# Half-lives of Proton Emission and Alpha Decay

Yongbeom Choi<sup>1</sup>, Youngman Kim<sup>2</sup>, Chang-Hwan Lee<sup>3</sup>, Toshitaka Kajino<sup>1,4,5</sup>

<sup>1</sup>School of Physics, Peng Huanwu Collaborative Center for Research and Education, and International Research Center for Big-Bang Cosmology and Element Genesis,

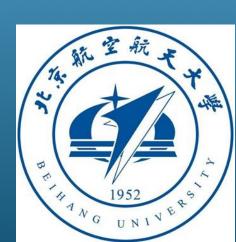
Beihang University, (37 Xueyuan Road, Haidian District,) Beijing 100191, P. R. China

<sup>2</sup>Center for Exotic Nuclear Studies, Institute for Basic Science, Daejeon 34126, Korea

<sup>3</sup>Department of Physics and Center for Innovative Physicist Education and Research, Pusan National University, Busan 46241, Korea

National Astronomical Observatory of Japan, 2–21–1 Osawa, Mitaka, Tokyo 181–8588, Japan

Center for Nuclear Study, The University of Tokyo, RIKEN campus, 2–1 Hirosawa, Wako, Saitama 351–0198, Japan



## Introduction

Understanding the half-lives of proton emission and alpha decay is important to understand nuclear structure near the proton drip line.

Relativistic continuum Hartree-Bogoliubov (RCHB) theory was developed to predict both exotic and stable nuclei by incorporating continuum effects into the self-consistent Hartree-Bogoliubov framework [1]. As an extension of the RCHB, Deformed relativistic Hartree-Boboliubov theory in continuum (DRHBc) was developed, which takes axial degrees of freedom into account [2-4]. In this study, we calculate the proton emission half-lives of  $71 \le Z \le 83$  nuclei predicted by DRHBc, using the semiclassical WKB approximation. We compare our results with available experimental data as well as previous calculations from RCHB [5], and discuss the differences. Finally, we calculate the alpha-decay half-lives of protonemitting nuclei, where possible, and compare them with the corresponding proton emission half-lives.

## WKB Framework

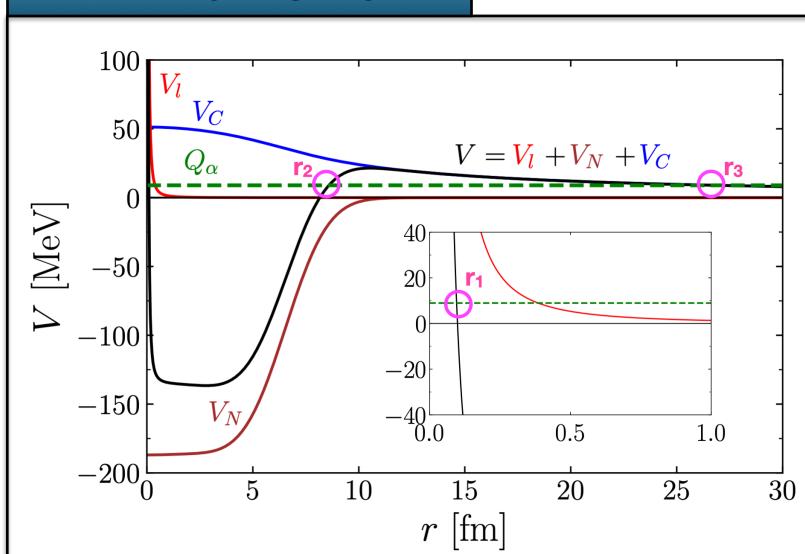


Fig. 1. Potential between daughter particle and a particle [6]. In WKB approximation framework, the half-live is calculated by considering the probability that the particles are formed and penetrates the Coulomb barrier. The time scale of the half-life is determined by the particle penetration probability of Coulomb barrier from  $r_2$  to  $r_3$ .

#### Formulae

$$T = \hbar \ln 2/\Gamma$$

$$\Gamma = P_x N_f(k) \frac{\hbar^2}{4\mu} P_{\text{penetration}}(k)$$

$$k = \sqrt{\frac{2\mu}{\hbar^2} |Q - (V_l + V_N + V_C)|}$$
DRHBc or RCHB

In proton emission, the  $V_{\rm N}$  is the nuclear potential of parent nucleus and  $P_x$  is the spectroscopic factor  $(S_{\rm p})$ . We estimate  $S_{\rm p}$  by decomposing the wave function into spherical components [7].

In alpha decay, the  $V_{\rm N}$  is the nucleus-nucleus double-folding potential and  $P_x$  is the probability of the alpha particle formation in the parent nucleus  $(P_{\alpha})$ . We estimate  $P_{\alpha}$  by adopting cluster formation model  $P_{\alpha} = P_{\alpha}(E_{\rm b})$  [8,9].

# Results

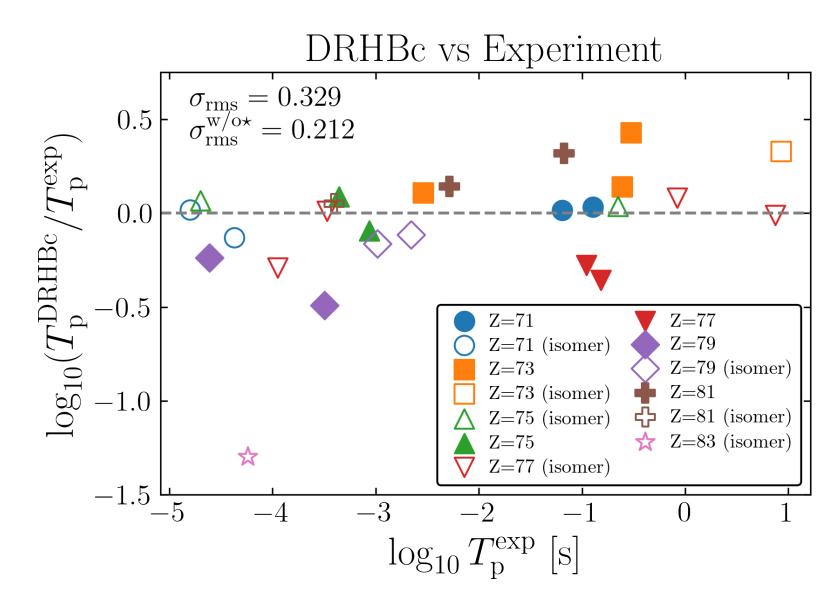


Fig. 2. Comparison of proton emission half-lives between DRHBc and experimental data [10]. Except <sup>185m</sup>Bi, the differences are all below 0.55.

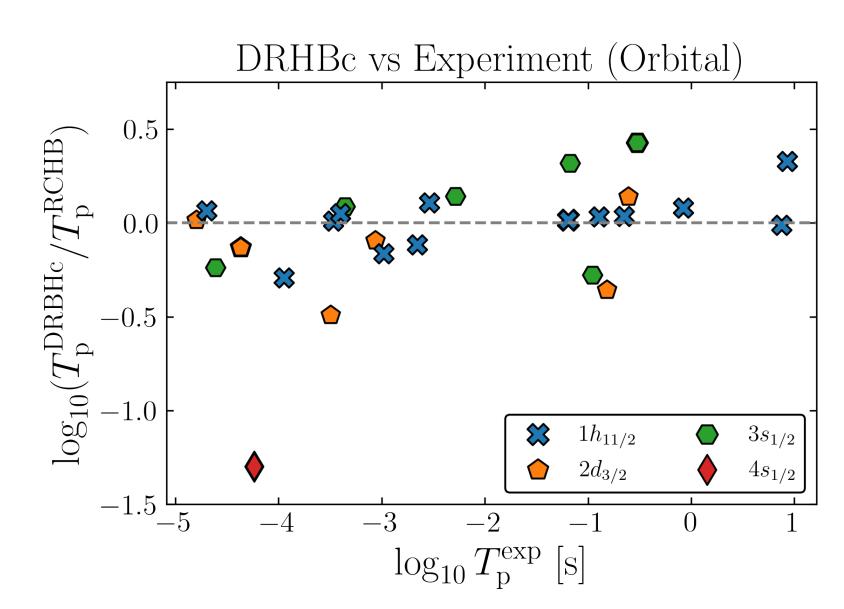


Fig. 5. Same as Fig. 2, but the markers are distinguished by color and shape according to the emitted proton orbital.

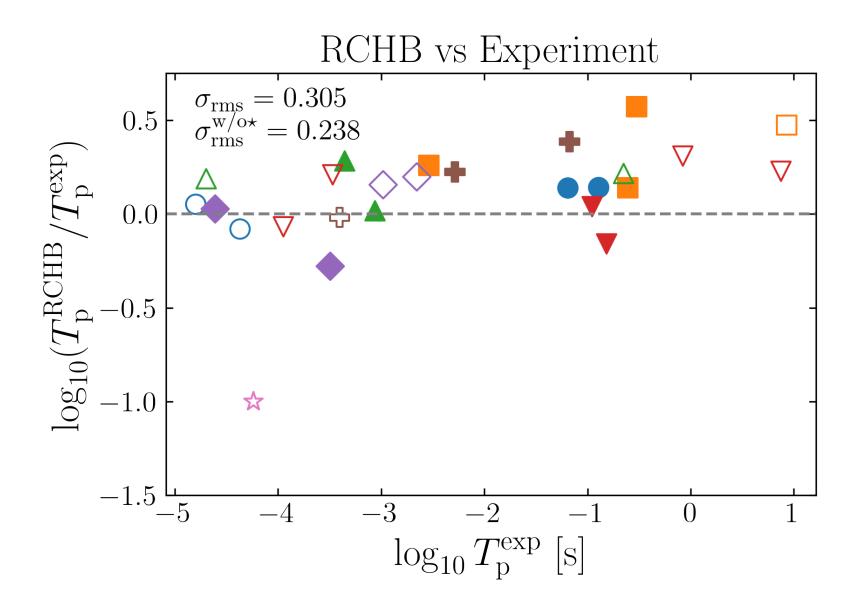


Fig. 3. Comparison of proton emission half-lives between RCHB and experimental data.

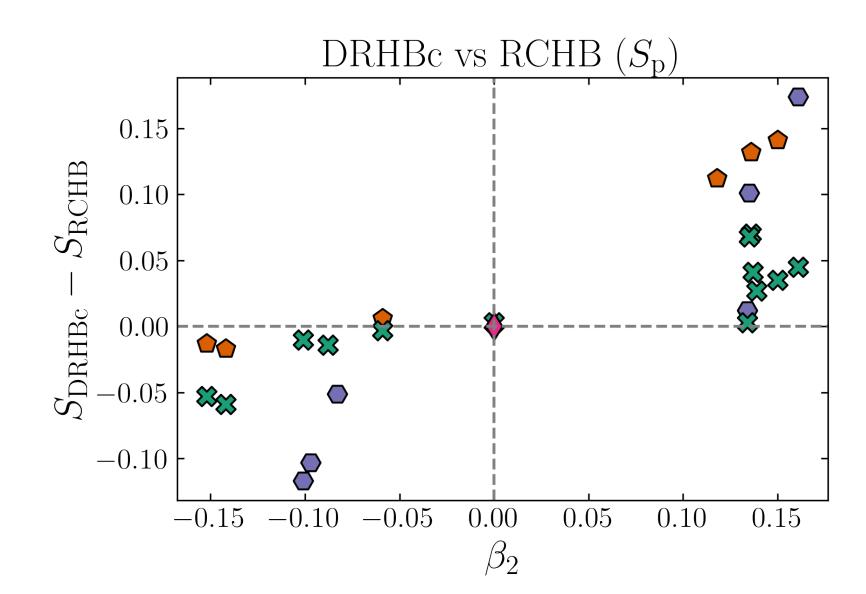


Fig. 6. Comparison of predicted spectroscopic factor from DRHBc and RCHB.

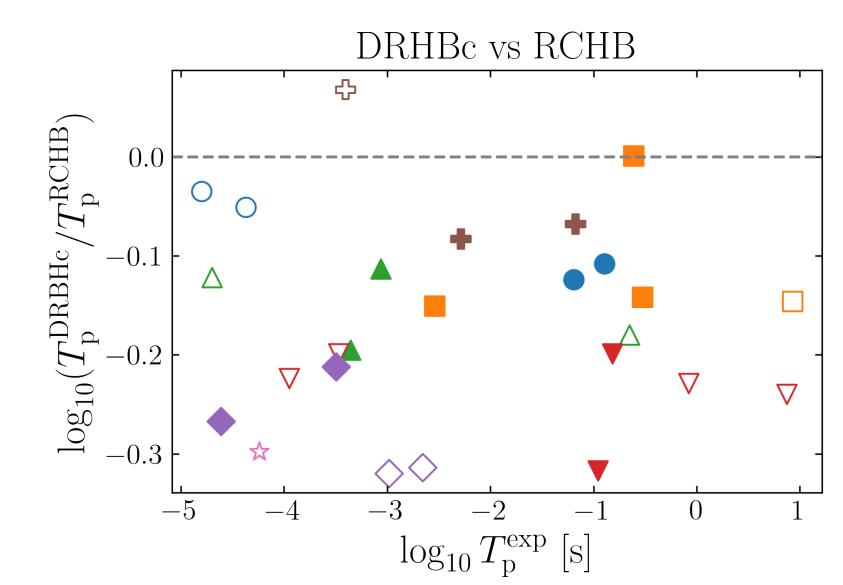


Fig. 4. Comparison of proton emission half-lives between DRHBc and RCHB. Half-lives calculated with DRHBc are shorter than those with RCHB.

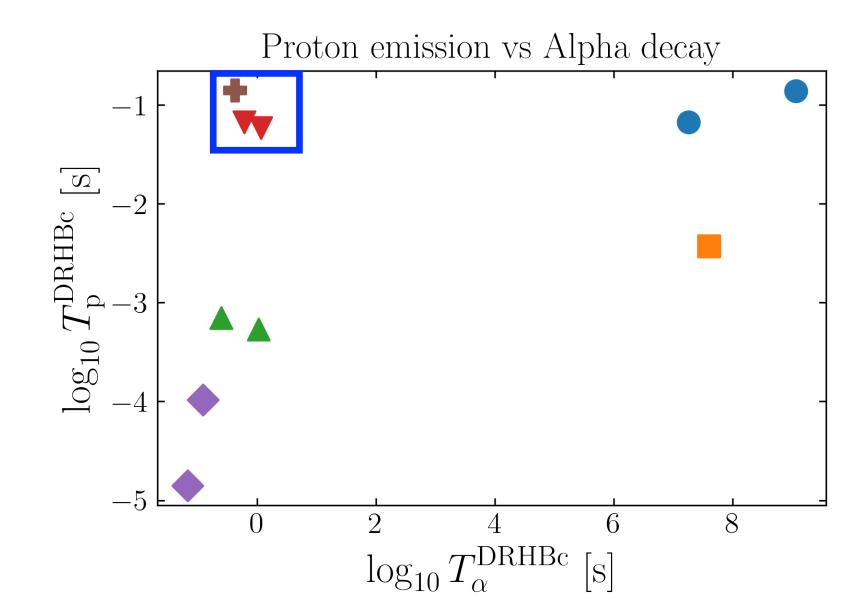


Fig. 7. Half-lives of proton emission vs alpha decay in DRHBc. The half-lives of <sup>166,167</sup>Ir and <sup>177</sup>Tl are comparable, as highlighted in the blue box.

#### Discussion

- Our results also show good agreement with experimental data for isomer states.
- Except for <sup>185m</sup>Bi (empty star marker), the half-lives with DRHBc agree better with experimental data than those with RCHB.
- The spectroscopic factor  $S_p = 1$  of  $^{185\text{m}}\text{Bi}$   $(4s_{1/2})$  may contribute to the discrepancy between calculated and experimental half-lives.
- The half-lives with DRHBc are shorter than those with RCHB.
- The calculated half-lives tend to be overestimated at large experimental values, possibly implying that the decay width is underestimated at small values.
- When comparing  $S_p$ , it tends to be higher for nuclei with a prolate shape  $(\beta_2 > 0)$ .
- For the selected nuclei, proton emission is generally more dominant than alpha decay, except for  $^{166,167}$ Ir and  $^{177}$ Tl.

# References

- [1] X. W. Xia *et al*, At. Data Nucl. Data Tables **121**, 1 (2018).
- [2] K. Zhang *et al*, Phys. Rev. C **102**, 024314 (2020).
- [3] C. Pan *et al*, Phys. Rev. C **106**, 014316 (2022).
- [4] P. Guo et al, At. Data Nucl. Data Tables 158, 101661 (2024).
- [5] Y. Lim *et al*, Phys. Rev. C **93**, 014314 (2016).
- [6] Y. Choi et al, Phys. Rev. C 109, 054310 (2024).
- [7] X.-X. Sun and S.-G. Zhou, Sci. Bull. **66**, 2072 (2021).
- [8] S. M. S. Ahmed *et al*, J. Phys. G **40**, 065105 (2013).
- [9] S. M. S. Ahmed *et al*, Eur. Phys. J. A **51**, 13 (2015).
- [10] B. Blank and M. Borge, Prog. Part. Nucl. Phys. **60**, 403 (2008).