Onset of *Quantum hydrodynamics* in high energy collisions

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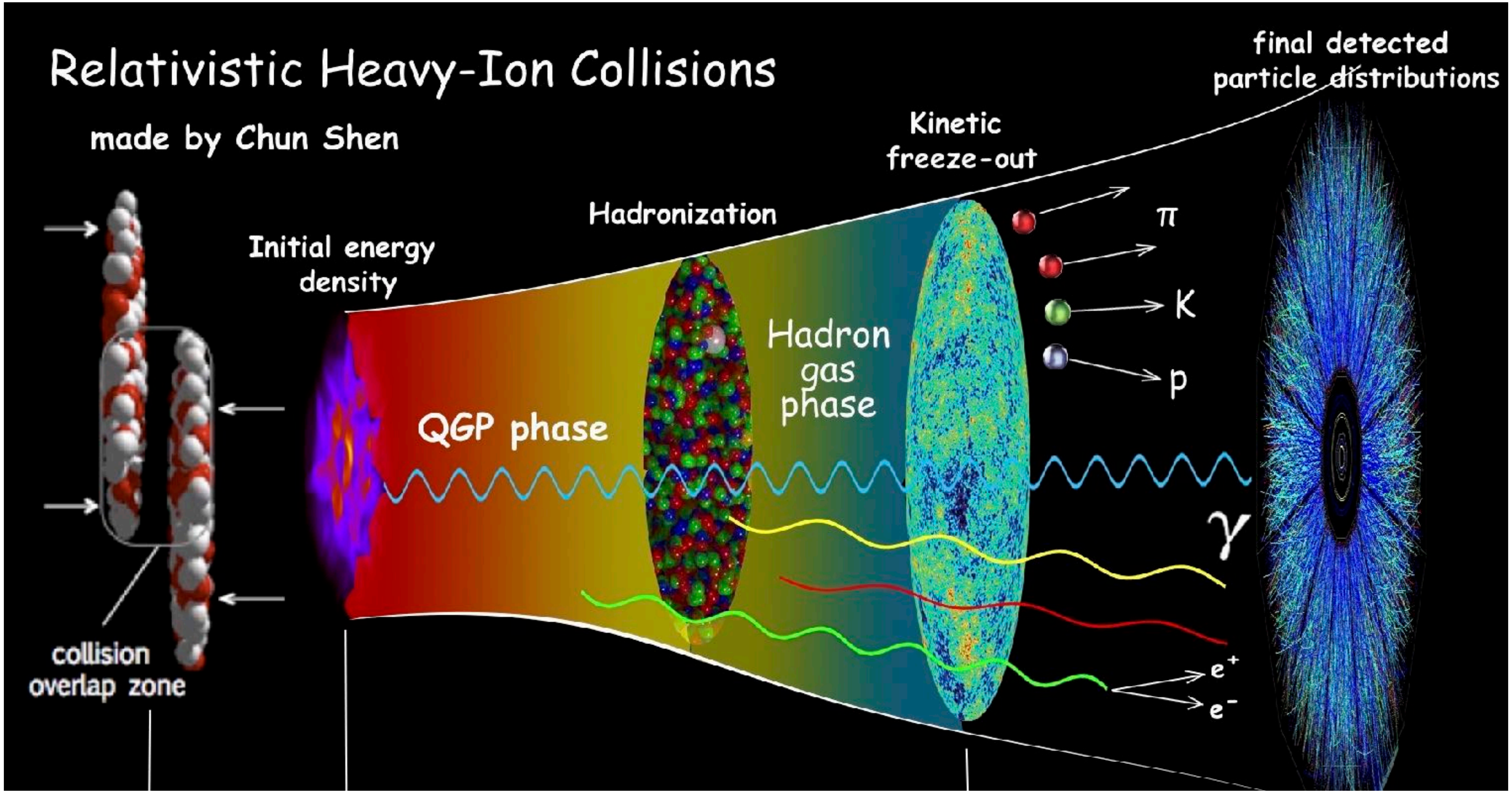
w/ Haiyang Shao (邵海洋) & Shile Chen (陈诗乐), 2506.xxxxx.



initial state

final state particles

$N \sim 10^{3-4}$

onset of hydrodynamics in Heavy-Ion Collisions

$$\partial_\mu T^{\mu\nu} = 0\,,$$

$$T^{\mu\nu} = (\varepsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}\,.$$

thermodynamic + kinetic quantities (EOS: QCD physics validated)

viscous (non-equilibrium) corrections

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Hydrodynamics $\Leftarrow$ near local-equilibrium (many body)

for ~ 10^{3-4} particles?

✓ mean free path (~0.5 fm) $\ll$ system size (~10 fm)

? rapid equilibration? (t ~ 0.5-1 fm/c)

? small system?

Hydrodynamic attractors $T^{\mu\nu} = (\varepsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$.

- focus on properties of *hydrodynamic equations*
- *common behavior* developed very rapidly

see e.g. Heller, Spalinski, Phys.Rev.Lett. 115 (2015) 072501

Thermalization in phase space distribution

- solve Boltzmann equations, compared with hydro
- usually *linearized* Boltzmann, e.g., Relaxation Time Approximation

$$p^\mu \partial_\mu f(x,p) = \mathscr{C}[f], \qquad T^{\mu\nu}(x) = \int_p p^\mu p^\nu f(x,p)$$

see e.g. Strickland, JHEP 12 (2018) 128

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Quark-Gluon Plasma:

A ***strongly coupled***, ***quantum***, (quasi)many-body system!

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Quark-Gluon Plasma:
A ***strongly coupled***, ***quantum***, (quasi)many-body system!

Ideally, full ***quantum simulation*** of QCD in 3+1D

Practically, strong coupling QED in 1+1D
— confinement, chiral condensate, etc.

Hamiltonian of Schwinger model

1+1D Schwinger model

$$H = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_x - g\gamma^1 A - m)\psi \right) \mathrm{d}x\,.$$

E: electric field

A: electric potential

$\psi, \bar{\psi}$: fermion field

$$L(t) = \int \left(-\frac{F^{\mu\nu}F_{\mu\nu}}{4} + \bar{\psi}(i\gamma^\mu \partial_\mu - g\gamma^\mu A_\mu - m)\psi \right) \mathrm{d}x\,.$$

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$|0\rangle$:empty

$|1\rangle$:occupied

$$\{\psi_a(x), \psi_b^\dagger(y)\} = \delta_{a,b}\delta(x-y)$$

field operators represented by matrices

$$\{\chi_n^\dagger, \chi_m\} = \delta_{nm}\,, \quad \{\chi_n^\dagger, \chi_m^\dagger\} = \{\chi_n, \chi_m\} = 0\,.$$

Jordan-Wigner

$$\chi_n = \frac{X_n - iY_n}{2} \prod_{m=1}^{n-1} (-iZ_m)$$

$$X_n \equiv I \otimes \cdots \otimes I \otimes \underbrace{\sigma_x}_{n^{\mathrm{th}}} \otimes I \otimes \cdots \otimes I$$

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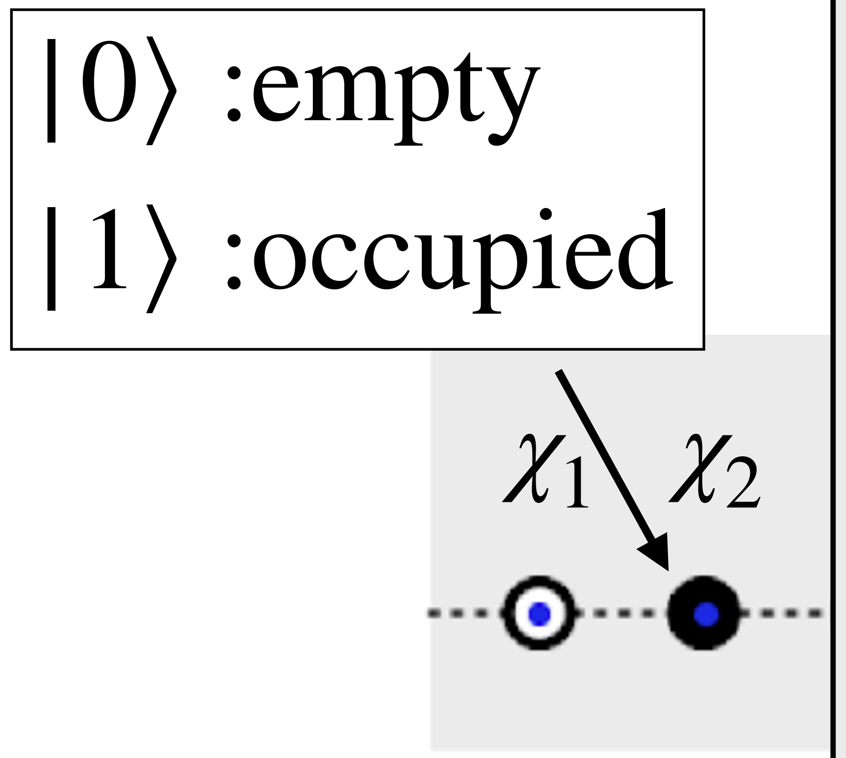
$$H = \frac{1}{4a} \sum_{n=1}^{N-1} (X_n X_{n+1} + Y_n Y_{n+1}) + \frac{m}{2} \sum_{n=1}^{N} (-1)^n Z_n + \frac{a\,g^2}{2} \sum_{n=1}^{N-1} L_n^2 \, .$$



Eigenstates of H: vacuum and (quasi)-particles

Time evolution:

$$\frac{\partial}{\partial t}|\psi(t)\rangle = -\,i\,H\,|\psi(t)\rangle$$

$$O(t) = \langle\psi(t)\,|\,\hat{O}\,|\,\psi(t)\rangle$$

$\delta_{a,b}\delta(x-y)$

d operators
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Thermalization in a pure state?

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Possible for local operators

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Quantum thermalization of Quark Gluon Plasma

May 26, 2025, 5:10 PM
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Contributed Oral Prese... | Quantum Computin... | Parallel Session

Shile Chen, (MON)

Thermalization of quantum distribution function

$$\hat{W}_{\alpha\beta}(t,z,p) = \int \bar{\psi}_\alpha(z_+)U(z_+,z_-)\psi_\beta(z_-)\, e^{i\frac{py}{\hbar}}\, \mathrm{d}y$$

Thermalization in a pure state?

Possible for local operators

Whether hydrodynamics develops in a pure quantum state?

initial state:

vacuum + [excitation @ center]

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

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$$\mathscr{L}[\psi, \bar{\psi}, A] \;\Rightarrow\; \hat{T}^{\mu\nu}[\psi, \bar{\psi}, A]$$

thermodynamic + kinetic quantities

$$\langle \Psi(t)|\hat{T}^{\mu\nu}(x)|\Psi(t)\rangle = (\varepsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu}$$

viscous (non-equilibrium) corrections

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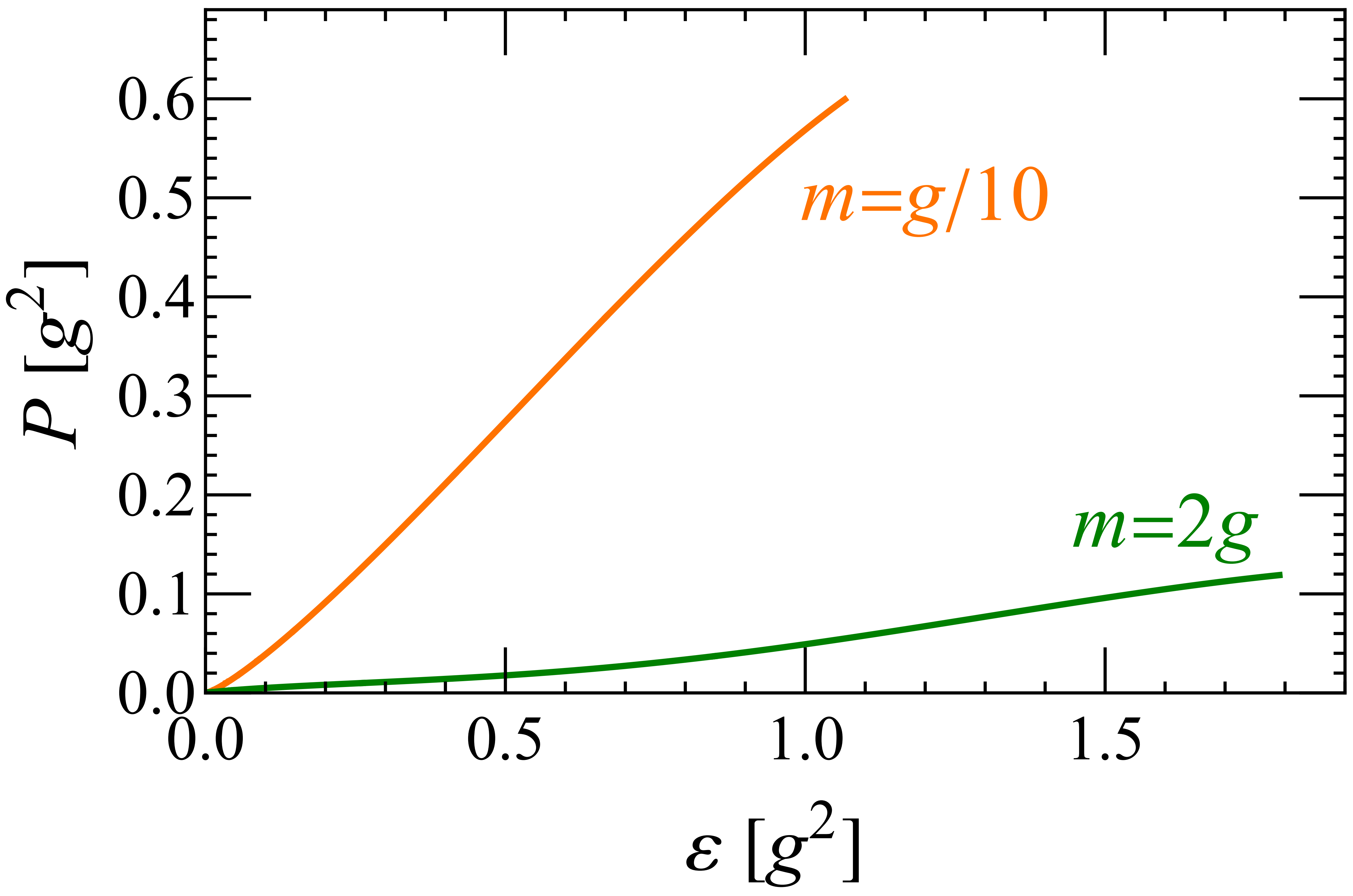

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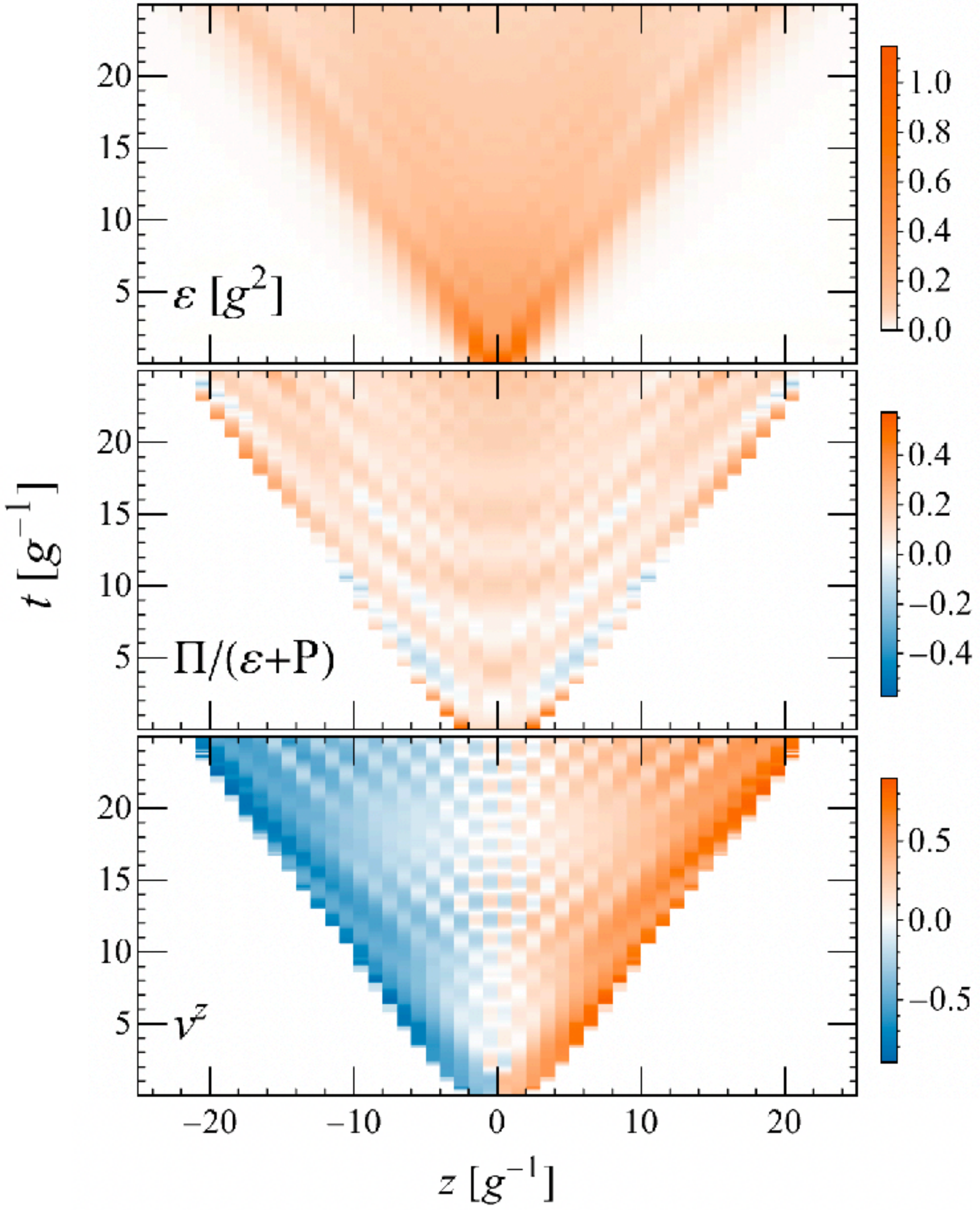
emergence of hydrodynamics



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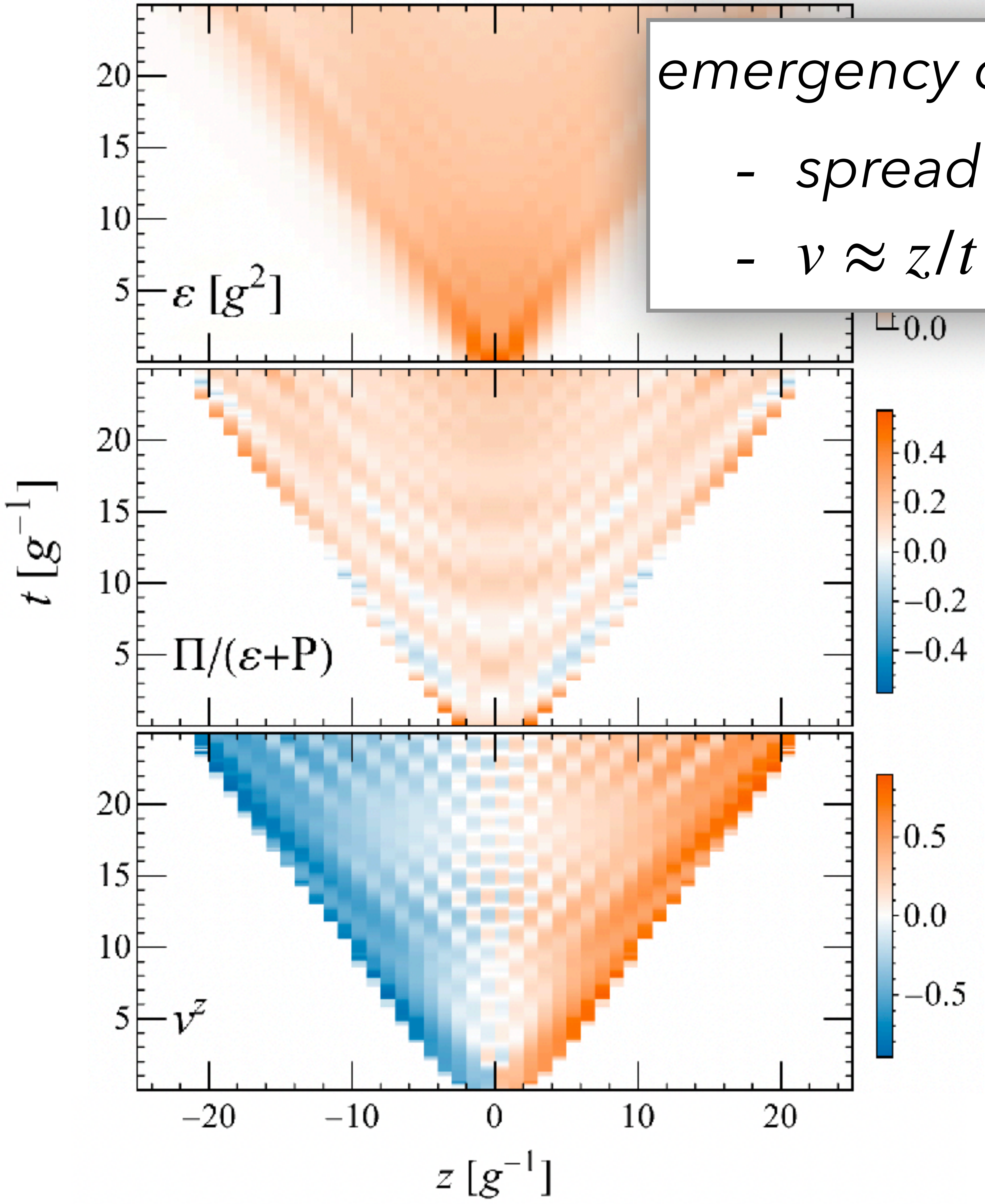
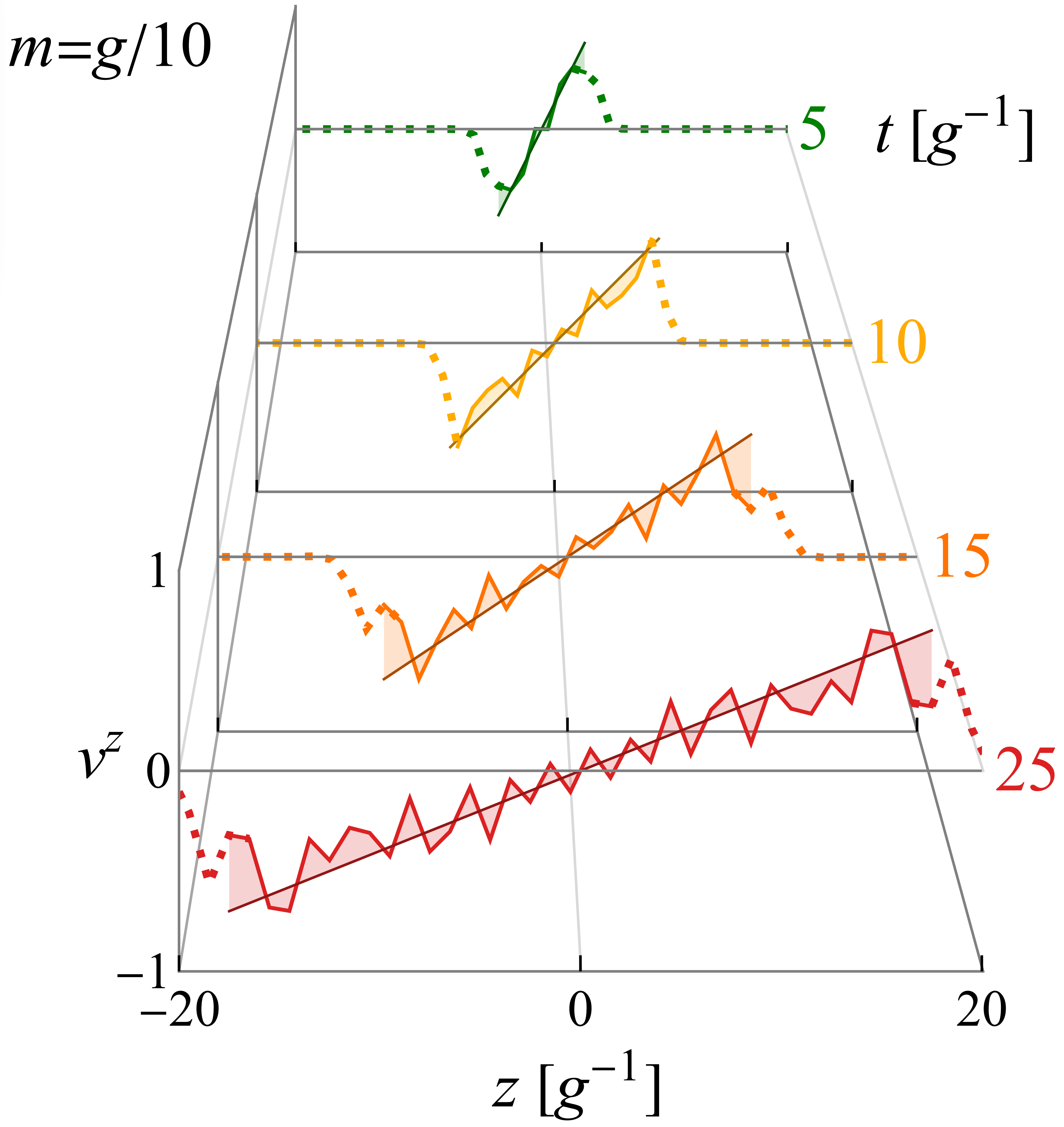


emergency of hydrodynamics

- *spread within light cone*
- $v \approx z/t$



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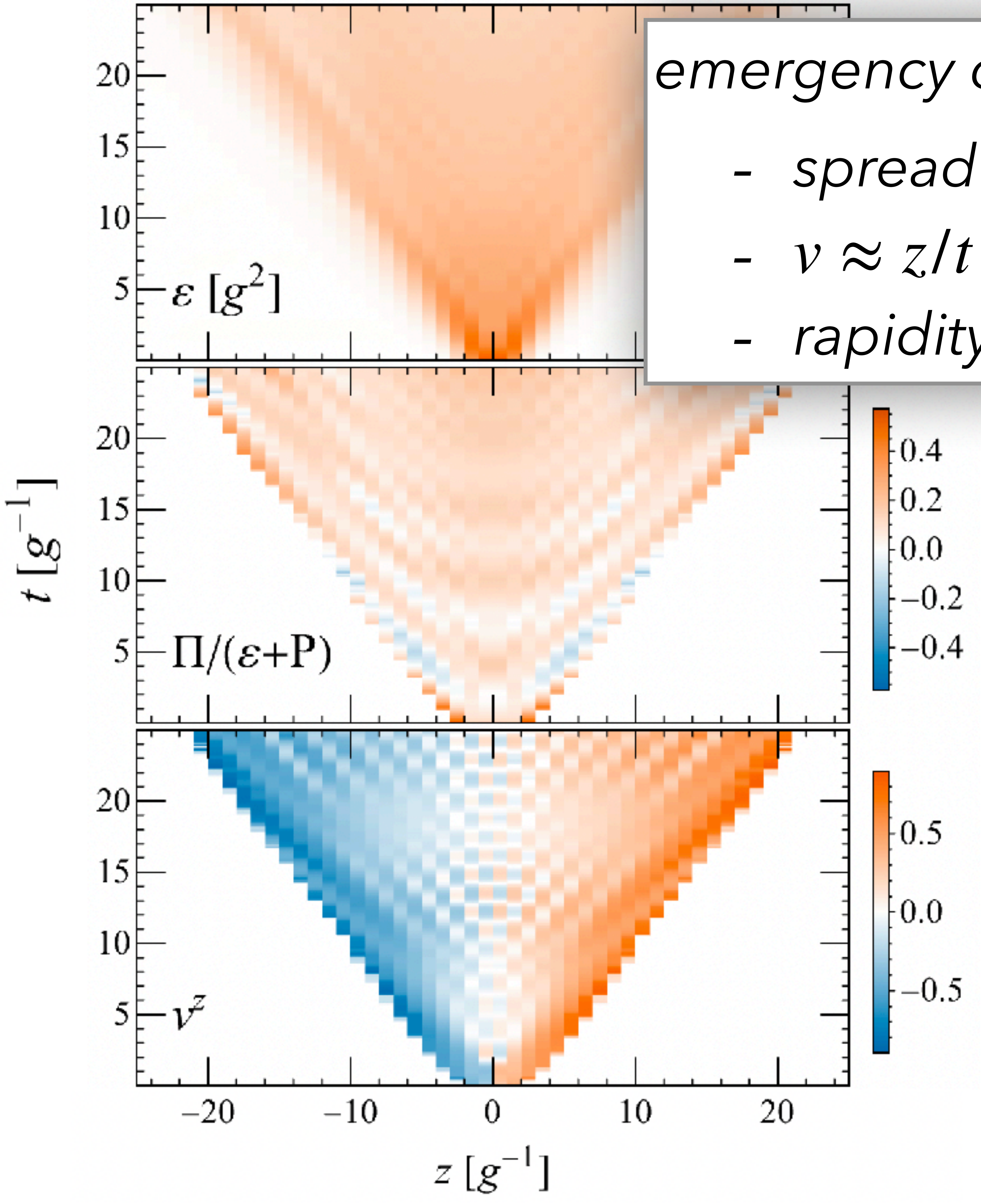
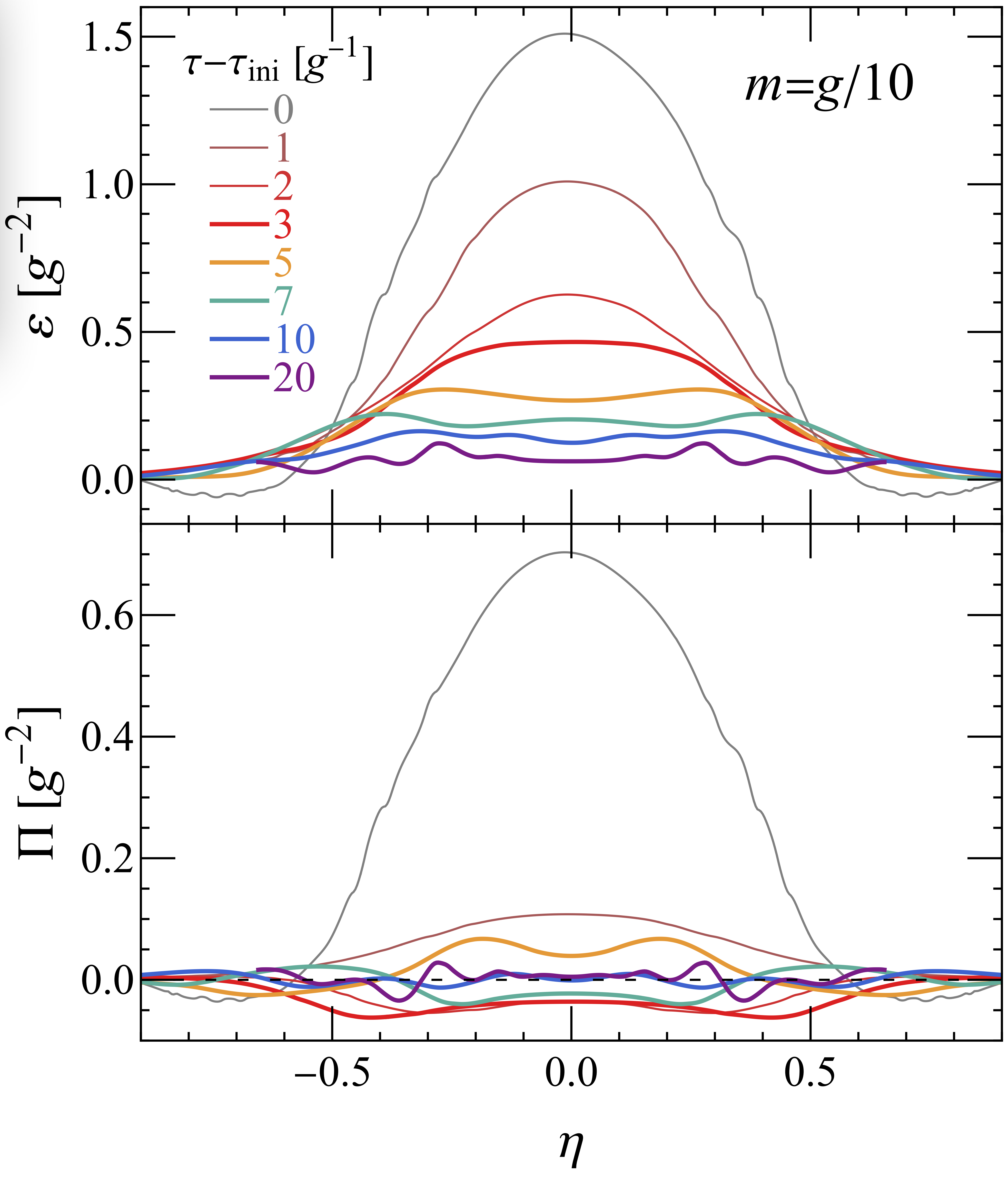
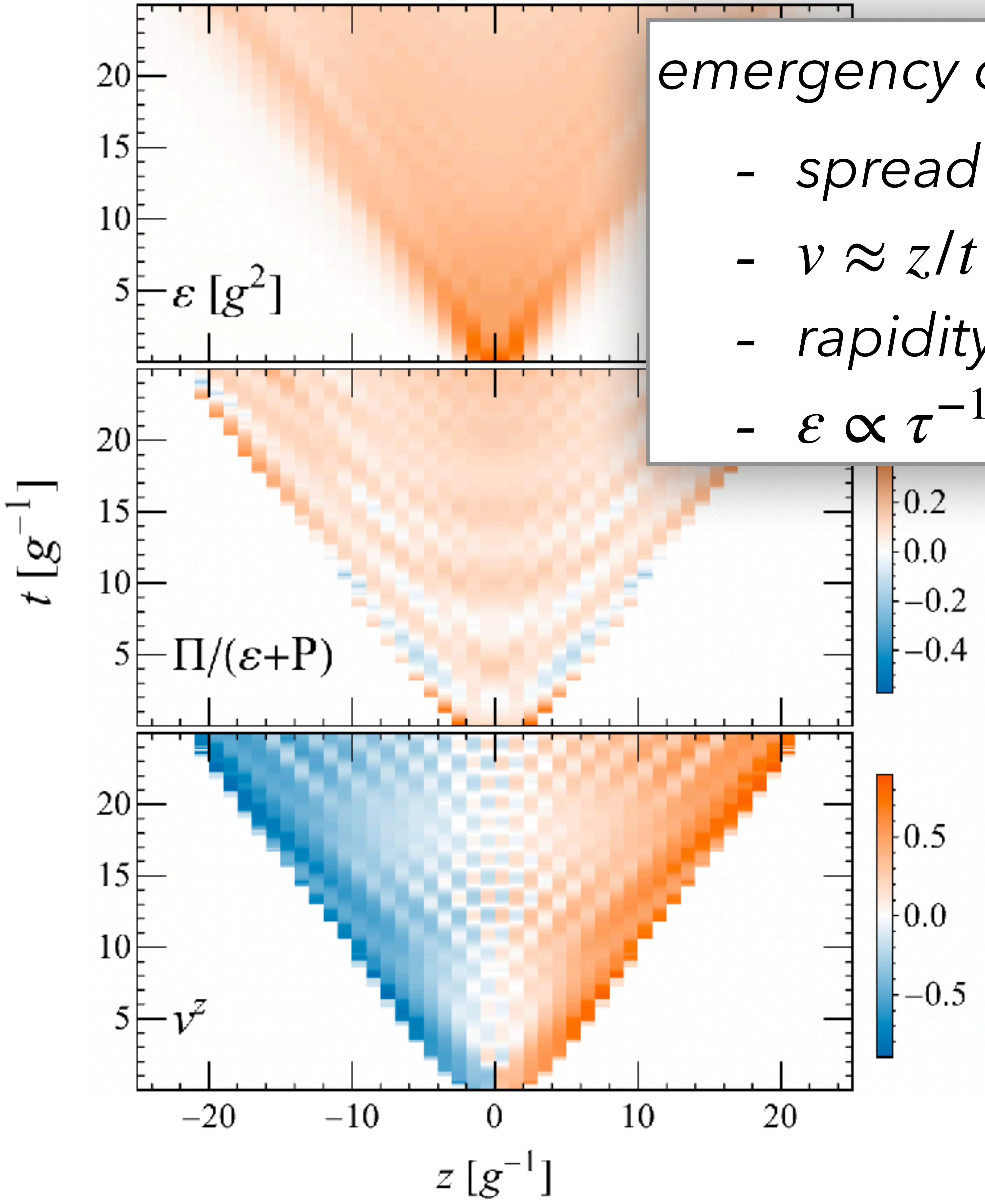


emergency of hydrodynamics

- *spread within light cone*
- $v \approx z/t$
- *rapidity structure*



emergence of hydrodynamics



emergency of hydrodynamics

- *spread within light cone*
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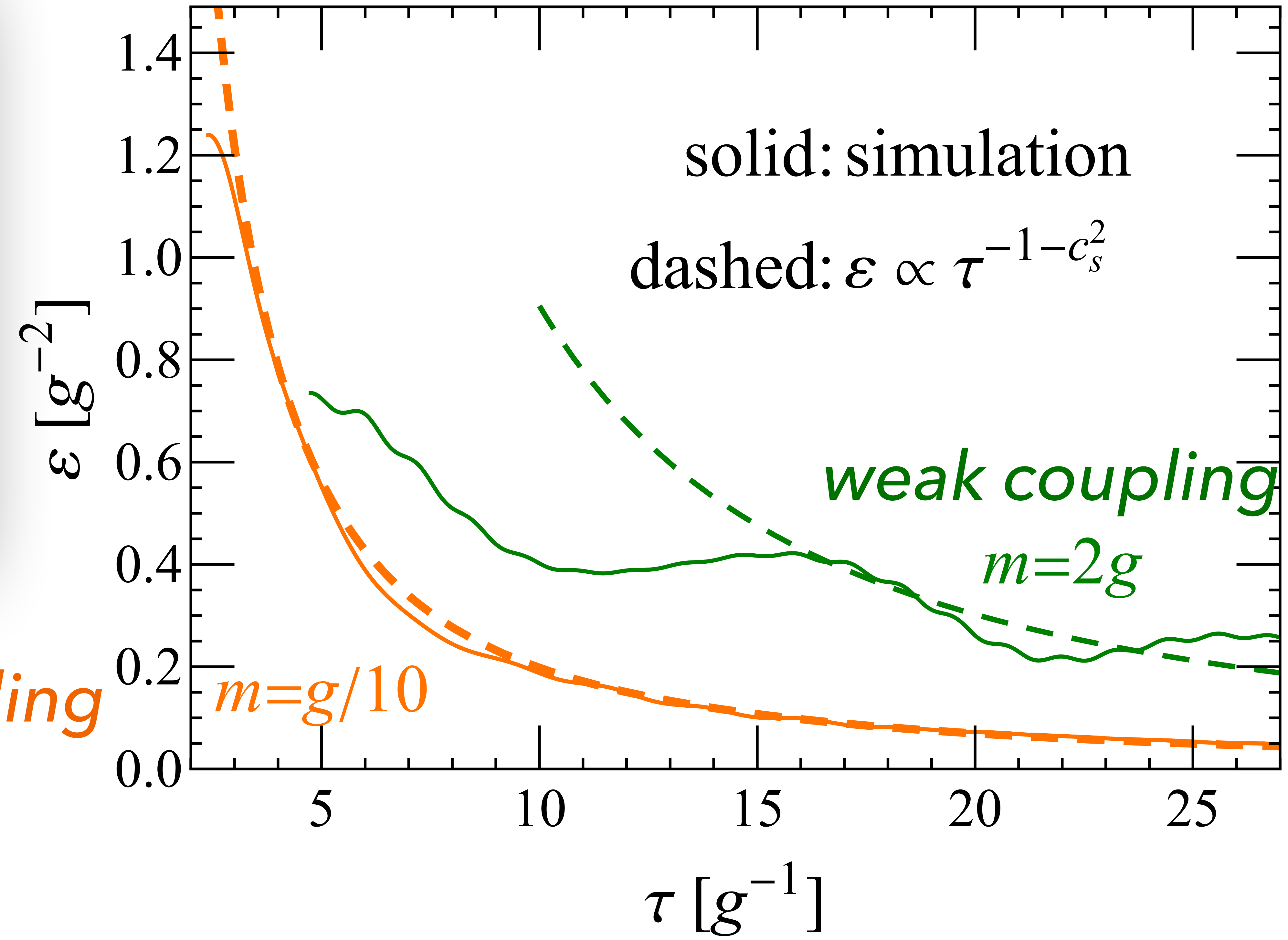
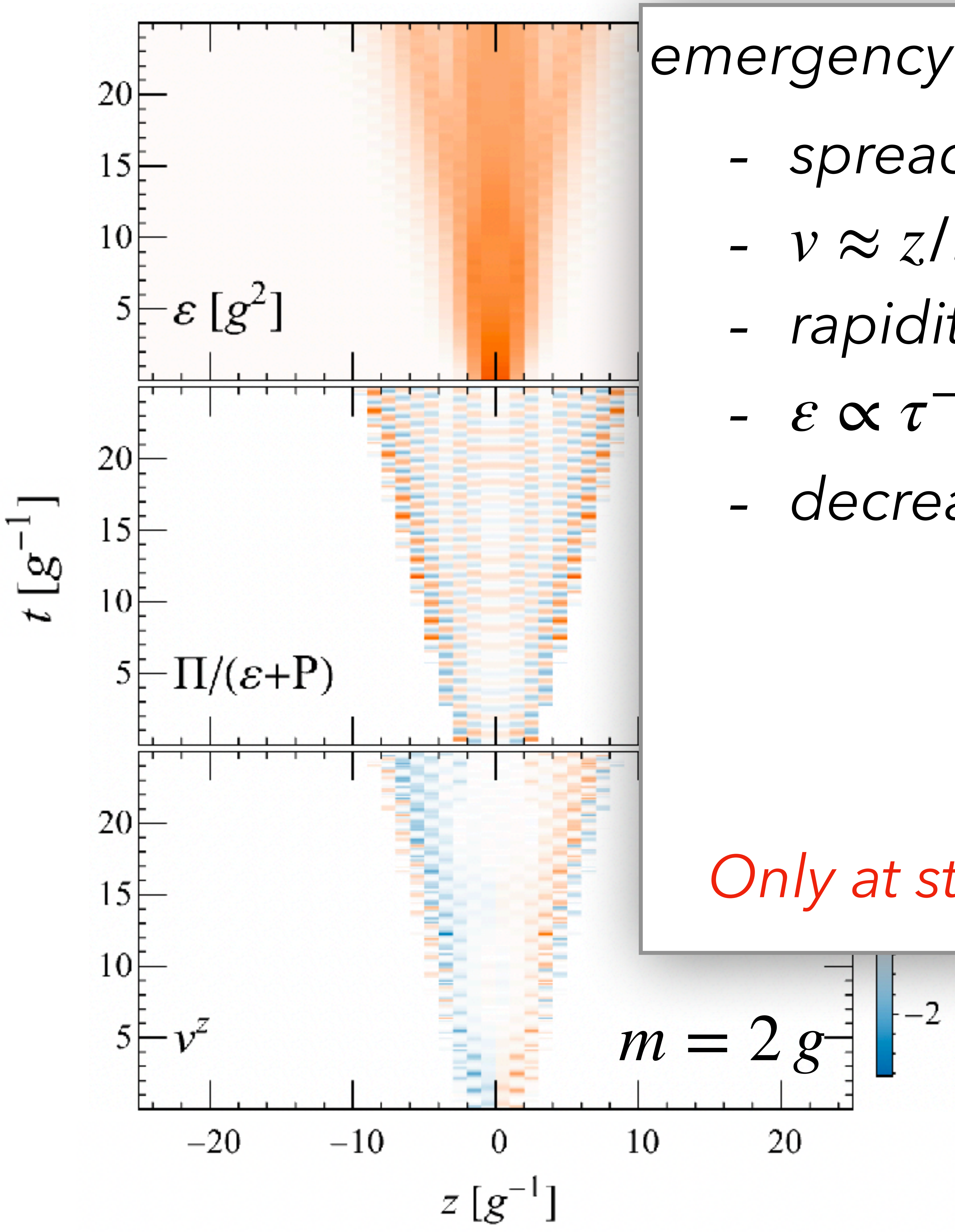
- $\varepsilon \propto \tau^{-1-c_s^2}$



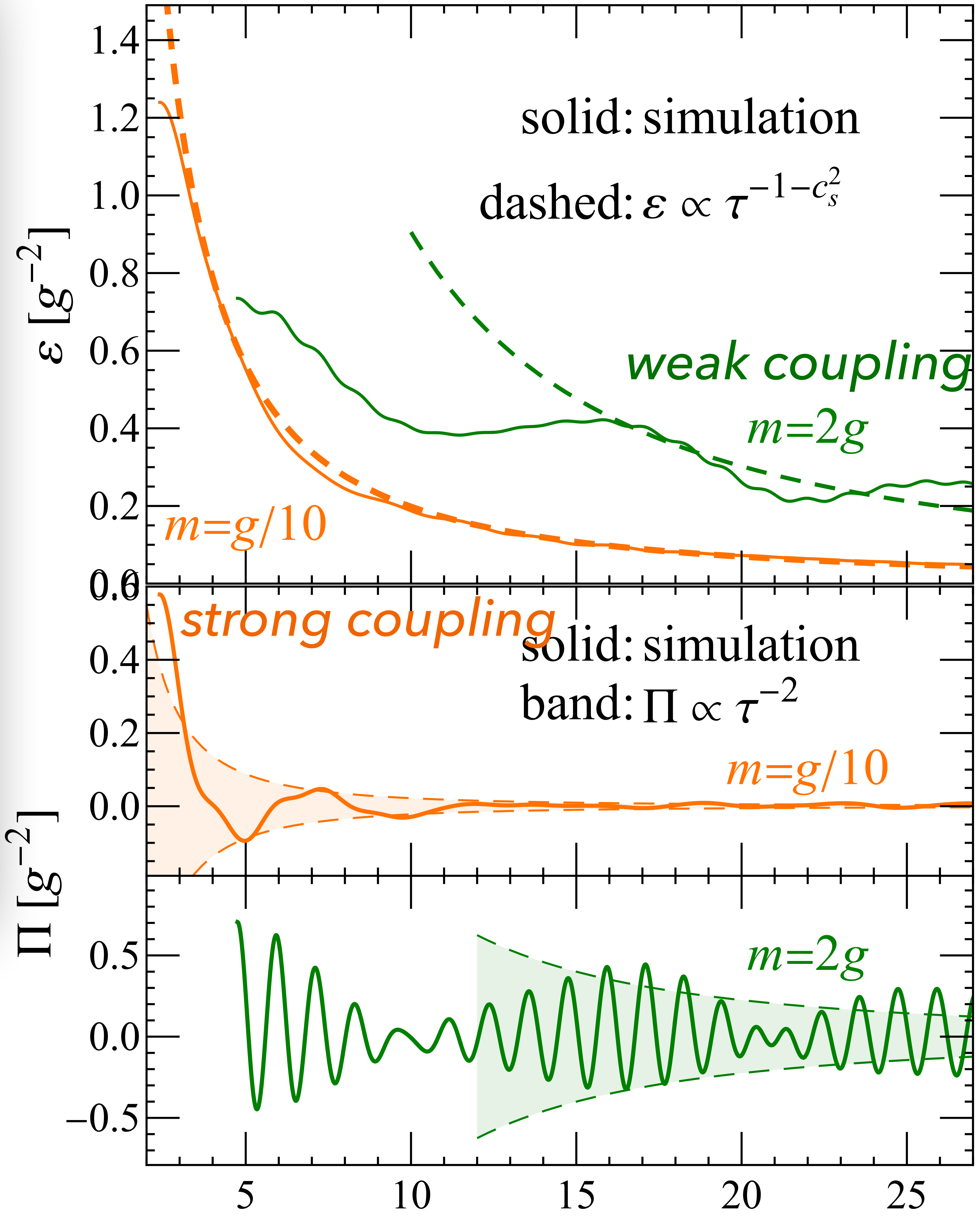
emergence of hydrodynamics



emergency of hydrodynamics

- *spread within light cone*
- $v \approx z/t$
- *rapidity structure*
- $\varepsilon \propto \tau^{-1-c_s^2}$
- *decreasing of* Π

Only at strong coupling!!!

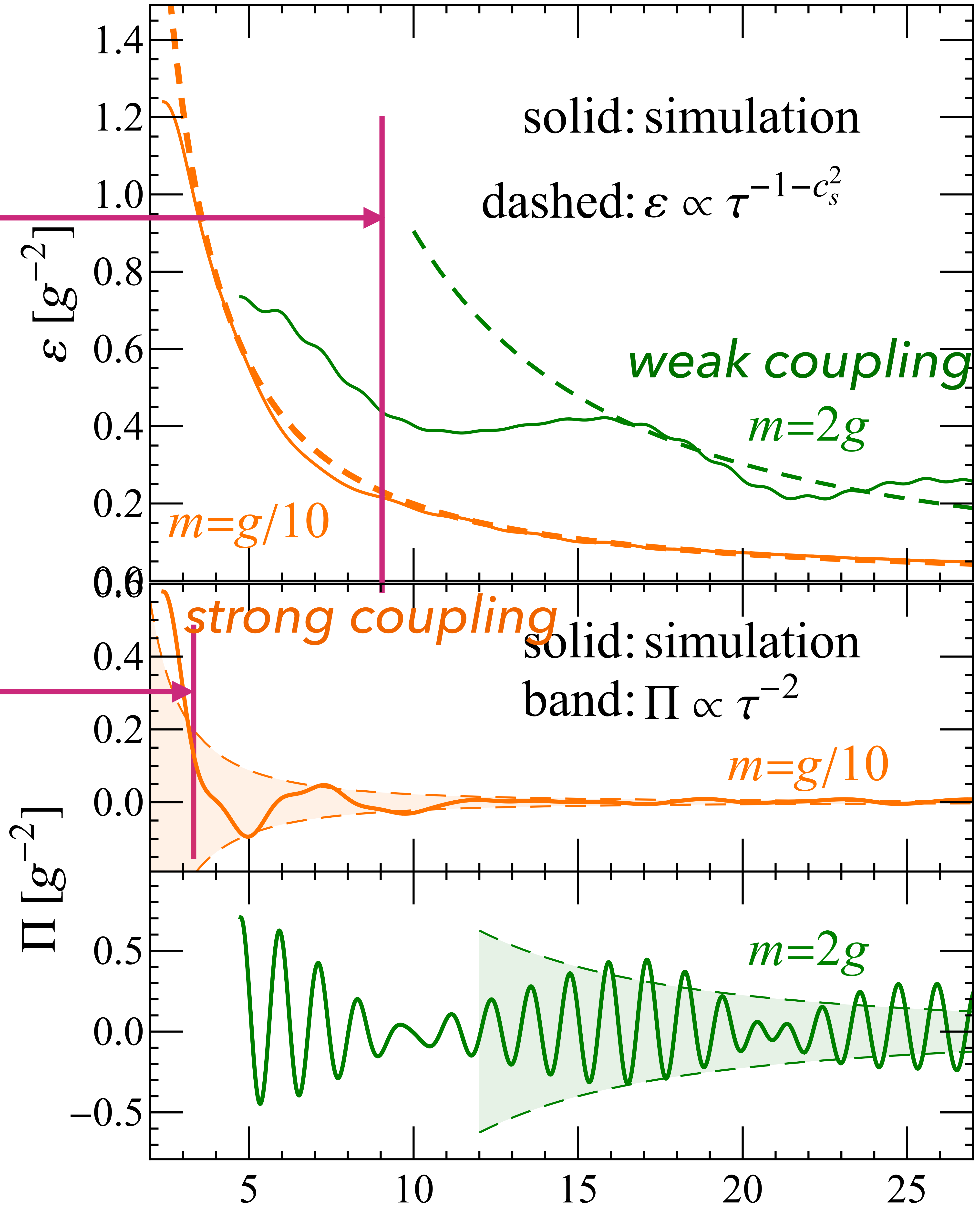


onset of local equilibrium

$$\tau - \tau_{\text{ini}} \sim 8/g \sim 4/M_\pi$$

onset of viscous hydro (Navier Stokes)

$$\tau - \tau_{\text{ini}} \sim 2/g \sim 1/M_\pi$$



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w/Haiyang Shao & Shile Chen, final preparation

summary

Emergence of Bjorken flow in Real-time non-perturbative quantum evolution

What is essential for hydrodynamization?

- Schwinger model: for *light particles* and/or when *coupling is strong*
- Other models