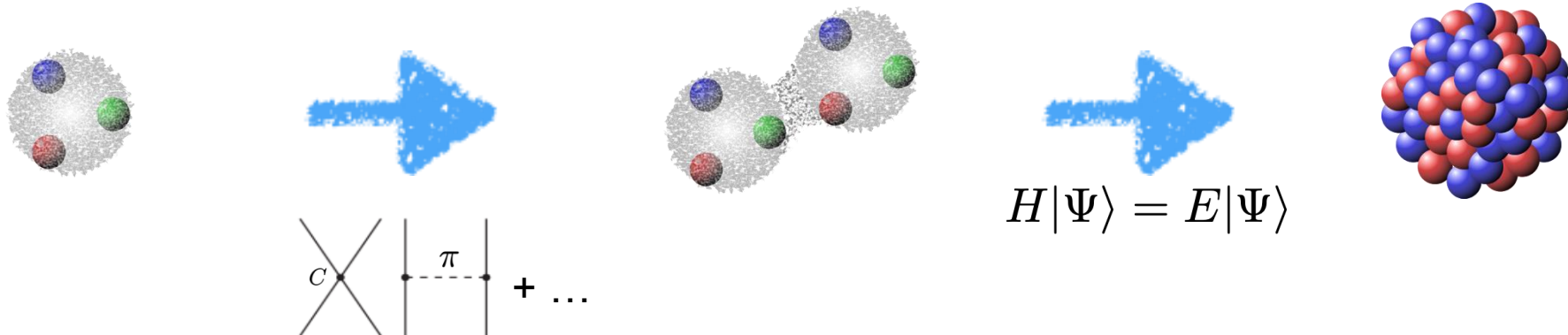


# Recent advances in ab initio calculations of electromagnetic observables



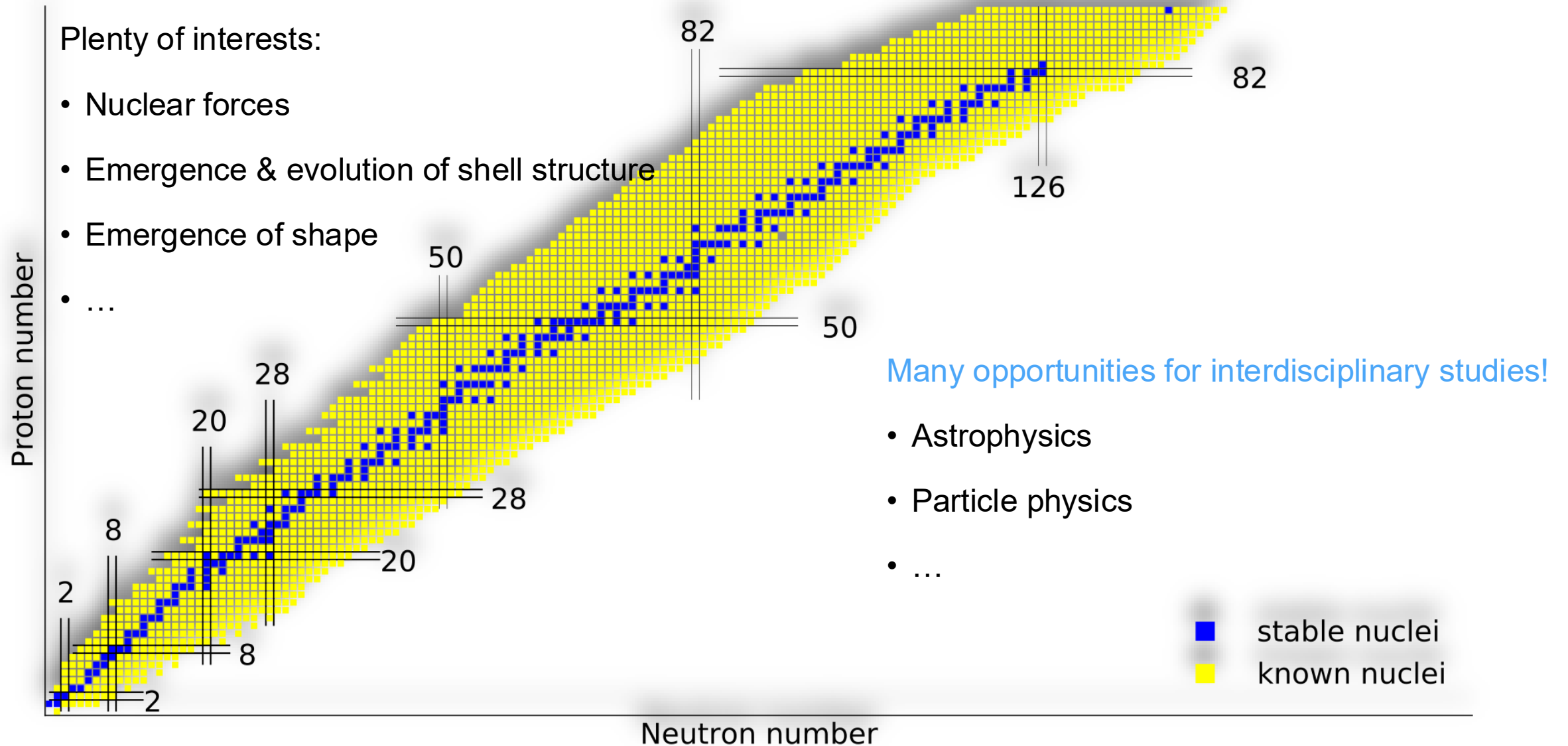
Supported by



Takayuki Miyagi (University of Tsukuba, Japan)

The 29th International Nuclear Physics Conference @Daejeon, Korea, May 26, 2025

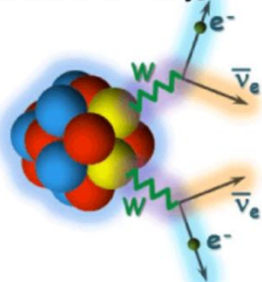
# Motivations



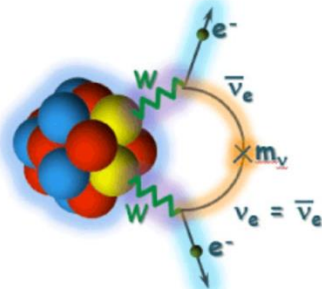
# Motivations

<https://wwwkm.phys.sci.osaka-u.ac.jp/en/research/r01.html>

[Double beta decay]

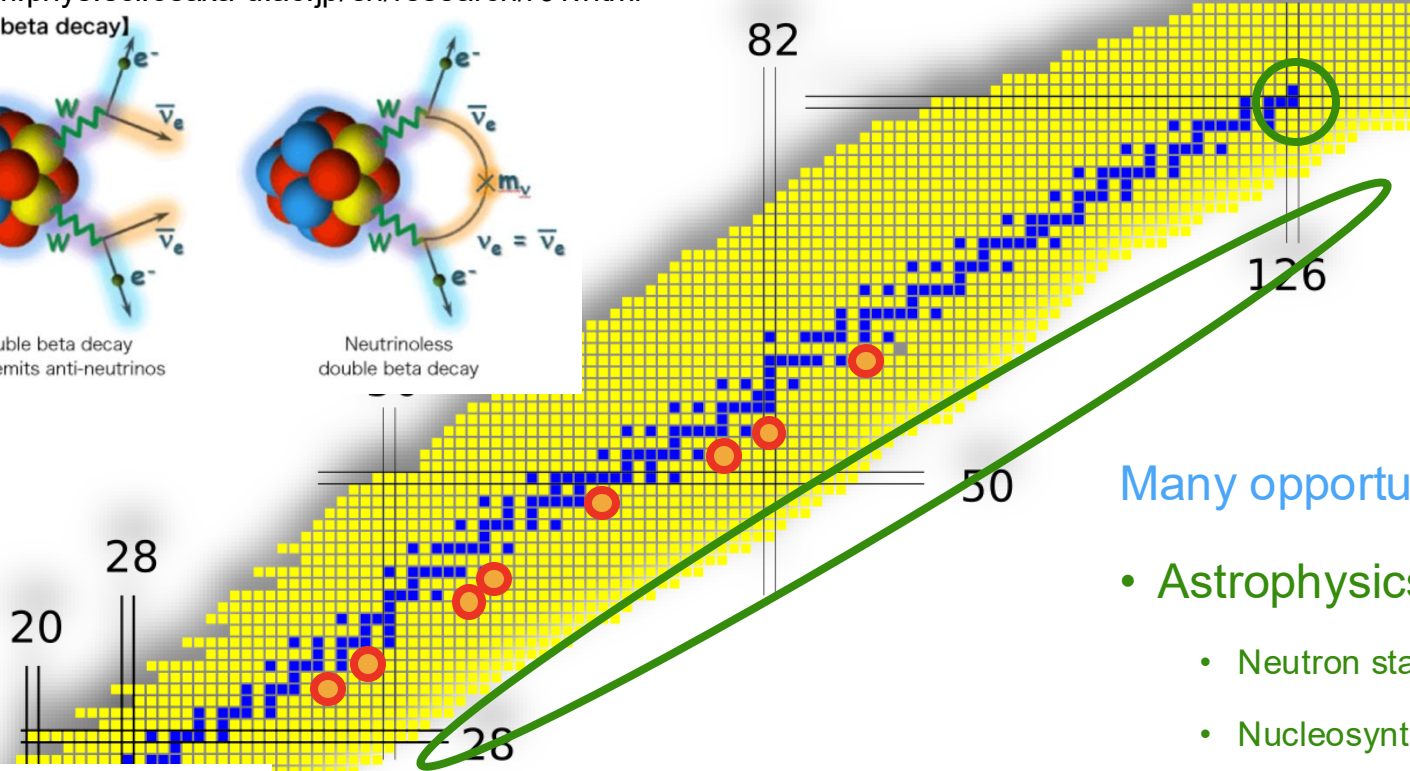


Double beta decay  
which emits anti-neutrinos

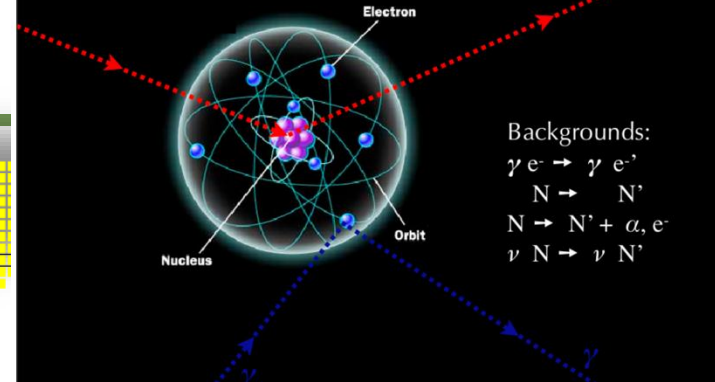


Neutrinoless  
double beta decay

Proton number



Direct Detection



Backgrounds:  
 $\gamma e^- \rightarrow \gamma e^-$   
 $N \rightarrow N'$   
 $N \rightarrow N' + \alpha, e^-$   
 $\nu N \rightarrow \nu N'$

F. S. Queiroz, arXiv:1605.08788.

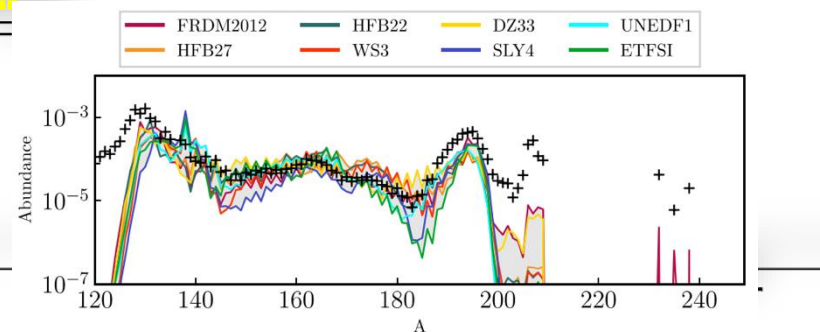
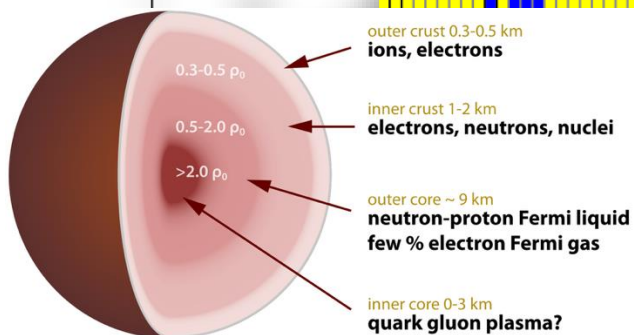
Many opportunities for interdisciplinary studies!

## • Astrophysics

- Neutron star
- Nucleosynthesis
- ....

## • Particle physics

- Neutrinoless double-beta decay
- WIMP neutrino-nucleus scattering
- ...

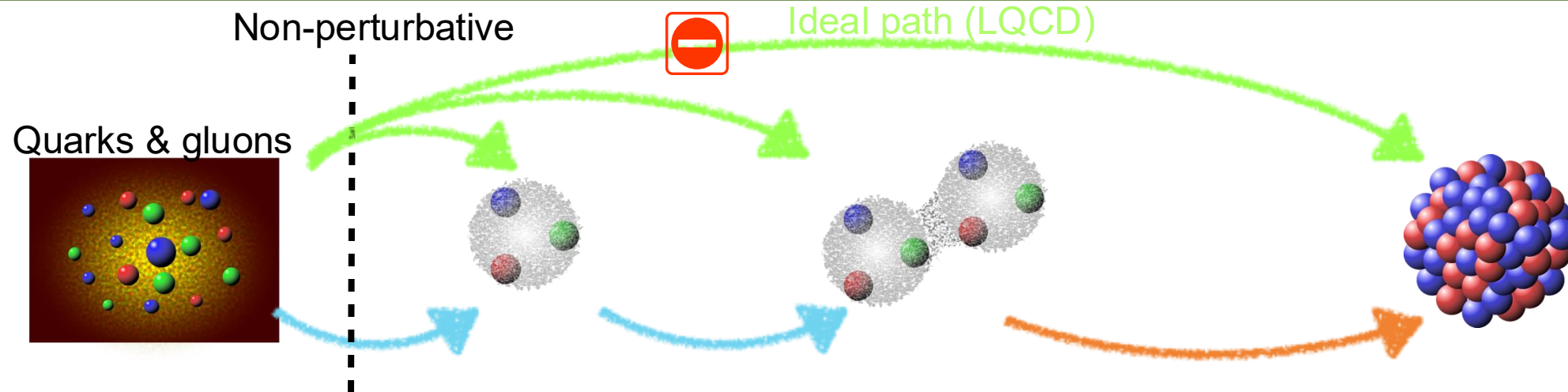


- EM observables can be used
  - ◆ to investigate nuclear structure (shell structure, shape, ...)
  - ◆ to test nuclear ab initio calculations
- To test ab initio calculations we need:
  - ◆ (precise) experimental data
  - ◆ reasonable starting nuclear Hamiltonian(s)
  - ◆ controllable many-body method(s)
  - ◆ higher-order contribution of EM operators

$$H|\Psi\rangle = E|\Psi\rangle$$
$$O_{\text{EM}}^{\text{exp.}} \sim \langle\Psi|\mathcal{O}_{\text{EM}}|\Psi\rangle$$

- Once the methods are tested, making predictions for the unknown are more convincing.

# Nuclear ab initio calculations



|                             | Two-nucleon force | Three-nucleon force | Four-nucleon force |
|-----------------------------|-------------------|---------------------|--------------------|
| LO ( $Q^0$ )                |                   | —                   | —                  |
| NLO ( $Q^2$ )               |                   | —                   | —                  |
| N <sup>2</sup> LO ( $Q^3$ ) |                   |                     | —                  |
| N <sup>3</sup> LO ( $Q^4$ ) |                   |                     |                    |
| N <sup>4</sup> LO ( $Q^5$ ) |                   |                     |                    |

## Nuclear many-body problem

- ♦ Green's function Monte Carlo
- ♦ No-core shell model
- ♦ Nuclear lattice effective field theory
- ♦ Self-consistent Green's function
- ♦ Coupled-cluster
- ♦ In-medium similarity renormalization group
- ♦ Many-body perturbation theory

# Chiral Effective Field Theory

Weinberg, van Kolck, Kaiser, Epelbaum, Glöckle, Meißner, Entem, Machleidt, ...

- Lagrangian construction
  - ◆ Chiral symmetry
  - ◆ Power counting
- Systematic expansion
  - ◆ Unknown LECs
  - ◆ Many-body interactions
  - ◆ Uncertainty estimation

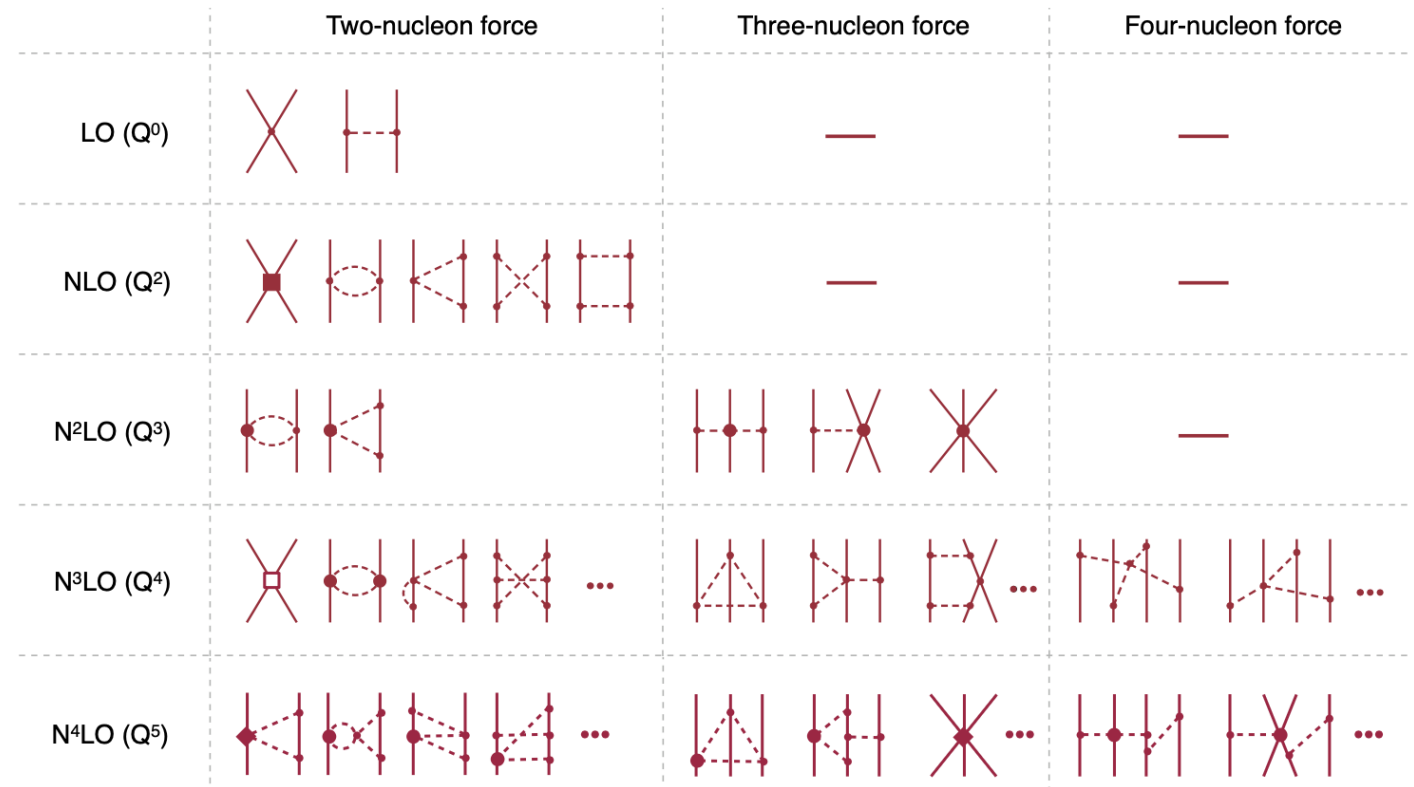
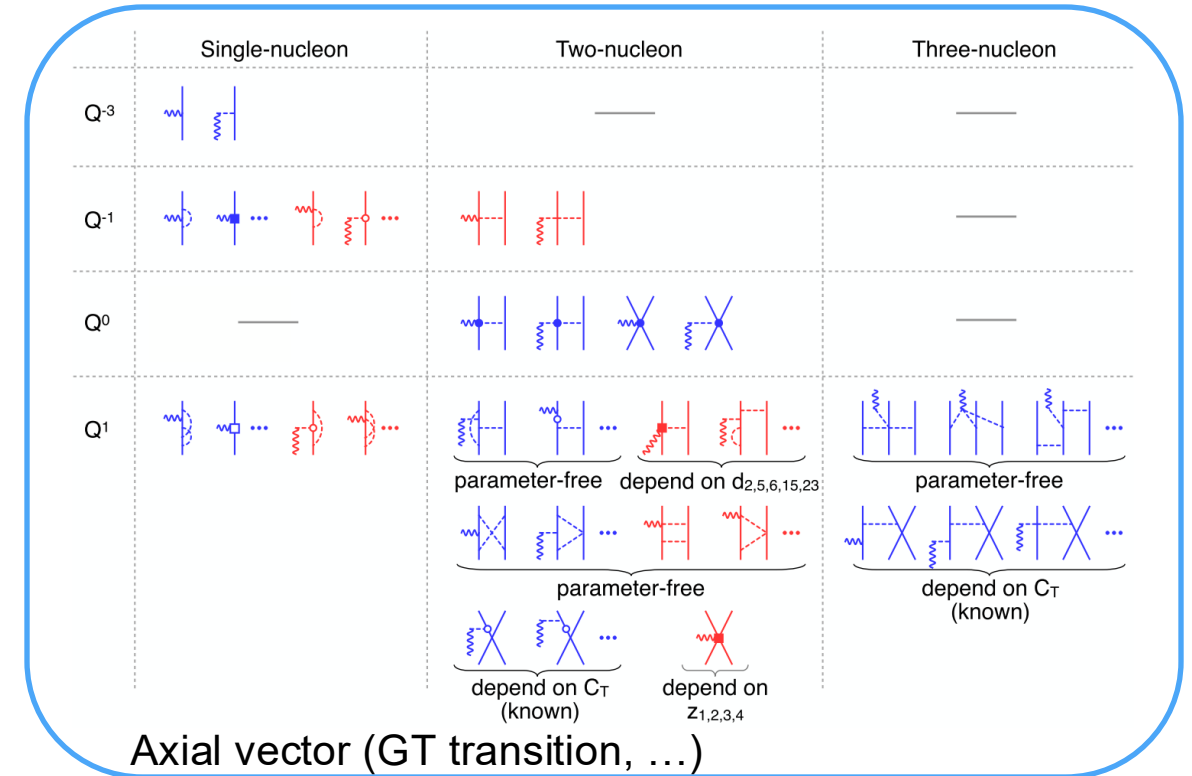
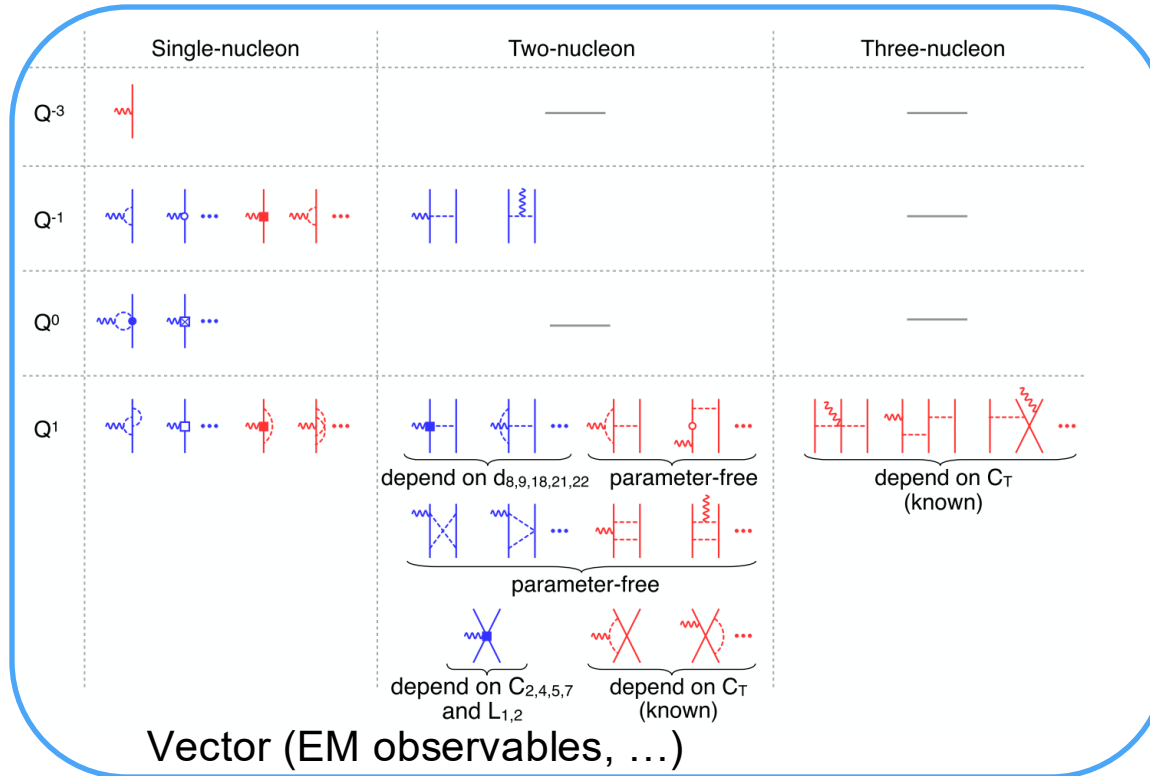


Figure is from E. Epelbaum, arXiv: 1510.07036

# Nuclear currents from ChEFT

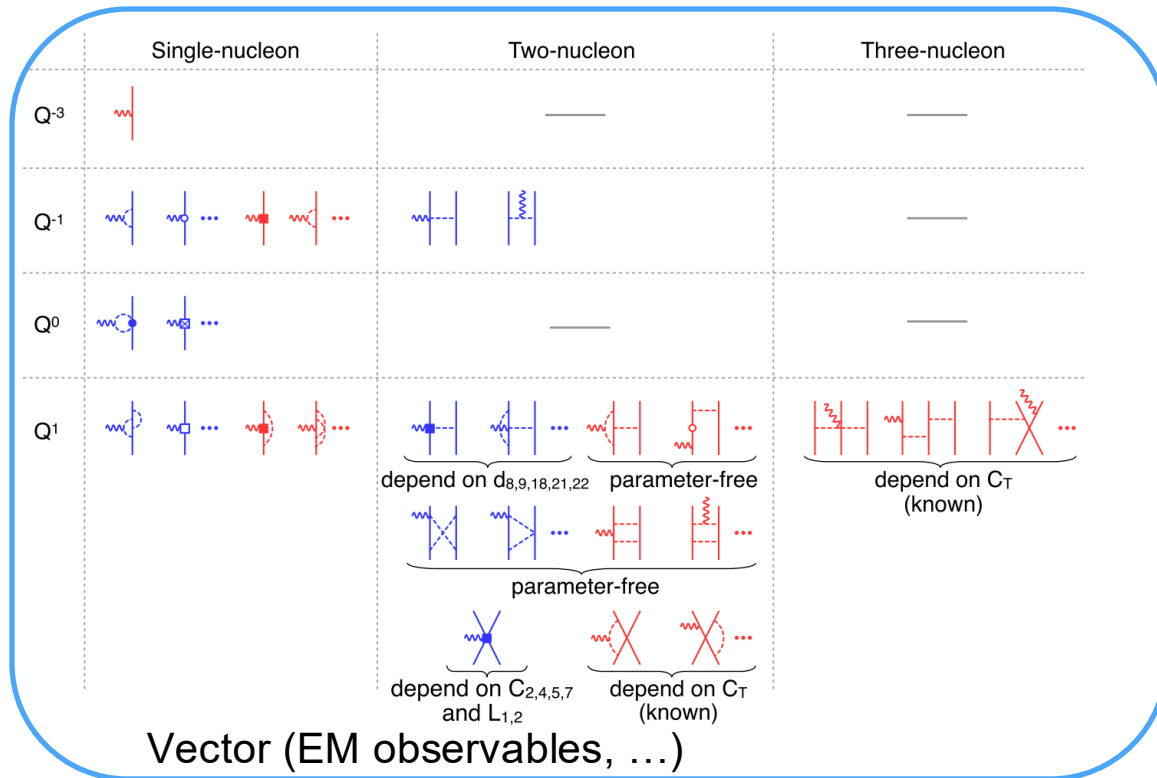
- Electroweak properties are related to current operators.
- Chiral EFT allows us a systematic expansion for **charge** and **current** operators.





# Nuclear currents from ChEFT

- Electroweak properties are related to current operators.
- Chiral EFT allows us a systematic expansion for **charge** and **current** operators.



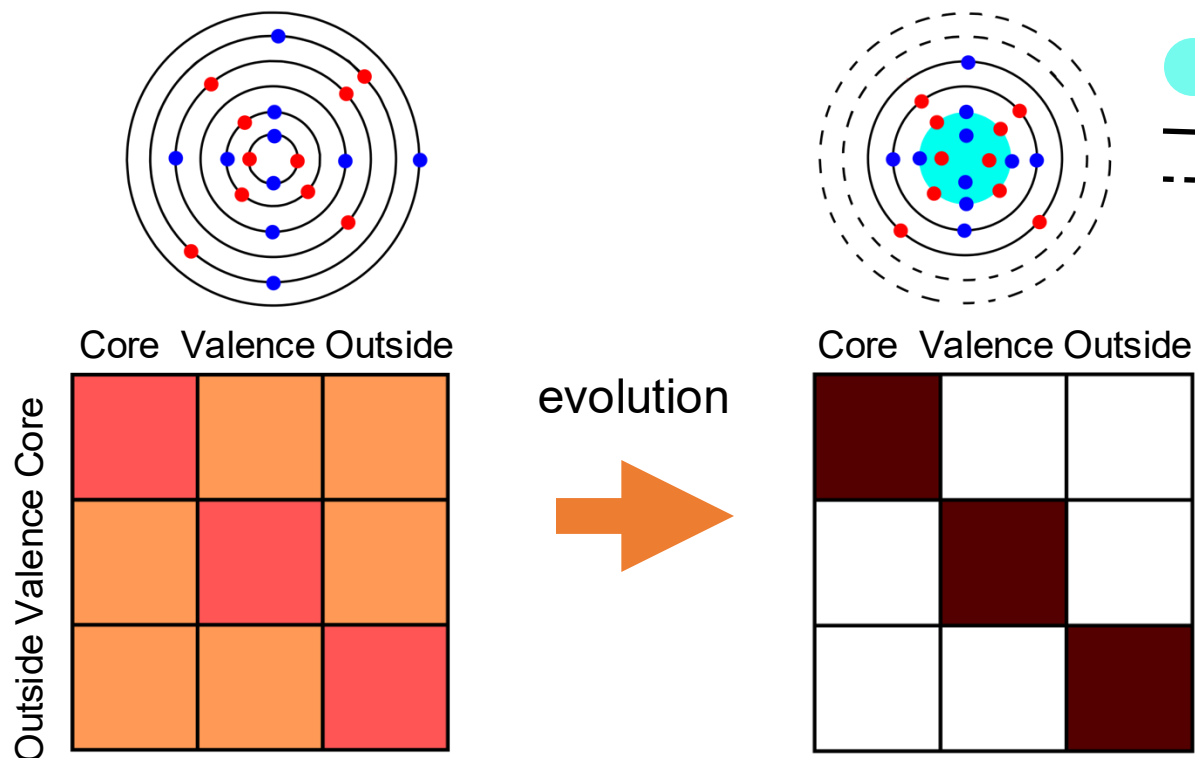
$$r_{ch}^2 = -\frac{6}{Z} \frac{1}{(4\pi)^{3/2}} \lim_{Q \rightarrow 0} \frac{d}{dQ^2} \int d\hat{Q} \tilde{\rho}(\mathbf{Q})$$

$$Q_{2\mu} = -\frac{15}{8\pi} \lim_{Q \rightarrow 0} \frac{d^2}{dQ^2} \int d\hat{Q} Y_{2\mu}(\hat{Q}) \tilde{\rho}(\mathbf{Q})$$

$$M_{1\mu} = -i \frac{3}{8\pi} \lim_{Q \rightarrow 0} \frac{d}{dQ} \int d\hat{Q} \{ [\mathbf{Q} \times \nabla_{\mathbf{Q}}] Y_{1\mu}(\hat{Q}) \} \cdot \tilde{\mathbf{j}}(\mathbf{Q})$$



# Valence-Space In-Medium Similarity Renormalization Group



● : frozen core  
— : valence  
--- : outside

$$\frac{d\Omega}{ds} = \eta(s) - \frac{1}{2}[\Omega(s), \eta(s)] + \dots$$

$$\eta(s) = \sum_{12} \eta_{12}(s) \{a_1^\dagger a_2\} + \sum_{1234} \eta_{1234}(s) \{a_1^\dagger a_2^\dagger a_4 a_3\}$$

$$\eta_{12} = \frac{1}{2} \arctan \left( \frac{2f_{12}}{f_{11} - f_{22} + \Gamma_{1212} + \Delta} \right)$$

$$\eta_{1234} = \frac{1}{2} \arctan \left( \frac{2\Gamma_{1234}}{f_{11} + f_{22} - f_{33} - f_{44} + A_{1234} + \Delta} \right)$$

$$A_{1234} = \Gamma_{1212} + \Gamma_{3434} - \Gamma_{1313} - \Gamma_{2424} - \Gamma_{1414} - \Gamma_{2323}$$

$f_{12}, \Gamma_{1234}$  : matrix elements to be suppressed

$H$

$$H(s) = e^{\Omega(s)} H e^{-\Omega(s)}$$

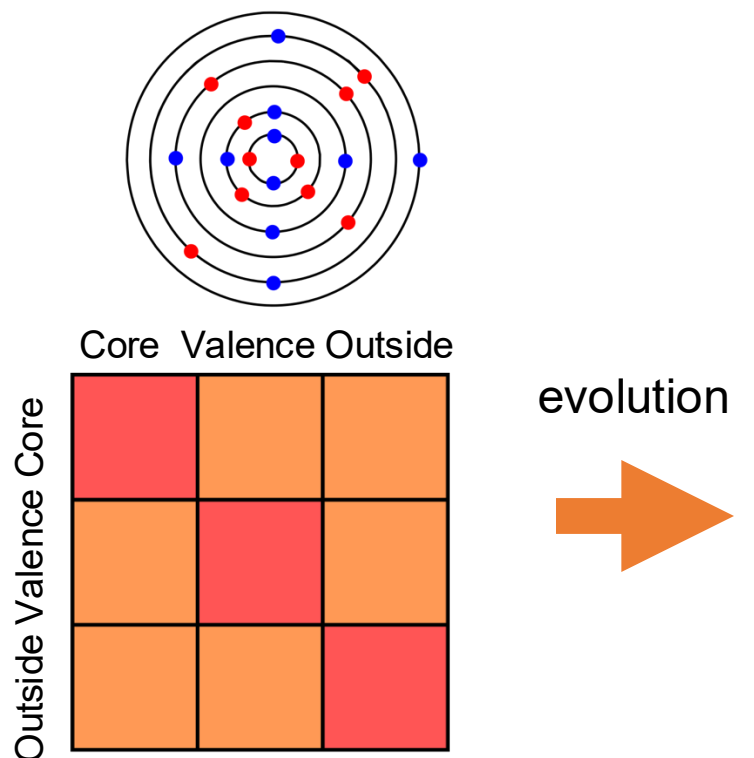
$$H(s) \approx E(s) + \sum_{12} f_{12}(s) \{a_1^\dagger a_2\} + \frac{1}{4} \sum_{1234} \Gamma_{1234}(s) \{a_1^\dagger a_2^\dagger a_4 a_3\} \quad \mathcal{O}(s) = e^{\Omega(s)} \mathcal{O} e^{-\Omega(s)} \approx \mathcal{O}^{[0]}(s) + \sum_{12} \mathcal{O}_{12}^{[1]}(s) \{a_1^\dagger a_2\} + \frac{1}{4} \sum_{1234} \mathcal{O}_{1234}^{[2]}(s) \{a_1^\dagger a_2^\dagger a_4 a_3\}$$

$s$ : flow parameter

H. Hergert, S. K. Bogner, T. D. Morris, A. Schwenk, and K. Tsukiyama, Phys. Rep. **621**, 165 (2016).

S. R. Stroberg, H. Hergert, S. K. Bogner, and J. D. Holt, Annu. Rev. Nucl. Part. Sci. **69**, 307 (2019).

# Valence-Space In-Medium Similarity Renormalization Group

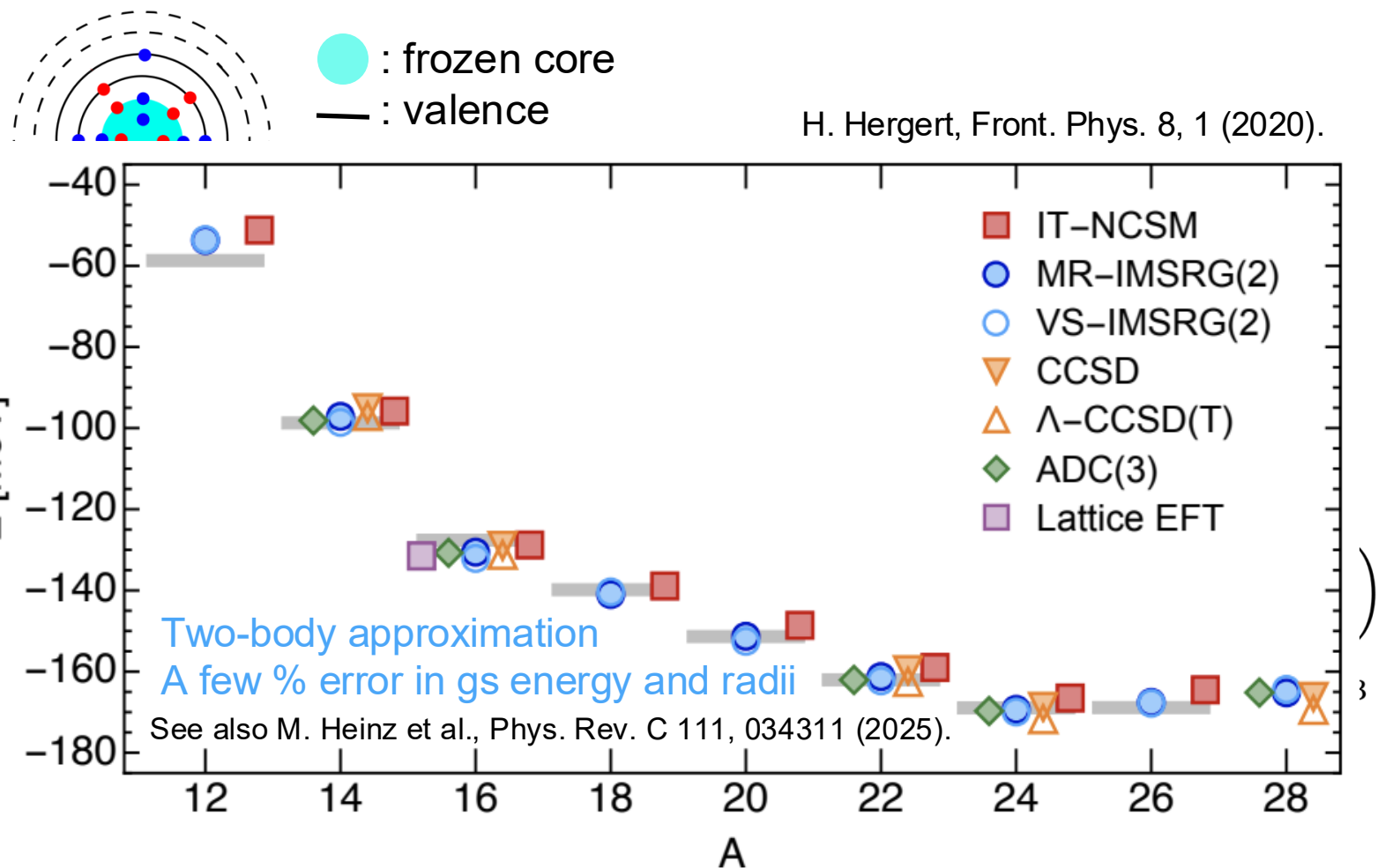


Similarity transform

$H$

$$H(s) \approx E(s) + \sum_{12} f_{12}(s) \{a_1^\dagger a_2\} + \frac{1}{4} \sum_{1234}$$

s: flow parameter



H. Hergert, S. K. Bogner, T. D. Morris, A. Schwenk, and K. Tsukiyama, Phys. Rep. **621**, 165 (2016).

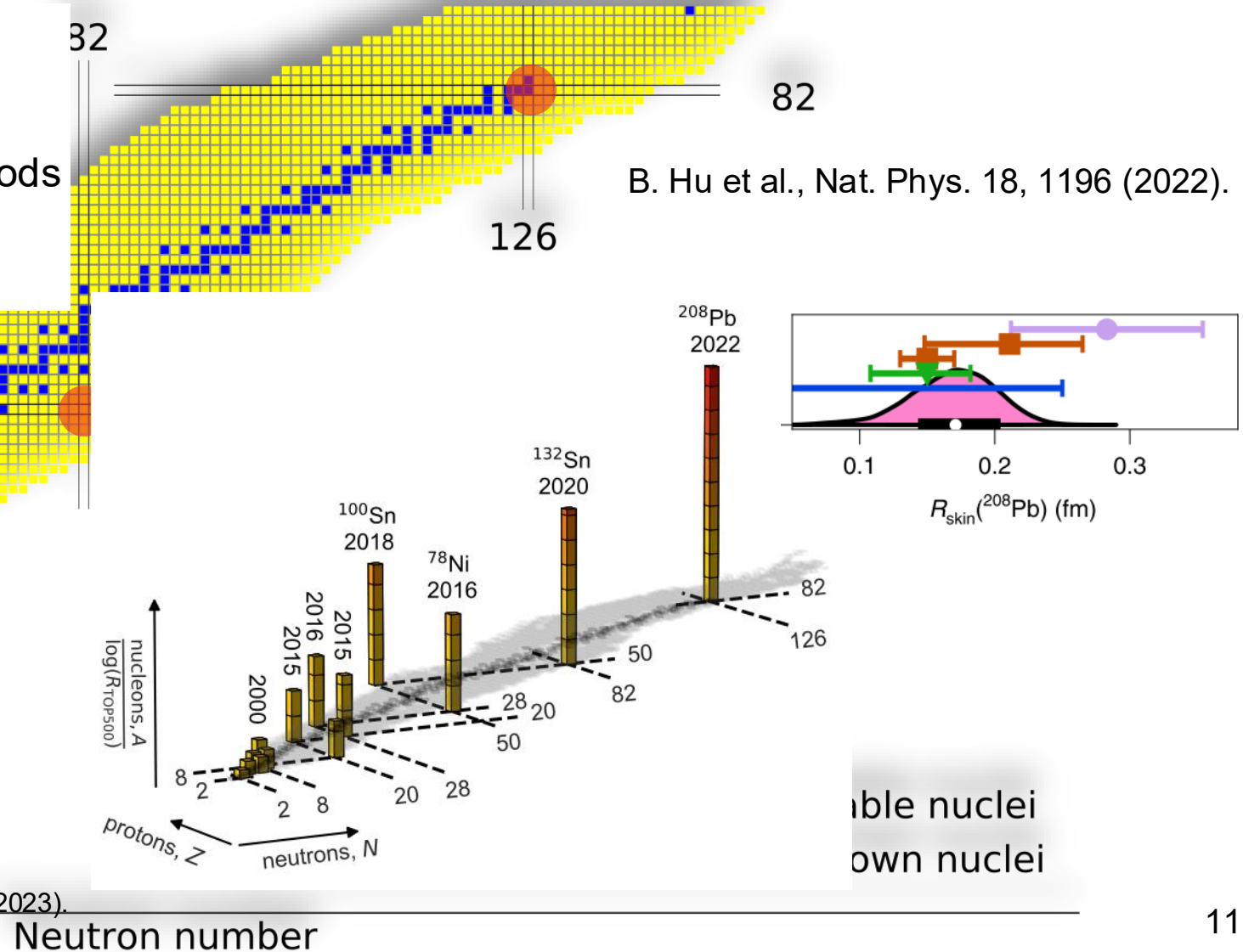
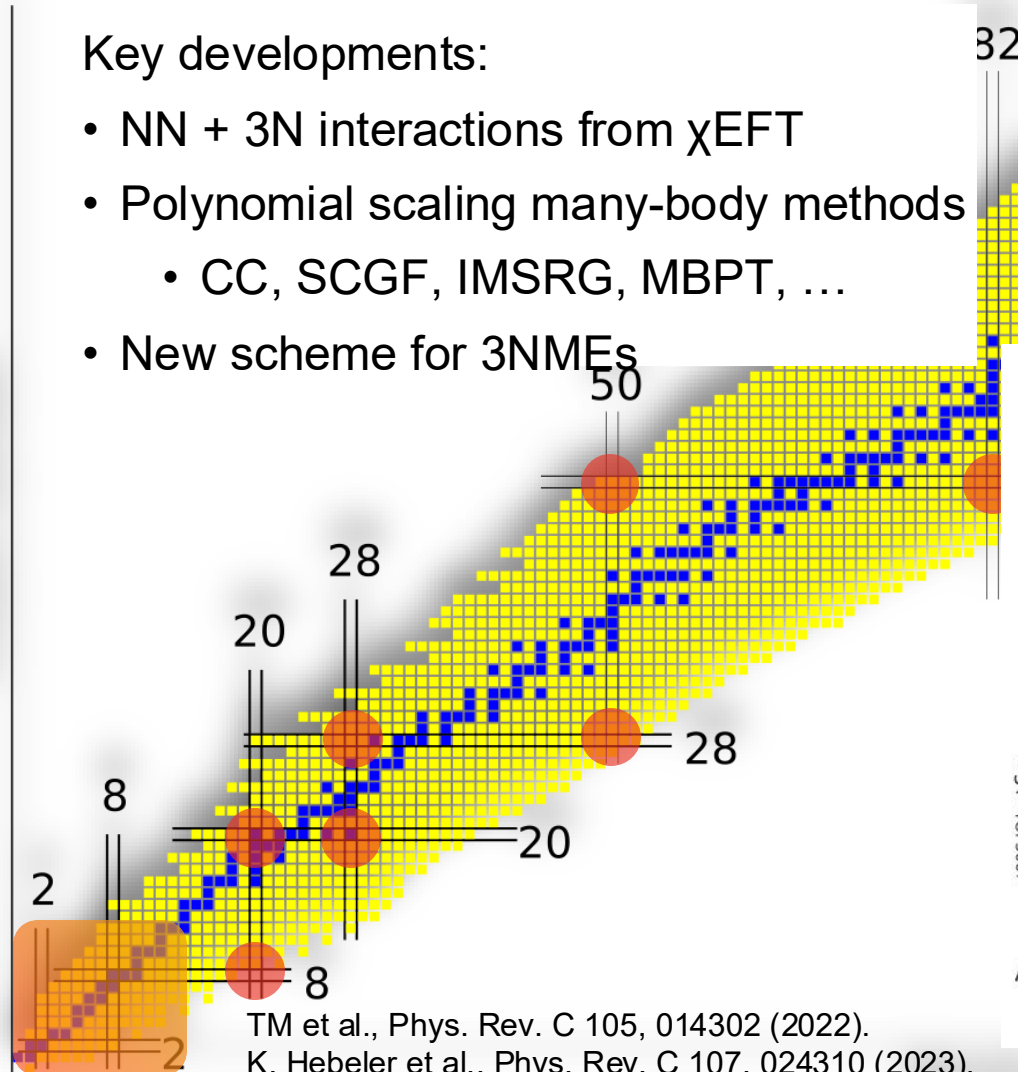
S. R. Stroberg, H. Hergert, S. K. Bogner, and J. D. Holt, Annu. Rev. Nucl. Part. Sci. **69**, 307 (2019).

# Range of applicability

Key developments:

- NN + 3N interactions from  $\chi$ EFT
- Polynomial scaling many-body methods
  - CC, SCGF, IMSRG, MBPT, ...
- New scheme for 3NMEs

Proton number

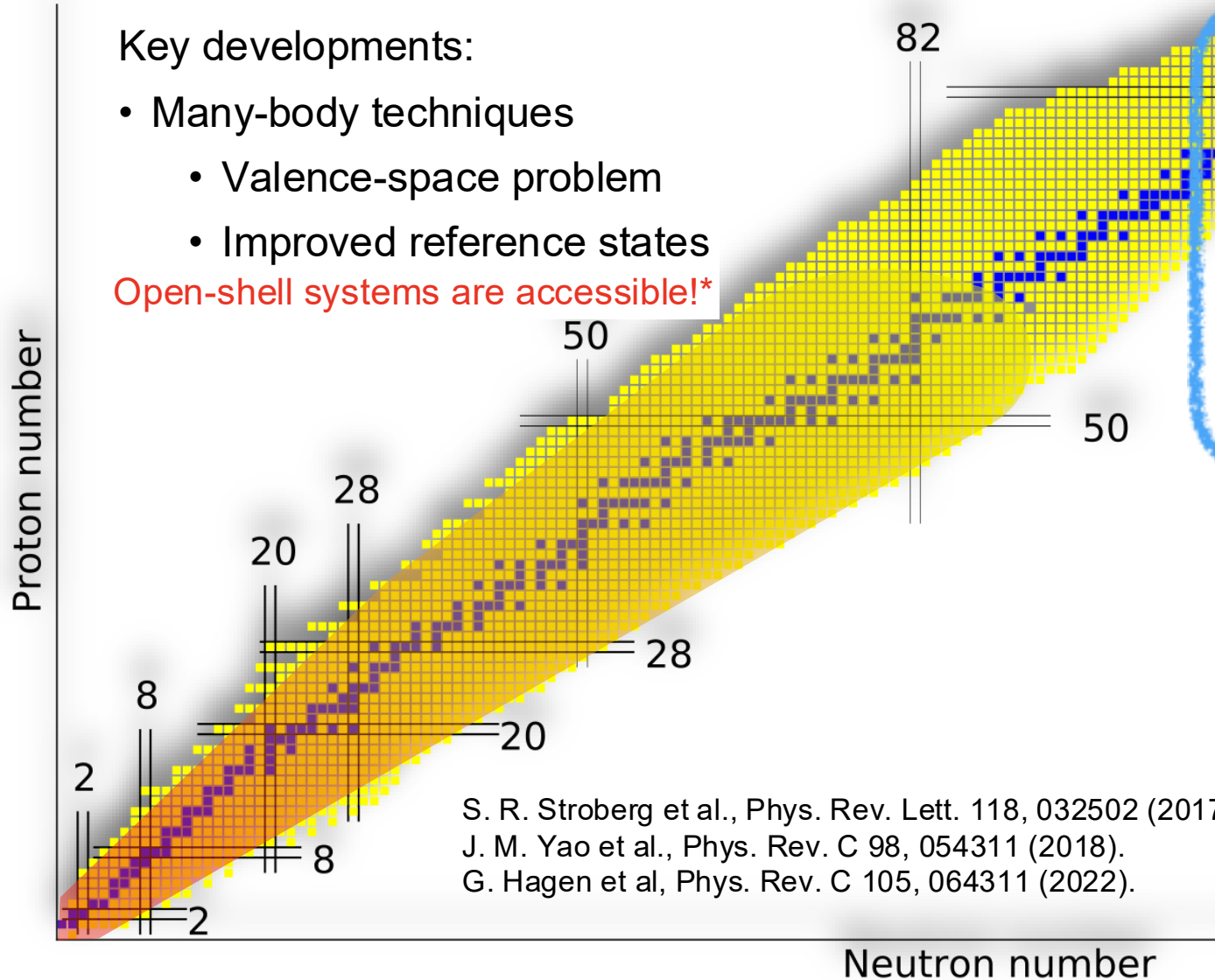


# Range of applicability

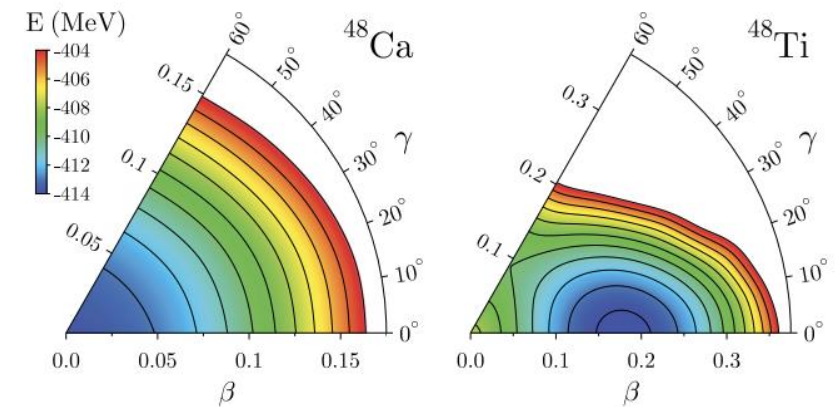
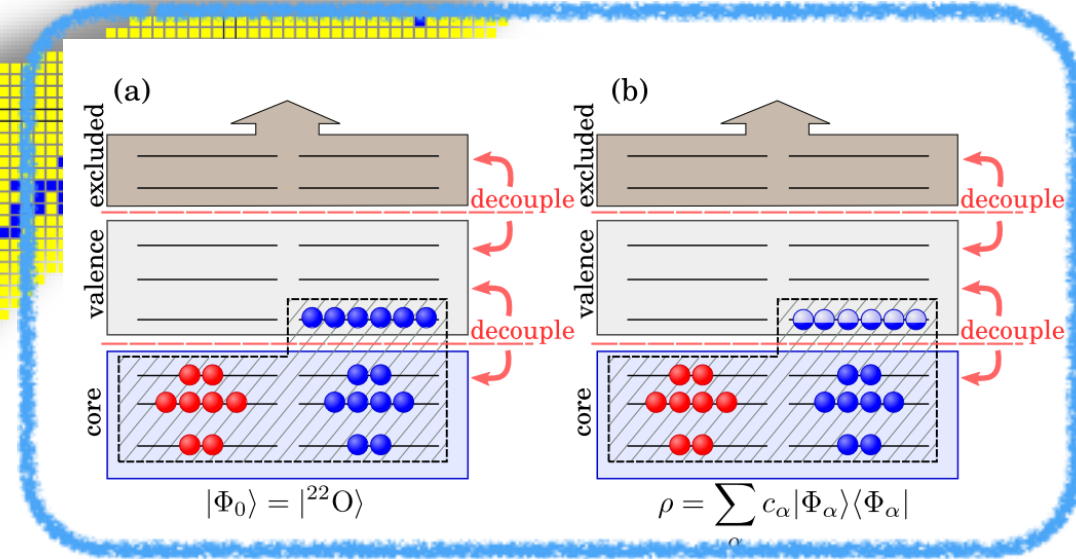
Key developments:

- Many-body techniques
  - Valence-space problem
  - Improved reference states

Open-shell systems are accessible!\*



S. R. Stroberg et al., Phys. Rev. Lett. 118, 032502 (2017).  
J. M. Yao et al., Phys. Rev. C 98, 054311 (2018).  
G. Hagen et al., Phys. Rev. C 105, 064311 (2022).

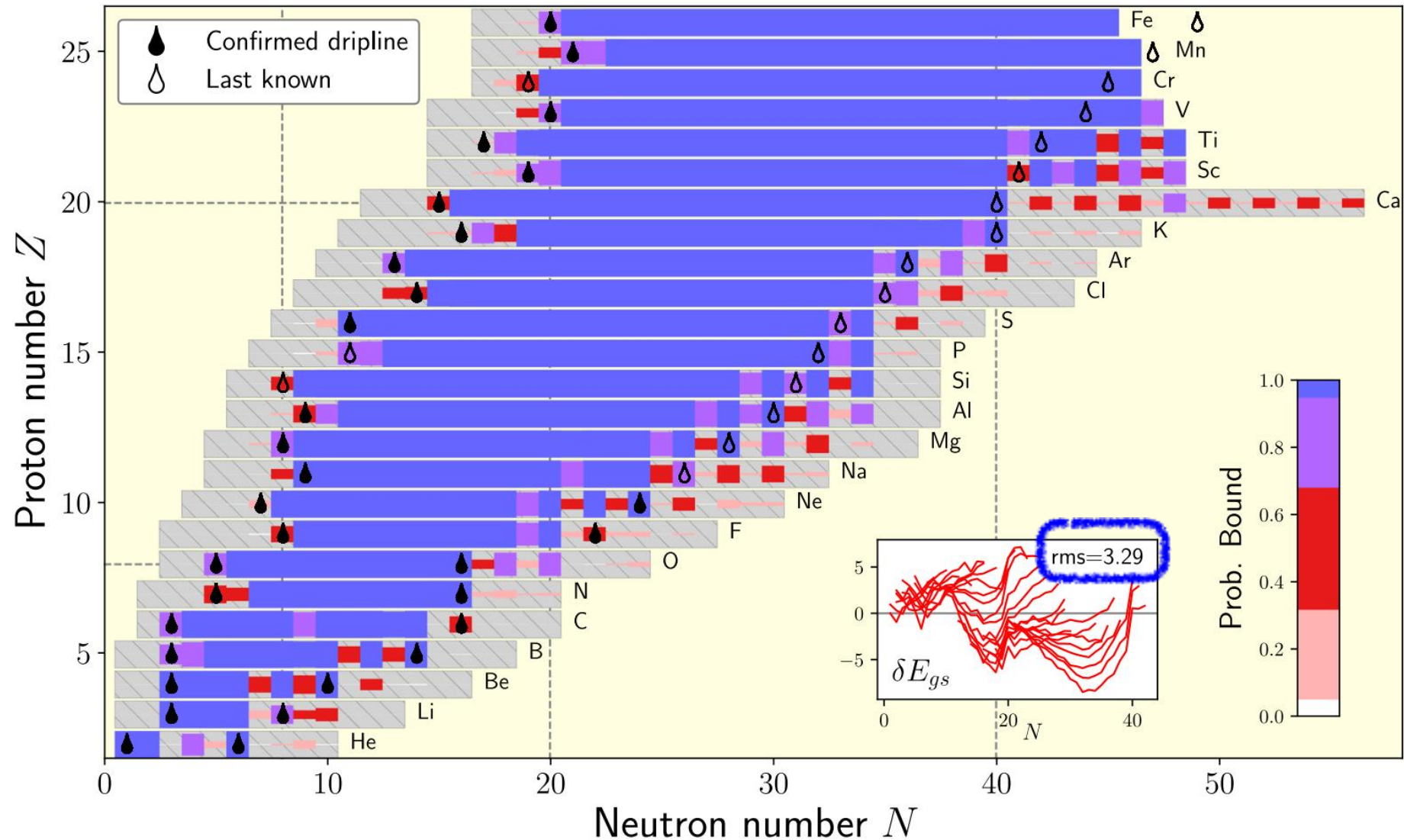


+ CC, IMSRG, ...

\*does not mean all nuclei are accessible

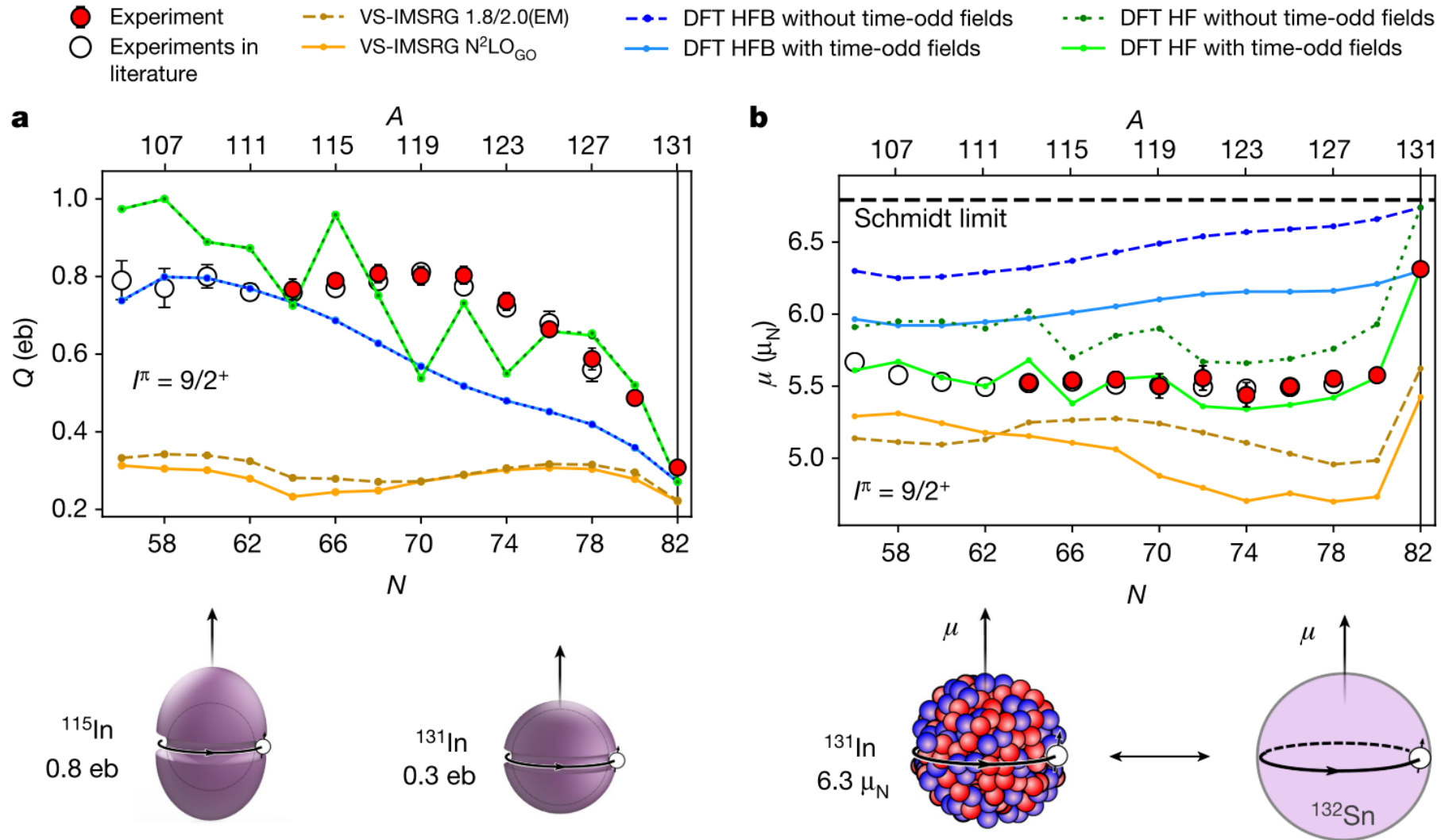
# Ground-state energies

S. R. Stroberg et al., Phys. Rev. Lett. 126, 022501 (2021).



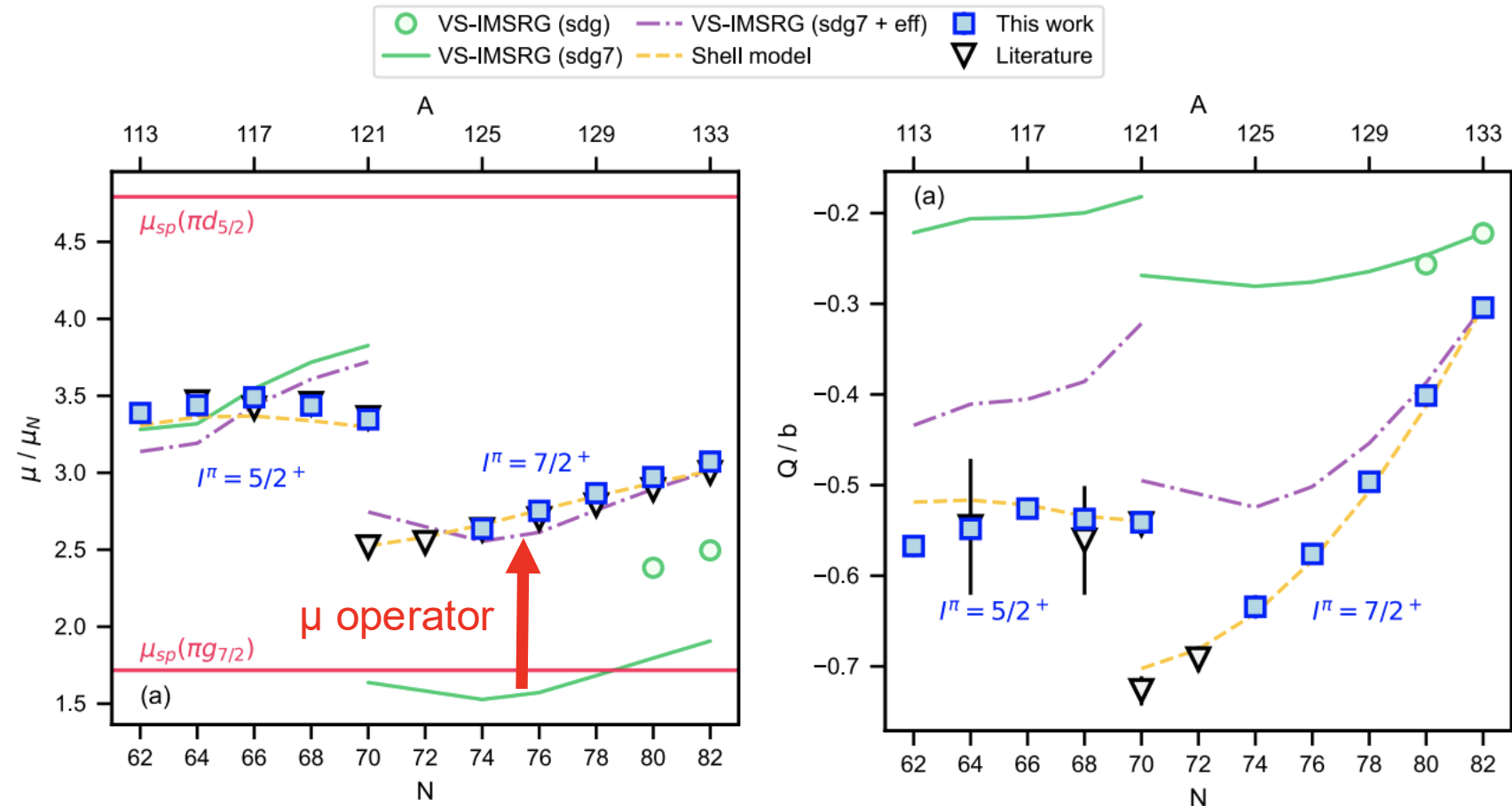


# Nuclear moments in In isotopes



# Nuclear moments in Sb isotopes

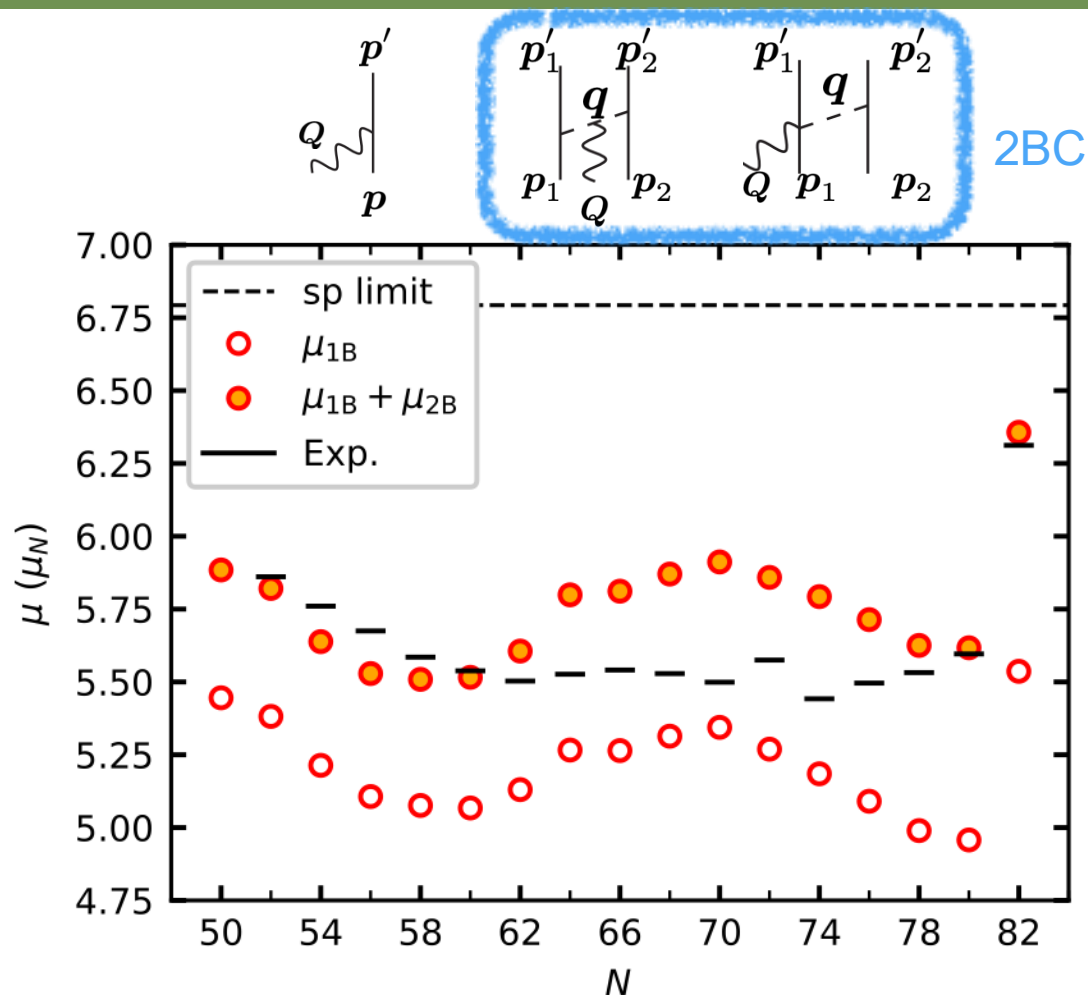
- Comparison with phenomenological CI results
- Magnetic moments
  - Operator (eff params.)
- Quadrupole moments
  - Operator (eff. params.)
  - Wave function



S. Lechner et al., Phys. Lett. B 847, 138278 (2023).

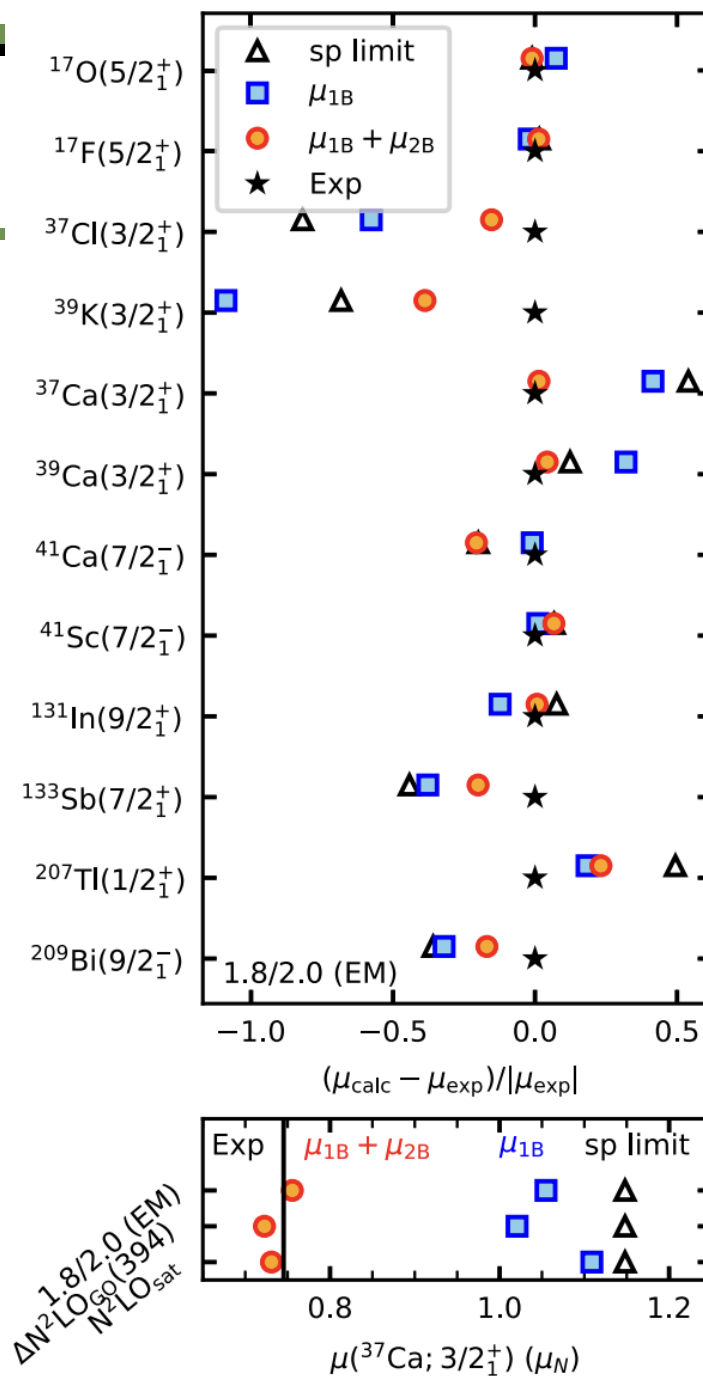


# Role of 2BCs: magnetic moments



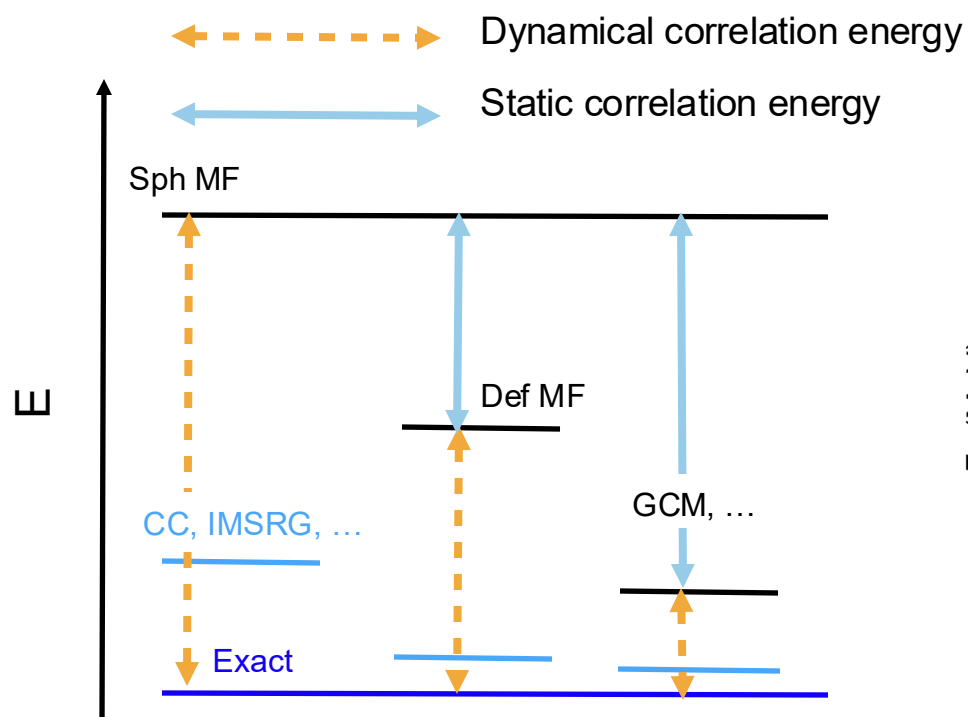
EM 2BC significantly improves magnetic moments.

TM et al., Phys. Rev. Lett. 132, 232503 (2024).

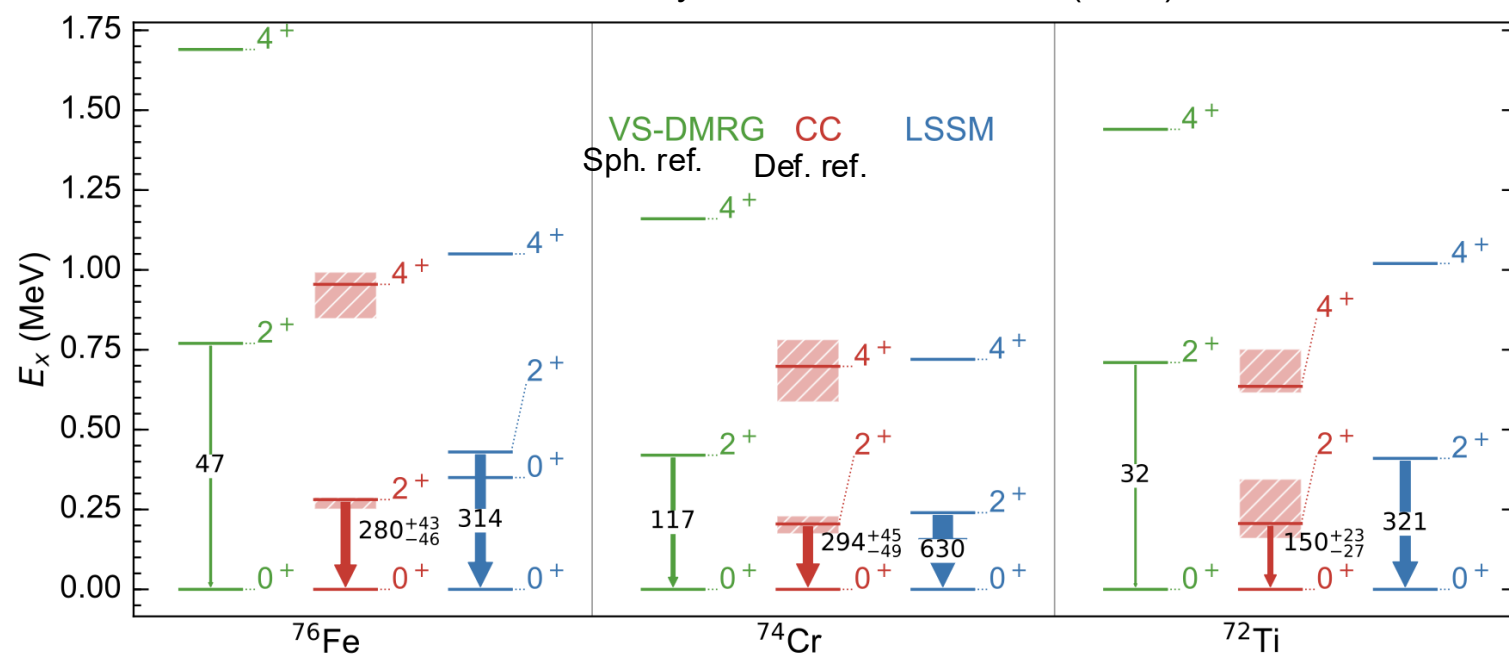


# Deformation

- Collective deformation cannot be captured by 1p1h and 2p2h excitations from the spherical reference.
- One can begin with a deformed reference.



B. S. Hu et al., Phys. Lett. B 858, 139010 (2024).



# Deformation

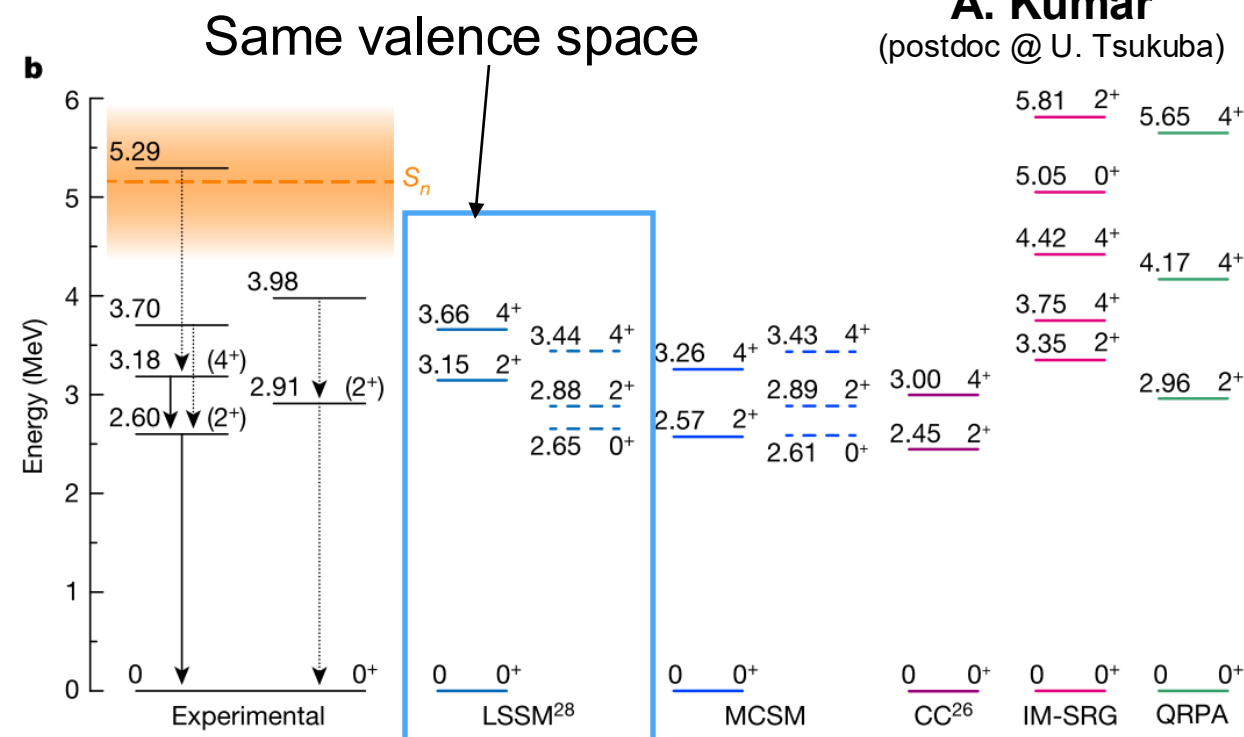
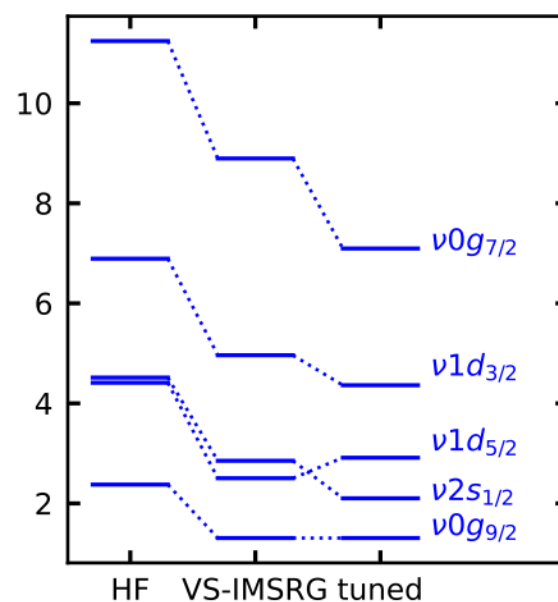
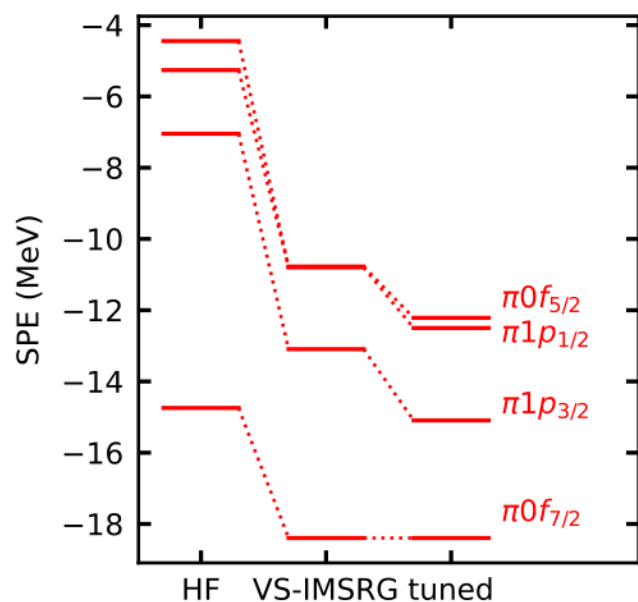


**A. Kumar**

(postdoc @ U. Tsukuba)

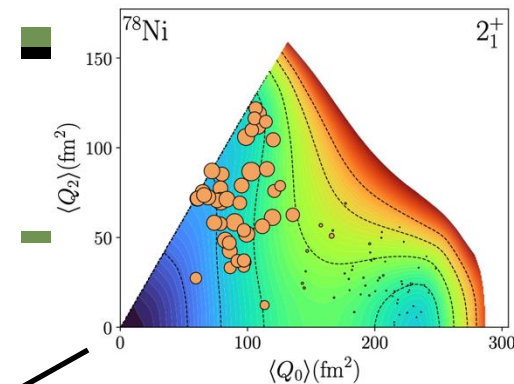
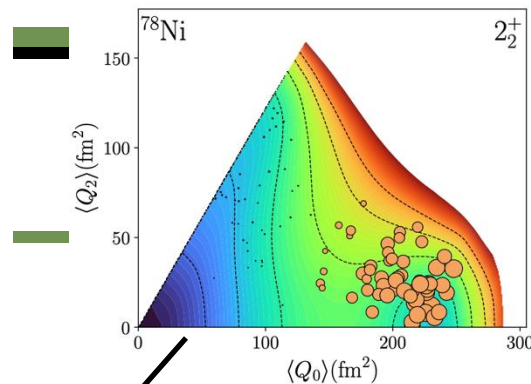
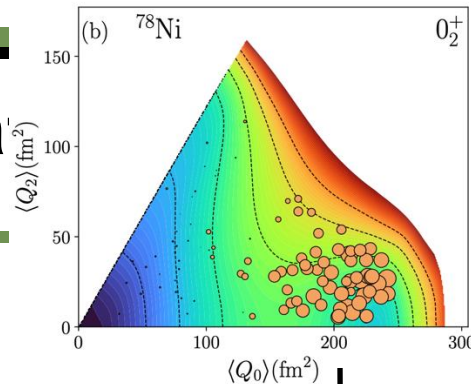
- A possible way to improve See also J. A. Purcell et al., Phys. Rev. C 111, 044313 (2025).

- ◆ As a first step, try to tune single-particle energies.
- ◆  $^{79}\text{Cu}$  and  $^{79}\text{Zn}$  low-lying spectra



R. Taniuchi et al., Nature 569, 53 (2019).

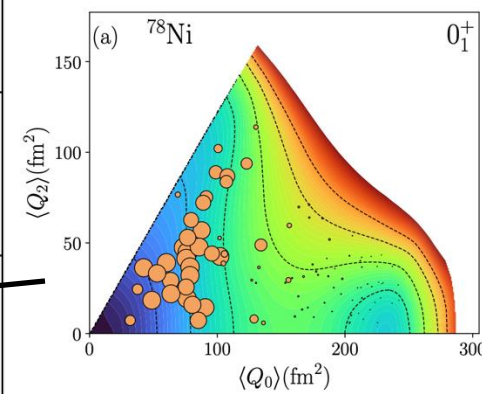
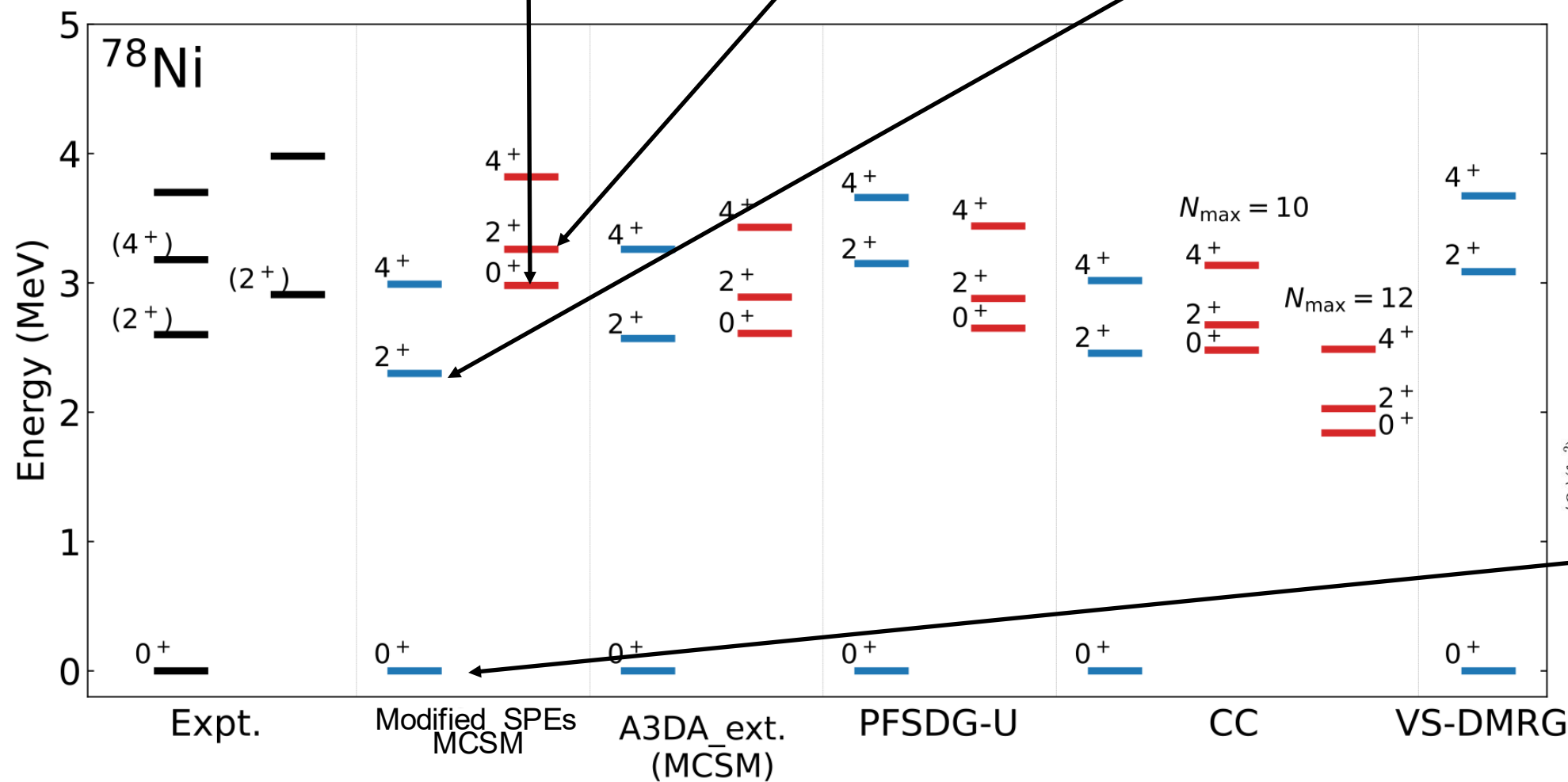
# Deforma



筑波大学  
計算科学  
Center for



**A. Kumar**  
(postdoc @ U. Tsukuba)

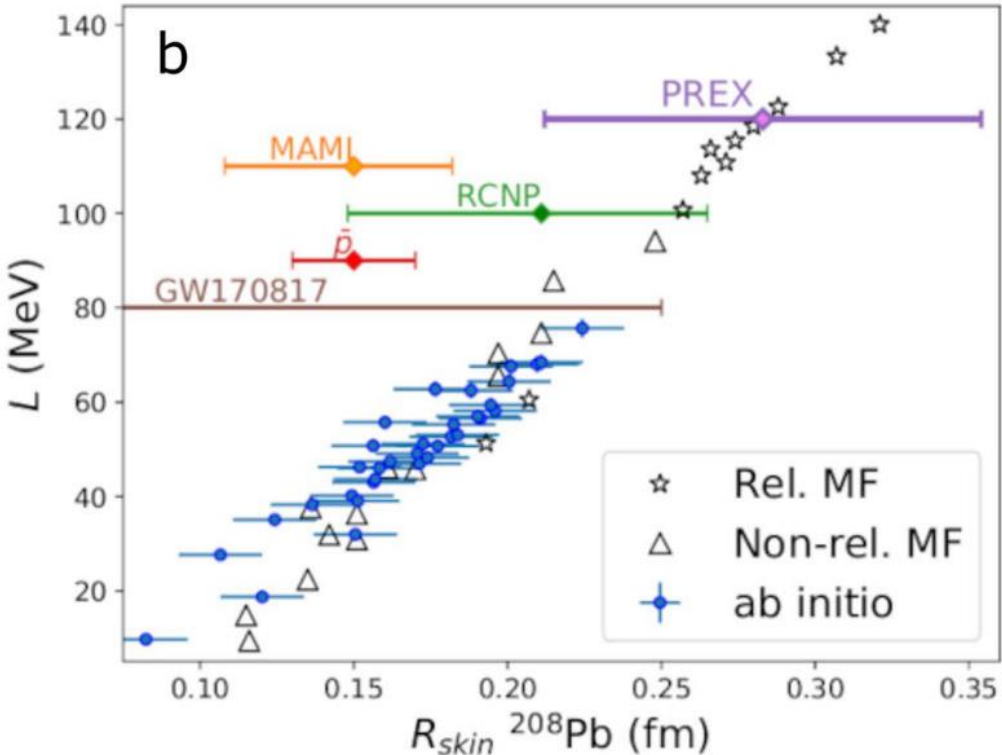


- Developments in ChEFT and many-body techniques enable us to perform nuclear ab initio calculations.
  - ◆ Heavier systems
  - ◆ Open-shell systems
- EM 2BCs are needed for the magnetic moment reproductions.
- E2 observables are significantly underestimated
  - ◆ Improvements in many-body methods are required.
- It seems a posteriori SPE tuning captures the deformation.
  - ◆ Perturbative correction of SPEs may solve the issue?





# Neutron skin of $^{208}\text{Pb}$



| Observable                         | Neutron skins |                |                |
|------------------------------------|---------------|----------------|----------------|
|                                    | median        | 68% CR         | 90% CR         |
| $R_{\text{skin}}(^{48}\text{Ca})$  | 0.164         | [0.141, 0.187] | [0.123, 0.199] |
| $R_{\text{skin}}(^{208}\text{Pb})$ | 0.171         | [0.139, 0.200] | [0.120, 0.221] |

B. Hu, W. Jiang, T. Miyagi, et al., Nat. Phys. 18, 1196 (2022).

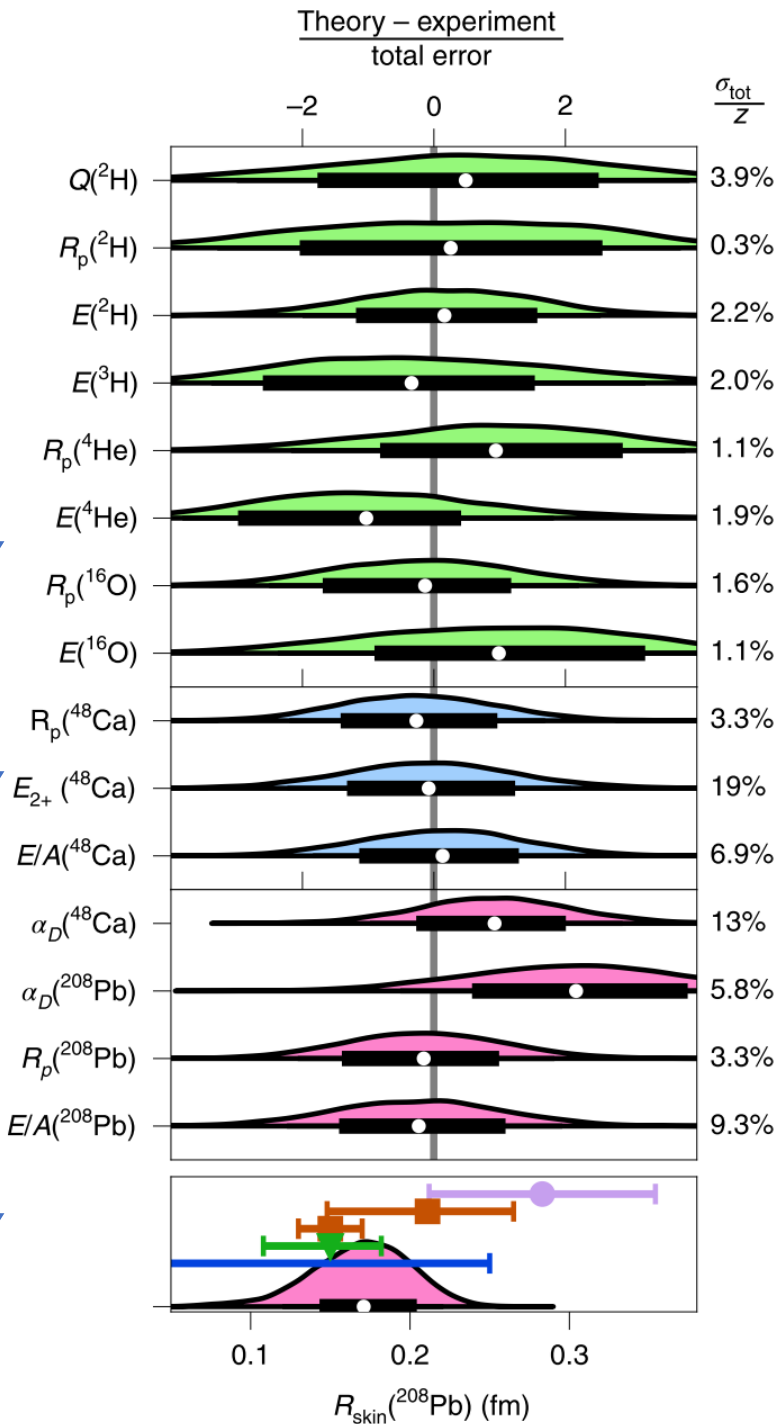
## History matching

17 parameters  
NN scattering  
few-body +  $^{16}\text{O}$   
 $10^9 \rightarrow 34$

## Calibration

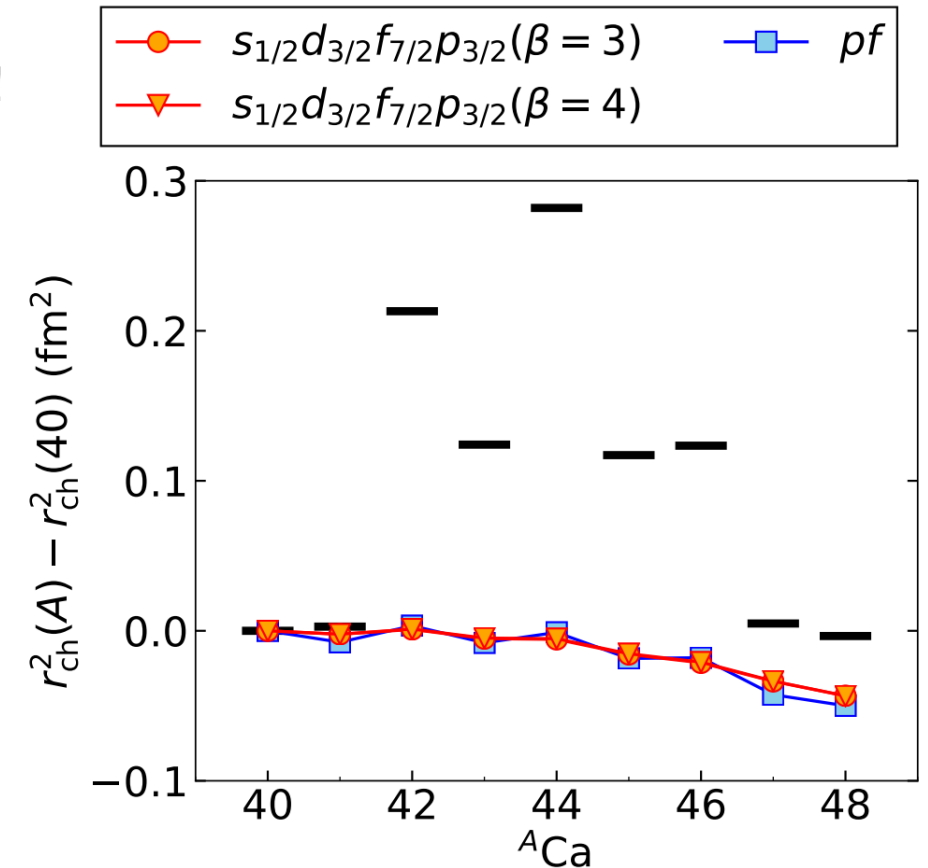
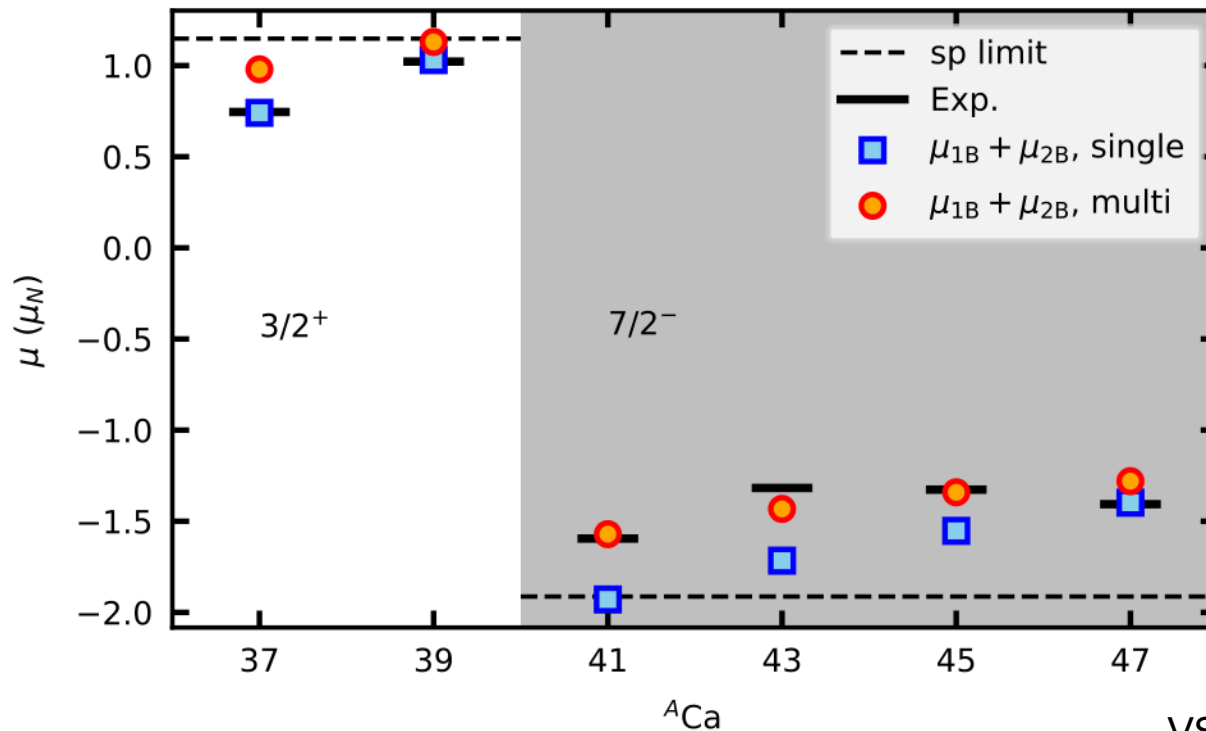
Sampling/Importance  
resampling

## Validation Prediction



# Is $^{40}\text{Ca}$ magic?

- 2BC makes agreement worse.
- Activating the  $^{40}\text{Ca}$  core explains the magnetic moments better.
- The radii are not explained. Further investigations are needed!



# Mass dependence of 2B contribution

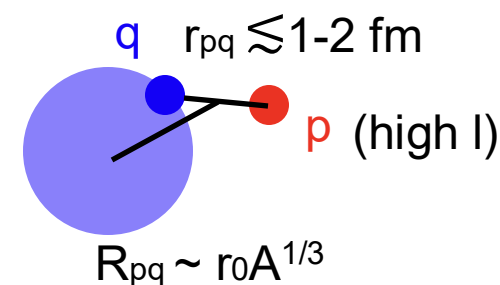
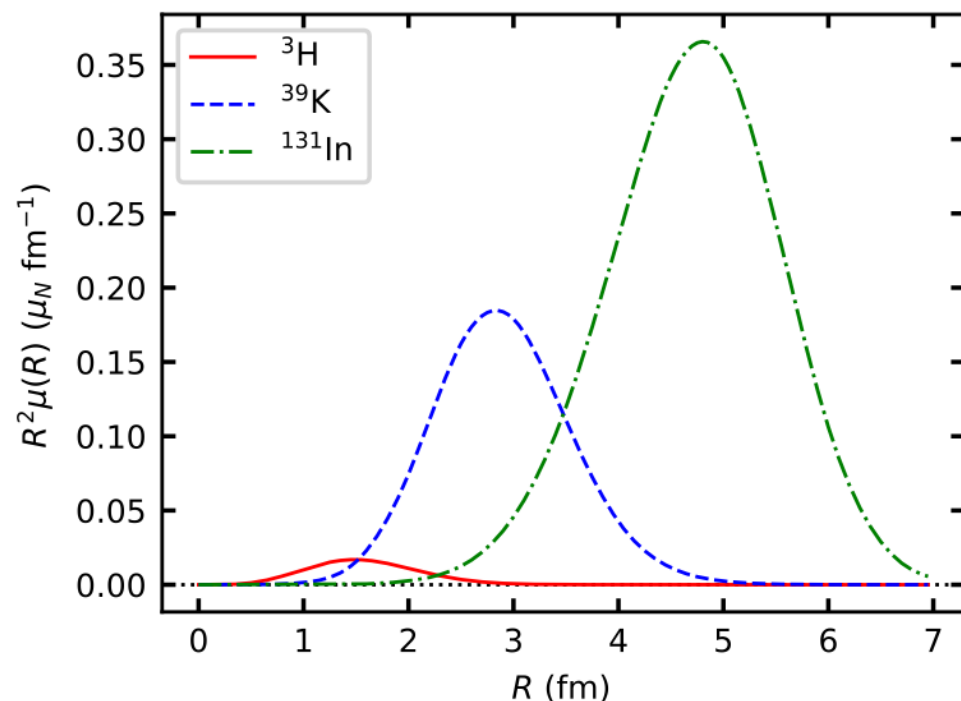
- The size of 2BC contribution is larger in heavier systems.
- The simplest configuration limit is  $0^+$  core + 1 particle (or hole)

$$\langle J || \mu || J \rangle \sim \sum_{q \in \text{core}} \sum_I f(j_p, j_q, I) \langle pq : I || \mu || pq : I \rangle$$

$$\mu^{2B} = \mu_{\text{intr}}^{2B} + \mu_{\text{Sachs}}^{2B} \quad \mu_{\text{Sachs}}^{2B} \propto \sum_{i < j} (\mathbf{R}_{ij} \times \mathbf{r}_{ij}) V^{\text{OPE}}(r_{ij})$$

Dominant in heavy systems

- In  $r \lesssim 1-2$  fm because of high exchange potential



$$\mu_{pq}^{\text{Sachs}} \propto (\mathbf{R}_{pq} \times \mathbf{r}_{pq}) V^{\pi}(r_{pq})$$

The peak position moves to larger  $R$  for heavier systems.