Soft dipole resonances in light neutron-rich and proton-rich nuclei

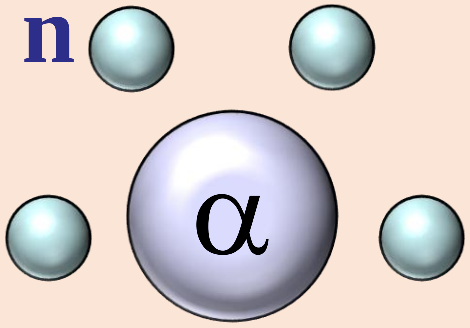
Takayuki Myo

OSAKA INSTITUTE OF TECHNOLOGY

科研費 KAKENHI

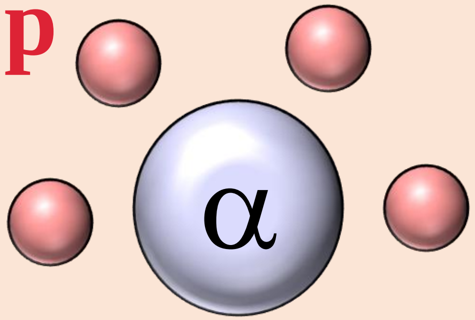
Outline

- Soft dipole resonance (SDR)
- Structures of ^{8}He (1^{-}) states
- Five-body cluster model
- Complex scaling
- Electric dipole strength
- Mirror nucleus ${}^{8}_{6}\mathrm{C}_{2}$
- Isospin symmetry



${}^{8}_{2}\mathrm{He}_{6}$

${}^{8}_{6}\mathrm{C}_{2}$



Collaborators

- Kiyoshi Katō, Hokkaido Univ.
- Myagmarjav Odsuren, National Univ. of Mongolia

Papers

- Physical Review C104 (2021) 044306 Energy spectra of ^{8}He and ^{8}C
- Physical Review C106 (2022) L021302 Soft dipole mode of ^{8}He (letter)
- Prog. Theor. Exp. Phys. 2022 (2022) 103D01 Soft dipole resonance in ^{8}He (full paper)
- Physical Review C107 (2023) 034305 Soft dipole resonance in ^{8}C

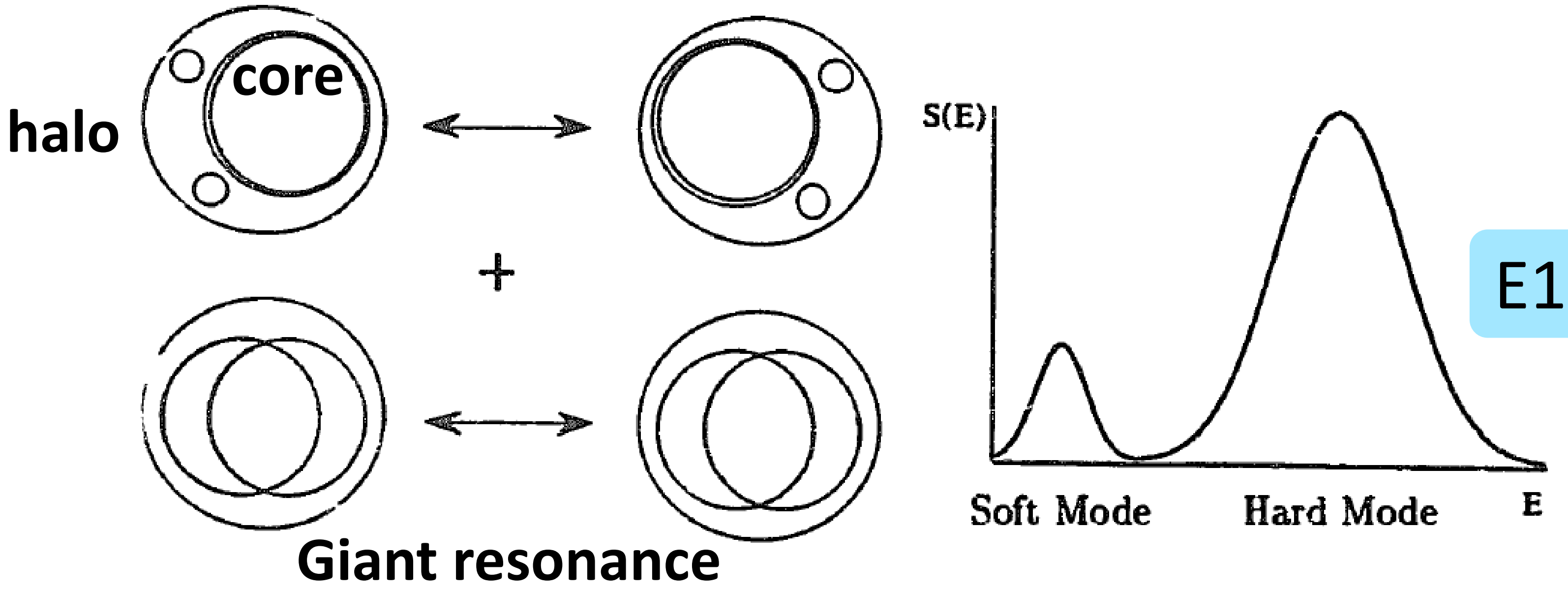
Idea of Soft Dipole Mode in neutron halo nuclei

P.G. Hansen and B. Jonson, Europhys. Lett. **4** (1987) 409.

Weak binding of the last neutrons leads to the appearance of a soft (electric) dipole mode at low excitation energies.

Kiyomi Ikeda, NPA538 (1992) 355

Dipole oscillation of valence neutrons against core

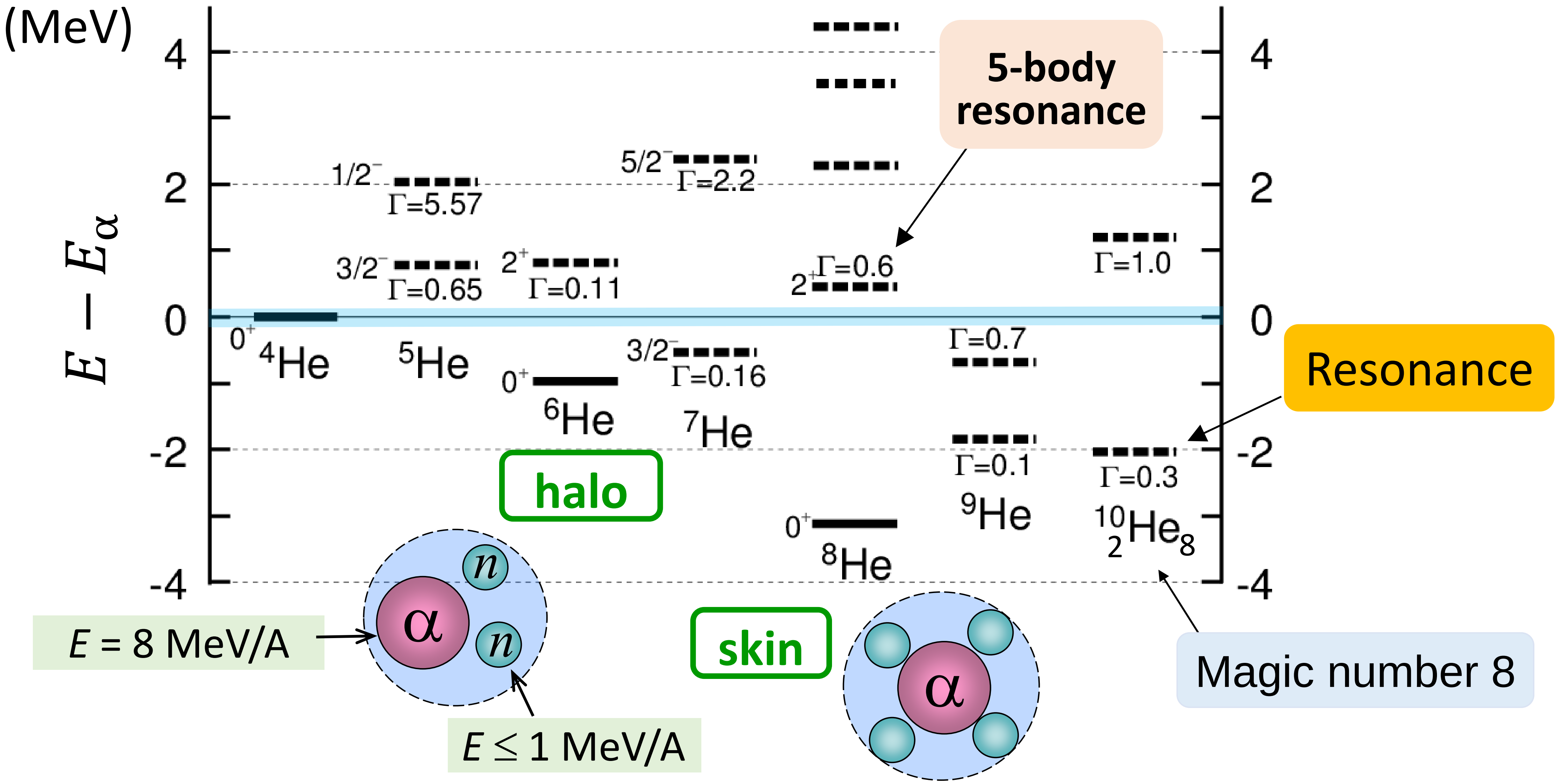


E1 strength

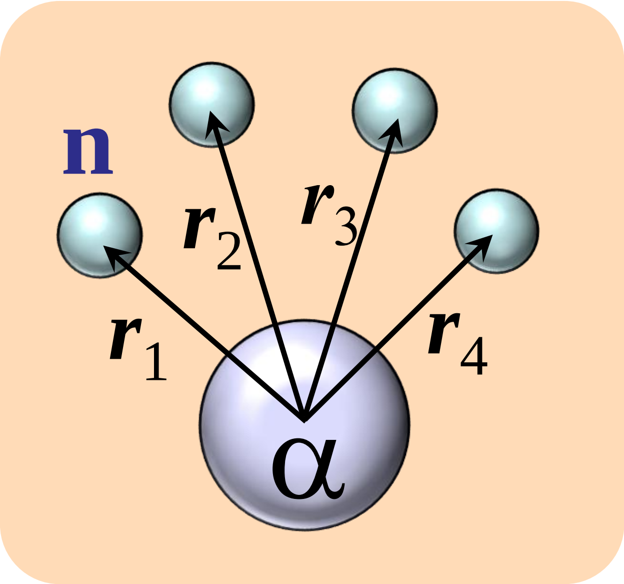
Prof. Kiyomi Ikeda

Neutron-rich He isotopes， Experiments

TUNL Nuclear Data Evaluation



Nuclear Model

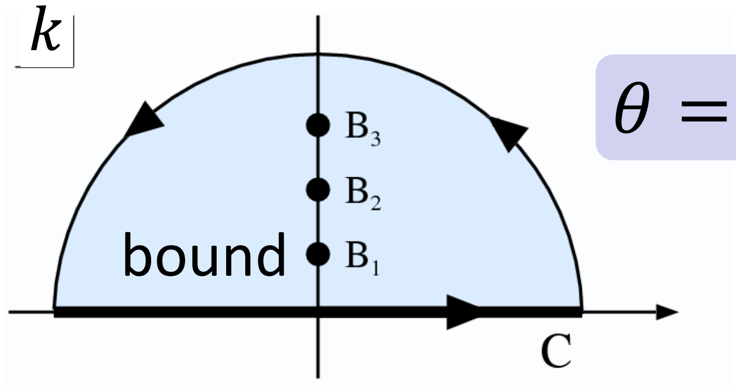


- Stable core nucleus + valence nucleons
  - ^{8}He = α+n+n+n+n (5-body problem)
  - Hamiltonian with $V_{\alpha N}$, V_{NN} (effective)
  - core nucleus is changeable such as ^{16}O
- Method
  - Single-particle picture with configuration mixing (s-wave, p-wave, ….)
  - Relative motion : few-body approach with Gaussian basis expansion
    - Description of strong/weak bindings of valence particles
  - Complex Scaling for resonances (Siegert condition) in many-body systems
    - $\boldsymbol{r}_i \to \boldsymbol{r}_i e^{i\theta}$, $\boldsymbol{k}_i \to \boldsymbol{k}_i e^{-i\theta}$ with scaling angle θ

Complex Scaling

$$H_\theta \Phi_\theta = E_\theta \Phi_\theta$$

$$U(\theta):\ \boldsymbol{r} \to \boldsymbol{r}\exp(i\theta),\quad \boldsymbol{k} \to \boldsymbol{k}\exp(-i\theta),\quad H_\theta = U(\theta) H U^{-1}(\theta),\quad \theta \in \mathbb{R}$$

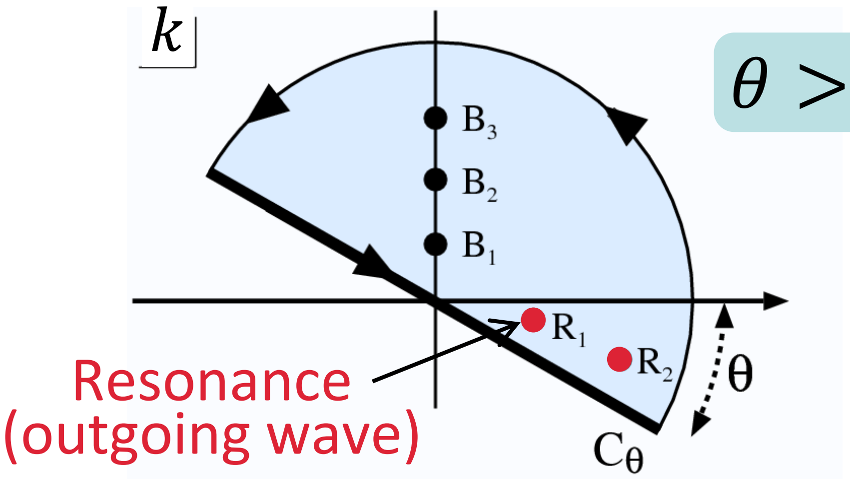


$\theta = 0$

Completeness relation

Textbook by R.G. Newton
J. Math. Phys. 1 (1960)319

$$1 = \sum_{\mathrm{B}} |\varphi_{\mathrm{B}}\rangle\langle\tilde{\varphi}_{\mathrm{B}}| + \int_C dk |\varphi_k\rangle\langle\tilde{\varphi}_k|$$



$\theta > 0$

Extended completeness relation

$$1 = \sum_{\mathrm{B}} |\varphi_{\mathrm{B}}\rangle\langle\tilde{\varphi}_{\mathrm{B}}| + \sum_{\mathrm{R}} |\varphi_{\mathrm{R}}\rangle\langle\tilde{\varphi}_{\mathrm{R}}| + \int_{C_\theta} dk_\theta |\varphi_{k_\theta}\rangle\langle\tilde{\varphi}_{k_\theta}|$$

Siegert state

$$E_{\mathrm{R}} = E_r - i\Gamma/2$$

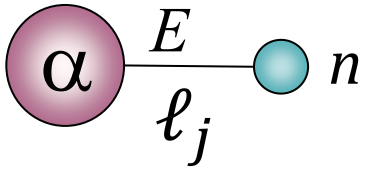
Bi-orthogonal states
T. Berggren, NPA109(1968)265.

J.Aguilar and J.M.Combes, Commun. Math. Phys.,22(1971)269.
E.Balslev and J.M.Combes, Commun. Math. Phys.,22(1971)280.

T. Myo, K. Katō, PTP98 (1997) 1275
B.G. Giraud, K. Katō, A. Ohnishi, J. Phys. A **37** (2004)11575

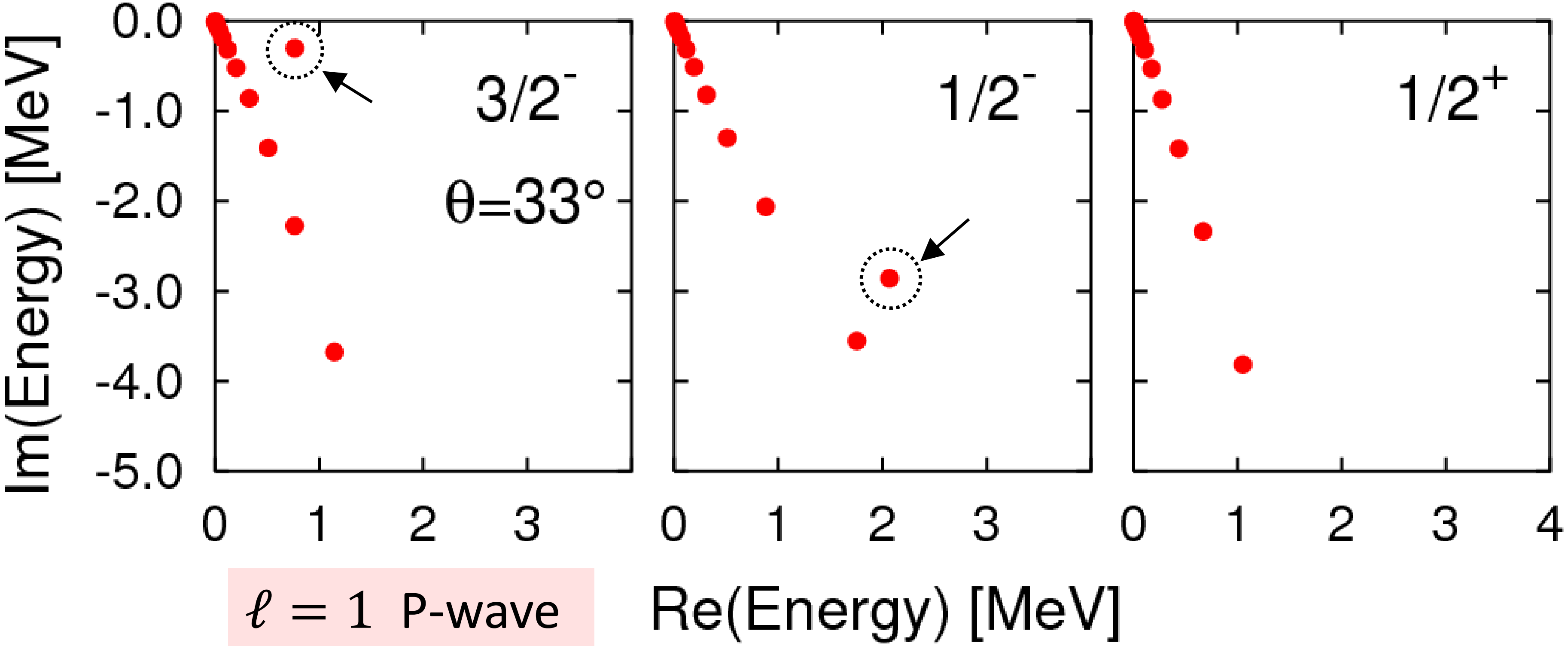
α+n system with discretized continuum states

Complex energy eigenvalues



$j = \ell \pm \frac{1}{2}$

$E_{\mathrm{R}} = E_r - i\Gamma/2$



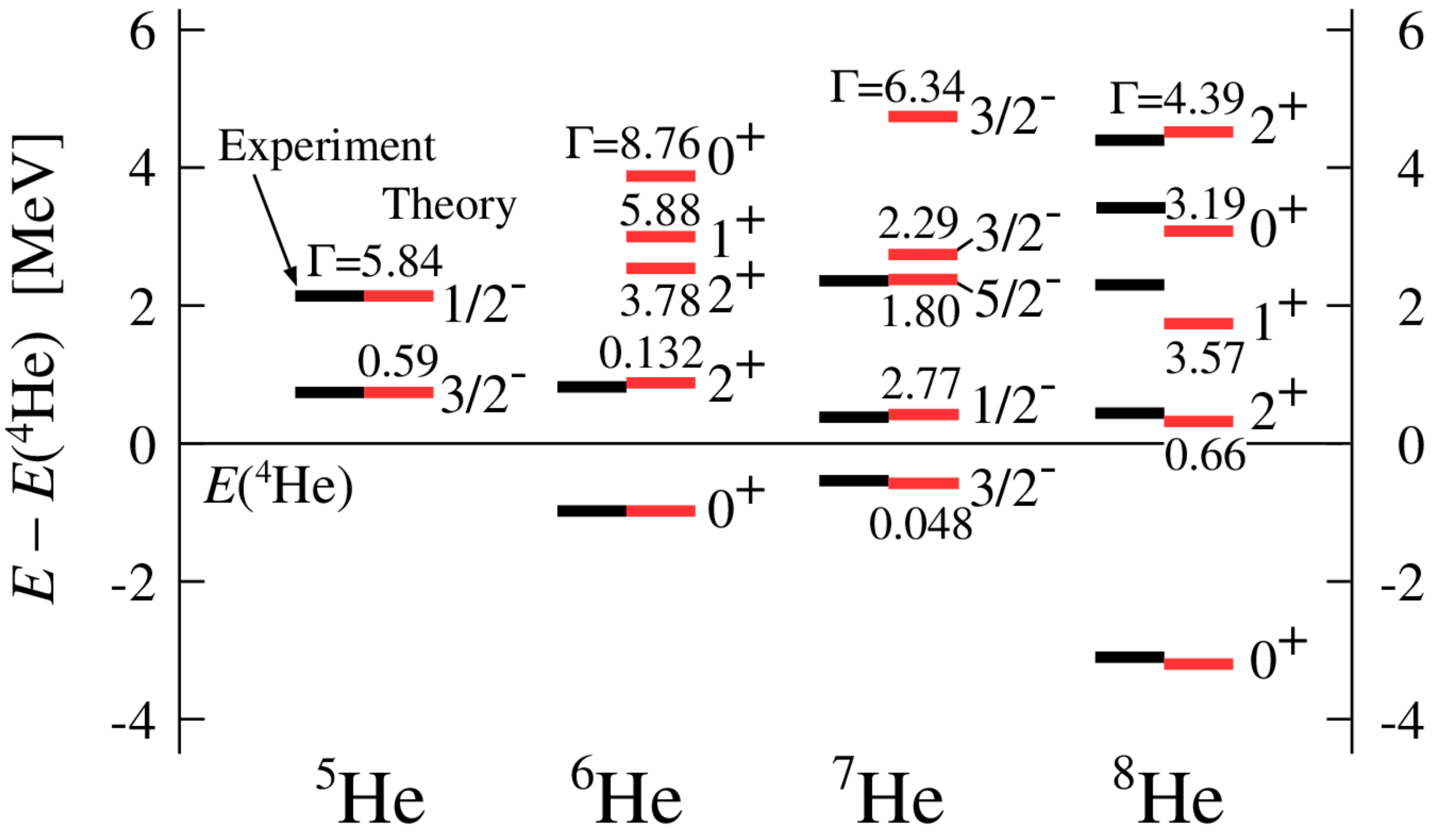
$\ell = 0$ S-wave

No resonance

$\ell = 1$ P-wave

States are discretized with **Gaussian functions** $\phi_\ell(\mathbf{r}) = \sum_n C_n \cdot r^\ell e^{-(r/b_n)^2} Y_\ell(\hat{\mathbf{r}})$

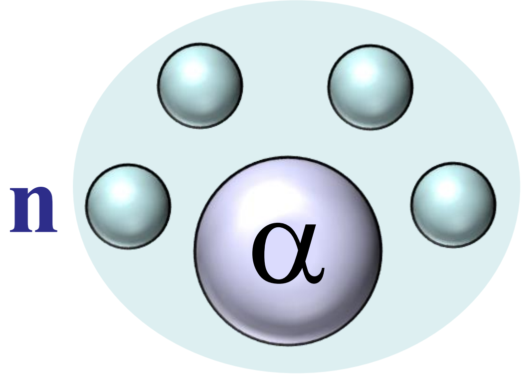
He isotopes : Experiment & Theory



α+n

α+n+n+n+n

α+n+n+n+n



- Solve the motions of valence neutrons precisely
- Reproduction & Prediction

TM. M. Odsuren, K. Katō, PRC104 (2021)

Dipole strength function of ^{8}He to α+n+n+n+n

- $H^\theta |1^-_{n,\theta}\rangle = E^\theta_n |1^-_{n,\theta}\rangle$

  n : state

Complex-scaled Schrödinger equation

- $B_\theta(E1,n) = \frac{1}{2J_0+1} \langle \tilde{1}^-_{n,\theta} \| O^\theta_{E1} \| 0^+_\theta \rangle^2$

E1 matrix element of state n

- $\frac{dB(E1)}{dE} = -\frac{1}{\pi} \mathrm{Im} \left[\sum_n \frac{B_\theta(E1,n)}{E - E^\theta_n} \right]$

Dipole Strength function

- Extended Completeness Relation

TM, Ohnishi, Katō, PTP99(1998)801

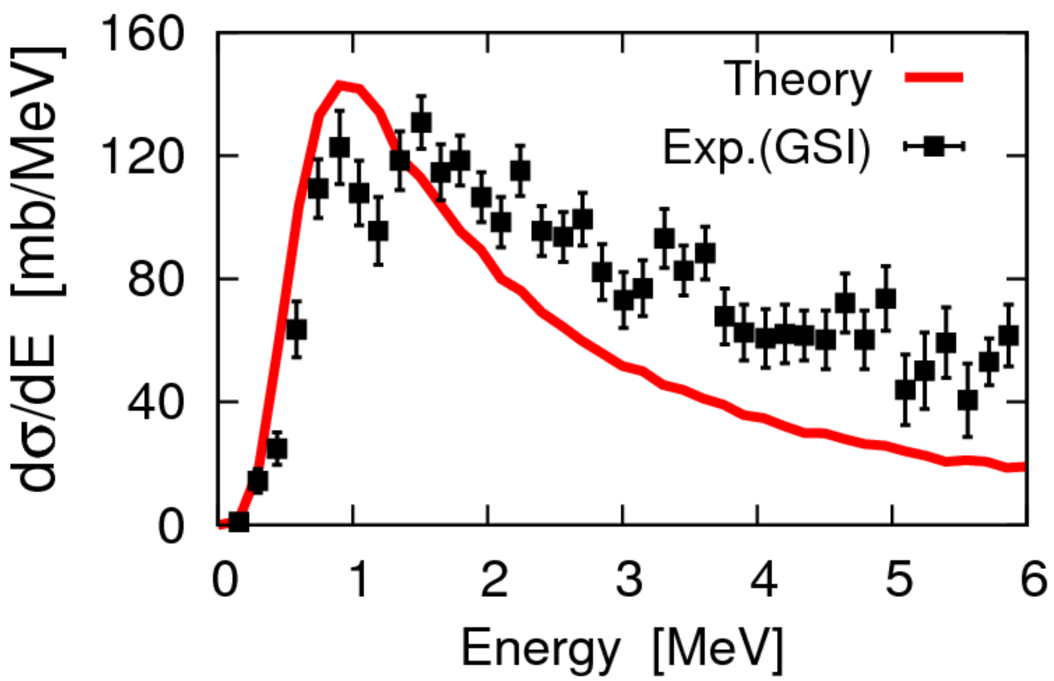
$$\mathbf{1} = \sum_n |1^-_{n,\theta}\rangle\langle\tilde{1}^-_{n,\theta}|$$

$$= \{\alpha + 4n\} + \{^5\mathrm{He} + 3n\} + \{^6\mathrm{He} + 2n\} + \{^7\mathrm{He} + n\} + \{^8\mathrm{He}\}$$

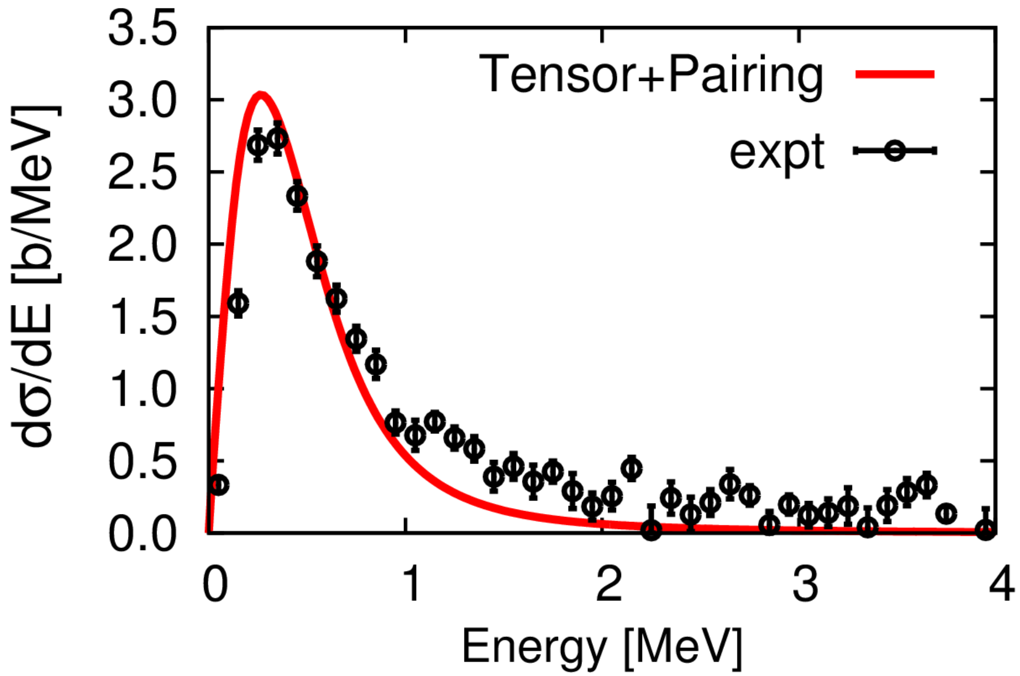
Coulomb breakup strengths of halo nuclei

- Dipole strength with **complex scaling** + Equivalent photon method
- **No three-body resonance: Threshold effect** explains the enhancements

$^{6}He \rightarrow \alpha+n+n$



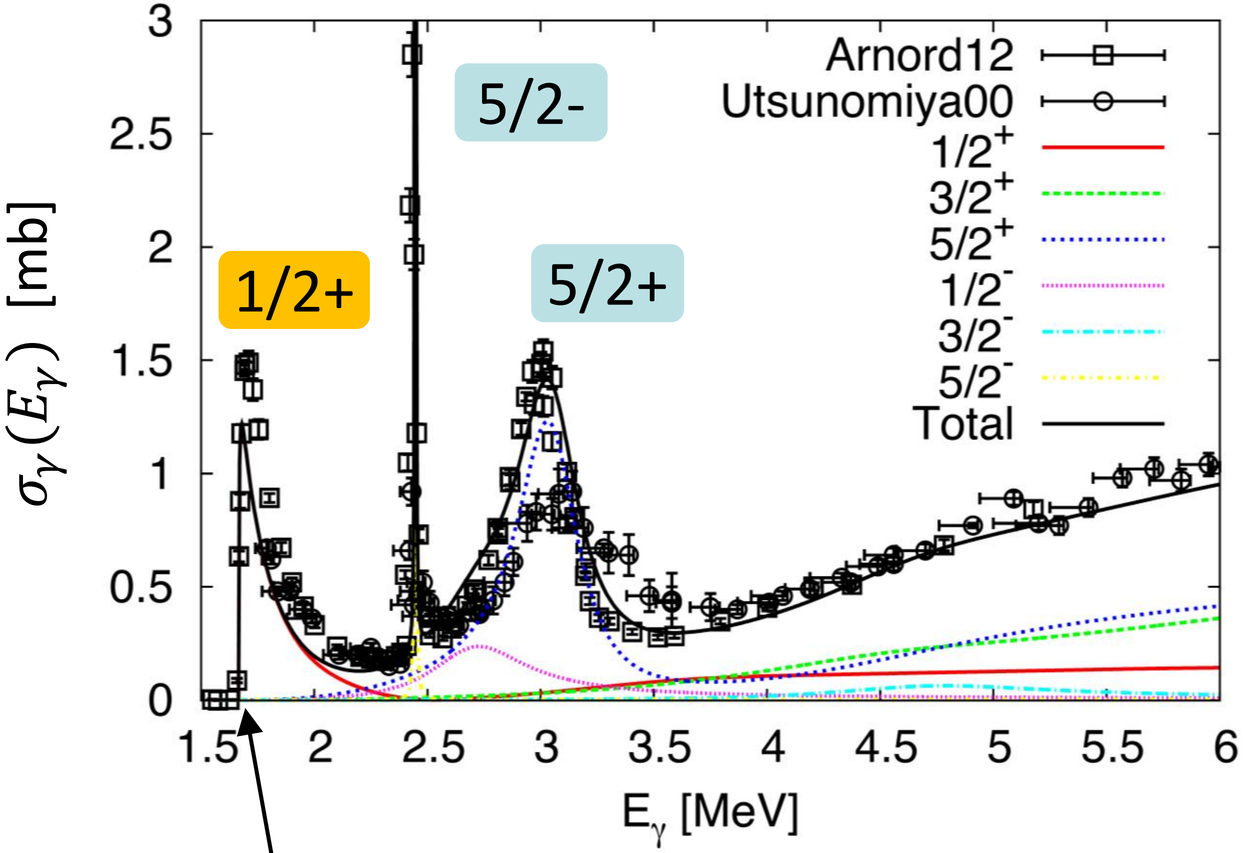
$^{11}Li \rightarrow {}^{9}Li+n+n$



^{6}He : T. Aumann et.al, PRC59 (1999) 1252
^{11}Li : T. Nakamura et al., PRL96 (2006) 252502

TM et al., PRC63 (2001) 054313, PRC76 (2007) 024305
Kikuchi, TM et al., PRC81 (2010) 044308

^{9}Be : Photodisintegration to α+α+n

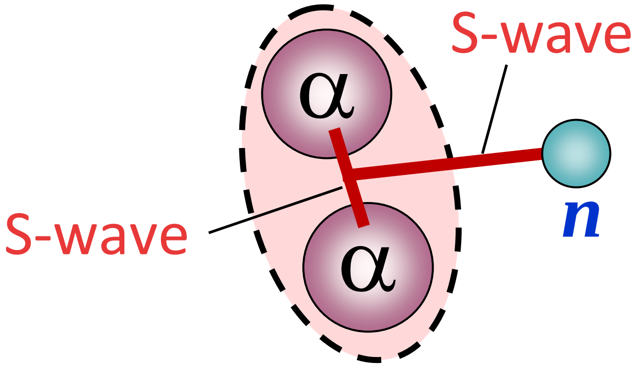


Low-lying enhancement suggests the ^{8}Be+n **S-wave virtual state** above the α+α+n threshold

- Contribute to reaction rate of α(αn,γ)^{9}Be in supernova
- $1/2^+$ state - resonance? / virtual state?

α+α+n with complex scaling

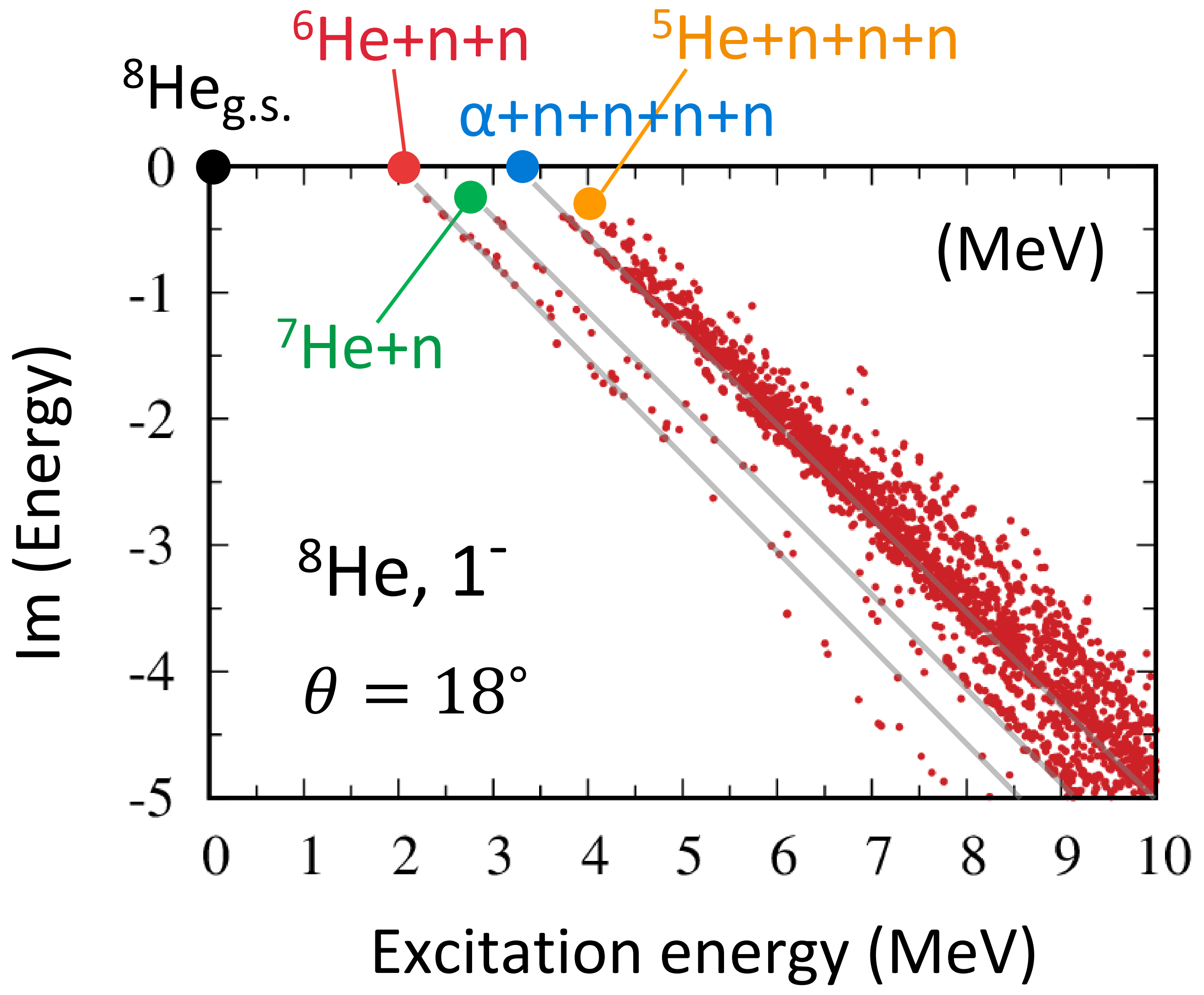
1/2+ : No 3-body resonance



Kikuchi, Katō, Odsuren, Myo, Aikawa
PRC 93 (2016) 054605

1^- states of ^{8}He : Energy eigenvalues

complex scaling

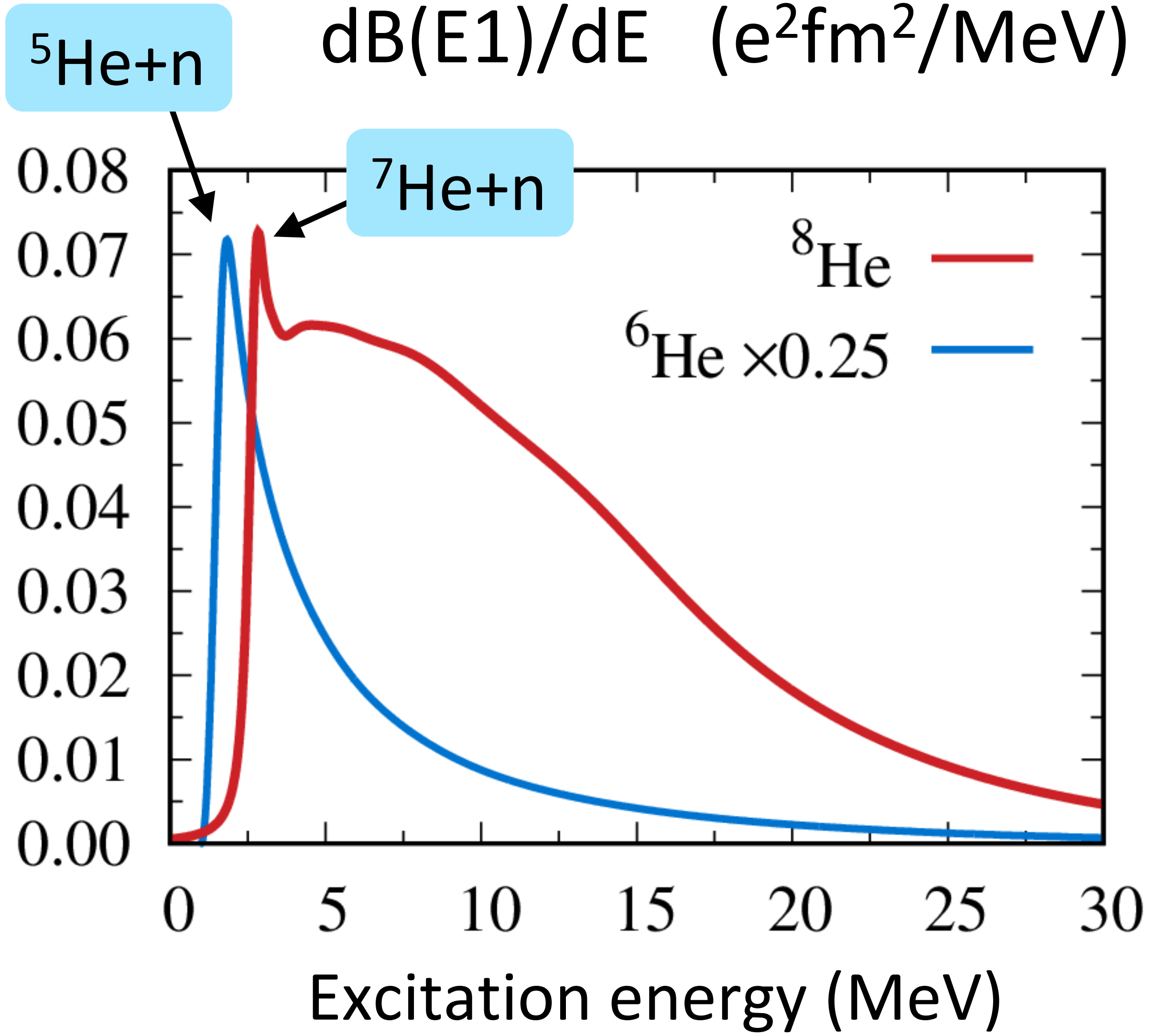


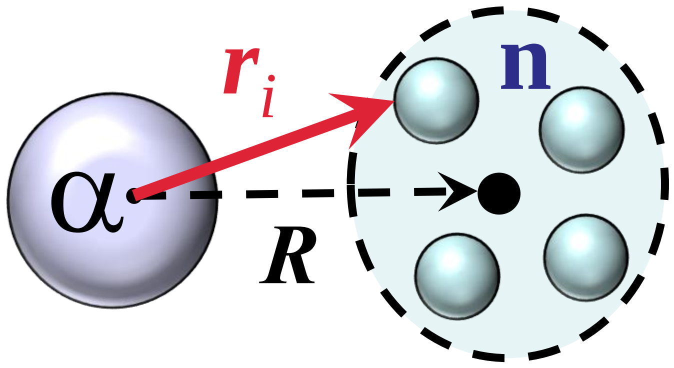
- Zero energy : ^{8}He$_{g.s.}$

$$\mathbf{1} = \sum_n |\Phi_n\rangle\langle\tilde{\Phi}_n|$$

$= \{^8\mathrm{He}\}$	Resonance
$+ \{^7\mathrm{He} + n\}$	Two-body continua
$+ \{^6\mathrm{He} + n + n\}$	Three-body
$+ \{^5\mathrm{He} + n + n + n\}$	Four-body
$+ \{\alpha + n + n + n + n\}$	Five-body

Dipole strength function of ^{8}He





- $O_{E1} = e\frac{Z_\alpha}{A}\sum_{i=1}^{4} r_i Y_1(\hat{\boldsymbol{r}}_i)$
- Cluster Sum-Rule Value

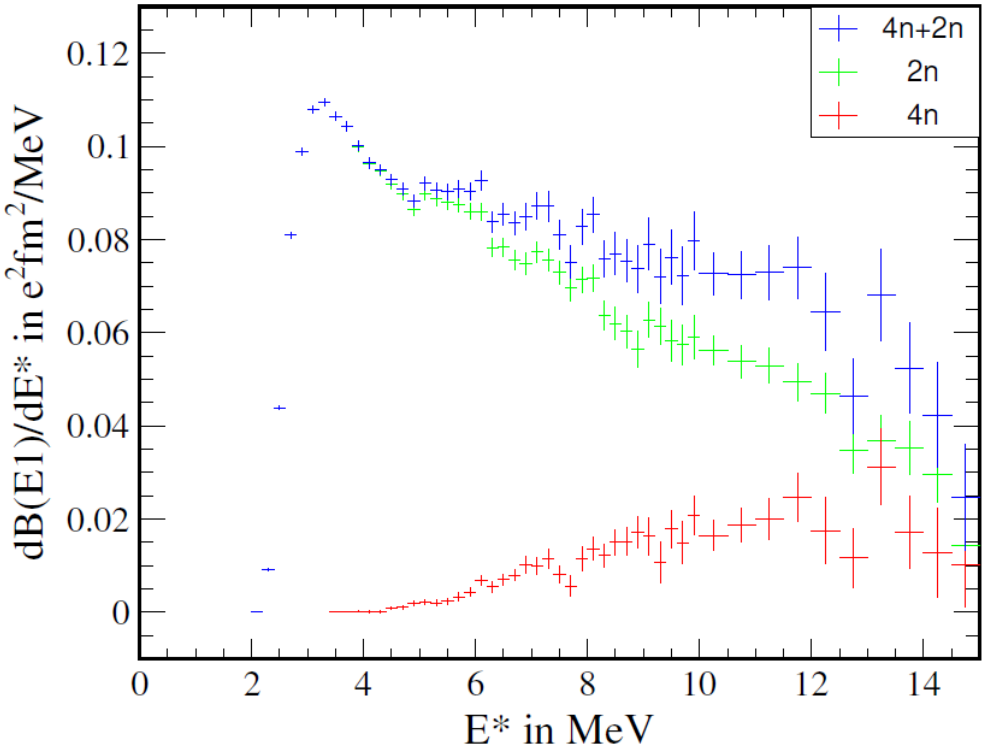
$$B_c(E1) = \frac{3e^2}{4\pi}\langle R^2_{\alpha-4n}\rangle_{\mathrm{G.S.}}$$
$$= 1.01\, e^2 \mathrm{fm}^2$$

- G.S. : α-4n distance = 2.05 fm
- Compare ^{8}He with ^{6}He

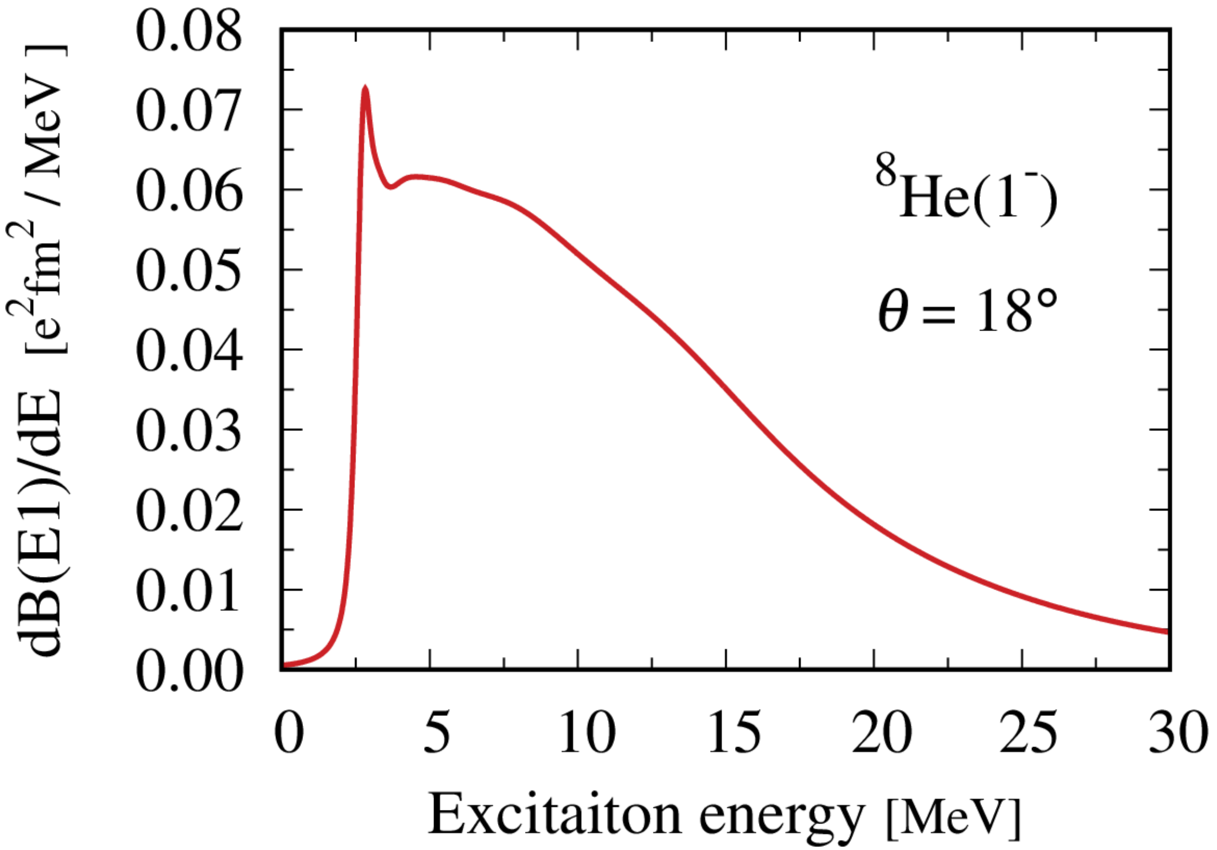
PhD Thesis by Christopher Lehr (TU Darmstadt)

- Coulomb breakup experiment of ^{8}He with Pb target with the help of C target

$B(E1)$ spectra for the ^{8}He breakup



5-body results



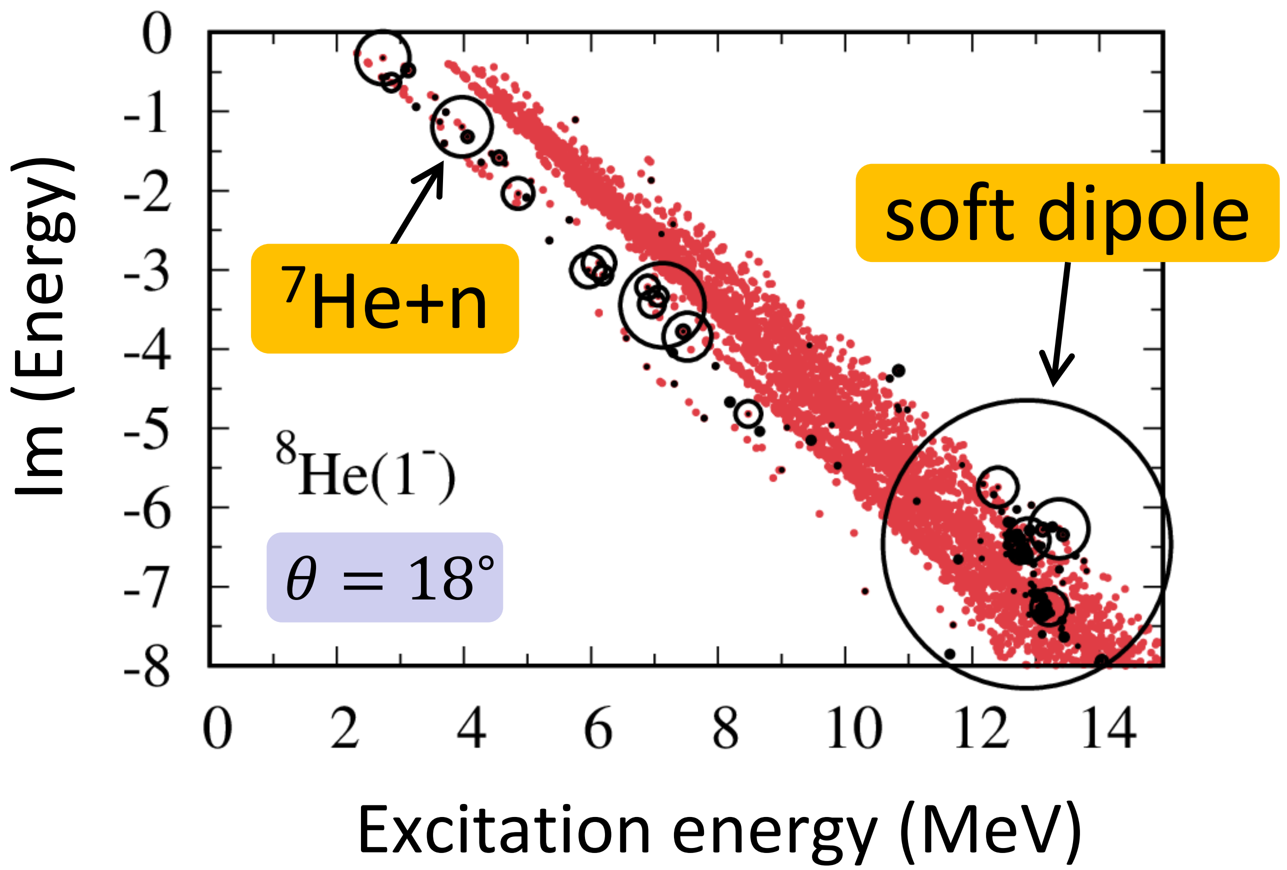
Dipole strength function of ^{8}He

- Zero energy : $^{8}He_{g.s.}$

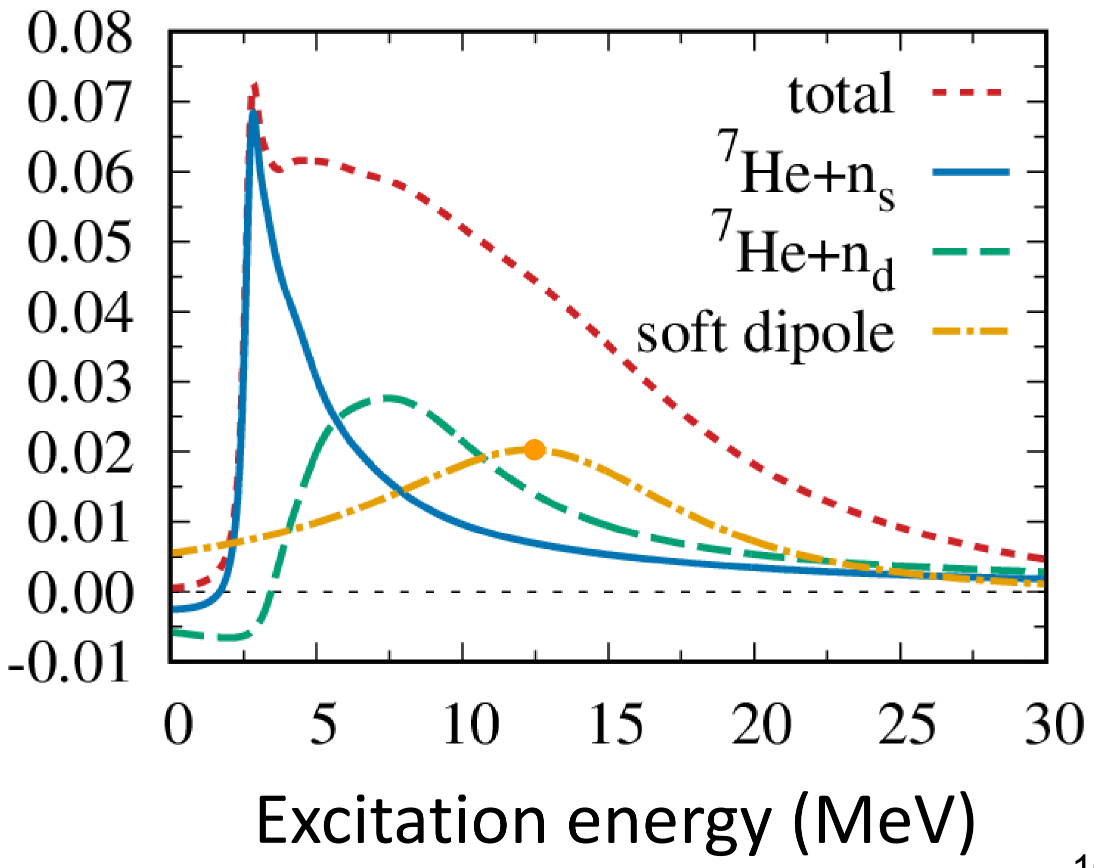
Circle size ∝ E1

complex scaling

Dipole matrix element (e^2fm^2)

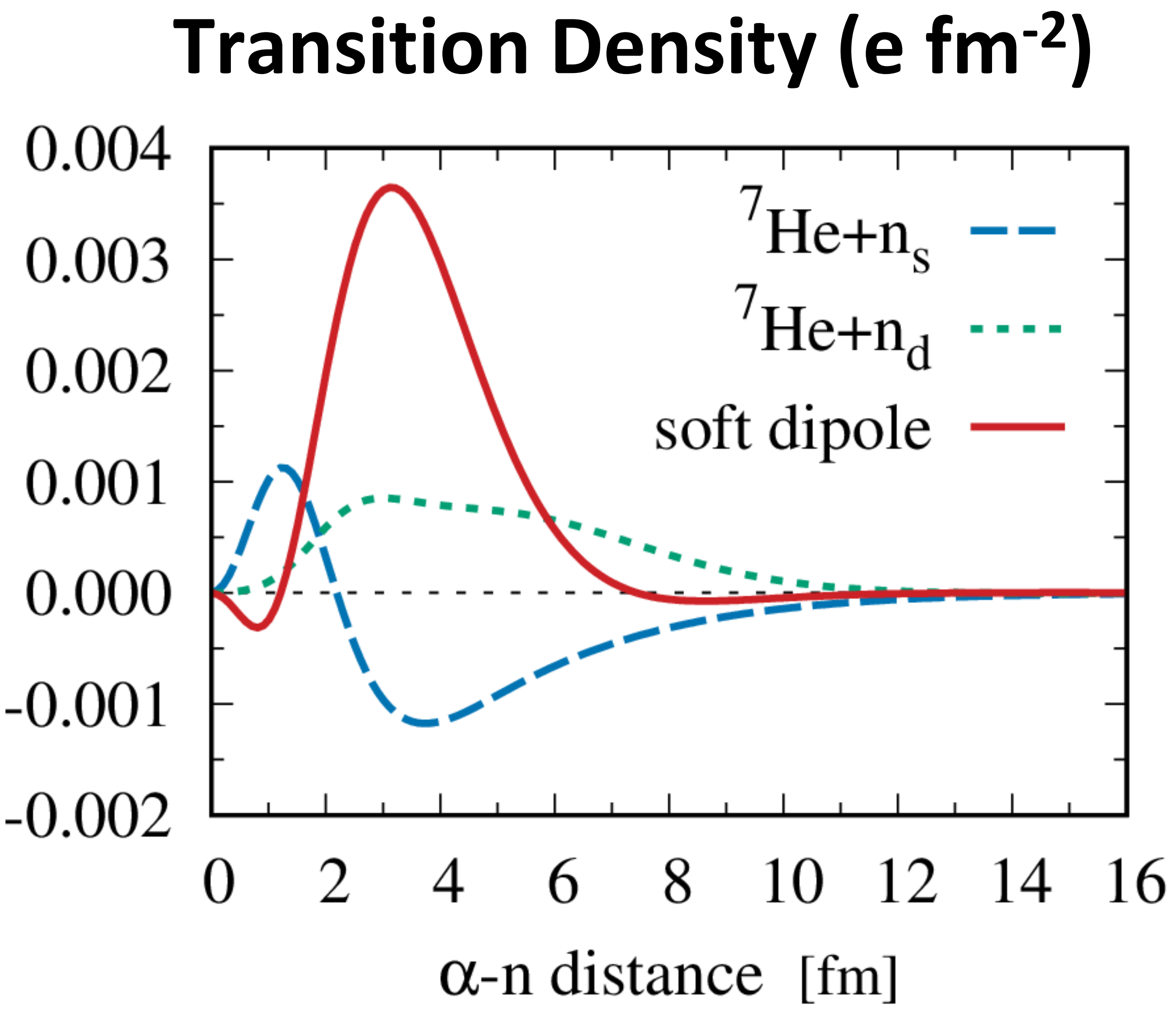


Dipole strength (e^2fm^2/MeV)

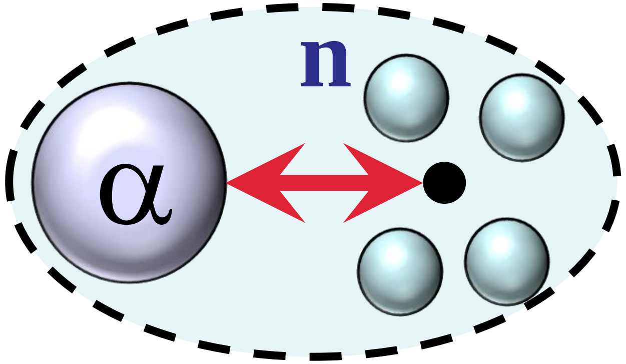


Transition density of ^{8}He

w/o scaling



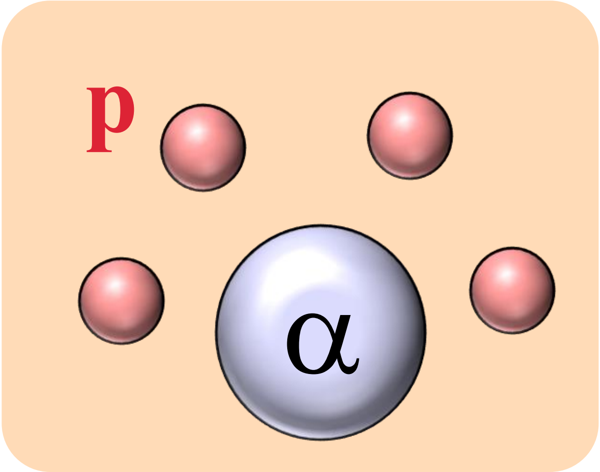
- Soft dipole mode of α+4n
- Dipole oscillation of 4n against α



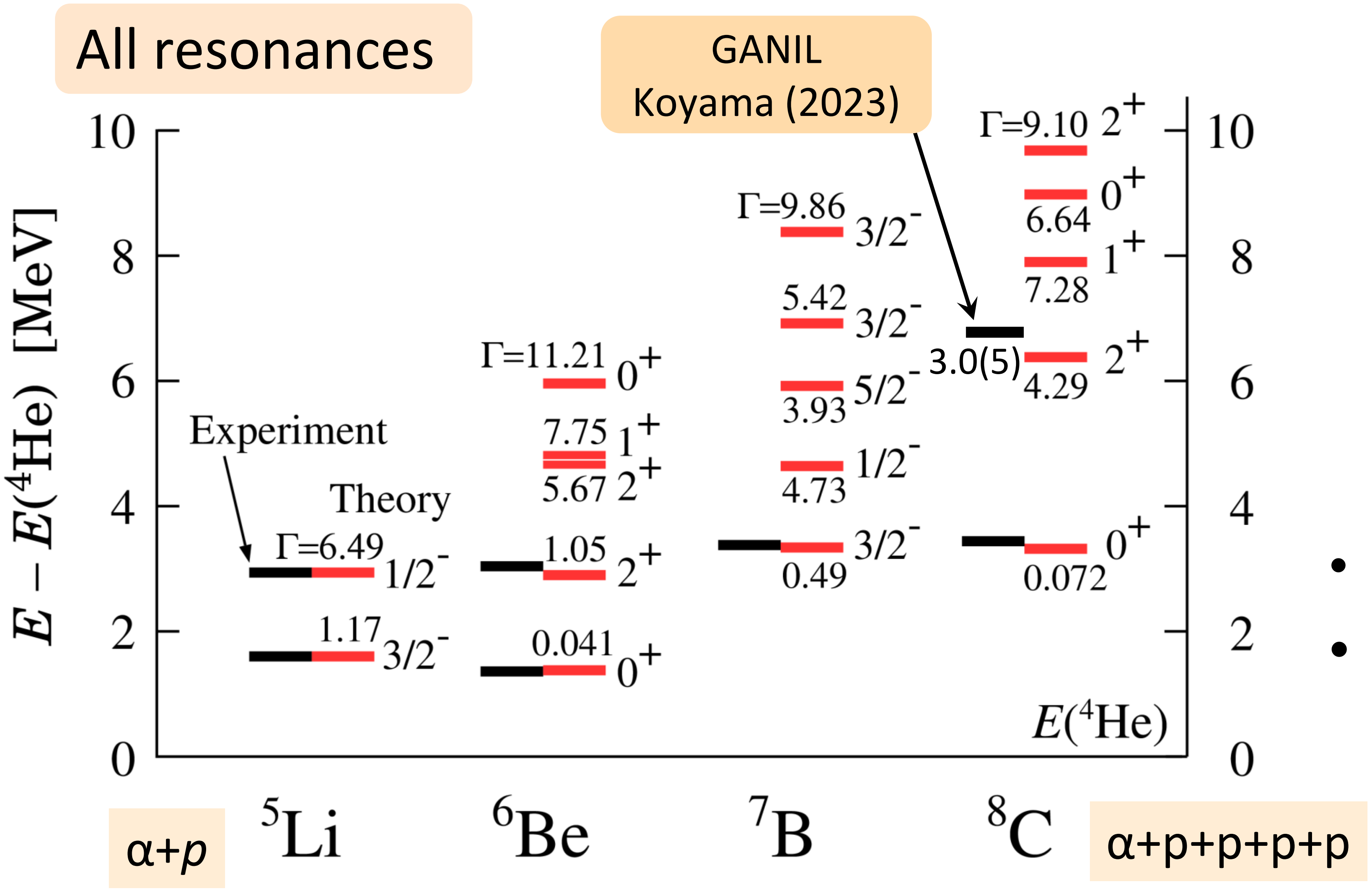
- $\langle \tilde{1}^- \| O_{E1} \| 0^+ \rangle = \int_0^\infty \boxed{\rho_{\rm tr}(r)}\, r^2 dr$
- $O_{E1} = e\,\frac{Z_\alpha}{A} \sum_{i=1}^{4} r_i Y_1(\hat{\boldsymbol{r}}_i)$
- r : α-n distance (fm)

^8C : mirror nucleus of ^8He

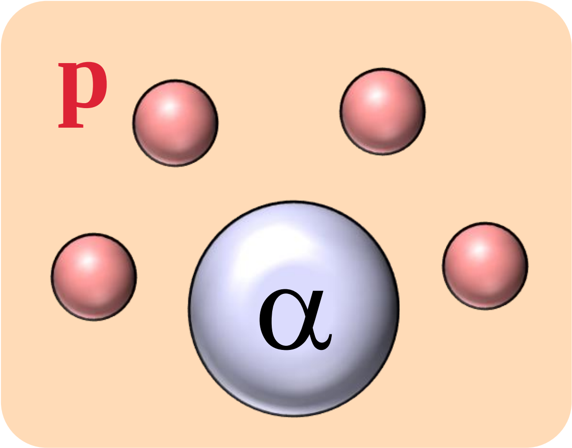
Physical Review C107 (2023) 034305



Proton-rich : α+p+p+p+p



α+p+p+p+p

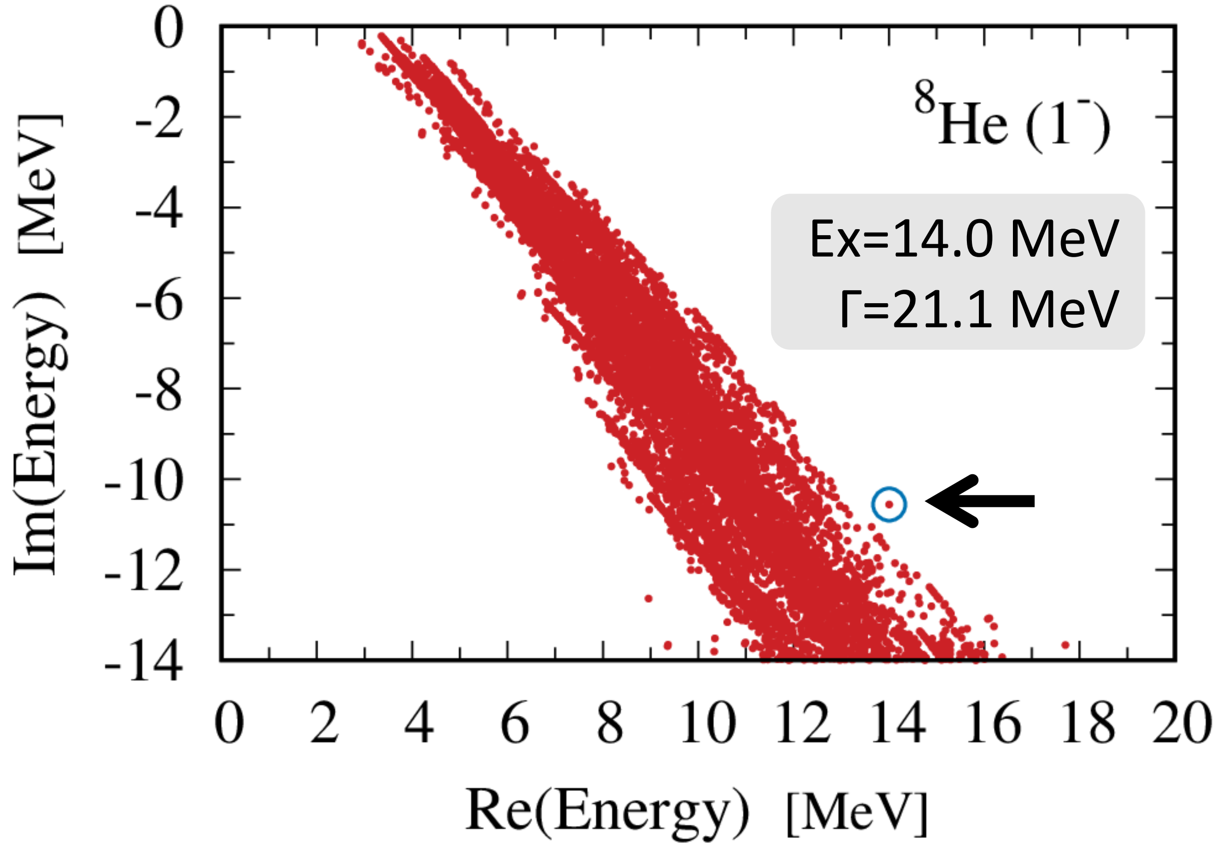


- Coulombic 5-body problem
- Discuss isospin symmetry between n-rich & p-rich

TM. M. Odsuren, K. Katō, PRC104 (2021)

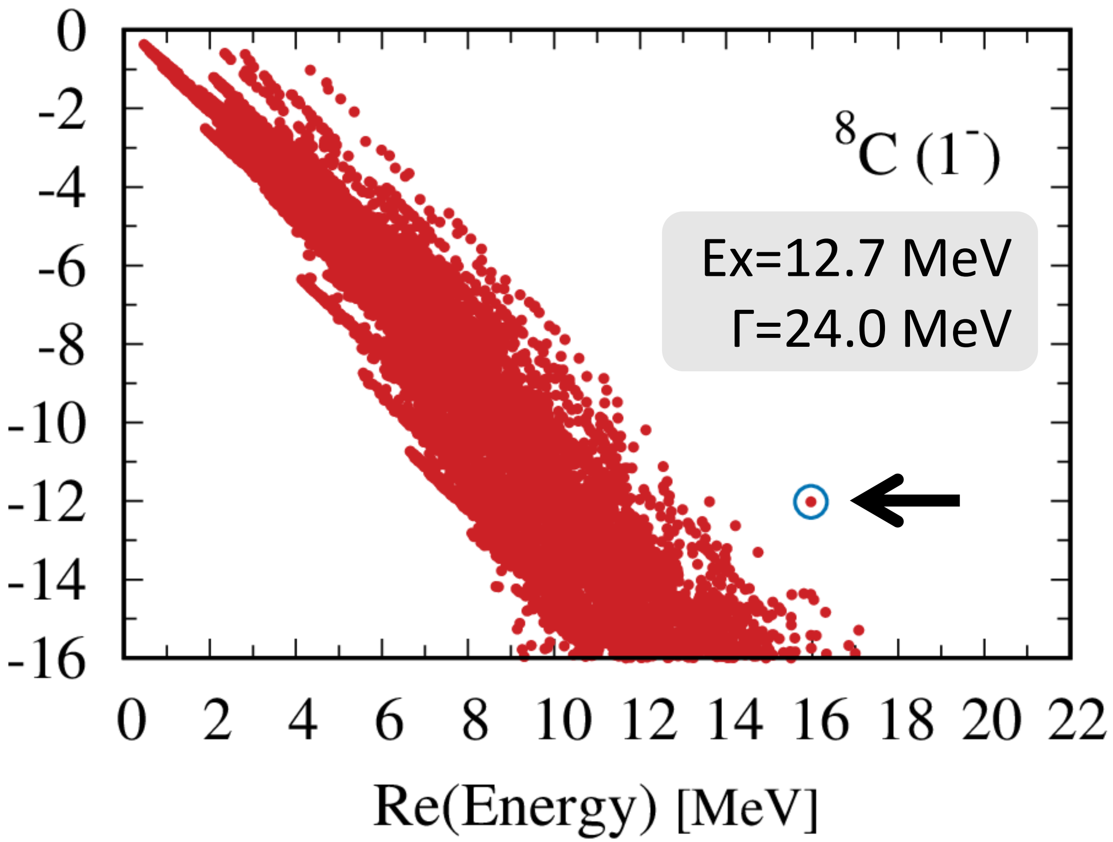
1^- resonances with complex scaling $\theta = 26^\circ$

^{8}He with α+n+n+n+n

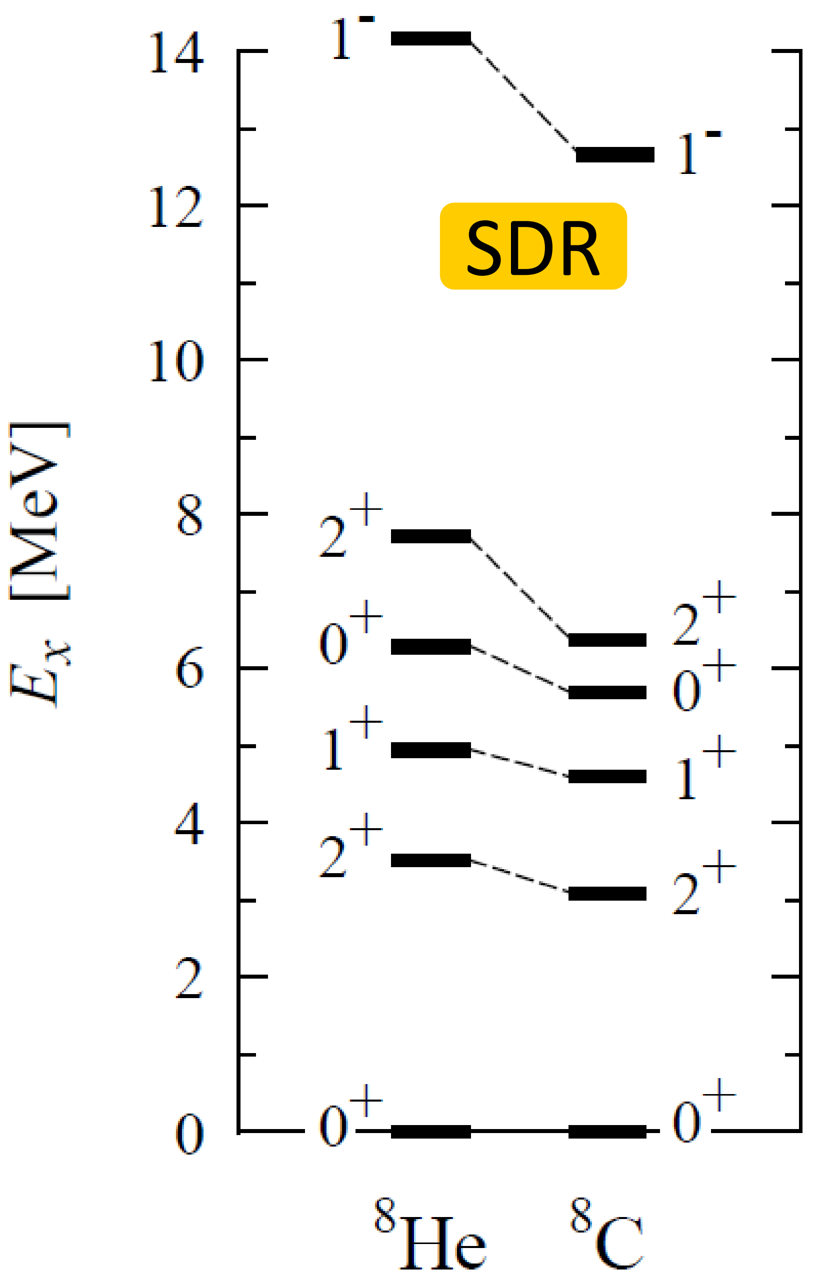


Excitation energy

^{8}C with α+p+p+p+p



Measured from α+p+p+p+p



Radii of ^{8}He & ^{8}C

complex scaling

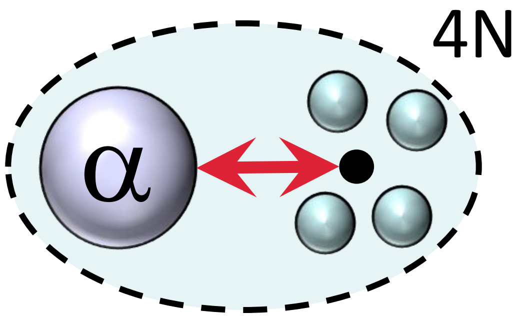
Radius (fm)	^{8}He	^{8}C
1- (SDR)	3.11	3.09
2+ 2nd	6.94	2.27
0+ 2nd	7.56	4.87
1+	6.03	4.59
2+ 1st	8.15	2.77
0+ 1st	2.53	2.81

close

increase

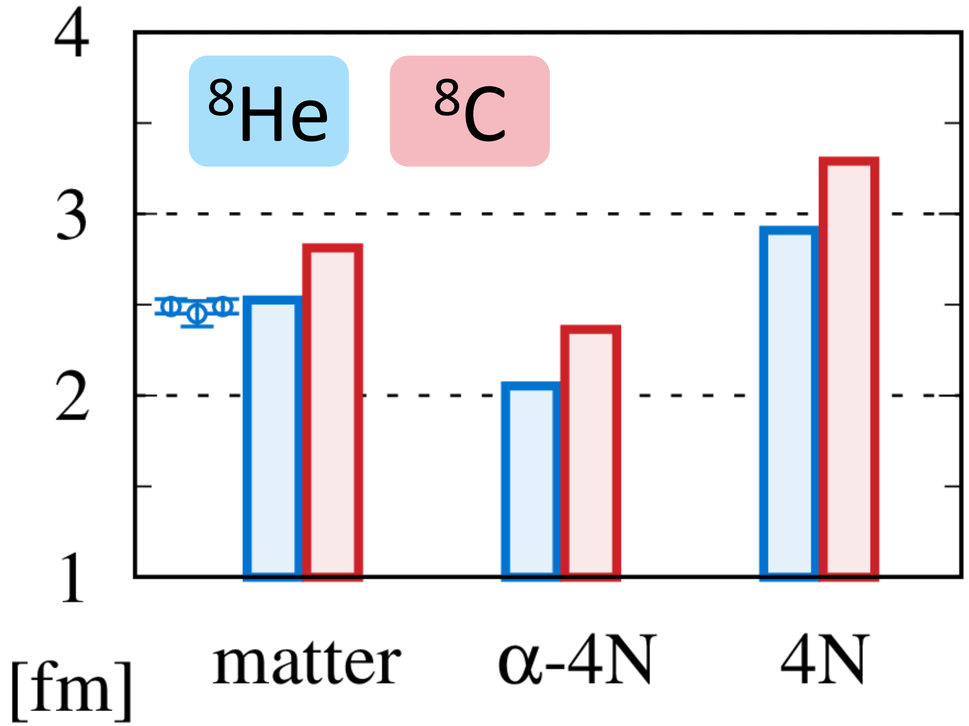
Coulomb repulsion & Coulomb barrier

Radii of SDR

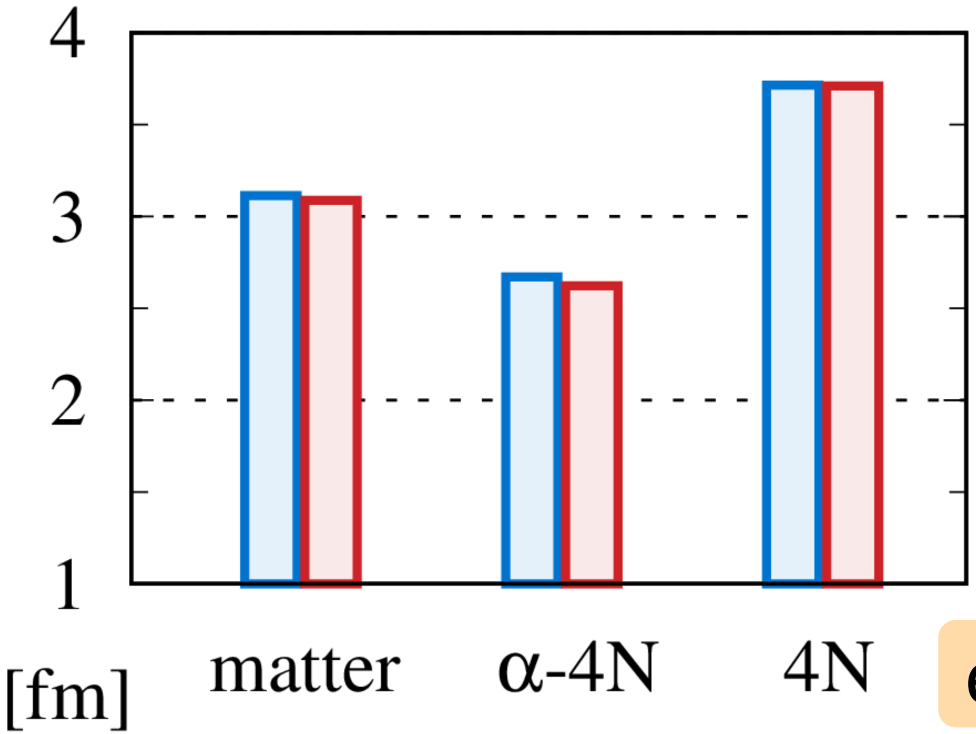


Burgers, Rost, J. Phys. B 29 (1996)
Doté, Inoue, Myo, PLB784 (2018)
Myo, Katō, PTEP2020 (2020)
Michel, Ploszajczak, LNP 983 (2022)
Myo, Katō, PRC107 (2023)

GS



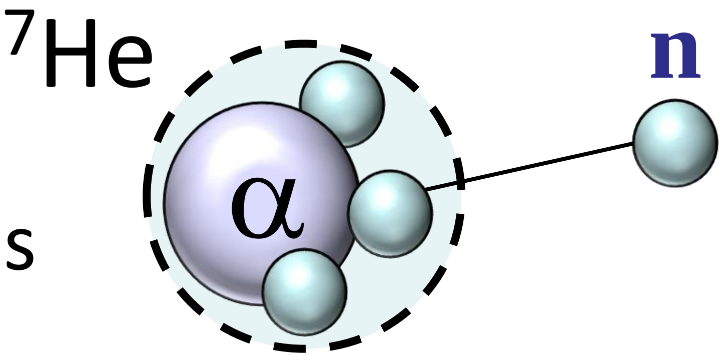
SDR



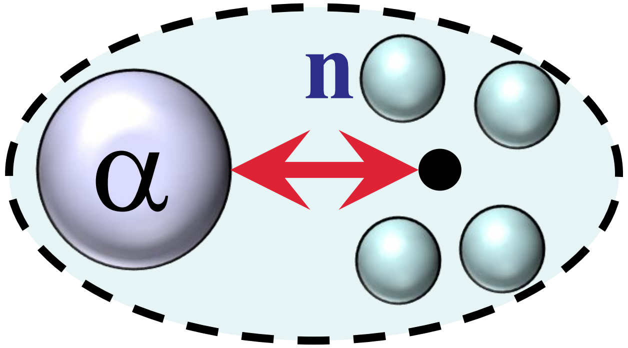
expand

- Ground states break the isospin symmetry by Coulomb repulsion
- SDR shows good isospin symmetry indicating α+4N oscillation

Summary



- Complex scaling for many-body unbound states
- ^{8}He : Electric dipole into α+n+n+n+n
  - Sequential breakup via ^{7}He+n into ^{6}He+n+n
  - Soft dipole resonance : $(E_x, \Gamma) = (14, 21)$ MeV
  - Collective excitation of 4n, oscillating against α

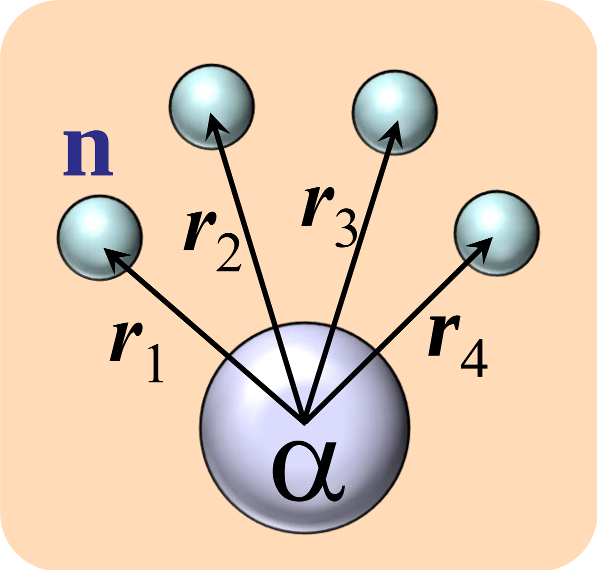


- ^{8}C : Predict soft dipole resonance : $(E_x, \Gamma) = (13, 24)$ MeV
- Good isospin symmetry in SDR in two nuclei with α+4N structure

Backup

Hamiltonian for He isotopes

- $V_{\alpha N}$: microscopic KKNN potential
  - s,p,d,f-waves of α-n scattering phase shifts
- V_{NN} : Minnesota central potential
  + Coulomb for *p*-rich nuclei

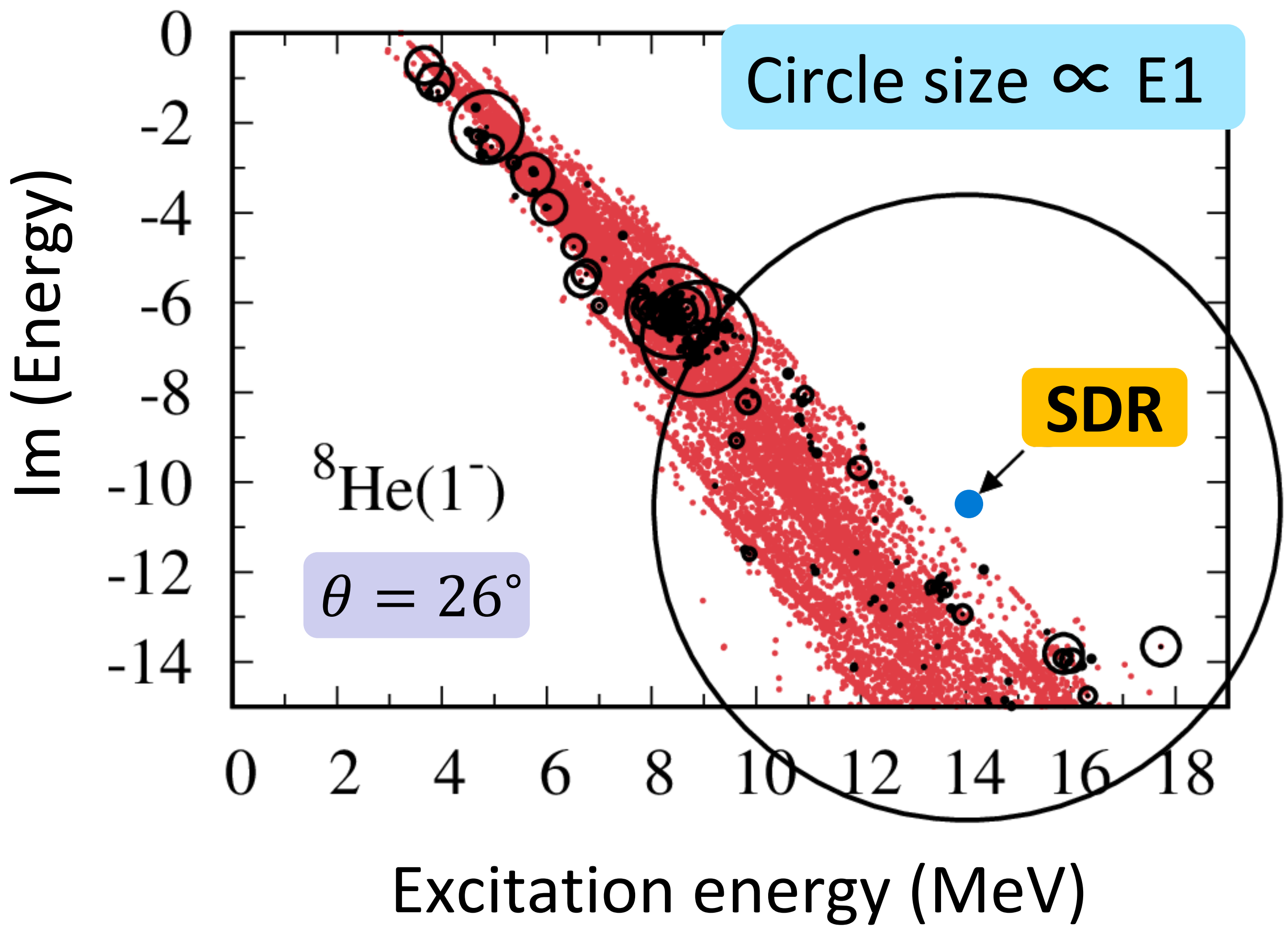


Fit energy of the ground state of ^{6}He

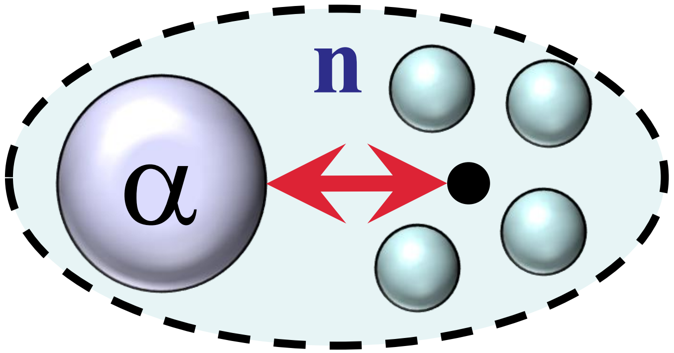
A. Csoto, PRC48(1993)165.
K. Arai, Y. Suzuki and R.G. Lovas, PRC59(1999)1432.
TM, S. Aoyama, K. Kato, K. Ikeda, PRC63(2001)054313.
TM et al. PTP113(2005)763.

1- Pole : Candidate of Soft Dipole Resonance

E1(pole) = 0.55 + i0.25 ($e^2 fm^2$)

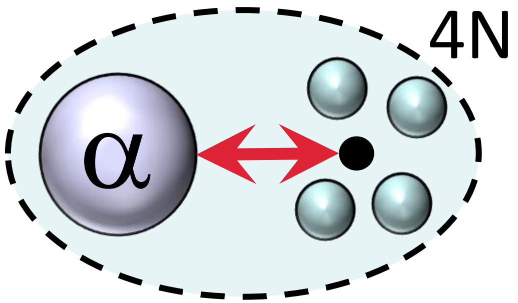


- Cluster Sum-Rule Value $B_c(E1) = 1.01\ e^2\mathrm{fm}^2$
- $(E_x, \Gamma) = (14, 21)$ MeV
- G.S. : α-4n distance = 2.05 fm
- $b_{\alpha-4n} = (\hbar/\mu\omega)^{1/2}$ = 1.10 fm
- $1\hbar\omega$ excitation (α-4n) = 17 MeV from a quanta $N_{\alpha\text{-}4n}$ =2 state



Radii of SDR

complex scaling



^{8}He

fm	0^+	SDR
Matter	2.53	3.11 + i0.86
α-4N	2.05	2.67 + i0.84
4N	2.91	3.72 + i1.14

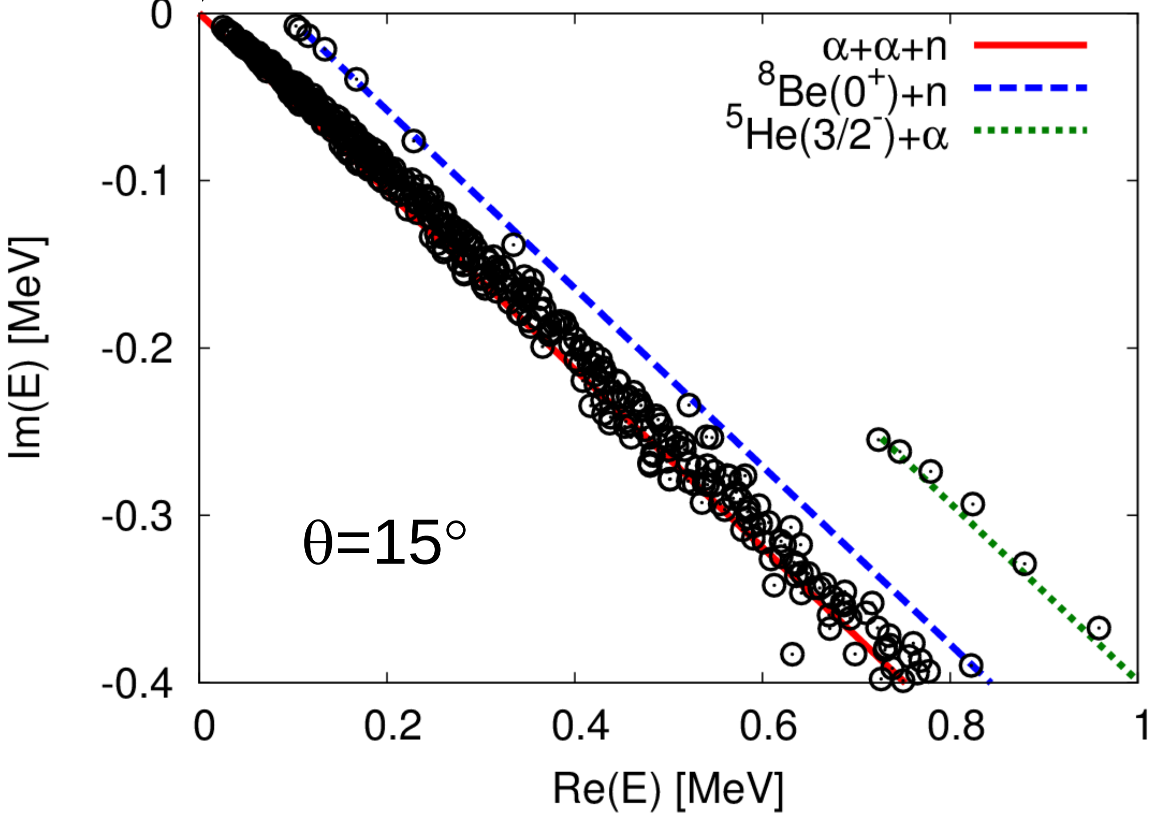
^{8}C

0^+	SDR
2.81 – i0.08	3.09 + i1.27
2.36 – i0.03	2.62 + i1.15
3.29 – i0.13	3.71 + i1.70

- Ground states break the isospin symmetry by Coulomb repulsion
- SDR shows good isospin symmetry indicating α+4N oscillation

^{9}Be : Property of $1/2^{+}$ state

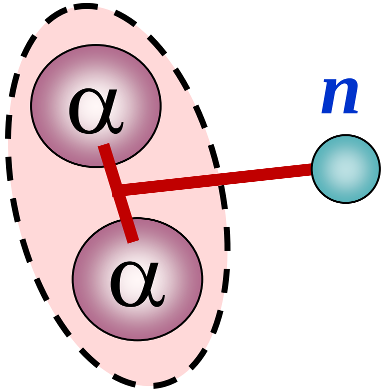
α+α+n threshold energy



- $1/2^{+}$ S-state - resonance ? / virtual state ?
- Photodisintegration with α+α+n 3-body approach

α+α+n model with complex scaling

No 3-body resonance

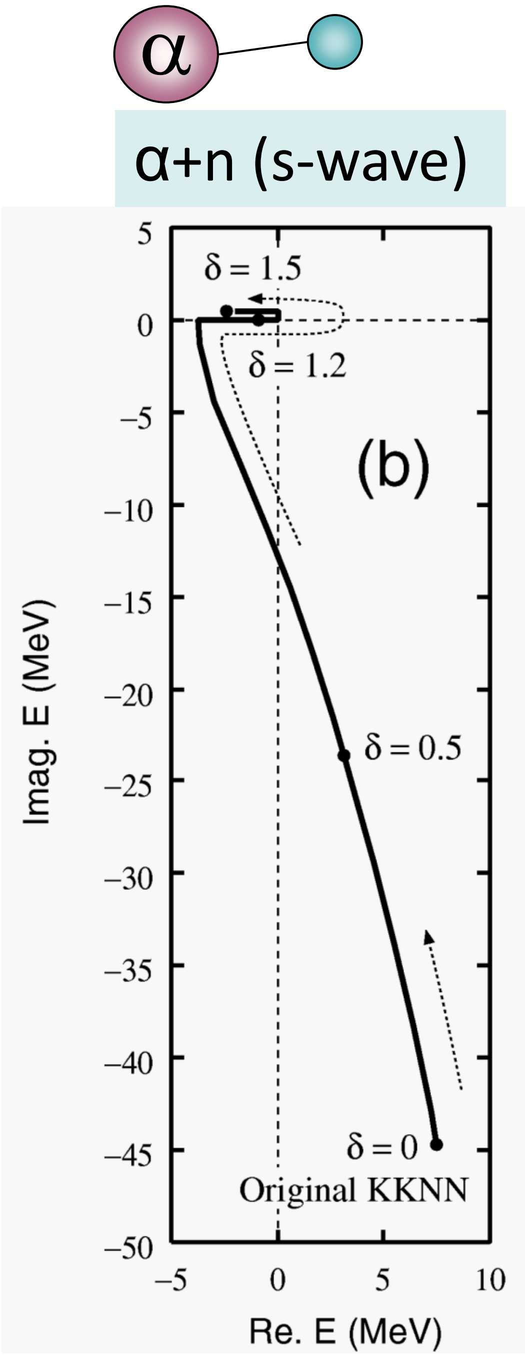


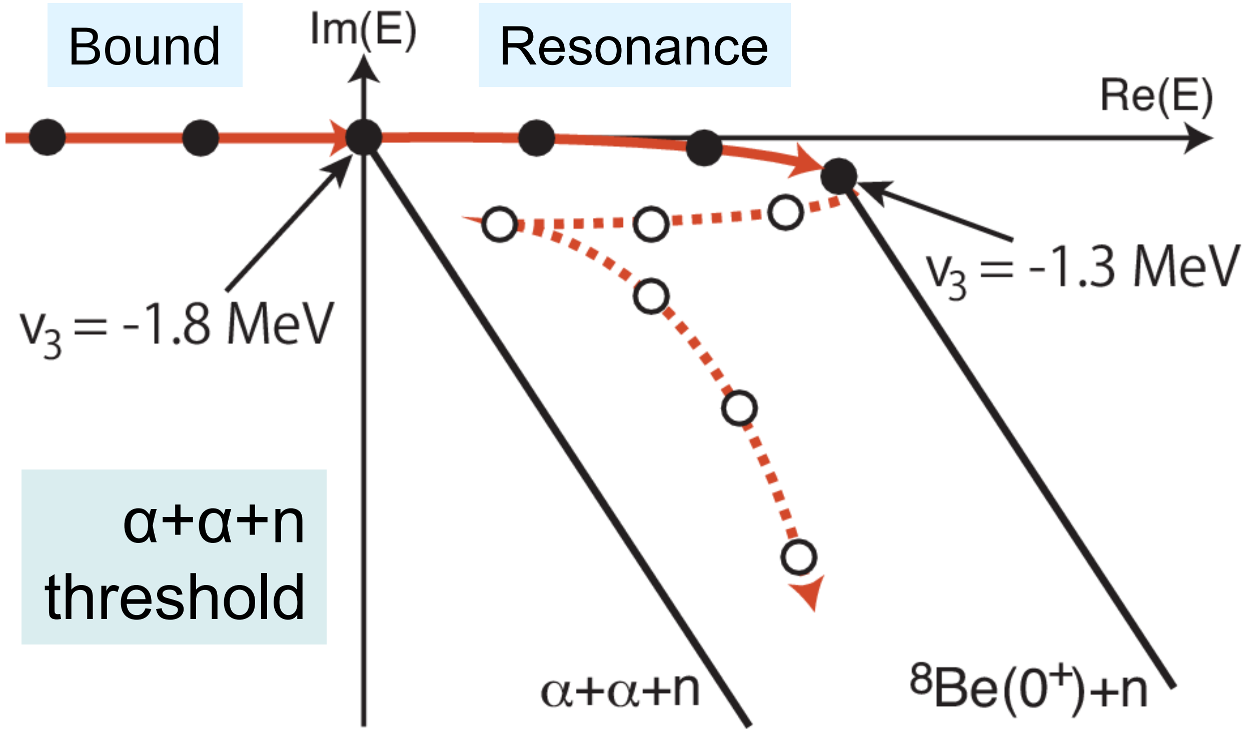
$$H(\,^{9}\mathrm{Be}) = T_R + T_r + V_{\alpha_1 n} + V_{\alpha_2 n} + V_{\alpha\alpha} + V_{\mathrm{PF}} + V_3$$

3-body potential
(1 range Gaussian
with hyper-radius)

M. Odsuren, Y. Kikuchi, T. Myo, M. Aikawa and K. Kato, Physical Review C (2015) 014322

Pole trajectory of ^{9}Be 1/2^{+} by analytical continuation of interaction





S-wave pole in α+n, ^{9}Li+n with Jost function method

H. Masui, S. Aoyama, T. Myo, K. Kato, K. Ikeda, Nucl. Phys. A 673 (2000) 207.

Hamiltonian & Wave function for *N*-rich systems

$$H_A = H_{\mathrm{core}} + H_{\mathrm{val.}} + H_{\mathrm{core-val.}}$$

valence neutron number : $N_V = 1, 2, \cdots$

$$\Phi_A = \mathcal{A}\{\,\Psi_{\mathrm{core}} \cdot \textstyle\sum_n^N C_n \Psi_{\mathrm{val},n}\,\}$$

n : configuration

$$\Psi_{\mathrm{val},n} = \mathcal{A}\{\phi_{n1}(\boldsymbol{r}_1)\phi_{n2}(\boldsymbol{r}_2)\phi_{n3}(\boldsymbol{r}_3)\cdots\}$$

single nucleon state, $s_{1/2}, p_{1/2}, p_{3/2}, \cdots$

$$\phi_{n\ell j}(\boldsymbol{r}) = r^{\ell} e^{-a_n r^2}\left[Y_\ell(\hat{\boldsymbol{r}}), \chi^{\sigma}_{1/2}\right]_j$$

Range parameter $a_1, a_2, \cdots, a_N$

$$H_A \Phi_A = E \Phi_A \qquad H_{\mathrm{core}}\Psi_{\mathrm{core}} = E_{\mathrm{core}}\Psi_{\mathrm{core}}$$

$\delta E = 0$: Solve eigenvalue problem

$$\sum_n^N \left\langle \Psi_{\mathrm{val},n'} \middle| \textstyle\sum_i^{N_V+\mathrm{core}} t_i - T_G + \sum_i^{N_V} V_i^{\mathrm{core}-N} + \sum_i^{N_V} V_{ij}^{NN} - (E - E_{\mathrm{core}}) \middle| \Psi_{\mathrm{val},n} \right\rangle C_n = 0$$

Orthogonality Condition Model

Y. Suzuki, K. Ikeda, PRC38(1988)410
H. Masui, K. Kato, K. Ikeda, PRC73(2006)034318

Strength function $S(E)$ with complex scaling

$$S(E)=\sum_n \langle\tilde{\Phi}_0|\hat{O}^\dagger|\varphi_n\rangle\langle\tilde{\varphi}_n|\hat{O}|\Phi_0\rangle\cdot\delta(E-E_n)$$

$$=-\frac{1}{\pi}\mathrm{Im}\left[R(E)\right]$$

$\hat{O}$: Transition operator

initial state

final state

$$R(E)=\sum_n \frac{\langle\tilde{\Phi}_0|\hat{O}^\dagger\boxed{|\varphi_n\rangle\langle\tilde{\varphi}_n|}\hat{O}|\Phi_0\rangle}{\boxed{E-E_n}}$$

Response function

- Complex-scaled Green's function

$$G^\theta(E)=\frac{1}{E-H_\theta}=\sum_n\frac{|\varphi_n^\theta\rangle\langle\tilde{\varphi}_n^\theta|}{E-E_n^\theta}$$

Bound+Resonance+Continuum

Reaction theory
- Lippmann–Schwinger-eq. (Kikuchi)
- CDCC (Matsumoto)
- Scatt. Amp. (Kruppa, Dote($K^{bar}N$))

Berggren, NPA109('68)265, Morimatsu, Yazaki, PPNP33('94), TM, Ohnishi, Kato, PTP99('98)801