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The largest periodic table in Europe

Volga River Embankment

Low-energy spectra of nobelium isotopes: Skyrme random-phase-approximation analysis

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[4]*Institute of Physics, Slovak Academy of Sciences, 84511 Bratislava, Slovakia*

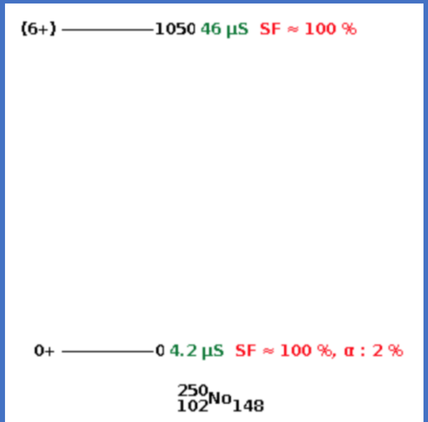
2025

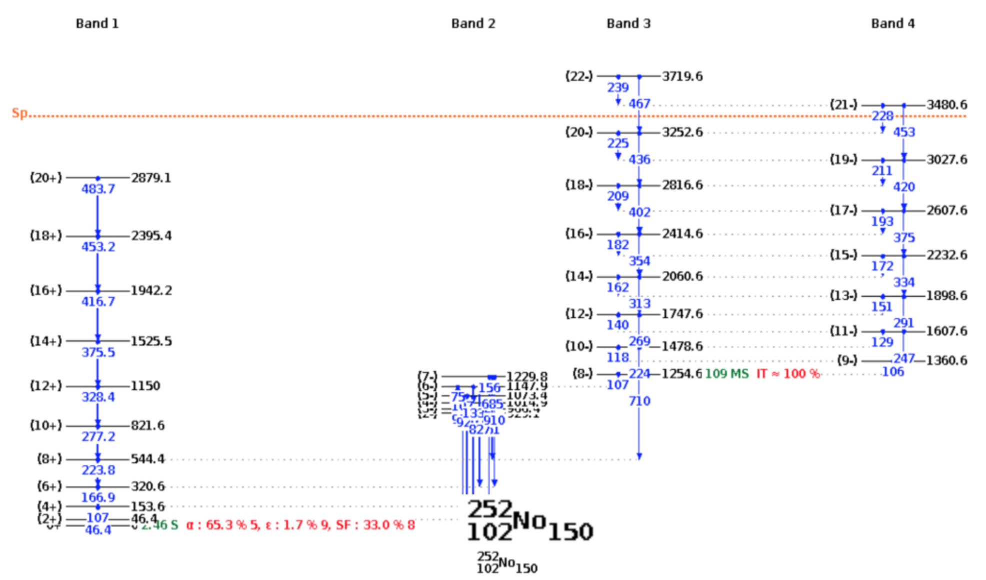
Spectroscopy of superheavy nuclei is hot research area

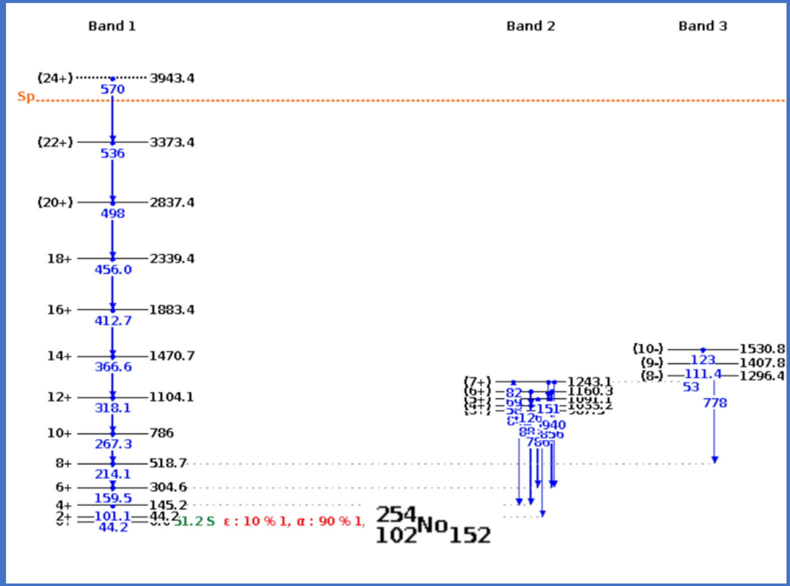
The largest periodic table in Europe(Dubna)

Perhaps, the most extensive experimental data are collected for transfermium region, in particular for nobelium isotopes

At the moment, there are experimental* spectroscopic data only for 3/7 nuclei: ^{250,252,254}No







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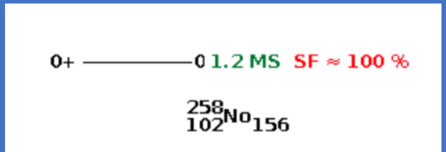
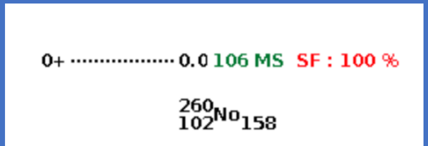
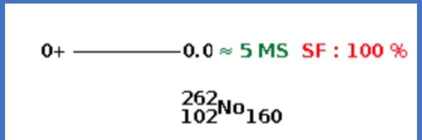


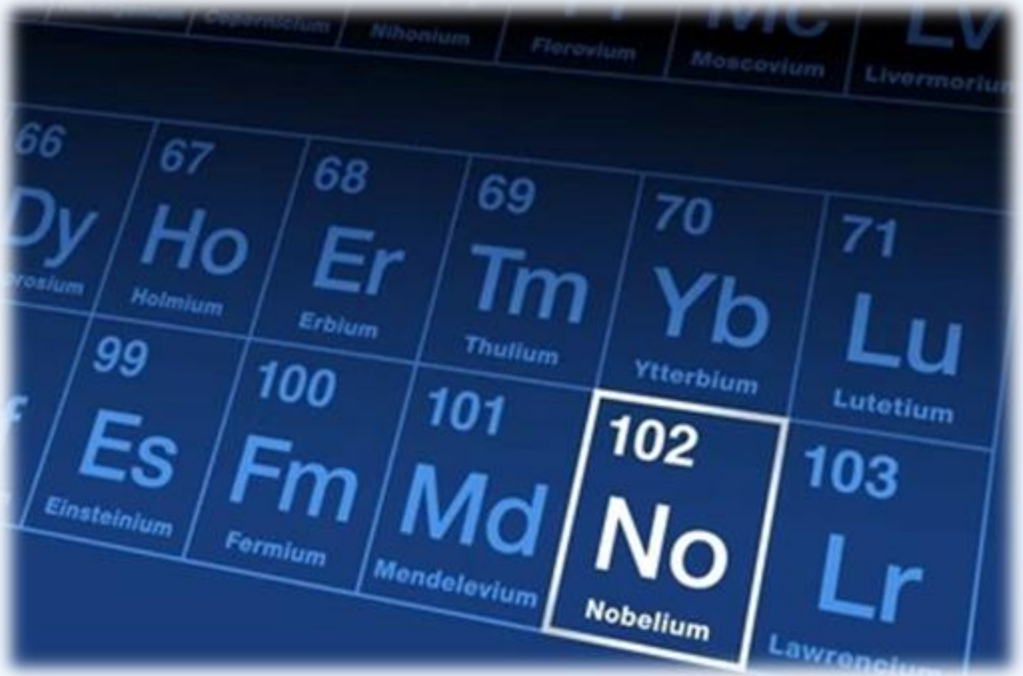




* NNDC data base

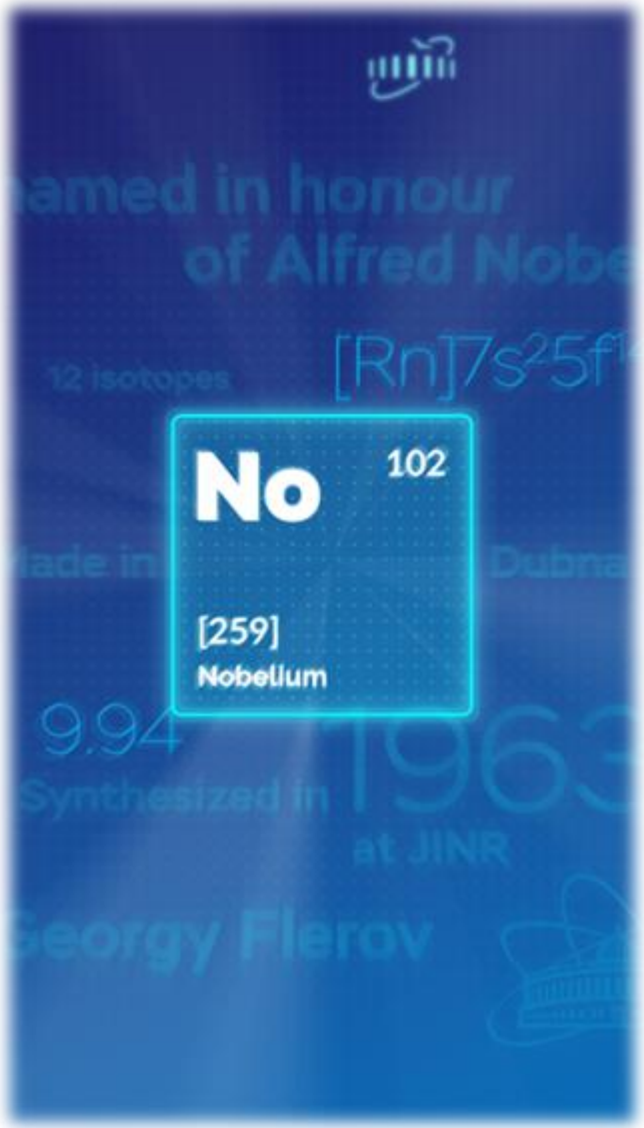
The chain of even-even Nobelium nuclei is one of the most studied superheavy nuclei:

- The low-lying spectrum of ^{250,252,254}No;
- Quadrupole moment of ^{252,254}No
- Dipole giant resonance in ^{252,254}No;
- The scissors mode of $^{250-256}$No;
- Single-particle properties and rotational bands in the ^{252,254}No;
- Spontaneous fission for the nuclei $^{250\text{-}260}$No



But, despite the great interest in these nuclei, the characteristics of the ground state of these isotopes are still poorly studied…

Despite an impressive theoretical effort:



- Even modern self-consistent models still give rather different results and exhibit troubles in description of shell structures and other features seen in experiment

- This work was partly done within QPM, IBM, double nuclear system, and cluster models (however, the above models are not self-consistent)

- **It is worth to enlarge the scope of calculated characteristics of superheavy nuclei and inspect, within the same self-consistent theory, a full set of low-energy vibrational states of main multipolarities:**
  $K^{\pi} = 0^{+}, 2^{+}, 3^{+}, 0^{-}, 1^{-}, 2^{-}, 8^{-}$

250No	251No	252No	253No	254No	255No	256No	257No	258No	259No	260No
4.6 μs	0.8 s	2.45 s	1.61 min	51.2 s	3.52 min	2.93 s	24.5 s	1.23 ms	58 min	107 ms
SF=100%	α=90%	α=65.3%	α=55%	α=90%	ε+β+=70%	α=99.47%	α=85%	SF=100%	α=75%	SF=100%
	SF=1.4e-3%	SF=33%	ε+β+=45%	ε=10%	α=30%	SF=0.53%	ε=15%		ε=25%	
	ε+β+>0%	ε+β+=1.7%		SF=0.17%			SF<1.5%		SF<10%	

The main attention is paid to ^{252,254}No where calculated:

- **K^π= 8$^-$ isomers**
  (at 1.361 MeV in ^{252}No and 1.747 MeV in ^{254}No)
- **Pairing vibrations K^π = 0$^+$**
- **States K^π=2$^+$**
- **Hexadecapole states with K^π = 3$^+$ and 4$^+$**
- **Octupole states with K^π = 0$^-$, 1$^-$, 2$^-$ and 3$^-$**

250No 4.6 μs SF=100%	251No 0.8 s α=90% SF=1.4e-3% ε+β+>0%	252No 2.45 s α=65.3% SF=33% ε+β+=1.7%	253No 1.61 min α=55% ε+β+=45%	254No 51.2 s α=90% ε=10% SF=0.17%	255No 3.52 min ε+β+=70% α=30%	256No 2.93 s α=99.47% SF=0.53%	257No 24.5 s α=85% ε=15% SF<1.5%	258No 1.23 ms SF=100%	259No 58 min α=75% ε=25% SF<10%	260No 107 ms SF=100%

The main attention is paid to ^{252,254}No where calculated:

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- **Pairing vibrations K^{π} = 0^+**
- **States K^{π}=2^+**
- **Hexadecapole states with K^{π} = 3^+ and 4^+**
- **Octupole states with K^{π} = 0^-, 1^-, 2^- and 3^-**

Skyrme forces

force	m/m*	kind of pairing
SVbas	0.90	surface
SkM*	0.79	volume
SLy6	0.69	volume

[P. Klupfel et al, PRC 79 034310 (2009)]

[J. Bartel et al, NPA 386, 79 (1982)]

[E. Chabanat et al, NPA, 635 231 (1998)]

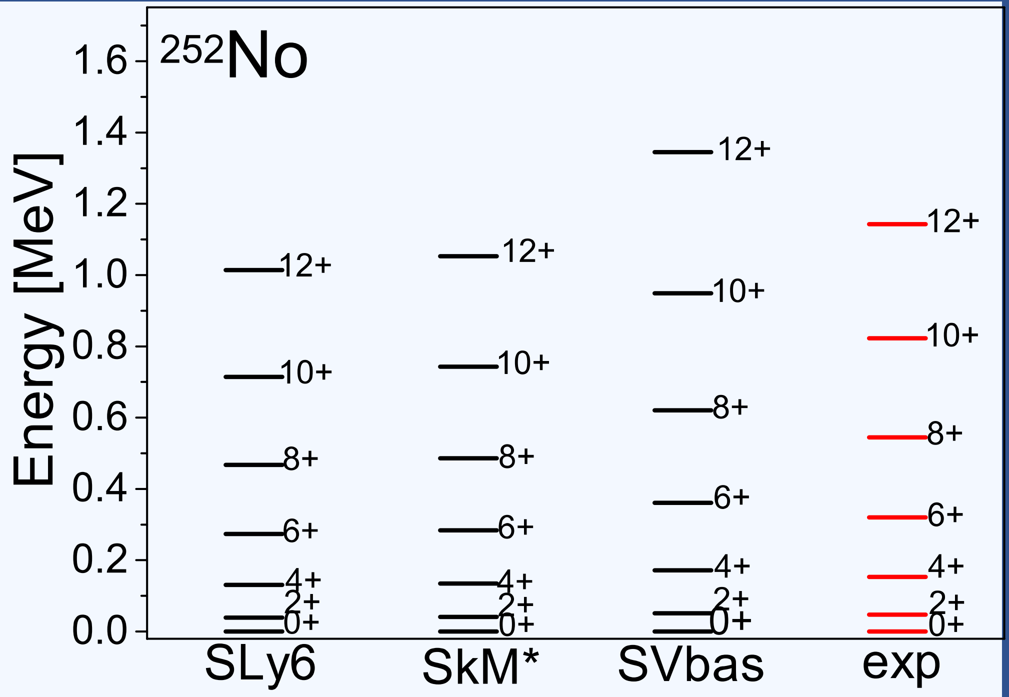
$$V^q_{\rm pair}(\mathbf{r},\mathbf{r}') = G_q\left[1-\eta\left(\frac{\rho(\mathbf{r})}{\rho_{\rm pair}}\right)\right]\delta(\mathbf{r}-\mathbf{r}')$$

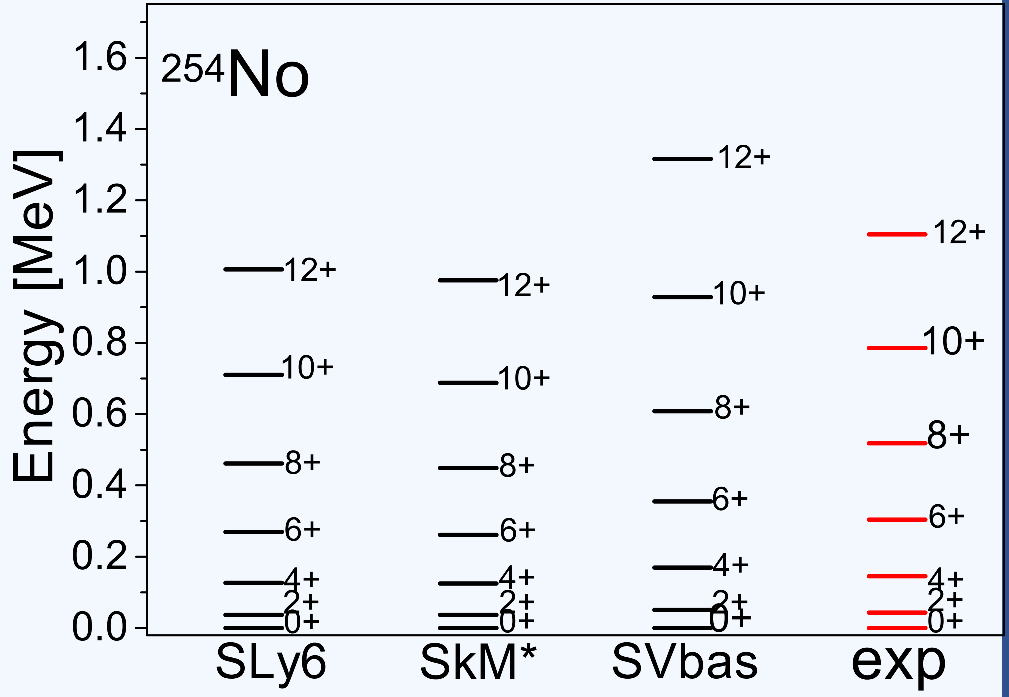
Where G_q are pairing strength constants ($q = p,n$).
We get so-called density-dependent surface pairing for η = 1 and volume pairing for η = 0

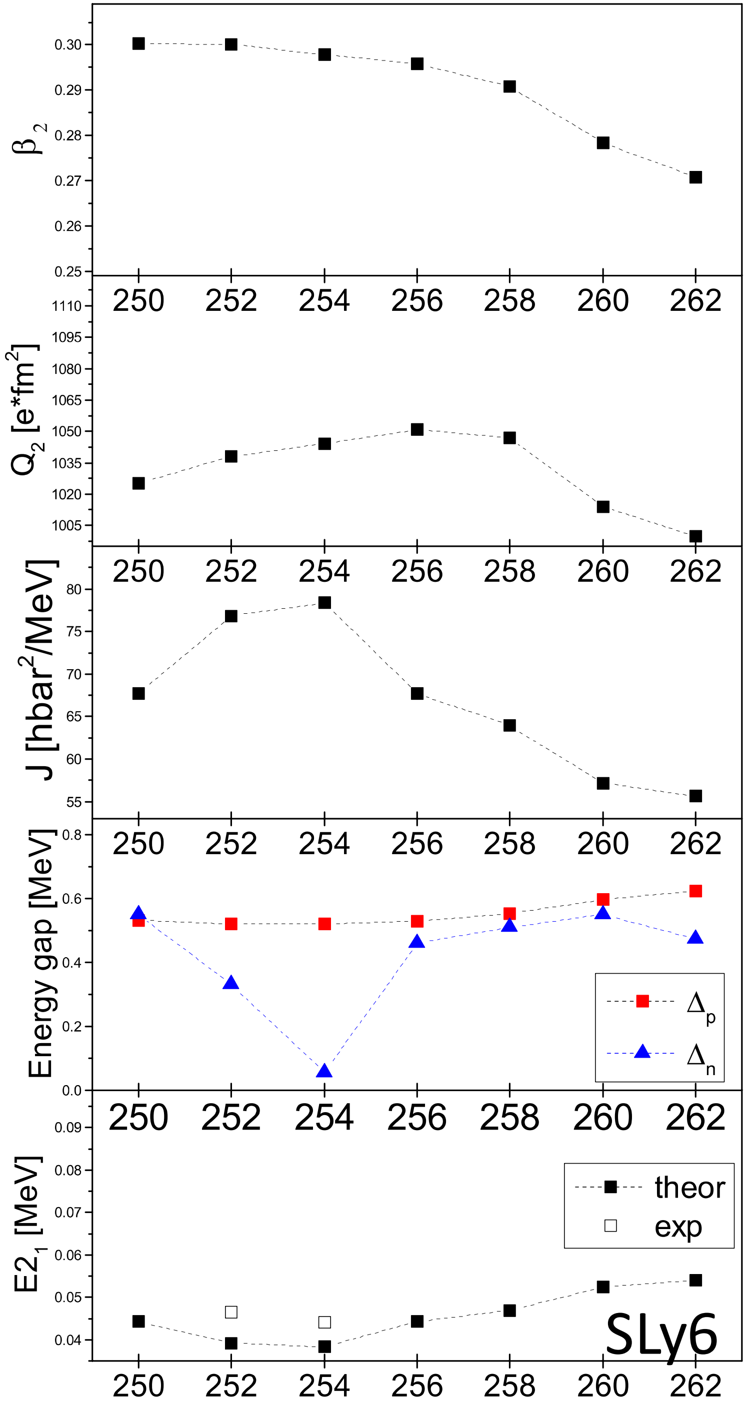
Calculation details:

Codes – SkyAx [P.-G. Reinhard et al, Comp. Phys. Communic. 258, 107603 (2021)]
QRPA [A. Repko et al, arXiv:1510.01248 (nucl-th), 2015]

- Accurate extraction of spurious admixtures
  [V. O. Nesterenko et al, Eur. Phys. J. A 55, 213 (2019)]
- 2D grid in cylindric coordinates
- All proton and neutron s-p levels up to +40 MeV







The characteristics of the ground states of $^{250-262}$No with increasing number neutrons

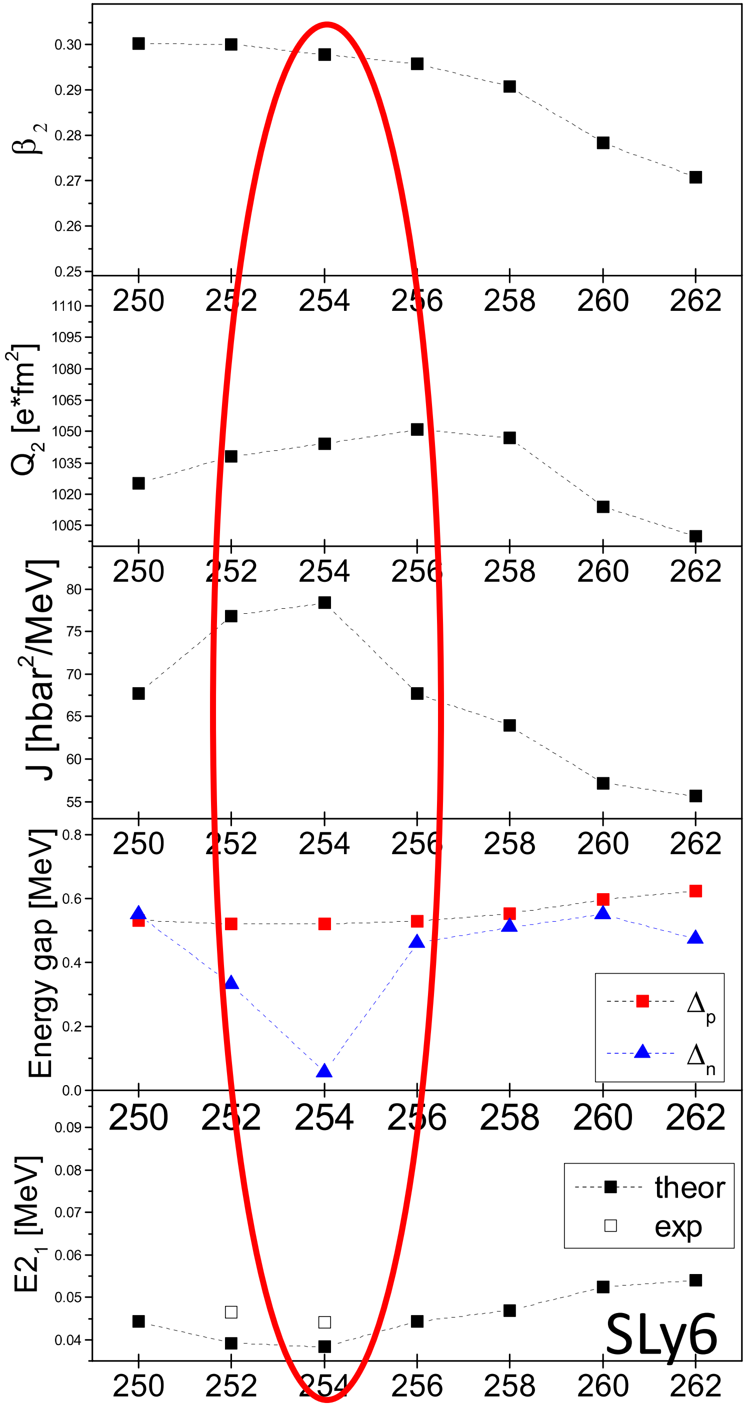
$$\beta_{20} = \frac{4\pi}{3}\frac{Q_{20}}{AR^2}\ ,\ R = R_0 A^{1/3}\ ,\ R_0 = 1.2\ \mathrm{fm}$$

$$J_{TV} = 2\sum_{\nu>0}\frac{|<\nu|J_x|0>|^2}{E_\nu - E_0}$$

$$V^q_{\mathrm{pair}}(\mathbf{r},\mathbf{r}') = G_q\left[1-\eta\left(\frac{\rho(\mathbf{r})}{\rho_{\mathrm{pair}}}\right)\right]\delta(\mathbf{r}-\mathbf{r}')$$

$$E_I = \frac{\hbar^2}{2\mathcal{J}}I(I+1),$$

Initially it was assumed that these characteristics would evolve monotonically, but we see irregularity at $^{252,\ 254}$No



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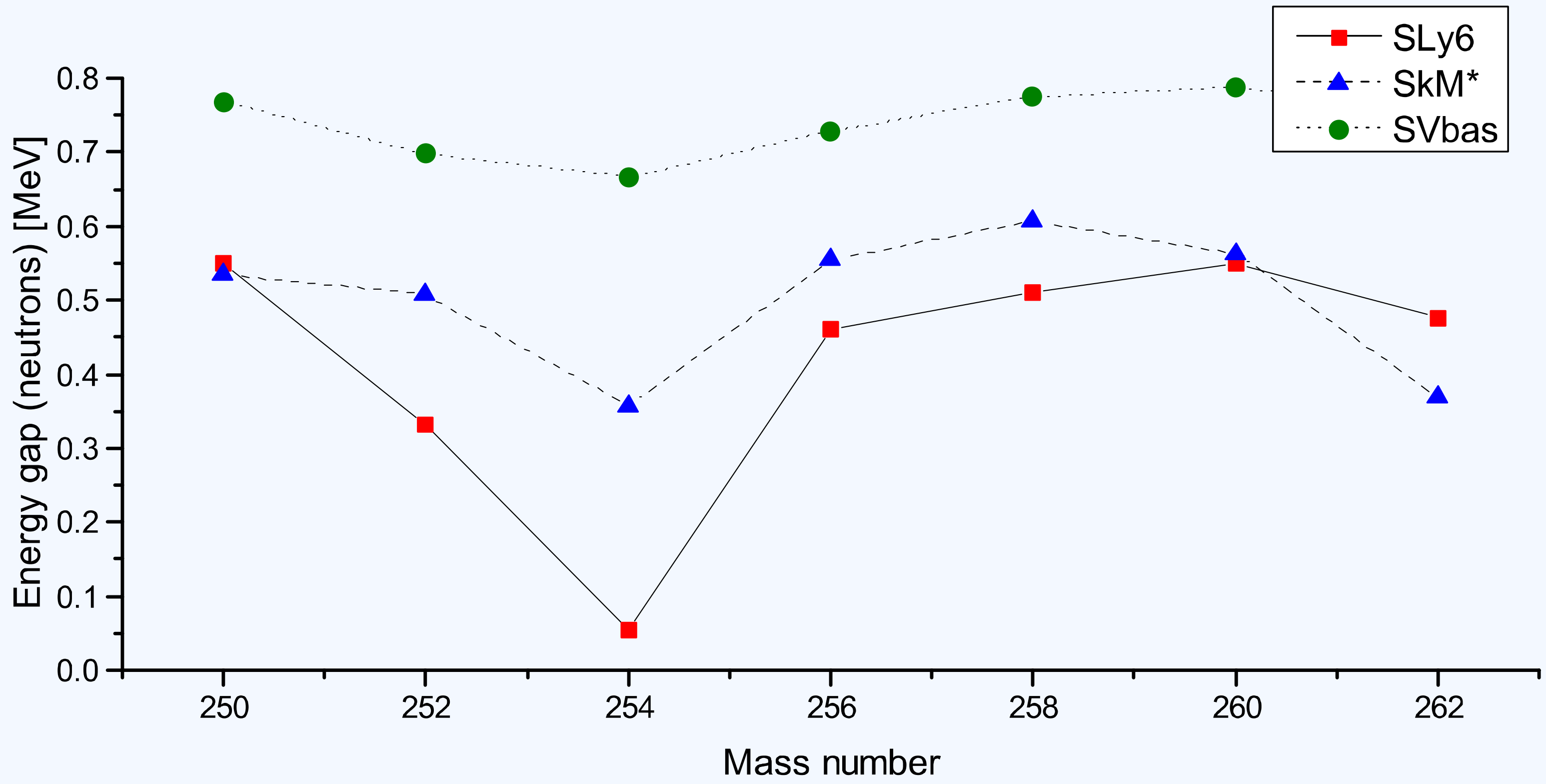
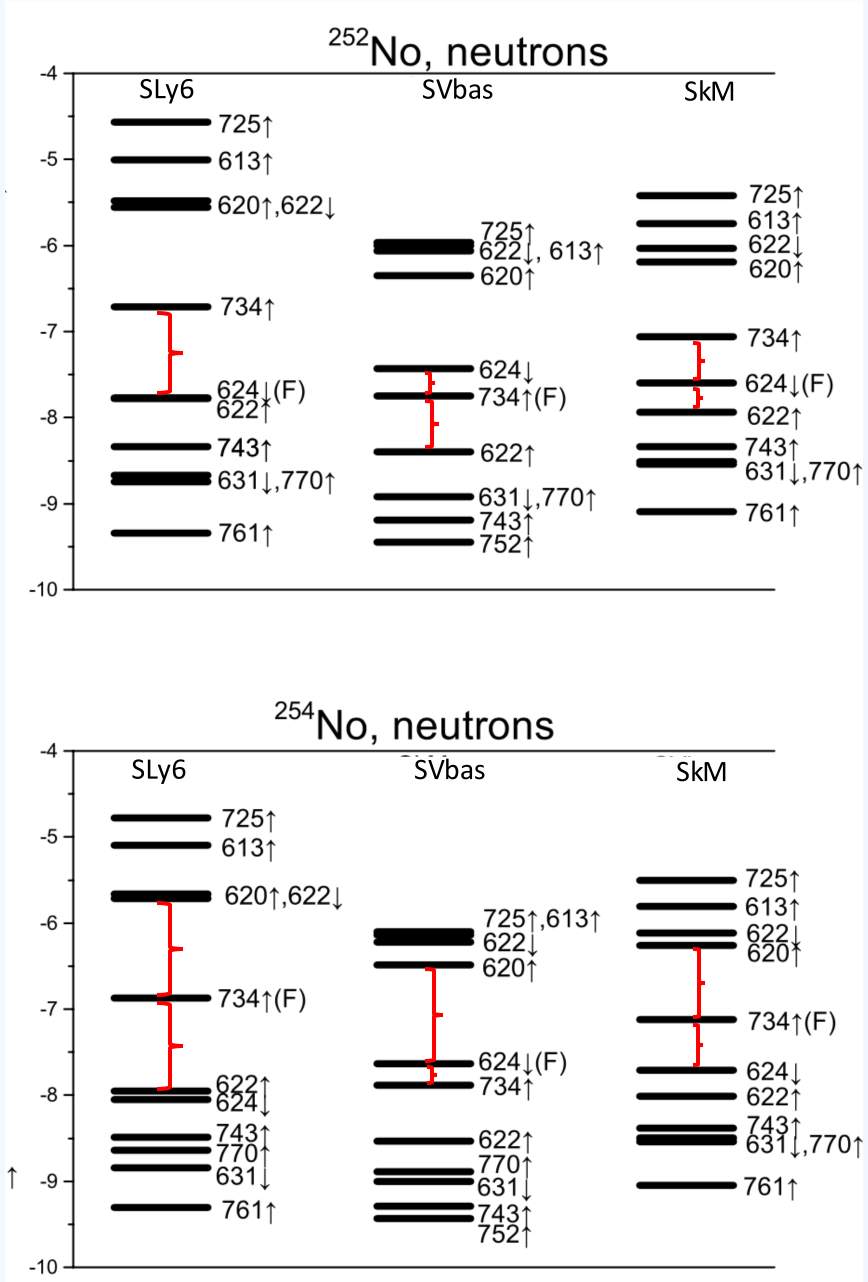
Does this irregularity (decline in neutron pairing) depend on the chosen Skyrme force?





- All 3 Skyrme forces support this irregularity;
- For ^{254}No the Fermi level isolated, so neutron pairing almost disappears;
- For ^{252}No the Fermi level is also quite far from neighboring levels, so pairing is also poorly developed

K^{π}= 8^- isomers

^{252}No: the 8^- state is usually assigned as neutron 2qp configuration nn[734 ↑, 624 ↓]

- R.-D. Herzberg and P.T. Greenlees, Prog. Part. Nucl. Phys. 61, 674 (2008)
- F.P. Heßberger, arXiv:2309.10468v2[nucl-ex].
- B. Sulignano et al, Eur. Phys. J. A 33, 327 (2007).

Force	$E_{\nu=1}$ [MeV]	$B(E98)$ [W.u.]	qq'	$\epsilon_{qq'}$ [MeV]	$N_{qq'}$	F-scheme
^{252}No, E_x=1.254 MeV						
SLy6	1.361	0.038	$nn[624\downarrow, 734\uparrow]$	1.317	0.996	F,F+1
SkM*	1.330	0.025	$nn[734\uparrow, 624\downarrow]$	1.198	0.992	F,F+1
SVbas	1.913	0.119	$nn[624\downarrow, 734\uparrow]$	1.751	0.912	F,F+1

Features of calculated 8^- states in ^{252,254}No:
QRPA excitation energies E_ν =1,
reduced transition probabilities B(E98),
the main 2qp component qq′, its energy $\varepsilon_{qq'}$, contribution to the state norm $N_{qq'}$ and F-scheme of 2qp excitation.

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^{254}No: forces predict different 2qp configurations nn[734 ↑, 613 ↓] and pp[514 ↓, 624 ↑]

- V.G. Soloviev, A.V. Sushkov, A.Yu. Shirikova, Sov. J. Nucl. Phys. 54, 748 (1991)
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SVbas	1.913	0.119	$nn[624\downarrow, 734\uparrow]$	1.751	0.912	F,F+1
^{254}No, E_{exp}=1.295 MeV						
SLy6	1.747	0.014	$nn[734\uparrow, 613\uparrow]$	1.780	0.994	F,F+3
SkM*	1.554	0.333	$pp[514\downarrow, 624\uparrow,]$	1.482	0.990	F+1,F+2
SVbas	1.994	0.370	$pp[514\downarrow, 624\uparrow,]$	1.751	0.791	F+1,F+2
			$nn[734\uparrow, 613\uparrow]$	2.026	0.169	F,F+3

Features of calculated 8^- states in ^{252,254}No: QRPA excitation energies E_ν =1, reduced transition probabilities B(E98), the main 2qp component qq′, its energy $\varepsilon_{qq'}$, contribution to the state norm $N_{qq'}$ and F-scheme of 2qp excitation.

K$^{\pi}$= 8^{-} isomers

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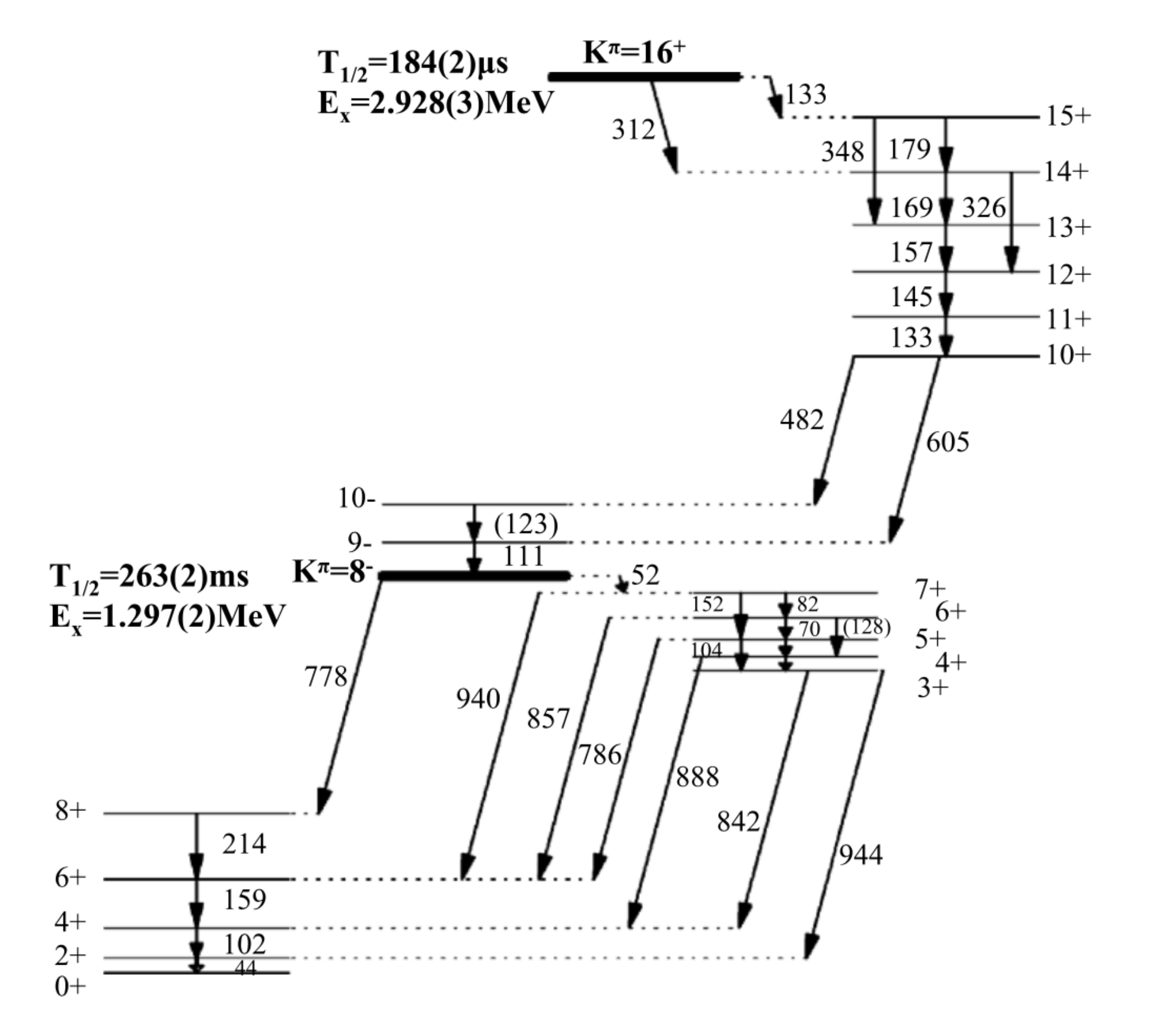

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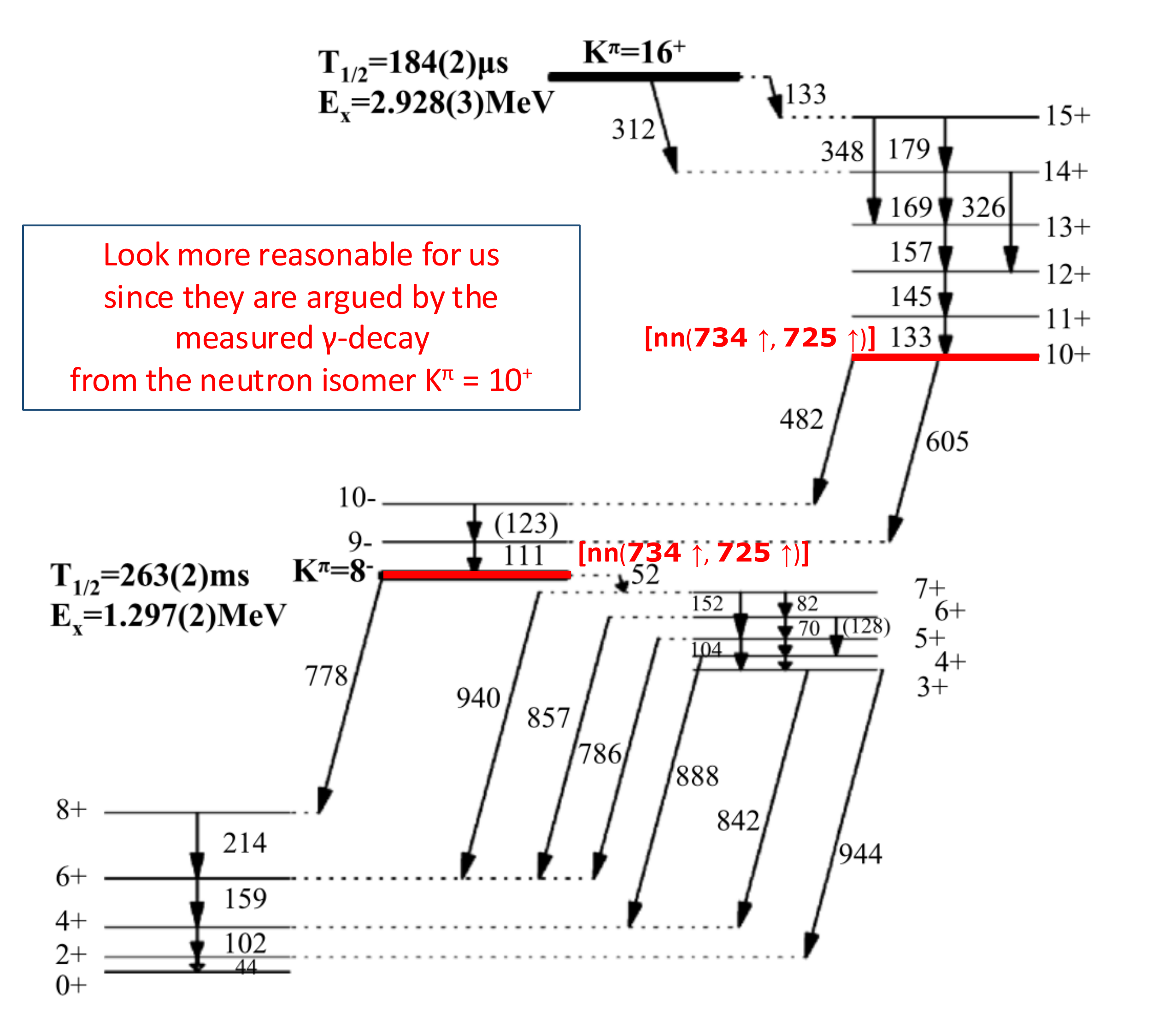

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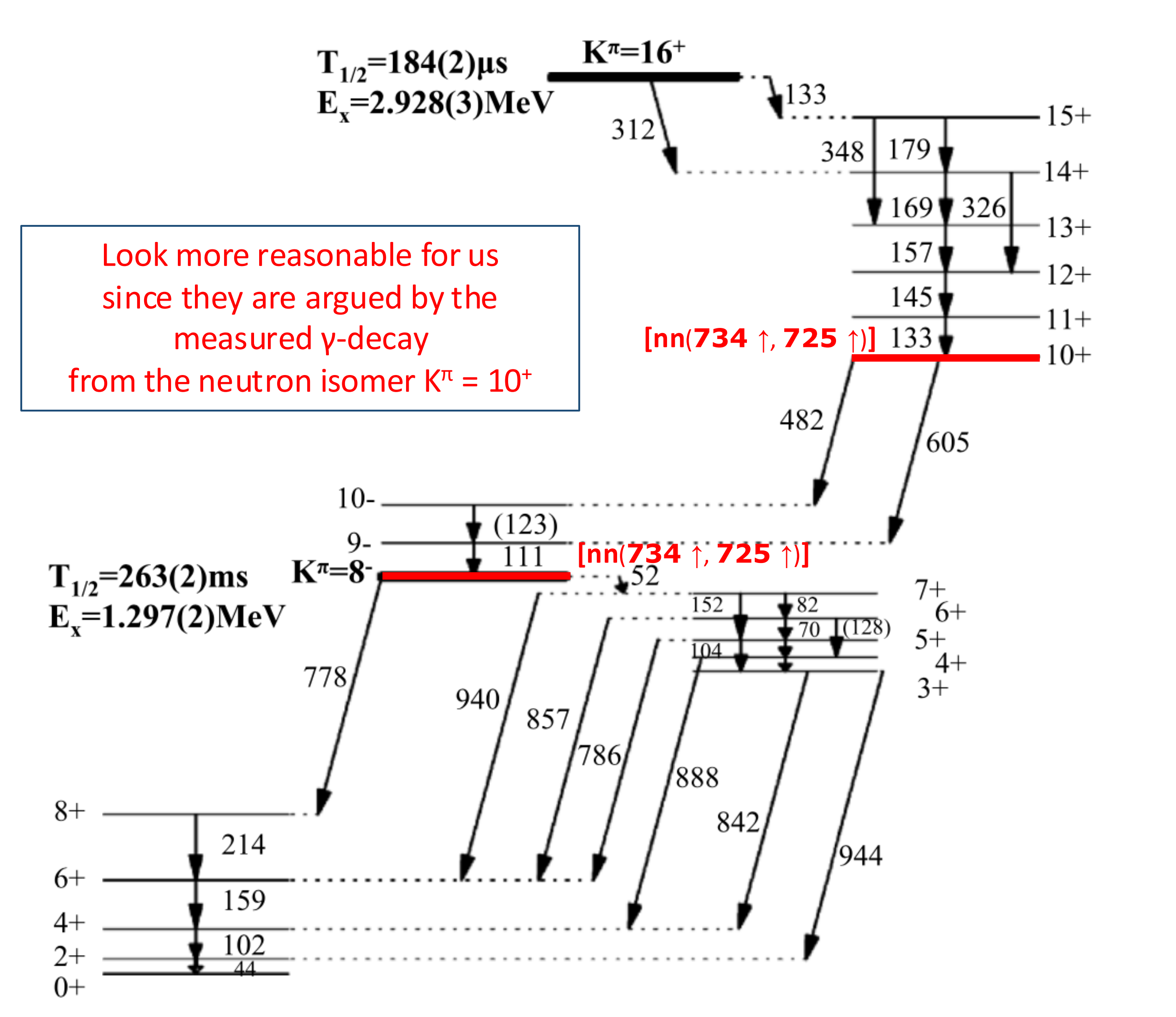

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Pairing vibrations $K^\pi = 0^+$

^{252}No:

Force	E (MeV)	B(E20) (W.u.)	qq’
SLy6	0.774	0.02	nn[734↑, 734↑]
SkM*	0.838	1.12	pp[521↓, 521↓]
SVbas	1.249	6.41	pp[514↓, 514↓]

^{254}No:

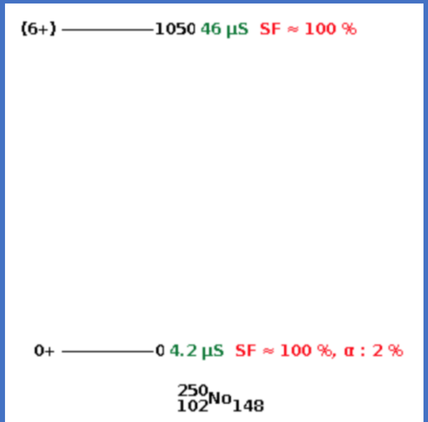
Force	E (MeV)	B(E20) (W.u.)	qq’
SLy6	0.224	0.002	nn[734↑, 734↑]
SkM*	0.767	0.17	nn[624↓, 624↓]
SVbas	1.236	6.36	pp[514↓, 514↓]

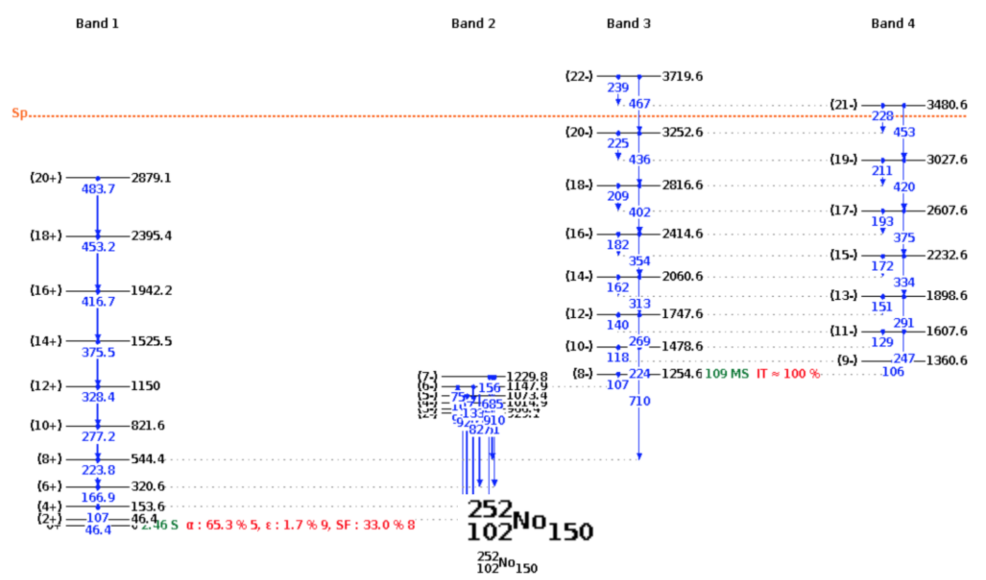
- Recent shell-model calculations with the projection after variation **also predicts $K^\pi = 0^+$ state with E=0.86 MeV as the lowest non-rotational state of ^{254}No** [D.D. Dao and F. Nowacki, Phys. Rev. C 105, 054314 (2022)]

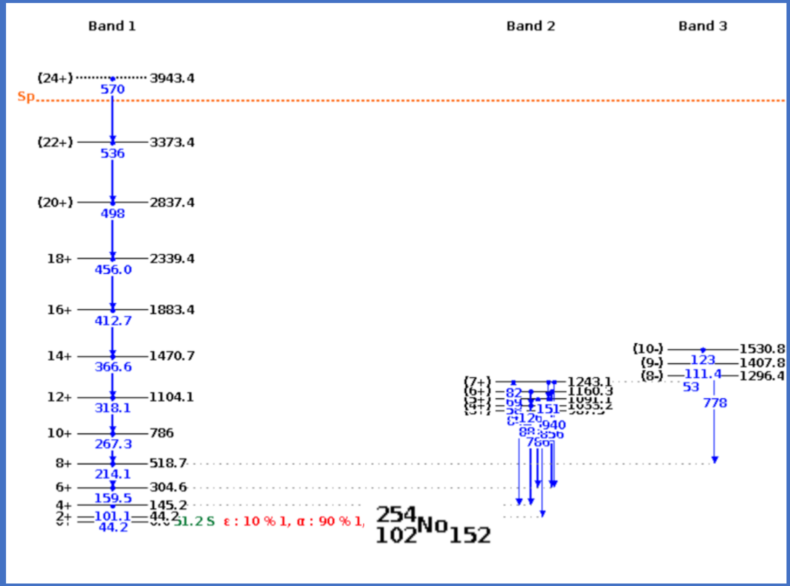
- [M. Forge et al., J. Phys.: Conf. Ser. 2586 012083 (2023)] **also predicts $K^\pi = 0^+$ state with E=0.89 MeV** (shape coexistence and superdeformations)

So, excited 0+ states below 1 MeV in superheavy nuclei are quite possible

At the moment, there are experimental* spectroscopic data only for 3/7 nuclei: $^{250,252,254}No$







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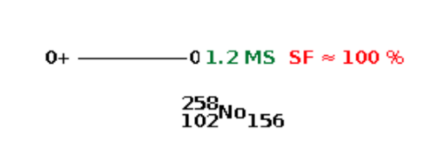

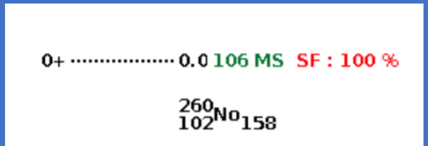

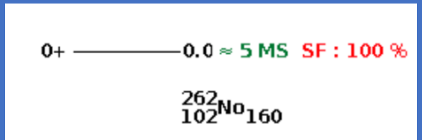

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So, excited 0+ states below 1 MeV in superheavy nuclei are quite possible

Hexadecapole states with $K^{\pi} = 3^{+}$ and 4^{+}

^{252}No

3^{+}:

Force	E (MeV)	B(E43) (W.u.)	qq’
SLy6	1.10	3.04	pp[521↓, 514↓]
SkM*	1.00	3.61	pp[521↓, 514↓]
SVbas	1.19	2.73	pp[521↓, 514↓]

4^{+}:

SLy6	1.16	$5.5*10^{-4}$	pp[521↓, 514↓]
SkM*	1.00	3.61	pp[521↓, 514↓]
SVbas	1.19	2.73	pp[521↓, 514↓]

- All the forces predict for this state **the proton 2qp configuration** pp[521 ↓, 514 ↓]

- **The calculated 4+ states in ^{252,254}No have the energies and structure very similar to 3^{+} states.**

This is not surprising since both kinds of states are basically formed by the same proton 2qp configuration pp[521↓, 514↓] with $|K1 - K2|=3$ and $K1 + K2=4$.

Hexadecapole states with $K^{\pi} = 3^{+}$ and 4^{+}

^{252}No

3⁺:

Force	E (MeV)	B(E43) (W.u.)	qq’
SLy6	1.10	3.04	pp[521↓, 514↓]
SkM*	1.00	3.61	pp[521↓, 514↓]
SVbas	1.19	2.73	pp[521↓, 514↓]

4⁺:

SLy6	1.16	$5.5*10^{-4}$	pp[521↓, 514↓]
SkM*	1.00	3.61	pp[521↓, 514↓]
SVbas	1.19	2.73	pp[521↓, 514↓]

^{254}No

3⁺:

Force	E (MeV)	B(E43) (W.u.)	qq’
SLy6	1.11	2.41	pp[521↓, 514↓]
SkM*	1.01	3.24	pp[521↓, 514↓]
SVbas	1.17	3.00	pp[521↓, 514↓]

4⁺:

SLy6	1.16	0.07	pp[521↓, 514↓]
SkM*	1.01	3.24	pp[521↓, 514↓]
SVbas	1.17	3.00	pp[521↓, 514↓]

- All the forces predict for this state **the proton 2qp configuration** pp[521 ↓, 514 ↓]

- **The calculated 4+ states in ^{252,254}No have the energies and structure very similar to 3^{+} states.**

This is not surprising since both kinds of states are basically formed by the same proton 2qp configuration pp[521↓, 514↓] with |K1 − K2|=3 and K1 + K2=4.

Hexadecapole states with $K^{\pi} = 3^{+}$ and 4^{+}

^{252}No

3^{+}:

Force	E (MeV)	B(E43) (W.u.)	qq'
SLy6	1.10	3.04	pp[521↓, 514↓]
SkM*	1.00	3.61	pp[521↓, 514↓]
SVbas	1.19	2.73	pp[521↓, 514↓]

4^{+}:

SLy6	1.16	$5.5*10^{-4}$	pp[521↓, 514↓]
SkM*	1.00	3.61	pp[521↓, 514↓]
SVbas	1.19	2.73	pp[521↓, 514↓]

^{254}No

Eexp=0.987 MeV

3^{+}:

Force	E (MeV)	B(E43) (W.u.)	qq'
SLy6	1.11	2.41	pp[521↓, 514↓]
SkM*	1.01	3.24	pp[521↓, 514↓]
SVbas	1.17	3.00	pp[521↓, 514↓]

4^{+}:

SLy6	1.16	0.07	pp[521↓, 514↓]
SkM*	1.01	3.24	pp[521↓, 514↓]
SVbas	1.17	3.00	pp[521↓, 514↓]

- All the forces predict for this state **the proton 2qp configuration** pp[521 ↓, 514 ↓]

- **The calculated 4+ states in ^{252,254}No have the energies and structure very similar to 3^{+} states.**

This is not surprising since both kinds of states are basically formed by the same proton 2qp configuration pp[521↓, 514↓] with |K1 − K2|=3 and K1 + K2=4.

Octupole states with K^{π} = 0^-, 1^-, 2^- and 3^- (SLy6)

- In agreement with the experimental analysis, all three Skyrme forces **suggest for the first 2^- state in ^{252}No --> nn[734 ↑, 622 ↑]**

- **In ^{254}No, our calculations for the first 2^- state give rather high energies (1.80-2.12 MeV) and essentially different structure and collectivity.**
  The difference in the excitation energy is caused by different neutron chemical potentials:
  λ_n=-7.09 MeV in ^{252}No and -6.35 MeV in ^{254}No.

^{252}No

K^{π}	E (MeV)	B(E3K) (W.u.)	qq'
0^-	1.24	9.1	pp[514↓, 633↑]
1^-	1.41	1.5	nn[734↑, 624↓]
2^-	0.95	11.5	nn[622↑, 734↑]
3^-	1.35	0.1	pp[633↑, 521↓]

^{254}No

K^{π}	E (MeV)	B(E3K) (W.u.)	qq'
0^-	1.25	11.2	pp[514↓, 633↑]
1^-	1.54	8.4	nn[734↑, 613↓]
2^-	2.12	0.6	nn[622↑, 734↑]
3^-	1.28	0.03	pp[734↑, 622↓]

Octupole states with K^{π} = 0^-, 1^-, 2^- and 3^- (SLy6)

- In agreement with the experimental analysis, all three Skyrme forces **suggest for the first 2^- state in ^{252}No --> nn[734 ↑, 622 ↑]**

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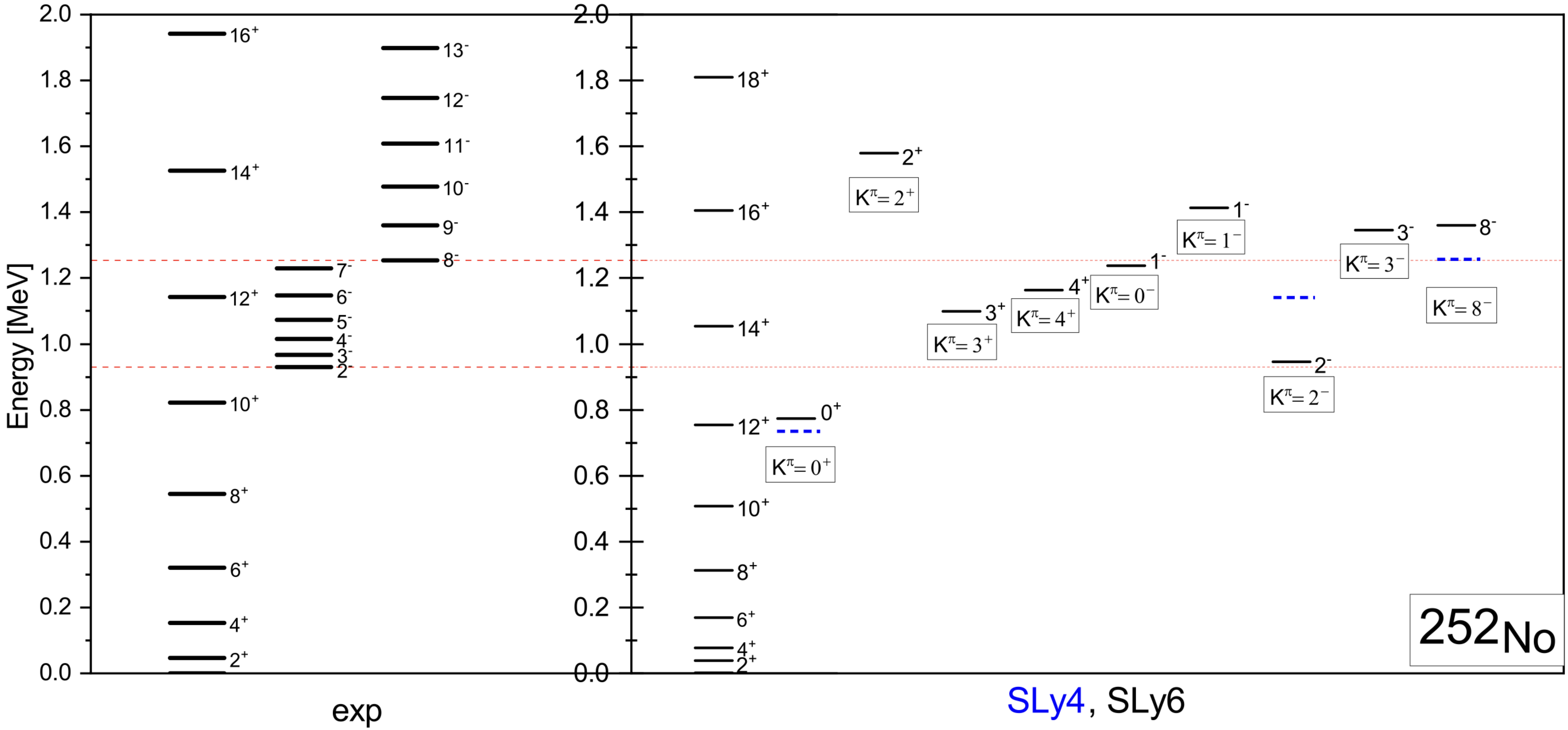
^{252}No

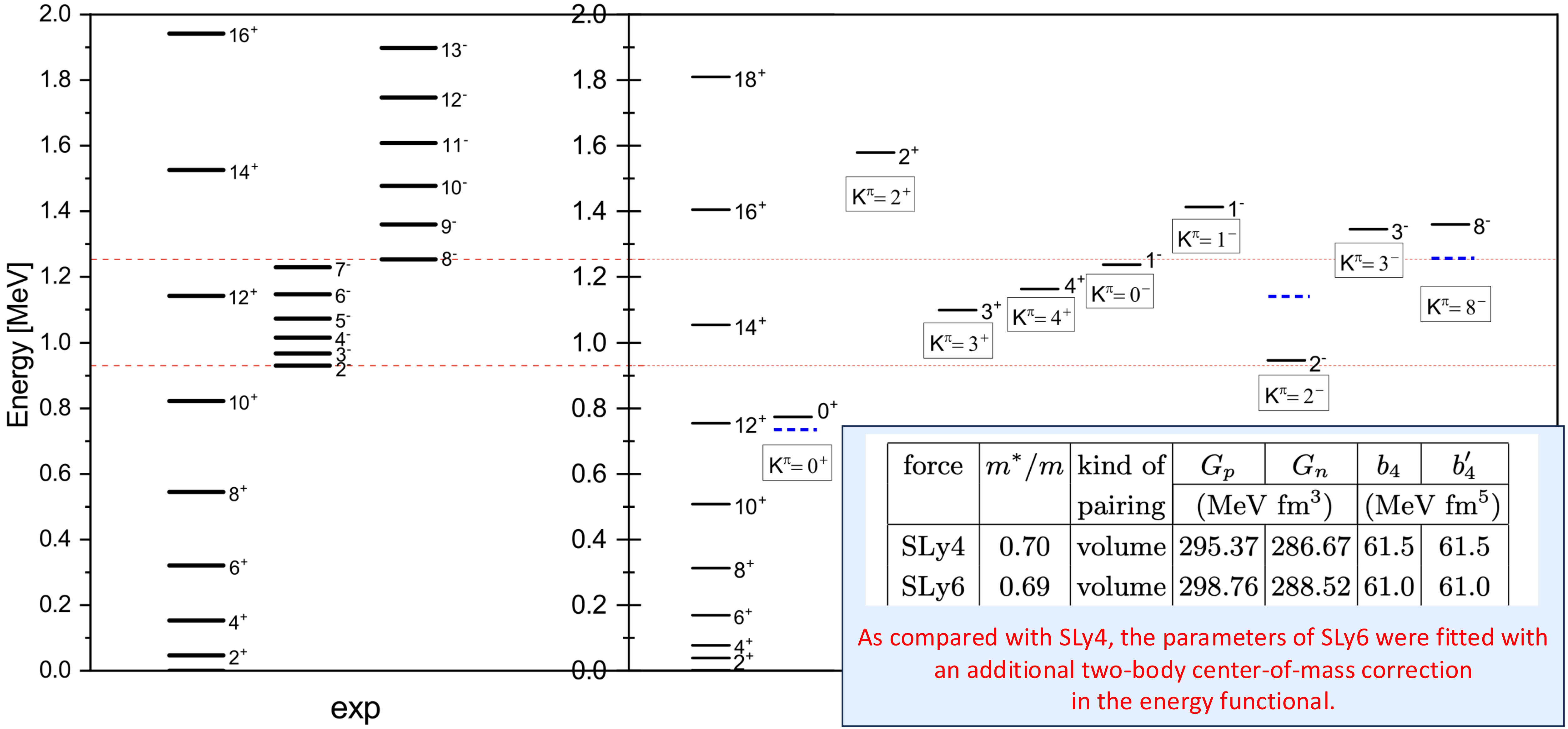
K^{π}	E (MeV)	B(E3K) (W.u.)	qq'
0^-	1.24	9.1	pp[514↓, 633↑]
1^-	1.41	1.5	nn[734↑, 624↓]
2^-	0.95	11.5	nn[622↑, 734↑]
3^-	1.35	0.1	pp[633↑, 521↓]

^{254}No

K^{π}	E (MeV)	B(E3K) (W.u.)	qq'
0^-	1.25	11.2	pp[514↓, 633↑]
1^-	1.54	8.4	nn[734↑, 613↓]
2^-	2.12	0.6	nn[622↑, 734↑]
3^-	1.28	0.03	pp[734↑, 622↓]



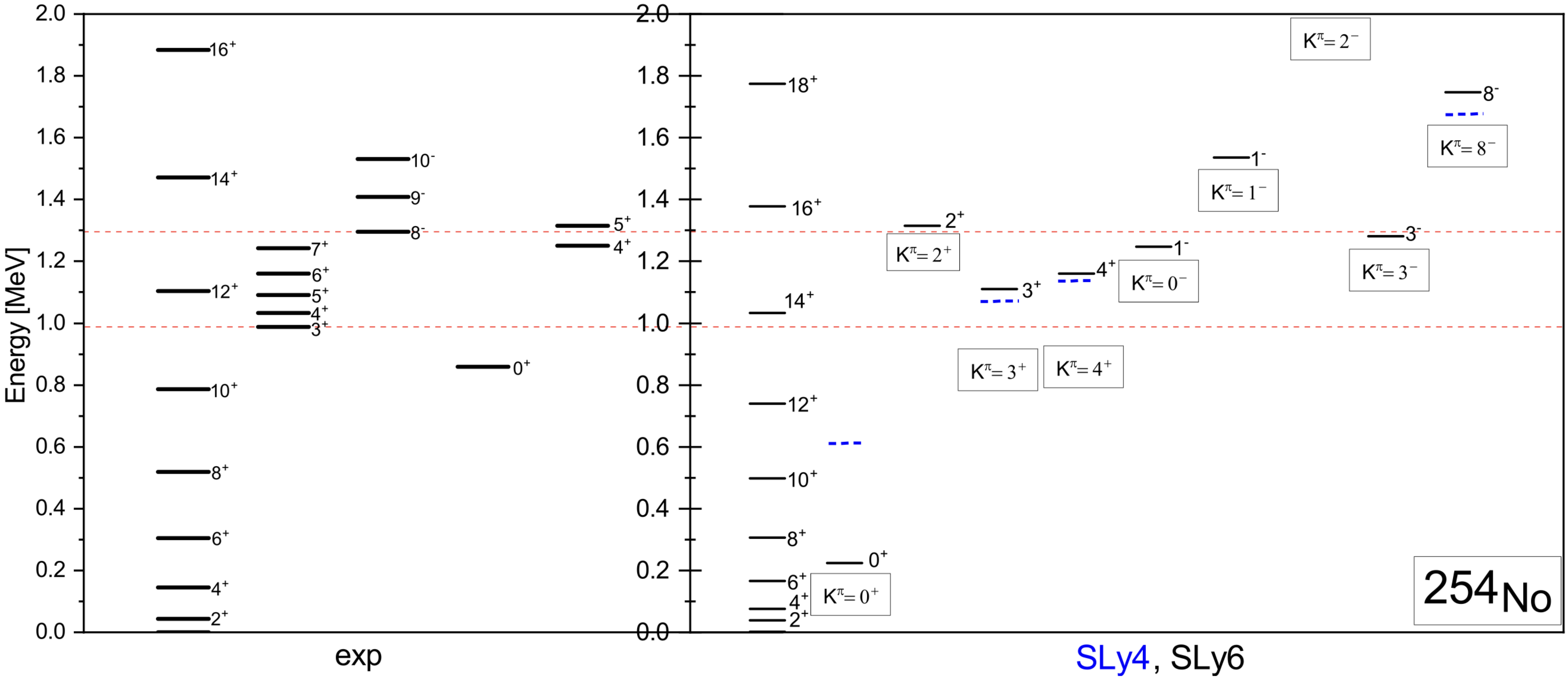
- The band of the ground state is slightly compressed
- The band, which built on state 2^- is described well and the two others bands are also described satisfactorily



force	m^*/m	kind of pairing	G_p (MeV fm^3)	G_n	b_4 (MeV fm^5)	b'_4
SLy4	0.70	volume	295.37	286.67	61.5	61.5
SLy6	0.69	volume	298.76	288.52	61.0	61.0

As compared with SLy4, the parameters of SLy6 were fitted with an additional two-body center-of-mass correction in the energy functional.

- The band of the ground state is slightly compressed
- The band, which built on state 2^- is described well and the two others bands are also described satisfactorily



We also describing the 3 experimental bands quite well and working to carry out the more detailed analyzes about the band starting with 8^-

Conclusion

- The low-energy spectra of the nobelium chain were studied within the framework of three Skyrme forces (SLy6, SkM*, SVbas) with different types of pairing
- All three Skyrme forces support this irregularity, despite different types of neutron pairing (volume/surface)
- For ^{252,254}No isotopes this irregularity associated with pairing effect and evolution of the single-particle spectrum
- The theoretically obtained bands for the lower spectrum for ^{252,254}No are in good agreement with experiment
- We also make the predictions about low-energy bands of different multipolarity ($K^{\pi} = 0^{+}, 2^{+}, 3^{+}, 0^{-}, 1^{-}, 2^{-}, 8^{-}$), some of then can be found experimentally

Thank you for your attention!

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Low-energy spectra of nobelium isotopes: Skyrme random-phase-approximation analysis

Authors: V. O. Nesterenko, M. A. Mardyban, A. Repko, R. V. Jolos, P. -G. Reinhard and A. A. Dzhioev

(29.05 accepted in PRC)

Backup slides

Obviously, so low energy is an artifact of the BCS description of a weak pairing. In this connection, SLy4 energy 0.611 MeV for 0^+_1 state is more realistic (here we have a drop but not collapse of the neutron pairing).

TABLE V. Features of the lowest excited QRPA 0^+ states in ^{252,254}No: QRPA excitation energies E, reduced transition probabilities $B(E20)$ and $\rho^2(E0)$, main two 2qp components qq', their energies $\epsilon_{qq'}$, contributions to the state norm $N_{qq'}$ and F-order. For the sake of discussion, in ^{254}No, two lowest 0^+ states are exhibited.

Force	E (MeV)	$B(E20)$ (W.u.)	$\rho^2(E0)$ (10^{-3})	qq'	$\epsilon_{qq'}$ (MeV)	$N_{qq'}$	F-order
	^{252}No						
SLy4	0.740	0.18	0.08	$nn[734\uparrow,734\uparrow]$	1.074	0.53	F+1, F+1
				$nn[624\downarrow,624\downarrow]$	1.348	0.30	F,F
SLy6	0.774	0.02	0.29	$nn[734\uparrow,734\uparrow]$	1.070	0.58	F+1, F+1
				$nn[624\downarrow,624\downarrow]$	1.563	0.17	F,F
SkM*	0.838	1.12	1.24	$pp[521\downarrow,521\downarrow]$	1.014	0.46	F,F
				$pp[514\downarrow,514\downarrow]$	1.093	0.42	F+1,F+1
SVbas	1.249	6.41	2.50	$pp[514\downarrow,514\downarrow]$	1.215	0.56	F+1,F+1
				$pp[521\downarrow,521\downarrow]$	1.186	0.36	F,F
	^{254}No						
SLy4	0.616	0.08	0.08	$nn[734\uparrow,734\uparrow]$	1.021	0.51	F, F
				$nn[620\uparrow,620\uparrow]$	1.156	0.25	F+1,F+1
	1.000	2.69	2.69	$pp[514\downarrow,514\downarrow]$	1.092	0.57	F+1,F+1
				$pp[521\downarrow,521\downarrow]$	1.163	0.30	F,F
SLy6	0.224	0.002	0.01	$nn[734\uparrow,734\uparrow]$	1.048	0.41	F, F
				$nn[620\uparrow,620\uparrow]$	1.267	0.27	F+1,F+1
	1.133	1.32	1.98	$pp[514\downarrow,514\downarrow]$	1.155	0.56	F+1,F+1
				$pp[521\downarrow,521\downarrow]$	1.152	0.45	F,F
SkM*	0.767	0.17	0.74	$nn[624\downarrow,624\downarrow]$	1.409	0.33	F,F
				$nn[620\uparrow,620\uparrow]$	1.362	0.23	F+1, F+1
	0.866	4.38	6.78	$pp[521\downarrow,521\downarrow]$	1.02	0.45	F,F
				$pp[514\downarrow,514\downarrow]$	1.08	0.43	F+1,F+1
SVbas	1.236	6.36	2.54	$pp[514\downarrow,514\downarrow]$	1.083	0.43	F+1,F+1
				$pp[521\downarrow,521\downarrow]$	1.017	0.38	F,F
	1.454	0.53	1.35	$pp[633\downarrow,633\downarrow]$	1.593	0.45	F-1,F-1
				$pp[521\downarrow,521\downarrow]$	1.186	0.25	F,F

The reduced probabilities ($\lambda > 1$)

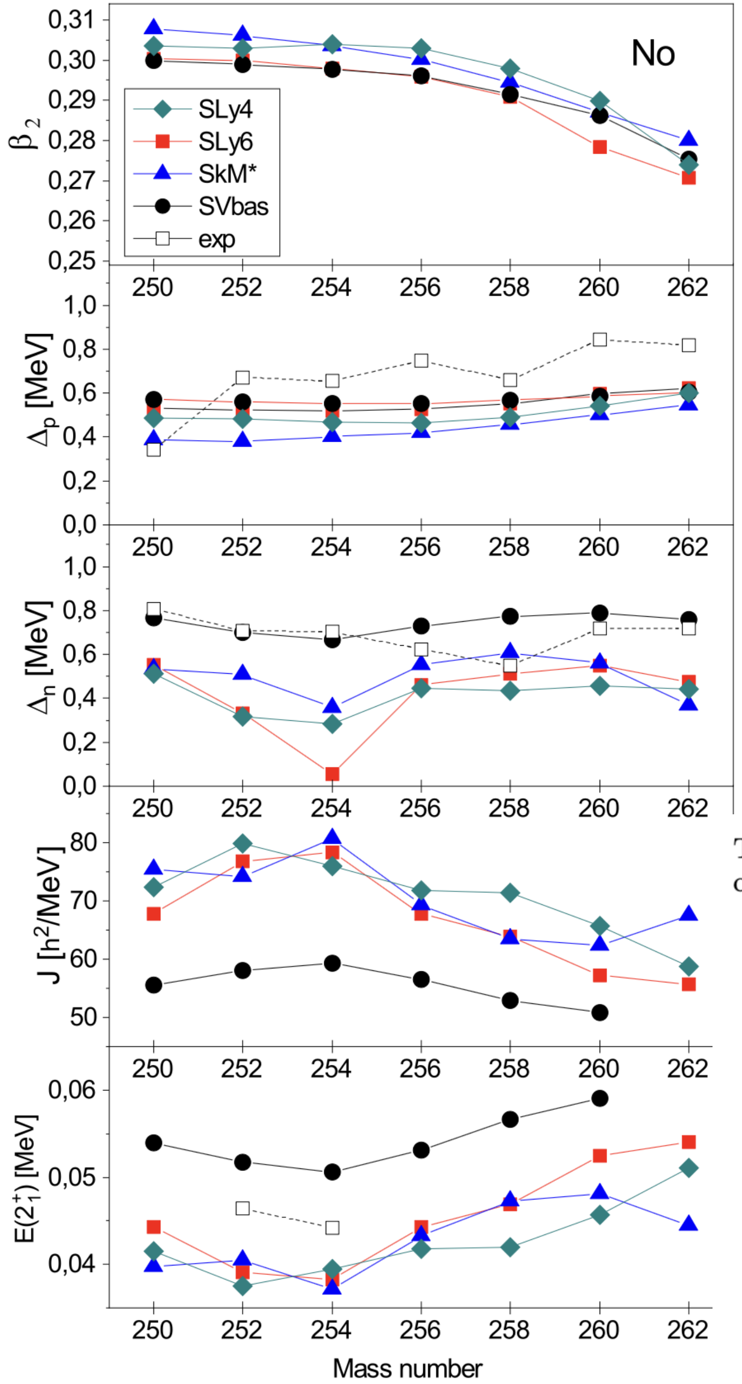
$$B(E\lambda\mu) = e^2(2-\delta_{\mu,0})|\langle\nu|F(E\lambda\mu)|0\rangle|^2 \quad (7)$$

for $E\lambda\mu$ -transitions from the ground state $|0\rangle$ with $I^\pi K = 0^+0_{\mathrm{gs}}$ to the QRPA state $|\nu\rangle$ with $I^\pi K$ ($I = \lambda, K = \mu, \pi = (-1)^\lambda$) are computed with the transition operator

$$F(E\lambda\mu) = \sum_{i=1}^{Z}[r^\lambda Y_{\lambda\mu}(\Omega)]_i. \quad (8)$$

TABLE II. The calculated and experimental [13] quadrupole deformations β, moments of inertia J, proton Δ_p and neutron Δ_n pairing spectral average gaps, energies $E(2_1^+)$ of the lowest rotational $I^\pi = 2^+$ states in ^{252,254}No.

Nucleus		SLy4	SLy6	SkM*	SVbas	exp
^{252}No	β	0.303	0.300	0.306	0.299	
	J ($\hbar^2$/MeV)	79.9	76.8	74.1	58.0	64.7
	Δ_p (MeV)	0.48	0.52	0.38	0.56	0.67
	Δ_n (MeV)	0.32	0.33	0.51	0.70	0.71
	$E(2_1^+)$ (keV)	38	39	40	52	46
^{254}No	β	0.304	0.298	0.304	0.298	
	J ($\hbar^2$/MeV)	76.0	78.3	80.8	59.3	67.9
	Δ_p (MeV)	0.47	0.52	0.40	0.55	0.66
	Δ_n (MeV)]	0.28	0.05	0.36	0.67	0.71
	$E(2_1^+)$ (keV)	39	38	37	50	44



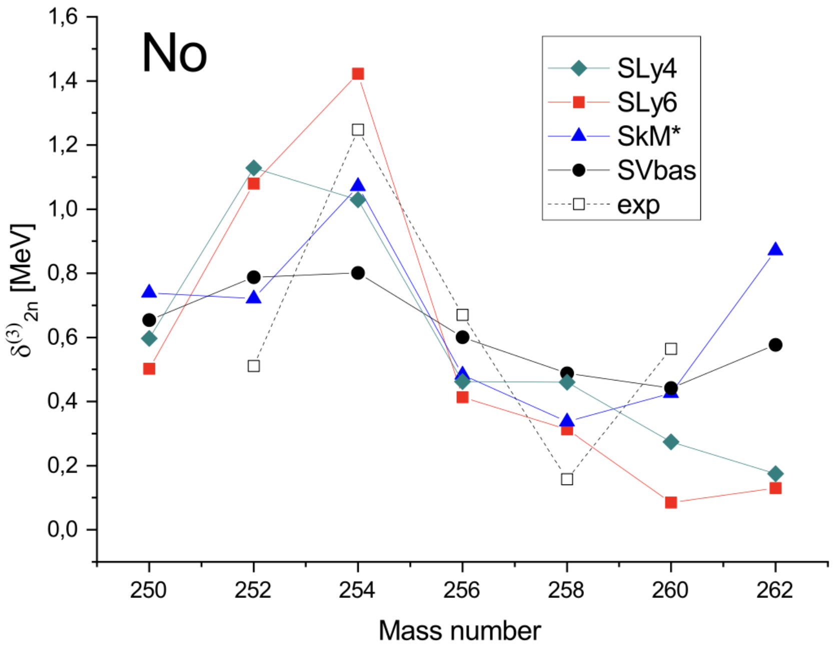
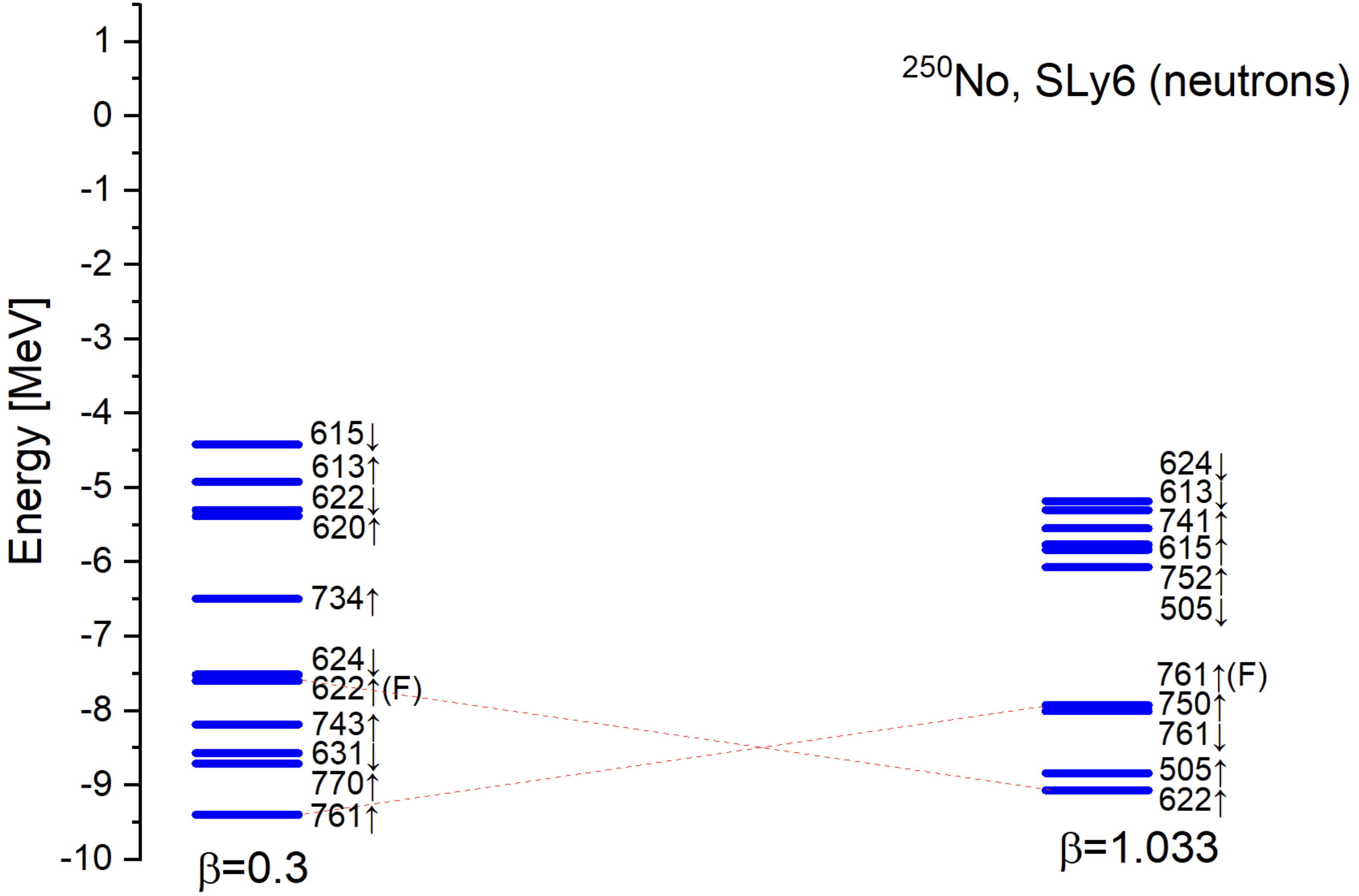


FIG. 4. SLy4, SLy6, SkM*, SVbas and experimental neutron values $\delta_{2n}^{(3)}$ in nobelium isotopes.

TABLE III. Experimental [13] and calculated ground and first excited non-rotational I^{π} states in Z-odd and N-odd neighbours of ^{252}No and ^{254}No. The experimental assignments in the parentheses are tentative.

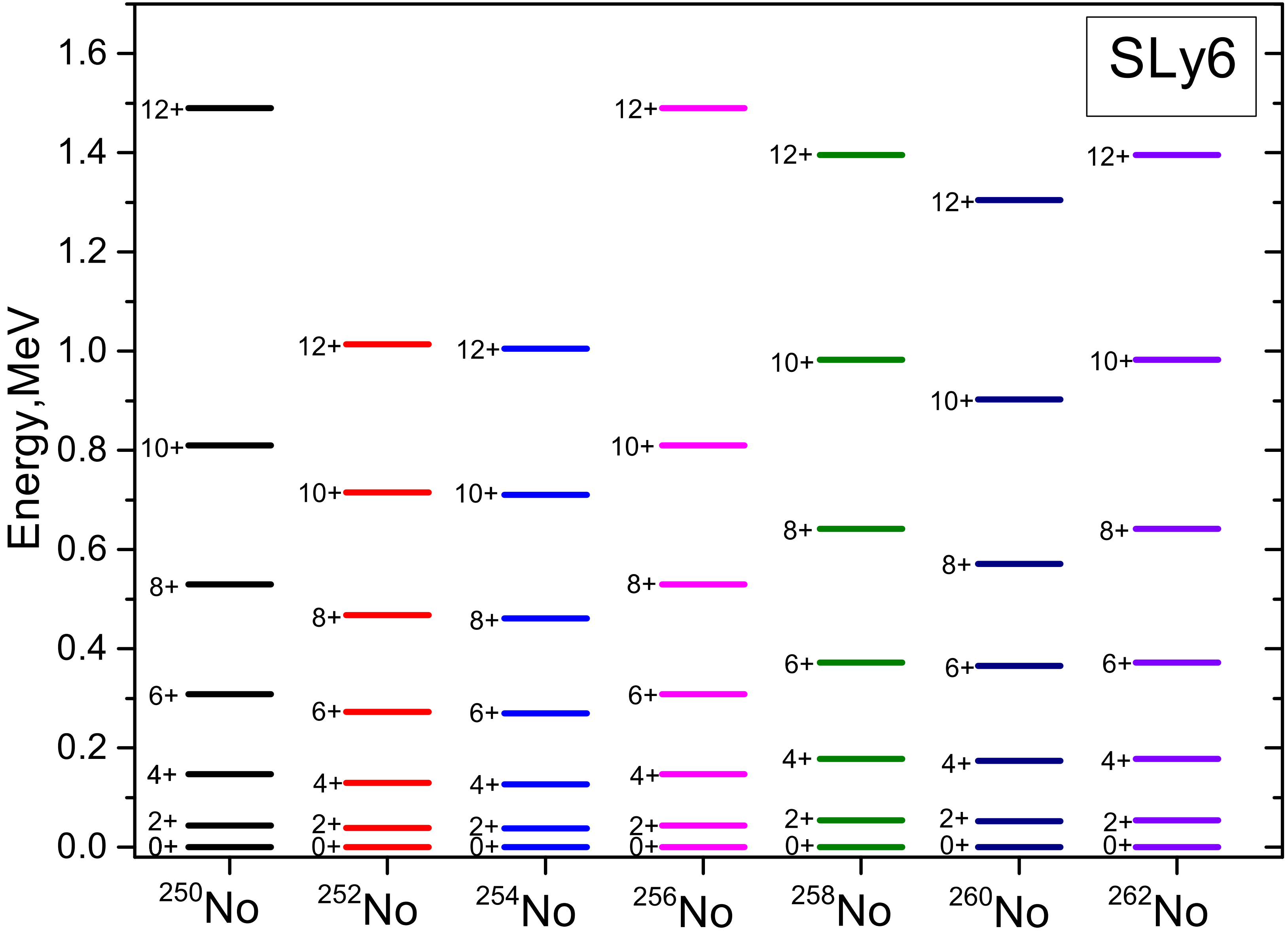
Nucl.	exp	SLy4	SLy6	SkM*	SVbas
^{252}No					
^{251}No	$(7/2^+)$, $(1/2^+)$	$7/2^+$, $5/2^+$	$7/2^+$, $5/2^+$	$9/2^-$, $7/2^+$	$7/2^+$, $5/2^+$
^{253}No	$(9/2^-)$, $5/2^+$	$9/2^-$, $1/2^+$	$9/2^-$, $7/2^+$	$7/2^+$, $9/2^-$	$9/2^-$, $7/2^+$
^{251}Md	$7/2^-$, $1/2^-$	$1/2^-$, $7/2^+$	$1/2^-$, $7/2^-$	$1/2^-$, $7/2^-$	$1/2^-$, $7/2^-$
^{253}Lr	$(7/2^-)$, $(1/2^-)$	$7/2^-$, $1/2^-$	$7/2^-$, $1/2^-$	$7/2^-$, $9/2^+$	$7/2^-$, $1/2^-$
^{254}No					
^{253}No	$(9/2^-)$, $5/2^+$	$9/2^-$, $1/2^+$	$9/2^-$, $5/2^+$	$7/2^+$, $9/2^-$	$9/2^-$, $7/2^+$
^{255}No	$(1/2^+)$	$1/2^+$, $3/2^+$	$1/2^+$, $3/2^+$	$1/2^+$, $3/2^+$	$1/2^+$, $3/2^+$
^{253}Md	$(7/2^-)$	$1/2^-$, $7/2^+$	$1/2^-$, $7/2^-$	$1/2^-$, $7/2^+$	$1/2^-$, $7/2^-$
^{255}Lr	$(1/2^-)$, $(7/2^-)$	$7/2^-$, $1/2^-$	$7/2^-$, $1/2^-$	$7/2^-$, $9/2^+$	$7/2^-$, $1/2^-$

Dynamic of neutron spectrum with β=0.3 (equilibrium deformation) and β=1



There is a gap near the Fermi level (in both case) which can give strong pairing effects

The irregularity in ^{252}No and ^{254}No at low-energy spectrum



<u>Our tasks are:</u>

- to analyze the occurrence of the irregularity for ^{252,254}No

- to make predictions not only for the ground state energy band, but for other bands too

States $K^{\pi} = 2$

- In most of the cases, if the first state is collective, then the next one is 2qp and vice versa, but:

- The first $K^{\pi} = 2^{+}$ - states are <u>γ-vibrational collective</u> in ^{252}No (SLy6, SV-bas) and in ^{254}No (SkM*, SV-bas)

- Instead, the first 2^{+} states are <u>purely 2qp</u> in ^{252}No (SkM*) and in ^{254}No (SLy6)

Anyway, all the calculated 2+ lie above the observed 2^{-} (^{252}No) and 3^{+} (^{252}No) K-isomers

- We know only IBM calculations
  [A. D. Efimov and I. N. Izosimov, Phys. Atom. Nucl. 84, 660 (2021)];
  [A. D. Efimov and I. N. Izosimov, JINR-E6-2022-19 (2022)]

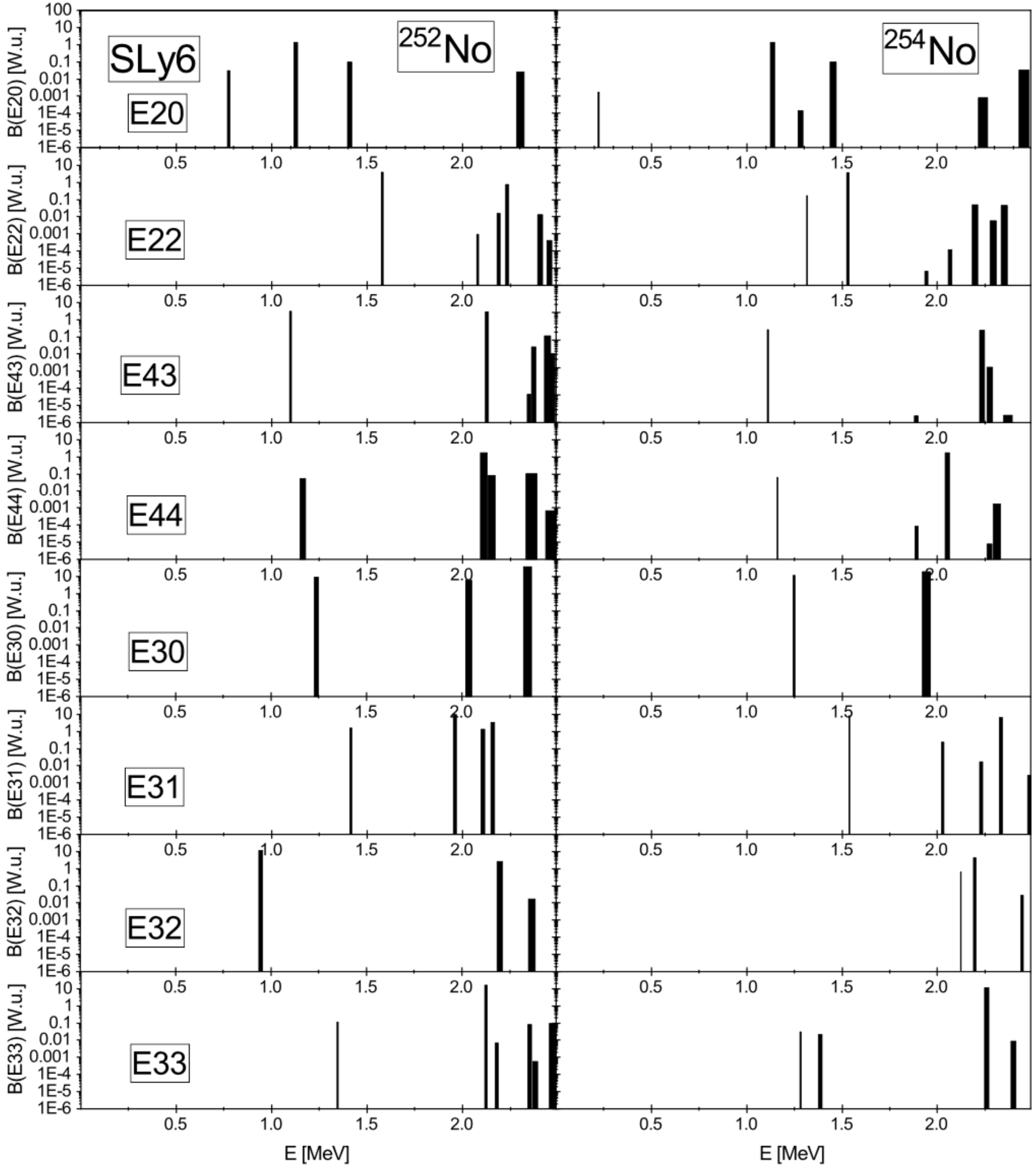
- In contrast to our results, calculations predict $K^{\pi} = 2^{+}$ states at 1.09 MeV (^{252}No) and 0.94 MeV (^{254}No).

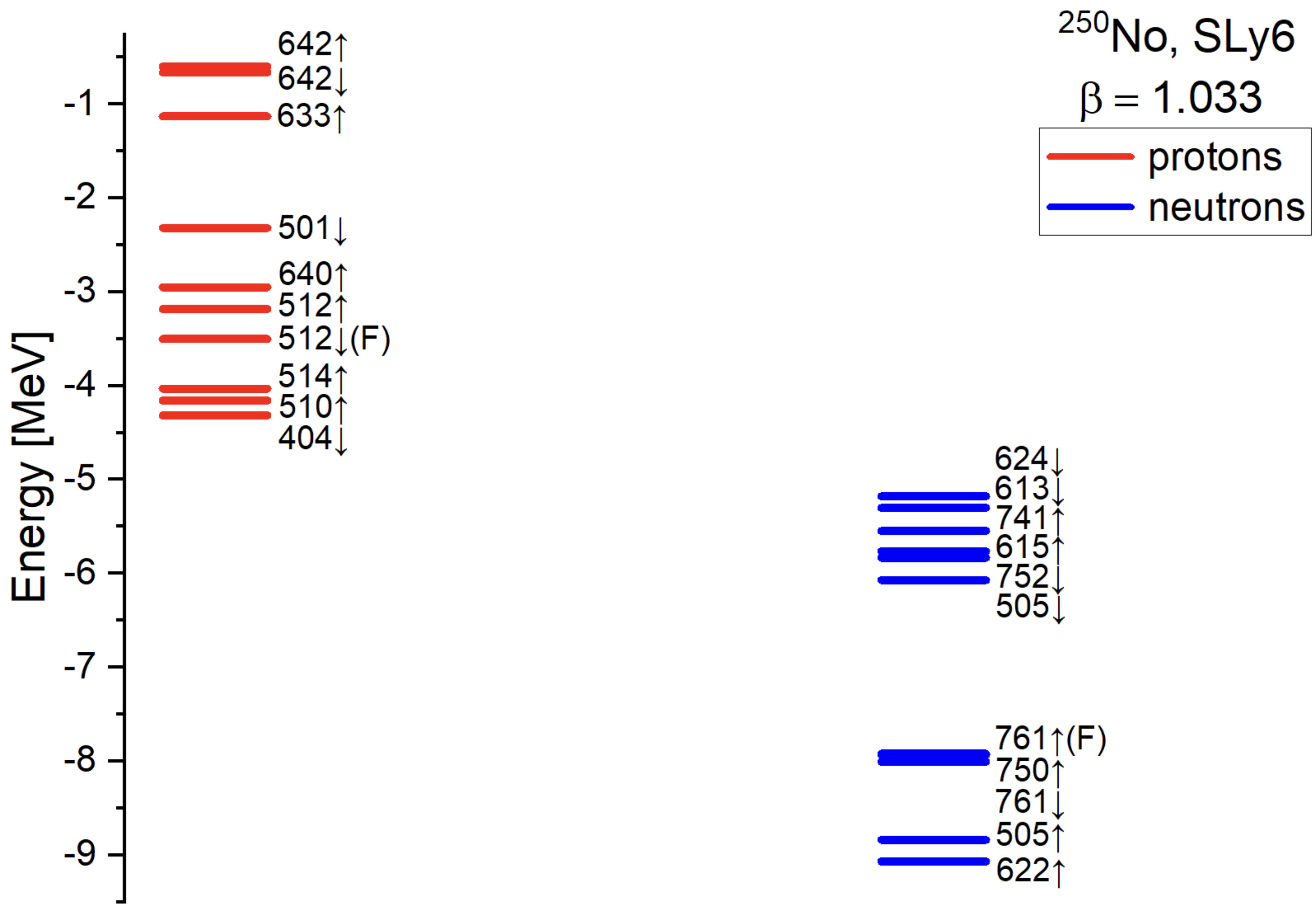
To estimate the true relevance of various theoretical results for No isotopes, the experimental data are necessary.

Force	E	$B(E22)$	qq'	$\epsilon_{qq'}$	$N_{qq'}$	F-struct
			^{252}No			
SLy6	1.58	3.87	nn[622 ↑, 620 ↑]	2.33	0.39	F-1,F+2
			pp[521 ↓, 521 ↑]	2.06	0.32	F,F-2
			nn[624 ↓, 622 ↓]	2.42	0.21	F,F+3
	2.08	-	pp[514 ↓], 521 ↑]	2.06	1.00	F+1,F-2
SkM*	1.70	0.06	pp[512 ↑, 521 ↓]	1.61	0.99	F+3,F
	1.78	-	nn[622 ↑], 620 ↑]	2.28	0.35	F-1,F+2
			nn[624 ↓], 622 ↓]	2.14	0.29	F+1,F+3
			pp[514 ↓], 521 ↑]	2.06	1.00	F+1,F-2
SVbas	1.62	2.72	pp[521 ↑], 521 ↓]	1.95	0.38	F-2,F
			nn[622 ↑, 620 ↑]	2.48	0.29	F-1,F+2
			nn[624 ↓], 622 ↓]	2.24	0.19	F,F+3
	1.89	-	pp[512 ↑, 521 ↓]	1.86	0.99	F+3,F
			^{254}No			
SLy6	1.31	0.17	nn[622 ↑, 620 ↑]	1.32	0.97	F-1,F+1
	1.53	-	nn[622 ↑, 620 ↑]	2.24	0.42	F-1,F+1
			pp[521 ↑], 521 ↓]	2.05	0.27	F-2,F
			nn[624 ↓], 622 ↓]	2.39	0.20	F-2,F+2
SkM*	1.32	2.62	nn[624 ↓, 622 ↓]	1.63	0.60	F,F+2
			nn[622 ↓], 620 ↑]	1.60	0.18	F+2,F+1
			nn[622 ↑], 620 ↑]	2.20	0.11	F-2,F+1
	1.62	-	nn[622 ↓], 620 ↑]	1.60	0.80	F-2,F+1
			nn[624 ↓, 622 ↓]	1.63	019	F,F+2
SVbas	1.45	4.46	nn[622 ↓, 620 ↑]	1.77	0.40	F+2,F+1
			pp[521 ↑], 521 ↓]	1.95	0.20	F-2,F
			nn[624 ↓], 622 ↓]	2.15	0.17	F-1,F+2
	1.87	-	nn[622 ↓], 620 ↑]	1.77	0.56	F-1,F+2
			pp[521 ↑], 521 ↓]	1.95	0.21	F-2,F
			nn[622 ↑, 620 ↑]	2.28	0.14	F+2,F+1

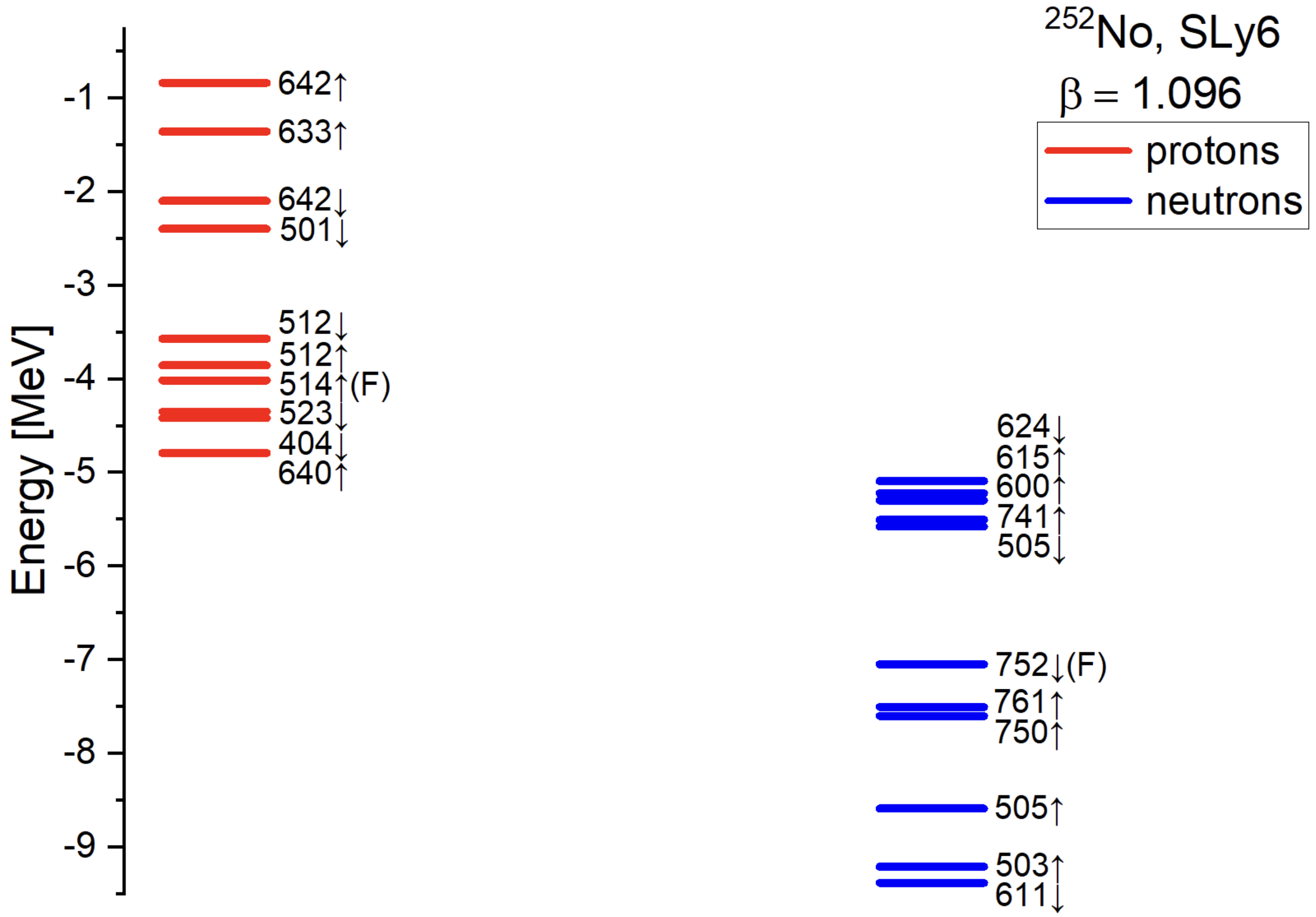
TABLE X. The lowest SLy6 neutron and proton 2qp configurations $K = K_1 + K_2$ and $K = |K_1 + K_2|$ in ^{252,254}No.

$\epsilon_{qq'}$	qq'	F-struct	K_1+K_2	K_1-K_2
		^{252}No		
1.16	$pp[521\downarrow, 514\downarrow]$	F,F+1	$\underline{4^+}$	$\underline{3^+}$
1.35	$pp[633\uparrow, 514\downarrow]$	F-1,F+1	7^-	$\underline{0^-}$
1.35	$pp[633\uparrow, 521\downarrow]$	F-1,F+1	4^-	$\underline{3^-}$
2.06	$pp[521\uparrow, 521\downarrow]$	F-2,F	$\underline{2^+}$	1^+
2.25	$pp[521\uparrow, 633\uparrow]$	F-2,F-1	5^-	$\underline{2^-}$
2.30	$pp[633\uparrow, 512\uparrow]$	F-1,F+3	6^-	$\underline{1^-}$
1.30	$nn[734\uparrow, 622\uparrow]$	F,F-2	7^-	$\underline{2^-}$
1.32	$nn[624\downarrow, 734\uparrow]$	F,F+1	$\underline{8^-}$	$\underline{1^-}$
2.08	$nn[624\downarrow, 743\uparrow]$	F,F-2	7^-	$\underline{0^-}$
2.33	$nn[622\uparrow, 620\uparrow]$	F-1,F+2	$\underline{3^+}$	$\underline{2^+}$
2.34	$nn[624\downarrow, 620\uparrow]$	F,F+2	$\underline{4^+}$	3^+
		^{254}No		
1.15	$pp[521\downarrow, 514\downarrow]$	F,F+1	$\underline{4^+}$	$\underline{3^+}$
1.38	$pp[633\uparrow, 514\downarrow]$	F-1,F+1	7^-	$\underline{0^-}$
1.38	$pp[633\uparrow, 521\downarrow]$	F-1,F	4^-	$\underline{3^-}$
2.05	$pp[521\uparrow, 521\downarrow]$	F-2,F	$\underline{2^+}$	1^+
2.27	$pp[521\uparrow, 633\uparrow]$	F-2,F-1	5^-	$\underline{2^-}$
2.43	$pp[633\uparrow, 512\uparrow]$	F-1,F+3	6^-	$\underline{1^-}$
1.21	$nn[734\uparrow, 622\downarrow]$	F,F+2	6^-	$\underline{3^-}$
1.32	$nn[622\uparrow, 620\uparrow]$	F-1,F+1	$\underline{2^+}$	1^+
1.78	$nn[734\uparrow, 613\uparrow]$	F,F+3	8^-	$\underline{1^-}$
1.89	$nn[620\uparrow, 613\uparrow]$	F+1,F+3	$\underline{4^+}$	$\underline{3^+}$
2.13	$nn[622\uparrow, 734\uparrow]$	F-1,F	7^-	$\underline{2^-}$
2.17	$nn[734\uparrow, 615\downarrow]$	F,F+5	9^-	$\underline{0^-}$

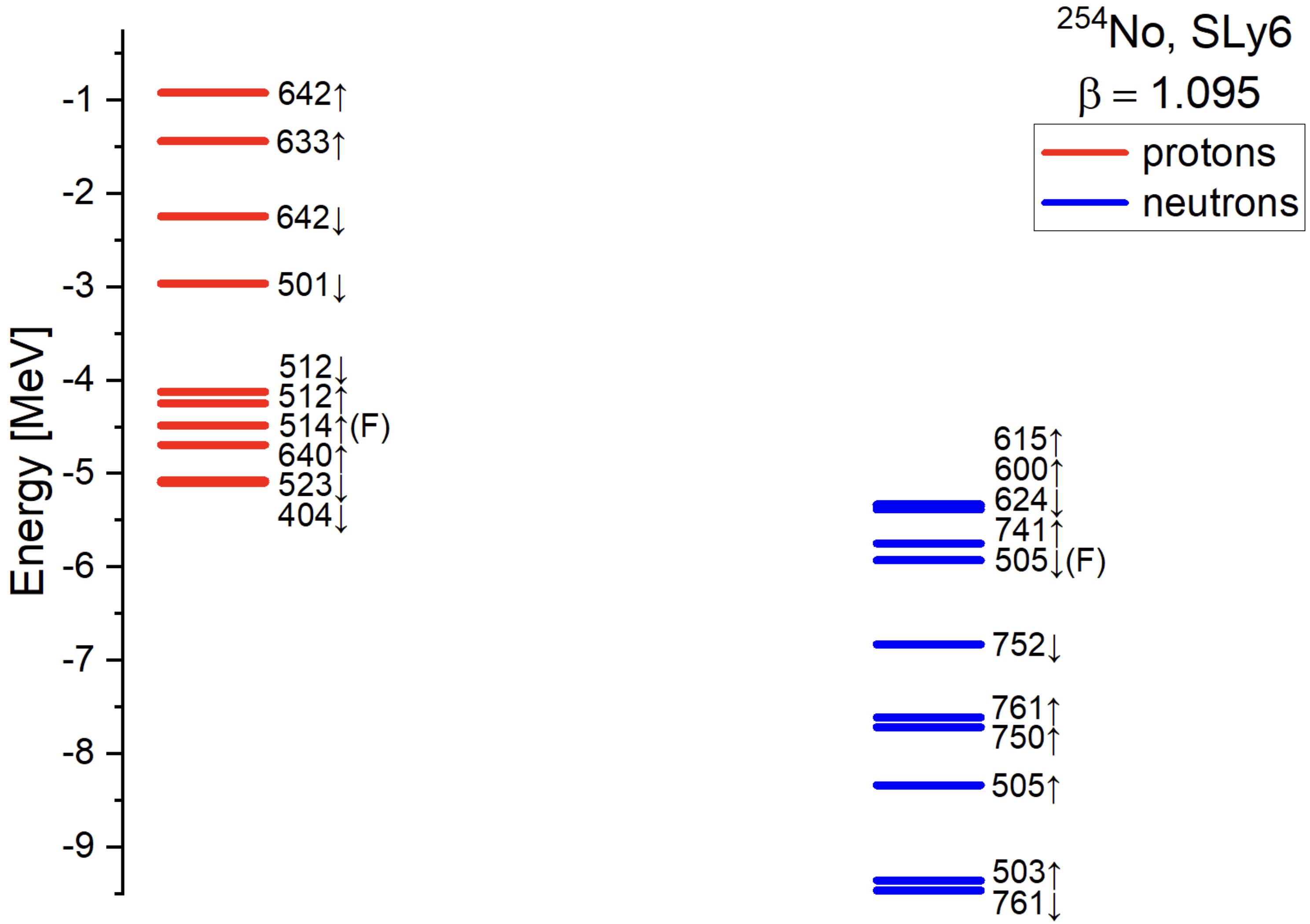




^{250}No, SLy6
$\beta = 1.033$
protons
neutrons
Energy [MeV]
-1
-2
-3
-4
-5
-6
-7
-8
-9
642↑
642↓
633↑
501↓
640↑
512↑
512↓(F)
514↑
510↑
404↓
624↓
613↓
741↑
615↑
752↓
505↓
761↑(F)
750↑
761↓
505↑
622↑



^{252}No, SLy6
β = 1.096
protons
neutrons
Energy [MeV]
-1
-2
-3
-4
-5
-6
-7
-8
-9
642↑
633↑
642↓
501↓
512↓
512↑
514↑(F)
523↓
404↓
640↑
624↓
615↑
600↑
741↑
505↓
752↓(F)
761↑
750↑
505↑
503↑
611↓



^{254}No, SLy6
β = 1.095
protons
neutrons
Energy [MeV]
-1
-2
-3
-4
-5
-6
-7
-8
-9
642↑
633↑
642↓
501↓
512↓
512↑
514↑(F)
640↑
523↓
404↓
615↑
600↑
624↓
741↑
505↓(F)
752↓
761↑
750↑
505↑
503↑
761↓