

INPC 2025

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In-medium mass shifts of $B_c^{(*)}$, $B_s^{(*)}$ and $D_s^{(*)}$ mesons

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Samuel L. P. G. Beres, Guilherme N. Zeminiani

- G. N. Zeminiani, S. L. P. G. Beres and K. Tsushima, Phys. Rev. D 110, 094045 (2024)
 - S. L. P. G. Beres, [arXiv:2407.03377 [hep-ph]] (Master Thesis)
 - K. T, PTEP 2022 043D02 (effective hadron mass parametrization)
- K. Saito, K. T, and A. W. Thomas, Prog. Part. Nucl. Phys. 58, 1 (2007) (QMC)

- 1 Motivation, Focus
- 2 Introduction: QMC Model
- 3 Effective Hadron Masses and Potentials (Parametrization)
- 4 Two-flavored heavy meson self-energies, mass shift
- 5 Summary, Discussion (two-flavored meson mass shift)

Motivation, Focus

- In-medium mass shifts of **two non-light quark flavor mesons** ($B_c, B_c^*, B_s, B_s^*, D_s$ and D_s^*)
 $\leftrightarrow$ **not directly with chiral symmetry**
- Effects of **chiral symm. partial restoration** on two-non-light quark flavor mesons ($B_c, B_c^*, B_s, B_s^*, D_s$ and D_s^*)
- B_c, B_c^* mesons (B_c^* not experimentally confirmed/found)
- (● **Density dependent parametrization: (for easy usage)**
 Baryon and Meson $m_{B,M}^*$ and vector $V_{\omega,\rho}^{q,B,M}$ potentials)

The QMC model P. Guichon, PLB 200, 235 (1988)

(For a review, PPNP 58, 1 (2007))

Light (u,d) quarks interact self-consistently with mean σ and ω fields

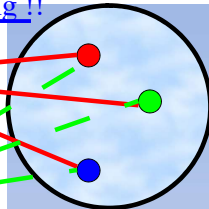
Nuclear Binding !!

$$m^*_q = m_q - g^q_\sigma \sigma = m_q - V^q_\sigma$$

↓ nonlinear in σ

$$M^*_N \approx M_N - g^N_\sigma \sigma + \frac{(d/2)}{2} (g^N_\sigma \sigma)^2$$

$$\langle \omega \rangle$$



$$M^*_N = M_N - V^N_\sigma$$

$$[i \gamma \cdot \partial - (m_q - V^q_\sigma) - \gamma_0 V^q_\omega] q = 0$$

1. Start

$$[i \gamma \cdot \partial - M^*_N - \gamma_0 V^N_\omega] N = 0$$

$$V^N_\omega = 3 V^q_\omega$$

Self-consistent !

(Applied quark model !)

K. Tsushima

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Mesons in nuclear medium in QMC

(For a review, PPNP 58, 1 (2007))

Light (u,d) quarks interact self-consistently with mean σ and ω fields

Nuclear Binding !!

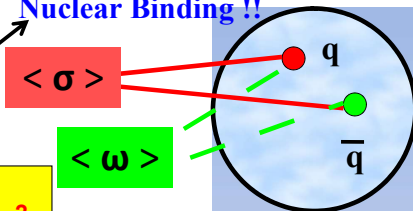
$$m^*_q = m_q - g^q_\sigma \sigma = m_q - V^q_\sigma$$

↓ **nonlinear in σ**

$$M^*_M \approx M_M - g^M_\sigma \sigma + (d^M/2) (g^M_\sigma \sigma)^2$$

$$[i\gamma \cdot \partial - (m_q - V^q_\sigma) - \gamma_0 V^q_\omega] q = 0$$

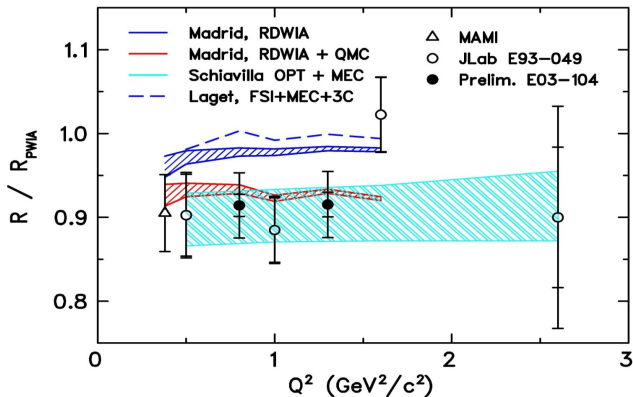
σ, ω fields: no couplings with **s,c,b** quarks!!



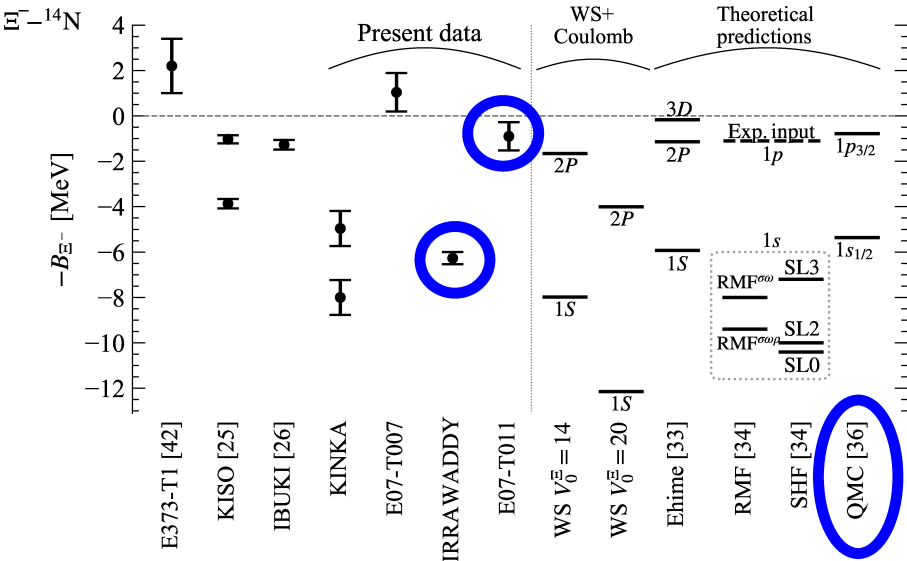
$$R = (p'_x / p'_z) = (G_E^p / G_M^p) : {}^4\text{He} / {}^1\text{H}$$

S. Malace, M. Paolone and S. Strauch, arXiv:0807.2251 [nucl-ex]

S. Strauch *et al.*, *Phys. Rev. Lett.* **91**, 052301 (2003)



${}_{-}^{14}\text{N}$ energy levels (Prog. Theor. Exp. Phys. 2021, 073D02)



QMC model 1: Hadron level

$$\begin{aligned}
 \mathcal{L} &= \bar{\psi}[i\gamma \cdot \partial - m_N^*(\sigma) - g_\omega \omega^\mu \gamma_\mu]\psi + \mathcal{L}_{\text{meson}}, \\
 m_N^*(\sigma) &\equiv m_N - g_\sigma(\sigma)\sigma \simeq m_N - g_\sigma[1 - (a_N/2)(g_\sigma\sigma)]\sigma \\
 g_\sigma &\equiv g_\sigma(\sigma = 0)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{L}_{\text{meson}} &= \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2 - \frac{1}{2}\partial_\mu\omega_\nu(\partial^\mu\omega^\nu - \partial^\nu\omega^\mu) \\
 &+ \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu,
 \end{aligned}$$

$$\begin{aligned}
 \rho_B &= \frac{4}{(2\pi)^3} \int d^3k \theta(k_F - |\vec{k}|) = \frac{2k_F^3}{3\pi^2}, \\
 \rho_s &= \frac{4}{(2\pi)^3} \int d^3k \theta(k_F - |\vec{k}|) \frac{m_N^*(\sigma)}{\sqrt{m_N^{*2}(\sigma) + \vec{k}^2}},
 \end{aligned}$$

QMC model 2: Quark level

$$\mathbf{x} = (t, \vec{r}) \quad (|\vec{r}| \leq \text{bag radius}), \quad \mathbf{V}_\sigma^q = g_\sigma^q \boldsymbol{\sigma}$$

$$\left[i\boldsymbol{\gamma} \cdot \partial_x - (m_q - \mathbf{V}_\sigma^q) \mp \gamma^0 \left(\mathbf{V}_\omega^q + \frac{1}{2} \mathbf{V}_\rho^q \right) \right] \begin{pmatrix} \psi_u(x) \\ \psi_{\bar{u}}(x) \end{pmatrix} = 0$$

$$\left[i\boldsymbol{\gamma} \cdot \partial_x - (m_q - \mathbf{V}_\sigma^q) \mp \gamma^0 \left(\mathbf{V}_\omega^q - \frac{1}{2} \mathbf{V}_\rho^q \right) \right] \begin{pmatrix} \psi_d(x) \\ \psi_{\bar{d}}(x) \end{pmatrix} = 0$$

$$[i\boldsymbol{\gamma} \cdot \partial_x - m_Q] \psi_Q(x) \text{ (or } \psi_{\bar{Q}}(x)) = 0$$

$$m_h^* = \sum_{j=q, \bar{q}, Q\bar{Q}} \frac{n_j \Omega_j^* - z_h}{R_h^*} + \frac{4}{3} \pi R_h^{*3} B, \quad \left. \frac{dm_h^*}{dR_h} \right|_{R_h=R_h^*} = 0$$

$$j_0(x_{q,Q}) = \beta_{q,Q}^* j_1(x_{q,Q}), \quad \beta_q^* = \sqrt{(\Omega_q^* - R_h^* m_q^*) / (\Omega_q^* + R_h^* m_q^*)}$$

$$\Omega_q^* = \Omega_{\bar{q}}^* = [x_q^2 + (R_h^* m_q^*)^2]^{1/2}, \text{ with } m_q^* = m_q - g_\sigma^q \sigma$$

$$\Omega_Q^* = \Omega_{\bar{Q}}^* = [x_Q^2 + (R_h^* m_Q^*)^2]^{1/2} \quad (Q = s, c, b)$$

QMC model 3: From quarks

$$\omega = \frac{g_\omega \rho_B}{m_\omega^2},$$

$$\sigma = \frac{g_\sigma}{m_\sigma^2} C_N(\sigma) \frac{4}{(2\pi)^3} \int d^3k \theta(k_F - |\vec{k}|) \frac{m_N^*(\sigma)}{\sqrt{m_N^{*2}(\sigma) + \vec{k}^2}}$$

$$= \frac{g_\sigma}{m_\sigma^2} C_N(\sigma) \rho_s \quad (g_\sigma \equiv g_\sigma(\sigma = 0)),$$

$$C_N(\sigma) = \frac{-1}{g_\sigma(\sigma = 0)} \left[\frac{dm_N^*(\sigma)}{d\sigma} \right], \leftrightarrow [j_0(x_q) = \beta_q^*(x_q, \sigma) j_1(x_q)]$$

$$E^{\text{tot}}/A - m_N = \frac{4}{(2\pi)^3 \rho_B} \int d^3k \theta(k_F - |\vec{k}|) \sqrt{m_N^{*2}(\sigma) + \vec{k}^2} \\ + \frac{m_\sigma^2 \sigma^2}{2\rho_B} + \frac{g_\omega^2 \rho_B}{2m_\omega^2} - m_N.$$

QMC model 4: Couplings etc.

$m_q(\text{MeV})$	$g_\sigma^2/4\pi$	$g_\omega^2/4\pi$	m_N^*	K	Z_N	$B^{1/4}(\text{MeV})$
5	5.39	5.30	754.6	279.3	3.295	170
220	6.40	7.57	698.6	320.9	4.327	148

$$\frac{dm_N^*(\sigma)}{d\sigma} = -3g_\sigma^q \int_{\text{bag}} d^3r \bar{\psi}_q(\vec{r}) \psi_q(\vec{r}) \quad \text{the lowest bag w.f.}$$

$$\equiv -3g_\sigma^q S_N(\sigma) = -\frac{\partial}{\partial \sigma} [g_\sigma(\sigma) \sigma],$$

$$C_N(\sigma) = \frac{-1}{g_\sigma(\sigma=0)} \left[\frac{dm_N^*(\sigma)}{d\sigma} \right],$$

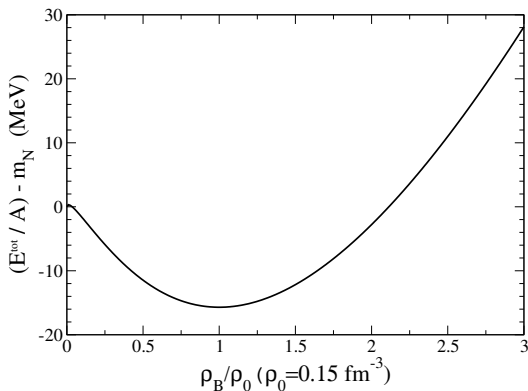
$$g_\sigma \equiv g_\sigma^N \equiv \underline{3g_\sigma^q S_N(\sigma=0)}.$$

Parameters

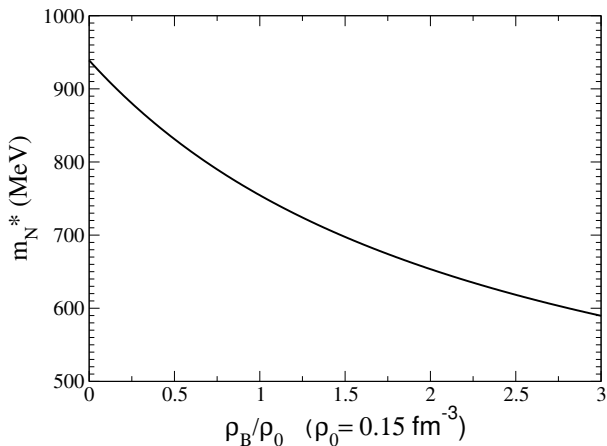
Table: Current quark mass values (inputs), quark-meson coupling constants and the bag constant B_p , obtained with the inputs: free nucleon bag radius $R_N = 0.8$ fm, empirical values $E^{\text{tot}}/A - m_N = -15.7$ MeV ($m_N = 939$ MeV) at the saturation density $\rho_0 = 0.15 \text{ fm}^{-3}$, and the symmetry energy, 35 MeV.

$m_{u,d}$	5 MeV	g_σ^q	5.69
m_s	250 MeV	g_ω^q	2.72
m_c	1270 MeV	g_ρ^q	9.33
m_b	4200 MeV	$B_p^{1/4}$	170 MeV

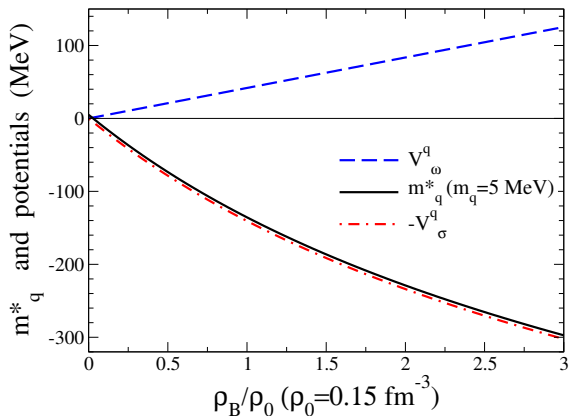
Results: Quark Meson Coupling Model (Standard)



- Symmetric Nuclear Matter - Binding Energy per Nucleon
- $m_q = 5 \text{ MeV}$, $K = 279.3 \text{ MeV}$



- Nucleon effective mass: $m_q = 5 \text{ MeV}$



- Effective mass of constituent quarks: $m_q = 5 \text{ MeV}$
- All the light-quarks in any hadrons feel the same potentials !!

Standard QMC, π, ρ in LF model parameters comparison

- **Motivation:** The present model works well (Symmetric Vertex)!

$m_q(\text{MeV})$	$g_\sigma^2/4\pi$	$g_\omega^2/4\pi$	m_N^*	K	Z_N	$B^{1/4}(\text{MeV})$
5	5.39	5.30	754.6	279.3	3.295	170
220	6.40	7.57	698.6	320.9	4.327	148
430	8.73	11.93	565.25	361.4	5.497	69.75

- **Refs. LF π, ρ in medium:** J.P.B.C. de Melo, K. T. et al.,
LF π model ($m_q = 220 \text{ MeV}$): Phys.Rev. C90 (2014) no.3, 035201;
 Phys.Lett. B766 (2017) 125; Few Body Syst. 58 (2017) no.2, 85
LF ρ model ($m_q = 430 \text{ MeV}$): Few Body Syst. 58 (2017) no.2, 82;
 Phys. Lett. B 788 (2019) 137
- **LF heavy mesons in medium ($m_q = 220 \text{ MeV}$):** Ahmad Jafar Arifi,
 Parada T. P. Hutaeruk, K. T., [Phys, Rev. D 107, 114010 \(2023\)](#)

QMC: Hadron masses in medium (Remind again!)

$x = (t, \vec{r})$ ($|\vec{r}| \leq \text{bag radius}$) , $V_\sigma^q = g_\sigma^q \sigma$

$$\left[i\gamma \cdot \partial_x - (m_q - V_\sigma^q) \mp \gamma^0 \left(V_\omega^q + \frac{1}{2} V_\rho^q \right) \right] \begin{pmatrix} \psi_u(x) \\ \psi_{\bar{u}}(x) \end{pmatrix} = 0$$

$$\left[i\gamma \cdot \partial_x - (m_q - V_\sigma^q) \mp \gamma^0 \left(V_\omega^q - \frac{1}{2} V_\rho^q \right) \right] \begin{pmatrix} \psi_d(x) \\ \psi_{\bar{d}}(x) \end{pmatrix} = 0$$

$$[i\gamma \cdot \partial_x - m_Q] \psi_Q(x) \text{ (or } \psi_{\bar{Q}}(x)) = 0$$

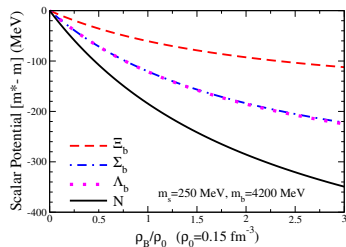
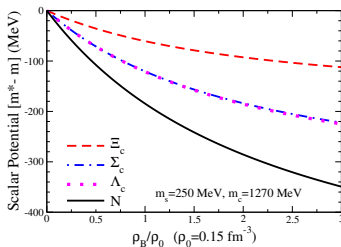
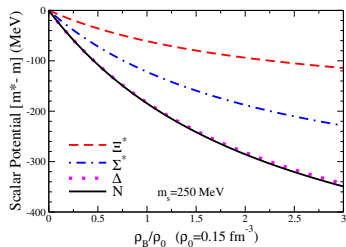
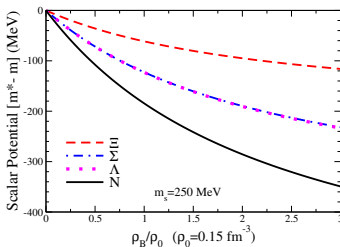
$$m_h^* = \sum_{j=q,\bar{q},Q\bar{Q}} \frac{n_j \Omega_j^* - z_h}{R_h^*} + \frac{4}{3} \pi R_h^{*3} B, \quad \left. \frac{dm_h^*}{dR_h} \right|_{R_h=R_h^*} = 0$$

$$j_0(x_{q,Q}) = \beta_{q,Q}^* j_1(x_{q,Q}), \quad \beta_q^* = \sqrt{(\Omega_q^* - R_h^* m_q^*) / (\Omega_q^* + R_h^* m_q^*)}$$

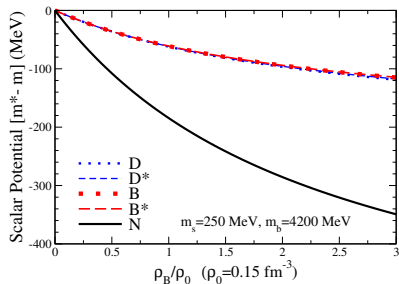
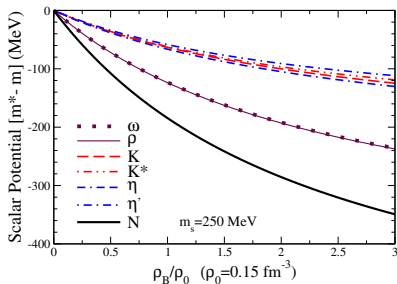
$$\Omega_q^* = \Omega_{\bar{q}}^* = [x_q^2 + (R_h^* m_q^*)^2]^{1/2}, \text{ with } m_q^* = m_q - g_\sigma^q \sigma$$

$$\Omega_Q^* = \Omega_{\bar{Q}}^* = [x_Q^2 + (R_h^* m_Q)^2]^{1/2} \quad (Q = s, c, b)$$

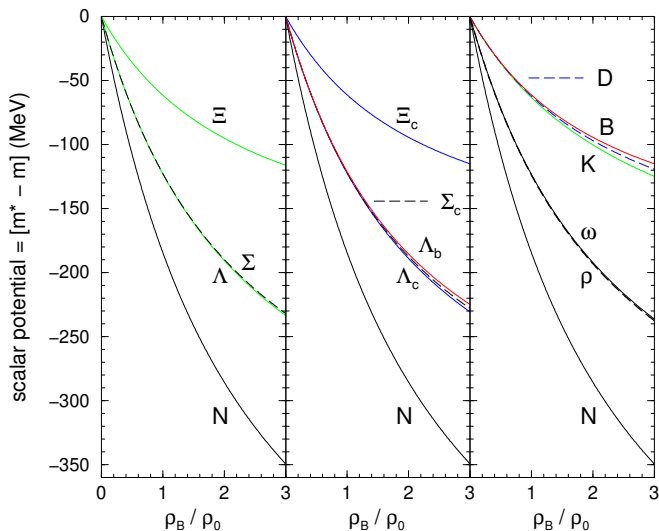
Baryon Scalar potential [$m_B^* - m_B$]



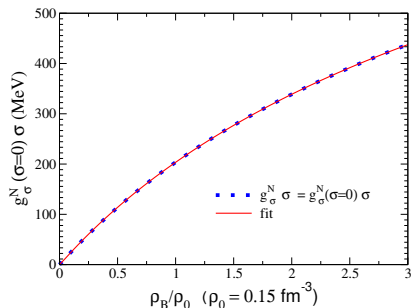
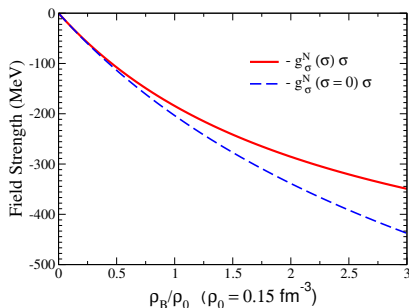
Meson Scalar potential [$m_M^* - m_M$]



Scalar potential: $m_{B,M}^* - m_{B,M}$



Scalar field σ and fit



$$x = (\rho_B/\rho_0)$$

$$(g_{\sigma}^N \sigma)(x) = 1.60828 - 23.9107 \sqrt{x} + 350.631 x - 144.309 x \sqrt{x} + 19.4750 x^2 \quad (x > 0, [x > 0.001])$$

$$(g_{\sigma}^N \sigma)(x) = 0 \quad (x = 0)$$

$M_{B,M}^*$: Parametrization

$$\begin{aligned}
 m_B^* &\simeq m_B - \frac{n_q}{3} g_\sigma^N \left[1 - \frac{a_B}{2} (g_\sigma^N \sigma) \right] \sigma, \\
 &= m_B - \frac{n_q}{3} \left[(g_\sigma^N \sigma) - \frac{a_B}{2} (g_\sigma^N \sigma)^2 \right], \\
 &\quad (B = N, \Lambda, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*, \Lambda_c, \Sigma_c, \Xi_c, \Lambda_b, \Sigma_b, \Xi_b).
 \end{aligned}$$

$$\begin{aligned}
 m_M^* &\simeq m_M - \frac{n_q^M}{3} g_\sigma^N \left[1 - \frac{a_M}{2} (g_\sigma^N \sigma) \right] \sigma, \\
 &= m_M - \frac{n_q^M}{3} \left[(g_\sigma^N \sigma) - \frac{a_M}{2} (g_\sigma^N \sigma)^2 \right], \\
 &\quad (M = \omega, \rho, K, K^*, \eta, \eta', D, D^*, B, B^*, \text{ with } n_q^M \rightarrow 1 \text{ for } \eta \text{ and } \eta').
 \end{aligned}$$

Vector Potentials: Parametrization

$$x = \rho_B / \rho_0 \quad (\rho_0 = 0.15 \text{ fm}^{-3})$$

$$V_\omega^B(x) = b_B x,$$

$$V_\omega^q(x) = 41.77 x$$

$$V_\omega^h(x) = V_\omega^h = (n_q - n_{\bar{q}}) V_\omega^q = (n_q - n_{\bar{q}}) \times 41.77 x$$

$$V_\omega^K(x) \simeq 1.96 \times 41.77 x$$

(baryon octet $\rightarrow$ Pauli potentials)

$$y \equiv \rho_3 / \rho_0 = (\rho_p - \rho_n) / \rho_0, \text{ isospin-third component of } h, I_3^h,$$

$$I_3^h V_\rho^h(y) = I_3^h \times 84.61 y$$

Parameters

Table: Effective mass slope parameter $a_{B,M}$, and b_B .
 $([m_{\Sigma}^* - m_{\Sigma}] + V_{\nu}^{\Sigma} \simeq +30 \text{ MeV at } \rho_0.)$

a_B	$\times 10^{-4} \text{ MeV}^{-1}$	a_B	$\times 10^{-4} \text{ MeV}^{-1}$	a_B	$\times 10^{-4} \text{ MeV}^{-1}$	a_B	$\times 10^{-4} \text{ MeV}^{-1}$
a_N	9.15	a_{Δ}	10.08	—	—	—	—
a_{Λ}	9.35	—	—	a_{Λ_c}	9.90	a_{Λ_b}	10.78
a_{Σ}	9.59	a_{Σ^*}	10.15	a_{Σ_c}	10.34	a_{Σ_b}	11.22
a_{Ξ}	9.52	a_{Ξ^*}	10.15	a_{Ξ_c}	9.99	a_{Ξ_b}	10.83
b_B	MeV	b_B	MeV	b_B	MeV	b_B	MeV
b_N	125.30	b_{Δ}	125.30	—	—	—	—
b_{Λ}	92.57	—	—	b_{Λ_c}	83.54	b_{Λ_b}	83.54
b_{Σ}	100.12	b_{Σ^*}	83.54	b_{Σ_c}	83.54	b_{Σ_b}	83.54
$\tilde{b}_{\Sigma}$	152.42	—	—	—	—	—	—
b_{Ξ}	46.29	b_{Ξ^*}	41.77	b_{Ξ_c}	41.77	b_{Ξ_b}	41.77
a_M	$\times 10^{-4} \text{ MeV}^{-1}$	a_M	$\times 10^{-4} \text{ MeV}^{-1}$	a_M	$\times 10^{-4} \text{ MeV}^{-1}$	a_M	$\times 10^{-4} \text{ MeV}^{-1}$
a_{ω}	8.73	a_K	6.66	a_D	8.61	a_B	9.92
a_{ρ}	8.70	a_{K^*}	8.60	a_{D^*}	9.09	a_{B^*}	10.04
—	—	$a_{\eta}(n_{\eta}^{\eta} \rightarrow 1)$	7.03	—	—	—	—
—	—	$a_{\eta'}(n_{\eta'}^{\eta'} \rightarrow 1)$	8.81	—	—	—	—

Two-nonlight flavor meson self-energies:

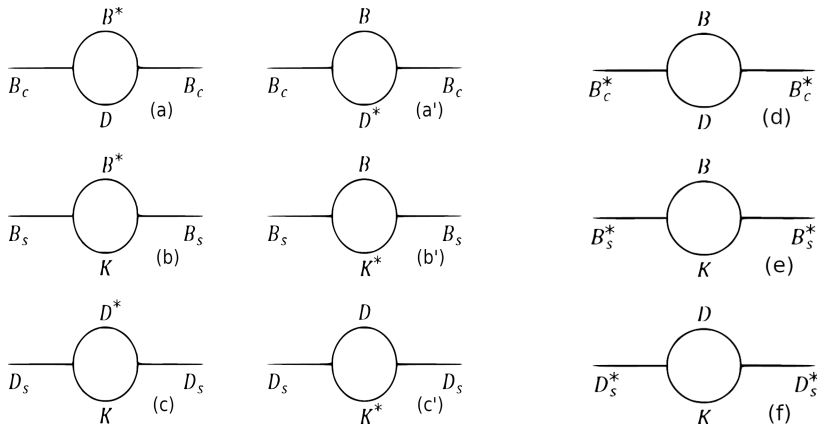


Figure: Self-energies of pseudoscalar mesons B_c , B_s and D_s , and vector mesons B_c^* , B_s^* and D_s^* (from top to bottom) in the lowest order.

B, B^*, D, D^*, K, K^* effective masses

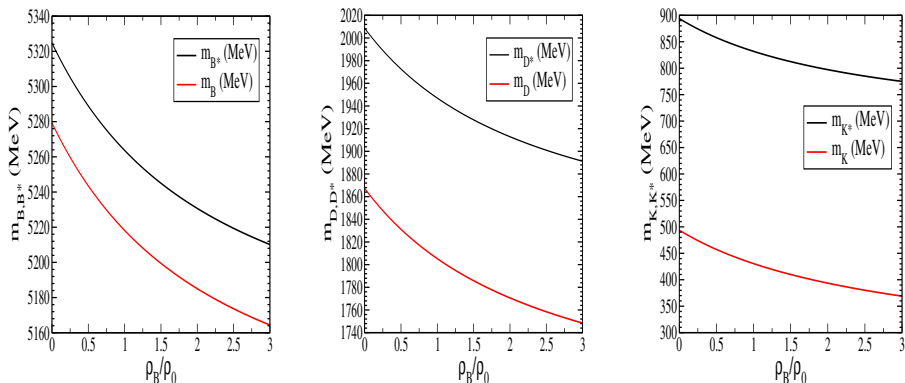


Figure: B and B^* (left), D and D^* (middle) and K and K^* (right) meson effective Lorentz-scalar masses in symmetric nuclear matter versus baryon density.

Effective Lagrangian

$$\mathcal{L}_0 = \text{Tr} \left(\partial_\mu P^\dagger \partial^\mu P \right) - \frac{1}{2} \text{Tr} \left(F_{\mu\nu}^\dagger F^{\mu\nu} \right)$$

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$$

$$\partial_\mu P \rightarrow \partial_\mu P - \frac{ig}{2} [V_\mu, P], \quad F_{\mu\nu} \rightarrow \partial_\mu V_\nu - \partial_\nu V_\mu - \frac{ig}{2} [V_\mu, V_\nu]$$

$$\begin{aligned} \mathcal{L} = \mathcal{L}_0 &+ ig \text{Tr}(\partial^\mu P [P, V_\mu]) - \frac{g^2}{4} \text{Tr}([P, V_\mu]^2) \\ &+ ig \text{Tr}(\partial^\mu V^\nu [V_\mu, V_\nu]) + \frac{g^2}{8} \text{Tr}([V_\mu, V_\nu]^2) \end{aligned}$$

Z. w. Lin and C. M. Ko, Phys. Lett. B 503, 104 (2001)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} P_1 & \pi^+ & K^+ & \bar{D}^0 & B^+ \\ \pi^- & P_2 & K^0 & D^- & B^0 \\ K^- & \bar{K}^0 & P_3 & D_s^- & B_s^0 \\ D^0 & D^+ & D_s^+ & P_4 & B_c^+ \\ B^- & \bar{B}^0 & \bar{B}_s^0 & B_c^- & P_5 \end{pmatrix}$$

$$P_1 = \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} + \frac{\eta_b}{\sqrt{20}}$$

$$P_2 = \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} + \frac{\eta_b}{\sqrt{20}}$$

$$P_3 = \frac{-2\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} + \frac{\eta_b}{\sqrt{20}}$$

$$P_4 = \frac{-3\eta_c}{\sqrt{12}} + \frac{\eta_b}{\sqrt{20}}$$

$$P_5 = \frac{-2\eta_b}{\sqrt{5}}$$

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} V_1 & \rho^+ & K^{*+} & \bar{D}^{*0} & B^{*+} \\ \rho^- & V_2 & K^{*0} & D^{*-} & B^{*0} \\ K^{*-} & \bar{K}^{*0} & V_3 & D_s^{*-} & B_s^{*0} \\ D^{*0} & D^{*+} & D_s^{*+} & V_4 & B_c^{*+} \\ B^{*-} & \bar{B}^{*0} & \bar{B}_s^{*0} & B_c^{*-} & V_5 \end{pmatrix}$$

$$\begin{aligned} V_1 &= \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} + \frac{J/\psi}{\sqrt{12}} + \frac{\Upsilon}{\sqrt{20}} \\ V_2 &= \frac{-\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} + \frac{J/\psi}{\sqrt{12}} + \frac{\Upsilon}{\sqrt{20}} \\ V_3 &= \frac{-2\omega}{\sqrt{6}} + \frac{J/\psi}{\sqrt{12}} + \frac{\Upsilon}{\sqrt{20}} \\ V_4 &= \frac{-3J/\psi}{\sqrt{12}} + \frac{\Upsilon}{\sqrt{20}} \\ V_5 &= \frac{-2\Upsilon}{\sqrt{5}} \end{aligned}$$

Effective lagrangian (example): B_c, B_c^*

$$\mathcal{L}_{B_c B^* D} = ig_{B_c B^* D} [(\partial_\mu B_c^-) D - B_c^- (\partial_\mu D)] B^{*\mu} + h.c.$$

$$\mathcal{L}_{B_c B D^*} = ig_{B_c B D^*} [(\partial_\mu B_c^+) \bar{B} - B_c^+ (\partial_\mu \bar{B})] \bar{D}^{*\mu} + h.c.$$

$$\mathcal{L}_{B_c^* B D} = -ig_{B_c^* B D} B_c^{*+\mu} [\bar{B} (\partial_\mu \bar{D}) - (\partial_\mu \bar{B}) \bar{D}] + h.c.$$

$$B = \begin{pmatrix} B^+ \\ B^0 \end{pmatrix}, \quad \bar{B} = \begin{pmatrix} B^- & \bar{B}^0 \end{pmatrix}, \quad B^* = \begin{pmatrix} B^{*+} \\ B^{*0} \end{pmatrix}, \quad \bar{B}^* = \begin{pmatrix} B^{*-} & \bar{B}^{*0} \end{pmatrix}.$$

$$\bar{D} = \begin{pmatrix} \bar{D}^0 \\ D^- \end{pmatrix}, \quad D = \begin{pmatrix} D^0 & D^+ \end{pmatrix}, \quad \bar{D}^* = \begin{pmatrix} \bar{D}^{*0} \\ D^{*-} \end{pmatrix}, \quad D^* = \begin{pmatrix} D^{*0} & D^{*+} \end{pmatrix}.$$

$$g_{\Upsilon BB} = \frac{5g}{4\sqrt{10}} \approx 13.2$$

$$g_{B_c B^* D} = g_{B_c B D^*} = g_{B_c^* B D} = \frac{g}{2\sqrt{2}} \approx 11.9$$

B_c self-energy (example): B^*D loop (only case)

$$V_{B_c} = \Delta m_{B_c} \equiv m_{B_c}^* - m_{B_c} \quad (m_{B_c}^* < m_{B_c}, \text{ Lorentz scalar}),$$

$$m_{B_c}^2 = \left(m_{B_c}^0\right)^2 - |\Sigma_{B_c}(k^2 = m_{B_c}^2)|,$$

$$m_{B_c}^{*2} = \left(m_{B_c}^0\right)^2 - |\Sigma_{B_c}^*(k^2 = m_{B_c}^{*2})|,$$

$$\Sigma_{B_c}^{B^*D}(m_{B_c}^*) = \frac{-4g_{B_cB^*D}^2}{\pi^2} \int d|k| |k|^2 I_{B^*D}(|k|) F_{B_cB^*D}(k^2),$$

$$I_{B_c}^{B^*D}(|k|) = \frac{m_{B_c}^{*2} (-1 + k_0^2/m_{B^*}^{*2})}{(k_0 - \omega_{B^*}^*)(k_0 - m_{B_c}^* + \omega_D^*)(k_0 - m_{B_c}^* - \omega_D^*)} \Big|_{k_0 = -\omega_{B^*}^*}$$

$$+ \frac{m_{B_c}^{*2} (-1 + k_0^2/m_{B^*}^{*2})}{(k_0 + \omega_{B^*}^*)(k_0 - \omega_{B^*}^*)(k_0 - m_{B_c}^* - \omega_D^*)} \Big|_{k_0 = m_{B_c}^* - \omega_D^*}$$

Total potential $V_s^{B_c} (B^*D + BD^*)$ loops

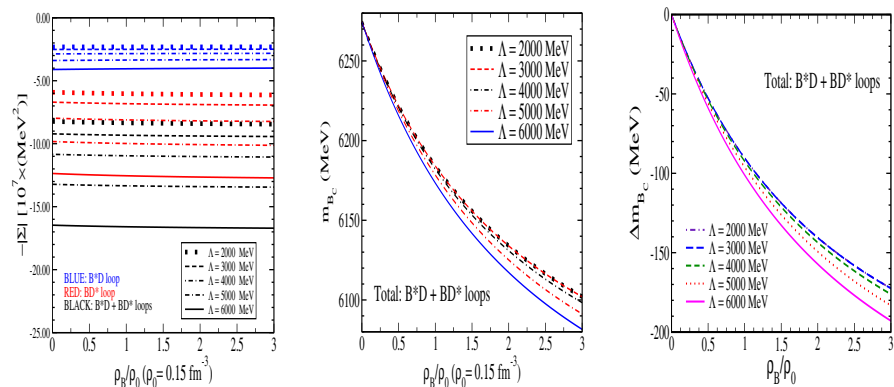


Figure: $|\Sigma_{B_c}|$ (left), $m_{B_c}^*$ (middle) and Δm_{B_c} (right)

Total potential $V_s^{B_c^*}$ (BD) loop

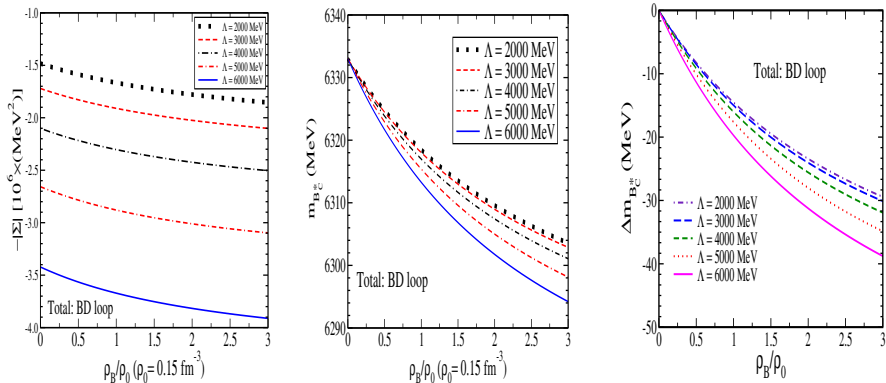


Figure: $|\Sigma_{B_c^*}|$ (left), $m_{B_c^*}^*$ (middle) and $\Delta m_{B_c^*}$ (right)

Total potential $V_s^{B_s^0}$ ($B^*K + BK^*$) loops

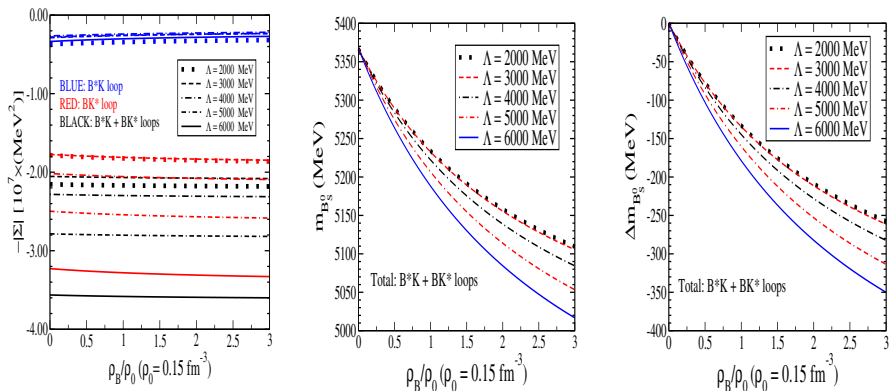


Figure: $|\Sigma_{B_s^0}|$ (left), $m_{B_s^0}^*$ (middle) and $\Delta m_{B_s^0}$ (right)

Total potential $V_s^{B_s^*}$ (BK) loop

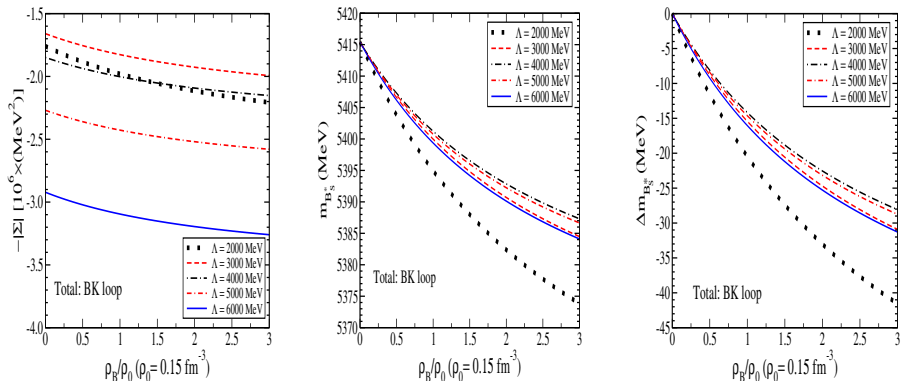


Figure: $|\Sigma_{B_s^*}|$ (left), $m_{B_s^*}^*$ (middle) and $\Delta m_{B_s^*}$ (right)

Total potential $V_s^{D_s} (D^*K + DK^*)$ loops

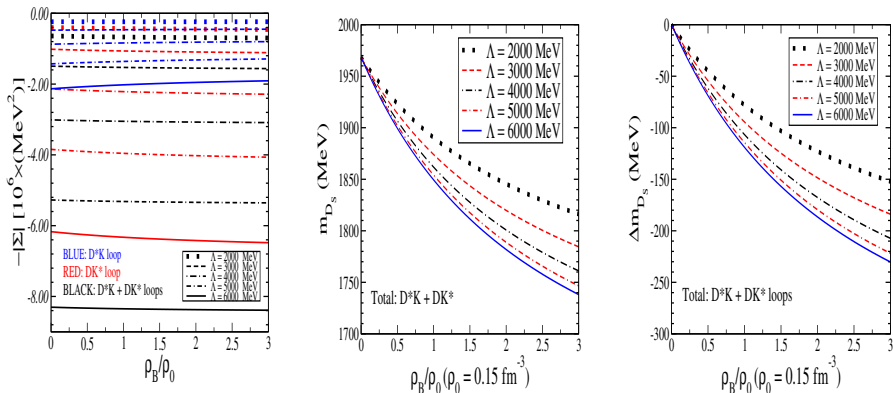


Figure: $|\Sigma_{D_s}|$ (left), $m_{D_s}^*$ (middle) and Δm_{D_s} (right)

Total potential $V_s^{D_s^*}$ (DK) loop

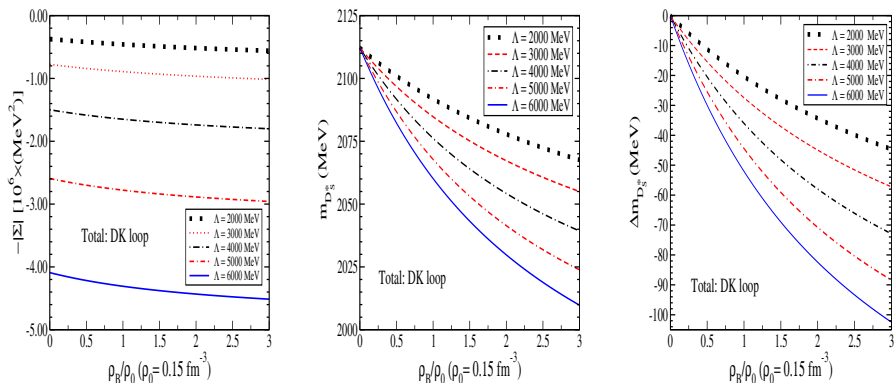


Figure: $|\Sigma_{D_s^*}|$ (left), $m_{D_s^*}^*$ (middle) and $\Delta m_{D_s^*}$ (right)

Total potentials: Δm_{B_c} , $\Delta m_{B_s^0}$, Δm_{D_s}

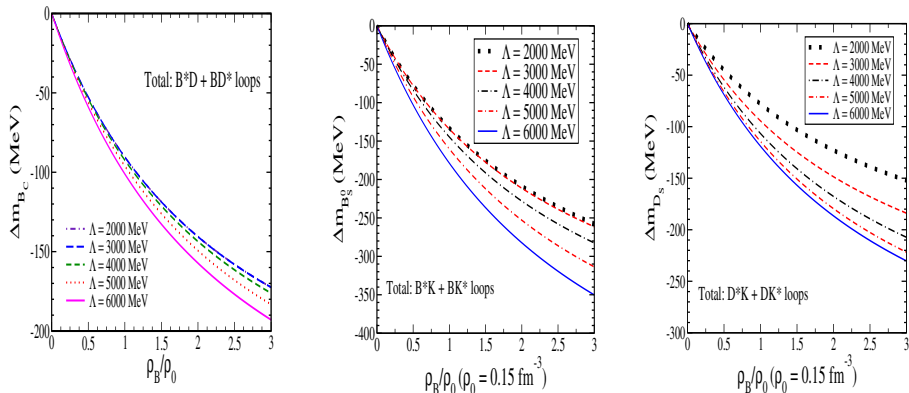


Figure: Δm_{B_c} (left), $\Delta m_{B_s^0}$ (middle) and Δm_{D_s} (right)

Total potentials: Δm_{η_b} , Δm_{B_c} , Δm_{η_c}

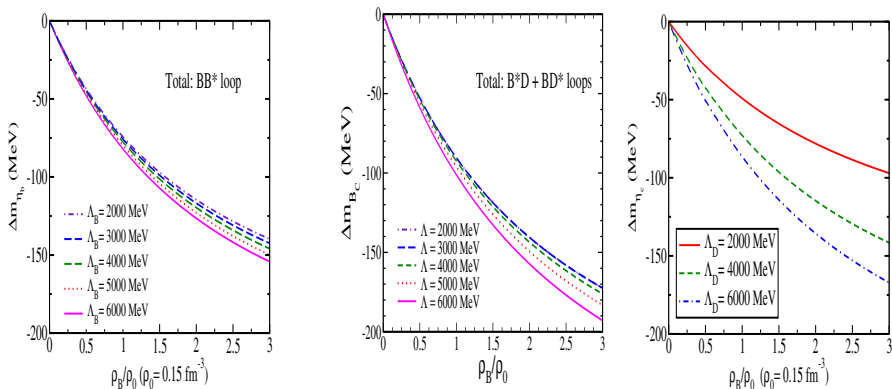


Figure: Δm_{η_b} (BB^*) (left), Δm_{B_c} ($B^*D + BD^*$) (middle) and Δm_{η_c} (DD^*) (right)

Total potentials: $\Delta m_{B_c^*}$, $\Delta m_{B_s^*}$, $\Delta m_{D_s^*}$

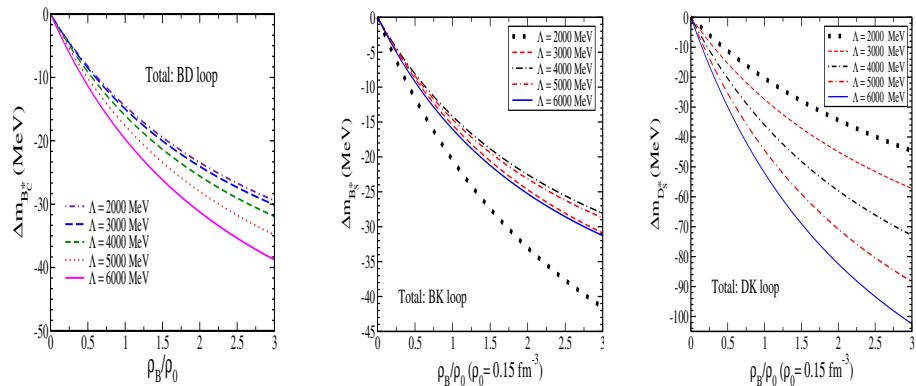


Figure: $\Delta m_{B_c^*}$ (left), $\Delta m_{B_s^*}$ (middle) and $\Delta m_{D_s^*}$ (right)

Total potentials: Δm_{Υ} , $\Delta m_{B_c^*}$, $\Delta m_{J/\psi}$

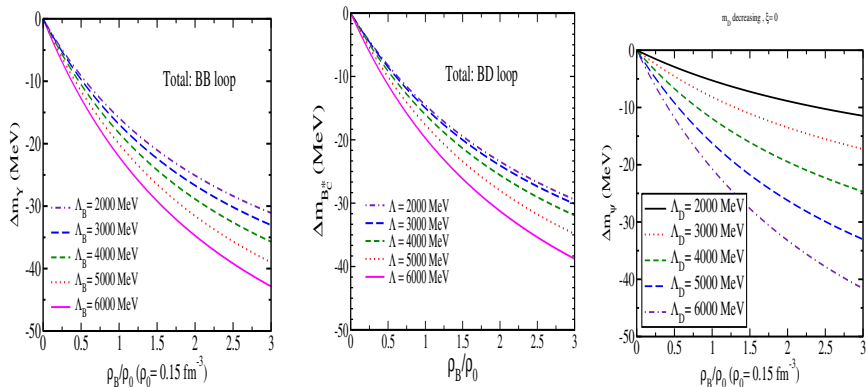


Figure: Δm_{Υ} (BB) (left), $\Delta m_{B_c^*}$ (BD) (middle) and $\Delta m_{J/\psi}$ (DD) (right)

Summary, Discussion:

- $|\Delta m_{B_c}| < |\Delta m_{B_s^0}| > |\Delta m_{D_s}|$

- $|\Delta m_{\eta_b}| < |\Delta m_{B_c}| > |\Delta m_{\eta_c}|$

- $|\Delta m_{B_c^*}| \simeq |\Delta m_{B_s^*}| < |\Delta m_{D_s^*}|$

- $|\Delta m_{\Upsilon}| \simeq |\Delta m_{B_c^*}| \simeq |\Delta m_{J/\psi}|$

- $B_c^\pm$ -nucleus bond states (^{12}C done, and more!)

Other studies (include KT below):

- A. W. Thomas, G. Krein, J. J. Cobos-Multinez: J/ψ - and η_c -nucleus bound states, *Prog. Part. Nucle. Phys.* **100**, 161 (2018)

- G. Zeminiani, J. J. Cobos-Multinez: Υ - and η_b -nucleus bound states, *Phys. Rev. C* **105**, 025204 (2022)

- J. J. Cobos-Multinez: η - and η' -nucleus bound states with widths, *Phys. Rev. C* **109**, 025202 (2024)

- $B_c^\pm$ - ^{12}C bound states! *Phys. Rev. C* **111**, 055202 (2025)

Thank You Very Much !!!

Bound quark Dirac spinor ($1s_{1/2}$)

Quark Dirac spinor in **a bound hadron**:

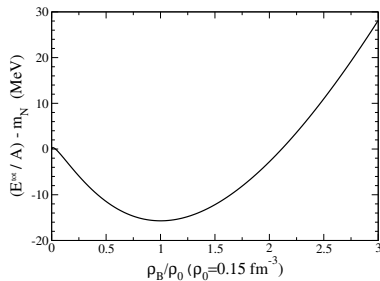
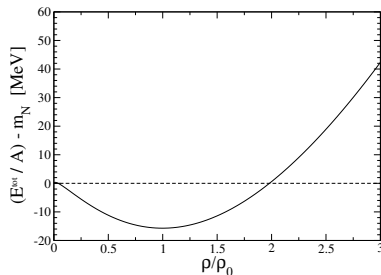
$$q_{1s}(\mathbf{r}) = \begin{pmatrix} U(\mathbf{r}) \\ i\hat{\boldsymbol{\sigma}} \cdot \mathbf{r} L(\mathbf{r}) \end{pmatrix} \chi$$

Lower component is **enhanced** !

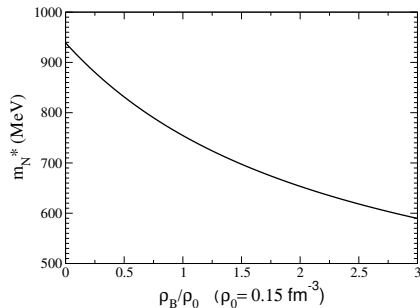
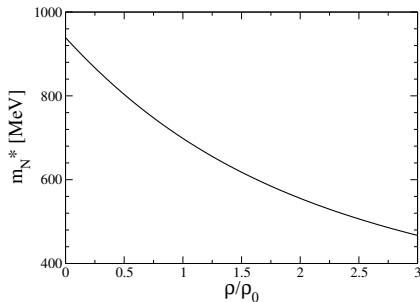
$$\Rightarrow g_A^* < g_A : \sim |U|^2 - (1/3) |L|^2,$$

$\Rightarrow$ **Decrease** of **scalar density** $\Rightarrow$

Comparison of Energy/nucleon



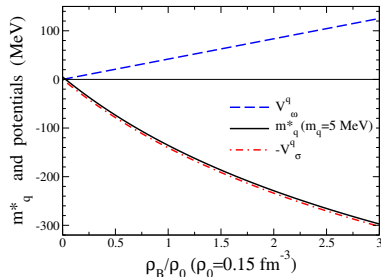
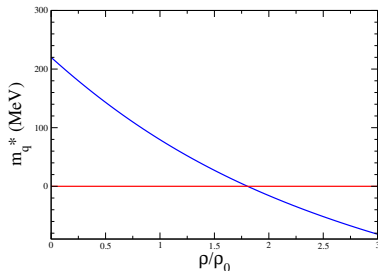
- **Symmetric Nuclear Matter - Binding Energy per Nucleon (scale !!)**
- **LF pion model (left):** $m_q = 220 \text{ MeV}$, $K = 320.9 \text{ MeV}$
- **Standard QMC (right):** $m_q = 5 \text{ MeV}$, $K = 279.3 \text{ MeV}$



Nucleon effective mass

- LF pion model (left: $m_q = 220 \text{ MeV}$)
- Standard QMC (right: $m_q = 5 \text{ MeV}$)

LF pion model and Standard QMC: m_q^* (potentials)



- Effective mass of constituent quarks, up and down
- LF pion model: $m_q = 220 \text{ MeV}$ (left)
- Standard QMC $m_q = 5 \text{ MeV}$ (right)