

# Coleman-Weinberg Higgs

IBS-Honam Focus Program  
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D Chway, R Dermisek, TH Jung, HDK, PRL (2014)  
D Chway, R Dermisek, TH Jung, HDK, JKPS (2016)  
D Chway, R Dermisek, TH Jung, HDK, in preparation

What if there is no mass term  
from the beginning?

$$m^2 = 0$$

This condition was shown to lead to symmetry breaking through radiative corrections. ... This argument is quite **speculative**, particularly since **no theory has even been found** in which the “zero bare mass” condition is really natural. But **it is interesting to pursue the consequences** of assuming that scalars satisfying this condition exist. ...

E. Witten (1981)

The final **blunder** was a claim that scalar elementary particles were unlikely to occur in elementary particle physics at currently measurable energies unless they were associated with some kind of broken symmetry. ... But this claim makes no sense when one becomes familiar with the history of physics. ...

This blunder **was potentially more serious**, if it caused any subsequent researchers to dismiss possibilities for very larger or very small values for parameters that now must be taken seriously. ... The lesson from history is that sometimes there is a need to **consider seriously a seemingly unlikely possibility**.

K. Wilson (2003)

What is important in science is not the solution of some particular scientific problems of one's own day, but understanding the world. ...

The Alexandrians **concentrated on understanding specific phenomena**, where **real progress** could be made. ...

Again and again, it has been an essential feature of scientific progress to understand which problems are ripe for study and which are not. ...

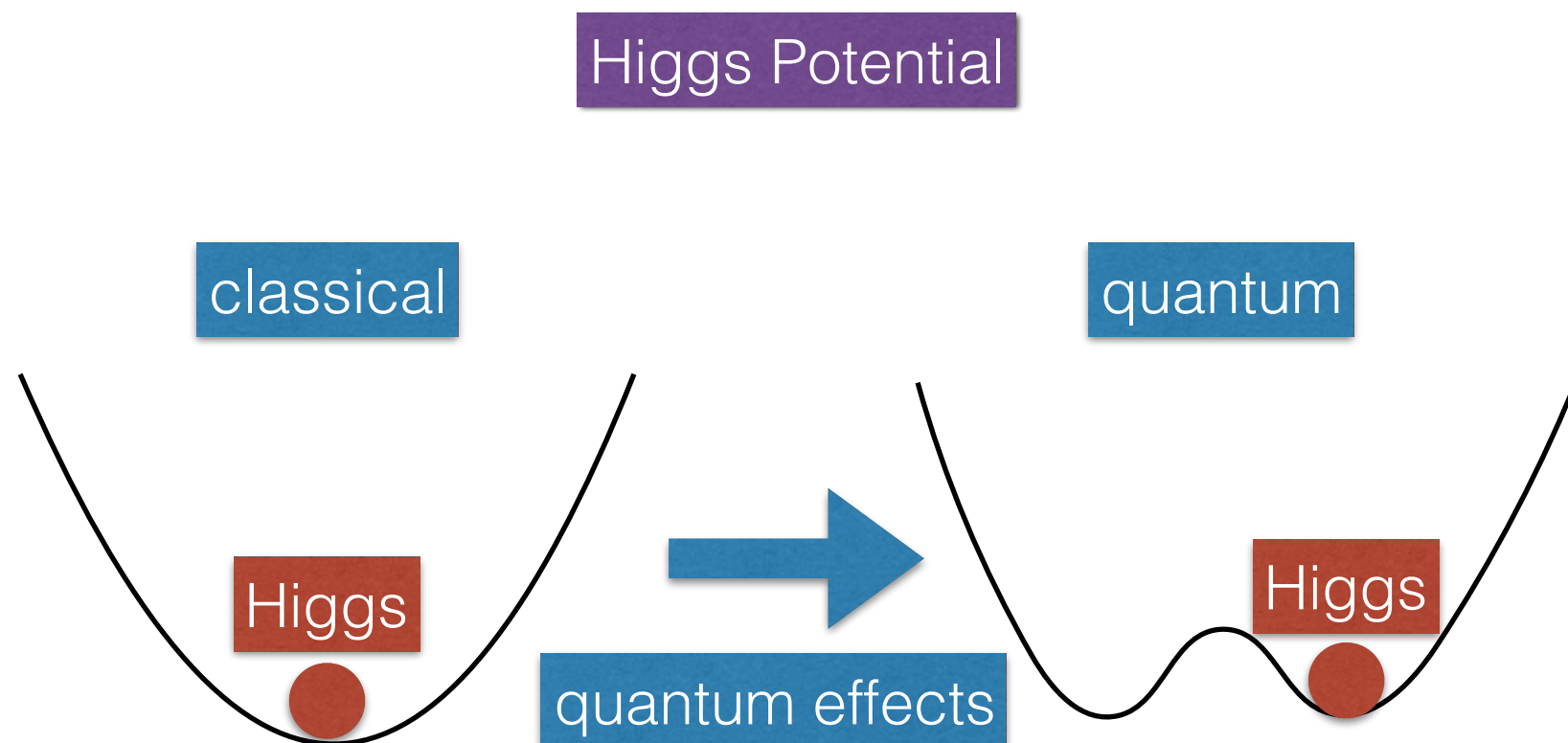
S. Weinberg (2015)

One simple question.  
Can we still make electroweak symmetry  
breaking possible without mass term?



# Coleman-Weinberg Higgs

with D Chway, R Dermisek and TH Jung, PRL(2014)



start from classically scale invariant theory

# Higgs self coupling in the SM

$$V(H) = -\mu^2 |H|^2 + \lambda |H|^4$$

$$\mu^2 = \lambda v^2 \quad \text{at the minimum of the potential}$$

$$m_h^2 = 2\lambda v^2$$

All three  
definitions  
give the  
same  
quartic  
coupling.

$$\lambda_{\text{eff}}^{(2)SM} = \frac{m^2}{2v^2} \longrightarrow \frac{1}{2!} \frac{d^2 V}{d\phi^2} \Big|_{\phi=v} \frac{1}{v^2}$$

$$\lambda_{\text{eff}}^{(3)SM} = \frac{1}{3!} \frac{d^3 V}{d\phi^3} \Big|_{\phi=v} \frac{1}{v}$$

$$\lambda_{\text{eff}}^{(4)SM} = \frac{1}{3!} \frac{d^4 V}{d\phi^4} \Big|_{\phi=v}$$

# Coleman-Weinberg mechanism

$$V(\phi) = m^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

$$m^2 = 0$$

(second derivative of  $V$  at the origin)

Spontaneous symmetry breaking can occur  
by radiative corrections.

Starting from scale invariant potential

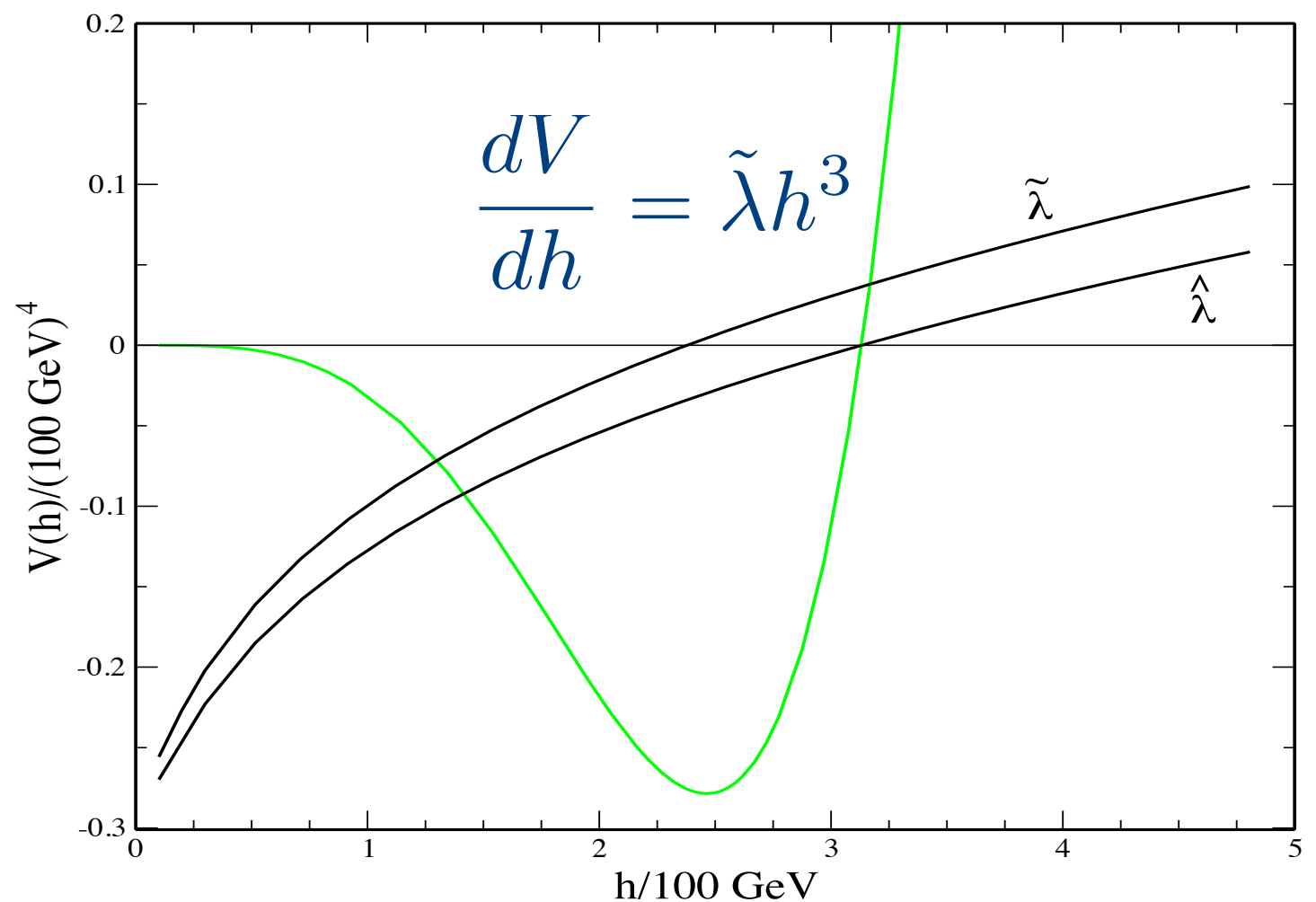
$$V(\phi) = \lambda(\phi^\dagger \phi)^2$$

RG improved effective potential is then

$$V(\phi) = \lambda(\phi)(\phi^\dagger \phi)^2 \qquad V(h) = \frac{\hat{\lambda}}{4} h^4$$

If the quartic changes  
sign at low energy,  
nontrivial minimum  
is developed

Espinosa and Quiros, PRD (2007)



# Scalar QED and Standard Model in 1970s

$$\frac{m_h^2}{m_V^2} = \frac{3}{2\pi} \frac{e^2}{4\pi} = \frac{3}{2\pi} \alpha \qquad m_V^2 = e^2 \langle \phi \rangle^2$$

$$m_h^2 = \frac{3}{32\pi^2} [2g^2 m_W^2 + (g^2 + g'^2) m_Z^2]$$

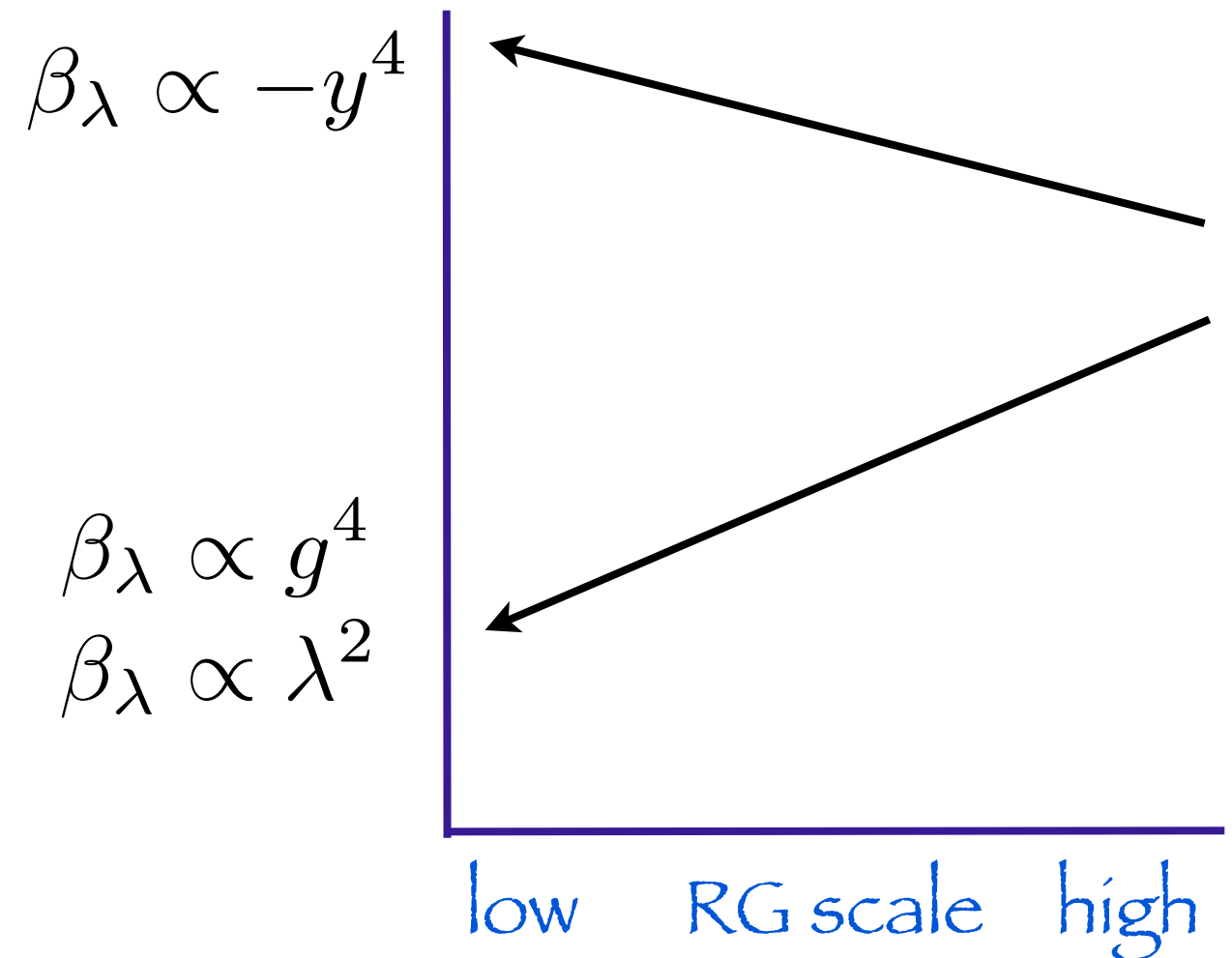
SM with W and Z (without top) :  $m_h \sim 10 \text{ GeV}$

Radiatively generated Higgs mass is one loop suppressed compared to the vector boson mass

Superconductor :

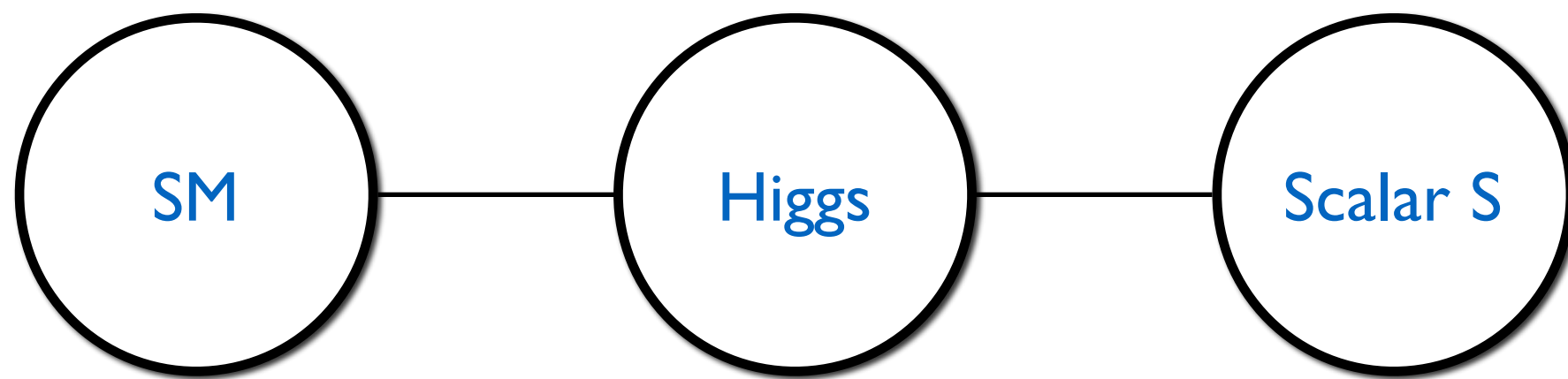
Coherence length is much longer than London penetration length

# Top Yukawa prevents CW mechanism in the SM



Radiative symmetry breaking is possible  
with gauge or mixed quartic interactions.

# New particles interacting with Higgs



$$V(h) \propto h^4 \log h$$

# Coleman-Weinberg Higgs

$$V(\phi) = \frac{\lambda(t)}{4} \phi^4$$

$$t = \log \phi$$

$$\frac{dV}{d\phi} = \frac{dt}{d\phi} \frac{\beta_\lambda}{4} \phi^4 + \frac{\lambda}{4} \cdot 4\phi^3$$

$$= (\lambda + \frac{\beta_\lambda}{4}) \phi^3|_{\phi=v} = 0$$

-75% (tree) + 175% (loop)

$$m^2 = \frac{d^2 V}{d\phi^2}|_{\phi=v} = (\beta_\lambda + \cancel{\frac{\beta'_\lambda}{4}}) v^2$$

$$\lambda_{\text{eff}}^{(2)} = \frac{1}{2} \frac{m^2}{v^2} \sim \frac{1}{8}$$

(precisely = 0.129)

$$\lambda_{\text{eff}}^{(3)} = \frac{5}{3} \lambda_{\text{eff}}^{(2)},$$

$$\lambda_{\text{eff}}^{(4)} = \frac{11}{3} \lambda_{\text{eff}}^{(2)}.$$

Scale dependence of the beta function is neglected here.

$$\beta_\lambda \sim \frac{1}{4}$$



## Higgs portal with extra scalar S

$$V = \lambda_h (H^\dagger H)^2 + \lambda_{hs} H^\dagger H S^\dagger S + \lambda_s (S^\dagger S)^2$$

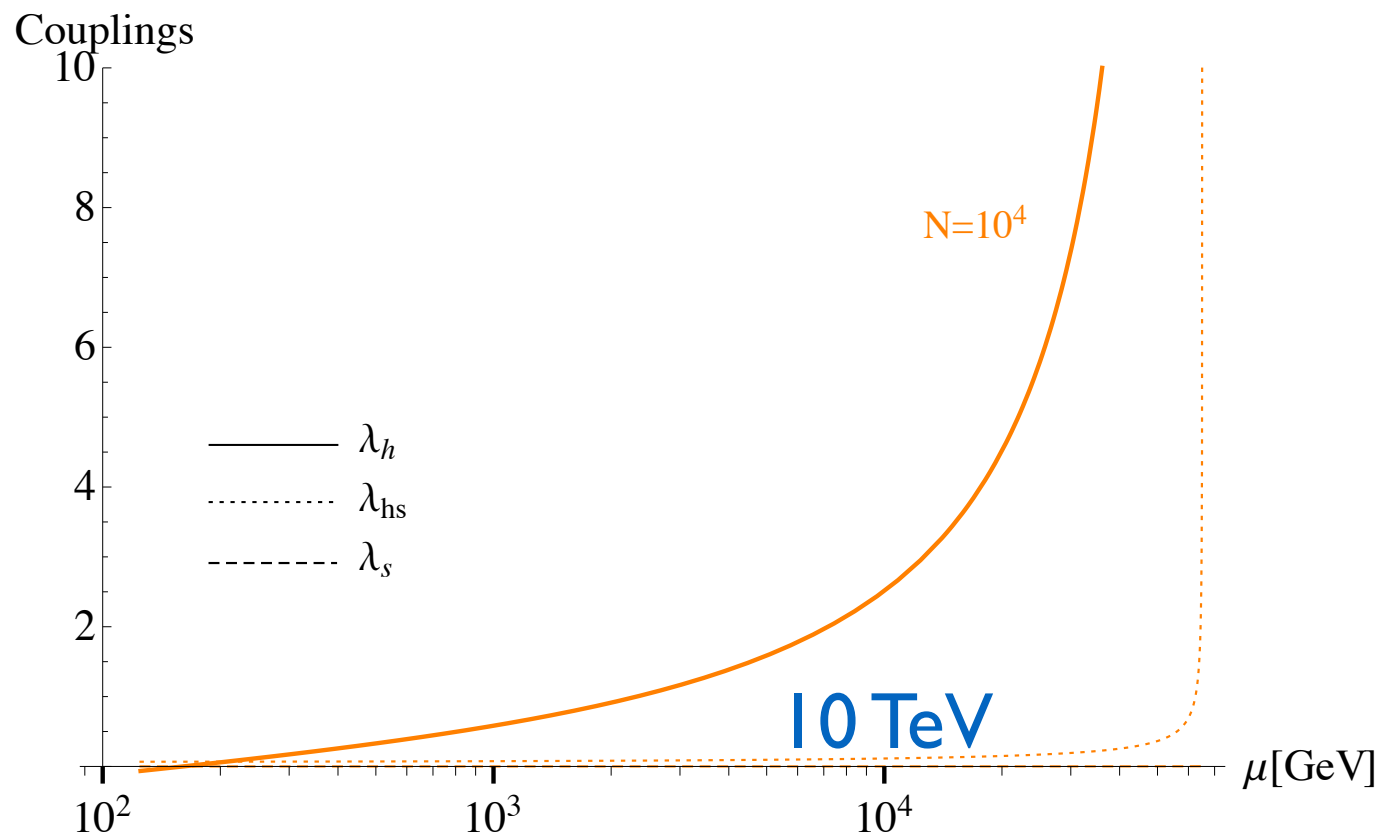
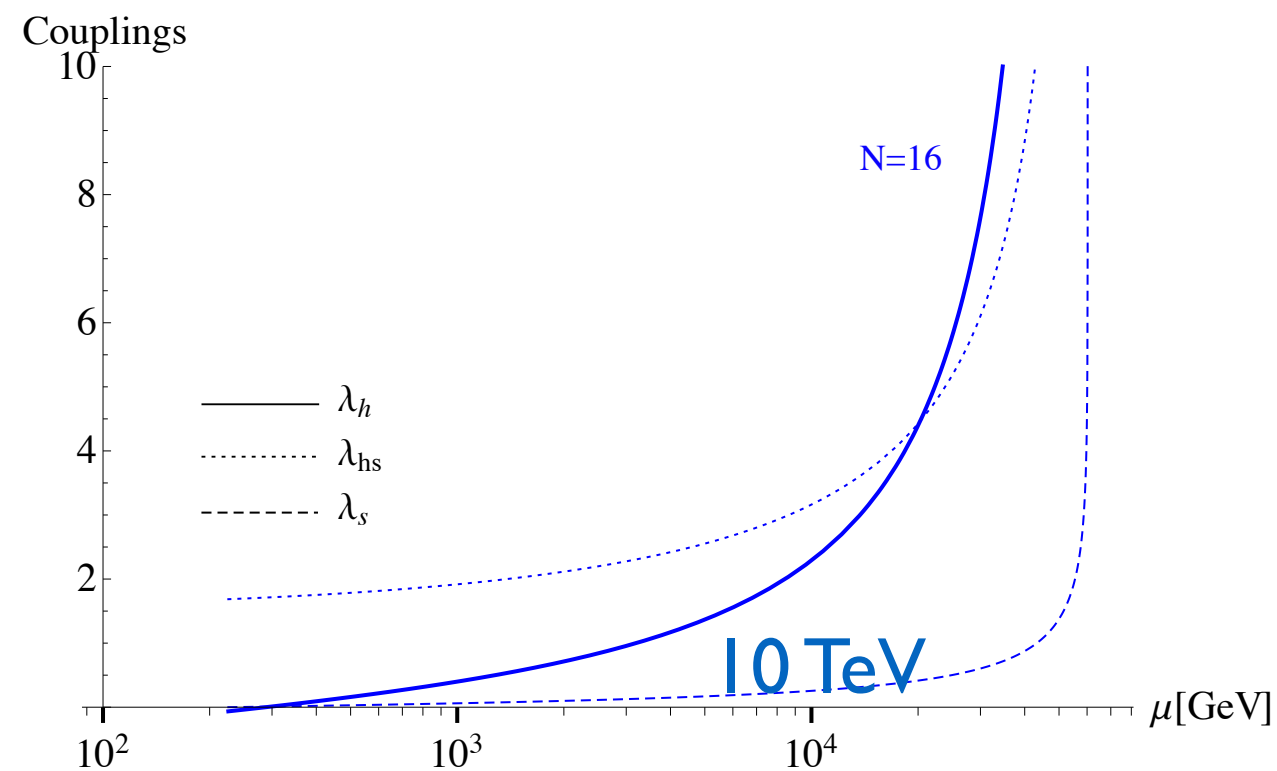
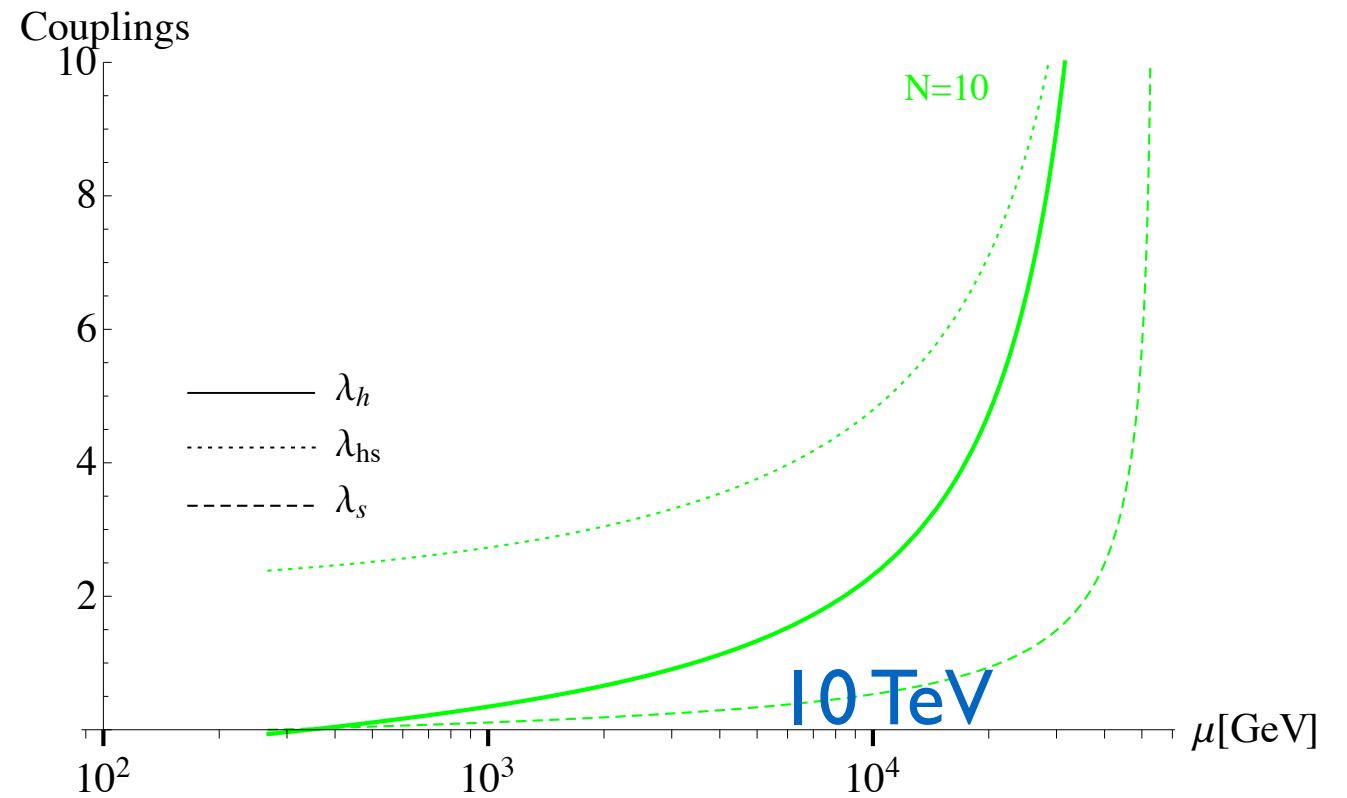
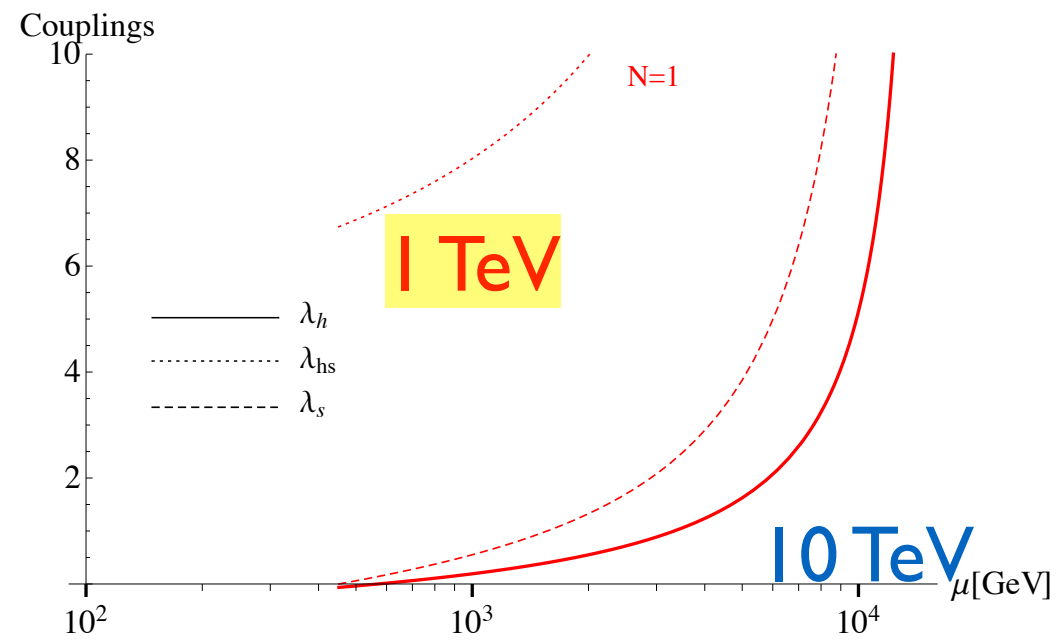
$$16\pi^2 \beta_{\lambda_h} = 24\lambda_h^2 + N\lambda_{hs}^2$$

$$16\pi^2 \beta_{\lambda_{hs}} = \lambda_{hs} [4\lambda_{hs} + 12\lambda_h + (4N + 4)\lambda_{hs}]$$

$$16\pi^2 \beta_{\lambda_s} = (16 + 4N)\lambda_s^2 + 2\lambda_{hs}^2$$

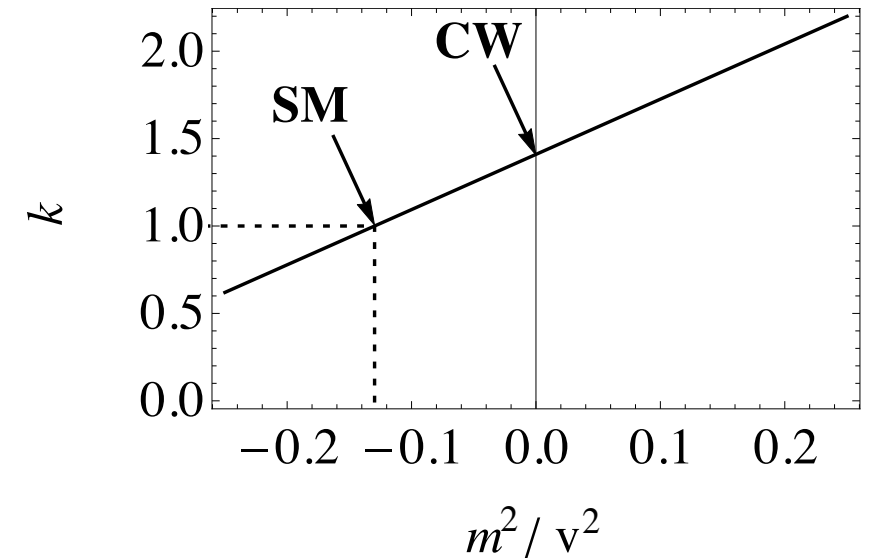
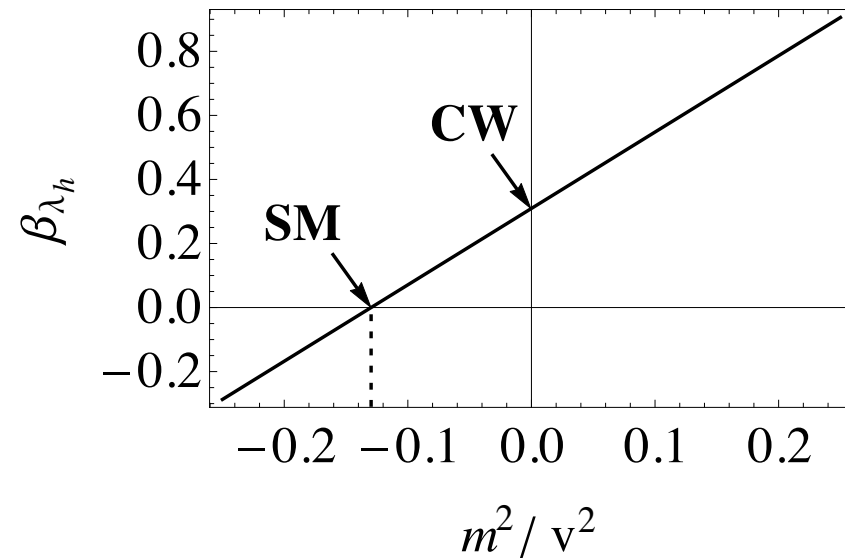
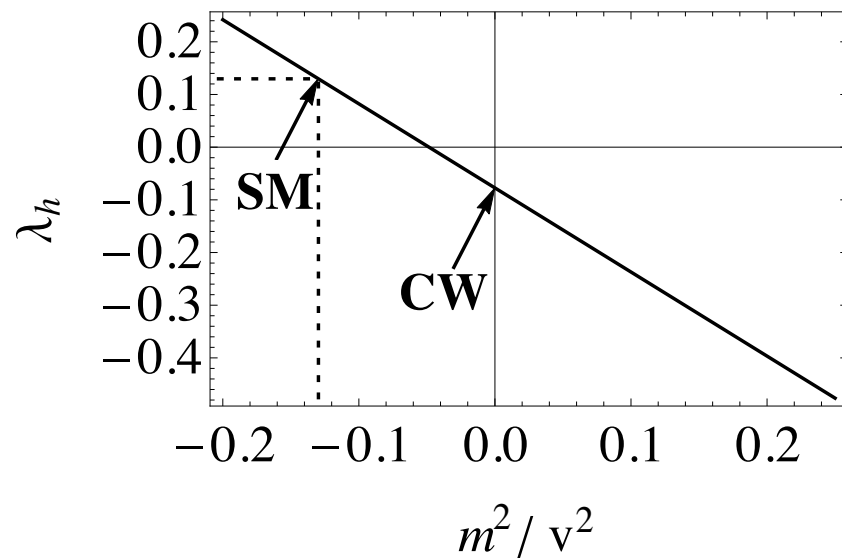
New mixed quartic raises Higgs quartic at high energy

# Non-perturbative at 20TeV for $N_s > 1$



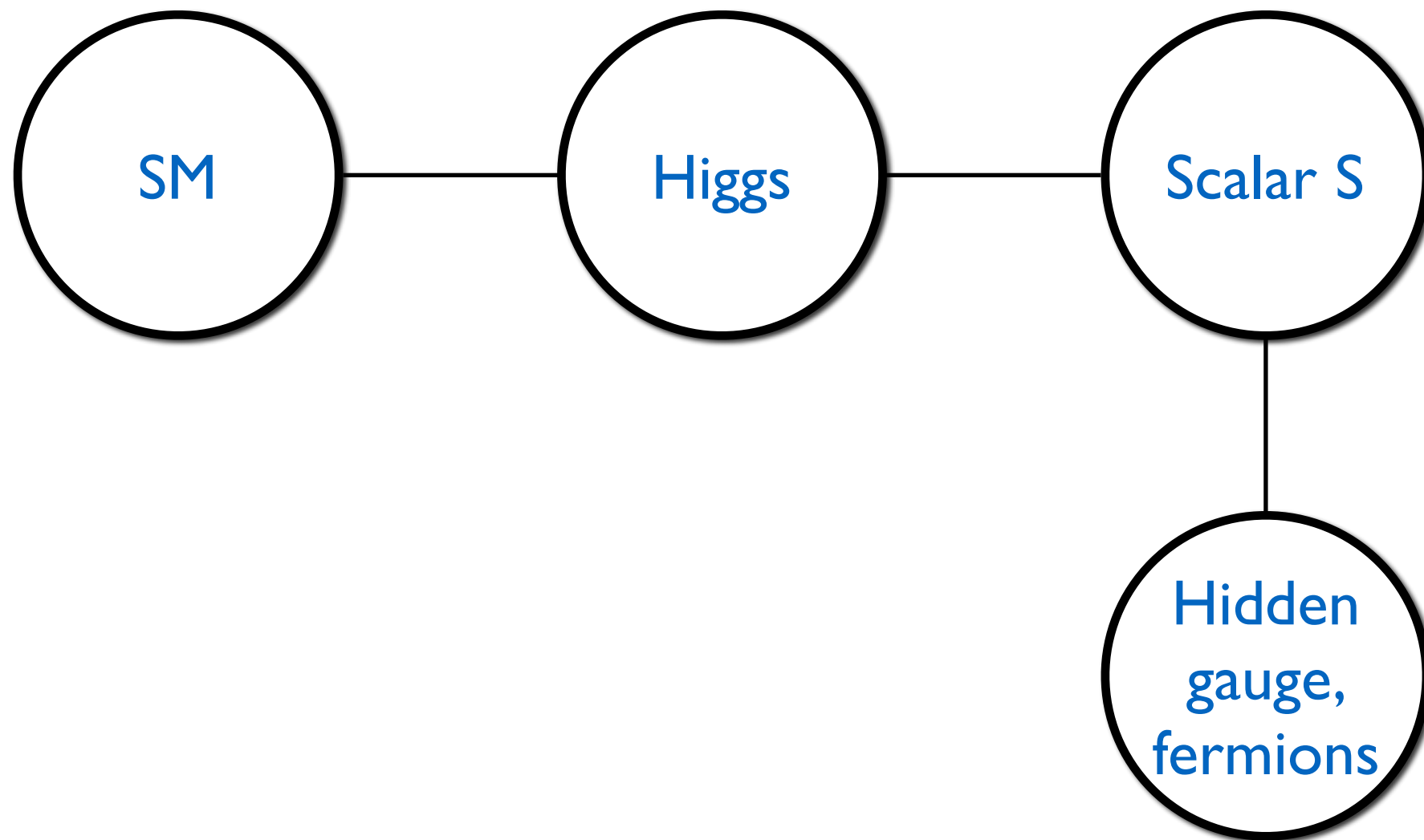
# New parameter space with running couplings

$m=0$  is a one point in the extended parameter space



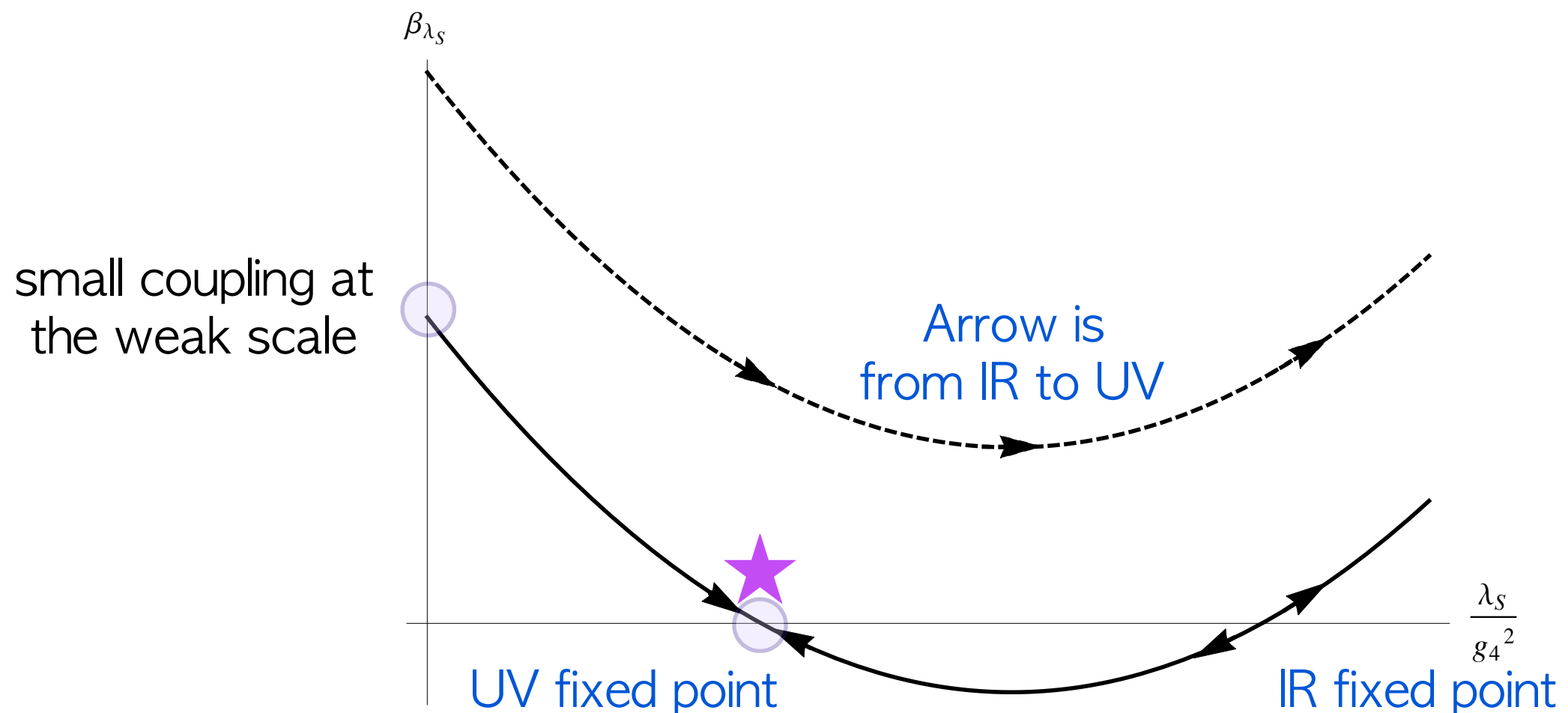
$k$ : Relative strength of Higgs cubic couplings with respect to SM

# Gauge extension of hidden sector

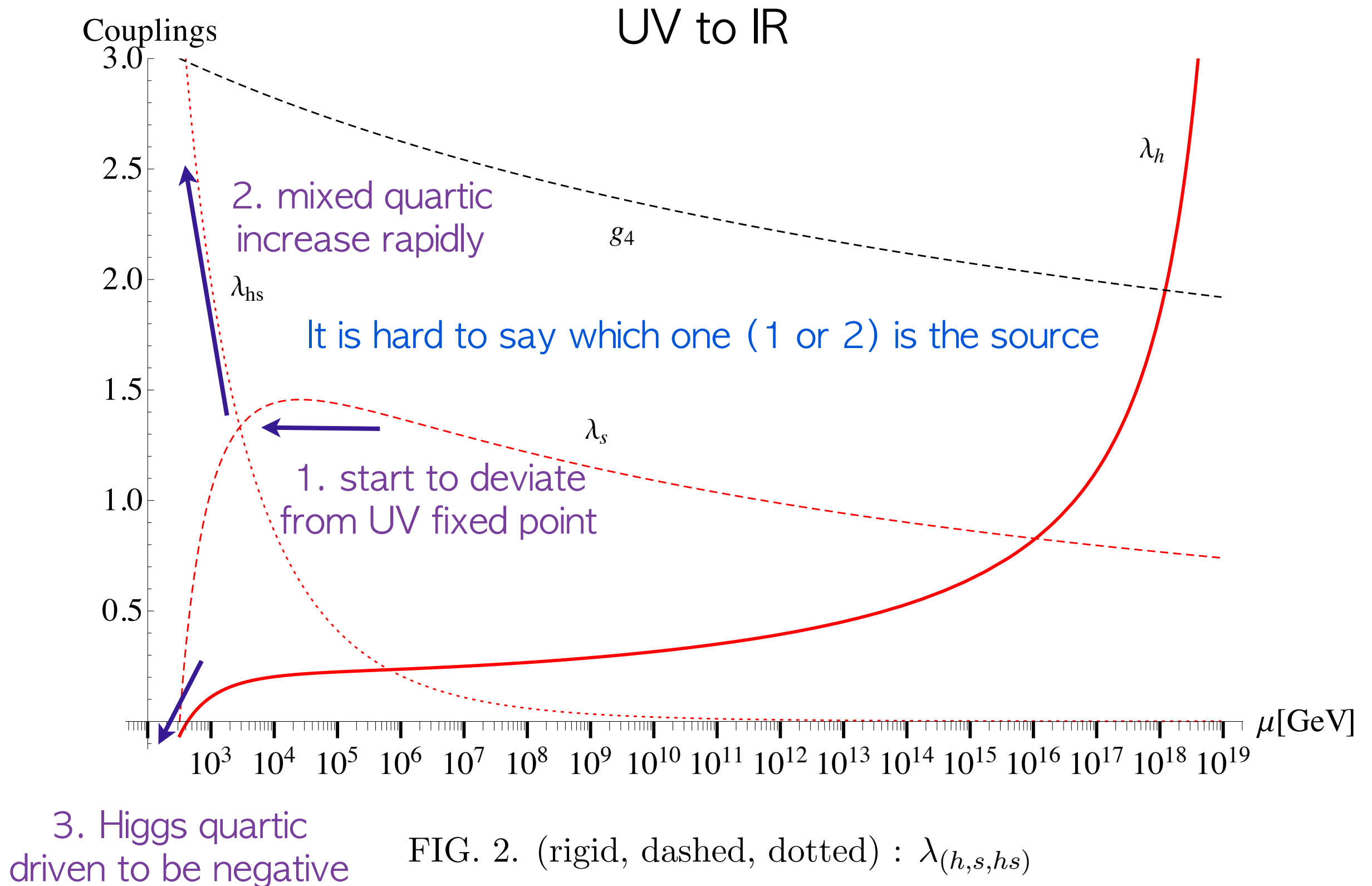


$$16\pi^2 \frac{d\lambda_s}{dt} = \frac{3}{4} \left( \frac{N_S^3 + N_S^2 - 4N_S + 2}{N_S} \right) g_4^4 - 6 \left( \frac{N_S^2 - 1}{N_S} \right) g_4^2 \lambda_s + 4(4 + N_S) \lambda_s^2 + 2\lambda_{hs}^2$$

$$16\pi^2 \frac{d\lambda_{hs}}{dt} = \lambda_{hs} \left[ 4\lambda_{hs} + 12\lambda_h + (4N_S + 4)\lambda_s - 3 \left( \frac{N_S^2 - 1}{N_S} \right) g_4^2 \right]$$

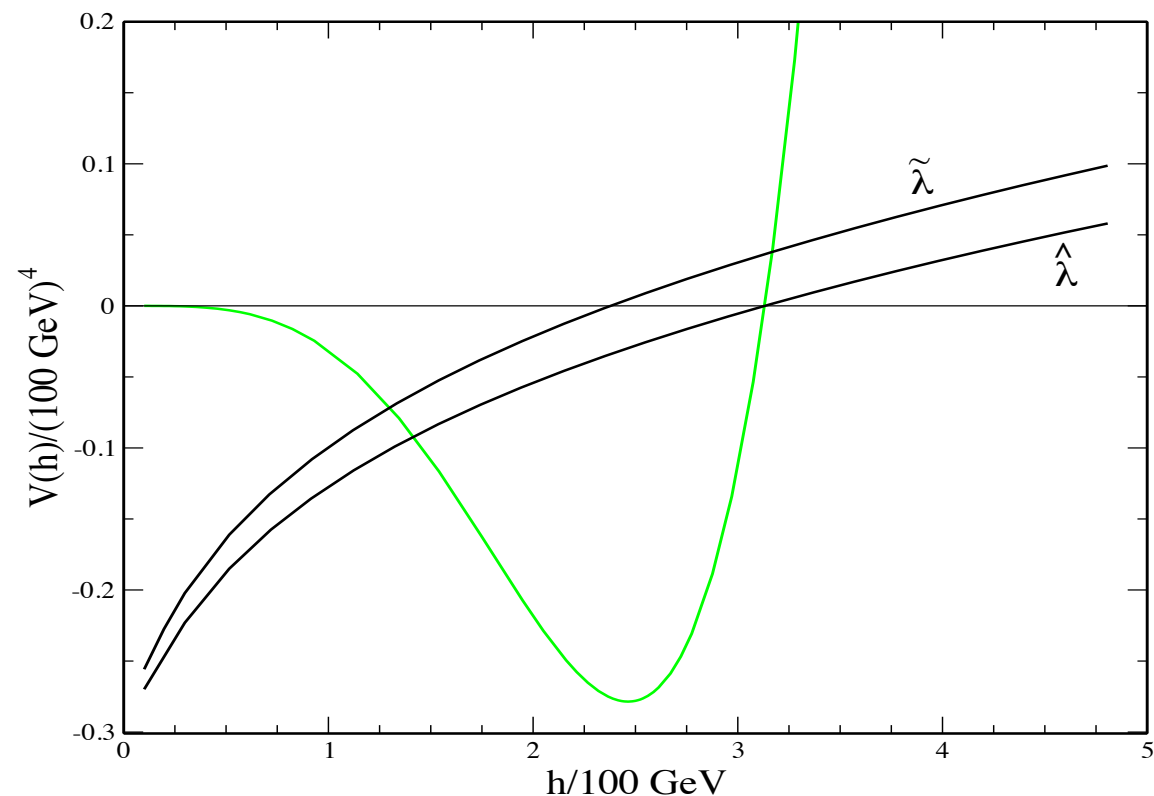


# Example : Scalar in 4 of SU(4) :

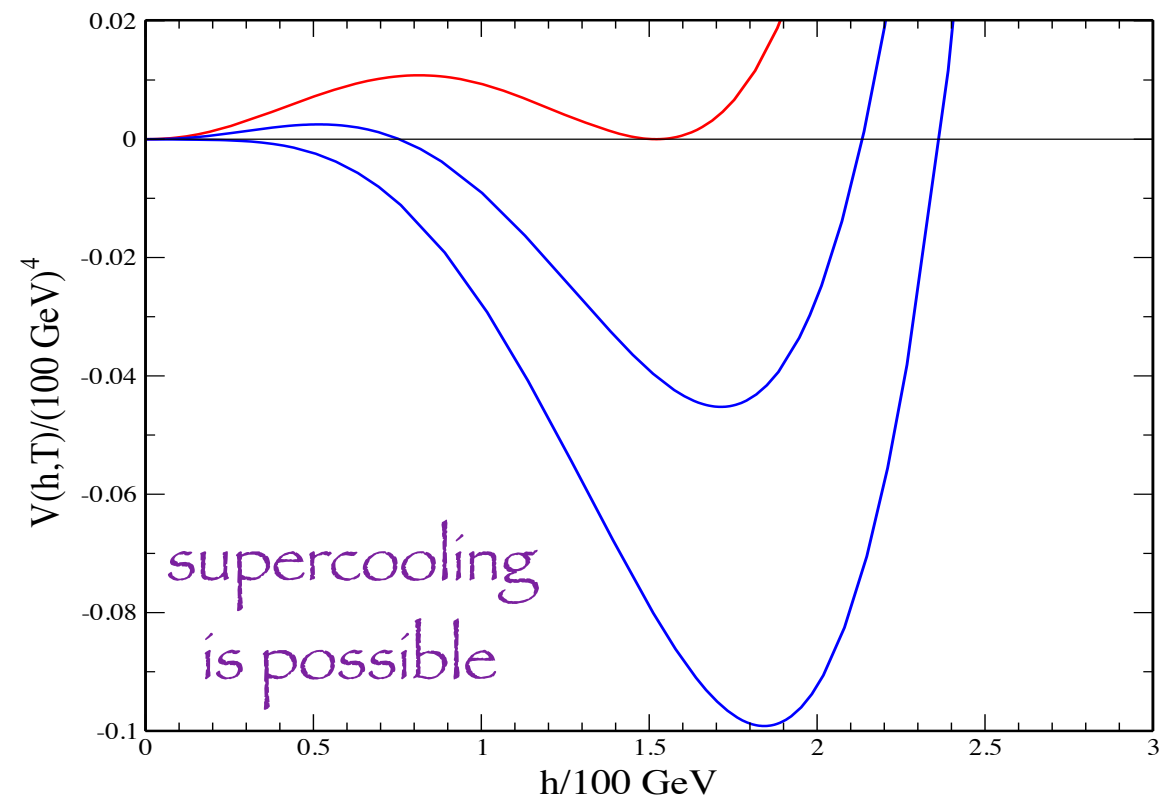


# Electroweak Baryogenesis

$$V = \lambda_h (H^\dagger H)^2 + \lambda_{hs} H^\dagger H S^\dagger S + \lambda_s (S^\dagger S)^2$$



Higgs potential

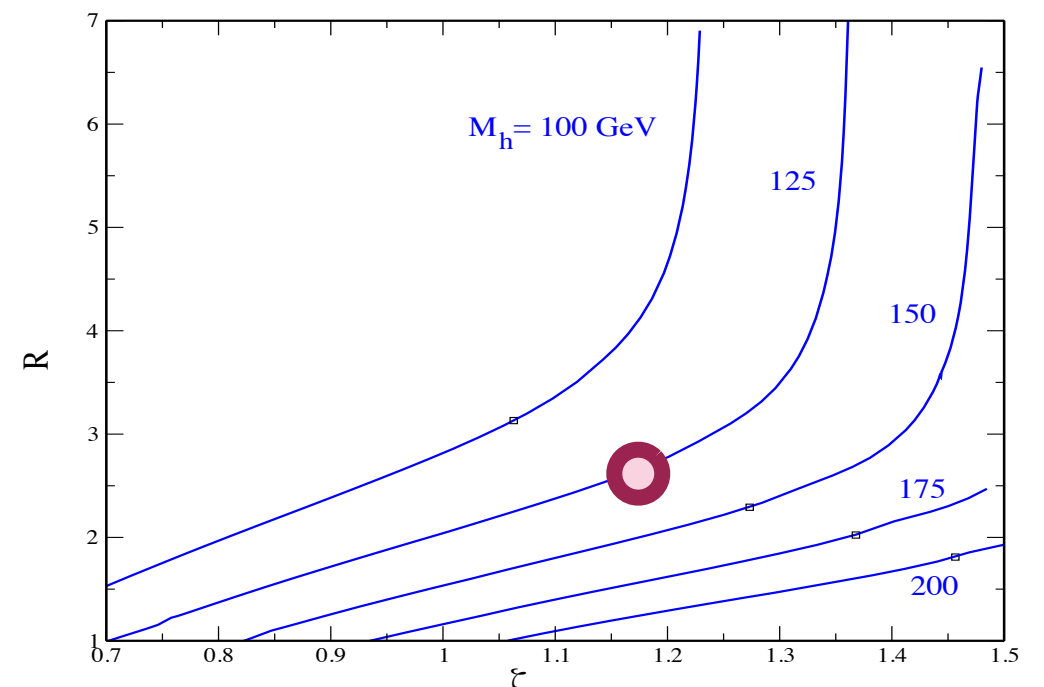


1. Strong 1st order phase transition
2. New source of CP violation exists

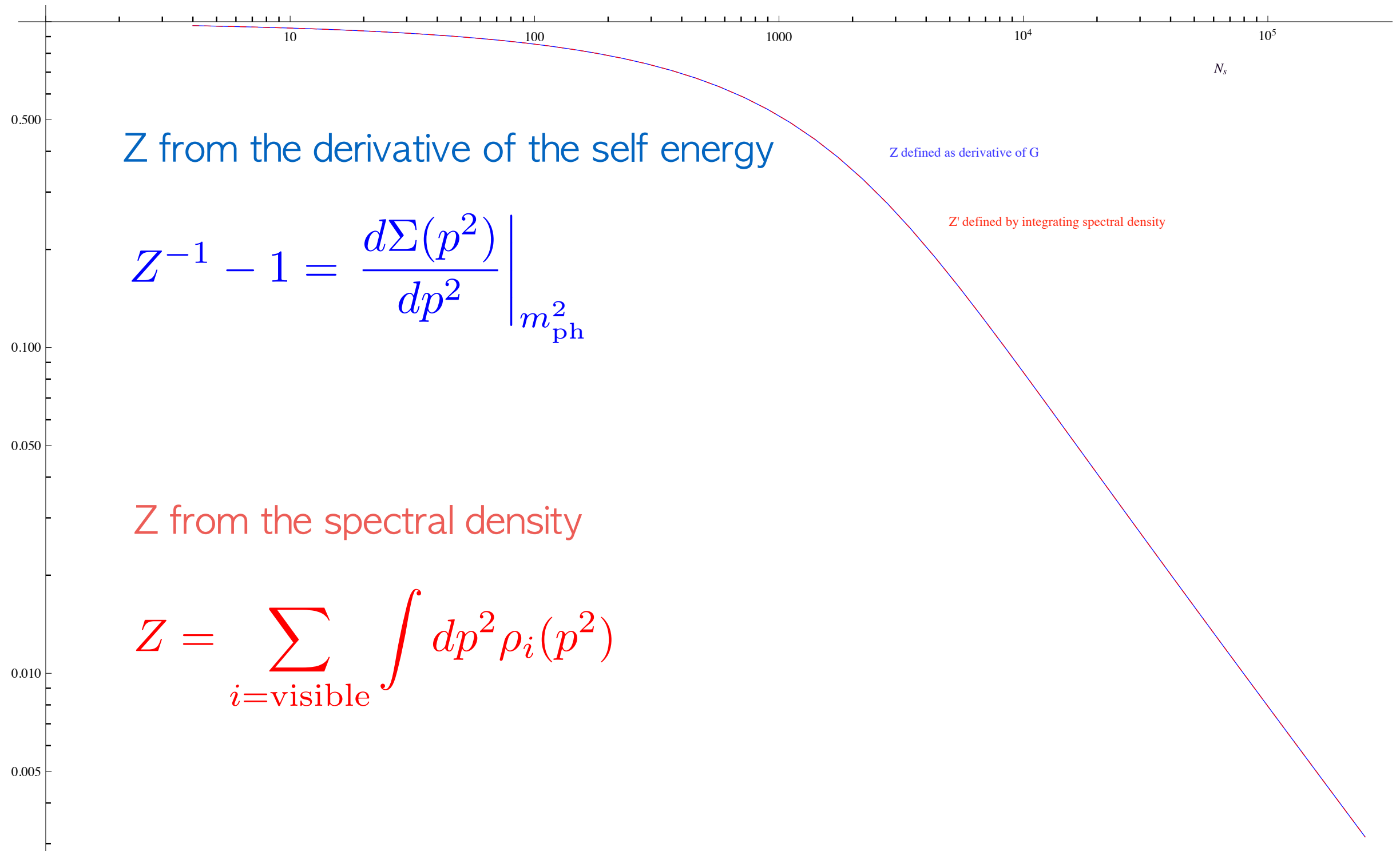
$$\lambda_{hs} H^\dagger H S^\dagger S + \theta G_{\mu\nu} \tilde{G}^{\mu\nu}$$

Espinosa and Quiros, PRD (2007)

$$R \equiv \langle h(T_c) \rangle / T_c$$



# Two definitions agree with each other





# Phenomenology of Higgs with singlets (no hidden gauge group)

Off-shell Higgs invisible decay

Suppression of Higgs couplings to all SM particles

Higgs self coupling

Electroweak precision

Higgs potential is stabilised by the balance of tree and one loop.

All one loop correction to Higgs mass should be kept.

There is only one parameter, the number of singlet scalars.

# Higgs portal with hidden scalar S

$$V = \lambda_h (H^\dagger H)^2 + \lambda_{hs} H^\dagger H S^\dagger S + \lambda_s (S^\dagger S)^2$$

$$16\pi^2 \beta_{\lambda_h} = 24\lambda_h^2 + N\lambda_{hs}^2 \sim 40$$

$$m_S \simeq \frac{440}{N_S^{1/4}} \text{ GeV}$$

Mass of hidden scalar S is entirely fixed from Higgs VEV and N.

Lower bound on hidden scalar mass exists when Higgs self energy is taken into account.

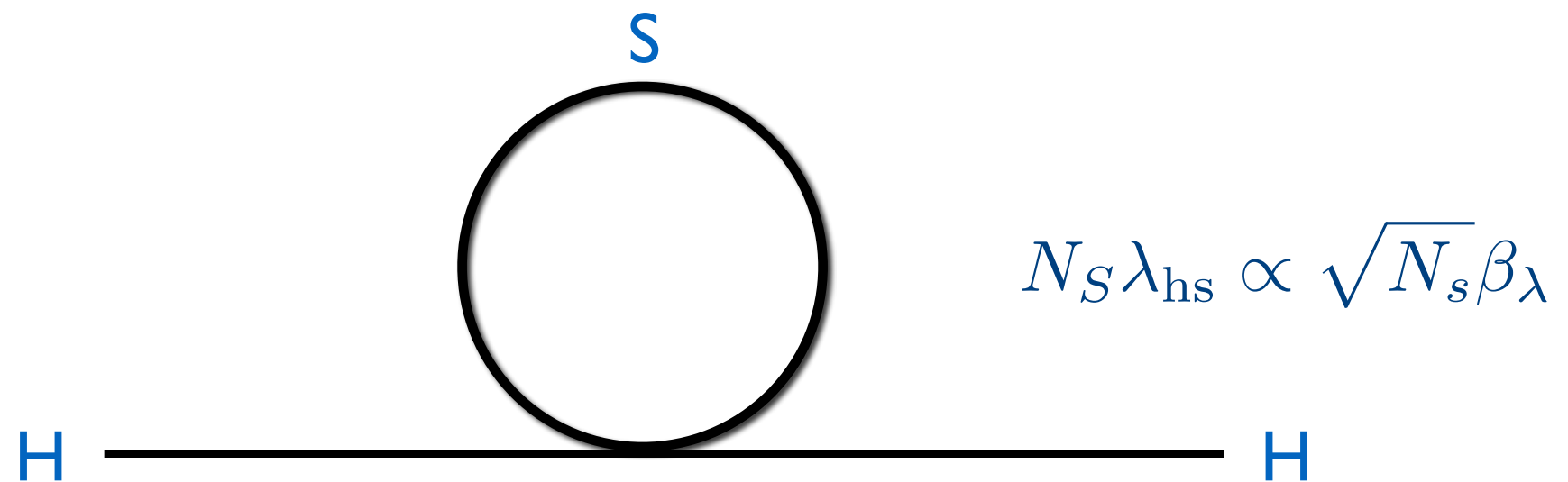
$$m_S = \left(1 + \frac{2}{\pi^2}\right) \frac{m_h}{2}$$

momentum dependent correction

$$G^{-1} = p^2 - \left[ \beta_\lambda v^2 - 2\beta_\lambda v^2 \left( 1 - \hat{\beta} \tan^{-1} \frac{1}{\hat{\beta}} \right) \right]$$

$$\hat{\beta} = \sqrt{\frac{4m_S^2}{p^2} - 1}$$

$$\geq \frac{1}{2}$$

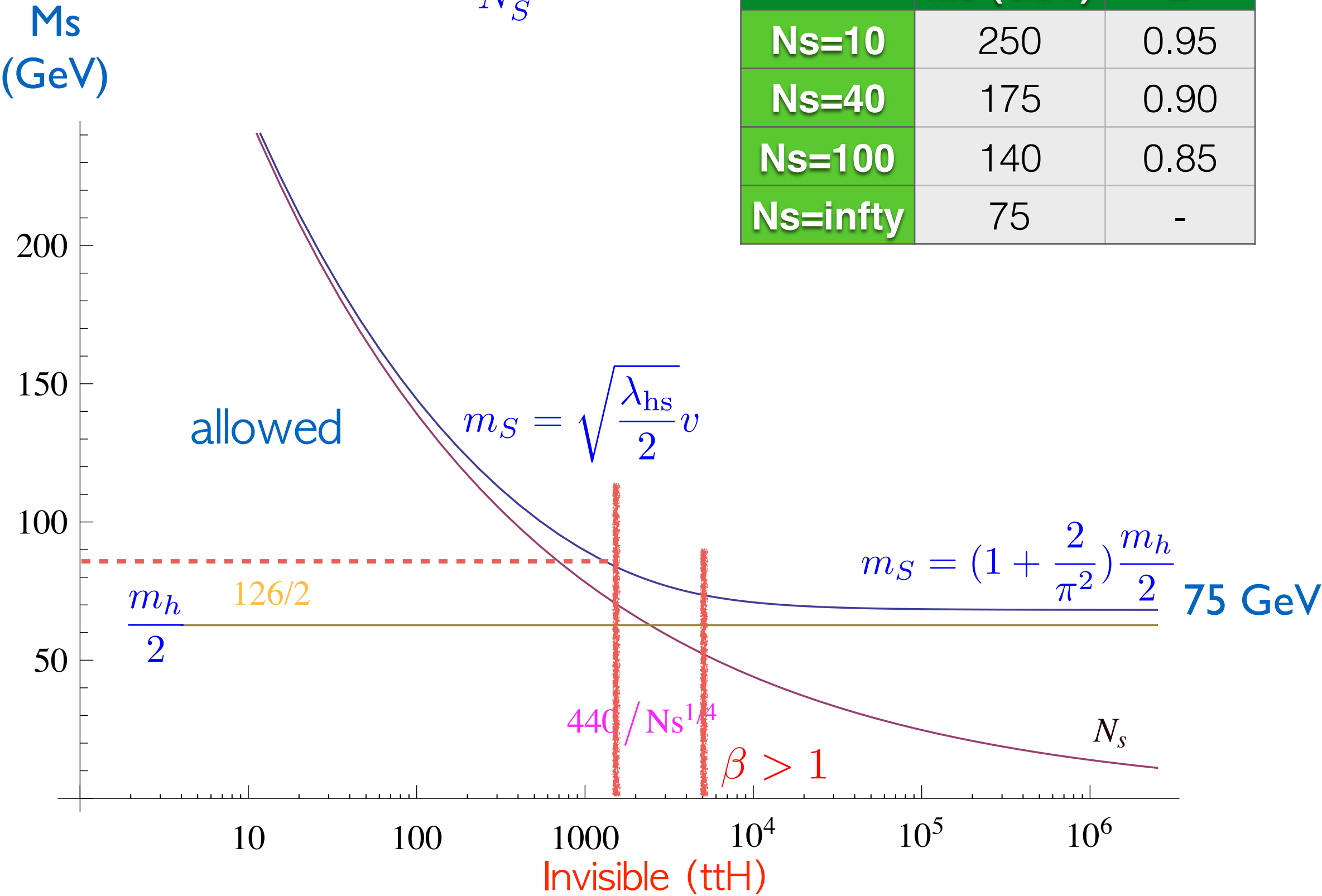


As  $N_s$  becomes larger, this diagram is more important and should be kept in all computations.

Hidden scalar mass

$$m_S \simeq \frac{440}{N_S^{1/4}} \text{ GeV}$$

	Ms (GeV)	Z
Ns=10	250	0.95
Ns=40	175	0.90
Ns=100	140	0.85
Ns=infty	75	-



# Measuring Higgs cubic coupling at the LHC

$$\mathcal{L}_{\text{eff}} = \frac{1}{4} \frac{\alpha_s}{3\pi} G_{\mu\nu}^a G^{a\mu\nu} \log\left(1 + \frac{h}{v}\right) \quad \text{from top loop}$$

$$\mathcal{L}_{\text{eff}} = \frac{1}{4} \frac{\alpha_s}{3\pi v} G_{\mu\nu}^a G^{a\mu\nu} h \ominus \frac{1}{4} \frac{\alpha_s}{6\pi v^2} G_{\mu\nu}^a G^{a\mu\nu} h^2 + \dots$$

$\downarrow \times \lambda v h^3$

destructive interference

$\downarrow$   
leading term

# Higgs cubic coupling at the LHC

$$\sigma_{hh}^{NLO} = 70y_t^4 - 50\lambda y_t^3 + 10\lambda^2 y_t^2$$

1      -0.1

Vainshtein theorem\*  
multi-Higgs production amplitudes  
vanish in the heavy top limit

The cross section only vary by 10% for order one change of Higgs self coupling.

$3 \text{ } ab^{-1}$

50% uncertainty at 14 TeV LHC  
(bb tau tau, bbWW, bb gamma gamma)

# Higgs cubic coupling at the ILC

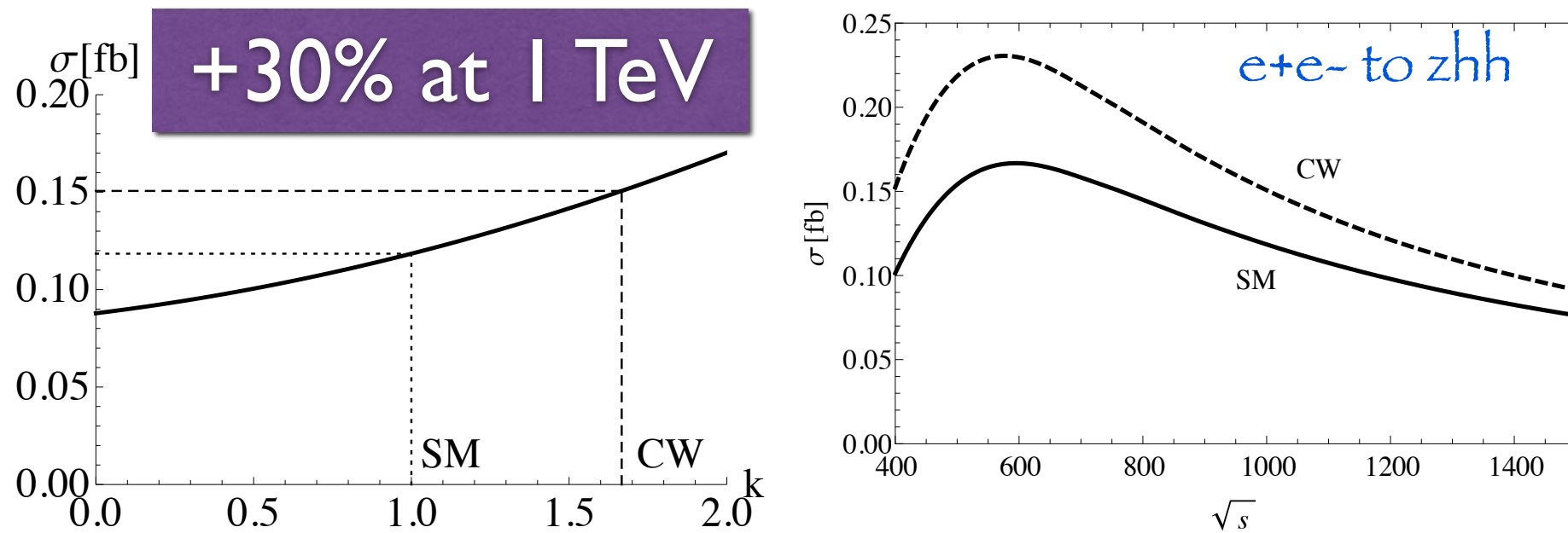


FIG. 7.  $e^+e^- \rightarrow hhZ$

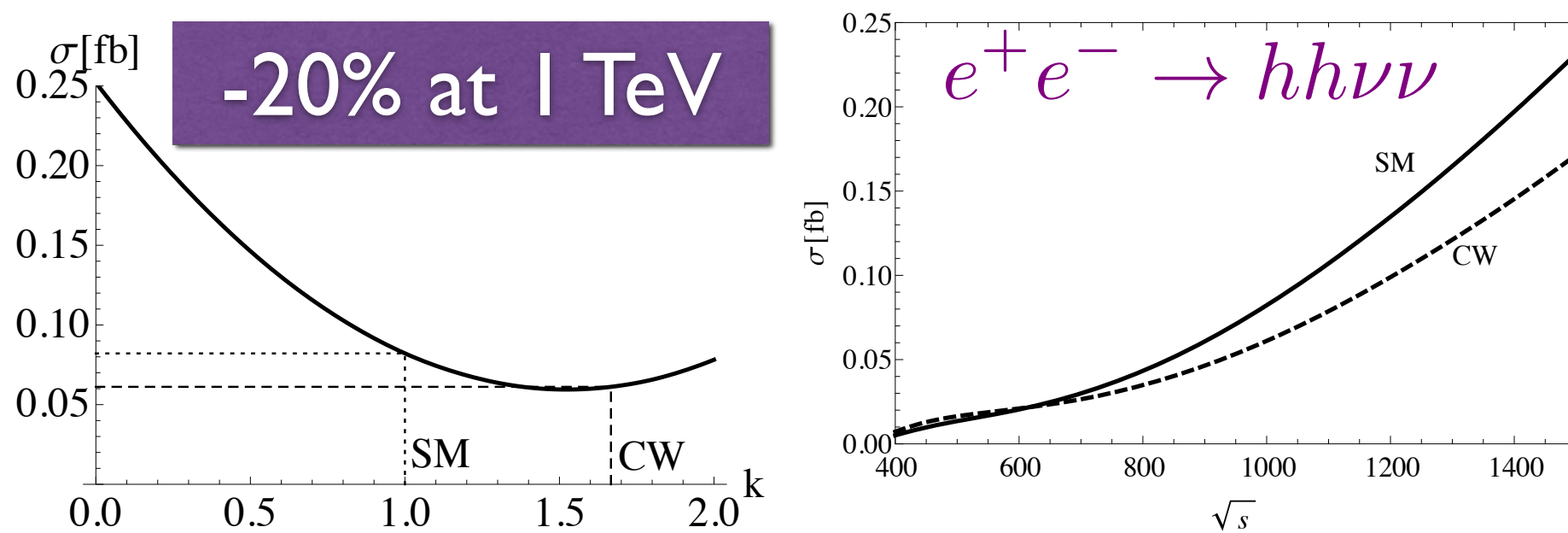
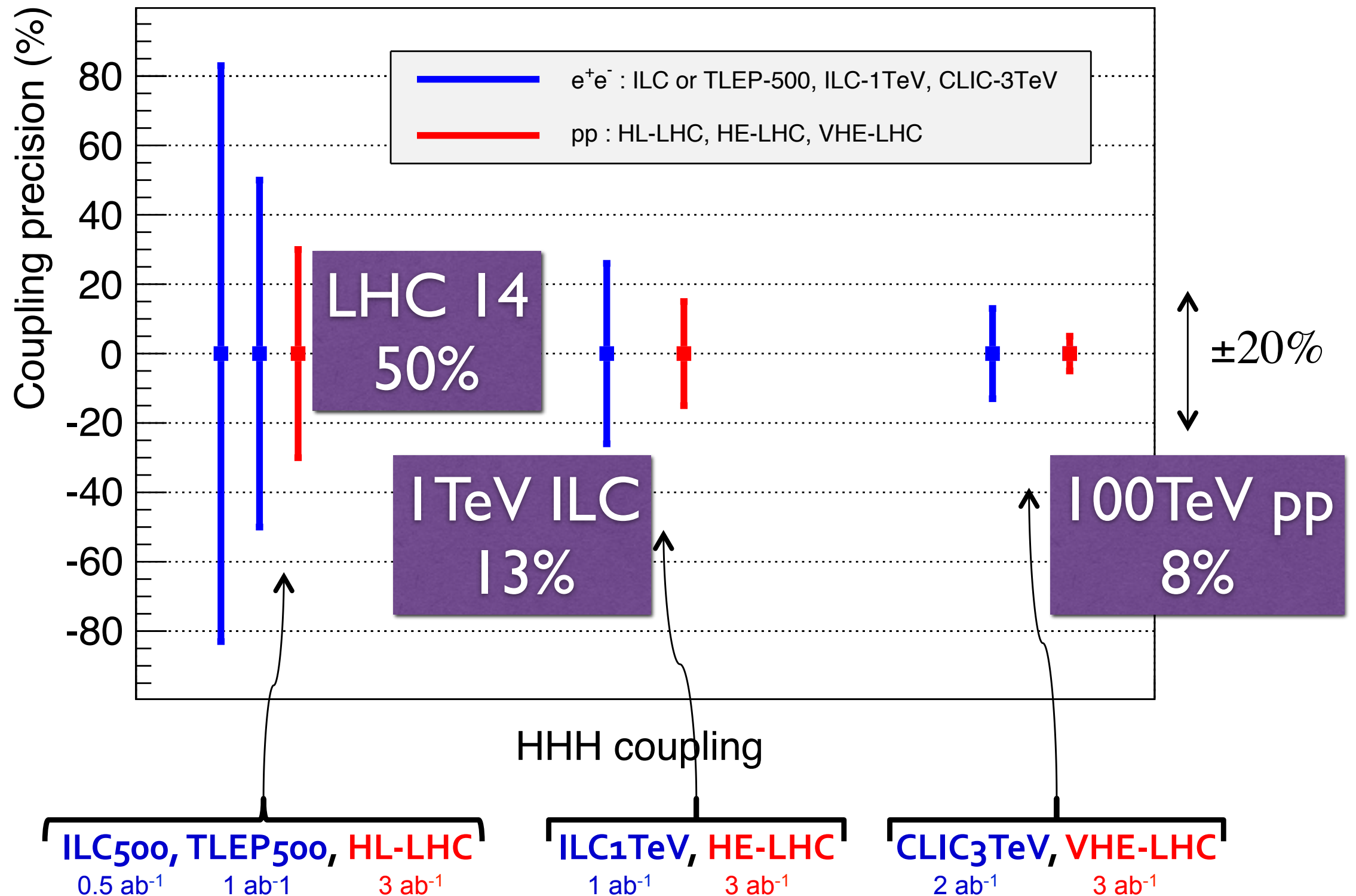


FIG. 8.  $e^+e^- \rightarrow hh\nu_e\bar{\nu}_e$

# Higgs cubic coupling measurement

TLEP design study working group, 1308.6176



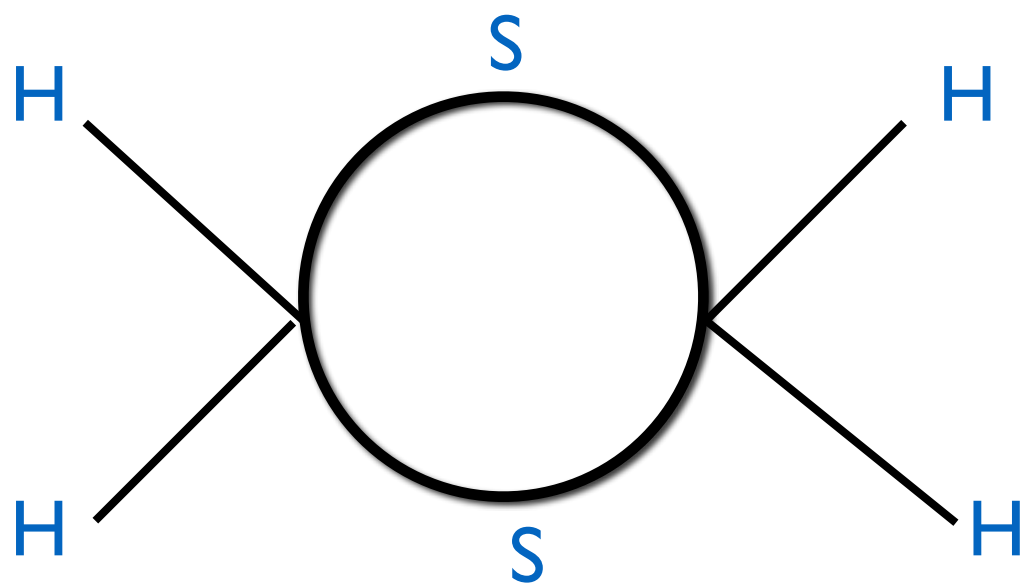


# Suppression of Higgs couplings to the SM

Expected precision for hZZ

LHC : 2% to 5%

Higgs factory 1% to 0.4%



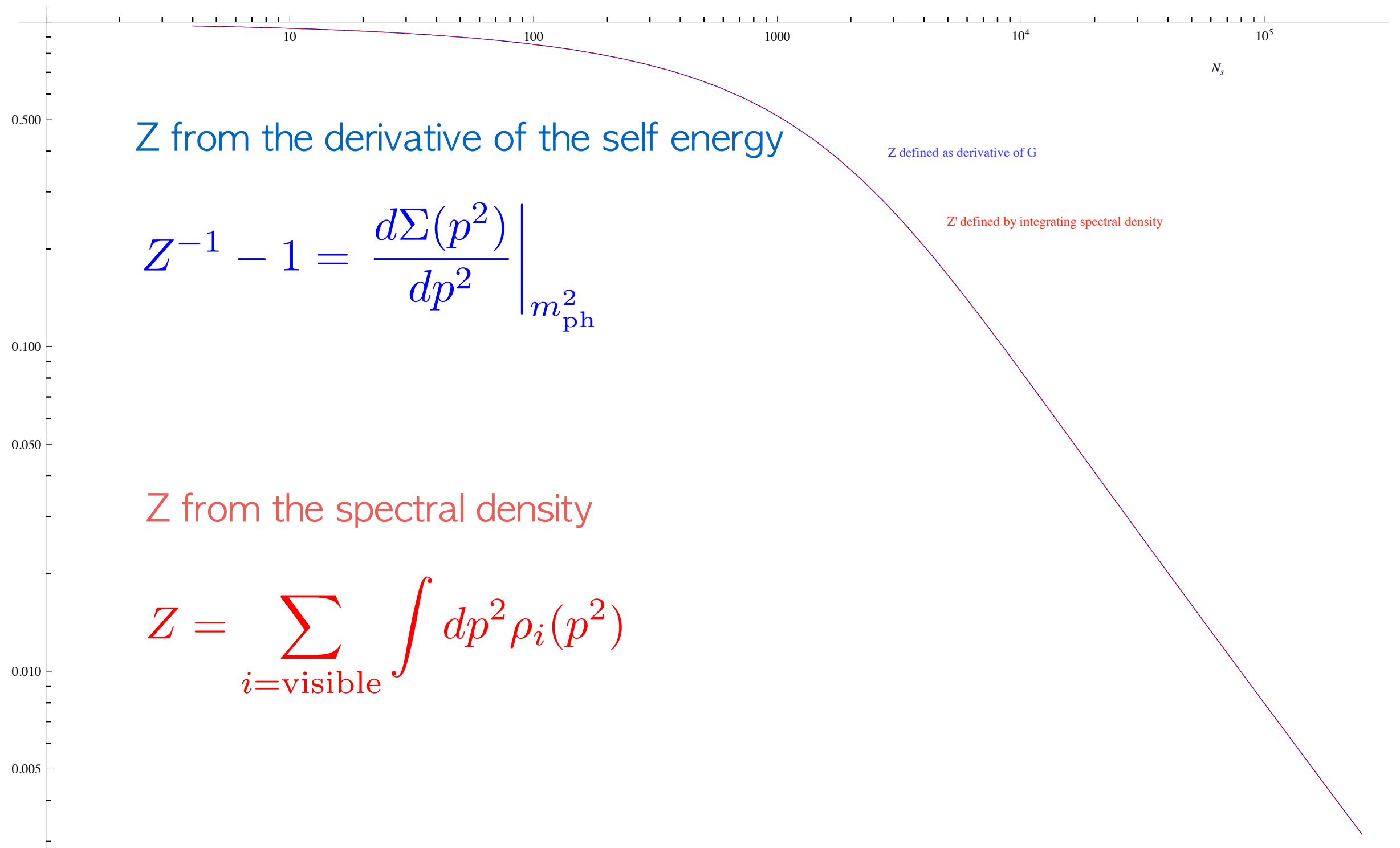
$$\begin{aligned} & \frac{1}{p^2 - m_h^2 + \Sigma(p^2)} \\ \simeq & \frac{Z}{p^2 - m_h^2 + (Z^{-1} - 1)(p^2 - m_h^2) + im_h\Gamma_h} \\ = & \frac{Z}{p^2 - m_h^2 + im_h Z\Gamma_h} \end{aligned}$$

expansion at the resonance  
can not be valid for off-shell

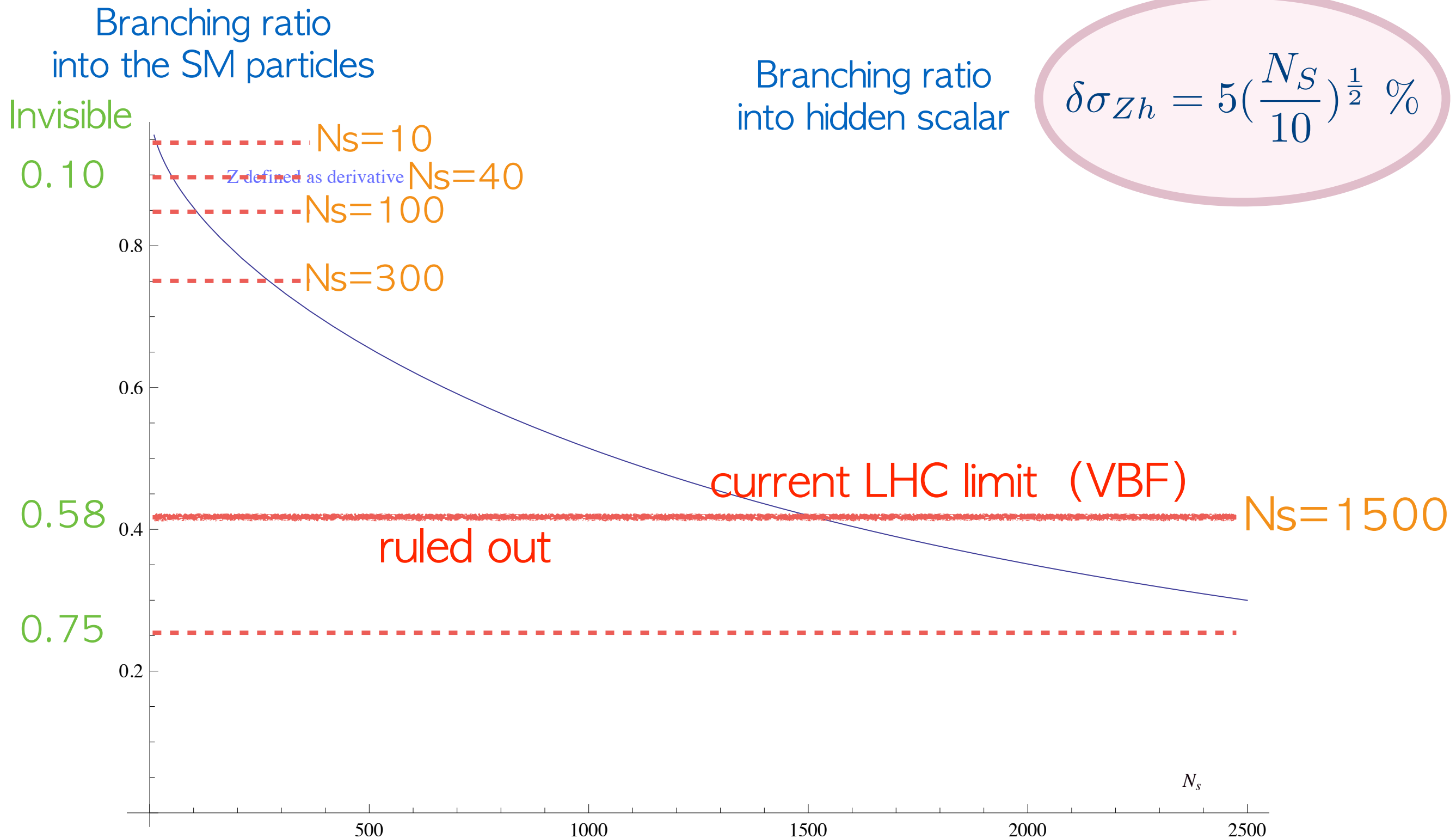
Generate dimension 6 operator

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \frac{c_H}{m_\phi^2} \left( \frac{1}{2} \partial_\mu |H|^2 \partial^\mu |H|^2 \right) + \dots$$

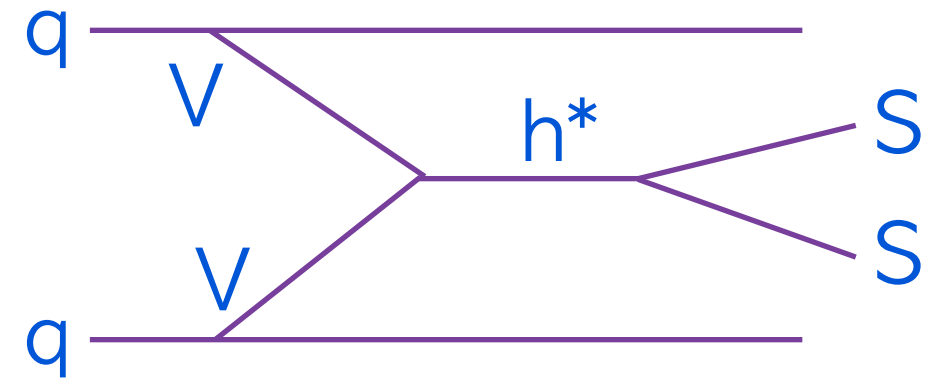
# Two definitions agree with each other



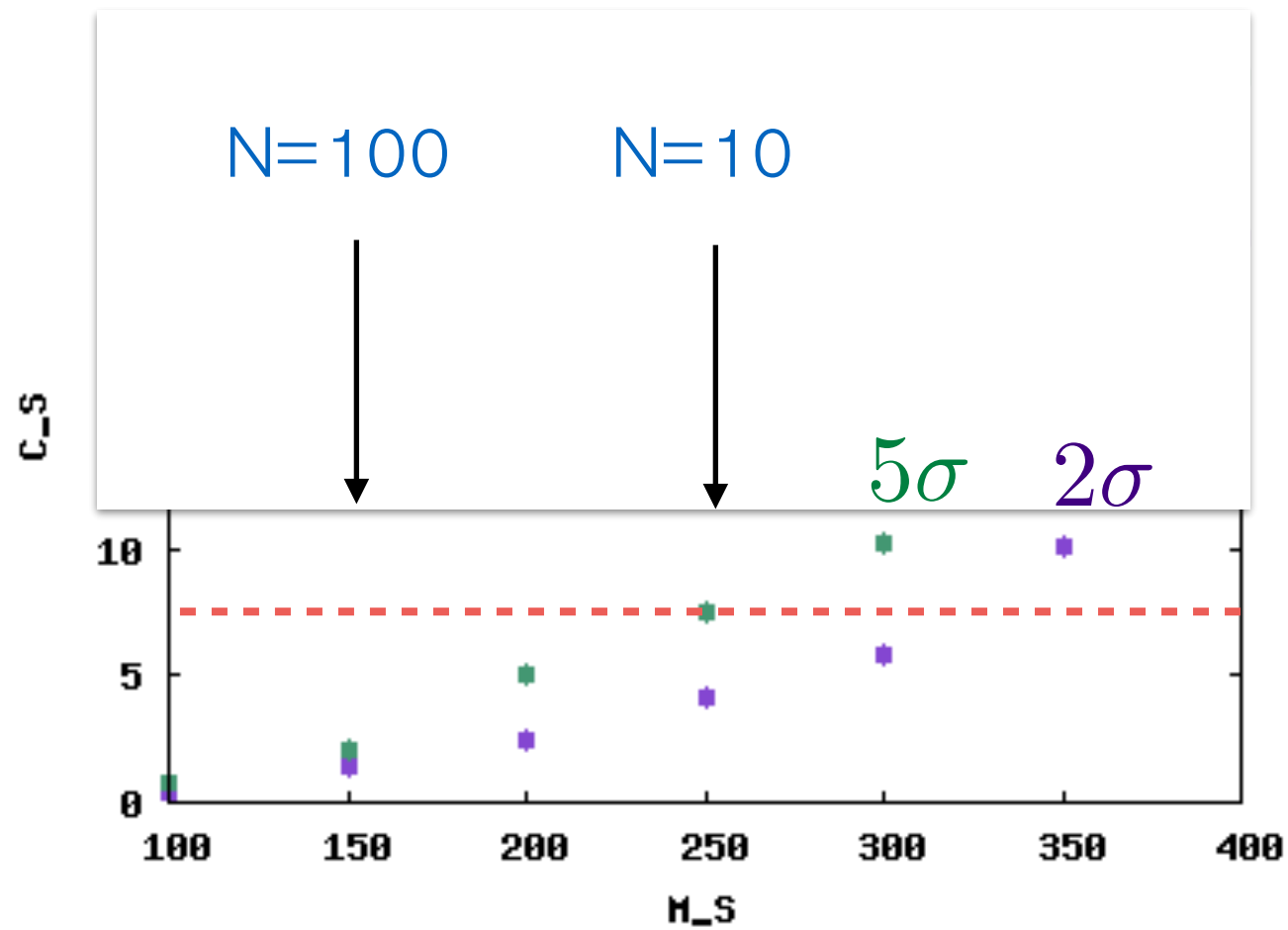
# Suppressed ZZh coupling as an invisible decay



# Off-shell Higgs decay : VBF (e.g., $N=10, M_S=250$ GeV)



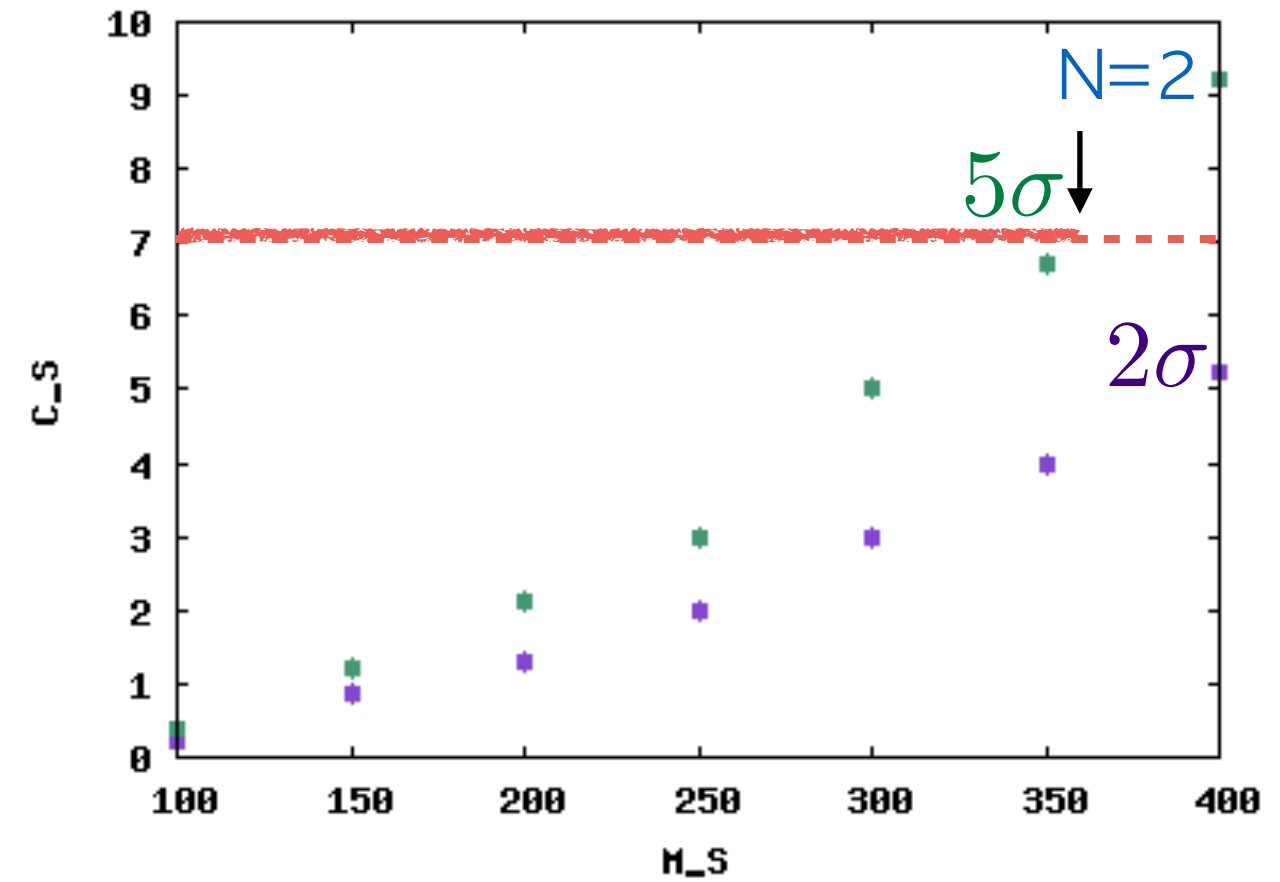
$$\sqrt{N_s} \lambda_{hs}$$



LHC 14 TeV

$$\sqrt{N_s} \lambda_{hs}$$

work done by DY Mo

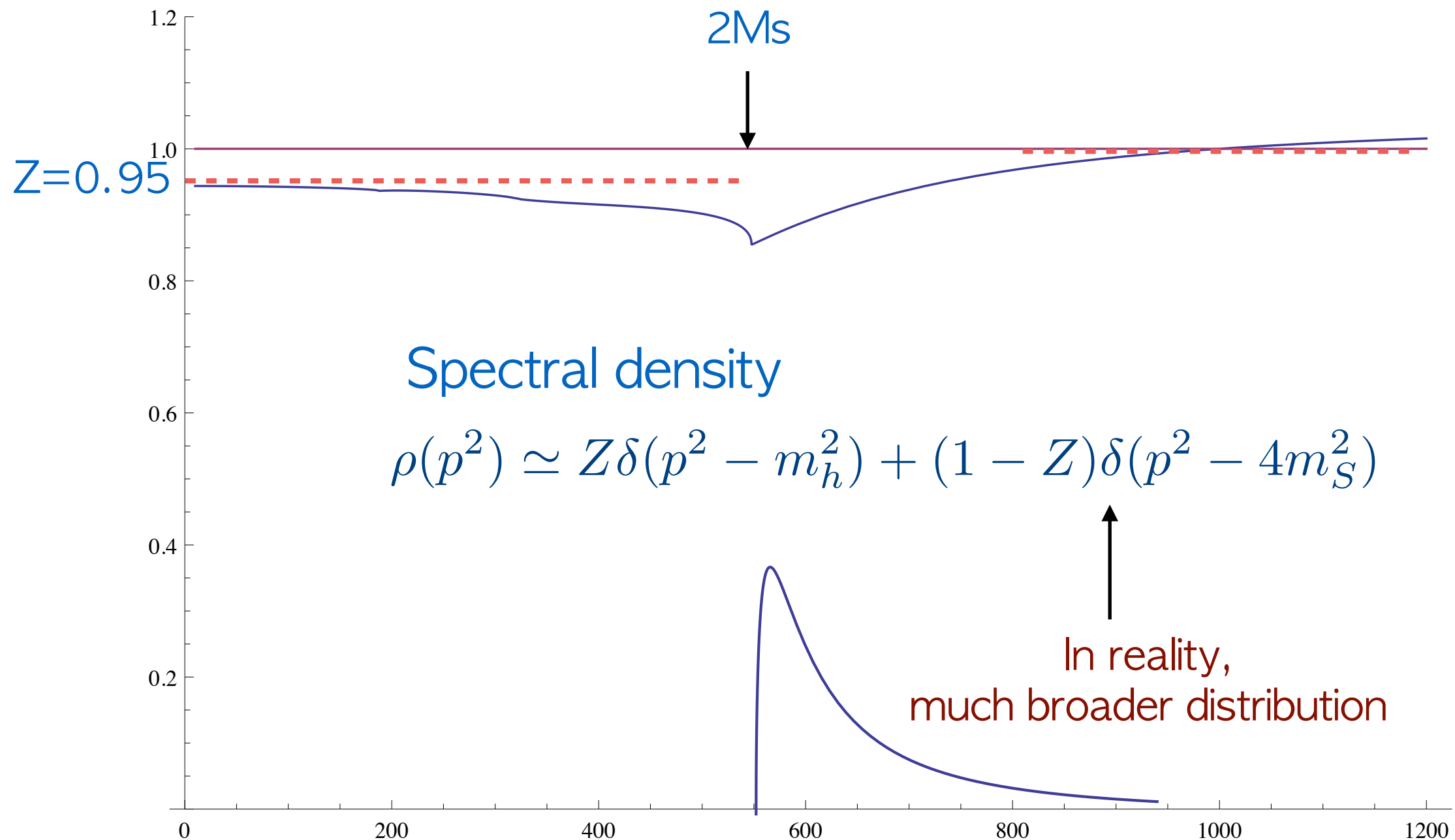


100 TeV

$\frac{v}{E}$  suppression for scalar cubic

# Relative suppression of the propagator

$$\frac{\frac{1}{p^2 - m_h^2 + \Sigma(p^2)}}{\frac{1}{p^2 - m_h^2 + i m_h \Gamma_h}}$$



(N=10,  $M_s=250$  GeV)


## Physics of Higgs with singlets using spectral density

$$\frac{1}{p^2 - m_h^2 + \Sigma(p^2)} = \int_0^\infty dq^2 \frac{\rho(q^2)}{p^2 - q^2 + i\epsilon}$$

$$\rho(p^2) \simeq Z\delta(p^2 - m_h^2) + (1 - Z)\delta(p^2 - 4m_S^2)$$

schematically the momentum dependence is captured by

$$\frac{Z}{p^2 - m_h^2 + i\epsilon} + \frac{1 - Z}{p^2 - 4m_S^2 + i\epsilon}$$



is approximated by delta function

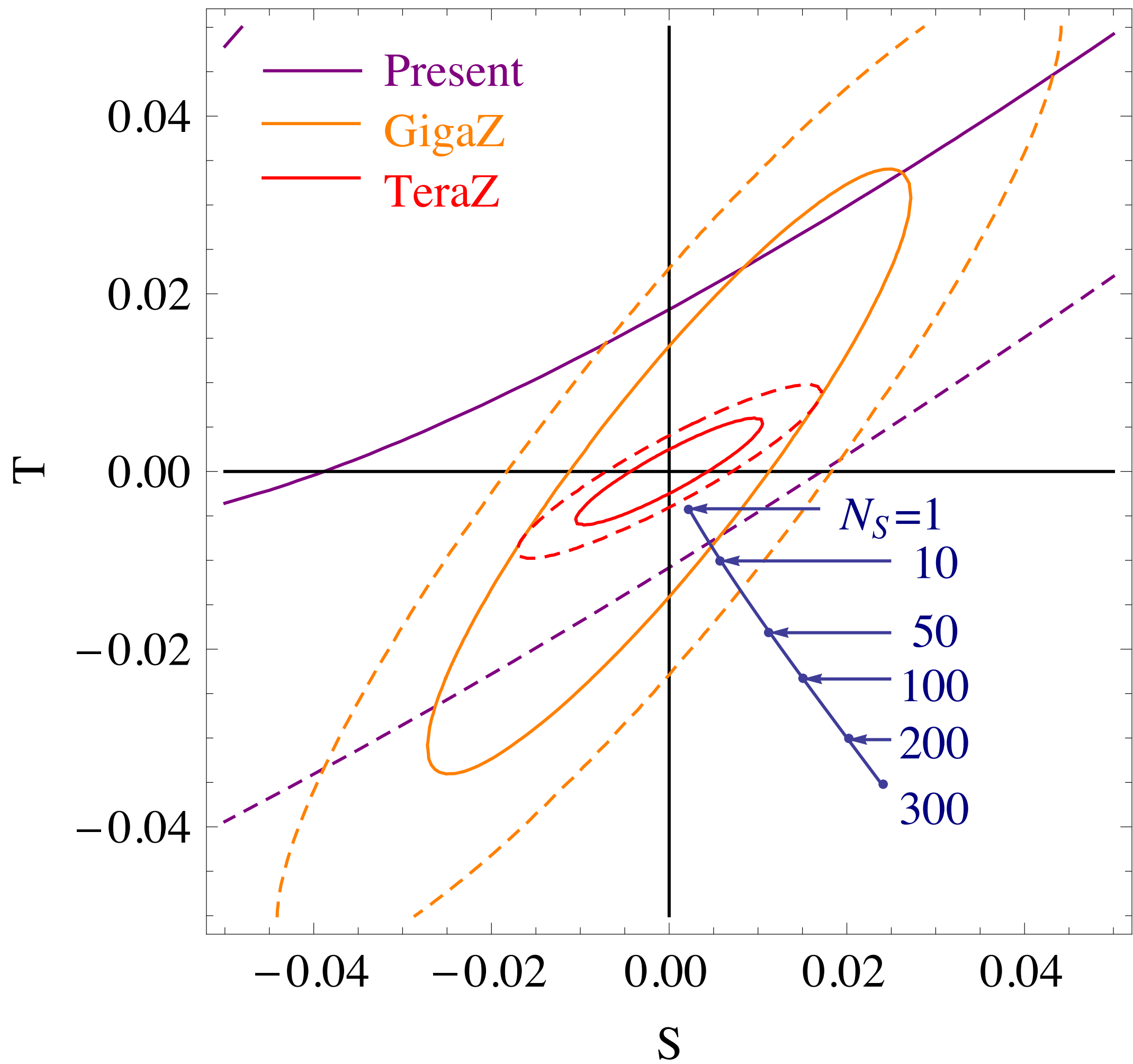
# Electroweak precision

$$\Delta S = (1 - Z) \frac{1}{6\pi} \log \frac{2m_S}{m_h}$$

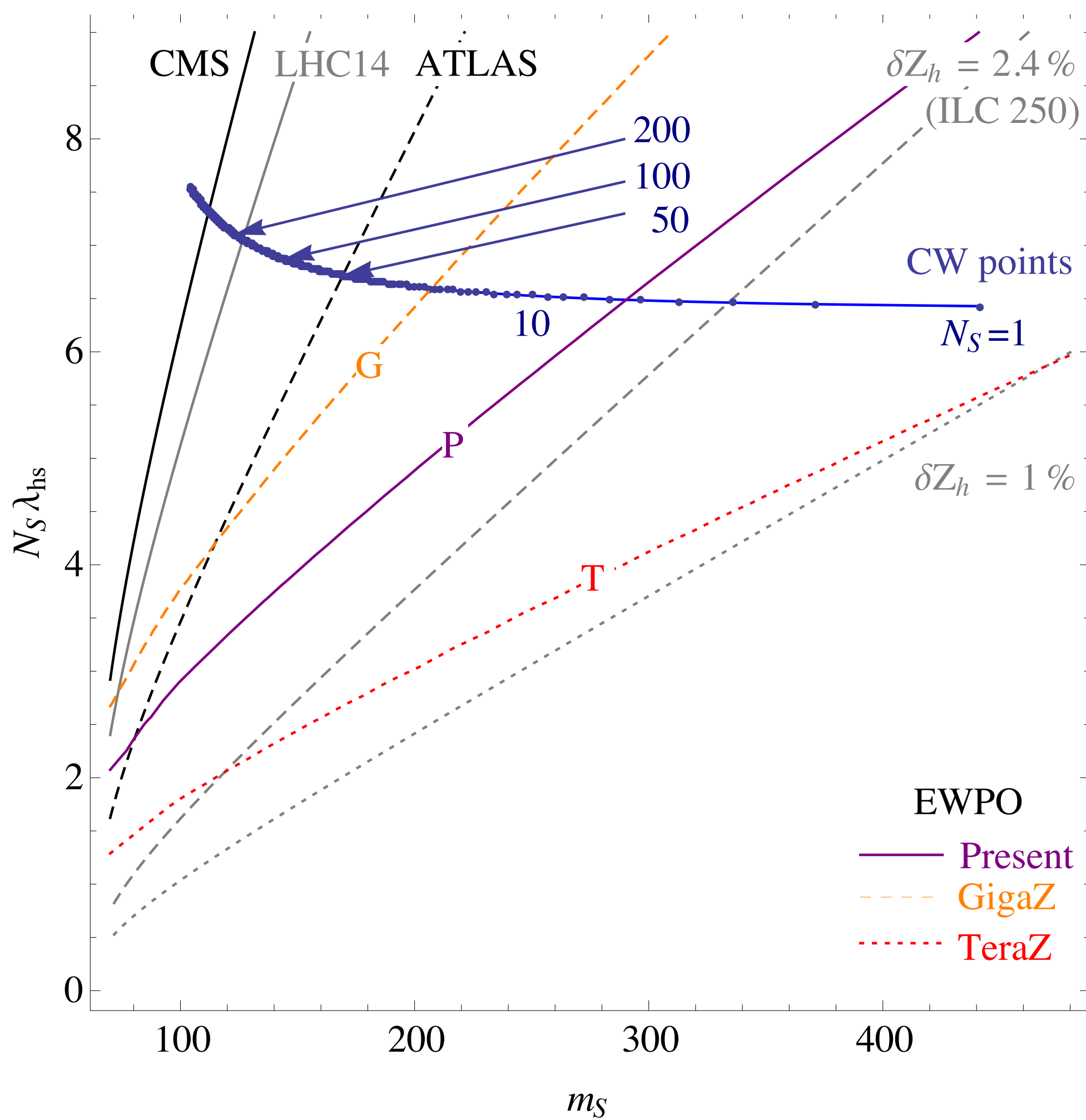
$$\Delta T = -(1 - Z) \frac{3}{8\pi \cos^2 \theta_W} \log \frac{2m_S}{m_h}$$

Preliminary result

	Ms	Z	1-Z	T
<b>Ns=10</b>	250	0.95	0.05	-0.011
<b>Ns=40</b>	175	0.90	0.10	-0.013
<b>Ns=100</b>	140	0.85	0.15	-0.020
<b>Ns=300</b>	120	0.75	0.25	-0.022







## Summary

Singlet mass

$$M_S = 250 \left( \frac{10}{N_S} \right)^{\frac{1}{4}} \text{ GeV}$$

$N_S \geq 10$   
reach : 250 GeV  
VBF from LHC 14

On-shell suppression

$$\delta\sigma_{Zh} = 5 \left( \frac{N_S}{10} \right)^{\frac{1}{2}} \%$$

$N_S \geq 10$  or 40  
95% CL on 4~10%  
from LHC 14  
all  $N_S$  (1~2% from ILC)

Higgs pair production

$$\lambda_{\text{CW}}^{(3)} = \frac{5}{3} \lambda_{\text{SM}}$$

50% from LHC14  
13% from ILC 1TeV

$$N_S \leq 40$$

Electroweak precision

$$\Delta T < -0.02$$

maximum at  $N_S=330$

# Conclusions

Quantum loop can explain the electroweak symmetry breaking.

Ginzburg-Landau

$$-\alpha\phi^2 + \beta\phi^4$$

vs

Coleman-Weinberg

$$\phi^4 \log \frac{\phi}{\langle \phi \rangle}$$

# Conclusions

Quantum loop can explain the electroweak symmetry breaking.

Ginzburg-Landau

$$-\alpha\phi^2 + \beta\phi^4$$

vs

Coleman-Weinberg

$$\phi^4 \log \frac{\phi}{\langle\phi\rangle}$$

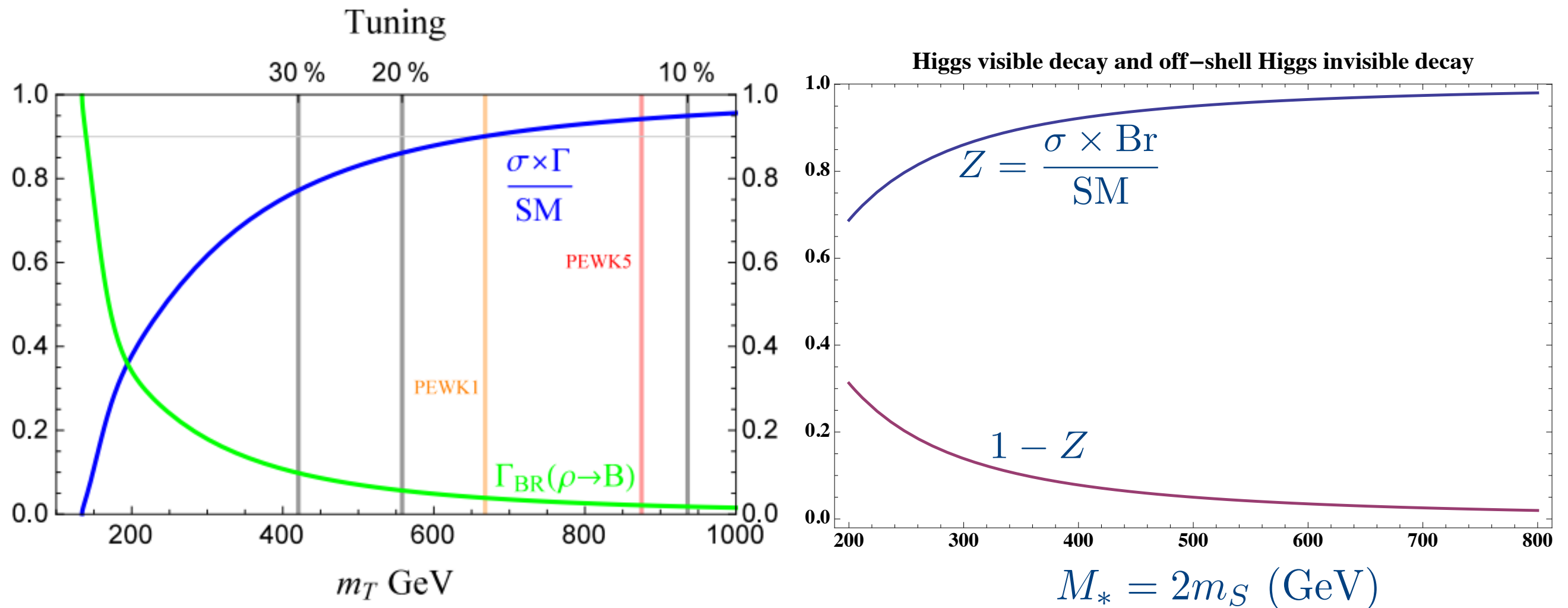
Even though there is no mixing with the singlets, the off-shell physics is very similar to mixing case. ( $2M_S$  instead of  $M_S$ )

LHC14 can cover  $N_s > 10$ , only the crazy parameter space.

Singlets coupled strongly to the Higgs would survive after LHC14.

# Coleman-Weinberg vs Twin phenomenology

Suppression of Higgs couplings disappear at high energy  $E > 2m_S$ .



Thank you

# Unstable particles

Unitarity of S-matrix requires only stable particles

R. Peierls 1955

J. Schwinger 1960

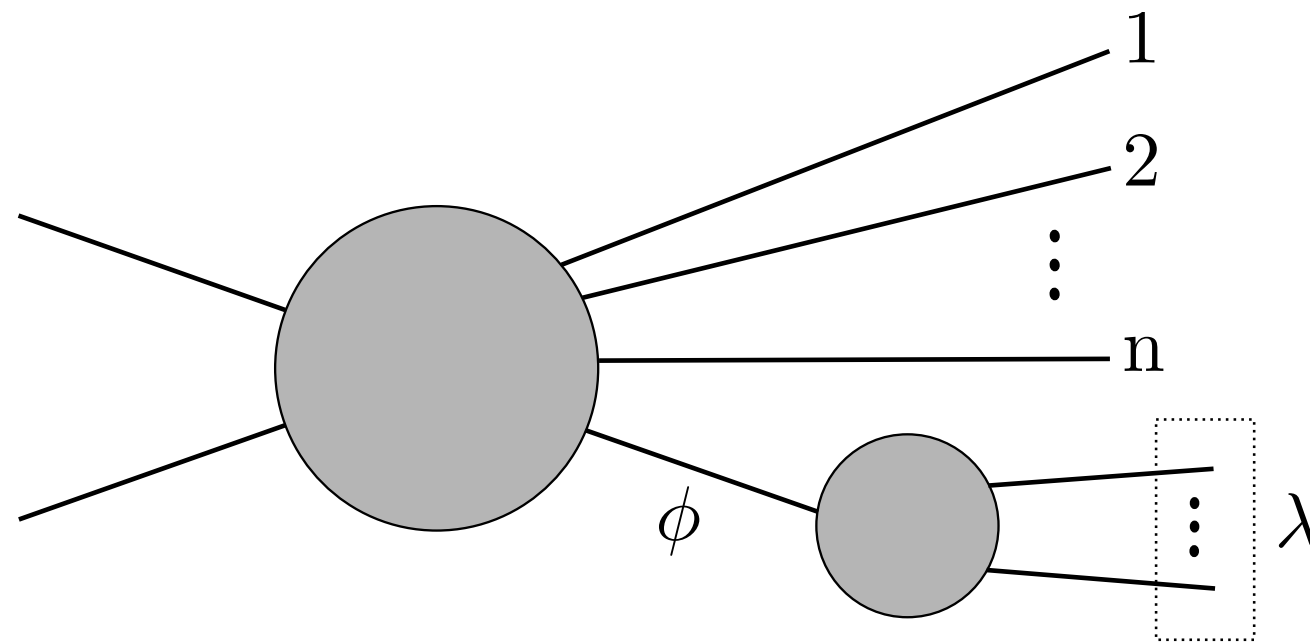
M. Veltman 1964

Narrow width approximation (NWA) :

Unstable particles are treated as if it is stable as long as the decay width is much smaller than its mass

NWA breaks down when the resonance is at a threshold

# Unstable particles : Factorization



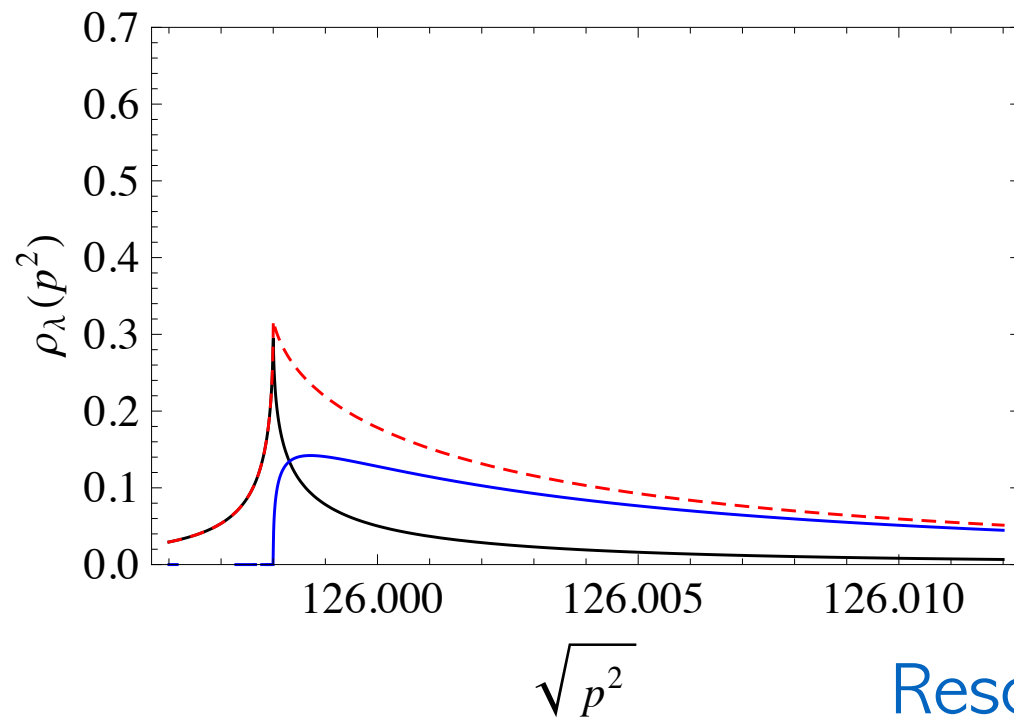
$$\begin{aligned} \sigma(\text{initial} \rightarrow 1, 2, \dots, n, \lambda) &= \int_{S_{\min}}^{S_{\max}} \sigma(\text{initial} \rightarrow m_1, m_2, \dots, m_n, \sqrt{S}) \rho_\lambda(S) \\ &\simeq \sigma(\text{initial} \rightarrow m_1, m_2, \dots, m_n, m_\lambda) \int_{S_{\min}}^{S_{\max}} \rho_\lambda(S) \end{aligned}$$



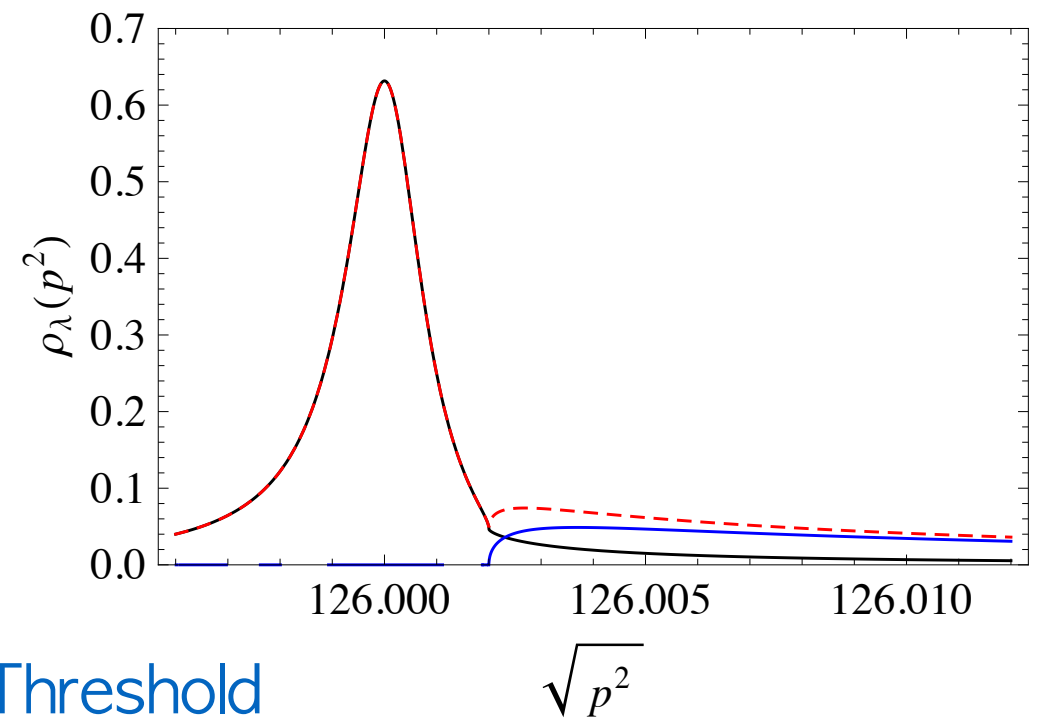
# Spectral density of toy example

## Higgs to bb and SS

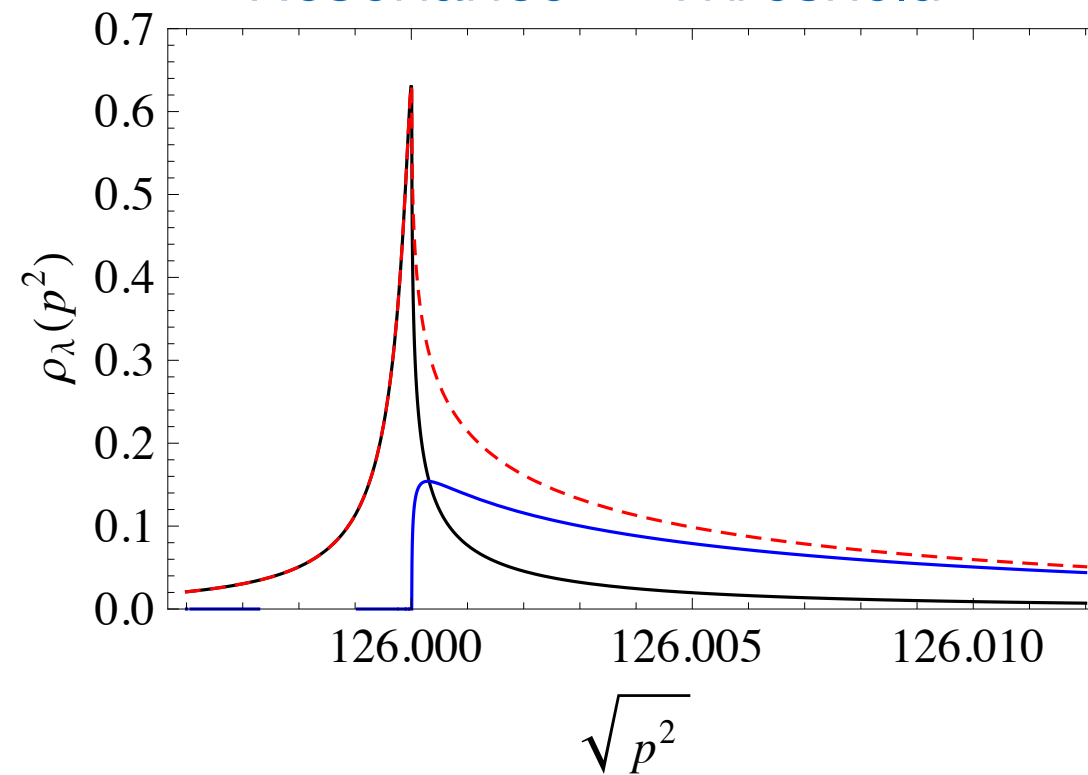
Resonance  $>$  Threshold



Resonance  $<$  Threshold



Resonance = Threshold



No definition of  
single width works

Flatte parametrization  
(1976)

# Wave function renormalisation vs. Branching ratio

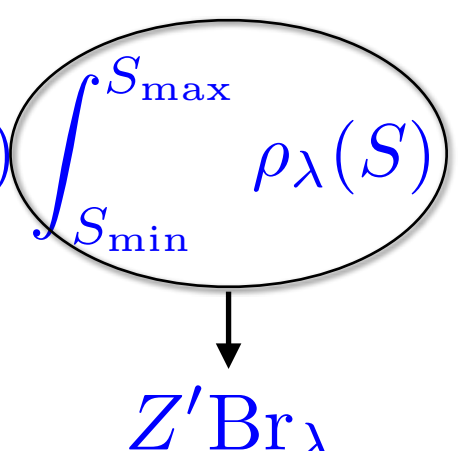
$$Z_i = \int_{S_{\min}}^{S_{\max}} \rho_i(S)$$

$$Z' = \sum_i Z_i$$

$i$  for relevant decay modes

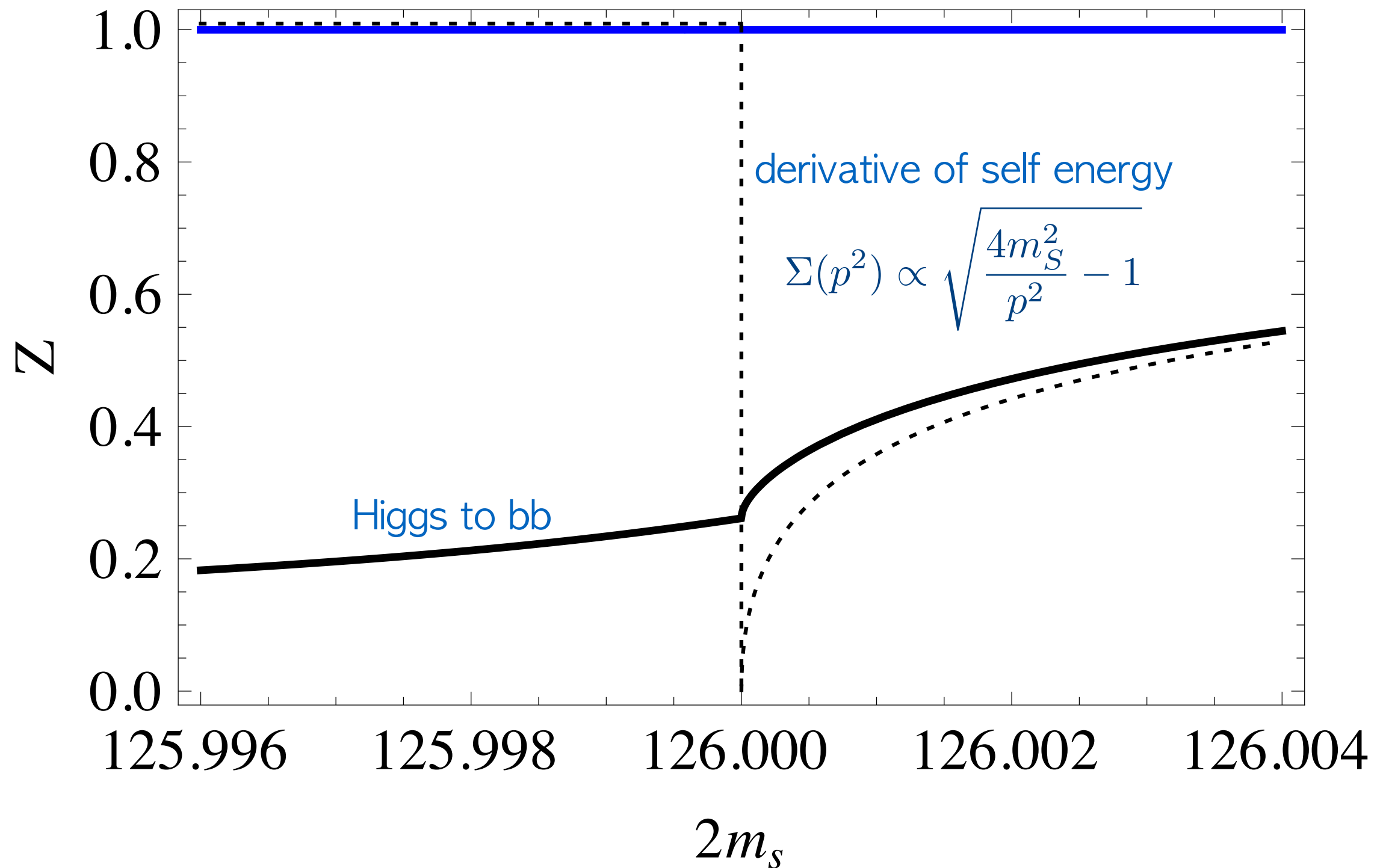
$$\text{Br}_\lambda = \frac{Z_\lambda}{Z'}$$

$$\begin{aligned} \sigma(\text{initial} \rightarrow 1, 2, \dots, n, \lambda) &= \int_{S_{\min}}^{S_{\max}} \sigma(\text{initial} \rightarrow m_1, m_2, \dots, m_n, \sqrt{S}) \rho_\lambda(S) \\ &\simeq \sigma(\text{initial} \rightarrow m_1, m_2, \dots, m_n, m_\lambda) \int_{S_{\min}}^{S_{\max}} \rho_\lambda(S) \end{aligned}$$

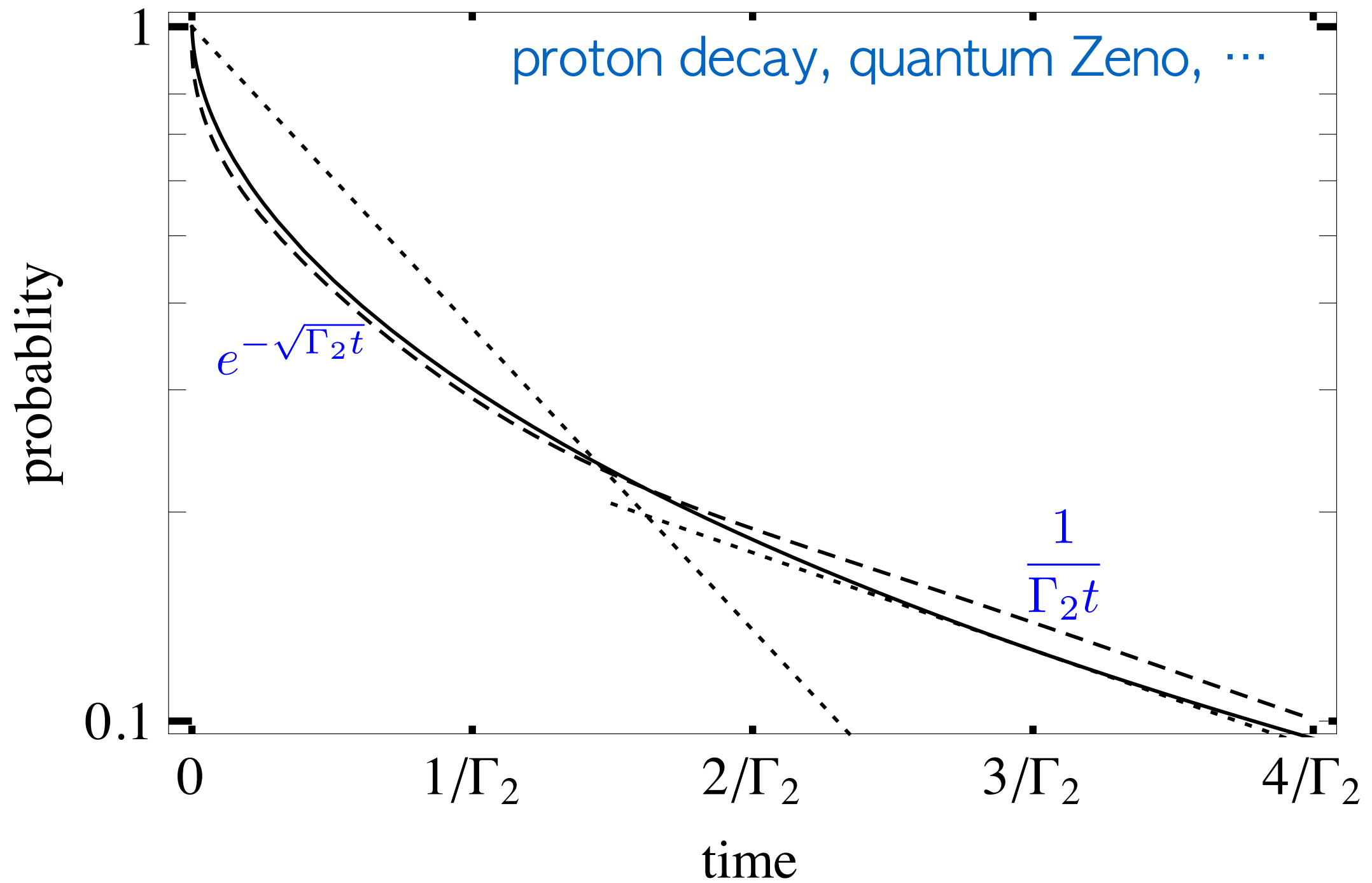


$Z' \text{Br}_\lambda$

# Branching ratios (threshold at resonance)



# Non-exponential decay



# Mass and width of unstable particles

## Complex mass/pole scheme

The Dyson resummed propagator has a pole in the unphysical second Riemann sheet.

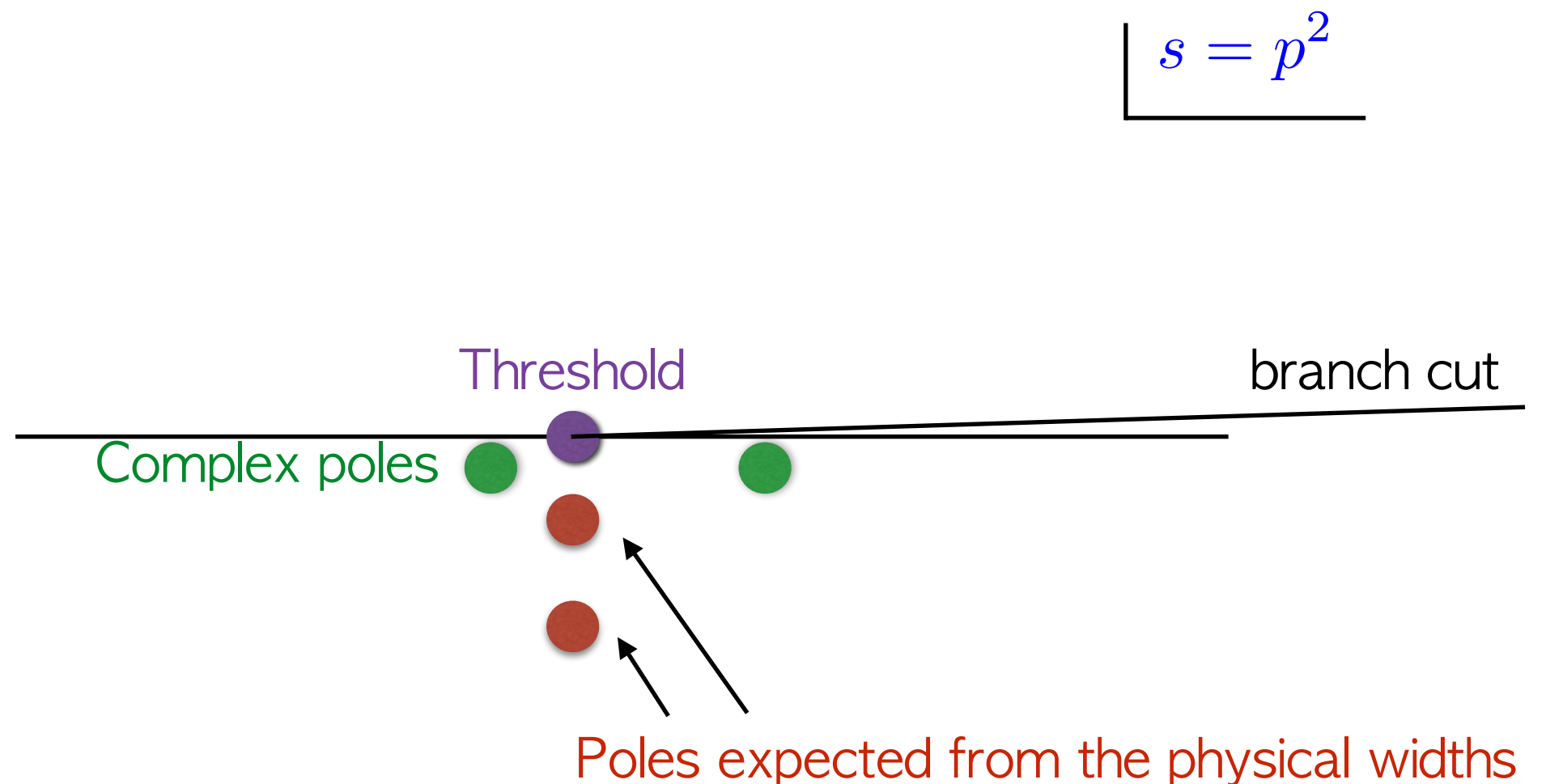
The real value is the mass and the imaginary value is the width.

$$S_* = M^2 - iM\Gamma$$

In the previous example, there are two solutions and two physical widths correspond to the shift in the real value rather than the imaginary value.

# Mass and width of unstable particles

## Complex mass/pole scheme



The mass and width obtained from the complex pole does not carry any physically meaningful information.