CP violation in kaons: precision tests of ε_K

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Contents

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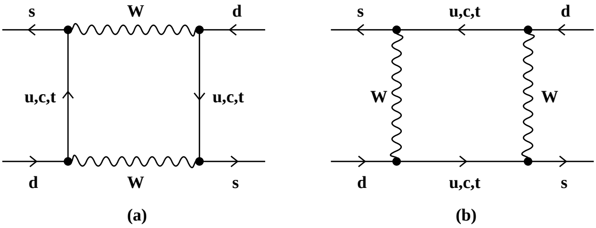
- Research Assistant Prof.: Dr. Jon Bailey
- 10 graduate students
- Secretary: Mrs. Sora Park.
- more details on `http://lgt.snu.ac.kr/`.

Group Photo (2014)

CP Violation and B_K

Kaon Eigenstates and ε

- Flavor eigenstates, $K^0 = (\bar{s}d)$ and $\bar{K}^0 = (s\bar{d})$ mix via box diagrams.



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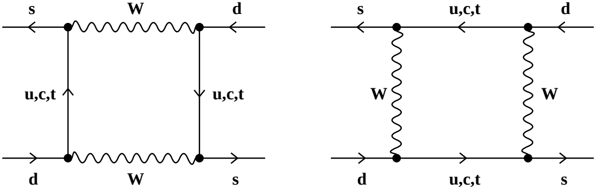

(a) (b)

- CP eigenstates K_1(even) and K_2(odd).

$$K_1 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0) \qquad K_2 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0)$$

Kaon Eigenstates and ε

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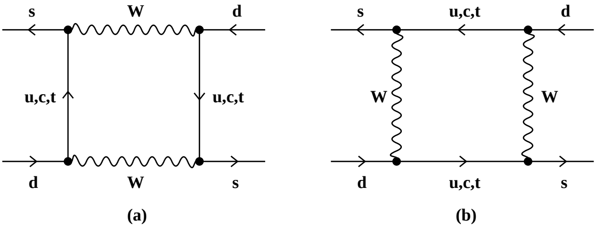


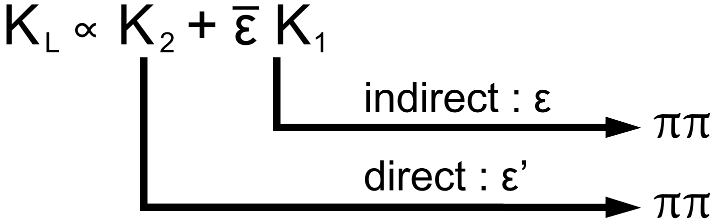
- CP eigenstates K_1(even) and K_2(odd).

$$K_1 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0) \qquad K_2 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0)$$

- Neutral Kaon eigenstates K_S and K_L.

$$K_S = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}(K_1 + \bar{\varepsilon}K_2) \qquad K_L = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}(K_2 + \bar{\varepsilon}K_1)$$

Indirect CP violation and direct CP violation



ε_K and $\hat{B}_K$, V_{cb} I

- Definition of ε_K

$$\varepsilon_K = \frac{A[K_L \to (\pi\pi)_{I=0}]}{A[K_S \to (\pi\pi)_{I=0}]}$$

- Master formula for ε_K in the Standard Model.

$$\varepsilon_K = \exp(i\theta)\,\sqrt{2}\,\sin(\theta)\left(C_\varepsilon\, X_{\mathsf{SD}}\, \hat{B}_K + \frac{\xi_0}{\sqrt{2}} + \xi_{\mathsf{LD}} \right) + \mathcal{O}(\omega\varepsilon') + \mathcal{O}(\xi_0 \Gamma_2/\Gamma_1)$$

$$X_{\mathsf{SD}} = \mathrm{Im}\lambda_t \Big[\mathrm{Re}\,\lambda_c\, \eta_{cc}\, S_0(x_c) - \mathrm{Re}\,\lambda_t\, \eta_{tt}\, S_0(x_t) - (\mathrm{Re}\,\lambda_c - \mathrm{Re}\,\lambda_t)\, \eta_{ct}\, S_0(x_c, x_t) \Big]$$

ε_K and $\hat{B}_K$, V_{cb} II

$$\lambda_i = V_{is}^* V_{id}, \qquad x_i = m_i^2/M_W^2, \qquad C_\varepsilon = \frac{G_F^2 F_K^2 m_K M_W^2}{6\sqrt{2}\,\pi^2 \Delta M_K}$$

$$\frac{\xi_0}{\sqrt{2}} = \frac{1}{\sqrt{2}} \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0} \approx -5\%$$

ξ_{LD} = Long Distance Effect $\approx 2\%$ $\longrightarrow$ systematic error

- Inami-Lim functions:

$$S_0(x_i) = x_i \left[\frac{1}{4} + \frac{9}{4(1-x_i)} - \frac{3}{2(1-x_i)^2} - \frac{3x_i^2 \ln x_i}{(1-x_i)^3} \right],$$

$$S_0(x_i, x_j) = \left\{ \frac{x_i x_j}{x_i - x_j} \left[\frac{1}{4} + \frac{3}{2(1-x_i)} - \frac{3}{4(1-x_i)^2} \right] \ln x_i - (i \leftrightarrow j) \right\} - \frac{3 x_i x_j}{4(1-x_i)(1-x_j)}$$

ε_K and $\hat{B}_K$, V_{cb} III

$$
\begin{aligned}
S_0(x_t) &\qquad \longrightarrow +70\% \\
S_0(x_c, x_t) &\qquad \longrightarrow +44\% \\
S_0(x_c) &\qquad \longrightarrow -14\%
\end{aligned}
$$

- Dominant contribution (≈70%) comes with $|V_{cb}|^4$.

$$
\begin{aligned}
\mathrm{Im}\lambda_t \cdot \mathrm{Re}\lambda_t &= \bar{\eta}\lambda^2 |V_{cb}|^4 (1-\bar{\rho}) \\
\mathrm{Re}\lambda_c &= -\lambda(1-\frac{\lambda^2}{2}) + \mathcal{O}(\lambda^5) \\
\mathrm{Re}\lambda_t &= -(1-\frac{\lambda^2}{2}) A^2\lambda^5 (1-\bar{\rho}) + \mathcal{O}(\lambda^7) \\
\mathrm{Im}\lambda_t &= \eta A^2 \lambda^5 + \mathcal{O}(\lambda^7)
\end{aligned}
$$

ε_K and $\hat{B}_K$, V_{cb} IV

- Definition of $\hat{B}_K$ in standard model.

$$B_K = \frac{\langle \bar{K}_0 | [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] | K_0 \rangle}{\frac{8}{3}\langle \bar{K}_0 | \bar{s}\gamma_\mu\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu\gamma_5 d | K_0 \rangle}$$

$$\hat{B}_K = C(\mu) B_K(\mu), \qquad C(\mu) = \alpha_s(\mu)^{-\frac{\gamma_0}{2b_0}} [1 + \alpha_s(\mu) J_3]$$

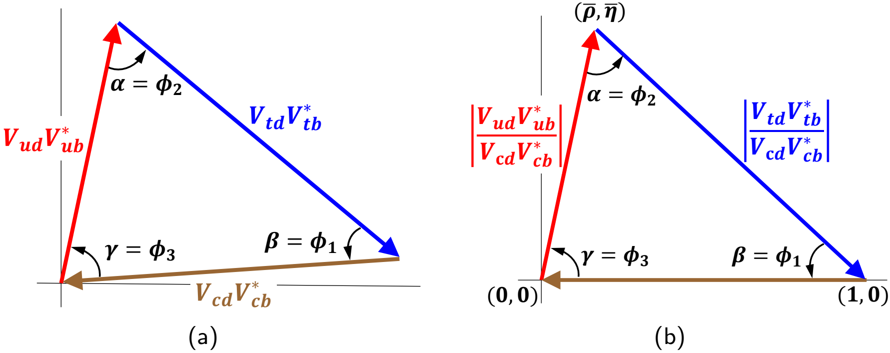
- Experiment:

$$\varepsilon_K = (2.228 \pm 0.011) \times 10^{-3} \times e^{i\phi_\varepsilon}$$

$$\phi_\varepsilon = 43.52(5)^\circ$$

ε_K on the lattice

Unitarity Triangle $\rightarrow (\bar{\rho}, \bar{\eta})$



Global UT Fit and Angle-Only-Fit (AOF)

Global UT Fit

- Input: $|V_{ub}|/|V_{cb}|$, Δm_d, $\Delta m_s/\Delta m_d$, ε_K, and $\sin(2\beta)$.
- Determine the UT apex $(\bar{\rho}, \bar{\eta})$.
- Take λ from

$$|V_{us}| = \lambda + \mathcal{O}(\lambda^7)\,,$$

  which comes from K_{l3} and $K_{\mu 2}$.
- Disadvantage: unwanted correlation between $(\bar{\rho}, \bar{\eta})$ and ε_K.

AOF

- Input: $\sin(2\beta)$, $\cos(2\beta)$, $\sin(\gamma)$, $\cos(\gamma)$, $\sin(2\beta+\gamma)$, $\cos(2\beta+\gamma)$, and $\sin(2\alpha)$.
- Determine the UT apex $(\bar{\rho}, \bar{\eta})$.
- Take λ from $|V_{us}| = \lambda + \mathcal{O}(\lambda^7)$, which comes from K_{l3} and $K_{\mu 2}$.
- Use $|V_{cb}|$ to determine A.

$$|V_{cb}| = A\lambda^2 + \mathcal{O}(\lambda^7)$$

- Advantage: NO correlation between $(\bar{\rho}, \bar{\eta})$ and ε_K.

Inputs of Angle-Only-Fit (AOF)

- $A_{\mathsf{CP}}(J/\psi\ K_s) \to S_{\psi K_s} = \sin(2\beta)$ with assumption of $S_{\psi K_s} \ggg C_{\psi K_s}$.
- $(B \to DK) + (B \to [K\pi]_D K) +$ (Dalitz method) give $\sin(\gamma)$ and $\cos(\gamma)$.
- $S(D^-\pi^+)$ and $S(D^+\pi^-)$ give $\sin(2\beta+\gamma)$ and $\cos(2\beta+\gamma)$.
- $(B^0 \to \pi^+\pi^-) + (B^0 \to \rho^+\rho^-) + (B^0 \to (\rho\pi)^0)$ give $\sin(2\alpha)$.
- Combining all of these gives β, γ, and α, which leads to the UT apex $(\bar\rho, \bar\eta)$.

Wolfenstein Parameters

Input Parameters for Angle-Only-Fit (AOF)

- ϵ_K, $\hat{B}_K$, and $|V_{cb}|$ are used as inputs to determine the UT angles in the global fit of `UTfit` and `CKMfitter`.
- Instead, we can use angle-only-fit result for the UT apex $(\bar{\rho}, \bar{\eta})$.
- Then, we can take λ independently from

$$|V_{us}| = \lambda + \mathcal{O}(\lambda^7)\,,$$

  which comes from K_{l3} and $K_{\mu 2}$.
- Use $|V_{cb}|$ instead of A.

$$|V_{cb}| = A\lambda^2 + \mathcal{O}(\lambda^7)$$

λ	$0.22537(61)$	[1] CKMfitter
	$0.2255(6)$	[1] UTfit
	$0.2253(8)$	[1] $\lvert V_{us}\rvert$ (AOF)
$\bar{\rho}$	$0.117(21)$	[1] CKMfitter
	$0.124(24)$	[1] UTfit
	$0.139(29)$	[2] UTfit (AOF)
$\bar{\eta}$	$0.353(13)$	[1] CKMfitter
	$0.354(15)$	[1] UTfit
	$0.337(16)$	[2] UTfit (AOF)

Input Parameters of B_K, V_{cb} and others

B_K

$\hat{B}_K$	$0.7625(97)$	[3] FLAG
	$0.7379(47)(365)$	[4] SWME
	$0.7499(24)(150)$	[5] RBC-UK

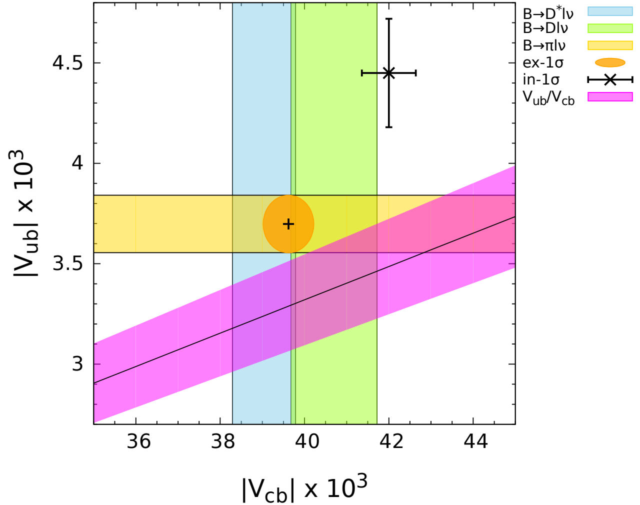
$\lvert V_{cb}\rvert \times 10^3$

$B \to X_c \ell \bar{\nu}$	$42.00(64)$	[6]
$B \to D^* \ell \bar{\nu}$	$39.04(49)(53)(19)$	[7]
$B \to D \ell \bar{\nu}$	$40.70(100)(20)$	[8]
ex-combined	$39.62(60)$	wlee

Others

G_F	$1.1663787(6) \times 10^{-5}$ GeV^{-2}	[1]
M_W	$80.385(15)$ GeV	[1]
$m_c(m_c)$	$1.2733(76)$ GeV	[9]
$m_t(m_t)$	$163.3(2.7)$ GeV	[10]
η_{cc}	$1.72(27)$	[11]
η_{tt}	$0.5765(65)$	[12]
η_{ct}	$0.496(47)$	[13]
θ	$43.52(5)^\circ$	[1]
m_{K^0}	$497.614(24)$ MeV	[1]
ΔM_K	$3.484(6) \times 10^{-12}$ MeV	[1]
F_K	$156.2(7)$ MeV	[1]

Current Status of $|V_{cb}|$ in ICHEP 2016



ξ_0
Indirect Method

$$\xi_0 = \frac{\mathrm{Re}A_0}{\mathrm{Im}A_0}$$

ξ_0	$-1.63(19) \times 10^{-4}$	RBC-UK-2015 [14]

- RBC-UKQCD calculated $\mathrm{Im}A_2$. $\mathrm{Im}A_2 \to \xi_0$

$$\mathrm{Re}\Big(\frac{\epsilon'_K}{\epsilon_K}\Big) = \frac{1}{\sqrt{2}|\epsilon_K|}\omega\Big(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \xi_0\Big)\,.$$

  Other inputs ω, ϵ_K and ϵ'_K/ϵ_K are taken from the experimental values.
- Here, we choose an approximation of $\cos(\phi_{\epsilon'} - \phi_\epsilon) \approx 1$.
- $\phi_\epsilon = 43.52(5)$, $\phi_{\epsilon'} = 42.3(1.5)$
- Isopspin breaking effect: (at most 20% of ξ_0) $\to$ (1% in ε_K) $\to$ neglected!

ξ_0
Direct Method

- RBC-UKQCD calculated $\mathrm{Im}A_0$. $\mathrm{Im}A_0 \to \xi_0$.

$$\xi_0 = \frac{\mathrm{Im}A_0}{\mathrm{Re}\,A_0} = -0.57(49) \times 10^{-4}$$

  Other input $\mathrm{Re}\,A_0$ is taken from the experimental value.
- RBC-UKQCD also calculated δ_0

$$\delta_0 = 23.8(49)(12)^\circ$$

  This value is 3.5σ away from the experimental value.
- This indicates that this method belongs to the category of exploratory study rather than precision measurement.
- Hence, we use the indirect method to determine ξ_0.

ξ_0
Comparison

Input Parameters: ξ_0

Method	Value	Reference
Indirect	$-1.63(19) \times 10^{-4}$	RBC-UK-2015 [14]
Direct	$-0.57(49) \times 10^{-4}$	RBC-UK-2015 [15]

ξ_{LD}

- Definition:

$$\xi_{\text{LD}} = \frac{m'_{\text{LD}}}{\sqrt{2}\,\Delta M_K}$$

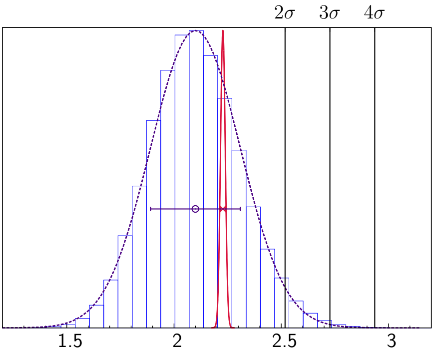
$$m'_{\text{LD}} = -\text{Im}\left[\mathcal{P}\sum_C \frac{\langle \overline{K}^0|H_{\text{w}}|C\rangle\langle C|H_{\text{w}}|K^0\rangle}{m_{K^0} - E_C}\right]$$

- Rough estimate in [PRD 88, 014508] gives

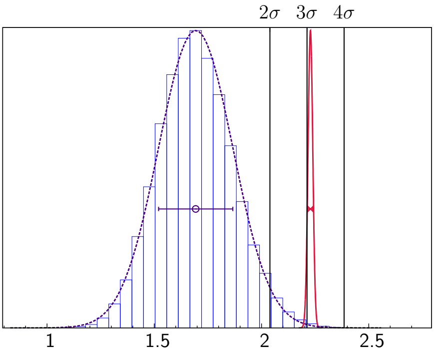
$$\xi_{\text{LD}} = (0 \pm 1.6)\%$$

- Precise lattice QCD calculation is not available yet.

ϵ_K: FLAG $\hat{B}_K$, AOF of $(\bar{\rho}, \bar{\eta})$, V_{us}



Inclusive V_{cb}

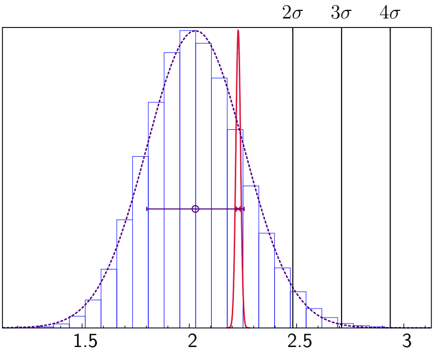


Exclusive V_{cb}

- With exclusive V_{cb}, it shows 3.2σ tension.

$$\epsilon_K^{Exp} = 2.228(11) \times 10^{-3}$$
$$\epsilon_K^{SM} = 1.69(17) \times 10^{-3}$$

ϵ_K: SWME $\hat{B}_K$, AOF of $(\bar{\rho}, \bar{\eta})$, V_{us}



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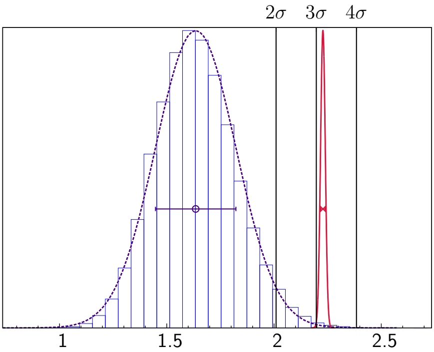


Exclusive V_{cb}

- With exclusive V_{cb}, it shows 3.1σ tension.

$$\epsilon_K^{Exp} = 2.228(11) \times 10^{-3}$$
$$\epsilon_K^{SM} = 1.63(19) \times 10^{-3}$$

Current Status of ε_K

- FLAG 2014: (in units of 1.0×10^{-3}, AOF)

$$\varepsilon_K = 1.69 \pm 0.17 \qquad \text{for Exclusive } V_{cb} \text{ (Lattice QCD)}$$
$$\varepsilon_K = 2.10 \pm 0.21 \qquad \text{for Inclusive } V_{cb} \text{ (QCD Sum Rule)}$$

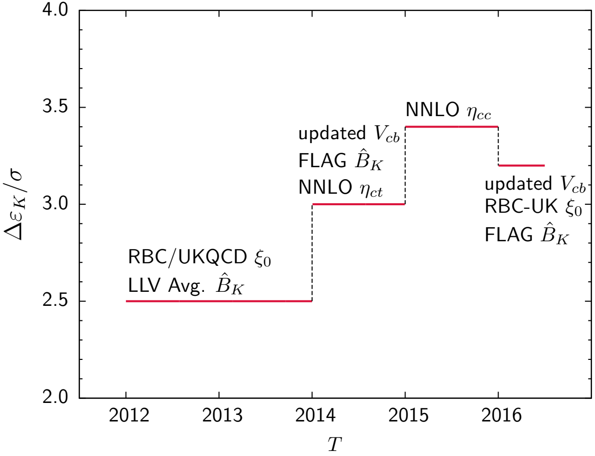
- Experiments:

$$\varepsilon_K = 2.228 \pm 0.011$$

- Hence, we observe 3.2 σ difference between the SM theory (Lattice QCD) and experiments.

- What does this mean? $\longrightarrow$ Breakdown of SM ?

Time Evolution of $\Delta\varepsilon_K$ on the Lattice

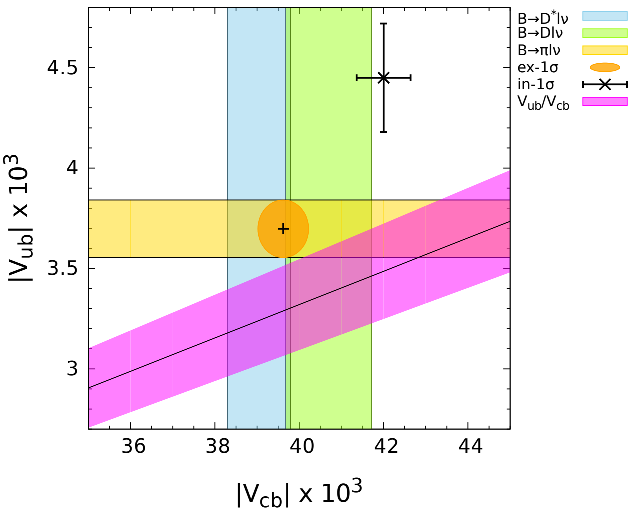


- $\Delta\varepsilon_K \equiv \varepsilon_K^{\text{exp}} - \varepsilon_K^{\text{SM}}$

Error Budget of Exclusive ε_K

source	error (%)	memo
V_{cb}	30.1	Exclusive Combined
$\bar{\eta}$	24.7	AOF
η_{ct}	19.5	$c-t$ Box
η_{cc}	8.2	$c-c$ Box
$\bar{\rho}$	6.6	AOF
m_t	3.0	top quark mass
ξ_{LD}	2.5	Long-distance
$\hat{B}_K$	1.8	FLAG
ξ_0	1.2	$\text{Im}(A_0)/\text{Re}(A_0)$
$\vdots$	$\vdots$	

Current Status of $|V_{cb}|$ in ICHEP 2016



V_{cb} on the lattice

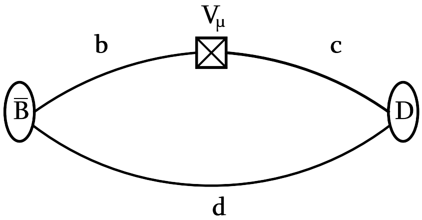
How to obtain V_{cb}

- Exclusive V_{cb} determination.
- $\bar{B} \to D + \ell + \bar{\nu}_\ell$
- $\bar{B} \to D^* + \ell + \bar{\nu}_\ell$

What to calculate on the lattice.

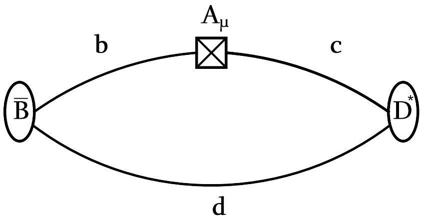
- $\langle D|Q_1|\bar{B}\rangle$ with $Q_1 = V_\mu,\ S$.

$$V_\mu = \bar{b}\gamma_\mu c$$
$$S = \bar{b}c$$



- $\langle D^*|Q_2|\bar{B}\rangle$ with $Q_2 = A_\mu,\ V_\mu,\ P$.

$$A_\mu = \bar{b}\gamma_\mu\gamma_5 c$$
$$P = \bar{b}\gamma_5 c$$



B_s meson mass

Motivation

- In heavy flavor physics, V_{cb} is of enormous interest.
- The dominant error in ε_K comes from V_{cb}.

$$\begin{cases} 40.1\% & \leftarrow V_{cb} \\ 1.5\% & \leftarrow \hat{B}_K \end{cases}$$

- 3.2σ tension is observed using most up to date input parameters.

$$|\varepsilon_K|^{\text{exp}} = 2.228(11) \times 10^{-3} \qquad \text{(PDG)}$$
$$|\varepsilon_K|^{\text{SM}} = 1.69(17) \times 10^{-3} \qquad (\text{FLAG } \hat{B}_K, \text{FNAL/MILC } V_{cb})$$

- More precise determination of V_{cb} might lead to larger tension.
- Because the dominant error in V_{cb} comes from heavy quark discretization effect, we plan to use the OK action for the form factor calculation of the semi-leptonic decays

$$\bar{B} \to D^* \ell \nu_\ell \,, \qquad \bar{B} \to D \ell \nu_\ell .$$

- Here, we will verify the improvement in B meson spectrum.

OK Action (mass form)

$$S_{\rm OK} = S_{\rm Fermilab} + S_{\rm new}\,, \qquad S_{\rm Fermilab} = S_0 + S_B + S_E$$

$$\begin{aligned} S_0 &= m_0 \sum_x \bar\psi(x)\psi(x) + \sum_x \bar\psi(x)\gamma_4 D_4 \psi(x) - \tfrac{1}{2} a \sum_x \bar\psi(x)\triangle_4\psi(x) \\ &+ \zeta \sum_x \bar\psi(x)\vec\gamma\cdot\vec D\psi(x) - \tfrac{1}{2} r_s \zeta a \sum_x \bar\psi(x)\triangle^{(3)}\psi(x) \\ &= \mathcal{O}(1) + \mathcal{O}(\lambda) \qquad [\lambda \sim a\Lambda,\ \Lambda/m_Q] \end{aligned}$$

$$S_B = -\tfrac{1}{2} c_B \zeta a \sum_x \bar\psi(x) i\vec\Sigma\cdot\vec B\psi(x) \to \mathcal{O}(\lambda)$$

$$S_E = -\tfrac{1}{2} c_E \zeta a \sum_x \bar\psi(x)\vec\alpha\cdot\vec E\psi(x) \to \mathcal{O}(\lambda^2) \qquad (c_E \neq c_B : \text{ OK action})$$

$$m_0 = \frac{1}{2\kappa_t} - (1 + 3 r_s\zeta + 18 c_4)$$

[M. B. Oktay and A. S. Kronfeld, PRD **78**, 014504 (2008)]

[A. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, PRD **55**, 3933 (1997)]

OK Action (mass form)

$$
\begin{aligned}
S_{\text{new}} = \mathcal{O}(\lambda^3) &= c_1 a^2 \sum_x \bar{\psi}(x) \sum_i \gamma_i D_i \triangle_i \psi(x) \\
&+ c_2 a^2 \sum_x \bar{\psi}(x) \{\vec{\gamma} \cdot \vec{D}, \triangle^{(3)}\} \psi(x) \\
&+ c_3 a^2 \sum_x \bar{\psi}(x) \{\vec{\gamma} \cdot \vec{D}, i\vec{\Sigma} \cdot \vec{B}\} \psi(x) \\
&+ c_{EE} a^2 \sum_x \bar{\psi}(x) \{\gamma_4 D_4, \vec{\alpha} \cdot \vec{E}\} \psi(x) \\
&+ c_4 a^3 \sum_x \bar{\psi}(x) \sum_i \triangle_i{}^2 \psi(x) \\
&+ c_5 a^3 \sum_x \bar{\psi}(x) \sum_i \sum_{j \neq i} \{\mathrm{i}\Sigma_i B_i, \triangle_j\} \psi(x)
\end{aligned}
$$

OK Action: Tadpole Improvement (hopping form)

$$
\begin{aligned}
& c_5 a^3 \bar{\psi}(x) \sum_i \sum_{j \neq i} \{ \mathrm{i}\Sigma_i B_{i\mathrm{lat}}, \triangle_{j\mathrm{lat}} \} \psi(x) \\
&= \mathrm{i}\frac{2\tilde{c}_5 \tilde{\kappa}_t}{4u_0^2} \bar{\psi}_x \sum_i \Sigma_i T_i^{(3)} \psi_x - \mathrm{i}\frac{32\tilde{c}_5 \tilde{\kappa}_t}{2u_0^3} \bar{\psi}_x \overrightarrow{\Sigma} \cdot \overrightarrow{B} \psi_x \\
&+ \mathrm{i}\frac{2\tilde{c}_5 \tilde{\kappa}_t}{u_0^4} \bar{\psi}_x \sum_i \Big(-\frac{1}{4} \Sigma_i T_i^{(3)} + \sum_{j \neq i} \{ \Sigma_i B_i, (T_j + T_{-j}) \} \Big) \psi_x
\end{aligned}
$$

$$
T_i^{(3)} \equiv \sum_{j,k=1}^{3} \epsilon_{ijk} \Big(T_{-k}(T_j - T_{-j})T_k - T_k(T_j - T_{-j})T_{-k} \Big)
$$

Measurement

Gauge Ensemble, Heavy Quark κ, Meson Momentum

- MILC asqtad $N_f = 2 + 1$

a(fm)	$N_L^3 \times N_T$	β	am'_l	am'_s	u_0	a^{-1}(GeV)	N_{conf}	$N_{t_{\text{src}}}$
0.12	$20^3 \times 64$	6.79	0.02	0.05	0.8688	1.683^{+43}_{-16}	500	6

- 11 momenta $|\boldsymbol{p}a| = 0,\ 0.099,\ \cdots,\ 1.26$

Measurement: Interpolating Operator

- Meson correlator

$$C(t,\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{x}} \langle \mathcal{O}^\dagger(t,\boldsymbol{x})\mathcal{O}(0,\mathbf{0})\rangle$$

- Heavy-light meson interpolating operator

$$\mathcal{O}_{\mathsf{t}}(x) = \bar{\psi}_\alpha(x)\Gamma_{\alpha\beta}\Omega_{\beta\mathsf{t}}(x)\chi(x)$$

$$\Gamma = \begin{cases} \gamma_5 & \text{(Pseudo-scalar)} \\ \gamma_\mu & \text{(Vector)} \end{cases} \quad , \ \Omega(x) \equiv \gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}\gamma_4^{x_4}$$

- Quarkonium interpolating operator

$$\mathcal{O}(x) = \bar{\psi}_\alpha(x)\Gamma_{\alpha\beta}\psi_\beta(x)$$

[Wingate *et al.*, PRD **67**, 054505 (2003) , C. Bernard *et al.*, PRD **83**, 034503 (2011)]

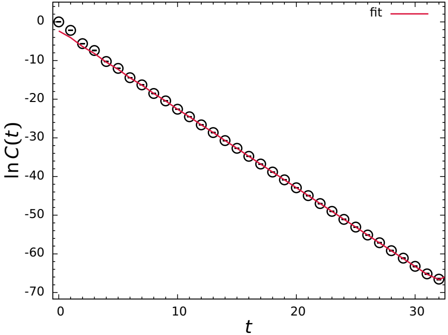
Correlator Fit

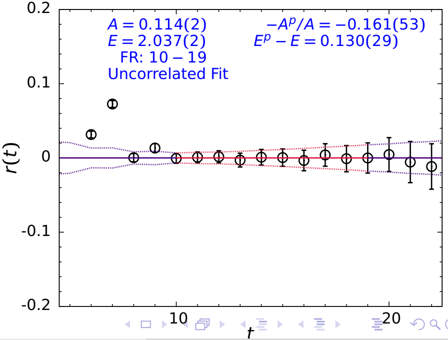
- fit function

$$f(t) = A\{e^{-Et} + e^{-E(T-t)}\} + (-1)^t A^p\{e^{-E^p t} + e^{-E^p(T-t)}\}$$

- fit residual

$$r(t) = \frac{C(t) - f(t)}{|C(t)|}, \text{where } C(t) \text{ is data.}$$





Correlator Fit: Effective Mass

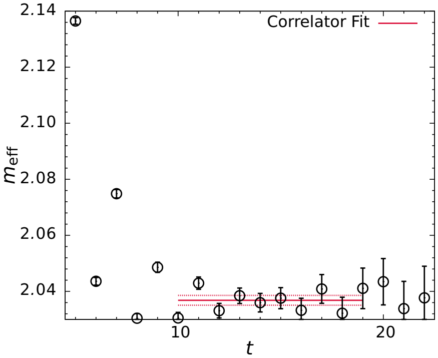
$$m_{\text{eff}}(t) = \frac{1}{2} \ln \left(\frac{C(t)}{C(t+2)} \right)$$

For small t,

$$\begin{aligned} C(t) \cong & A(e^{-Et} + \beta e^{-(E+\Delta E)t}) \\ = & A e^{-Et}(1 + \beta e^{-(\Delta E)t}) \ , \end{aligned}$$

$$\begin{cases} \beta > 0 & \text{(excited state)} \\ \beta \sim -(-1)^t & \text{(time parity state)} \end{cases}$$

$$m_{\text{eff}} \approx E + \beta(\Delta E) e^{-(\Delta E)t}$$

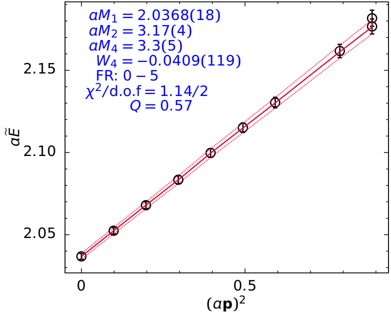


$[\overline{Q}q$, PS, $\kappa = 0.041$, $\boldsymbol{p} = \boldsymbol{0}]$

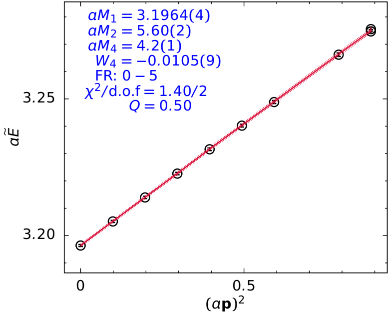
Dispersion Relation

$$E = M_1 + \frac{\boldsymbol{p}^2}{2M_2} - \frac{(\boldsymbol{p}^2)^2}{8M_4^3} - \frac{a^3 W_4}{6}\sum_i p_i^4$$

$$\widetilde{E} = E + \frac{a^3 W_4}{6}\sum_i p_i^4 \,, \quad \boldsymbol{n} = (2,2,1)\,,(3,0,0)$$



$[\overline{Q}q$, PS, $\kappa = 0.041]$



$[\overline{Q}Q$, PS, $\kappa = 0.041]$

Improvement Test: Inconsistency Parameter

$$I \equiv \frac{2\delta M_{\overline{Q}q} - (\delta M_{\overline{Q}Q} + \delta M_{\overline{q}q})}{2M_{2\overline{Q}q}} = \frac{2\delta B_{\overline{Q}q} - (\delta B_{\overline{Q}Q} + \delta B_{\overline{q}q})}{2M_{2\overline{Q}q}}$$

$$M_{1\overline{Q}q} = m_{1\overline{Q}} + m_{1q} + B_{1\overline{Q}q} \qquad \delta M_{\overline{Q}q} = M_{2\overline{Q}q} - M_{1\overline{Q}q}$$

$$M_{2\overline{Q}q} = m_{2\overline{Q}} + m_{2q} + B_{2\overline{Q}q} \qquad \delta B_{\overline{Q}q} = B_{2\overline{Q}q} - B_{1\overline{Q}q}$$

[S. Collins *et al.*, NPB **47**, 455 (1996) , A. S. Kronfeld, NPB **53**, 401 (1997)]

- Inconsistency parameter I can be used to examine the improvements by $\mathcal{O}(\boldsymbol{p}^4)$ terms in the action. OK action is designed to improve these terms and matched at tree-level.
- Binding energies B_1 and B_2 are of order $\mathcal{O}(\boldsymbol{p}^2)$. Because the kinetic meson mass M_2 appears with a factor $\boldsymbol{p}^2$, the leading contribution of binding energy B_2 generated by $\mathcal{O}(\boldsymbol{p}^4)$ terms in the action.

$$E = M_1 + \frac{\boldsymbol{p}^2}{2M_2} + \cdots = M_1 + \frac{\boldsymbol{p}^2}{2(m_{2\overline{Q}} + m_{2q})}\left[1 - \frac{B_{2\overline{Q}q}}{(m_{2\overline{Q}} + m_{2q})} + \cdots\right] + \cdots$$

Improvement Test: Inconsistency Parameter

$$I \cong \frac{2\delta M_{\overline{Q}q} - \delta M_{\overline{Q}Q}}{2M_{2\overline{Q}q}} \cong \frac{2\delta B_{\overline{Q}q} - \delta B_{\overline{Q}Q}}{2M_{2\overline{Q}q}}$$

- Considering non-relativistic limit of quark and anti-quark system, for S-wave case ($\mu_2^{-1} = m_{2\overline{Q}}^{-1} + m_{2q}^{-1}$),

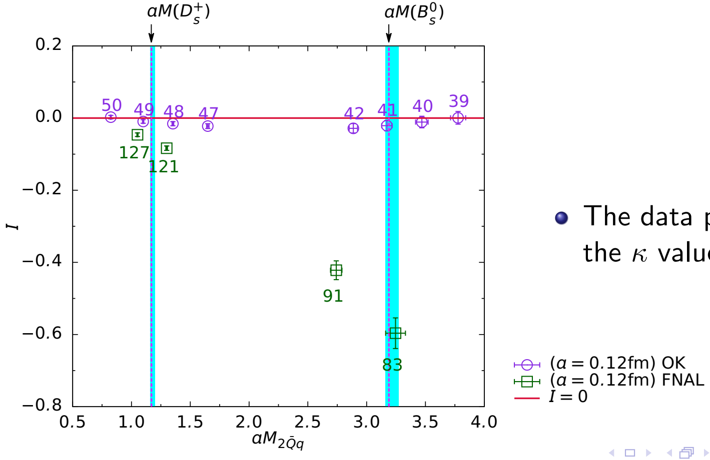
$$\begin{aligned} \delta B_{\overline{Q}q} \quad &= \frac{5}{3}\frac{\langle \boldsymbol{p}^2 \rangle}{2\mu_2}\left[\mu_2\left(\frac{m_{2\overline{Q}}^2}{m_{4\overline{Q}}^3} + \frac{m_{2q}^2}{m_{4q}^3}\right) - 1\right] \qquad (m_4 : c_1, c_3) \\ &+ \frac{4}{3}a^3\frac{\langle \boldsymbol{p}^2 \rangle}{2\mu_2}\mu_2(w_{4\overline{Q}}m_{2\overline{Q}}^2 + w_{4q}m_{2q}^2) \quad (w_4 : c_2, c_4) \\ &+ \mathcal{O}(p^4) \end{aligned}$$

[A. S. Kronfeld, NPB **53**, 401 (1997) , C. Bernard *et al.*, PRD **83**, 034503 (2011)]

- Leading contribution of $\mathcal{O}(\boldsymbol{p}^2)$ in δB vanishes when $w_4 = 0$, $m_2 = m_4$, not only for S-wave states but also for higher harmonics.

Improvement Test: Inconsistency Parameter

- The coarse ($a = 0.12$fm) ensemble data covers the B_s^0 mass and shows significant improvement compared to the Fermilab action.



- The data point labels denote the κ values.

Improvement Test: Hyperfine Splitting Δ

$$\Delta_1 = M_1^* - M_1\,,\ \Delta_2 = M_2^* - M_2$$

Recall,

$$\begin{aligned}
M^{(*)}_{1\overline{Q}q} &= m_{1\overline{Q}} + m_{1q} + B^{(*)}_{1\overline{Q}q} \\
M^{(*)}_{2\overline{Q}q} &= m_{2\overline{Q}} + m_{2q} + B^{(*)}_{2\overline{Q}q} \\
\delta B^{(*)} &= B_2^{(*)} - B_1^{(*)}
\end{aligned}$$

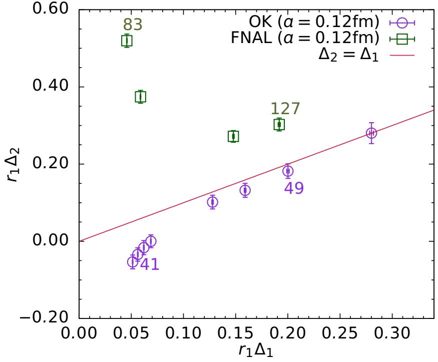
Then,

$$\Delta_2 = \Delta_1 + \delta B^* - \delta B$$

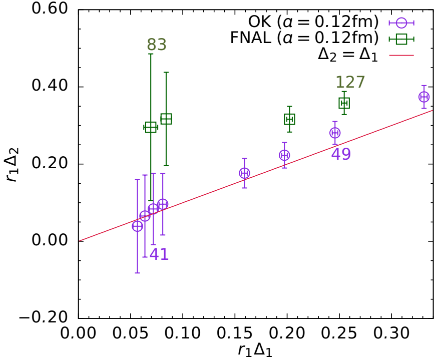
- The difference in hyperfine splittings $\Delta_2 - \Delta_1$ also can be used to examine the improvement from $\mathcal{O}(\boldsymbol{p}^4)$ terms in the action.

Improvement Test: Hyperfine Splitting Δ

$$\Delta_2 = \Delta_1 + \delta B^* - \delta B$$



Quarkonium



Heavy-light

Conclusion and Outlook

- Inconsistency parameter shows that the OK action clearly improves $\mathcal{O}(\boldsymbol{p}^4)$ terms.
- Hyperfine splitting shows that the OK action clearly improves the higher dimension magnetic effects for the quarkonium.
- We plan to calculate V_{cb} with the highest precision possible.
- Improved current relevant to the decay $\bar{B} \to D^* l\nu$ at zero recoil is needed. (Jon A. Bailey and J. Leem)
- We plan to calculate the 1-loop coefficients for c_B and c_E in the OK action. (Y.C. Jang)
- Highly optimized inverter using QUDA will be available soon. (Y.C. Jang)

BSM Corrections to ε_K

BSM Four Fermion Operators I

- New $\Delta S = 2$ four-fermion operators that contribute to Kaon Mixing

$$
\begin{aligned}
Q_1 &= [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] \quad \rightarrow \quad B_K \\
Q_2 &= [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1-\gamma_5)d^b] \\
Q_3 &= [\bar{s}^a\sigma_{\mu\nu}(1-\gamma_5)d^a][\bar{s}^b\sigma_{\mu\nu}(1-\gamma_5)d^b] \\
Q_4 &= [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1+\gamma_5)d^b] \\
Q_5 &= [\bar{s}^a\gamma_\mu(1-\gamma_5)d^a][\bar{s}^b\gamma_\mu(1+\gamma_5)d^b]
\end{aligned}
$$

BSM Four Fermion Operators II

- In general, there are additional operators that can be obtained from $Q_{1,2,3}$ by changing $L \to R$, but we do not consider. (Matrix elements for left- and right-handed operators are the same for Kaon mixing.)

- $$\mathcal{H}_{eff}^{\Delta S=2} = \sum_{i=1}^{5} C_i(\mu) Q_i(\mu)$$

- With the constraint from experiment, calculating corresponding hadronic matrix elements

$$\langle \bar{K}_0 | Q_i | K_0 \rangle$$

can impose strong constraints on BSM models.

Lattice Calculation : BSM B-parameters

- **B-parameters**

$$B_K = \frac{\langle \bar{K}_0 | Q_1 | K_0 \rangle}{8/3 \langle \bar{K}_0 | \bar{s}\gamma_0\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_0\gamma_5 d | K_0 \rangle} \qquad \text{SM, BSM}$$

$$B_i = \frac{\langle \bar{K}_0 | Q_i | K_0 \rangle}{N_i \langle \bar{K}_0 | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | K_0 \rangle} \qquad \text{BSM}$$

Where, $i = 2, 3, 4, 5$ and $(N_2, N_3, N_4, N_5) = (5/3, 4, -2, 4/3)$

- **Golden Combinations :** G_i

$$G_{23} \equiv \frac{B_2}{B_3} \qquad G_{45} \equiv \frac{B_4}{B_5}$$

$$G_{24} \equiv B_2 \cdot B_4 \qquad G_{21} \equiv \frac{B_2}{B_K}$$

1. Advantage: no SU(2) chiral logs at NLO order in G_i (Golden Combinations)

$N_f = 2+1$ QCD: MILC asqtad lattices

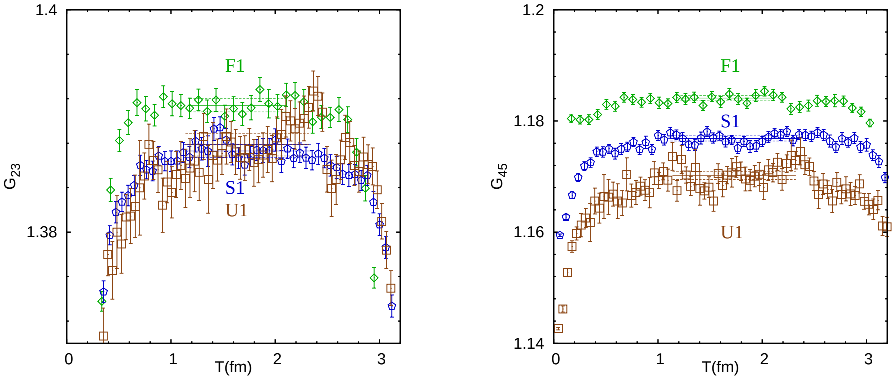
a (fm)	am_l/am_s	geometry	ens×meas	ID	Status
0.09	0.0062/0.0310	$28^3 \times 96$	995×9	F1	
0.09	0.0031/0.0310	$40^3 \times 96$	959×9	F2	
0.09	0.0093/0.0310	$28^3 \times 96$	949×9	F3	
0.09	0.0124/0.0310	$28^3 \times 96$	1995×9	F4	
0.09	0.00465/0.0310	$32^3 \times 96$	651×9	F5	
0.09	0.0062/0.0186	$28^3 \times 96$	950×9	F6	New
0.09	0.0031/0.0186	$40^3 \times 96$	701×9	F7	New
0.09	0.00155/0.0310	$64^3 \times 96$	790×9	F9	New
0.06	0.0036/0.018	$48^3 \times 144$	749×9	S1	
0.06	0.0025/0.018	$56^3 \times 144$	799×9	S2	
0.06	0.0072/0.018	$48^3 \times 144$	593×9	S3	
0.06	0.0054/0.018	$48^3 \times 144$	582×9	S4	
0.06	0.0018/0.018	$64^3 \times 144$	572×9	S5	New
0.045	0.0030/0.015	$64^3 \times 192$	747×1	U1	

Data Analysis

- **Calculate raw data**
  Calculate B_K and G_i for different valence quark mass combinations for each gauge ensemble. ($\overline{\text{MS}}$ scheme with NDR.)
- **Chiral fitting (valence quarks)**
  X-fit: Fix valence strange quark mass and extrapolate the light quark mass m_x to physical down quark mass.
  Y-fit: Extrapolate m_y to physical strange quark mass.
- **RG Evolution**
  Obtain results at μ_f =2GeV or 3GeV by running from $\mu_i = 1/a$.
- **Continuum-chiral extrapolation (sea quarks)**
  Perform [1–3] for different lattices and extrapolate to $a = 0$ and to physical sea quark masses.

Raw Data of G_{23} and G_{45}

- We compare three ensembles which have the same ratio of sea quark mass $m_\ell/m_s = 1/5$.



(a) (b)

SchPT X-fit and Y-fit of B_K

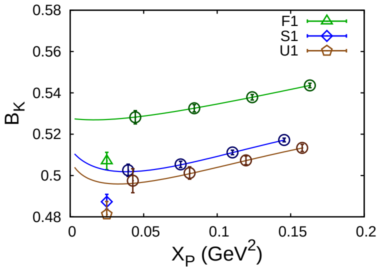
- NNNLO X-fit

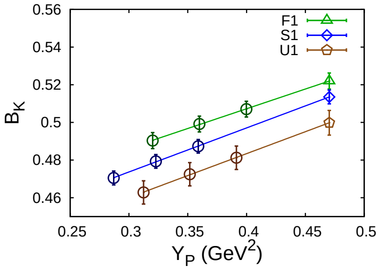
$$\begin{aligned} B_K & \quad \text{(NNNLO)} \\ = & \quad c_1 F_0 + c_2 X + c_3 X^2 \\ + & \quad c_4 X^2 (\ln(X))^2 \\ + & \quad c_5 X^2 \ln(X) + c_6 X^3 \end{aligned}$$

  Here, F_0 contains the chiral logs. Bayesian constraints on $c_{4-6} = 0 \pm 1$.

- Y-fit(U1 ensemble)

$$B_K(\text{Y-fit}) = b_1 + b_2 Y_P$$





SchPT X-fit and Y-fit of G_{23}

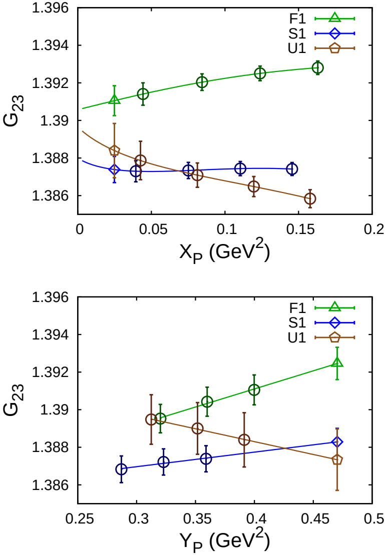
- NNNLO X-fit

$$\begin{aligned} G_{23} & \quad \text{(NNNLO)} \\ = & \quad c_1 + c_2 X + c_3 X^2 \\ + & \quad c_4 X^2 (\ln(X))^2 \\ + & \quad c_5 X^2 \ln(X) + c_6 X^3 \end{aligned}$$

Bayesian constraints on $c_{4-6} = 0 \pm 1$.

- Y-fit(U1 ensemble)

$$G_{23}(\text{Y-fit}) = b_1 + b_2 Y_P$$



SchPT X-fit and Y-fit of G_{45}

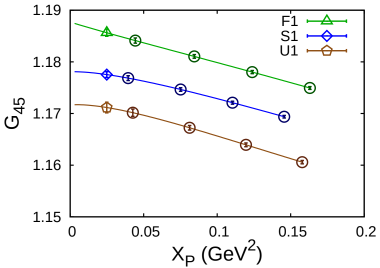
- NNNLO X-fit

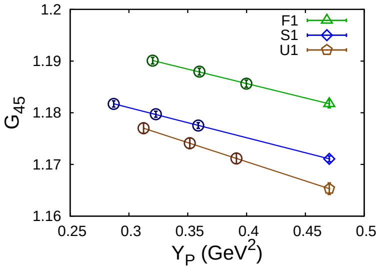
$$\begin{aligned} G_{45} & \quad \text{(NNNLO)} \\ = & \quad c_1 + c_2 X + c_3 X^2 \\ + & \quad c_4 X^2 (\ln(X))^2 \\ + & \quad c_5 X^2 \ln(X) + c_6 X^3 \end{aligned}$$

  Bayesian constraints on $c_{4-6} = 0 \pm 1$.

- Y-fit(U1 ensemble)

$$G_{45}(\text{Y-fit}) = b_1 + b_2 Y_P$$





Chiral-Continuum Fit

- We use 14 data points from 14 MILC ensembles in the fitting. We extrapolate the results to physical point $a = 0$, $L_P = m^2_{\pi_0}$, and $S_P = m^2_{s\bar{s},\text{phys}}$.
- Fitting functional forms come from the SU(2) SChPT theory.
- $\Delta S_P \equiv S_P - m^2_{s\bar{s},\text{phys}}$

fit type	fitting functional form	Bayesian Constraints
F_B^1	$d_1 + d_2 \frac{L_P}{\Lambda_\chi^2} + d_3 \frac{\Delta S_P}{\Lambda_\chi^2} + d_4 (a\Lambda_Q)^2$	$d_2 \cdots d_4 = 0 \pm 2$
F_B^2	$F_B^1 + d_5 (a\Lambda_Q)^2 \frac{L_P}{\Lambda_\chi^2} + d_6 (a\Lambda_Q)^2 \frac{\Delta S_P}{\Lambda_\chi^2}$	$d_2 \cdots d_6 = 0 \pm 2$
F_B^4	$F_B^2 + d_7 (a\Lambda_Q)^2 \alpha_s + d_8 \alpha_s^2 + d_9 (a\Lambda_Q)^4$	$d_2 \cdots d_9 = 0 \pm 2$

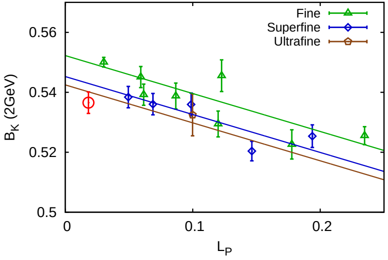
Chiral-Continuum Fit : Fitting quality

- We can see that the χ^2 values for fitting functional forms get saturated as we add higher order terms in the fitting functional forms. We choose F_B^1-fit results as central values for B_K, G_{24}, and G_{21}. For G_{23} and G_{45}, we choose those of F_B^4 as the central values.

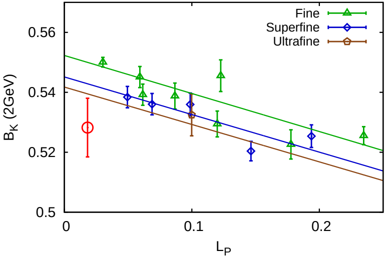
fit type	B_K	G_{23}	G_{45}	G_{24}	G_{21}
F_B^1	1.49	2.01	4.06	1.08	1.25
F_B^2	1.49	1.86	3.75	1.02	1.22
F_B^3	1.48	1.42	1.53	0.93	1.19
F_B^4	1.48	1.32	1.38	0.91	1.18
F_B^5	1.48	1.30	1.33	0.90	1.17
F_B^6	1.48	1.22	1.15	0.88	1.13

Chiral-Continuum Fit of B_K

- The result of Chiral-Continuum fit. The straight line in the plots represents the value of fitting function at fixed S_P and a^2 for fine ($a \approx 0.09$fm), superfine ($a \approx 0.06$fm), and ultrafine ($a \approx 0.045$fm) gauge ensembles.

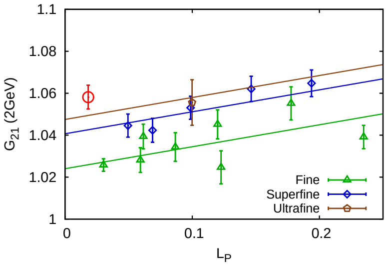


(c) F_B^1, $\chi^2/\text{dof} = 1.49$

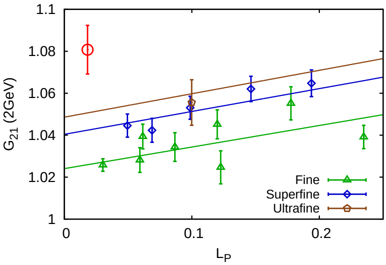


(d) F_B^4, $\chi^2/\text{dof} = 1.48$

Chiral-Continuum Fit of $G_{21} = B_2/B_K$

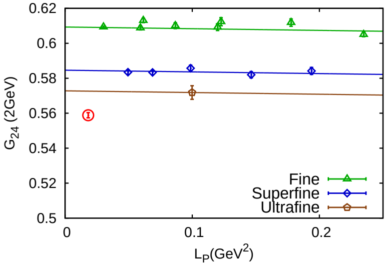


(e) F_B^1 , χ^2/dof $= 1.25$

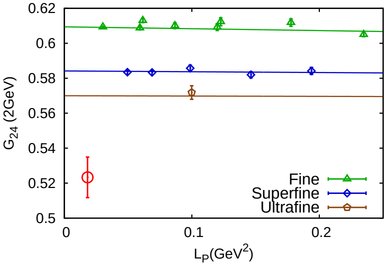


(f) F_B^4 , χ^2/dof $= 1.18$

Chiral-Continuum Fit of $G_{24} = B_2 \cdot B_4$



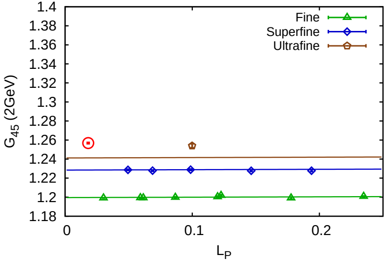
(g) F_B^1 , $\chi^2/\text{dof} = 1.08$



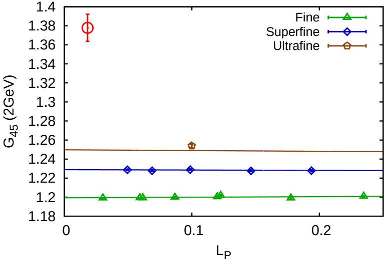
(h) F_B^4 , $\chi^2/\text{dof} = 0.91$

Chiral-Continuum Fit of $G_{45} = B_4/B_5$

- The fitting quality for F_B^1 fit is terribly poor.
- Hence, we have to choose the F_B^4 fit as our central value.



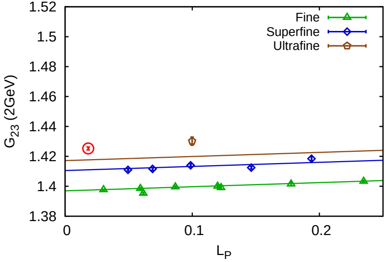
(i) F_B^1 , $\chi^2/\text{dof} = 4.07$



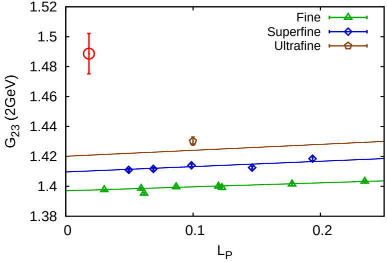
(j) F_B^4 , $\chi^2/\text{dof} = 1.39$

Chiral-Continuum Fit of $G_{23} = B_2/B_3$

- The fitting quality for F_B^1 fit is poor.
- Hence, we have to choose the F_B^4 fit as our central value.



(k) F_B^1 , χ^2/dof $= 2.01$



(l) F_B^4 , χ^2/dof $= 1.33$

Historical Progress

- 2013(PRD)[1] $\rightarrow$ 2014(ens) : Add more gauge ensembles.
- 2014(ens) $\rightarrow$ 2014(A.D.) : Correct two-loop contribution to pseudoscalar anomalous dimension.
- 2014(A.D.) $\rightarrow$ 2014(final) : Change fit type from F_B^1 to F_B^4 for G_{23} and G_{45}.

$\mu = 3$GeV	2013(PRD)[1]	2014(ens)	2014(A.D.)	2014(final)
B_K	0.519(7)(23)	0.518(3)	0.518(4)	0.518(4)(24)
B_2	0.549(3)(28)	0.547(1)	0.525(1)	0.525(1)(25)
B_3^{Buras}	0.390(2)(17)	0.390(1)	0.375(1)	0.358(4)(23)
B_3^{SUSY}	0.790(30)	0.783(2)	0.750(2)	0.774(6)(34)
B_4	1.033(6)(46)	1.024(1)	0.981(3)	0.981(3)(71)
B_5	0.855(6)(43)	0.853(3)	0.817(2)	0.748(9)(76)

[1] SWME Collaboration, Phys.ReV. **D88**,071503(2013)

Comparison in 2014

- We obtain B_i from results of G_i and B_K.

$$\text{Dominant error} \begin{cases} \text{Perturbative matching : } 4.4\% \\ \text{Chiral-continuum extrap : } 1.3 \sim 10.1\% \end{cases}$$

- RBC-UKQCD and ETM use RI-MOM for matching.

	SWME14	RBC-UK12	ETM12
B_K	0.518(4)(24)	0.53(2)	0.51(2)
B_2	0.525(1)(25)	0.43(5)	0.47(2)
B_3^{Buras}	0.358(4)(23)	N.A.	N.A.
B_3^{SUSY}	0.774(6)(34)	0.75(9)	0.78(4)
B_4	0.981(3)(71)	0.69(7)	0.75(3)
B_5	0.748(9)(76)	0.47(6)	0.60(3)

Comparison in 2015

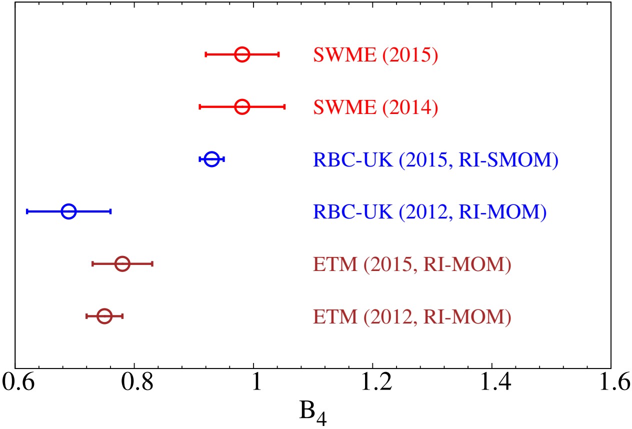
- We obtain B_i from G_i and B_K using the perturbative matching.
- RBC-UKQCD: RI-MOM (2012) $\rightarrow$ RI-SMOM (2015)
- ETM still uses RI-MOM for matching.

	SWME14	RBC-UK12	RBC-UK15	ETM12	ETM15
B_K	0.518(4)(24)	0.53(2)	0.53(2)	0.51(2)	0.51(2)
B_2	0.525(1)(25)	0.43(5)	0.54(3)	0.47(2)	0.46(3)
B_3^{Buras}	0.358(4)(23)	N.A.	N.A.	N.A.	N.A.
B_3^{SUSY}	0.774(6)(34)	0.75(9)	0.79(7)	0.78(4)	0.79(5)
B_4	0.981(3)(71)	0.69(7)	0.93(2)	0.75(3)	0.78(5)
B_5	0.748(9)(76)	0.47(6)	0.68(5)	0.60(3)	0.49(4)

※ RBC-UK15: the errors are preliminary (the error budget is incomplete).

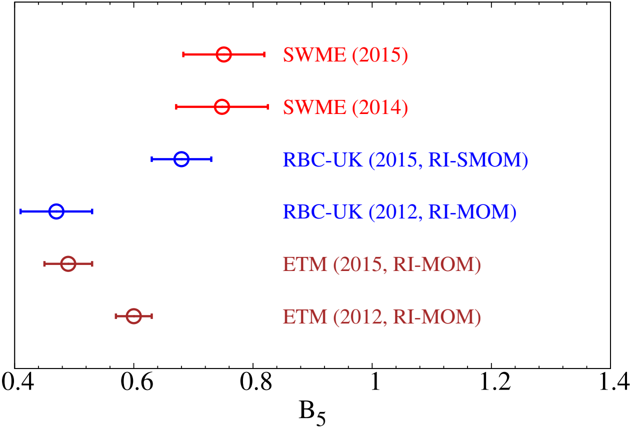
Comparison of B_4

- SWME agree with RBC-UKQCD (2015, RI-SMOM)
- SWME do NOT agree with RBC-UKQCD (2012, RI-MOM) and ETM.



Comparison of B_5

- SWME agree with RBC-UKQCD (2015, RI-SMOM)
- SWME do NOT agree with RBC-UKQCD (2012, RI-MOM) and ETM.



Conclusion

- Our results (SWME) agrees with those of RBC-UKQCD (2015, RI-SMOM).
- Our results of B_4 and B_5 do NOT agree with the results of RBC-UKQCD (2012, RI-MOM) and ETM collaborations.
- The large difference between RBC-UKQCD (2015, RI-SMOM) and RBC-UKQCD (2012, RI-MOM) indicates that the systematic uncertainty in non-perturbative renormalization might be underestimated for BSM B-parameters.
- We plan to use RI-MOM and RI-SMOM to obtain the matching factors in near future.

Grand Challenges in the front

Tentative Goals (1)

1. We would like to determine B_K directly from the standard model with its systematic and statistical error $\leq 2\%$.

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3. Basically, we need to accumulate at least 9 times more statistics using the SNU GPU cluster machine.
※ statistical error $<$ 0.5%

4. In addition, we need to obtain the matching factor using NPR (Jangho Kim) and using the two-loop perturbation theory ($\cdots$).
※ matching error $<$ 1.0%

Tentative Goals (2)

1. V_{cb}, we need to calculate the following semi-leptonic form factors:

$$\bar{B} \to D\ell\nu \tag{1}$$

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4. Our goal is to determine V_{cb} with its statistical and systematic error $\leq 1.1\%$. [1.9% (2016) $\to$ 1.1% (2018$\sim$9)]

Tentative Goals (3)

1. For ξ_0 and ξ_2, we need to calculate the following kaon decays:

$$K \to \pi\pi(I = 2) \tag{3}$$

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4. Our first goal is to obtain the $(I = 2)$ $\pi - \pi$ state in this way.

Tentative Goals (4)

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2. Here, the precision goal is only $10 \sim 15\%$.
3. We need $N_f = 2+1+1$ calculation on the lattice. MILC provides HISQ ensembles with $N_f = 2+1+1$.
4. As a by-product, a substantial gain is that the charm quark mass dependence might be under control in this way. (Brod and Gorbahn)

Ultimate Goals

1. For the time being, we will focus on exclusive V_{cb}, and ξ_2.
2. As a result, we hope to discover a breakdown of the standard model for the ε_K channel in the level of 5σ or higher precision.

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2. As a result, we hope to discover a breakdown of the standard model for the ε_K channel in the level of 5σ or higher precision.
3. As a result, we would like to provide a crucial clue to the physics beyond the standard model.
4. As a result, we would like to guide the whole particle physics community into a new world beyond the standard model.

Thank God for your help !!!

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