GENERATING COSMOLOGICAL PERTURBATIONS AT HORNDESKI BOUNCE

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Motivation

1. The search of non-singular alternatives to inflation seems as an important problem;
2. We study bounce epoch as such alternative/completion to/of inflation.

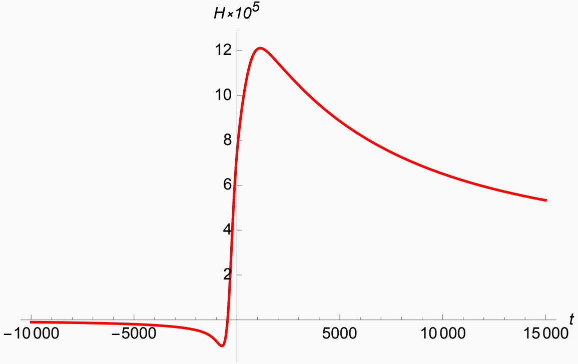


Figure 1: Hubble parameter: bounce

Qui'2011,2013; Easson'2011; Cai'2012;
Osipov'2013; Koehn'2013; Battarra'2014; Ijjas'2016

Null Energy Condition

Realization of non-singular evolution within classical field theory requires the violation of the Null Energy Condition (NEC) $T_{\mu\nu}n^\mu n^\nu > 0$ (or Null Convergence Condition (NCC) $R_{\mu\nu}n^\mu n^\nu > 0$ for modified gravity).

$$T_{00} = \rho, \quad T_{ij} = a^2\gamma_{ij}p,$$

$$\dot{H} = -4\pi G(\rho + p) + \sout{\text{curvature term}}.$$

Let us use $n_\mu = (1, a^{-1}\nu^i)$ with $\gamma_{ij}\nu^i\nu^j = 1$ and then NEC leads to

$$T_{\mu\nu}n^\mu n^\nu > 0 \to \rho + p \geq 0 \to \dot{H} \leq 0.$$

Penrose theorem: singularity in the past.

Horndeski theory

Violation of NEC/NCC without obvious pathologies is possible in the class of Horndeski theories [*Horndeski'74*]:

$$\begin{aligned}\mathcal{L}_H = {} & G_2(\phi, X) - G_3(\phi, X)\Box\phi + \\ & G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ & + G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi \\ & - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right],\end{aligned}$$

where $X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ and $\Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi$. For our purposes it is enough to study

$$\mathcal{L}_H = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi)R.$$

In the framework of this theory one can (quite straightforwardly) obtain healthy bounce epoch.

Another problem arises if one considers the whole evolution $(-\infty < t < +\infty)$ of such a singularity-free universe: instabilities show up at some moment in the history $\rightarrow$ No-Go theorems. *M. Libanov, S. Mironov, V. Rubakov'2016; T. Kobayashi'2016; S. Mironov, V. Rubakov, V. Volkova'2018*.

Let us consider the following perturbed ADM metric:

$$ds^2 = -N^2 dt^2 + \gamma_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right),$$

$$\gamma_{ij} = a^2 e^{2\zeta}(\delta_{ij} + h_{ij} + \ldots), \quad N = N_0(1+\alpha), \quad N_i = \partial_i \beta.$$

Here α and β are not physical. We work with unitary gauge $\delta\phi = 0$.

The quadratic actions for ζ and h_{ij} are given, respectively:

$$\mathcal{L}_{\zeta\zeta} = a^3\left[\mathcal{G}_S \frac{\dot{\zeta}^2}{N^2} - \frac{\mathcal{F}_S}{a^2}\zeta_{,i}\zeta_{,i}\right], \; \mathcal{L}_{hh} = \frac{a^3}{8}\left[\mathcal{G}_T \frac{\dot{h}_{ij}^2}{N^2} - \frac{\mathcal{F}_T}{a^2} h_{ij,k} h_{ij,k}\right].$$

Remind that bounce solution is $a(t) \to \infty$ as $t \to -\infty$. No-Go works if

$$\int_{-\infty}^{t} a(t)(\mathcal{F}_T + \mathcal{F}_S)dt = \infty,$$

$$\int_{t}^{+\infty} a(t)(\mathcal{F}_T + \mathcal{F}_S)dt = \infty.$$

No-Go: $\mathcal{F}_{S,T} < 0$ at some moment of time, instability.

No-Go theorem

- One way is to go beyond Horndeski and DHOST [Cai et.al.' 2016, Creminelli et.al.'2016, Kolevatov et.al.'2017, Cai, Piao'2017]
- Another way to avoid No-Go theorem for Horndeski is to obtain such a model/solution that $\mathcal{F}_{S,T}$ coefficients have asymptotics

$$\mathcal{F}_{S,T} \to 0 \text{ as } t \to -\infty, \text{ where } \mathcal{F}_T = 2G_4.$$

- This means that

$$G_4 \to 0 \text{ as } t \to -\infty.$$

- Effective Planck mass goes to zero and it signalizes that we may have strong coupling at $t \to -\infty$.

Solution: no SC regime at $t \to -\infty$ in some region of lagrangian parameters.

With the appropriate choice of lagrangian functions, the bounce solution is given by

$$N = \text{const}\ , \qquad a = d(-t)^{\chi}\ ,$$

where $\chi > 0$ is a constant and $Nt \to t$ is cosmic time, so that $H = \chi/t$. Coefficients from quadratic actions are

$$\mathcal{G}_T = \mathcal{F}_T = \frac{g}{(-t)^{2\mu}},$$

and

$$\mathcal{G}_S = g\frac{g_S}{2(-t)^{2\mu}}\ , \qquad \mathcal{F}_S = g\frac{f_S}{2(-t)^{2\mu}},$$

$$u_T^2 = \frac{\mathcal{F}_T}{\mathcal{G}_T} = 1, \quad u_S^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S} = \frac{f_S}{g_S} \neq 1.$$

To avoid No-Go:

$$1 > \chi > 0,\ 2\mu > \chi + 1.$$

To avoid SC regime ($t \to -\infty$):

$$\mu < 1.$$

Power spectrum

Spectra are given by

$$\mathcal{P}_\zeta \equiv \mathcal{A}_\zeta \left(\frac{k}{k_*}\right)^{n_s-1} \quad , \qquad \mathcal{P}_T \equiv \mathcal{A}_T \left(\frac{k}{k_*}\right)^{n_T} \quad ,$$

where k_* is pivot scale, the spectral tilts are

$$n_S - 1 = n_T = 2 \cdot \left(\frac{1-\mu}{1-\chi}\right),$$

$$n_S = 0.9649 \pm 0.0042.$$

The amplitudes in our model are

$$\mathcal{A}_\zeta = \frac{C}{g}\frac{1}{g_S u_S^{2\nu}} \, , \; \mathcal{A}_T = \frac{8C}{g},$$

where

$$\nu = \frac{1+2\mu-3\chi}{2(1-\chi)} = \frac{3}{2} + \frac{1-n_S}{2} \;\approx \frac{3}{2},$$

approximate flatness is ensured in our set of models by choosing $\mu \approx 1$, while the slightly red spectrum is found for $\mu > 1$.

The problem №1: red-tilted spectrum requires $\mu > 1$, while absence of strong coupling $\mu < 1$!

Solution: consider time-dependent μ: changes from $\mu < 1$ to $\mu > 1$ (time runs as $-\infty < t < \infty$).

Try to escape from SC and generate spectrum, consistent with experiment. Horizon exit must occur in weak coupling regime!

The problem №2: r-ratio is small:

$$r = \frac{\mathcal{A}_T}{\mathcal{A}_\zeta} \approx 8 g_S u_S^3 < 0.032.$$ *Tristram'2022*

Solution: choose $u_S \ll 1$. *Mukhanov'1999, 2000*, k-inflation

Cubic action for tensors

$$\mathcal{S}^{(3)}_{TTT} = \int dt\, a^3 d^3x \left[\frac{\mathcal{F}_T}{4a^2}\left(h_{ik}h_{jl} - \frac{1}{2}h_{ij}h_{kl}\right)h_{ij,kl}\right] .$$

Corresponding SC and classical scales are

$$E^{TTT}_{strong} \sim \frac{\mathcal{G}_T^{3/2}}{\mathcal{F}_T} = \frac{g^{1/2}}{|t|^{\mu}} , \quad E_{cl} \sim H \sim |t|^{-1},$$

thus we obtain for $E^{TTT}_{strong} > E_{cl}$:

$$|t|^{2\mu-2} < g .$$

Tensors exit (effective) horizon:

$$t_f^{(T)}(k) \sim \left(\frac{d}{k}\right)^{\frac{1}{1-\chi}}$$

so the absence of SC at $t = t_f$

$$\frac{1}{g}\left(\frac{d}{k}\right)^{2\frac{\mu-1}{1-\chi}} \sim \mathcal{A}_T \ll 1.$$

Cubic action for scalars

$$\mathcal{S}^{(3)}_{\zeta\zeta\zeta} = \int dt\, d^3x \Lambda_\zeta \partial^2\zeta \left(\partial_i\zeta\right)^2 \ ,$$

$$E^{\zeta\zeta\zeta}_{strong} \sim \Lambda_\zeta (\mathcal{G}_S)^{-3/2} u_S^{-11/2} \sim \frac{1}{|t|}\left(\frac{g^{1/2}u_S^{11/2}}{|t|^{\mu-1}}\right)^{1/3} \ ,$$

thus we obtain for $E^{\zeta\zeta\zeta}_{strong} > E_{cl}$:

$$\left(\frac{g u_S^{11}}{|t|^{2(\mu-1)}}\right)^{1/6} > 1\,.$$

Scalars exit (effective) horizon:

$$t_f^{2(\mu-1)} \sim g\mathcal{A}_\zeta u_S^3\,.$$

$$\left(\frac{g u_S^{11}}{|t_f(k_{min})|^{2(\mu-1)}}\right)^{1/6} \sim \left(\frac{u_S^8}{\mathcal{A}_\zeta}\right)^{1/6} \sim \left(\frac{r^{8/3}}{\mathcal{A}_\zeta}\right)^{1/6} ,$$

$$\left(\frac{r^{8/3}}{\mathcal{A}_\zeta}\right)^{1/6} > 1\,.$$

Strong coupling and r-ratio

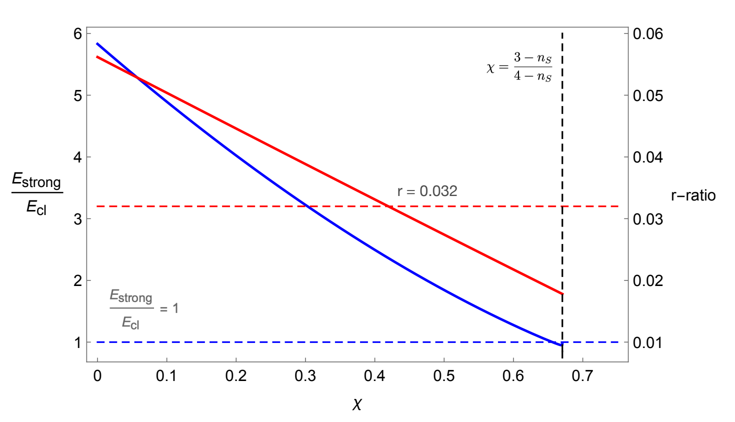


Figure 2: The r-ratio (red line) and ratio $E_{strong}(k_*)/E_{cl}(k_*)$ (blue line) as functions of χ for the central value $n_S = 0.9649$.

- We construct the model of bounce, within one can generate nearly flat (red-tilted) power spectrum of scalar perturbations. But it is not so automatic as in inflation!
- In such models the requirement of strong coupling absence leads to the fact that the r-ratio cannot be arbitrarily small and, moreover, it is close to the boundary $r < 0.032$ suggested by the observational data.

Thank you for attention!

Coefficients $\mathcal{F}_S, \mathcal{G}_S, \mathcal{F}_T, \mathcal{G}_T$ are given by:

$$\mathcal{F}_T = 2G_4 + ..., \quad \mathcal{G}_T = 2G_4 + ...,$$

and

$$\mathcal{F}_S = \frac{1}{a}\frac{d}{dt}\left(\frac{a}{\Theta}\mathcal{G}_T^2\right) - \mathcal{F}_T, \quad \mathcal{G}_S = \frac{\Sigma}{\Theta^2}\mathcal{G}_T^2 + 3\mathcal{G}_T,$$

where Σ and Θ are some cumbersome expression of G_2, G_3, G_4 and H.
Stability conditions are:

$$\mathcal{G}_T \geq \mathcal{F}_T > 0, \quad \mathcal{G}_S \geq \mathcal{F}_S > 0.$$

Denote $\xi = a\mathcal{G}_T^2/\Theta$, we rewrite $\mathcal{F}_S$ as

$$\mathcal{F}_S = \frac{1}{a}\frac{d\xi}{dt} - \mathcal{F}_T \to \frac{d\xi}{dt} > a\mathcal{F}_T > 0$$

$$\frac{d\xi}{dt} > a\mathcal{F}_T > 0, \quad \xi = a\mathcal{G}_T^2/\Theta,$$

Here $|\Theta| < \infty$ everywhere and it is smooth function of time (as it is function of ϕ and H), so ξ can never vanish (except $a = 0$) $\rightarrow$ thus we demand non-singular model. Integrating from some t_i to t_f, we obtain:

$$\xi(t_f) - \xi(t_i) > \int_{t_i}^{t_f} a(t)\mathcal{F}_T dt,$$

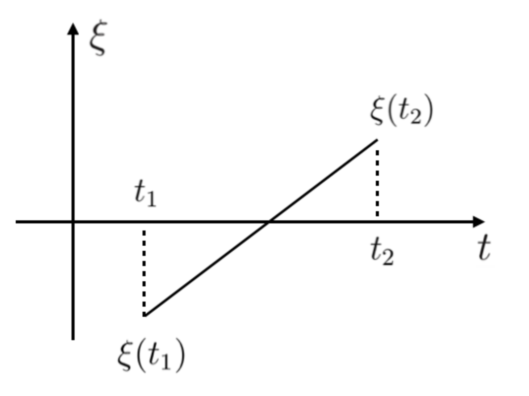
where $a > const > 0$ for $t \rightarrow -\infty$ and it is increasing with $t \rightarrow +\infty$.

$$\xi(t_f) - \xi(t_i) > \int_{t_i}^{t_f} a(t)\mathcal{F}_T dt,$$

- Let $\xi_i < 0$, so
$$-\xi_f < |\xi_i| - \int_{t_i}^{t_f} a\mathcal{F}_T dt,$$
where RHS $\to$ negative with $t_f \to +\infty$. So therefore $\xi_f > 0$. And it means that $\xi = 0$ at some moment of time - singularity! So we should demand $\xi > 0$ for all times.
- But on the other had, again just rewritting:
$$-\xi_i > -\xi_f + \int_{t_i}^{t_f} a\mathcal{F}_T dt,$$
and now RHS $\to$ positive with $t_i \to -\infty$ and ξ_i must be negative. Again contradiction...

Thus we have two important features here:

$$1.\xi \neq 0,$$
$$2.d\xi/dt > a\mathcal{F}_T > 0.$$



$$\xi(t_f) - \xi(t_i) > \int_{t_i}^{t_f} a(t)\mathcal{F}_T dt,$$

$$G_2 = A_2 - 2XF_\phi,$$
$$G_3 = -2XF_X - F,$$
$$G_4 = B_4,$$

where $F(\phi, X)$ is an auxiliary function, such that

$$F_X = -\frac{A_3}{(2X)^{3/2}} - \frac{B_{4\phi}}{X},$$

with

$$N^{-1} d\phi/dt = \sqrt{2X}.$$

EoMs are

$$(NA_2)_N + 3NA_{3N}H + 6N^2(N^{-1}A_4)_N H^2 = 0,$$
$$A_2 - 6A_4H^2 - \frac{1}{N}\frac{d}{d\hat{t}}(A_3 + 4A_4H) = 0\ .$$

Let us move to ADM formalism now:

$$\mathcal{L} = A_2(t,N) + A_3(t,N)K + A_4(K^2 - K_{ij}^2) + B_4(t,N)R^{(3)}.$$

We remind that we have unitary gauge $\phi = \phi(t)$. ${}^{(3)}R_{ij}$ is the Ricci tensor made of γ_{ij}, $\sqrt{-g} = N\sqrt{\gamma}$, $K = \gamma^{ij}K_{ij}$, ${}^{(3)}R = \gamma^{ij}\ {}^{(3)}R_{ij}$ and

$$K_{ij} \equiv \frac{1}{2N}\left(\frac{d\gamma_{ij}}{dt} - {}^{(3)}\nabla_i N_j - {}^{(3)}\nabla_j N_i\right),$$

At $t \to -\infty$

$$A_2(t,N) = g(-t)^{-2\mu-2} \cdot a_2(N), \quad a_2(N) = c_2 + \frac{d_2}{N}$$
$$A_3(t,N) = g(-t)^{-2\mu-1} \cdot a_3(N), \quad a_3(N) = c_3 + \frac{d_3}{N},$$
$$A_4(t) = -B_4(t) = -\frac{g}{2}(-t)^{-2\mu} \ .$$

$$f_S = \frac{2(2 - 4\mu + N^2 a_{3N})}{2\chi - N^2 a_{3N}},$$

$$g_S = 2\left[\frac{2\Big(2N^3 a_{2N} + N^4 a_{2NN} - 3\chi(2\chi + N^3 a_{3NN})\Big)}{(N^2 a_{3N} - 2\chi)^2} + 3\right],$$

$$f_S = -2\left(\frac{4\mu - 2 + d_3}{2\chi + d_3}\right),$$

$$g_S = \frac{6d_3^2}{(2\chi + d_3)^2}.$$

$$d_3 = -2,$$

$$f_S = \frac{4(\mu - 1)}{1 - \chi} = 2(1 - n_S),$$

$$g_S = \frac{6}{(1 - \chi)^2}.$$

$$\zeta = \frac{1}{(2\mathcal{G}_S a^3)^{1/2}} \cdot \psi,$$

$$\mathcal{S}^{(2)}_{\psi\psi} = \int d^3x dt \left[\frac{1}{2}\dot{\psi}^2 + \frac{1}{2}\frac{\ddot{\alpha}}{\alpha}\psi^2 - \frac{u_S^2}{2a^2}(\vec{\nabla}\psi)^2\right],$$

$$\alpha = \left(2\mathcal{G}_S a^3\right)^{1/2} = \frac{\text{const}}{(-t)^{\frac{2\mu-3\chi}{2}}} \, .$$

$$\psi_{WKB} = \frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{2\omega}} \cdot \mathrm{e}^{-i\int \omega dt} = \frac{1}{(2\pi)^{3/2}}\sqrt{\frac{d}{2u_S k}}(-t)^{\chi/2} \cdot \mathrm{e}^{i\frac{u_S}{d}\frac{k}{1-\chi}(-t)^{1-\chi}},$$

$$\omega = \frac{u_S k}{a} = \frac{u_S \cdot k}{d(-t)^\chi}.$$

Power spectra

$$\zeta = \mathfrak{C} \cdot (-t)^{\delta} \cdot H_{\nu}^{(2)} \left(\beta(-t)^{1-\chi}\right),$$

$$\delta = \frac{1 + 2\mu - 3\chi_1}{2},$$
$$\beta = \frac{u_S k}{d(1-\chi)},$$
$$\nu = \frac{\delta}{\gamma} = \frac{1 + 2\mu - 3\chi}{2(1-\chi)},$$

$$\mathfrak{C} = \frac{1}{(g g_S)^{1/2}} \frac{1}{2^{5/2}\pi(1-\chi)^{1/2}} \frac{1}{d^{3/2}},$$
$$\zeta = (-i)\frac{\mathfrak{C}}{\sin(\nu\pi)} \frac{(1-\chi)^{\nu}}{u_S^{\nu}\Gamma(1-\nu)} \left(\frac{2d}{k}\right)^{\nu},$$
$$\mathcal{P}_{\zeta} = 4\pi k^3 \zeta^2.$$

Space of parameters n_S and χ

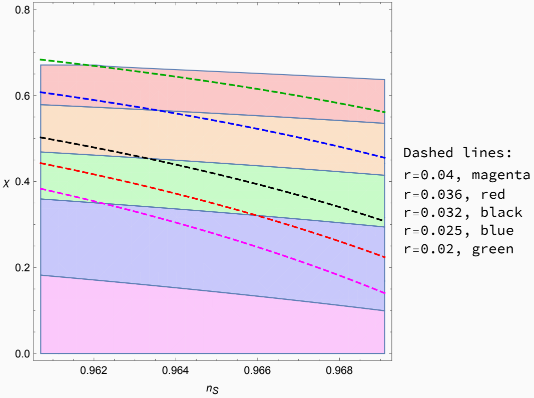


Figure 3: Space of parameters n_S and χ. Colored strips correspond to different ratios of strong coupling scale to classical scale:
$1 < E_{strong}(k_*)/E_{cl}(k_*) < 1.5$ (red), $1.5 < E_{strong}(k_*)/E_{cl}(k_*) < 2.2$ (orange), $2.2 < E_{strong}(k_*)/E_{cl}(k_*) < 3$ (green), $3 < E_{strong}(k_*)/E_{cl}(k_*) < 4.5$ (blue), $4.5 < E_{strong}(k_*)/E_{cl}(k_*)$ (magenta).

SPACE OF PARAMETERS ϵ AND χ: $\mu = 1$

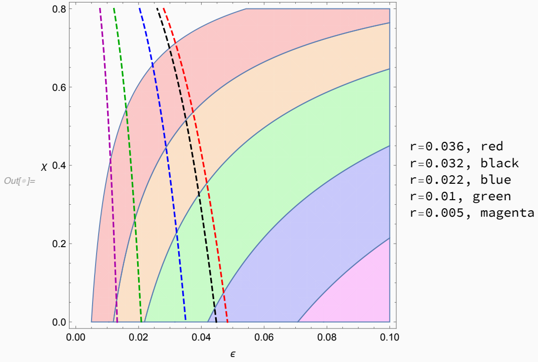


Figure 4: Space of parameters ϵ and χ in the case $\mu = 1$. Colored strips correspond to different ratios of strong coupling scale to classical scale: $1 < E_{strong}/E_{cl} < 1.8$ (red), $1.8 < E_{strong}/E_{cl} < 2.7$ (orange), $2.7 < E_{strong}/E_{cl} < 4.2$ (green), $4.2 < E_{strong}/E_{cl} < 6$ (blue), $6 < E_{strong}/E_{cl}$ (magenta).