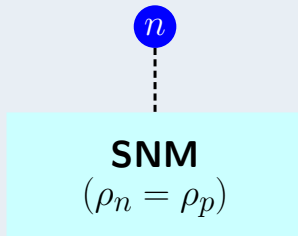
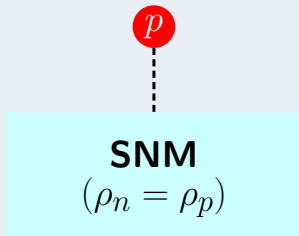
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Simplest model systems for ONS anomaly

- The simplest model towards ONS anomaly is “SNM + p/n ”
- Only the nuclear interaction is considered
- Nucleon mass in medium depends on density due to chiral symmetry breaking and its restoration
 $\rightarrow \Delta_{np} = M_n - M_p$ also depends on ρ
- Therefore, the energy difference except mass diff. can be regarded as “Okamoto-Nolen-Schiffer anomaly” $\Delta E = \delta - \Delta_{np} (\rho = 0)$



QSR approach for Δ_{np}

- In-medium self-energy Σ_τ can be calculated by QSR

$$\Delta_{np}(\rho) = \omega_n - \omega_p \simeq \Sigma_n^S - \Sigma_p^S$$

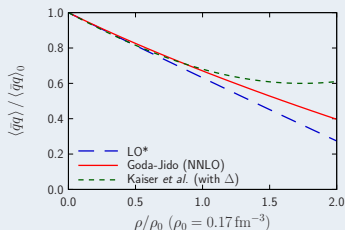
- Using the Borel sum and QSR,

$$\Delta_{np}(\rho) = C_1 \left(\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} \right)^{1/3} - C_2 \quad C_1 = -a\gamma \quad \gamma = \frac{\langle \bar{d}d \rangle_0}{\langle \bar{u}u \rangle_0} - 1$$

(ω_τ : Time-component of 4-momentum, Σ_τ^S : Scalar self-energy, $C_1 = 5.24_{-1.21}^{+2.48}$ MeV)

Hatsuda, Høgaasen, and Prakash. *Phys. Rev. Lett.* **66**, 2851 (1991)

Estimation of in-medium chiral condensation



$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} \simeq 1 + k_1 \frac{\rho}{\rho_0} + k_2 \left(\frac{\rho}{\rho_0} \right)^{5/3}$$

$$k_1 = -\frac{\sigma_{\pi N} \rho_0}{f_\pi^2 m_\pi^2} \quad k_2 = -k_1 \frac{3k_{F0}^2}{10m_N^2}$$

$\sigma_{\pi N}$: π -N sigma term, f_π : pion decay const.

Goda and Jido. *Phys. Rev. C* **88**, 065204 (2013)

Estimation of Δ_{np}

- Eventually, the ONS anomaly $\delta_{\text{QSR}} = \Delta_{np}(\rho = 0) - \Delta_{np}(\rho)$ is

$$\begin{aligned}\delta_{\text{QSR}} &= C_1 \left[1 - \left(\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} \right)^{1/3} \right] \\ &= C_1 \left[\frac{\sigma_{\pi N}}{3f_\pi^2 m_\pi^2} \rho - \left(\frac{3\pi^2}{2} \right)^{2/3} \frac{\sigma_{\pi N}}{10f_\pi^2 m_N^2 m_\pi^2} \rho^{5/3} + \dots \right]\end{aligned}$$

Sagawa, Naito, Roca-Maza, and Hatsuda. *Phys. Rev. C* **109**, L011302 (2024)

Effective interaction approach

- We introduce Skyrme-type CSB interaction

$$v_{\text{Sky}}^{\text{CSB}}(\mathbf{r}) = \left\{ s_0 (1 + y_0 P_\sigma) \delta(\mathbf{r}) + \frac{s_1}{2} (1 + y_1 P_\sigma) \left[\mathbf{p}^{\dagger 2} \delta(\mathbf{r}) + \delta(\mathbf{r}) \mathbf{p}^2 \right] \right. \\ \left. + s_2 (1 + y_2 P_\sigma) \mathbf{p}^{\dagger} \cdot \delta(\mathbf{r}) \mathbf{p} \right\} \frac{\tau_{1z} + \tau_{2z}}{4}$$

- Contribution of $v_{\text{Sky}}^{\text{CSB}}$ to nuclear EoS is

$$\frac{E^{\text{CSB}}}{A} = \left[\frac{\tilde{s}_0}{8} \rho + \frac{1}{20} \left(\frac{3\pi^2}{2} \right)^{2/3} (\tilde{s}_1 + 3\tilde{s}_2) \rho^{5/3} \right] \frac{\rho_n - \rho_p}{\rho}$$

$$\tilde{s}_0 = s_0 (1 - y_0), \quad \tilde{s}_1 = s_1 (1 - y_1), \quad \tilde{s}_2 = s_1 (1 + y_2)$$

- Therefore,

$$\delta_{\text{Skyrme}} \simeq -\frac{\tilde{s}_0}{4} \rho - \frac{1}{10} \left(\frac{3\pi^2}{2} \right)^{2/3} (\tilde{s}_1 + 3\tilde{s}_2) \rho^{5/3}$$

$$\text{since } (\rho_n - \rho_p) / \rho \simeq (N - Z) / A$$

Sagawa, [Naito](#), Roca-Maza, and Hatsuda. *Phys. Rev. C* **109**, L011302 (2024)

Comparison of δ obtained by two methods

$$\delta_{\text{QSR}} \simeq C_1 \left[\frac{\sigma_{\pi N}}{3f_\pi^2 m_\pi^2} \rho - \left(\frac{3\pi^2}{2} \right)^{2/3} \frac{\sigma_{\pi N}}{10f_\pi^2 m_\pi^2 m_N^2} \rho^{5/3} \right]$$

$$\delta_{\text{Skyrme}} \simeq -\frac{\tilde{s}_0}{4} \rho - \frac{1}{10} \left(\frac{3\pi^2}{2} \right)^{2/3} (\tilde{s}_1 + 3\tilde{s}_2) \rho^{5/3}$$

These two results should be identical; therefore

$$\begin{aligned} \tilde{s}_0 &\simeq -\frac{4}{3} \frac{C_1 \sigma_{\pi N}}{f_\pi^2 m_\pi^2} \\ &= -15.5_{-12.5}^{+8.8} \text{ MeV fm}^3 \end{aligned}$$

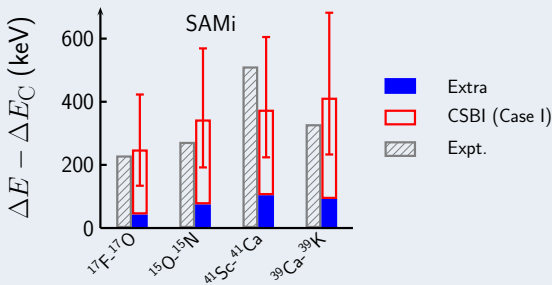
$$\begin{aligned} \tilde{s}_1 + 3\tilde{s}_2 &\simeq \frac{1}{m_N^2} \frac{C_1 \sigma_{\pi N}}{f_\pi^2 m_\pi^2} \\ &= 0.52_{-0.29}^{+0.42} \text{ MeV fm}^5 \end{aligned}$$

$\sigma_{\pi N}$ has the large uncertainty $\sigma_{\pi N} = 45 \pm 15 \text{ MeV}$ (conservative estimation)

Sagawa, [Naito](#), Roca-Maza, and Hatsuda. *Phys. Rev. C* **109**, L011302 (2024)

QCD Sum Rule Approach for CSB EDF

Application for actual Skyrme Hartree-Fock calculation



- “Extra” contribution is not enough to describe ΔE
 - Higher-order correction for the Coulomb interaction
 - Change of kinetic energy due to $m_p \neq m_n$
- QCD-CSB interaction describe ΔE quite nicely
→ ONS anomaly may be solved?

Sagawa, Naito, Roca-Maza, and Hatsuda. *Phys. Rev. C* **109**, L011302 (2024)

Oppositely...

- If one extracts the CSB strength in the consistent way, we may determine $\langle \bar{q}q \rangle_\rho$ throughout this method
→ This may be an alternative method to pin down $\langle \bar{q}q \rangle_\rho$
- For instance, pionic atoms are used to determine $\langle \bar{q}q \rangle_\rho$ now
→ Probably, we can access $\langle \bar{q}q \rangle_\rho$ at different density?
- However, extracting the CSB strength independently from other interactions is still difficult because of its weak
 - Mass difference of mirror nuclei

Bączyk *et al.* *Phys. Lett. B* **778**, 178 (2018)

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 - Mass difference of mirror nuclei
 - “Mirror-skin thickness” $\Delta R_{\text{mirror}}(Z, N) = R_p(Z, N) - R_n(N, Z)$

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Conclusion

- CSB and CIB terms contribute to ΔR_{np} of ^{208}Pb in -0.02 fm (12 MeV in L value)
- CSB is related to chiral symmetry breaking and its restoration
- QCD sum rule approach gives CSB EDF
- Perspectives
QCD-based CIB $\leftarrow$ originating from pion pairing, deformation, reveal the open problem, (Q)RPA calc., ...
- Ultimate goal
 - **“Complete & accurate” nuclear EDF**
 - **Can we understand “in-medium effect” from QCD?**

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Thank you for attention!!