Anatomy of critical fluctuations in hadronic matter

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References:

[1] **MM**, K Redlich, C. Sasaki PRD 107, (2023) 5, 054046

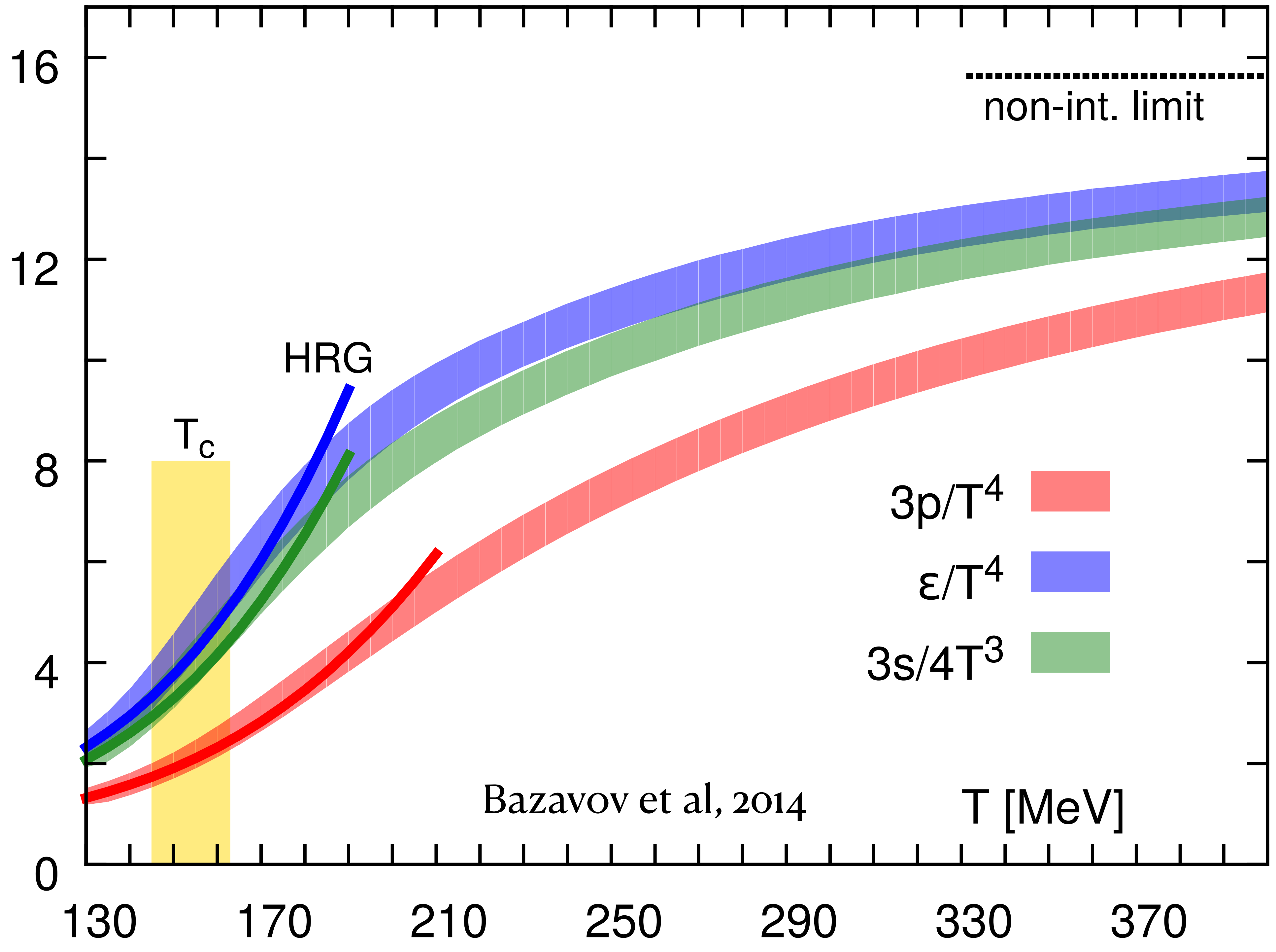
[2] V. Koch, **MM**, K Redlich, C. Sasaki, PRD 109 (2024) 1, 014033

[3] **MM**, PRD 110 (2024) 1, 014018

[4] **MM**, K Redlich, C. Sasaki arXiv:2410.21746 (2024)

26.05.2025, International Nuclear Physics Conference, Daejeon, Korea

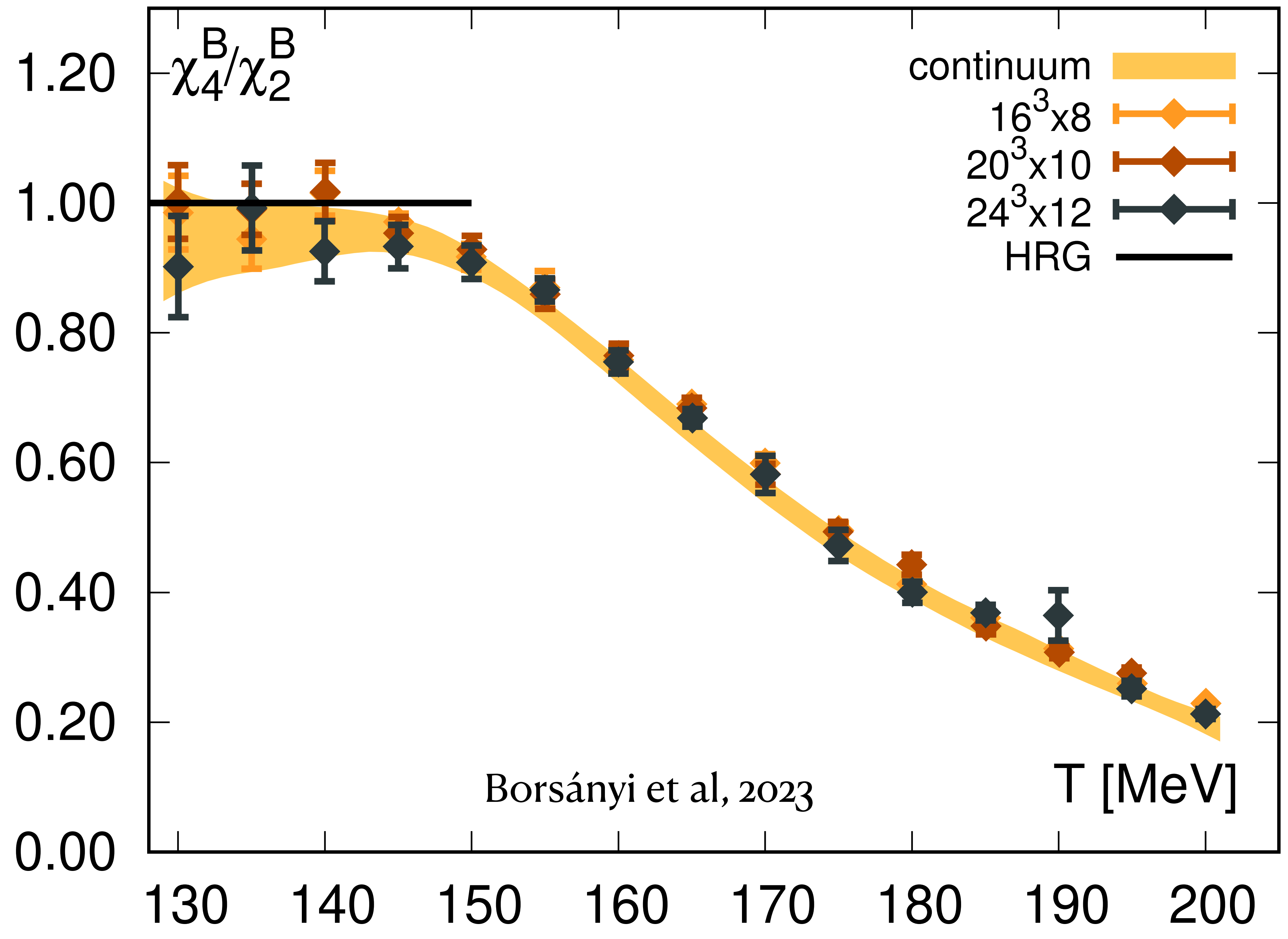
Lattice QCD vs Hadron Resonance Gas



Pressure in the HRG model

$$P^{\mathrm{HRG}} = \sum_{i \in \mathrm{had}} P^{\mathrm{id}}\left(T, \mu_i; m_i\right)$$

Excellent agreement with LQCD EoS up to $\simeq T_c$

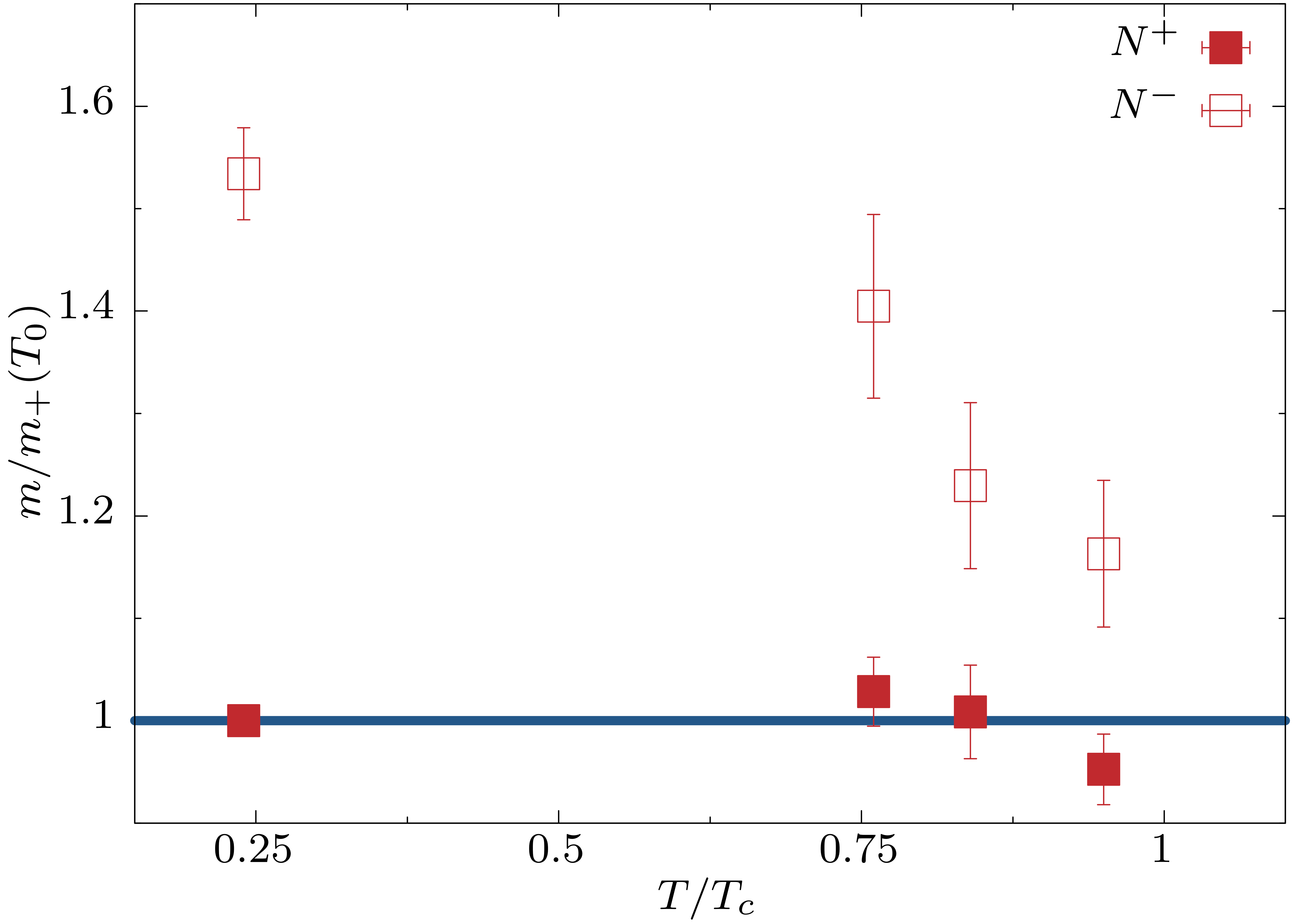


Taylor expansion of LQCD EoS

$$\frac{P}{T^4} = \sum_{k=0}^{\infty} \left(\frac{\mu_B}{T}\right)^k \frac{\chi_k^B}{k!}, \text{ where } \chi_k^B = \frac{\partial^k P/T^4}{\partial \left(\mu_B/T\right)^k}$$

Kurtosis: $\chi_4^B/\chi_2^B \sim B^2$

Parity Doubling in Lattice QCD Aarts et al, 2017, 2019



- N^+ nucleon stays nearly unchanged
- N^- chiral partner drops mass towards T_c
- Chiral partners $N^\pm$ degenerate at T_c
- Chiral partners stay massive
- Seen for octet and decouplet of baryons

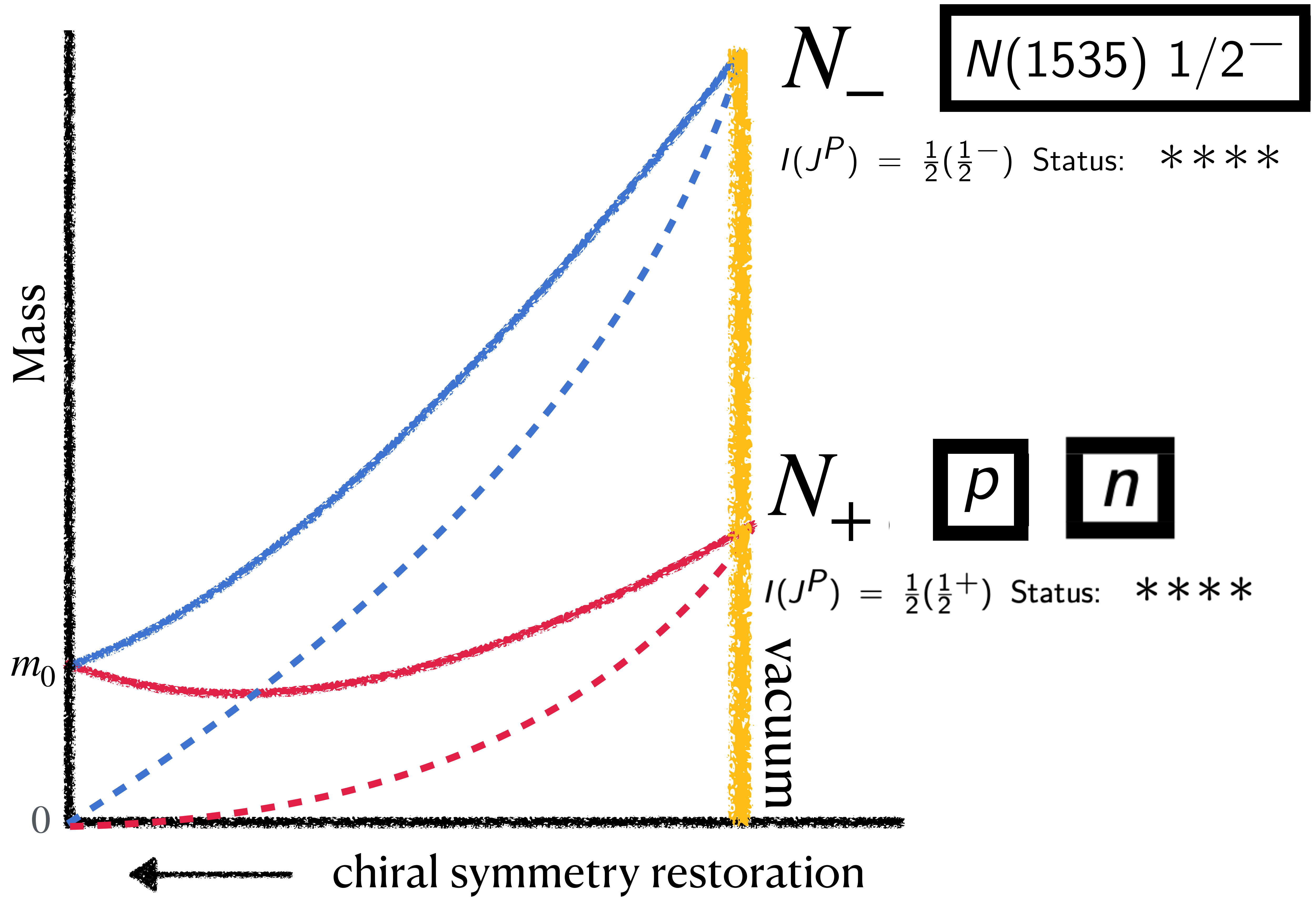
Imprint of chiral symmetry restoration in the baryonic sector

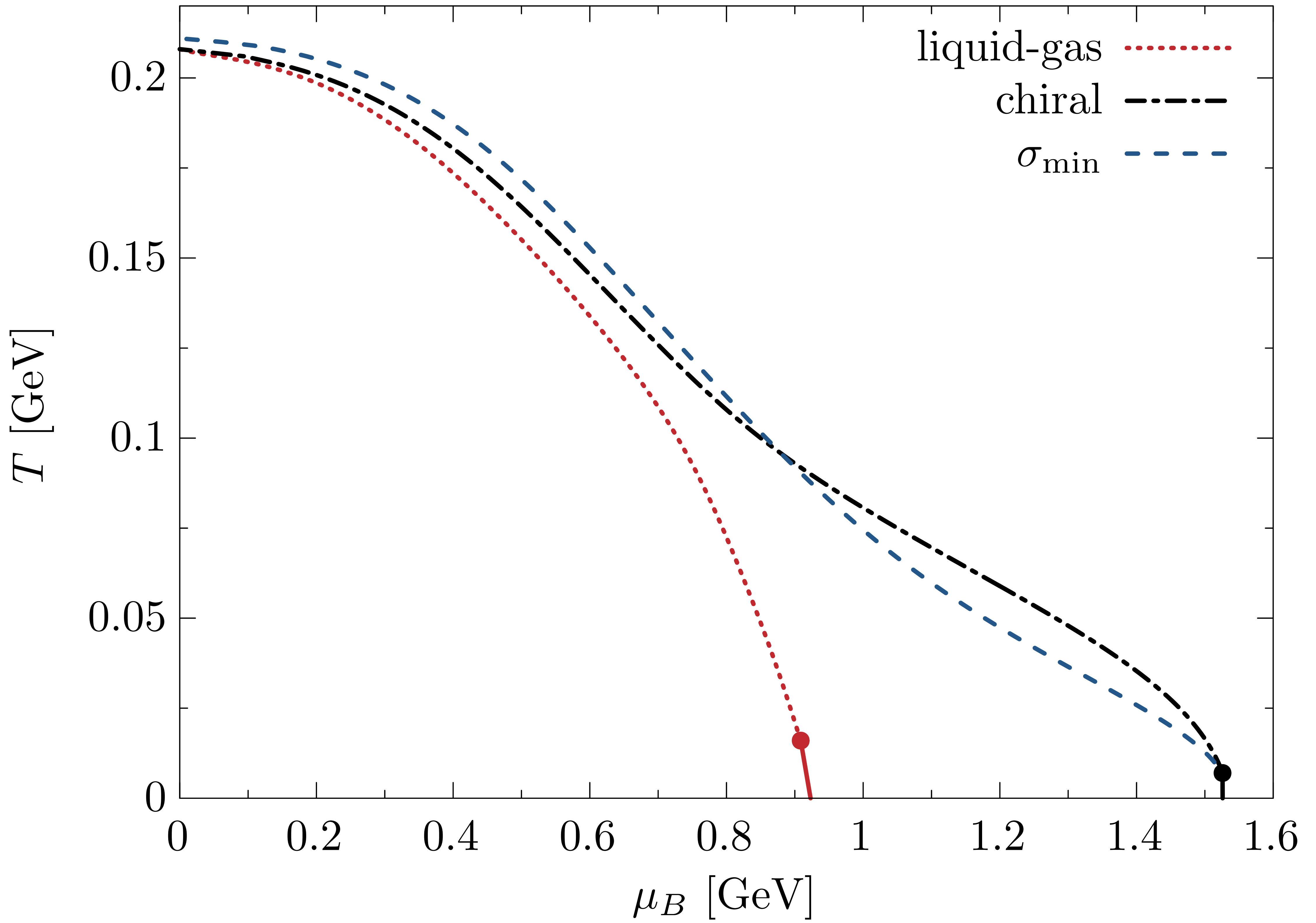
LQCD results still obtained with heavy m_π far from continuum limit

Parity Doublet Model a'la DeTar, Kunihiro 1989

- SU(2) chiral transformation of 2 nucleons → how to assign 2 independent rotation to them?

$$\mathscr{L}_{\text{mass}} \sim m_0\left(\bar{\psi}_1\gamma_5\psi_2 + \bar{\psi}_2\gamma_5\psi_1\right) \Longrightarrow M_{\pm} = \frac{1}{2}\left(\sqrt{4m_0^2 + a^2\sigma^2} \mp b\sigma\right) \xrightarrow{\sigma\to 0} m_0$$





For multiplicity $N_B = N_+ + N_-$ Net-baryon number: $\langle N_B \rangle = \langle N_+ \rangle + \langle N_- \rangle$

Second-order fluctuations of the net-baryon number:

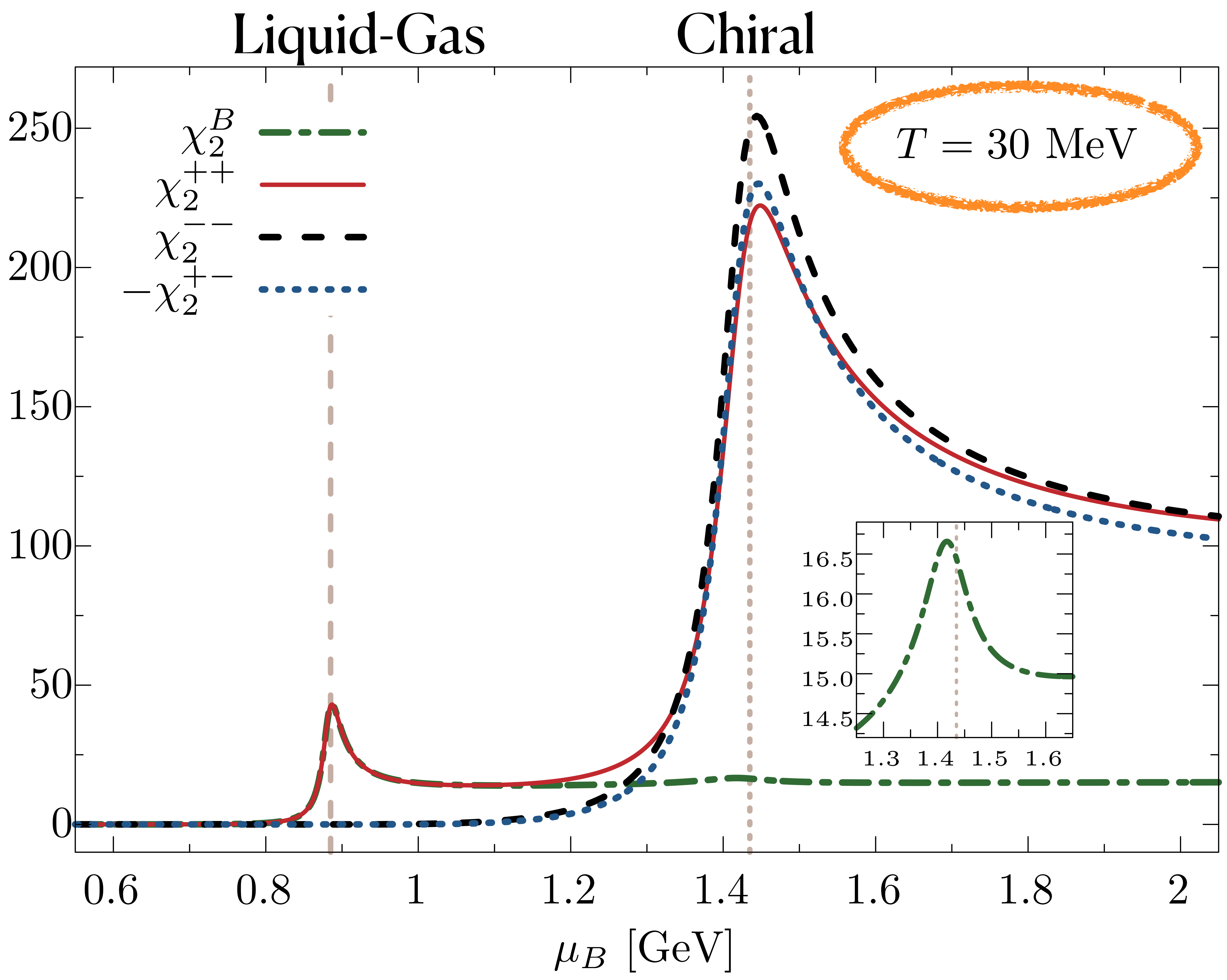
$$\langle \delta N_B \delta N_B \rangle = \langle (\delta N_+)^2 \rangle + \langle (\delta N_-)^2 \rangle + 2 \langle \delta N_+ \delta N_- \rangle$$

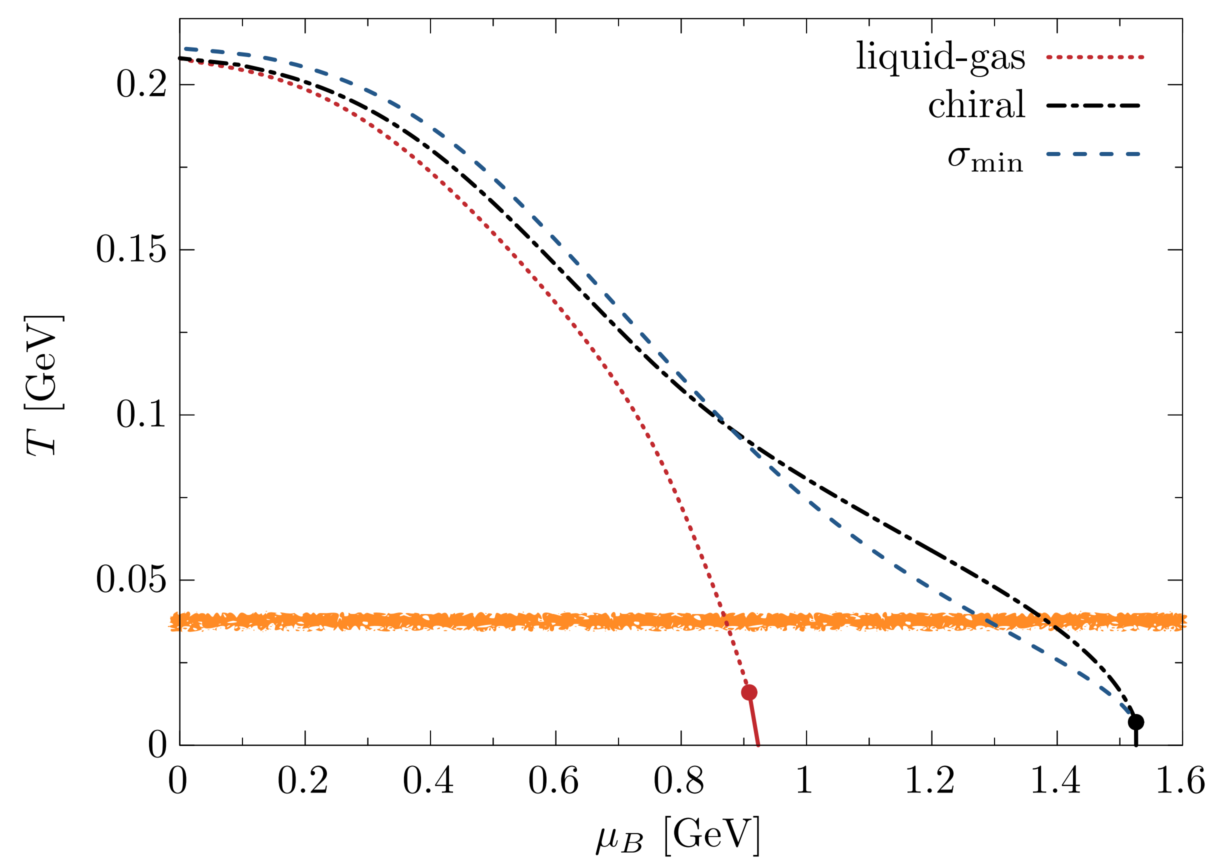
$$\langle \delta N_\alpha \delta N_\beta \rangle = V T^3 \chi_n^{\alpha\beta} \longleftrightarrow \chi_2^{\alpha\beta} = \frac{d^2 P / T^4}{d(\mu_\alpha / T) d(\mu_\beta / T)}$$

$$\chi_2^B = \chi_2^{++} + \chi_2^{--} + 2\chi_2^{+-}$$

- What are the individual contributions of parity partners N_+ and N_-?
- What is the strength and sign of the correlation χ_2^{+-}?
- Is net-proton a good proxy for net-baryon fluctuations? $\chi_2^B = \chi_2^{++} + \cancel{\chi_2^{--} + 2\chi_2^{+-}}$

Fluctuations at liquid-gas and chiral transitions



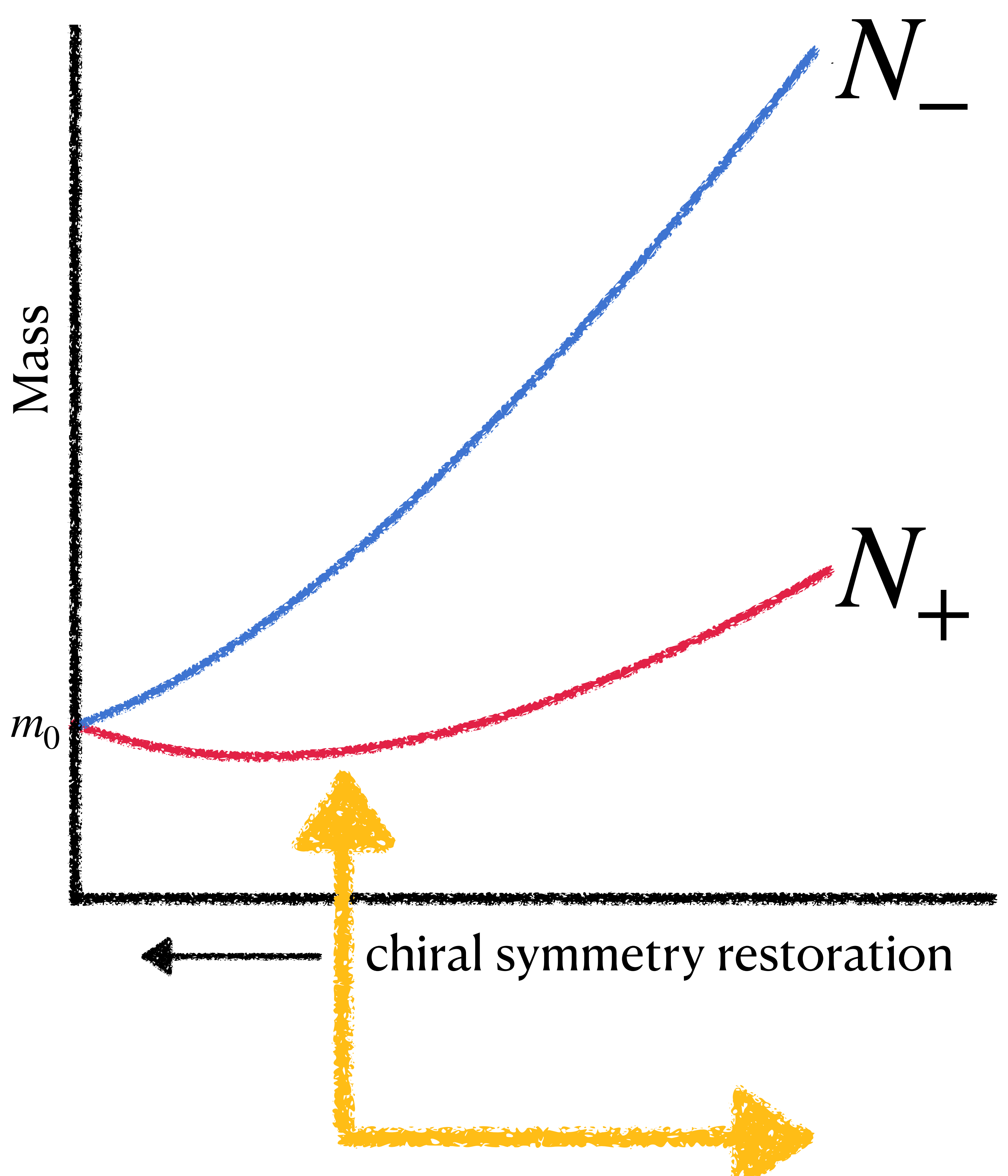


- Liquid-Gas dominated by χ_2^{++}
- Chiral dominated by χ_2^{++} and χ_2^{--}
- Peaks diminished by negative χ_2^{+-}

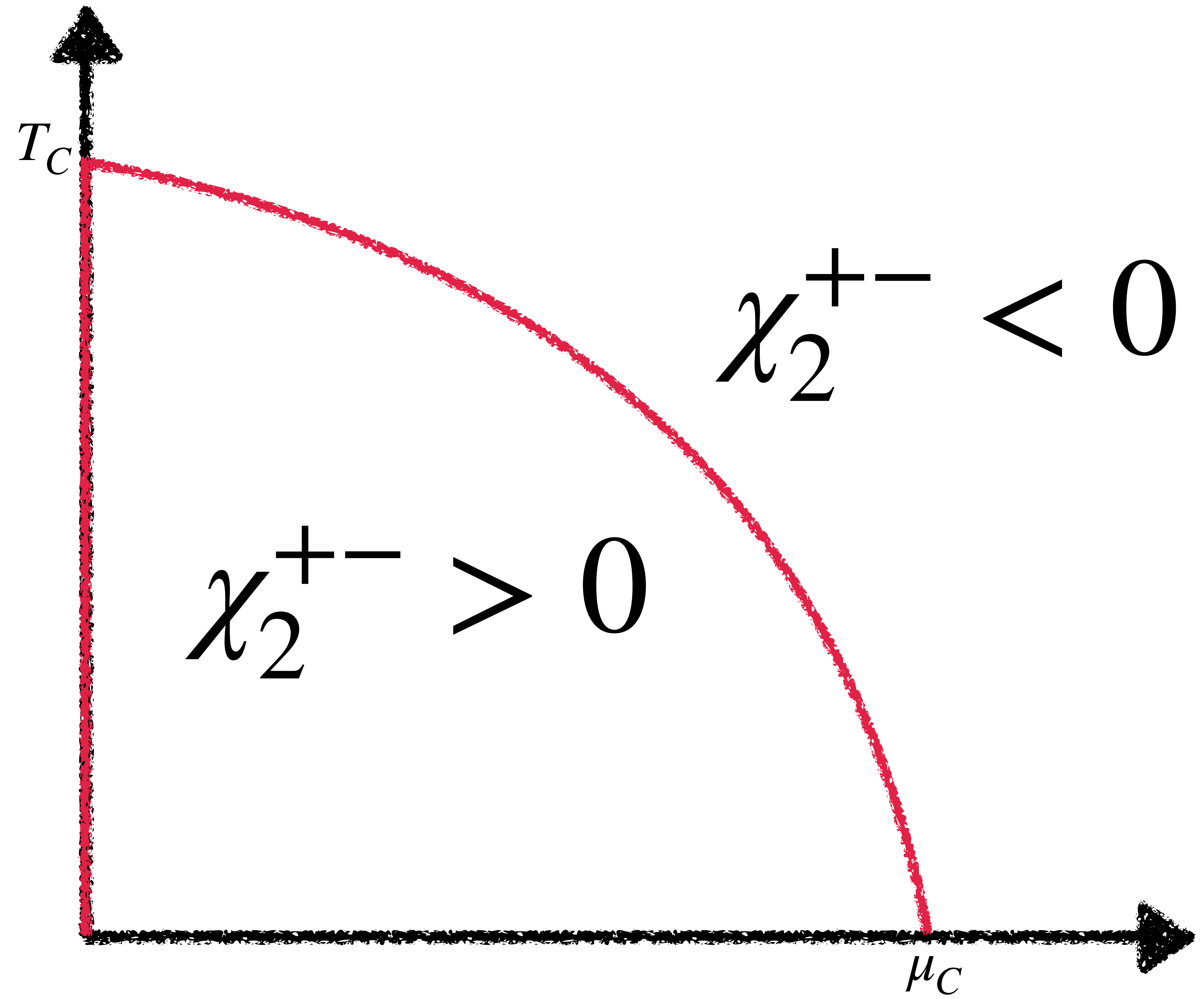
$$\chi_2^B = \chi_2^{++} + \chi_2^{--} + 2\chi_2^{+-}$$

weak signal in χ_2^B

Idealized behavior of the χ_2^{+-} correlator $\longrightarrow$ no repulsive forces



Min of M_+



$$\chi_2^{+-} \sim \frac{\partial m_+}{\partial \sigma} \frac{\partial m_-}{\partial \sigma}$$

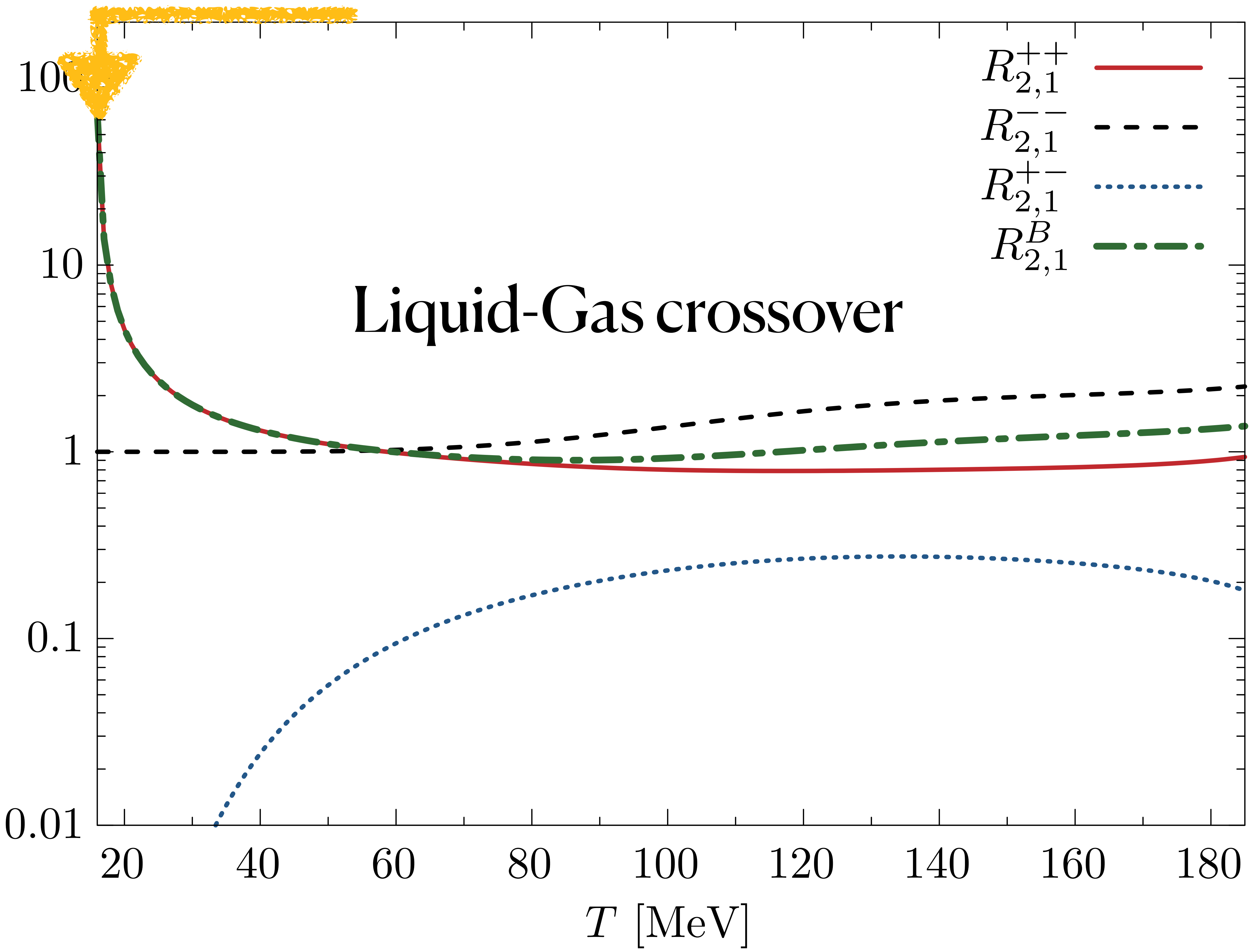
but also repulsion

Correlations of between different baryon species e.g., $N^{\pm}\Delta^{\mp}$, behave similarly

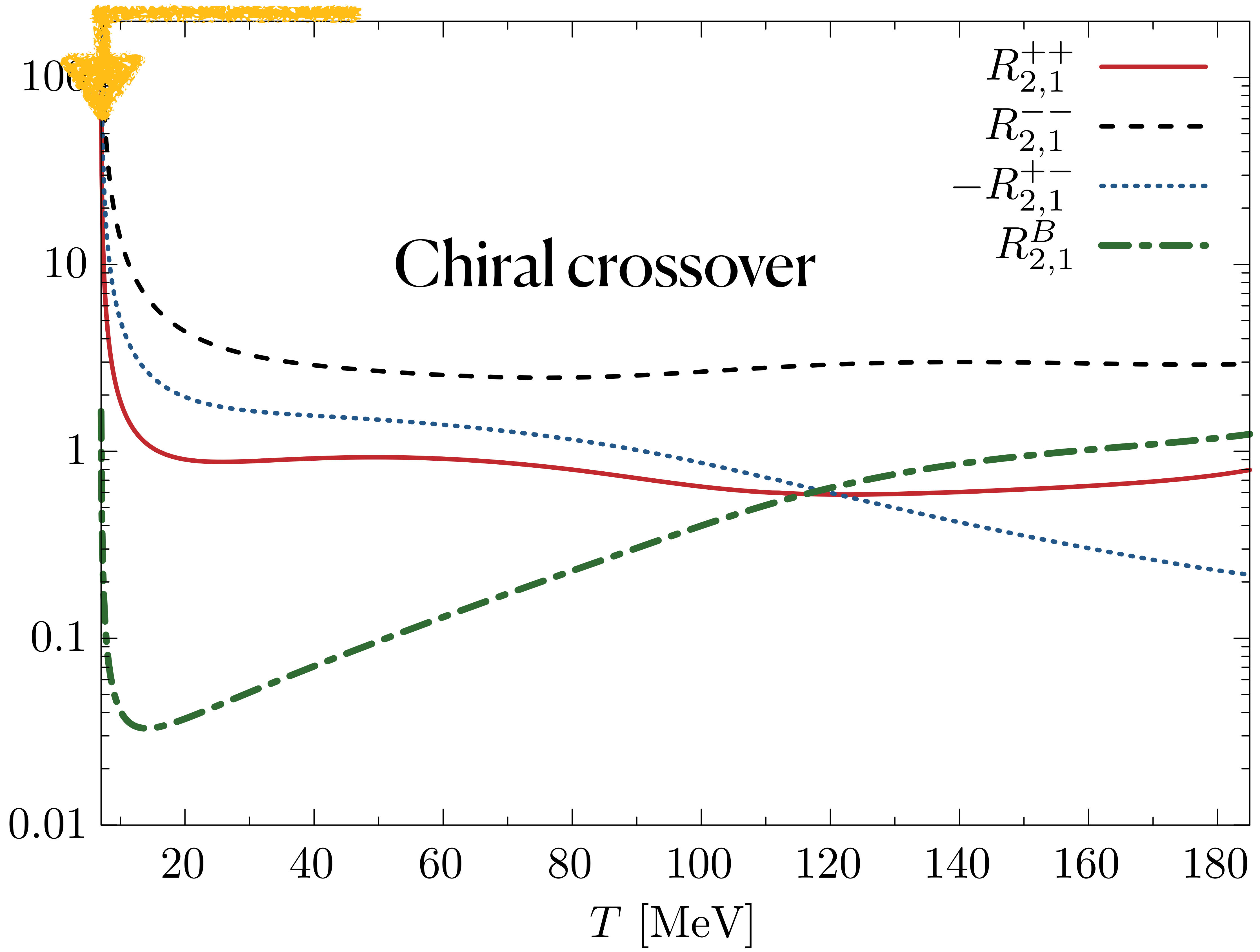
Change of the sign of χ_2^{+-} linked to the chiral phase boundary $\longrightarrow$ interesting quantity to calculate in LQCD

$R_{2,1} = \chi_2/\chi_1$ along phase boundary

Liquid-Gas CP



Chiral CP



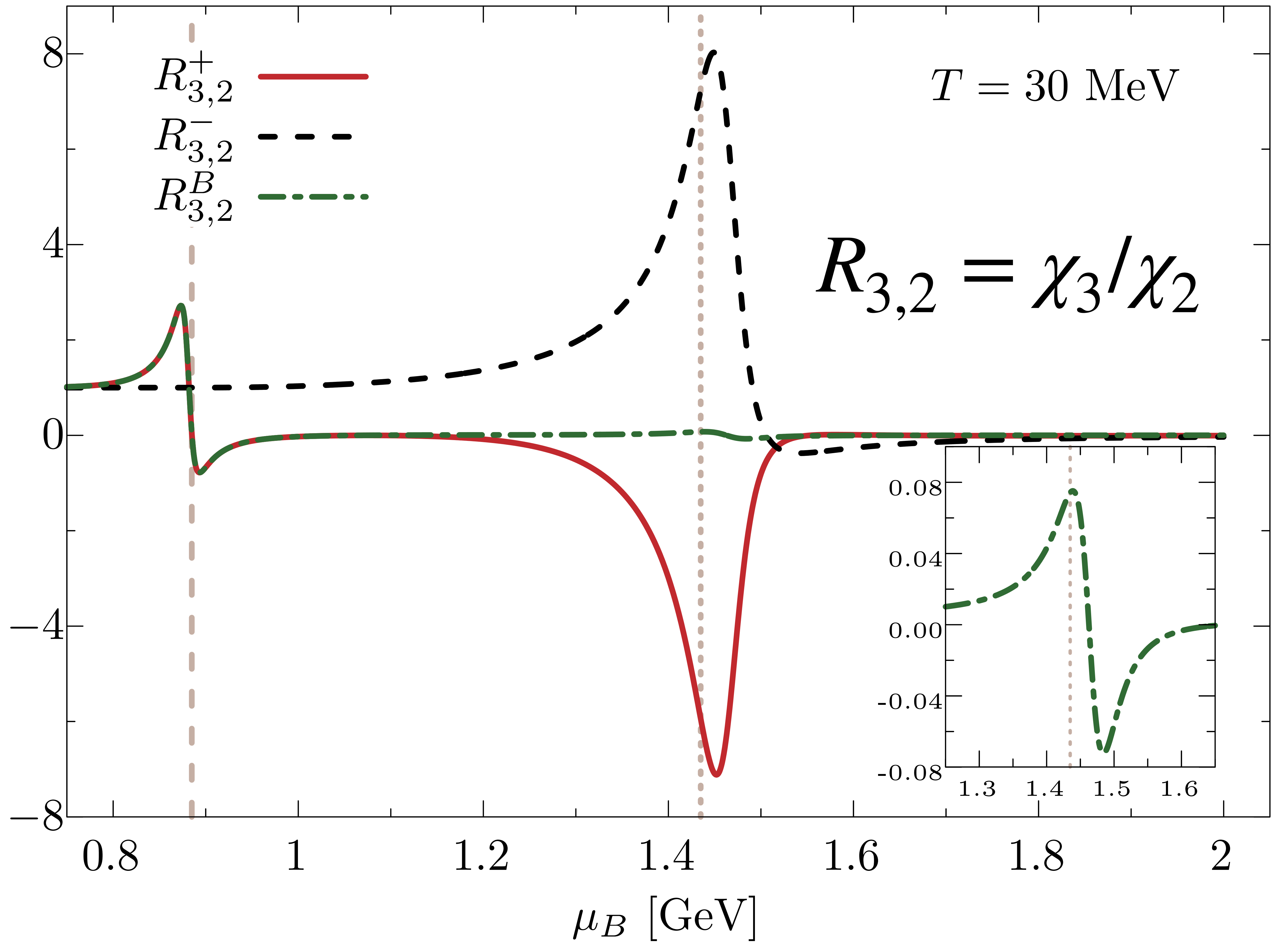
Fluctuations dominated by positive parity

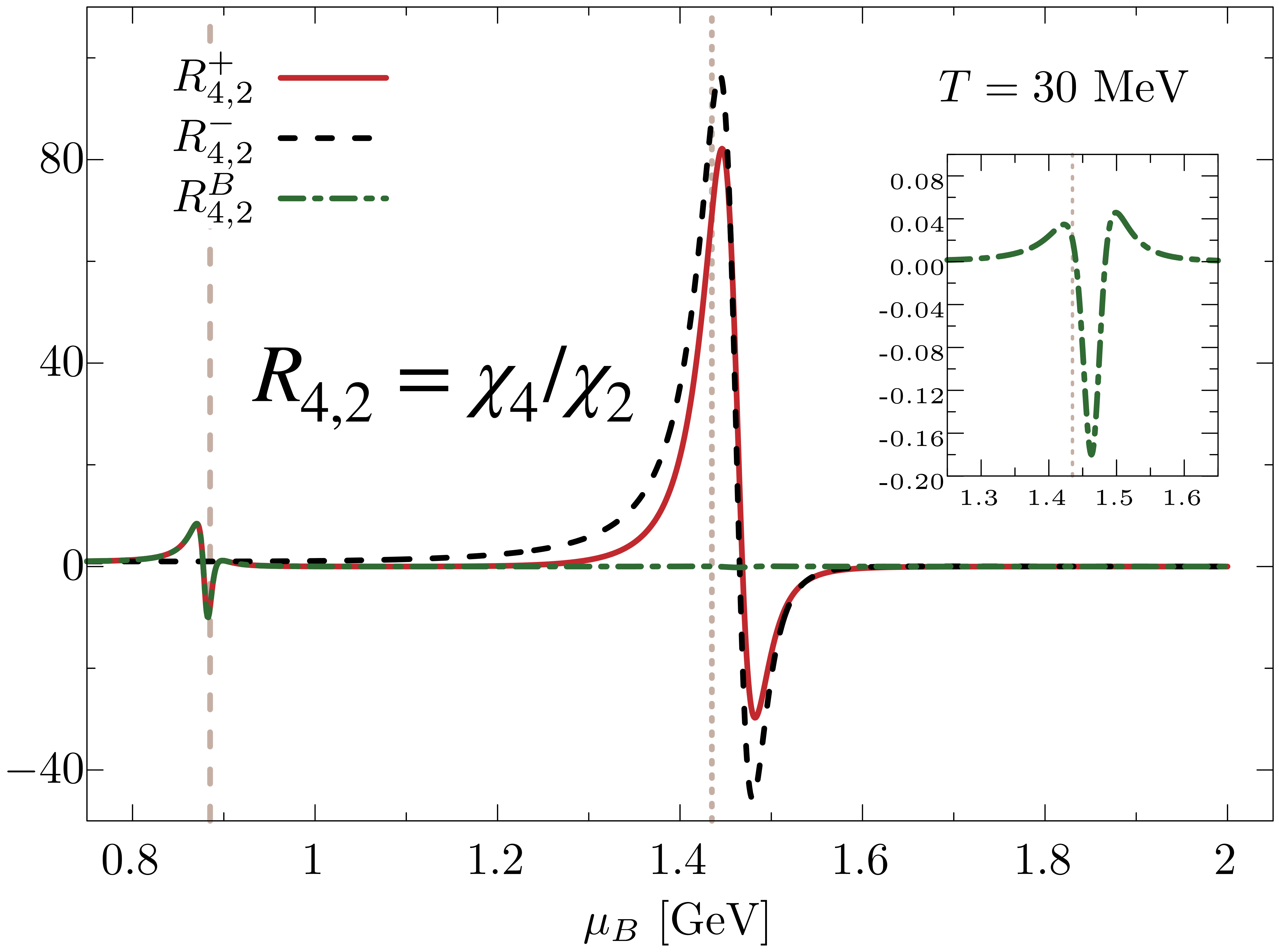
Net-baryon ~ Net-nucleon

Presence of chiral partners + correlations

Net-baryon ≪ Net-nucleon

Higher-Order Fluctuations of Parity Partners

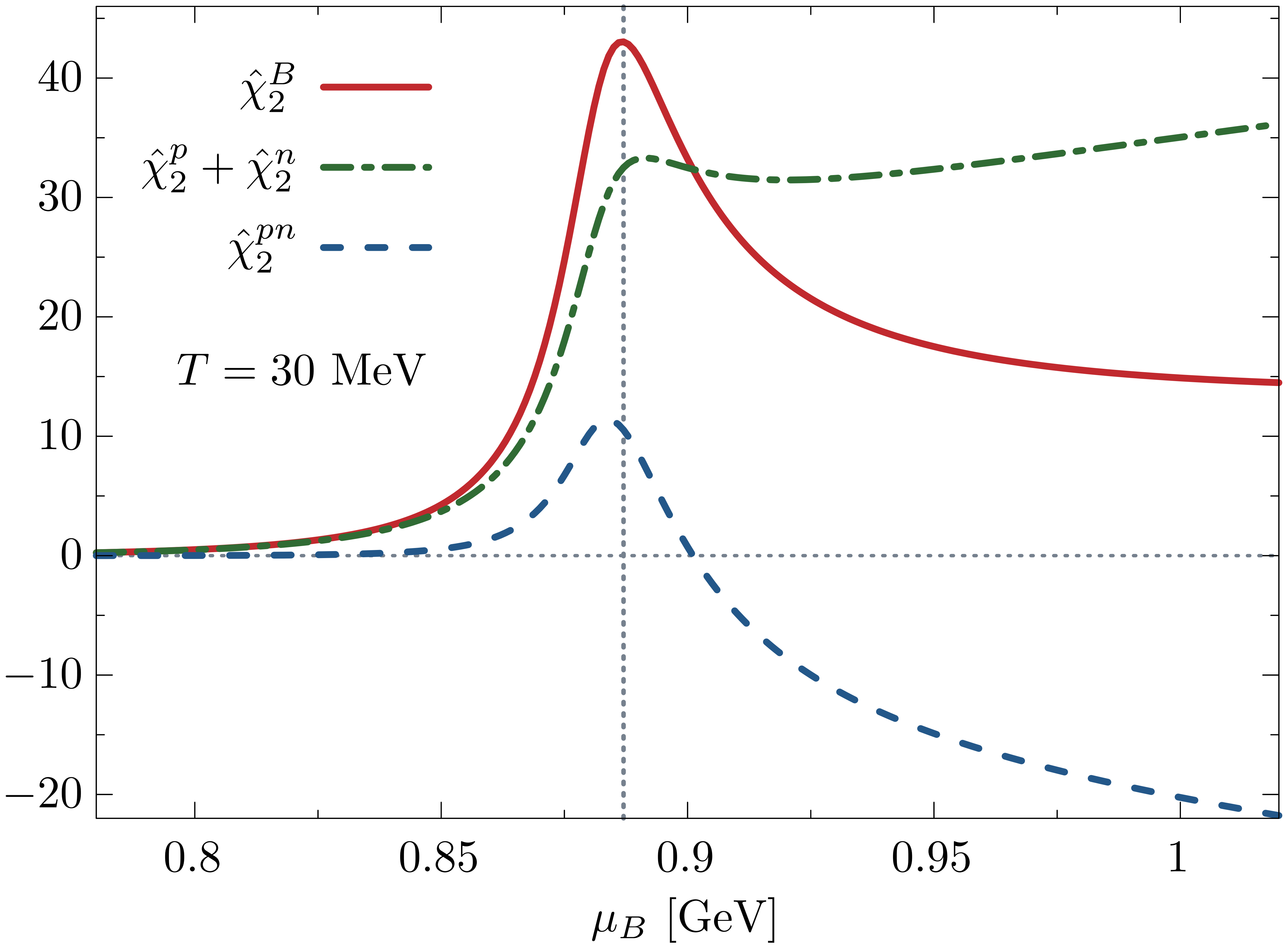


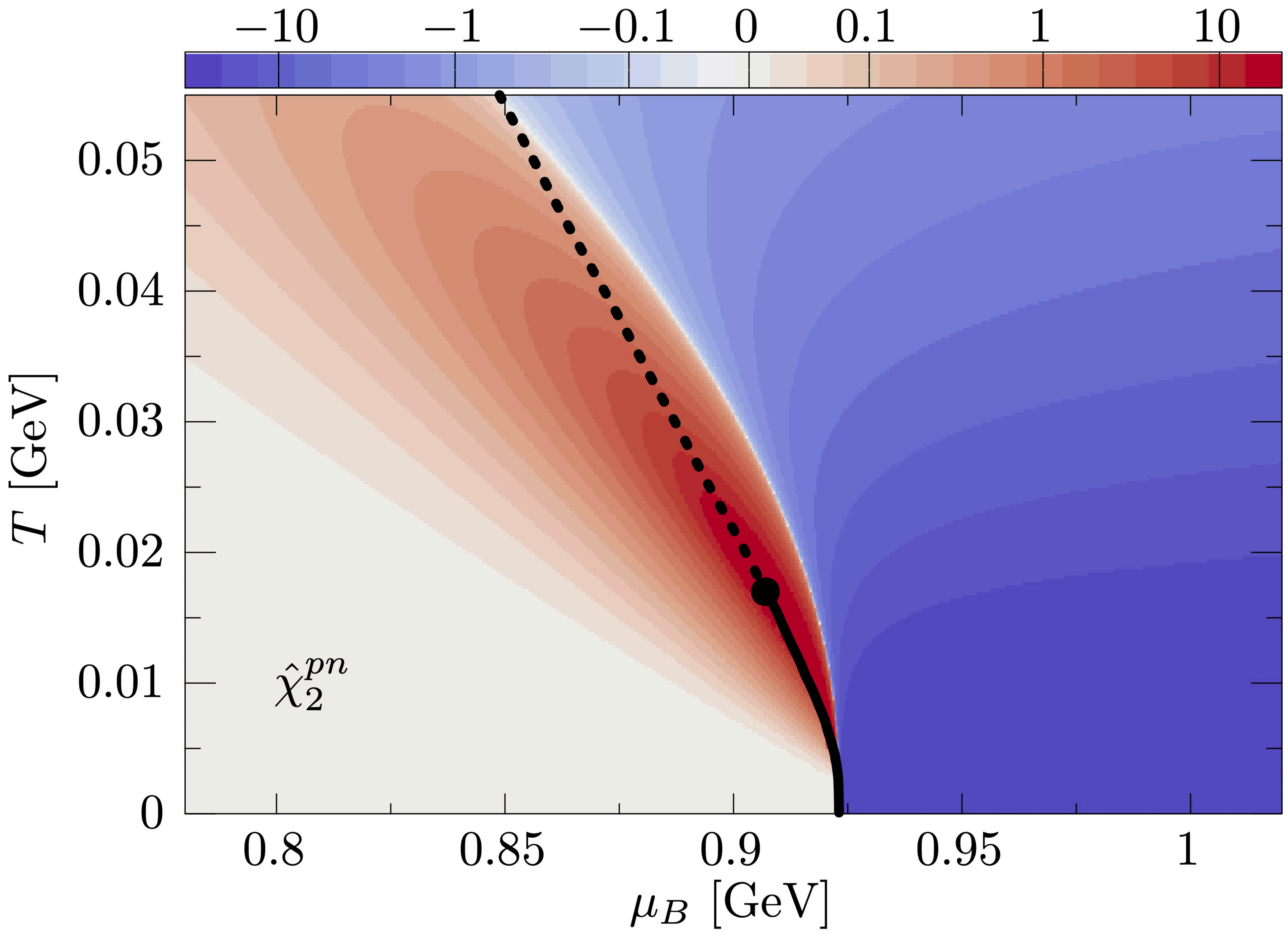


The net-proton fluctuations do not necessarily reflect the net-baryon fluctuations at the chiral phase boundary

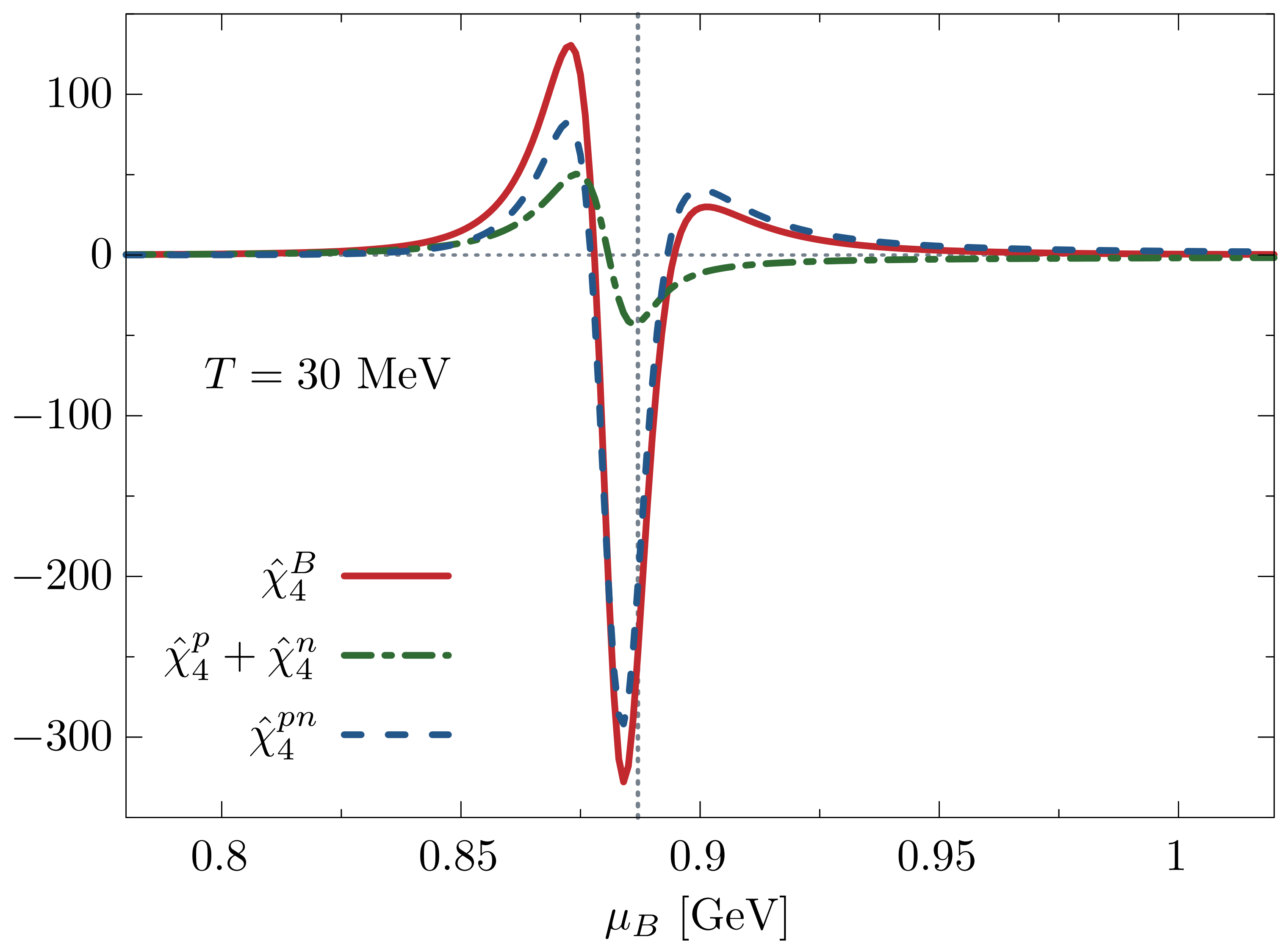
Isospin Correlations Near the Liquid-Gas Transition

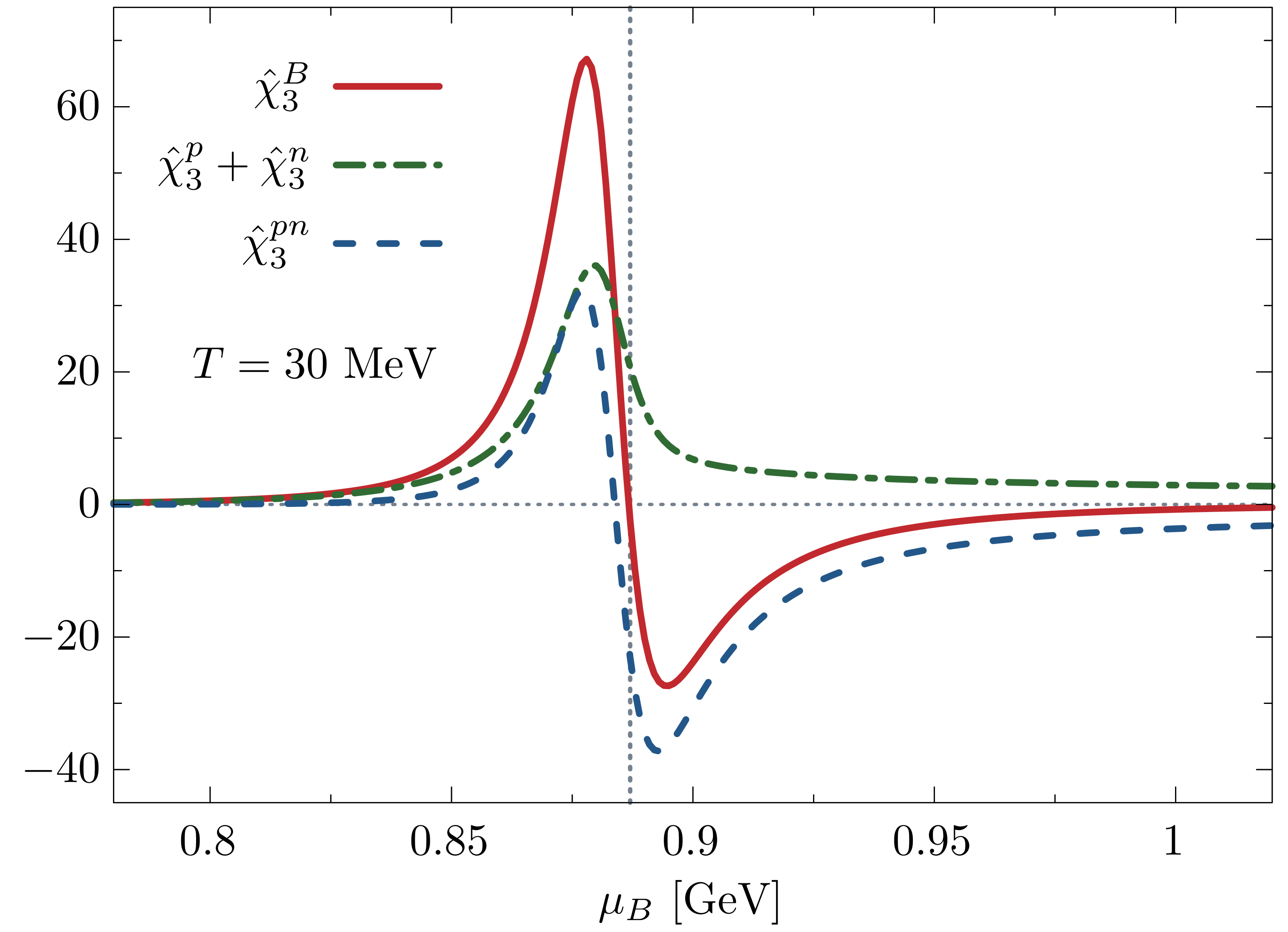
$$\chi_2^B = \chi_2^{++} + \ldots \simeq \chi_2^p + \chi_2^n + \chi_2^{pn} \neq 2\chi_2^p$$

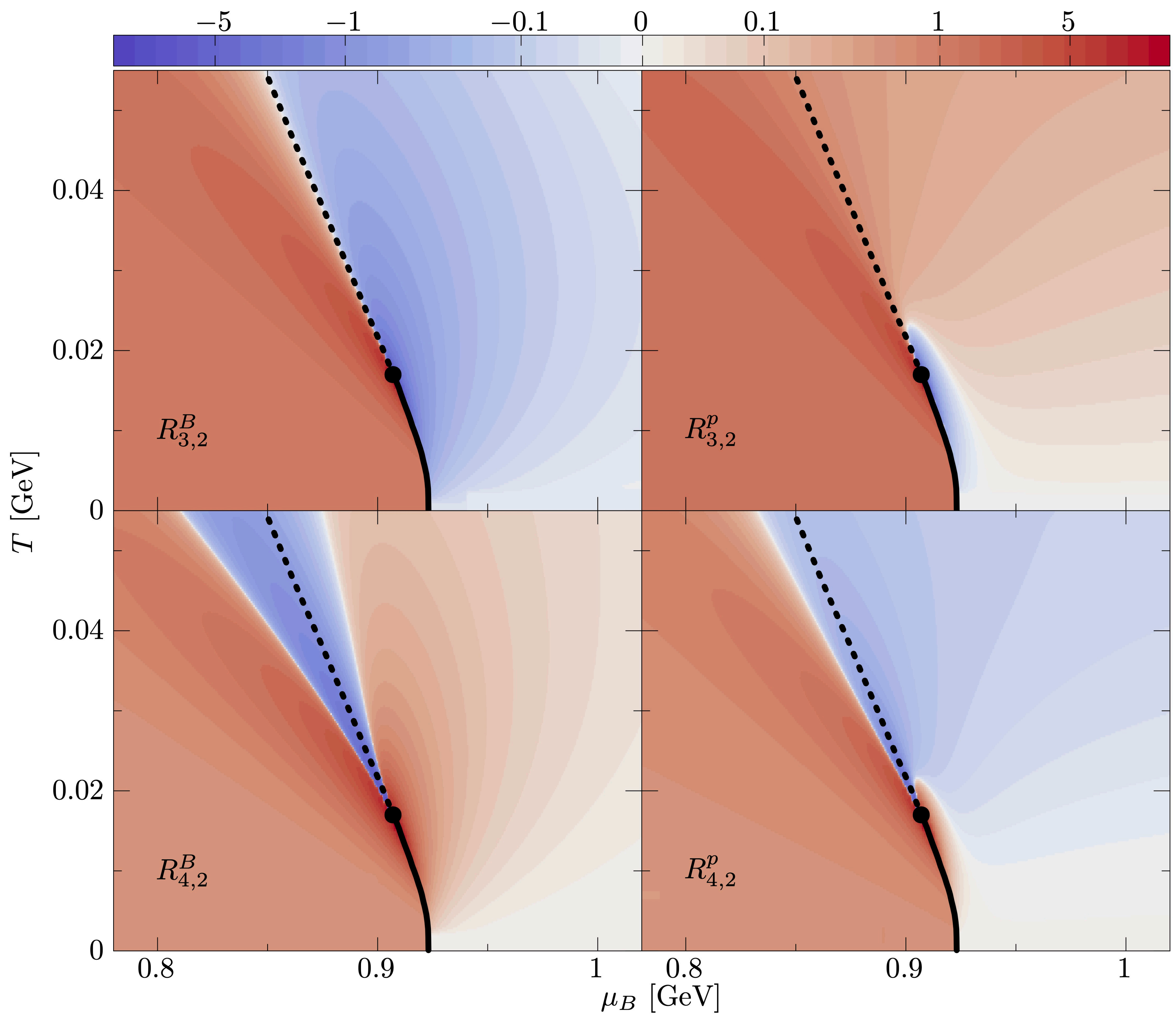




Isospin Correlations Near the Liquid-Gas Transition







Differences clearly visible for higher-order fluctuations

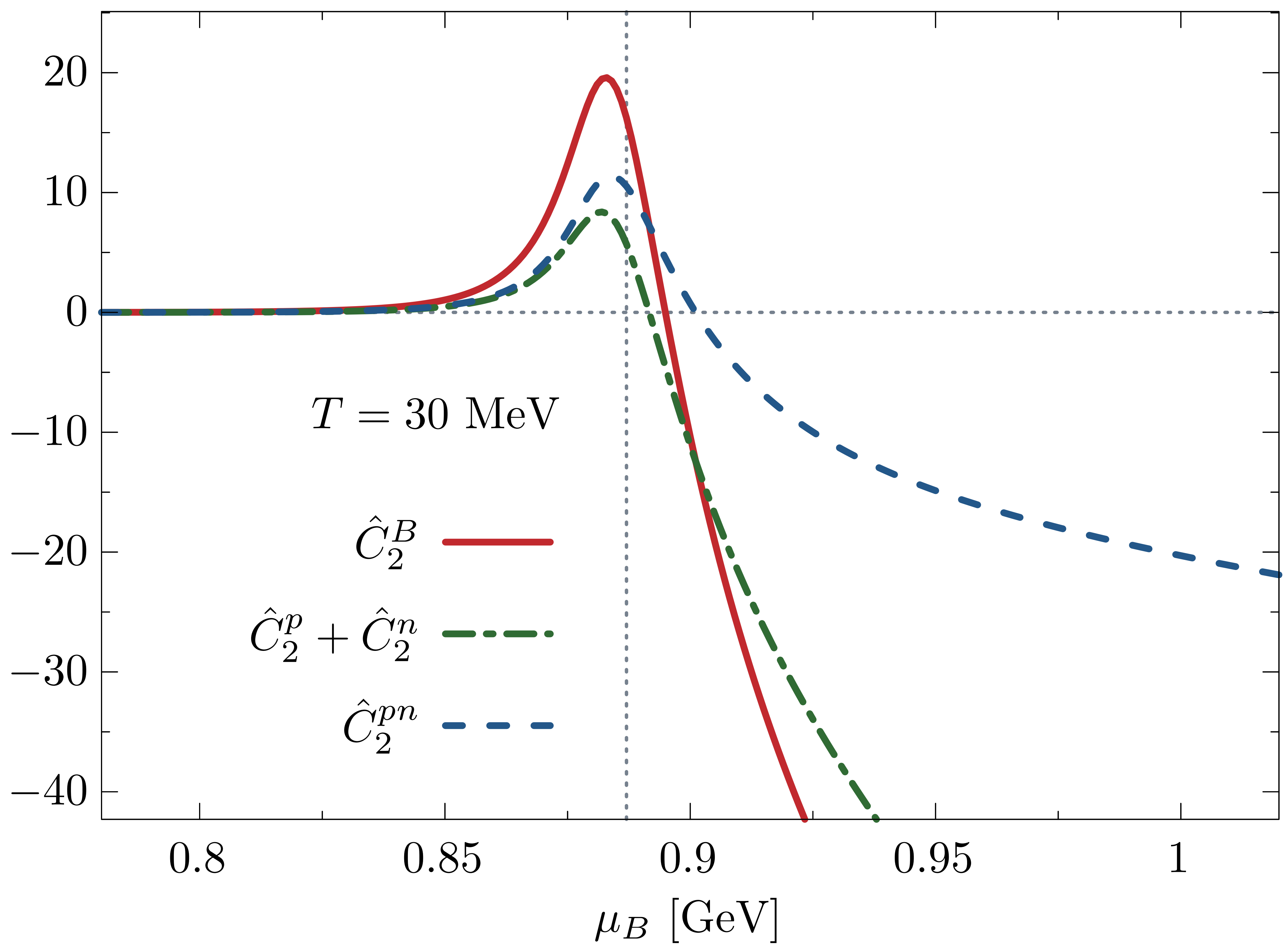
Isospin Correlations: Factorial Cumulants

Factorial Cumulants

$$\hat{C}_1^\alpha = \hat{\chi}_1^\alpha$$
$$\hat{C}_2^\alpha = \hat{\chi}_2^\alpha - \hat{\chi}_1^\alpha$$
$$\hat{C}_3^\alpha = \hat{\chi}_3^\alpha - 3\hat{\chi}_2^\alpha + 2\hat{\chi}_1^\alpha$$
$$\hat{C}_4^\alpha = \hat{\chi}_4^\alpha - 6\hat{\chi}_3^\alpha + 11\hat{\chi}_2^\alpha - 6\hat{\chi}_1^\alpha$$

"Factorial Correlations"

$$\hat{C}_1^{pn} \equiv \hat{C}_1^B - 2\hat{C}_1^p = 0$$
$$\hat{C}_2^{pn} \equiv \hat{C}_2^B - 2\hat{C}_2^p = \hat{\chi}_2^{pn}$$
$$\hat{C}_3^{pn} \equiv \hat{C}_3^B - 2\hat{C}_3^p = \hat{\chi}_3^{pn} - 3\hat{\chi}_2^{pn}$$
$$\hat{C}_4^{pn} \equiv \hat{C}_4^B - 2\hat{C}_4^p = \hat{\chi}_4^{pn} - 6\hat{\chi}_3^{pn} + 11\hat{\chi}_2^{pn}$$



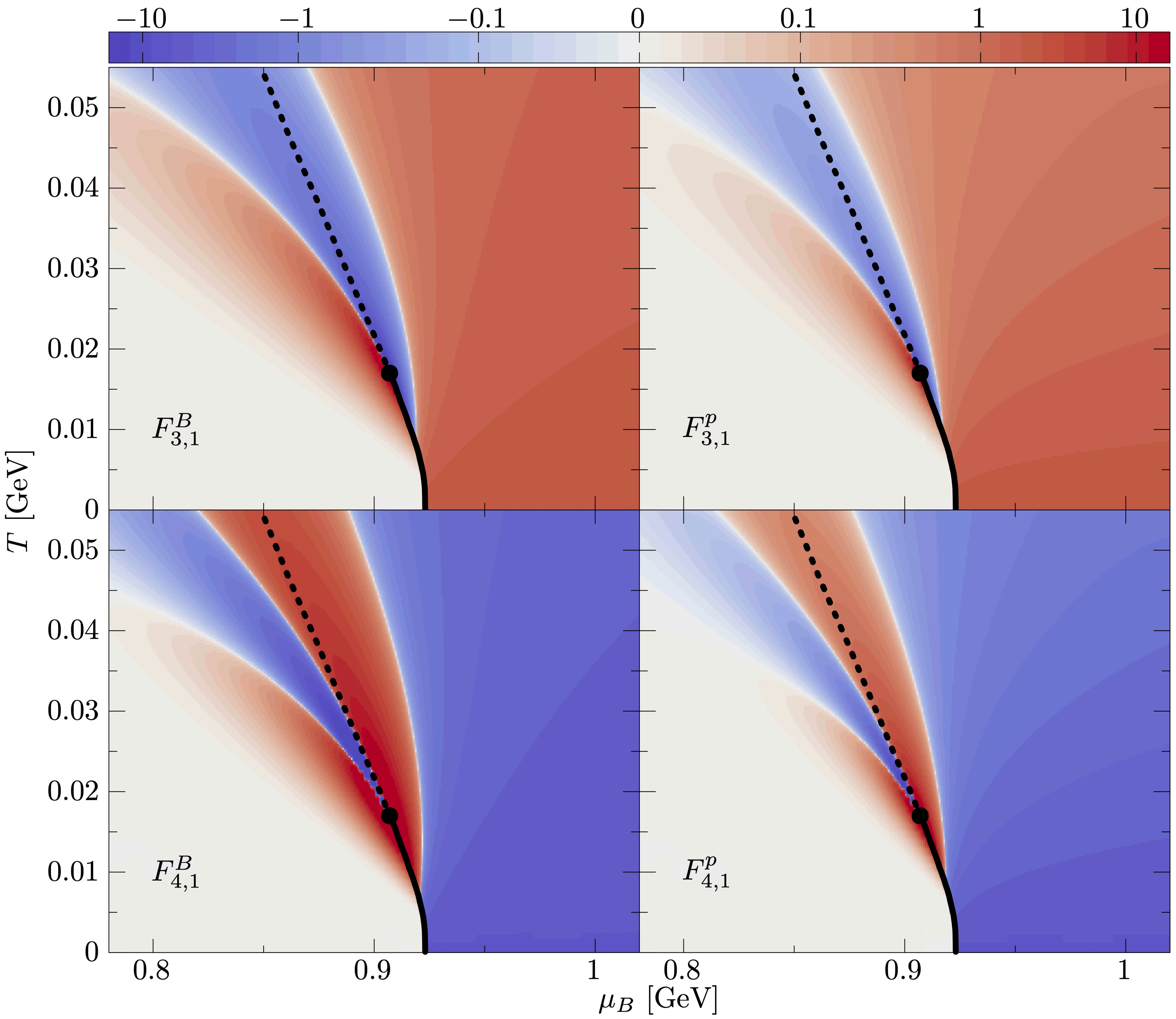
baryons vs protons:

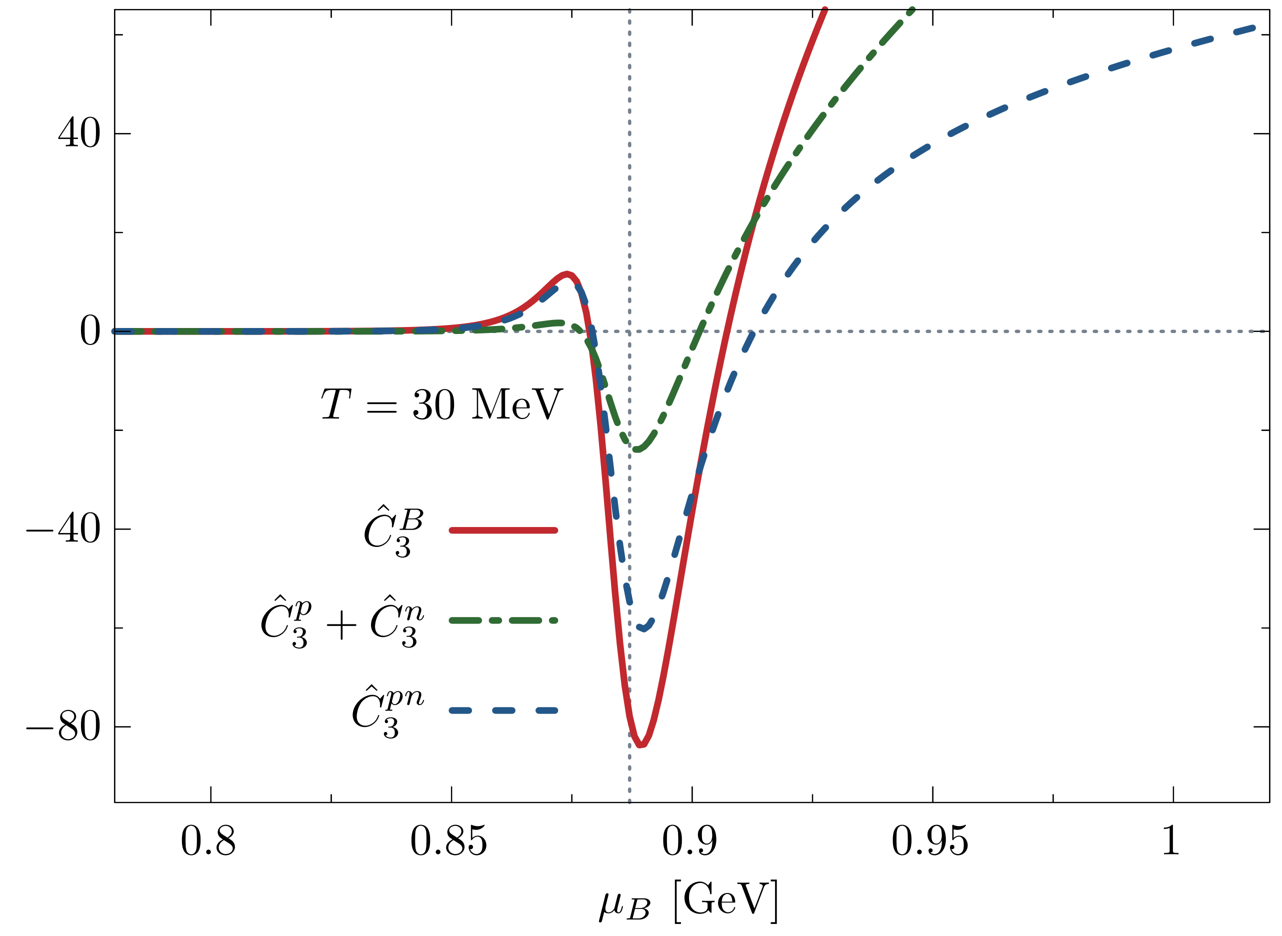
$$\hat{C}_n^B \sim 2^n \hat{C}_n^p$$

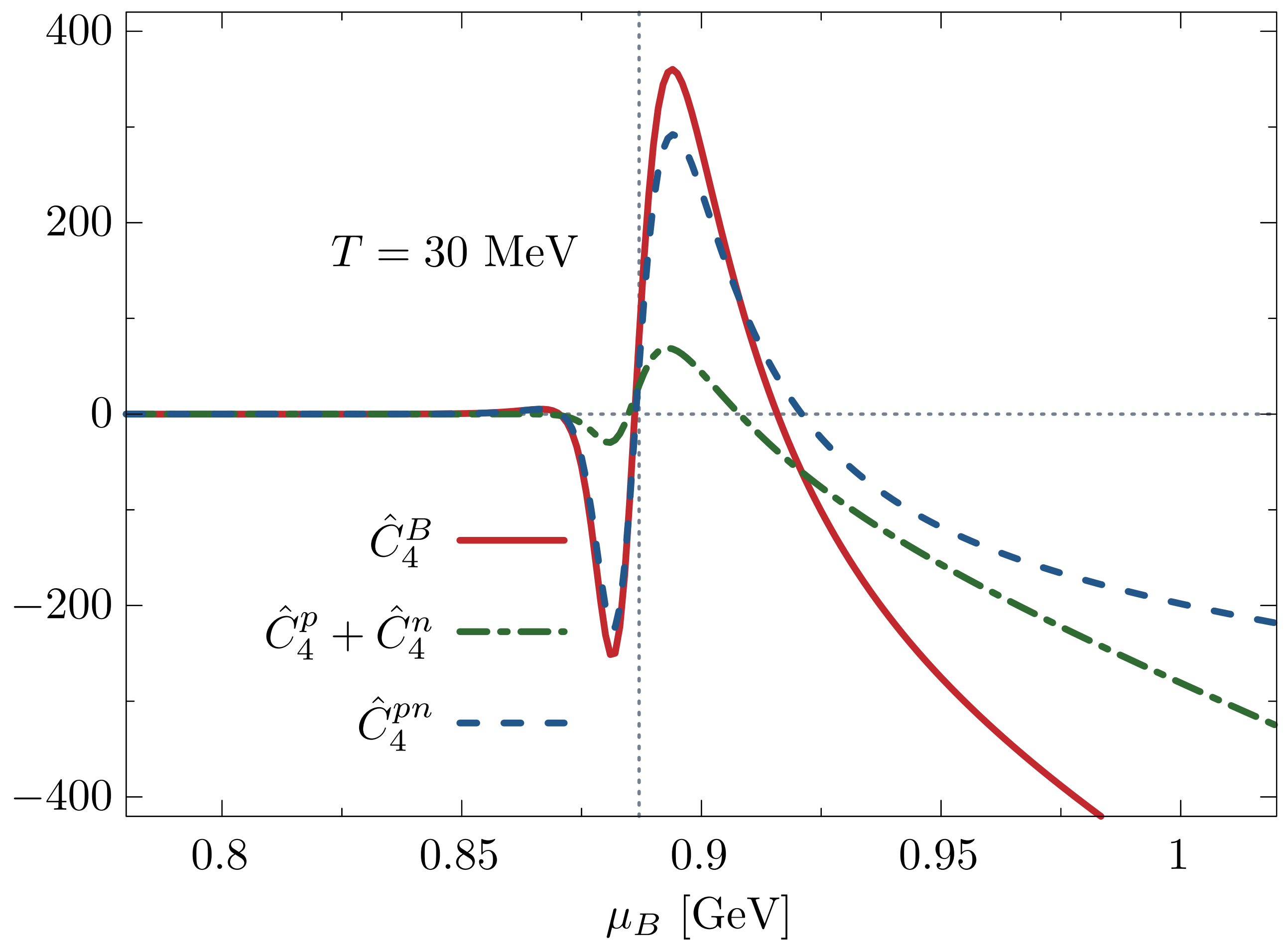
Kitazawa, Asakawa (2012)

Same Sign?

Isospin Correlations: Factorial Cumulants







In general $\hat{C}^B_n \sim 2^n \hat{C}^p_n$ scaling broken by non-trivial correlations near LG transition

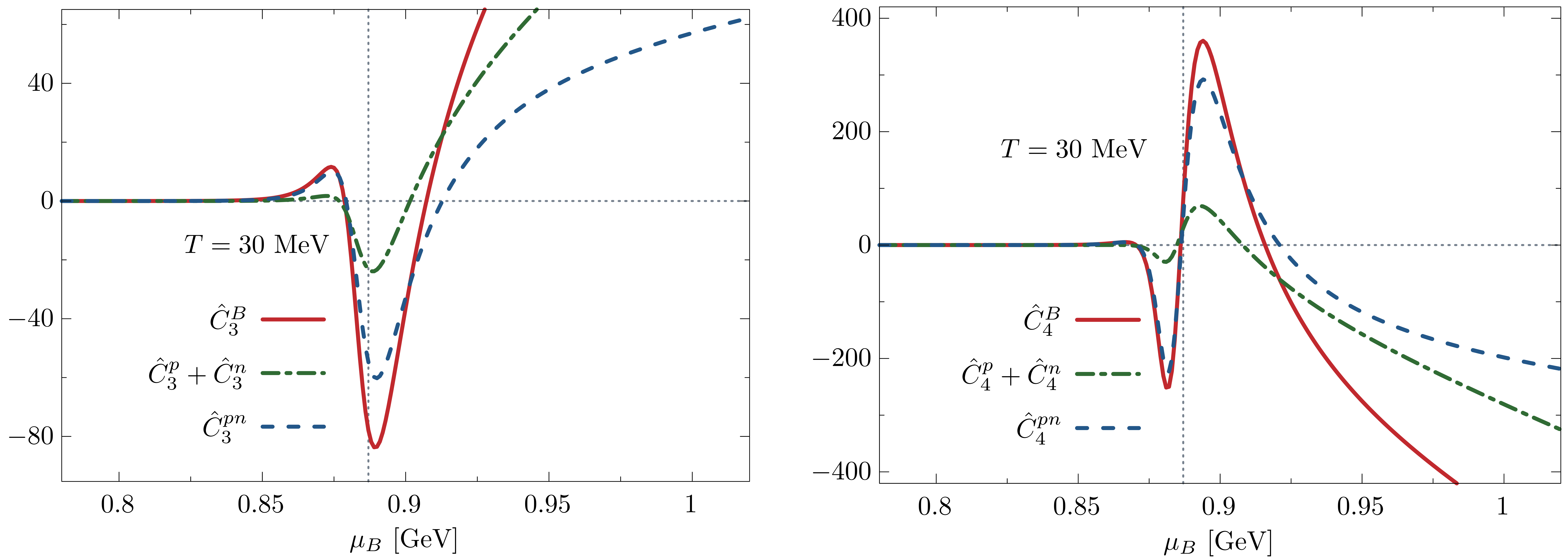
Summary

Non-trivial correlations between baryonic chiral partners

Interesting to calculate χ_2^{+-} in other non-perturbative approaches

χ_2^{proton} may not reflect χ_2^B at the chiral or LG phase boundary

Thank You



40
0
−40
−80
$T = 30$ MeV
$\hat{C}_3^B$
$\hat{C}_3^p + \hat{C}_3^n$
$\hat{C}_3^{pn}$
0.8
0.85
0.9
0.95
1
μ_B [GeV]
400
200
0
−200
−400
$T = 30$ MeV
$\hat{C}_4^B$
$\hat{C}_4^p + \hat{C}_4^n$
$\hat{C}_4^{pn}$
0.8
0.85
0.9
0.95
1
μ_B [GeV]

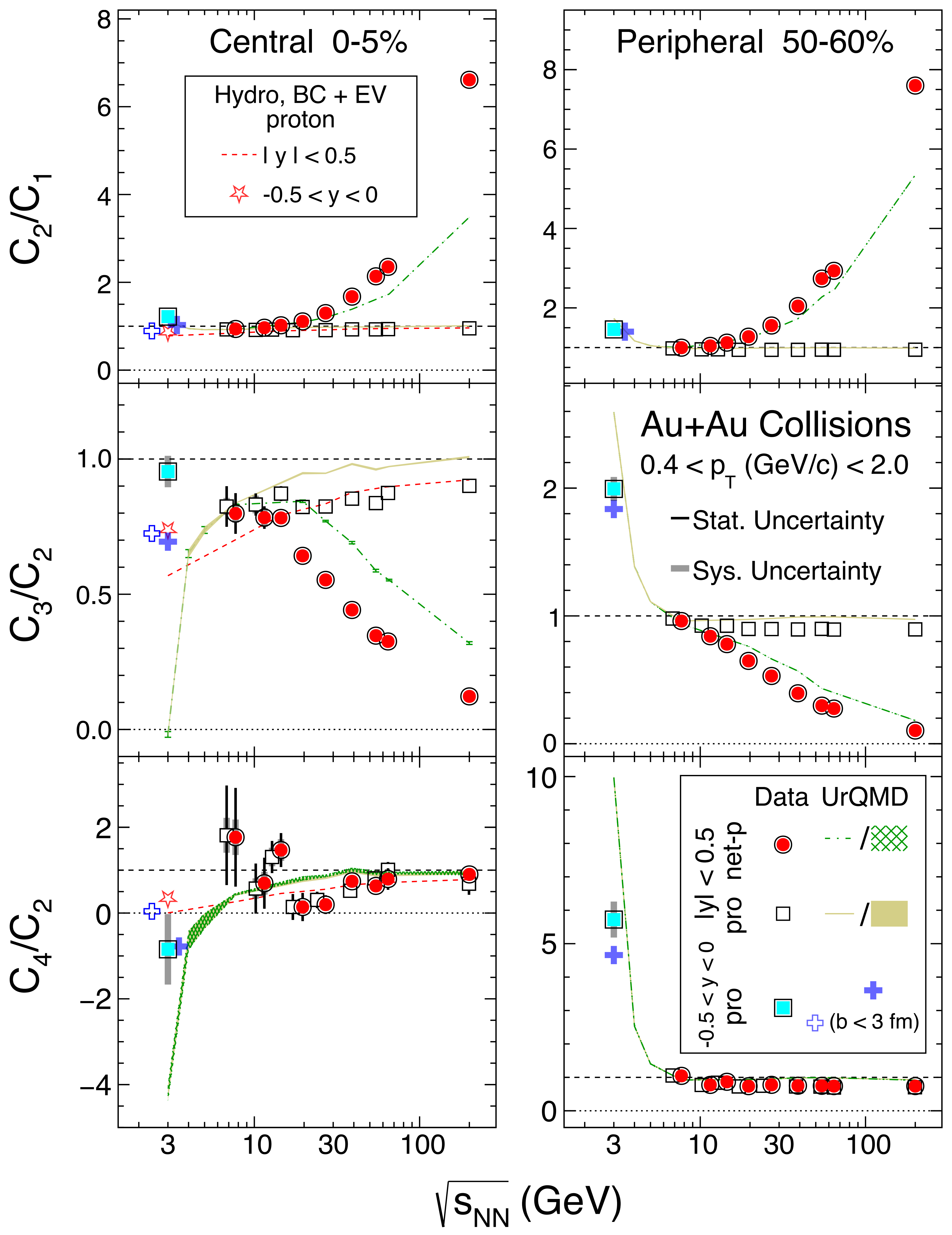
Cumulants vs Susceptibilities

Mean: M	$\langle N_B \rangle$	C_1
Variance: σ^2	$\langle (\delta N_B)^2 \rangle$	C_2
Skewness: S	$\langle (\delta N_B)^3 \rangle / \sigma^3$	$C_3 / C_2^{3/2}$
Kurtosis: K	$\langle (\delta N_B)^4 \rangle / \sigma^3 - 3$	C_4 / C_2^2

$$C_n \equiv VT^3 \frac{d^n P/T^4}{d\left(\mu_B/T\right)^n}\bigg|_T \longleftrightarrow \chi_n^B \equiv \frac{d^n P/T^4}{d(\mu_B/T)^n}\bigg|_T$$

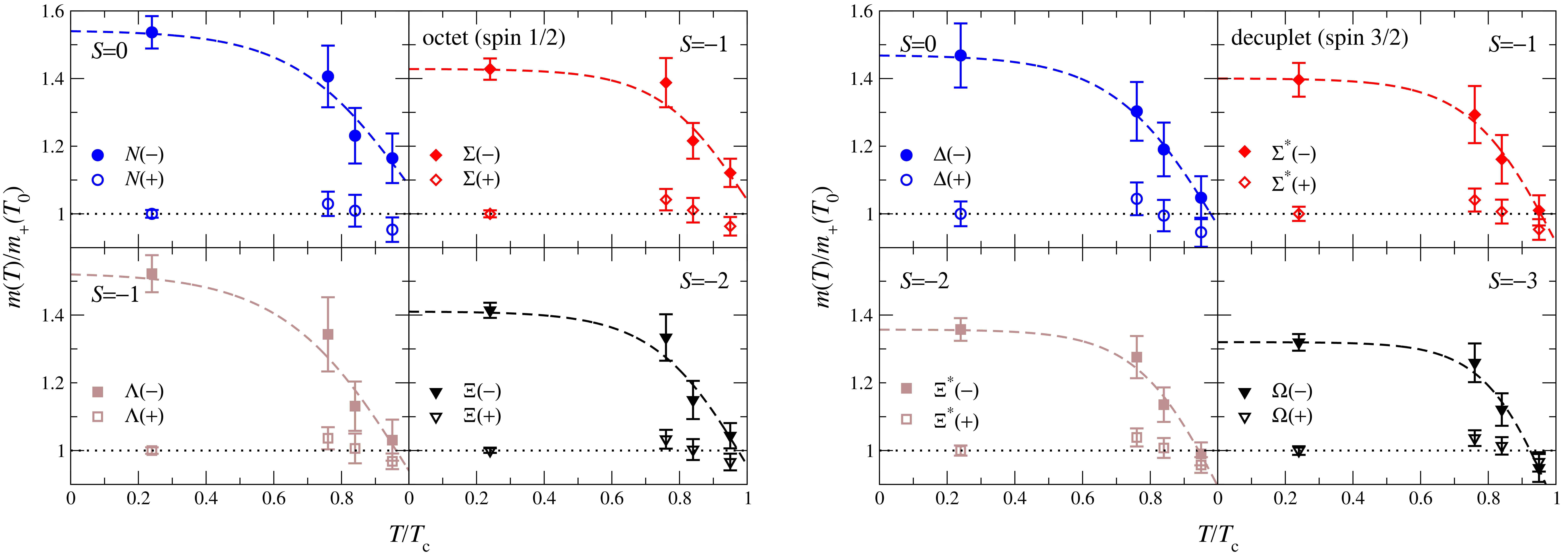
$$\boxed{C_n = VT^3 \chi_n^B}$$

STAR, 2023



Imprint of chiral symmetry restoration in the baryonic sector

Aarts et al, 2019



Clear evidence for partial restoration of chiral symmetry in the strange baryon sector