

Neutrinoless double beta decay of hyperons in covariant chiral perturbation theory

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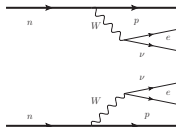
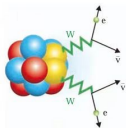


- 1 Introduction
- 2 Calculation in BChPT
- 3 Summary and Outlook

Nuclear double beta decays

- $2\nu 2\beta$ -decay

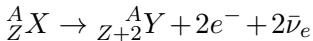
[M.G.Mayer, Phys.Rev.48,512(1935)]



$$T_{\frac{1}{2}} \sim 10^{18-24} \text{ yr}$$

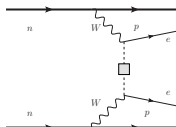
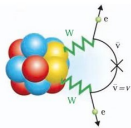
[A.s.Barabash, Phys.Rev.C81,035501(2010)]

✓ Experimentally Observed



- $0\nu 2\beta$ -decay

[Furry, Phys.Rev.56,1184(1939)]



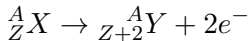
$$T_{\frac{1}{2}} > 3.8 \times 10^{26} \text{ yr}$$

[KamLAND-Zen

Collaboration, arXiv:2406.11438

[hep-ex] (2024)]

✓ Lepton number violation
✓ Majorana nature of neutrinos
✓ Neutrino mass scale and hierarchy



Half-life of $0\nu\beta\beta$

Nuclear matrix elements (NMEs) encode the impact of the nuclear structure on the decay half-life, crucial to interpreting the experimental limits on the effective neutrino mass.

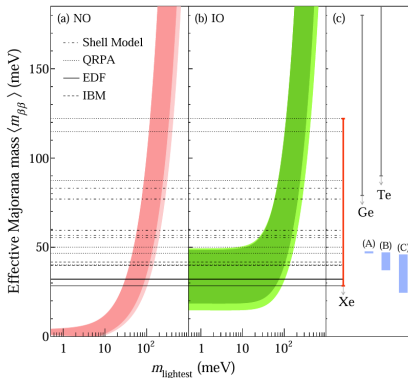
$$(T_{\frac{1}{2}}^{0\nu})^{-1} = G^{0\nu} |M^{0\nu}|^2 |m_{\beta\beta}|^2$$

(Light Majorana neutrino exchange mechanism)

NMEs:

$$M^{0\nu} = \langle \Psi_f | \hat{O}^{0\nu} | \Psi_i \rangle$$

- Depending on decay operator $\hat{O}^{0\nu}$
- Differences between different nuclear models for $\Psi_{f,i}$
 $\Rightarrow$ Sizable uncertainties



Theoretical

Due to the lepton number violation(LNV) in the neutrinoless double beta decay process, there must be new physics to the process that goes beyond the Standard Model.

- Top-down approach: Study signals in explicit BSM
- **Bottom-up approach**: Work with effective field theories(EFT)

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i^{(5)}}{\Lambda} \mathcal{O}_i^{(5)} + \sum_j \frac{C_j^{(6)}}{\Lambda^2} \mathcal{O}_j^{(6)} + \sum_k \frac{C_k^{(7)}}{\Lambda^3} \mathcal{O}_k^{(7)} + \sum_j \frac{C_j^{(8)}}{\Lambda^4} \mathcal{O}_j^{(8)} + \sum_j \frac{C_j^{(9)}}{\Lambda^5} \mathcal{O}_j^{(9)} + \dots$$

$$\mathcal{O}^{(5)} = \epsilon_{kl} \epsilon_{mn} (L_k^T C^{(5)} L_m) H_l H_n$$

[Phys. Rev. Lett. 43 (1979) 1566]
[JHEP 11(2016)043]

$$\psi^2 H^4 : \quad \epsilon_{ij} \epsilon_{mn} (L_i^T C L_m) H_j H_n (H^\dagger H)$$

$$\psi^4 D : \quad \epsilon_{ij} (\bar{d} \gamma_\mu u) (L_i^T C D^\mu L_j)$$

$$\psi^2 H^2 D^2 : \quad \epsilon_{ij} \epsilon_{mn} (L_i^T C D_\mu L_j) H_m (D^\mu H)_n$$

$$\psi^4 H : \quad \epsilon_{ij} \epsilon_{mn} (\bar{e} L_i) (L_j^T C L_m) H_n$$

$$\epsilon_{im} \epsilon_{jn} (L_i^T C D_\mu L_j) H_m (D^\mu H)_n$$

$$\epsilon_{ij} \epsilon_{mn} (\bar{d}_L Q_i^T C L_m) H_n$$

$$\psi^2 H^3 D : \quad \epsilon_{ij} \epsilon_{mn} (L_i^T C \gamma^\mu e) H_m (D_\mu H)_n$$

$$\epsilon_{ij} \epsilon_{mn} (\bar{d}_L Q_j^T C L_m) H_n$$

$$\psi^2 H^2 X : \quad \epsilon_{ij} \epsilon_{mn} g' (L_i^T C \sigma^{\mu\nu} L_m) H_j H_n B_{\mu\nu}$$

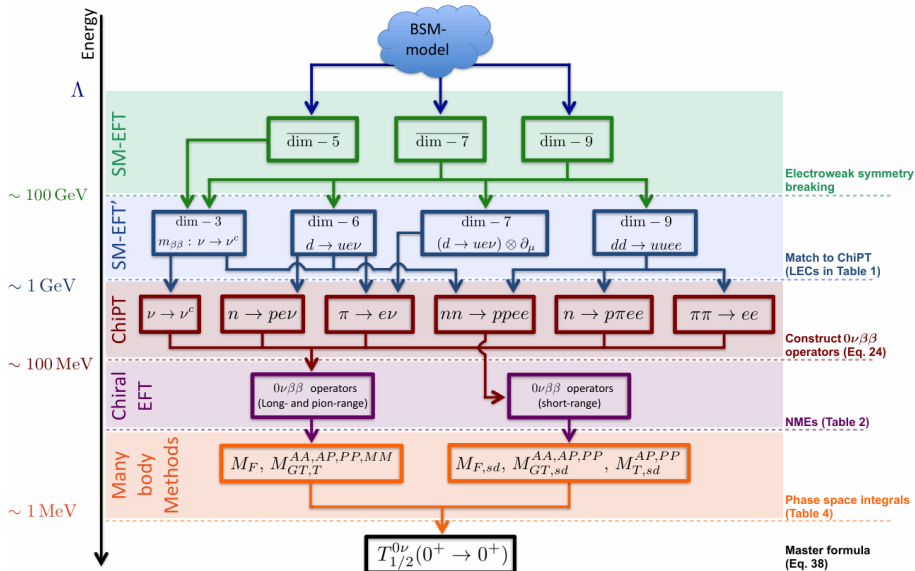
$$\epsilon_{ij} (\bar{u}_L Q_m^T C L_i) H_j$$

$$\epsilon_{ij} (\epsilon \tau^I)_{mn} g (L_i^T C \sigma^{\mu\nu} L_m) H_j H_n W_{\mu\nu}^I$$

$$\epsilon_{ij} (\bar{e} \bar{d}_R) (Q_j^T C u_R) H_i$$

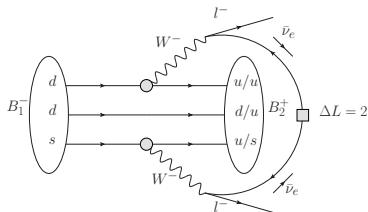
EFT at various energy scales

[Cirigliano, et al. JHEP12(2018)]

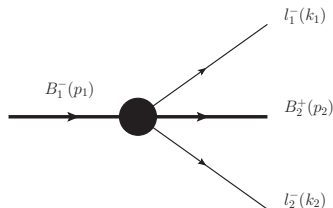


Hyperon $0\nu 2\beta$ decay in BChPT

- **ChPT** is an effective field theory of QCD at low energy based on chiral symmetry at hadronic level.
- A first glance: from quarks to hadrons



Quark level



Hadron level

The Lorentz invariant Mandelstam variables:

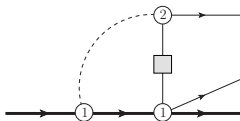
$$s = (k_1 + k_2)^2 = (p_1 - p_2)^2$$

$$t = (k_1 + p_2)^2 = (p_1 - k_2)^2$$

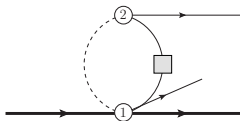
$$u = (k_2 + p_2)^2 = (p_1 - k_1)^2$$

Hyperon $0\nu 2\beta$ decay in BChPT

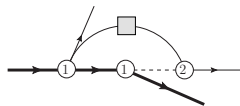
- Description of $0\nu 2\beta$ decay of hyperons at one-loop level in ChPT



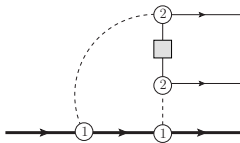
(a)



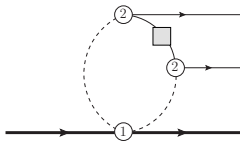
(b)



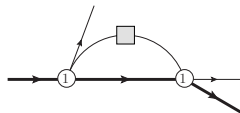
(c)



(d)



(e)



(f)

Chiral effective Lagrangian

The chiral effective Lagrangian relevant to the decay process:

$$\mathcal{L}_{eff} = \mathcal{L}_{MB}^{(1)} + \mathcal{L}_M^{(2)} + \mathcal{L}_{\Delta L=2}$$

- The LO baryon-meson interaction Lagrangian is given by

$$\mathcal{L}_{MB}^{(1)} = \text{Tr} [\bar{B} (i \not{D} - m) B] - \frac{D}{2} \langle \bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\} \rangle - \frac{F}{2} \langle \bar{B} \gamma^\mu \gamma_5 [u_\mu, B] \rangle$$

- The LO chiral Lagrangian for purely mesonic interaction reads

$$\mathcal{L}_M^{(2)} = \frac{F_0^2}{4} \text{Tr}[(D_\mu U)^\dagger D^\mu U] + \frac{F_0^2}{4} \text{Tr}[U^\dagger \chi + U \chi^\dagger]$$

- Low-energy dimension-5 operator for the $\Delta L=2$ Lagrangian

$$\mathcal{L}_{\Delta L=2} = -\frac{1}{2} (m_{\beta\beta})_{ij} (\nu_{L,i}^T C \nu_{L,j} + \bar{\nu}_{L,i} C^\dagger \bar{\nu}_{L,j}^T)$$

Building block

- The covariant derivative:

$$[D_\mu, X] = \partial_\mu X + [\Gamma_\mu, X]$$

- Chiral connection:

$$\Gamma_\mu = \frac{1}{2} \left\{ u^\dagger (\partial_\mu - i r_\mu) u + u (\partial_\mu - i l_\mu) u^\dagger \right\}$$

- The so-called chiral vielbein:

$$u_\mu = i \left\{ u^\dagger (\partial_\mu - i r_\mu) u - u (\partial_\mu - i l_\mu) u^\dagger \right\}$$

- Left-handed current:

$$l_\mu = -2\sqrt{2}G_F T_+ [\bar{\nu}_L \gamma_\mu \ell_L] + \text{h.c.}$$

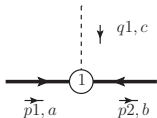
- Right-handed current:

$$r_\mu = 0$$

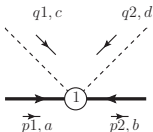
$$T_+ = \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \frac{V_{ud}}{2}(\lambda_1 + i\lambda_2) + \frac{V_{us}}{2}(\lambda_4 + i\lambda_5)$$

Feynman Rules

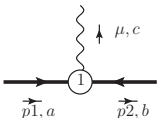
Due to the presence of the λ matrices in the Lagrangian, we can express the vertex rules as a product of the Gell-Mann part and the Dirac part



$$= \frac{1}{F_0} (D d_{abc} + i F f_{abc}) \not{q}_1 \cdot \gamma_5$$



$$= \frac{i}{2F_0^2} f_{abe} f_{cde} (\not{q}_1 - \not{q}_2)$$



$$= \frac{1}{2} \{ -\gamma^\mu f_{abc} + \gamma^\mu \gamma_5 (i D d_{abc} - F f_{abc}) \}$$

Amplitude structure

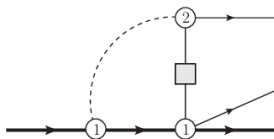
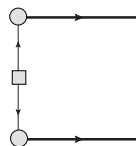


Diagram (a)



leptonic part

$$\mathcal{M} = L^{\mu\nu} H_{\mu\nu} = \underbrace{[\bar{u}_{eL}(k_2)\gamma^\mu\gamma^\nu C\bar{u}_{eL}^T(k_1)]T_{\text{lept}}}_{\text{leptonic}} \underbrace{H_{\mu\nu}}_{\text{hadronic}}$$

$$\Delta S = 0$$

$$\Delta S = 1$$

$$\Delta S = 2$$

Process

$$\Sigma^- \rightarrow \Sigma^+ e^- e^-$$

$$\Sigma^- \rightarrow p e^- e^-$$

$$\Xi^- \rightarrow p e^- e^-$$

$$\Xi^- \rightarrow \Sigma^+ e^- e^-$$

$$T_{\text{lept}}$$

$$8m_{\beta\beta}G_F^2V_{ud}^2$$

$$8m_{\beta\beta}G_F^2V_{ud}V_{us}$$

$$8m_{\beta\beta}G_F^2V_{us}^2$$

Lorentz decomposition of hadronic tensor

$$H_{\mu\nu} = \bar{u}(p_2) \left\{ \sum_{i=1}^{34} (\mathcal{H}_V^i \mathcal{O}_{V,\mu\nu}^i + \mathcal{H}_A^i \mathcal{O}_{A,\mu\nu}^i) \right\} u(p_1)$$

$\mathcal{O}_{V,\mu\nu}^1 = g_{\mu\nu}$
$\mathcal{O}_{V,\mu\nu}^2 = k_1 g_{\mu\nu}$
$\mathcal{O}_{V,\mu\nu}^3 = \gamma_\mu \gamma_\nu \not{k}_1$
$\mathcal{O}_{V,\mu\nu}^4 = \gamma_\mu \gamma_\nu$
$\mathcal{O}_{V,\mu\nu}^5 = k_{1\mu} k_{2\nu}$
$\mathcal{O}_{V,\mu\nu}^6 = k_{1\nu} k_{2\mu}$
$\mathcal{O}_{V,\mu\nu}^7 = k_{1\mu} k_{1\nu}$
$\mathcal{O}_{V,\mu\nu}^8 = k_{2\mu} k_{2\nu}$
$\mathcal{O}_{V,\mu\nu}^9 = p_{1\mu} k_{2\nu}$
$\mathcal{O}_{V,\mu\nu}^{10} = k_{1\nu} p_{1\mu}$
$\mathcal{O}_{V,\mu\nu}^{11} = p_{1\mu} k_{2\nu}$
$\mathcal{O}_{V,\mu\nu}^{12} = k_{1\mu} p_{1\nu}$

$\mathcal{O}_{V,\mu\nu}^{13} = p_{1\nu} k_{2\mu}$
$\mathcal{O}_{V,\mu\nu}^{14} = k_{1\mu} k_{2\nu} \not{k}_1$
$\mathcal{O}_{V,\mu\nu}^{15} = k_{1\nu} k_{2\mu} \not{k}_1$
$\mathcal{O}_{V,\mu\nu}^{16} = k_{1\mu} k_{1\nu} \not{k}_1$
$\mathcal{O}_{V,\mu\nu}^{17} = k_{2\mu} k_{2\nu} \not{k}_1$
$\mathcal{O}_{V,\mu\nu}^{18} = p_{1\mu} p_{1\nu} \not{k}_1$
$\mathcal{O}_{V,\mu\nu}^{19} = k_{1\nu} p_{1\mu} \not{k}_1$
$\mathcal{O}_{V,\mu\nu}^{20} = p_{1\mu} k_{2\nu} \not{k}_1$
$\mathcal{O}_{V,\mu\nu}^{21} = k_{1\mu} p_{1\nu} \not{k}_1$
$\mathcal{O}_{V,\mu\nu}^{22} = p_{1\nu} k_{2\mu} \not{k}_1$
$\mathcal{O}_{V,\mu\nu}^{23} = p_{1\mu} \gamma_\nu$

$\mathcal{O}_{V,\mu\nu}^{24} = p_{1\nu} \gamma_\mu$
$\mathcal{O}_{V,\mu\nu}^{25} = k_{1\mu} \gamma_\nu$
$\mathcal{O}_{V,\mu\nu}^{26} = k_{2\mu} \gamma_\nu$
$\mathcal{O}_{V,\mu\nu}^{27} = k_{1\nu} \gamma_\mu$
$\mathcal{O}_{V,\mu\nu}^{28} = k_{2\nu} \gamma_\mu$
$\mathcal{O}_{V,\mu\nu}^{29} = p_{1\mu} \gamma_\nu \not{k}_1$
$\mathcal{O}_{V,\mu\nu}^{30} = p_{1\nu} \gamma_\mu \not{k}_1$
$\mathcal{O}_{V,\mu\nu}^{31} = k_{1\mu} \gamma_\nu \not{k}_1$
$\mathcal{O}_{V,\mu\nu}^{32} = k_{2\nu} \gamma_\mu \not{k}_1$
$\mathcal{O}_{V,\mu\nu}^{33} = k_{2\mu} \gamma_\nu \not{k}_1$
$\mathcal{O}_{V,\mu\nu}^{34} = k_{1\nu} \gamma_\mu \not{k}_1$

Hadronic amplitude

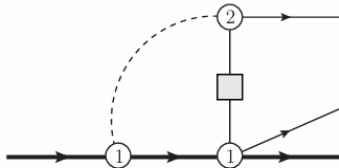


Diagram (a)

$$\begin{aligned} \mathcal{H}_{ij,ab}^{\mu\nu} = & f_{lja}(iD d_{ilb} - F f_{ilb}) \times \frac{i}{8} \int \frac{d^d k}{(2\pi)^d} \frac{\gamma^\nu (k^2 + m_l \not{k} + \not{p}_1 \not{k}) \gamma_5 k^\mu}{[k^2 - M_b^2][(k + p_1)^2 - m_l^2][(k + k_1)^2 - m_{\beta\beta}^2]} \\ & + (iD^2 d_{ilb} d_{lja} - D F d_{ilb} f_{lja} - D F f_{ilb} d_{lja} - iF^2 f_{ilb} f_{lja}) \times \left(-\frac{i}{8}\right) \\ & \int \frac{d^d k}{(2\pi)^d} \frac{\gamma^\nu \gamma_5 (k^2 + m_l \not{k} + \not{p}_1 \not{k}) k^\mu}{[k^2 - M_b^2][(k + p_1)^2 - m_l^2][(k + k_1)^2 - m_{\beta\beta}^2]} \end{aligned}$$

Decay width

Casimir's trick

$$|\overline{\mathcal{M}^2}| = |T_{\text{lept}}|^2 \text{Tr}[\Gamma_1(\not{k}_2 - m_l)\bar{\Gamma}_2(\not{k}_1 + m_l)] \text{Tr}[\Gamma^3(\not{p}_1 + m_{B_1})\bar{\Gamma}^4(\not{p}_2 + m_{B_2})]$$

$$\gamma^\mu \gamma^\nu C = \Gamma_1$$

$$\gamma^\alpha \gamma^\beta C = \Gamma_2$$

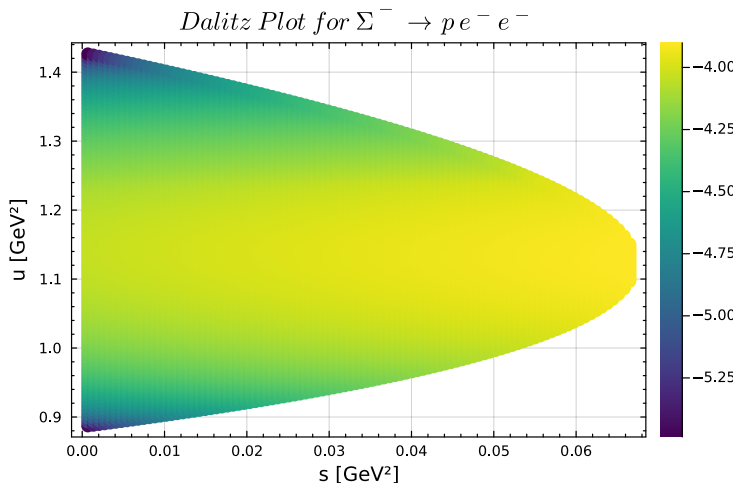
$$\sum_{i=1}^{34} (\mathcal{H}_i \mathcal{O}_{\mu\nu}^i + \mathcal{H}_i^A \mathcal{O}_{\mu\nu}^{Ai}) = \Gamma^3$$

$$\sum_{j=1}^{34} (\mathcal{H}_j \mathcal{O}_{\alpha\beta}^j + \mathcal{H}_j^A \mathcal{O}_{\alpha\beta}^{Aj}) = \Gamma^4$$

Decay width

$$\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32M^3} \int_{u_{\min}}^{u_{\max}} \int_{4m_l^2}^{(m_{B_1} - m_{B_2})^2} |\overline{\mathcal{M}}|^2 ds du$$

Dalitz plot



preliminary result: $\mathcal{B}(\Sigma^- \rightarrow p e^- e^-) = 1.06 \times 10^{-25}$

Comparison

- ① Measurement by BESIII Collaboration: [\[Phys.Rev.D103,052011\(2021\)\]](#)

$$\mathcal{B}(\Sigma^- \rightarrow pe^-e^-) < 6.7 \times 10^{-5}, \quad \mathcal{B}(\Sigma^- \rightarrow \Sigma^+ X) < 1.2 \times 10^{-4}$$

- ② Based on loops involving virtual baryon and Majorana neutrino states:

$$\mathcal{B}(\Sigma^- \rightarrow pe^-e^-) \sim 10^{-31}, \quad \mathcal{B}(\Sigma^- \rightarrow \Sigma^+ X) \sim 10^{-35}$$

[\[H.F Li Phys.Rev.D76, 116008 \(2007\)\]](#)

- ③ Based on the MIT bag model: [\[H.F Li Phys.Rev.D87,036010 \(2013\)\]](#)

$$\mathcal{B}(\Sigma^- \rightarrow pe^-e^-) \sim 10^{-23}$$

Advantage:

- Nuclear $0\nu 2\beta$ decays $\Delta S = 0$; Hyperon $0\nu 2\beta$ decays $\Delta S \neq 0$.
- Hyperon factory @ BESIII & STCF: $J/\Psi \rightarrow \Sigma \bar{\Sigma}$.

Summary and Outlook

Summary

- The prediction of the decay amplitude of $\Sigma^- \rightarrow pe^-e^-$ can be obtained based on **chiral perturbation theory**
- Counter terms still need to be constructed for renormalization with **EOMS** scheme

Outlook

- We will extend the study to the $SU(3)$ particles with $\text{spin-}\frac{3}{2}$ with ChPT
- We are currently using a **dimension-5** operator for $\Delta L = 2$, and as the next step will calculate the decay width in the presence of **dimension-7** operators

Thank you very much for your patience!