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Probing fluctuating color fields of QCD matter from spin alignment in relativistic heavy ion collisions

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(INPC2025, May 29th, 2025)

primary references :

A. Kumar, B. Müller, DY, PRD 108, 016020 (2023), arXiv:2304.04181
A. Kumar, B. Müller, DY, PRD 107, 076025 (2023), arXiv:2212.13354
DY, PRD 110, 056005 (2025), arXiv:2411.14822

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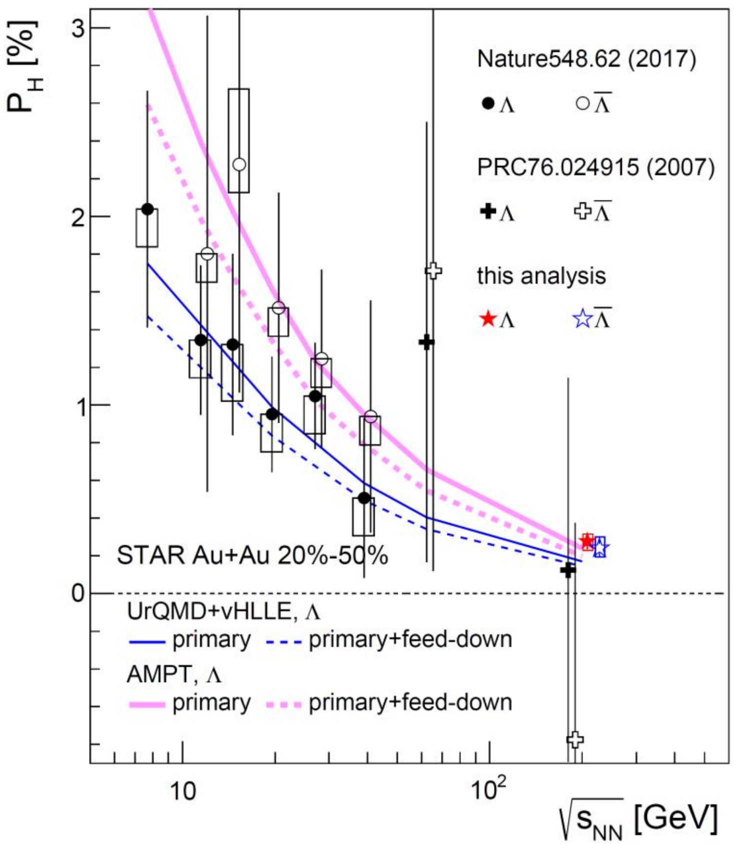
NSTC 國家科學及技術委員會 National Science and Technology Council

Global Λ polarization in HIC

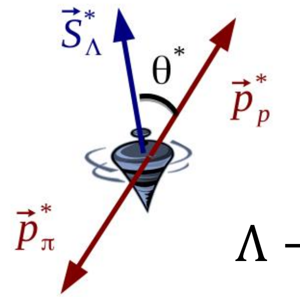
- The measurement of Global polarization of Λ hyperons revealed the spin-orbit interaction & strong vorticity in heavy ion collisions. Z.-T. Liang and X.-N. Wang, PRL. 94, 102301 (2005)

(relativistic Barnett effect)



L. Adamczyk et al. (STAR), Nature 548, 62 (2017)

❖ Self-analyzing via the weak decay :



$$\Lambda \to p + \pi^-$$

T. Niida, QM18

❖ In global equilibrium : F. Becattini, et al., Ann. Phys. 338, 32 (2013) R. Fang, et al., PRC 94, 024904 (2016)

$$\mathcal{P}^\mu(\boldsymbol{p}) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\nu \frac{\int d\Sigma \cdot p\,\omega_{\rho\sigma} f_p^{(0)}(1-f_p^{(0)})}{\int d\Sigma \cdot p f_p^{(0)}},$$

$$\boxed{\omega_{\mu\nu} = \frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu).}$$

thermal vorticity $(\beta^\mu \equiv u^\mu/T)$

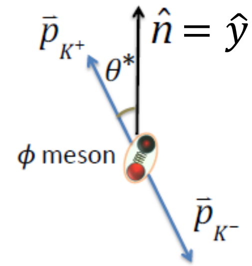
❖ Indication of strong (kinetic) vorticity :

$$P_{\Lambda(\bar\Lambda)} \simeq \frac{1}{2}\frac{\omega}{T} \pm \frac{\mu_\Lambda B}{T} \Longrightarrow \omega \sim 10^{22}\,\mathrm{s}^{-1}$$

F. Becattini et al., PRC95, 054902 (2017)

Spin alignment of vector mesons

- Production of the decay daughter w.r.t the quantization axis :

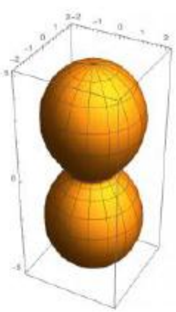


$$\frac{dN}{d\cos\theta^*} \propto [1-\rho_{00}+\cos^2\theta^*(3\rho_{00}-1)]$$

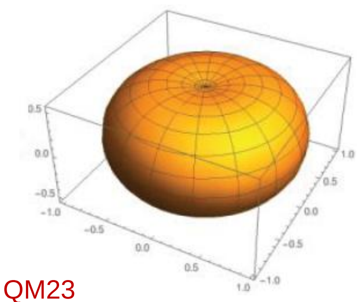
$$\boxed{\rho_{00} = \frac{1-\langle \mathcal{P}_q^y \mathcal{P}_{\bar{q}}^y \rangle}{3+\langle \mathcal{P}_q^y \mathcal{P}_{\bar{q}}^y \rangle}}$$

Z.-T. Liang and X.-N. Wang, PLB 629, 20 (2005)

$\rho_{00} > 1/3$:

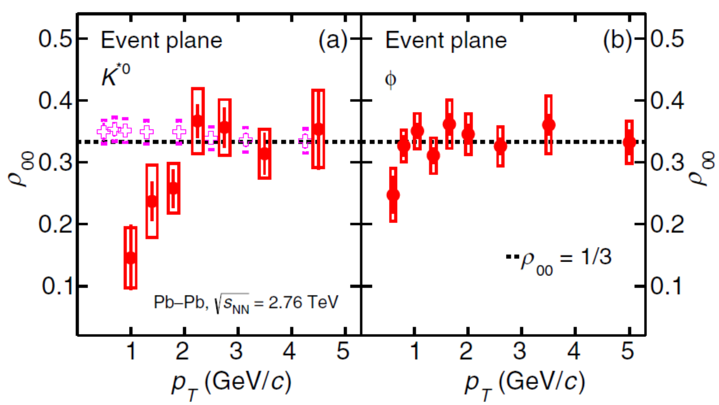


$\rho_{00} < 1/3$:

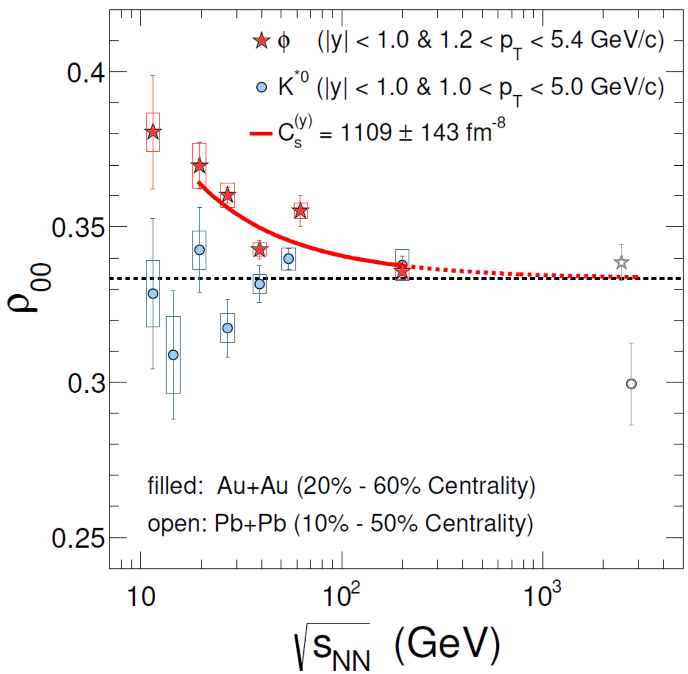


B. Xi, QM23

$\rho_{00} \neq 1/3$: spin correlation



S. Acharya et al. (ALICE), PRL.125, 012301 (2020)



M.S. Abdallah et al. (STAR), Nature 614 (2023) 7947, 244-248,

Spin polarization beyond subatomic swirls?

- Spin alignment puzzle : the deviation of ρ_{00} from 1/3 is unexpectedly large

$$\text{e.g. } \rho_{00} \approx \frac{1}{3} - \left(\frac{\omega}{T}\right)^2, \ \frac{\omega}{T} \sim 0.1\% \text{ at LHC energy.}$$ (from Λ pol.~ s quark pol.)

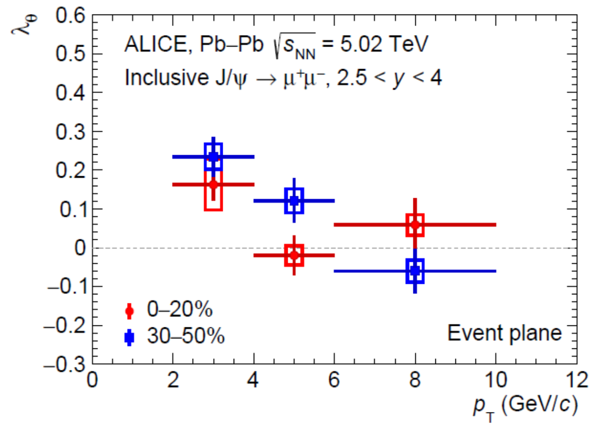
- Flavor & collision energy dep. :

	ϕ	K^{*0}
ALICE	$\rho_{00} < 1/3 \ (p_T \leq 1 \text{ GeV})$	$\rho_{00} < 1/3$
STAR	$\rho_{00} > 1/3$	$\rho_{00} \approx 1/3$

- Spin alignment for J/ψ :

S. Acharya et al. , PRL 131,042303 (2023)

$$\lambda_\theta = \frac{1 - 3\rho_{00}}{1 + \rho_{00}} > 0 \Rightarrow \rho_{00} < \frac{1}{3}$$

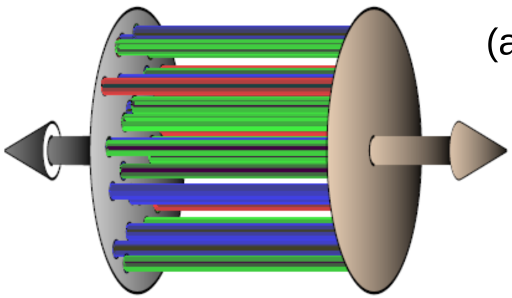


- Other sources for the spin correlation (alignment) beyond vorticity?

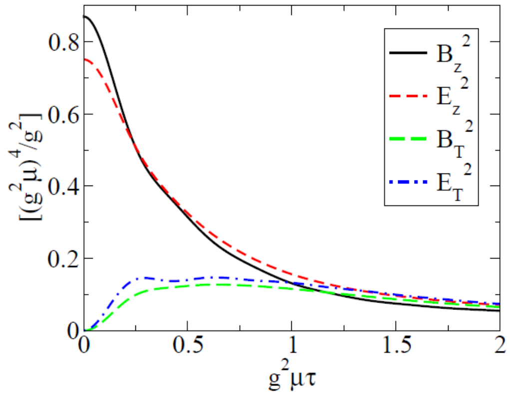
❑ Electromagnetic fields can polarize the spin. How about gluonic fields in QCD matter? $\mathcal{P}^i(\boldsymbol{p}) \propto c_1 B^i + c_2 \epsilon^{ijk} p_j E_k$

Color-field induced spin alignment

- Small-x gluons may form a glasma state characterized by mostly longitudinal chromo-electromagnetic fields in early times of HIC from CGC.



(at top RHIC & LHC energies)



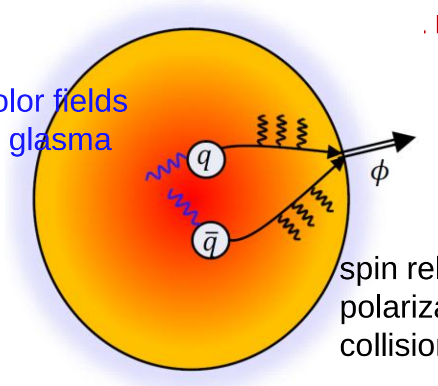
Reviews : F. Gelis et al., Ann.Rev.Nucl.Part.Sci.60:463-489,2010

J. Berges et al., Rev. Mod. Phys. 93 (2021) 3, 035003

. Lappi, PLB 643 (2006) 11-16

❖ Why glasma fields ?
(1) intrinsic saturation scale $Q_s \gg \omega$
(2) fluctuating
(3) intrinsic anisotropy

color fields in glasma



spin relaxation or polarization from collisions

- An order of magnitude estimate for the glasma effect ?

- How to track the spin evolution of massive quarks in phase space?
quantum kinetic theory from Wigner functions

Review : Y. Hidaka S. Pu, Q, Wang, DY, PPNP 127, 103989 (2022)

DY, JHEP 06, 140 (2022)
B. Müller, DY, PRD 105, L011901 (2022)

Transverse spin alignment spectra

- Spin alignment from color-field correlations : A. Kumar, B. Müller, DY, PRD 107, 076025 (2023)

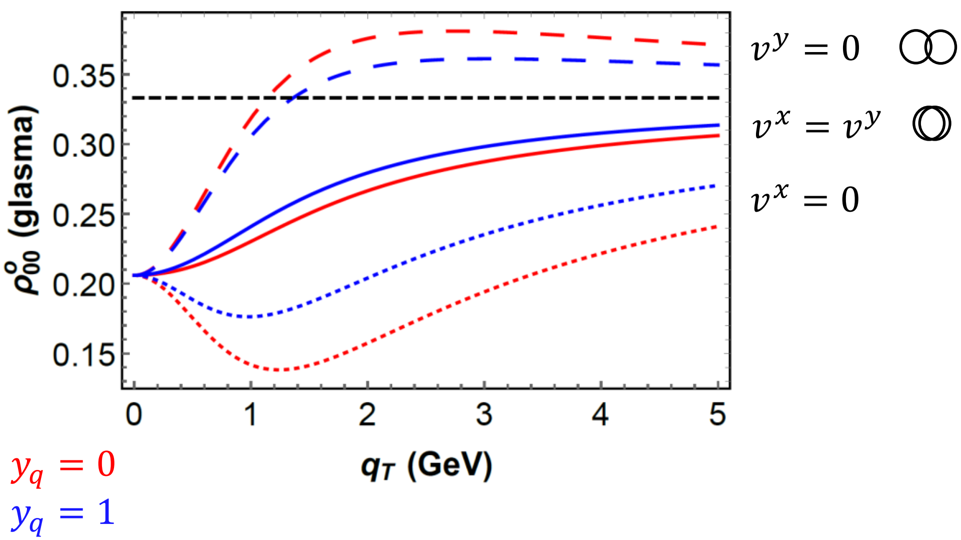
e.g. $$\delta\rho_{00} = \rho_{00} - \frac{1}{3} \propto \langle B^{az}(x) B^{az}(x') \rangle e^{-2\Delta t/\tau_R}$$

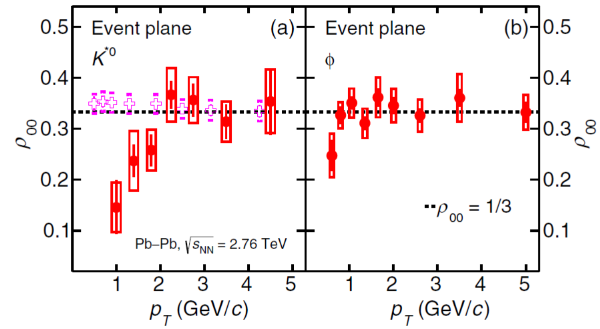
glasma effect relaxation in QGP

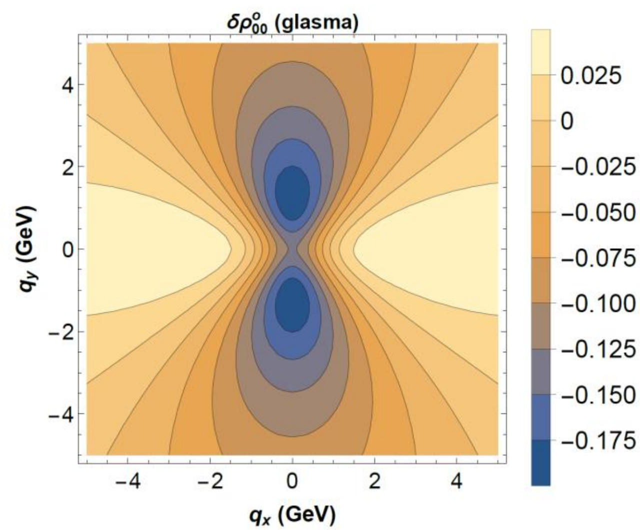
❖ Two sources of anisotropy : DY, PRD 110, 056005 (2025)

longitudinal fields v.s. momentum anisotropy

- Out-of-plane spin alignment :





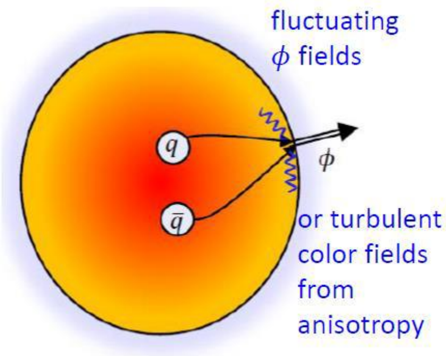


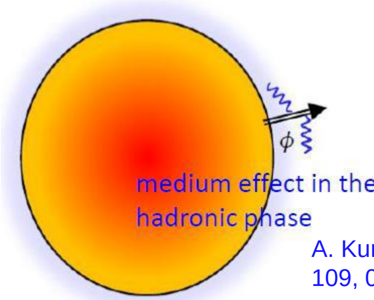
(not yet weighted by pT (=qT) spectra)

Color fields in the QGP phase

- Late-time effects :

X.-L. Sheng et al., PRD 109, 036004, (2024)
PRL 131, 042304 (2023)
B. Müller, DY, PRD 105, L011901 (2022)
DY, JHEP 06, 140 (2022)



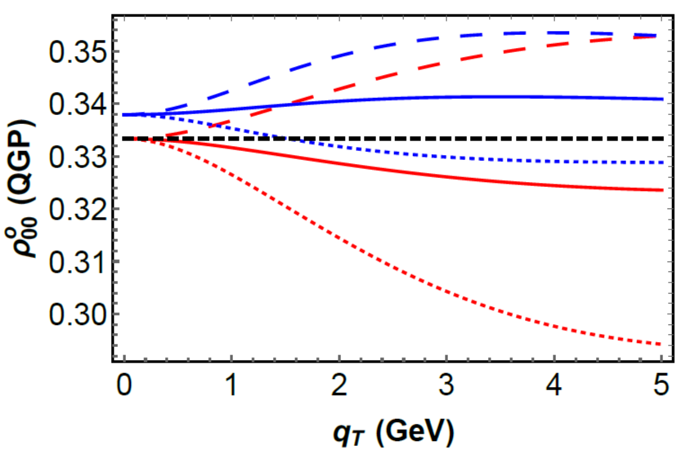


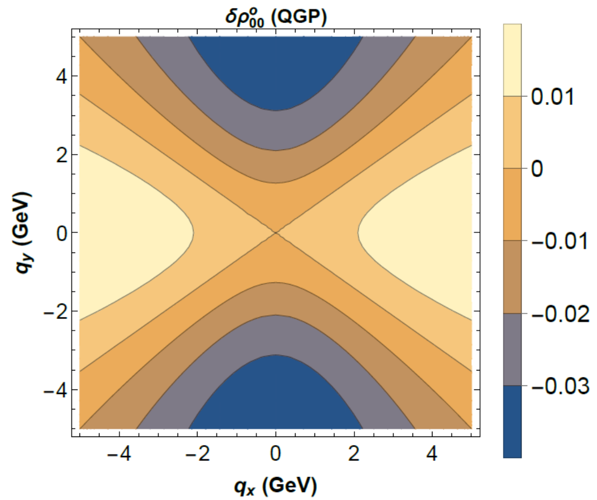
A. Kumar, Philipp Gubler, DY, PRD 109, 054038 (2024)
D. Wagner, N. Weickgenannt, E. Speranza,PRR 5, 013187 (2023)
F. Li, S. Liu, arXiv:2206.11890

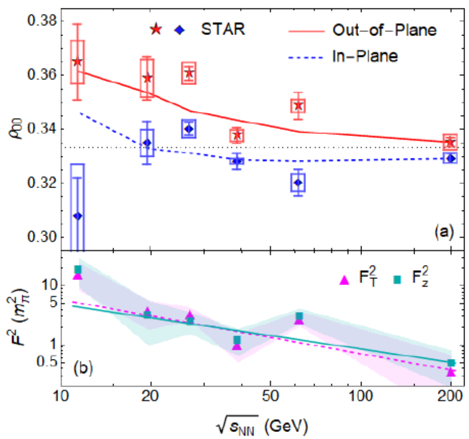
- From isotropic color fields (soft thermal gluons in QGP)

➢ Out-of-plane spin alignment (qualitative) : DY, PRD 110, 056005 (2025)

X.-L. Sheng et al., PRL 131, 042304 (2023)

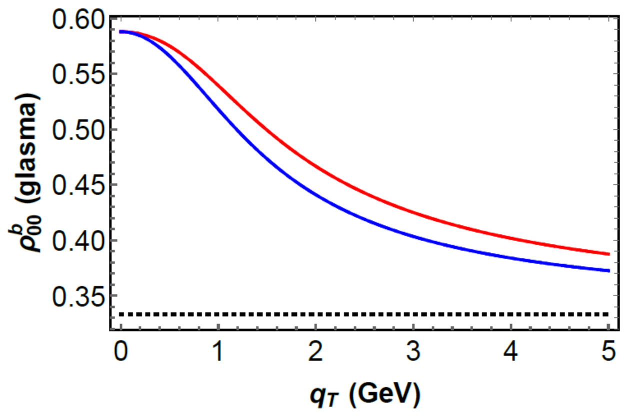


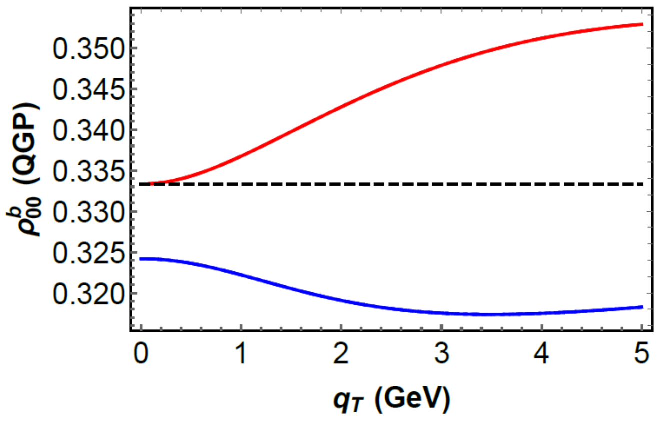


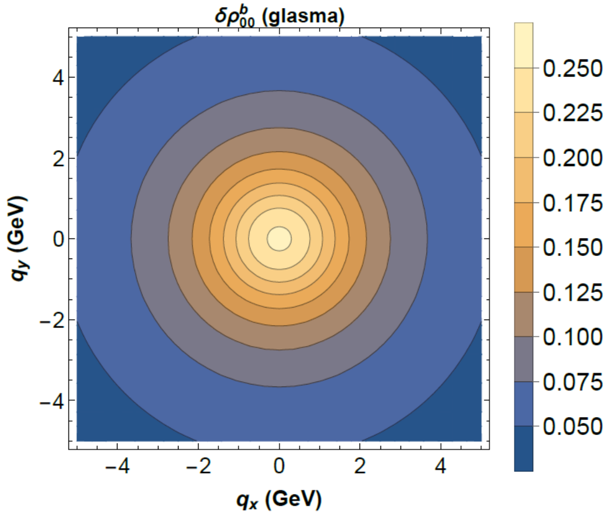


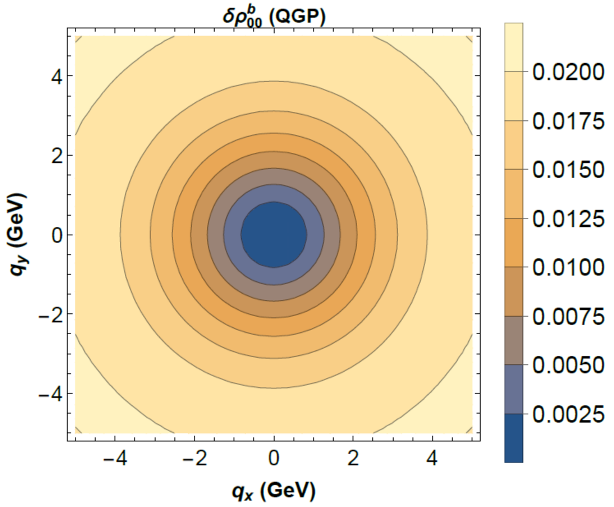
Longitudinal spin alignment

- Spin alignment along the beam direction : DY, PRD 110, 056005 (2025)









Summary & outlook

❑ Summary :

- ✓ In general, spin alignment may be a useful probe for strong interaction forces led by gluons in QCD matter.
- ✓ The glasma and QGP effects may be responsible for the spin alignment in low and mid transverse momenta, respectively.
- ✓ However, the order-of-magnitude estimate is sensitive to the initial distribution function of quarks, spin relaxation time, and subject to the weak color fields in QKT.

❑ Outlook :

- ➢ Spin alignment of quarkonia like J/ψ : more applicable for the kinetic theory in a medium.
- ➢ Scale separation : EFT + open quantum system approach

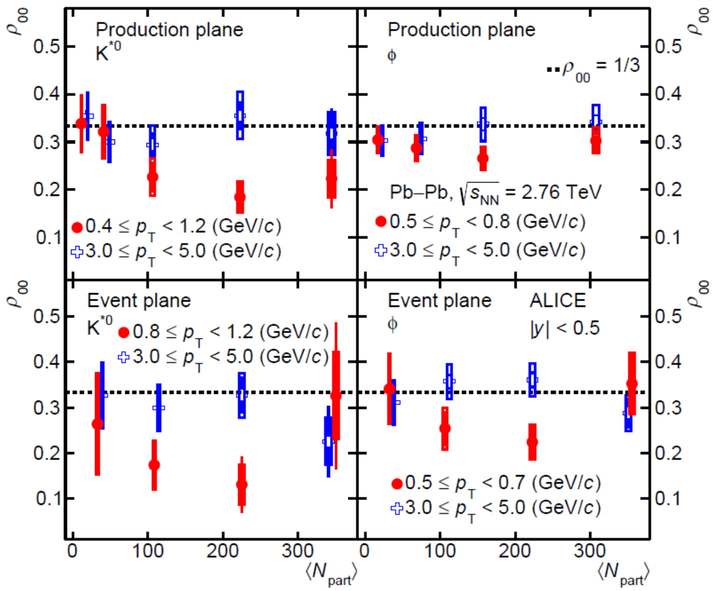
  E.g., DL & X. Yao, "Quarkonium Polarization in Medium from Open Quantum Systems and Chromomagnetic Correlators", PRD 110, 074037 (2024).
- ➢ "High-multiplicity" events in proton-nucleus collisions?

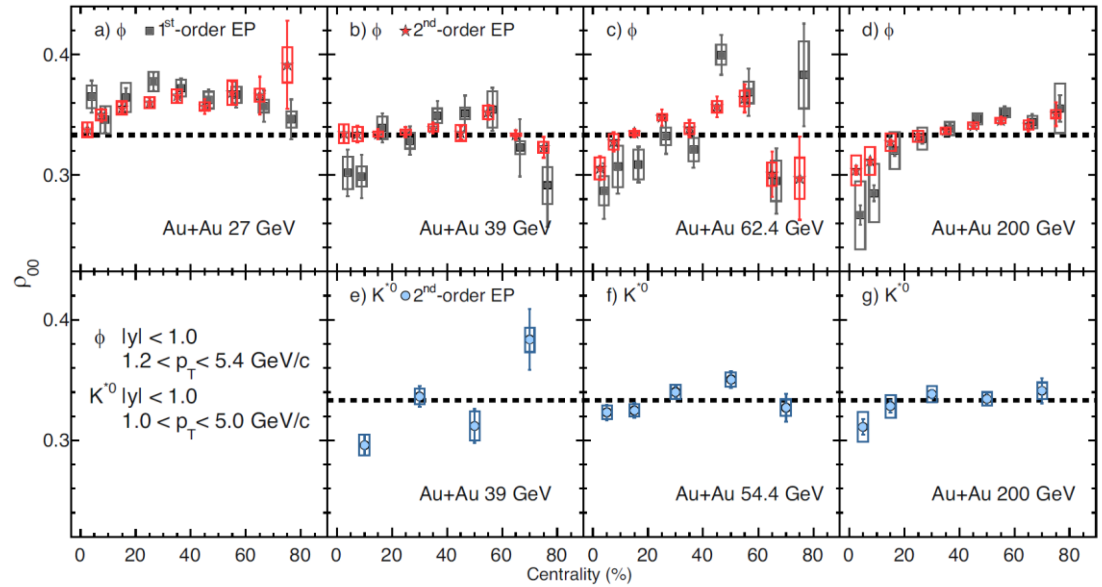
  NRQCD+CGC : (low multiplicity) Y.-Q. Ma, T. Stebel, R. Venugopalan, JHEP 12 (2018) 057
  T. Stebel, K. Watanabe, PRD 104, 034004 (2021)

Thank you!

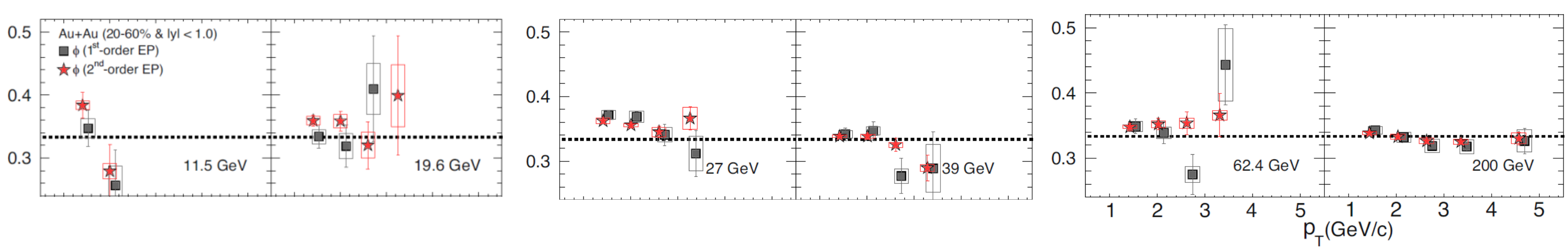
More experimental data

❖ Centrality dependence :





❖ pT dependence in RHIC :



From spin correlations to spin alignment

- Spin alignment is led by spin correlations : $\langle\mathcal{P}_q^i\mathcal{P}_{\bar{q}}^i\rangle \neq \langle\mathcal{P}_q^i\rangle\langle\mathcal{P}_{\bar{q}}^i\rangle$

  $\Rightarrow$ $\rho_{00} \neq 1/3$ with $\langle\mathcal{P}_{q/\bar{q}}^i\rangle = 0$ is possible

  spin polarization of Λ could be unaffected

  (the sources for spin alignment may be fluctuating)

- Spin quantization axis needs not be parallel to the spin polarization (or correlation)
- Anisotropic spin correlation is needed :

X.-L. Sheng et al., PRD 109, 036004, (2024)
A. Kumar, B. Müller, DY, PRD 108, 016020 (2023)

$$\rho_{00}(q) = \frac{1 - \mathrm{Tr}_c\langle\hat{\mathcal{P}}_q^y(\boldsymbol{q}/2)\hat{\mathcal{P}}_{\bar{q}}^y(\boldsymbol{q}/2)\rangle_{\boldsymbol{q}=0}}{3 - \sum_{i=x,y,z}\mathrm{Tr}_c\langle\hat{\mathcal{P}}_q^i(\boldsymbol{q}/2)\hat{\mathcal{P}}_{\bar{q}}^i(\boldsymbol{q}/2)\rangle_{\boldsymbol{q}=0}}$$

(quark model & kinetic equation of vector mesons in the non-relativistic limit)

$$\xrightarrow[|\langle\mathcal{P}_q^i\mathcal{P}_{\bar{q}}^i\rangle| \ll 1]{} \quad \rho_{00} \approx \frac{1}{3} + \frac{1}{9}\mathrm{Tr}_c\left(\langle\hat{\mathcal{P}}_q^x\hat{\mathcal{P}}_{\bar{q}}^x\rangle + \langle\hat{\mathcal{P}}_q^z\hat{\mathcal{P}}_{\bar{q}}^z\rangle - 2\langle\hat{\mathcal{P}}_q^y\hat{\mathcal{P}}_{\bar{q}}^y\rangle\right)$$

$$\rho_{00} = 1/3 \ \text{ when } \langle\mathcal{P}_q^j\mathcal{P}_{\bar{q}}^j\rangle \neq 0 \text{ is isotropic.}$$

- Electromagnetic fields can polarize the spin. How about gluonic fields in QCD matter?

$$\mathcal{P}^i(\boldsymbol{p}) \propto c_1 B^i + c_2\epsilon^{ijk}p_j E_k$$

Axial kinetic theory with color fields

- Incorporation of background color fields into Wigner functions and kinetic equations.
- Color decomposition : $O = O^s I + O^a t^a$

U. W. Heinz, Phys. Rev. Lett. 51, 351 (1983)
H. T. Elze, M. Gyulassy, D. Vasak, Nucl. Phys. B276, 706 (1986).

e.g., $\mathcal{A}^\mu(\boldsymbol{p},x) = \mathcal{A}^{s\mu}(\boldsymbol{p},x)I + \mathcal{A}^{a\mu}(\boldsymbol{p},x)\,t^a,\quad f_V(\boldsymbol{p},x) = f_{\rm V}^{\rm s}(\boldsymbol{p},x)I + f_{\rm V}^a(\boldsymbol{p},x)\,t^a,$

$$\tilde{a}^\mu(\boldsymbol{p},x) = \tilde{a}^{s\mu}(\boldsymbol{p},x)I + \tilde{a}^{a\mu}(\boldsymbol{p},x)\,t^a.$$

- Kinetic equations : DY, JHEP 06, 140 (2022)

SKEs : $p^\rho\left(\partial_\rho f_{\rm V}^{\rm s} + \frac{g}{2N_c}F^a_{\nu\rho}\partial_p^\nu f_{\rm V}^a\right) = \mathcal{C}_{\rm s},\quad p^\rho\left(\partial_\rho f_{\rm V}^a + gF^a_{\nu\rho}\partial_p^\nu f_{\rm V}^{\rm s} + \frac{d^{bca}}{2}gF^b_{\nu\rho}\partial_p^\nu f_{\rm V}^c\right) = \mathcal{C}_{\rm o}^a,$

diffusion

dynamical spin polarization

AKEs : $p^\rho\partial_\rho\tilde{a}^{s\mu} + \frac{g}{2N_c}\left(p^\rho F^a_{\nu\rho}\partial_p^\nu\tilde{a}^{a\mu} + F^{a\nu\mu}\tilde{a}^a_\nu\right) - \frac{\hbar}{4N_c}\epsilon^{\mu\nu\rho\sigma}p_\rho(\partial_\sigma gF^a_{\beta\nu})\partial_p^\beta f_{\rm V}^a = \mathcal{C}_{\rm s}^\mu,$

$$p^\rho\partial_\rho\tilde{a}^{a\mu} + g\left(p^\rho F^a_{\nu\rho}\partial_p^\nu\tilde{a}^{s\mu} + F^{a\nu\mu}\tilde{a}^{\rm s}_\nu\right) + \frac{d^{bca}}{2}g\left(p^\rho F^b_{\nu\rho}\partial_p^\nu\tilde{a}^{c\mu} + F^{b\nu\mu}\tilde{a}^c_\nu\right) - \frac{\hbar}{2}\epsilon^{\mu\nu\rho\sigma}p_\rho(\partial_\sigma gF^a_{\beta\nu})\partial_p^\beta f_{\rm V}^{\rm s} = \mathcal{C}_{\rm o}^{a\mu}.$$

Axial Wigner functions :

$$\mathcal{A}^{s\mu}(\boldsymbol{p},x) = \frac{1}{2\epsilon_{\boldsymbol{p}}}\left[\tilde{a}^{s\mu} - \frac{\hbar}{4N_c}\tilde{F}^{a\mu\nu}\left(\partial_{p\nu}f_{\rm V}^a - \frac{\epsilon_{\boldsymbol{p}}}{2}\partial_{p_\perp\nu}(f_{\rm V}^a/\epsilon_{\boldsymbol{p}})\right)\right]_{p_0=\epsilon_{\boldsymbol{p}}},$$

$$\mathcal{A}^{a\mu}(\boldsymbol{p},x) = \frac{1}{2\epsilon_{\boldsymbol{p}}}\left[\tilde{a}^{a\mu} - \frac{\hbar}{2}\tilde{F}^{a\mu\nu}\left(\partial_{p\nu}f_{\rm V}^{\rm s} - \frac{\epsilon_{\boldsymbol{p}}}{2}\partial_{p_\perp\nu}(f_{\rm V}^{\rm s}/\epsilon_{\boldsymbol{p}})\right)\right]_{p_0=\epsilon_{\boldsymbol{p}}}.$$

dynamical (w/ memory effect) non-dynamical (w/o memory effect)

Spin polarization: $\mathcal{P}^\mu(\boldsymbol{p}) = \frac{\int d\Sigma\cdot p\,{\rm Tr_c}\mathcal{A}^\mu(\boldsymbol{p},x)}{2m\int d\Sigma\cdot p\,(2\epsilon_{\boldsymbol{p}})^{-1}f_{\rm V}^{\rm s}(\boldsymbol{p},x)} = \frac{\int d\Sigma\cdot p\,\mathcal{A}^{s\mu}(\boldsymbol{p},x)}{2m\int d\Sigma\cdot p\,(2\epsilon_{\boldsymbol{p}})^{-1}f_{\rm V}^{\rm s}(\boldsymbol{p},x)}.$

Axial kinetic theory

- Axial kinetic theory : scalar/axial-vector kinetic eqs. (SKE/AKE)

K. Hattori, Y. Hidaka, DY, PRD 100, 096011 (2019)
DY, K. Hattori, Y. Hidaka, JHEP 20, 070 (2020)
Z. Wang, X. Guo, P. Zhuang, Eur. Phys. J. C 81, 799 (2021)

➢ SKE : $p\cdot\Delta f_V = \mathcal{C}[f_V], \quad \Delta_\mu = \partial_\mu + \underbrace{eF_{\nu\mu}}_{\text{EM fields}}\partial_p^\nu.$

standard Vlasov eq.

➢ AKE : $p\cdot\Delta\tilde{a}^\mu + eF^{\nu\mu}\tilde{a}_\nu - \frac{e}{2}\hbar\epsilon^{\mu\nu\rho\sigma}p_\rho(\partial_\sigma F_{\beta\nu})\partial_p^\beta f_V = \underbrace{\hat{L}^{\mu\nu}\tilde{a}_\nu}_{\text{spin relaxation}} + \underbrace{\hbar\hat{H}^{\mu\nu}\partial_\nu f_V}_{\text{dynamical spin pol. from spin-orbit int.}}.$

($\tilde{a}^\mu(p,x)$: effective spin four vector)
($\hbar$: gradient correction in phase space)

(entangled f_V & $\tilde{a}^\mu$)

➢ Axial Wigner functions :

$$\mathcal{A}^\mu(\boldsymbol{p},x) = \frac{1}{2\epsilon_{\boldsymbol{p}}}\Big[\tilde{a}^\mu - \frac{\hbar}{2}\tilde{F}^{\mu\nu}\Big(\partial_{p\nu}f_V - \frac{\epsilon_{\boldsymbol{p}}}{2}\partial_{p_\perp\nu}(f_V/\epsilon_{\boldsymbol{p}})\Big)\Big]_{p_0=\epsilon_{\boldsymbol{p}}=\sqrt{|\boldsymbol{p}|^2+m^2}}.$$

dynamical (w/ memory effect) non-dynamical (w/o memory effect)

$$\Rightarrow \quad \mathcal{P}^\mu(\boldsymbol{p}) = \frac{\int d\Sigma\cdot p\,\mathcal{A}^\mu(\boldsymbol{p},x)}{2m\int d\Sigma\cdot p\,(2\epsilon_{\boldsymbol{p}})^{-1}f_V(\boldsymbol{p},x)}.$$

- Relaxation-time approx. & weak coupling :

$$p\cdot\partial\tilde{a}^\mu - \frac{e}{2}\hbar\epsilon^{\mu\nu\rho\sigma}p_\rho(\partial_\sigma F_{\beta\nu})\partial_p^\beta f_V = -\frac{p_0\delta\tilde{a}^\mu}{\tau_{\mathrm{R}}}, \quad \delta\tilde{a}^\mu = \tilde{a}^\mu - \tilde{a}^\mu_{\mathrm{eq}}.$$

$$\Rightarrow \quad \delta\tilde{a}^\mu(\boldsymbol{p},x) = \frac{\hbar e}{2}\int_{t_{\mathrm{i}}}^{t_{\mathrm{f}}} dx_0' e^{-(x_0-x_0')/\tau_{\mathrm{R}}}\epsilon^{\mu\nu\rho\sigma}\hat{p}_\rho\big(\partial_{x'\sigma}F_{\beta\nu}(x')\big)\partial_p^\beta f_V(p,x')$$

Spin alignment from glasma

- Generalized AKT with color fields : $\tilde{a}^{\mu}(\boldsymbol{p},x)=\tilde{a}^{s\mu}(\boldsymbol{p},x)I+\boxed{\tilde{a}^{a\mu}(\boldsymbol{p},x)}\,t^{a}.$

  DY, JHEP 06, 140 (2022)
  B. Müller, DY, PRD 105, L011901 (2022)

  (more dominant in the perturbative approach)

- Dynamical spin polarization from glasma fields : A. Kumar, B. Müller, DY, PRD 108, 016020 (2023)

suppressed

$$\tilde{a}^{a\mu}(\boldsymbol{p},x)\approx-\frac{\hbar g}{2}e^{-(t_{\rm f}-t_{\rm i})/\tau_{\rm R}^{\rm o}}\left(B^{a\mu}(t_{\rm i})\partial_{\epsilon_{\boldsymbol{p}}}f_{\rm V}^{\rm s}(\epsilon_{\boldsymbol{p}},t_{\rm i})-\cancel{B^{a\mu}(t_{\rm f})\partial_{\epsilon_{\boldsymbol{p}}}f_{\rm V}^{\rm s}(\epsilon_{\boldsymbol{p}},t_{\rm f})}\right)$$

- Correlation of initial color magnetic fields : $g^{2}\langle B^{az}(x)B^{az}(x)\rangle_{x_0=t_{\rm i}}\sim\dfrac{Q_s^4(N_c^2-1)}{2N_c}$

  K. J. Golec-Biernat and M. Wusthoff, Phys. Rev. D 59, 014017 (1998)
  P. Guerrero-Rodr´ıguez and T. Lappi, Phys. Rev. D 104, 014011 (2021)

- Initial quark distribution function : $f_{\rm V}^{\rm s}(\epsilon_{\boldsymbol{p}},t_{\rm i})=1/\left(e^{\epsilon_{\boldsymbol{p}}/Q_s}+1\right)$

❖ spin correlation : $\langle\mathcal{P}_q^z\mathcal{P}_{\bar{q}}^z\rangle\sim\dfrac{Q_s^2}{m_q m_{\bar{q}}}e^{-2(t_{\rm f}-t_{\rm i})/\tau_{\rm R}^{\rm o}}$

❖ Order-of-magnitude estimation (for ϕ) : $\rho_{00}\sim\dfrac{1}{3+10\,e^{-2(t_{\rm f}-t_{\rm i})/\tau_{\rm R}^{\rm o}}}<\dfrac{1}{3}$

$Q_s\approx 1\sim 2\ \mathrm{GeV}$

glasma effect　　relaxation effect

❖ Heavy-quark approx. : $\tau_{\rm R}^{\rm o}\approx\left(\dfrac{g^2C_2(F)m_D^2T}{6\pi m^2}\ln g\right)^{-1}\approx 5\,\mathrm{fm/c}\;\Rightarrow\;\rho_{00}\approx 0.24$

M. Hongo et al., JHEP 08, 263 (2022)

(model dependent)

Spin correlations from color fields

- Spin density matrix can be directly related to Wigner functions of the coalesced quark and antiquark through the quark-meson interaction.

❖ Kinetic theory of vector mesons :

$$q\cdot\partial f^{\phi}_{\lambda}=\epsilon^{*}_{\mu}(\lambda,\boldsymbol{q})\epsilon_{\nu}(\lambda,\boldsymbol{q})\left[\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(q,x)(1+f^{\phi}_{\lambda})-\mathcal{C}^{\mu\nu}_{\mathrm{diss}}(q,x)f^{\phi}_{\lambda}\right]\approx\boxed{\epsilon^{*}_{\mu}(\lambda,\boldsymbol{q})\epsilon_{\nu}(\lambda,\boldsymbol{q})\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(q,x)}$$

quark-meson int. :

$$\mathcal{L}_{\mathrm{int}}=g_{\phi}\bar{\psi}\gamma^{\mu}V_{\mu}\psi$$

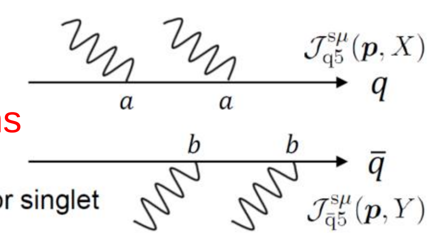
$$\rho_{00}(q)=\frac{\int d\Sigma_X\cdot q f^{\phi}_{0}(q,X)}{\int d\Sigma_X\cdot q\big(f^{\phi}_{0}(q,X)+f^{\phi}_{+1}(q,X)+f^{\phi}_{-1}(q,X)\big)}$$
$$=\frac{1-\mathrm{Tr}_c\langle\hat{\mathcal{P}}^{y}_{q}(\boldsymbol{q}/2)\hat{\mathcal{P}}^{y}_{\bar{q}}(\boldsymbol{q}/2)\rangle_{\boldsymbol{q}=0}}{3-\sum_{i=x,y,z}\mathrm{Tr}_c\langle\hat{\mathcal{P}}^{i}_{q}(\boldsymbol{q}/2)\hat{\mathcal{P}}^{i}_{\bar{q}}(\boldsymbol{q}/2)\rangle_{\boldsymbol{q}=0}}$$

❖ Spin correlations in the non-relativistic limit :

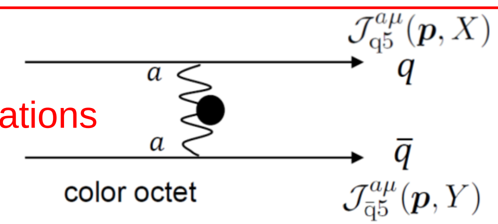
$$\mathrm{Tr}_c\langle\hat{\mathcal{P}}^{i}_{q}(\boldsymbol{p})\hat{\mathcal{P}}^{i}_{\bar{q}}(\boldsymbol{p})\rangle\approx\frac{4\int d\Sigma_X\cdot p\big(\langle\mathcal{A}^{si}_{q}(\boldsymbol{p},X)\mathcal{A}^{si}_{\bar{q}}(\boldsymbol{p},X)\rangle+\langle\mathcal{A}^{ai}_{q}(\boldsymbol{p},X)\mathcal{A}^{ai}_{\bar{q}}(\boldsymbol{p},X)\rangle/(2N_c)\big)}{\int d\Sigma_X\cdot p f^{\mathrm{s}}_{\mathrm{V}q}(\boldsymbol{p},X)f^{\mathrm{s}}_{\mathrm{V}\bar{q}}(\boldsymbol{p},X)}$$

➢ Weak coupling :

4-field correlations $\propto g^4$



2-field correlations $\propto g^2$



A. Kumar, B. Müller, DY, PRD 107, 076025 (2023)

A. Kumar, B. Müller, DY, PRD 108, 016020 (2023)

Color fields from the glasma

- Solving linearized Yang-Mills eqs. : $[D_\mu, F^{\mu\nu}] = J^\nu$

P. Guerrero-Rodrguez, T. Lappi, PRD 104, 014011 (2021)

- Color-field correlators in the glasma :

evolution in time

e.g. $$\langle E_T^{ai}(X')E_T^{aj}(X'')\rangle = -\bar{N}_c \epsilon^{in}\epsilon^{jm}\int_{\perp;q,u}^{X'}\int_{\perp;l,v}^{X''}\Omega_-(u_\perp,v_\perp)\frac{q^n l^m}{ql}\times \boxed{J_1(qX_0')J_1(lX_0'')},$$

$$\langle B_T^{ai}(X')B_T^{aj}(X'')\rangle = -\bar{N}_c \epsilon^{in}\epsilon^{jm}\int_{\perp;q,u}^{X'}\int_{\perp;l,v}^{X''}\Omega_+(u_\perp,v_\perp)\frac{q^n l^m}{ql}\times J_1(qX_0')J_1(lX_0''),$$

$$\bar{N}_c \equiv \frac{1}{2}g^2 N_c(N_c^2-1),$$

$$\Omega_\mp(u_\perp,v_\perp) = [G_1(u_\perp,v_\perp)\boxed{G_2(u_\perp,v_\perp)} \mp h_1(u_\perp,v_\perp)\boxed{h_2(u_\perp,v_\perp)}],$$

$$\int_{\perp;q,u}^{X'} \equiv \int\frac{d^2q_\perp}{(2\pi)^2}\int d^2u_\perp e^{iq_\perp(X'-u)_\perp}.$$

unpolarized & linearly polarized
Gluon distribution functions

- Golec-Biernat Wusthoff (GBW) distribution : K. J. Golec-Biernat and M. Wustho, PRD 59, 014017 (1998)

$$\Omega_\pm(u_\perp,v_\perp) = \Omega(u_\perp,v_\perp) = \frac{Q_s^4}{g^4 N_c^2}\left(\frac{1-e^{-Q_s^2|u_\perp-v_\perp|^2/4}}{Q_s^2|u_\perp-v_\perp|^2/4}\right)^2$$