

Understanding Galactic Dark Matter:

Galactic Dynamics meets AI and Machine Learning



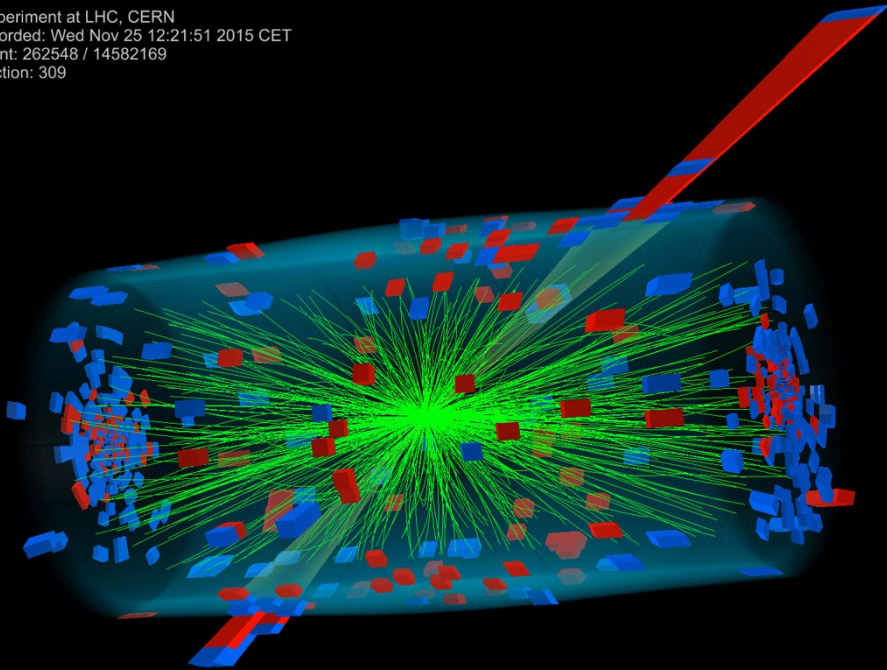
AI+HEP
in East Asia
Feb. 2025

Sung Hak Lim
CTPU-PTC, IBS

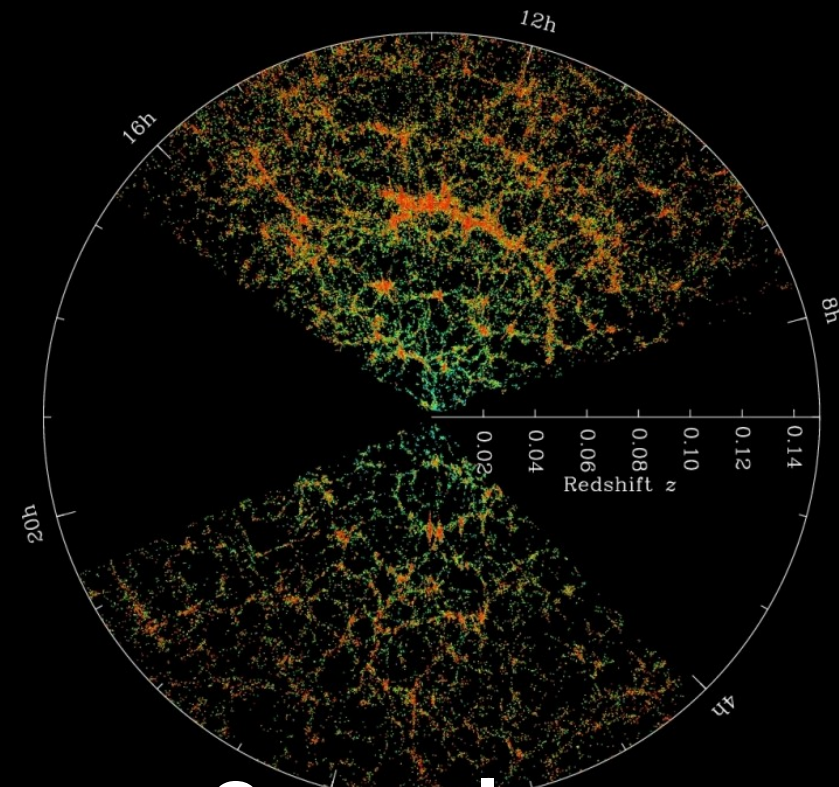
ibS Institute for Basic Science

So far we have been discussed applications of machine learning in...

Experiment at LHC, CERN
recorded: Wed Nov 25 12:21:51 2015 CET
Event: 262548 / 14582169
section: 309



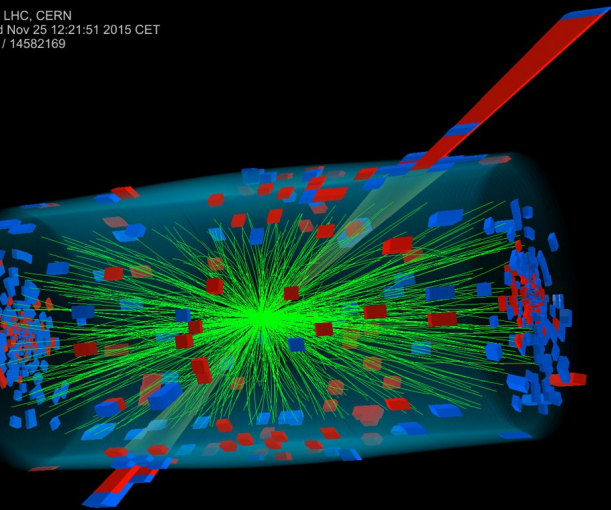
Particle Physics
(small length scale)



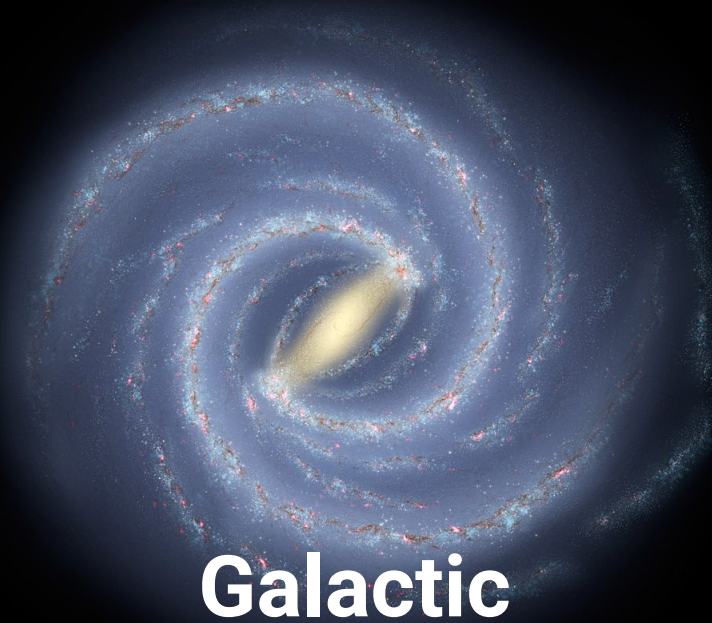
Cosmology
(large length scale)

Let's discuss something in between!

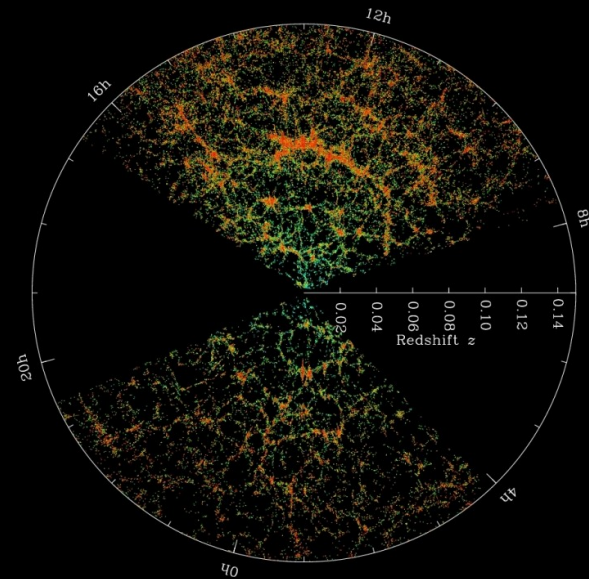
LHC, CERN
3 Nov 25 12:21:51 2015 CET
/ 14582169



Particle Physics
(small length scale)



**Galactic
Archaeology of
the Milky Way**
(medium length scale?)



Cosmology
(large length scale)

Galactic Archaeology?

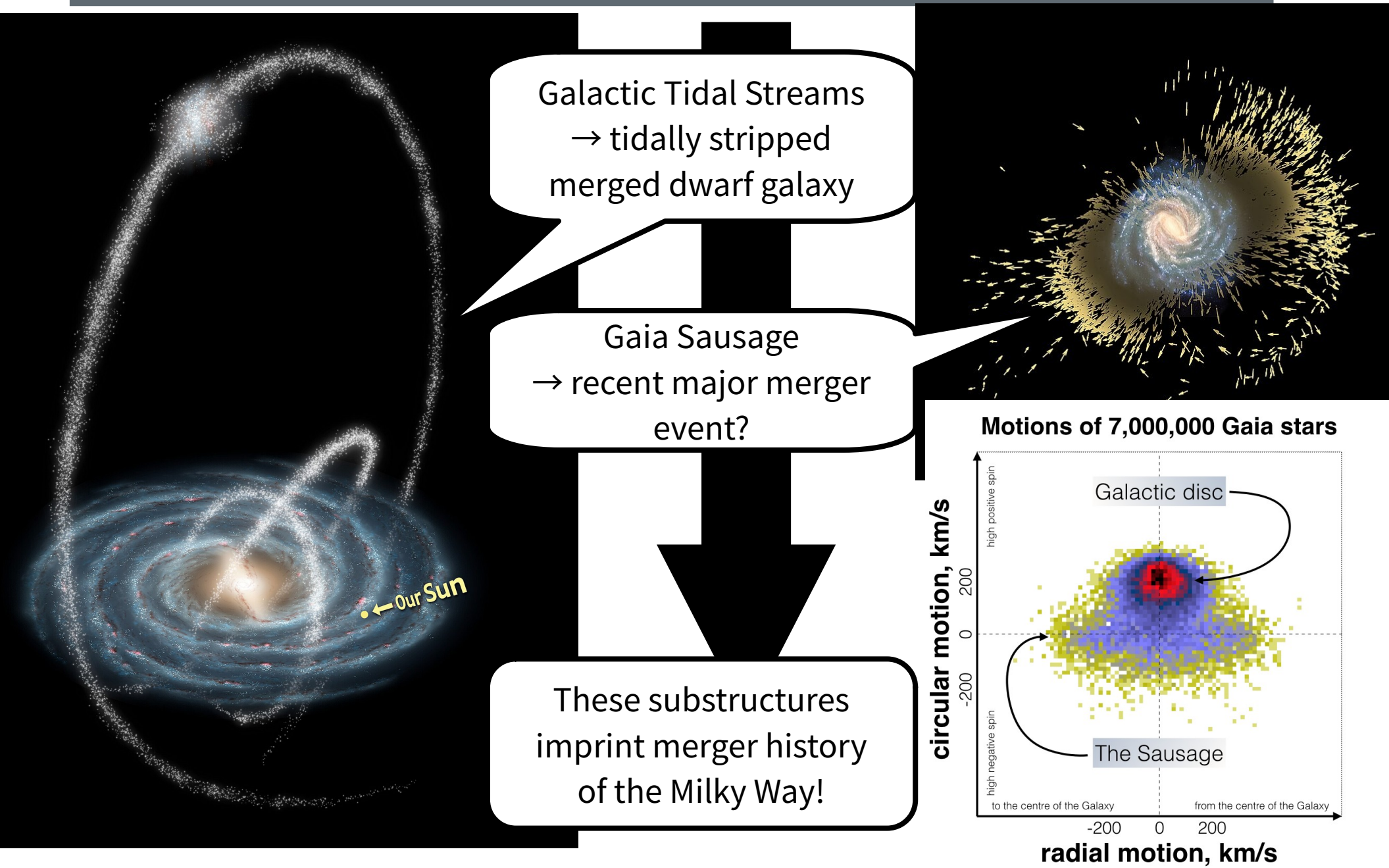


What is Galactic Archaeology?

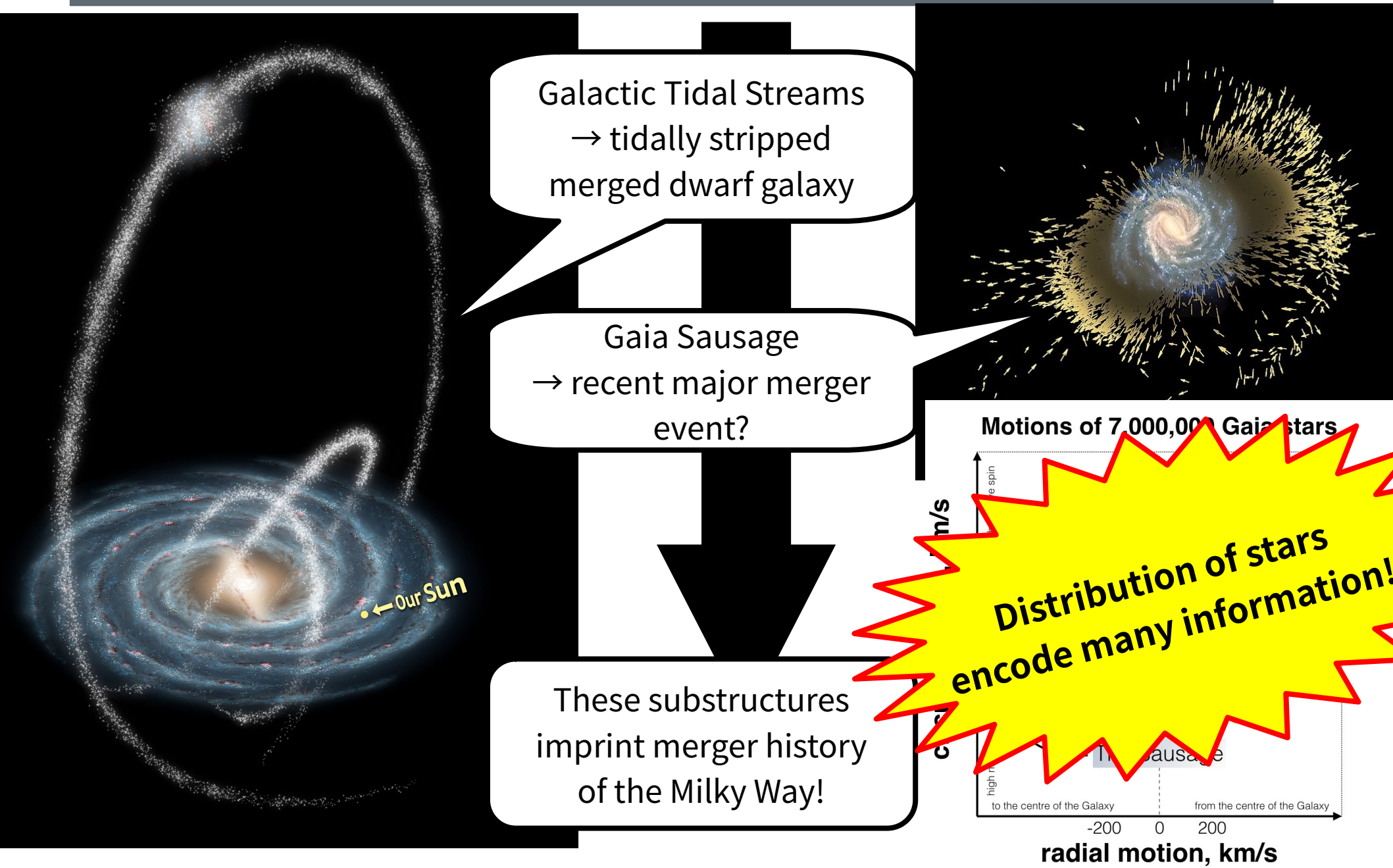
Galactic Archaeology is the study of the formation and evolutionary history of galaxies—especially the Milky Way—by analyzing the properties of their stars and stellar populations. The field operates under the assumption that galaxies preserve imprints of their formation processes in the chemical compositions, kinematics, and ages of their stars, much like how layers of rock on Earth preserve geological history.

- ChatGPT :)

Example: Stellar Distribution and Merger History of the Milky Way



Example: Stellar Distribution and Merger History of the Milky Way

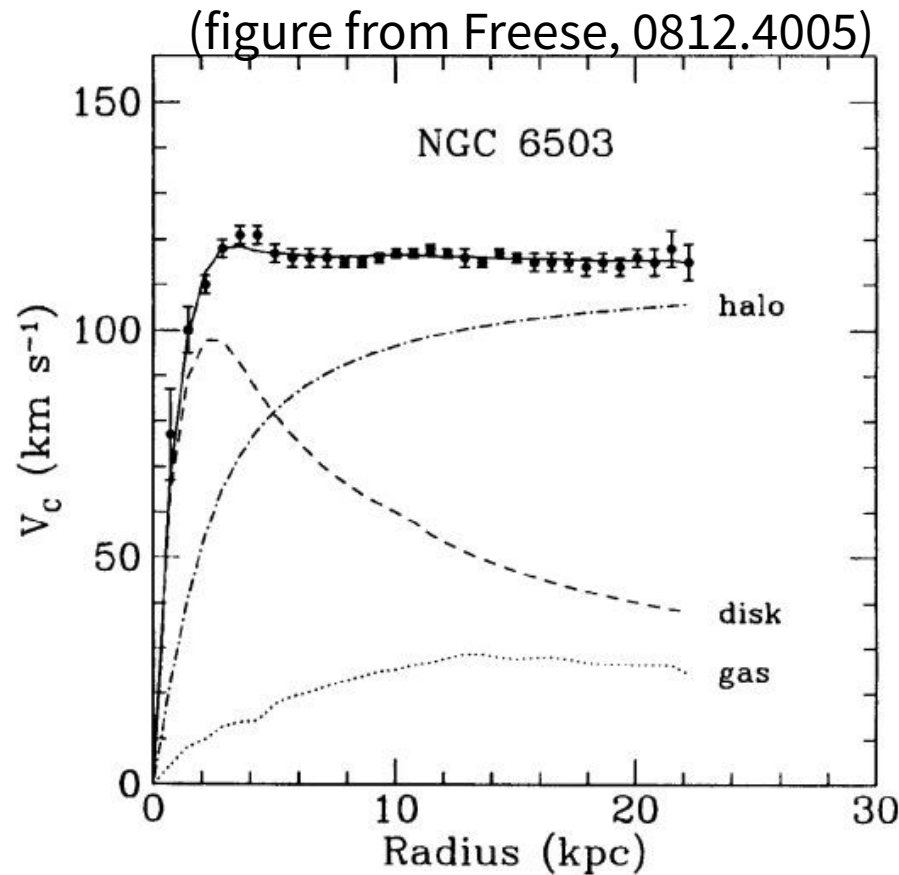


**There are many interesting things but...
perhaps, most interested topic in galactic archaeology
to high energy particle physicists is...**

**There are many interesting things but...
perhaps, most interested topic in galactic archaeology
to high energy particle physicists is...
about dark matter.**



Galaxy rotation curve and dark matter



NGC 6503 from NASA Hubble telescope

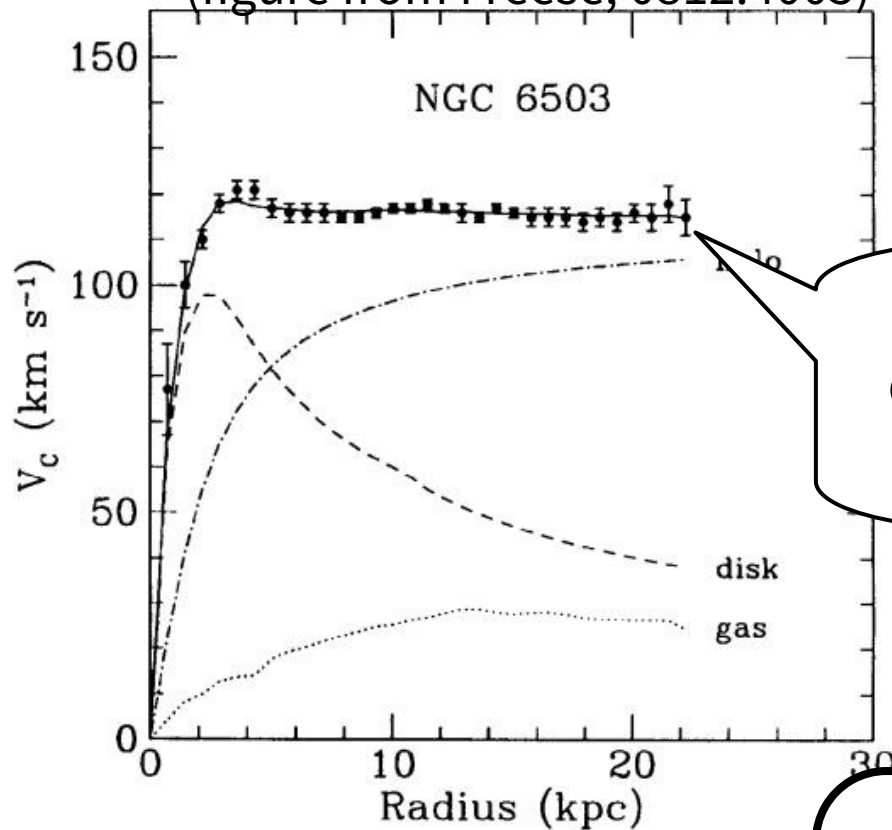


$$v_{\text{circ}}(R) = \sqrt{\frac{GM(R)}{R}}$$

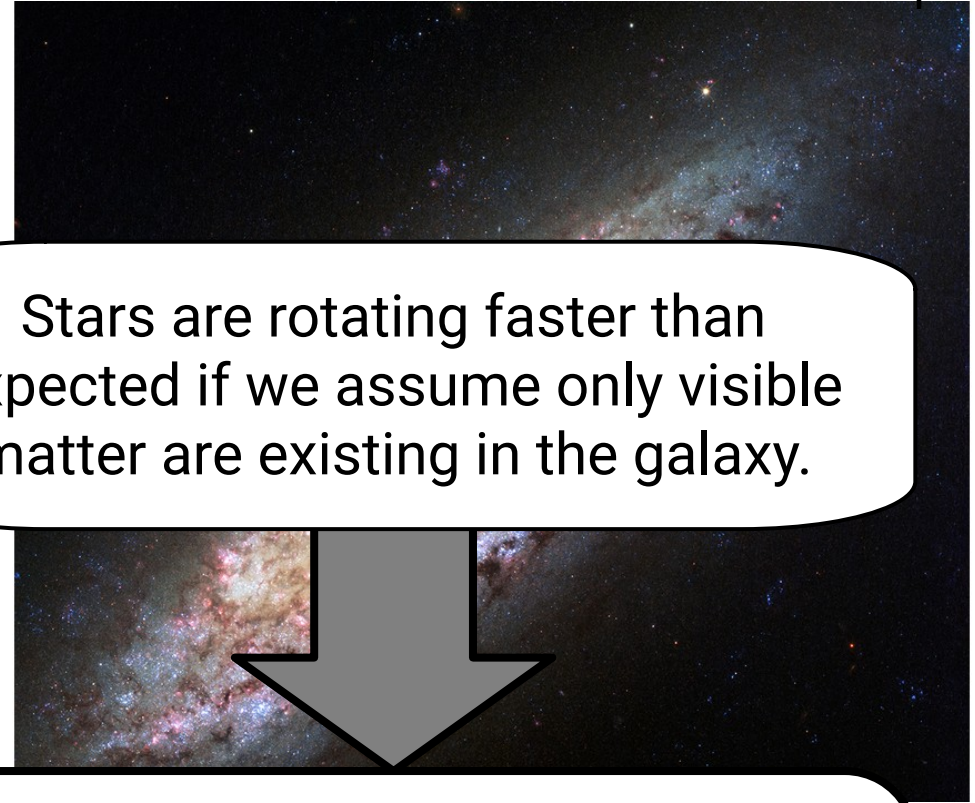
Obtain mass density
from enclosed mass
 $M(R)$

Galaxy rotation curve and dark matter

(figure from Freese, 0812.4005)



NGC 6503 from NASA Hubble telescope



Stars are rotating faster than expected if we assume only visible matter are existing in the galaxy.

$$v_{\text{circ}}(R) = \sqrt{\frac{GM(R)}{R}}$$

DARK MATTER!

Probably, you heard about the dark matter density on Earth...

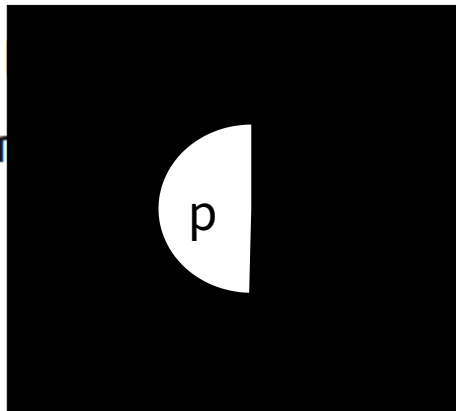
How much is the dark matter density at the Solar location?

The local dark matter density at the Solar location (i.e., in the vicinity of the Sun within the Milky Way) is typically estimated to be around:

$$\rho_{\text{DM},\odot} \approx 0.3 \text{ GeV}/\text{cm}^3$$

This value is based on dynamical studies of the Milky Way's rotation curve and stellar kinematics. However, there is some uncertainty, and estimates range from about **0.2 to 0.6 GeV/cm³** depending on the specific model and data used.

Would you
observation



About half-proton
in a cubic centimeter box!

Galaxy rotation curve of Milky Way

Ou, et. al., 2303.12838

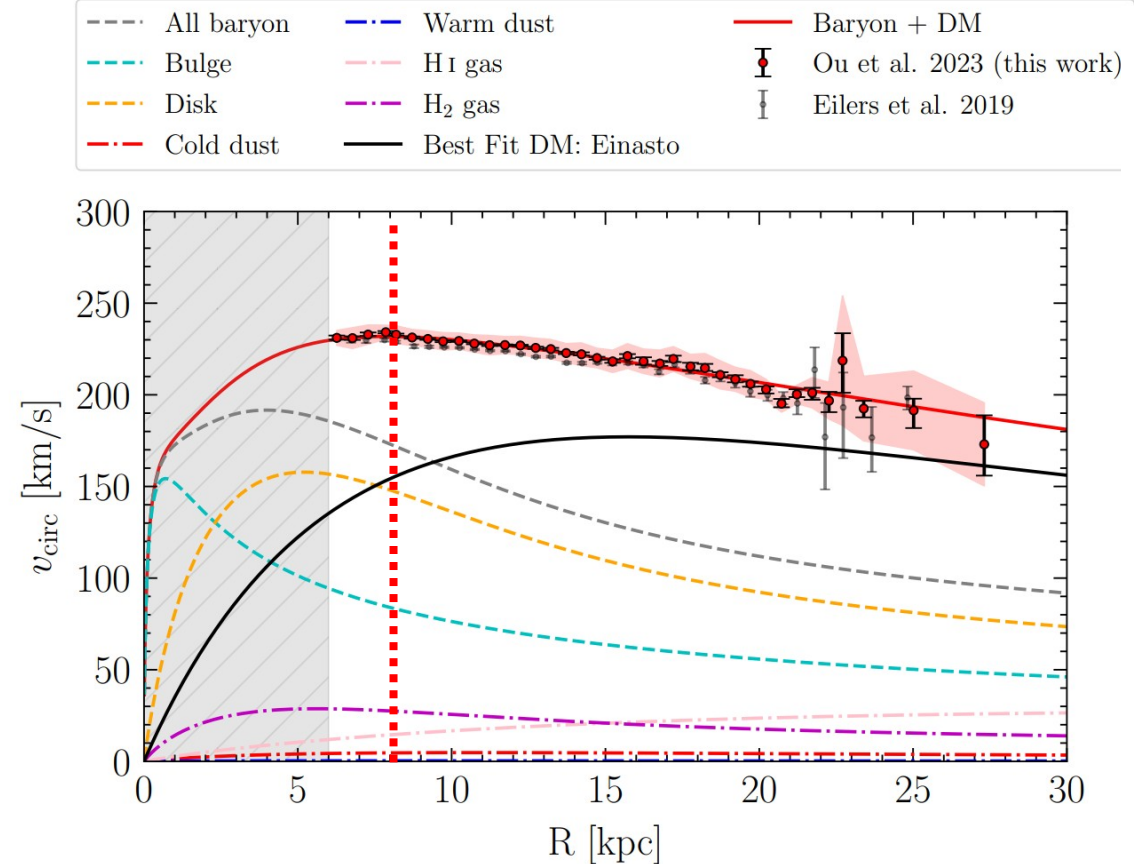


Figure 4. Comparison between the circular velocity curve measured from [Eilers et al. \(2019\)](#) (black) and this work (red). The best-fit Einasto DM profile, with the baryonic model from [de Salas et al. \(2019\)](#), is also shown here. The grey shaded region represents the bulge region, which we do not model due to the non-axisymmetric potential near the galactic bar. The red shaded region represents the total uncertainty estimate from the dominating systematic sources, as shown in [Figure 5](#).

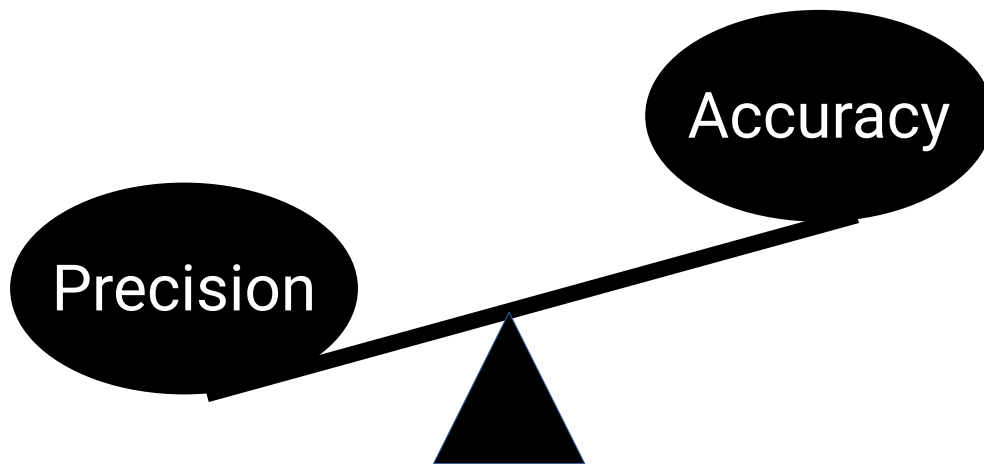
	Einasto
Normalization Mass (M_0)	$0.62^{+0.12}_{-0.11} \times 10^{11} M_\odot$
Scale Radius (r_s)	$3.86^{+0.35}_{-0.38} \text{ kpc}$
Slope Parameter (α, β)	$0.91^{+0.04}_{-0.05}$
Virial Mass (M_{200})	$1.81^{+0.06}_{-0.05} \times 10^{11} M_\odot$
Virial Radius (r_{200})	$119.35^{+1.37}_{-1.21} \text{ kpc}$
Concentration (c_{200})	$13.02^{+0.11}_{-0.10}$
Local Dark Matter Density ($\rho_{\text{DM}, \odot}$)	$0.447^{+0.004}_{-0.004} \text{ GeV cm}^{-3}$
J-factor ($J(\theta < 15^\circ)$)	$15.8^{+1.98}_{-0.93} \times 10^{22} \text{ GeV}^2 \text{ cm}^{-2}$
χ^2 per d.o.f. (χ^2_ν)	2.97

^a Fitted in logarithmic scale.

Very precise
DM density
measurement!

Measuring DM density in the Solar Neighborhood

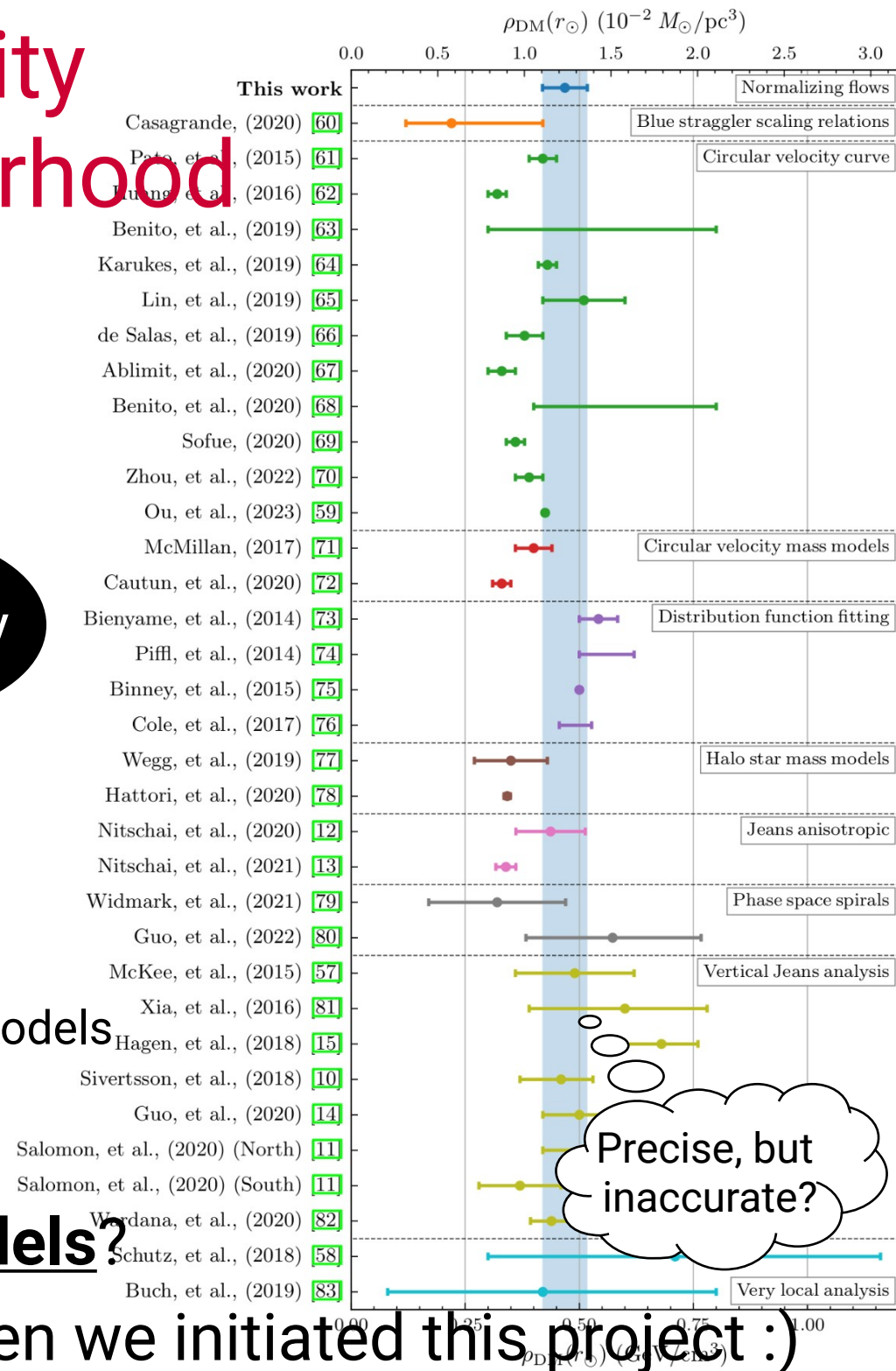
Thanks to recent progress in observing stars in the Milky Way, we can measure **the dark matter density in the Solar neighborhood** in very high precision using model-based analyses.



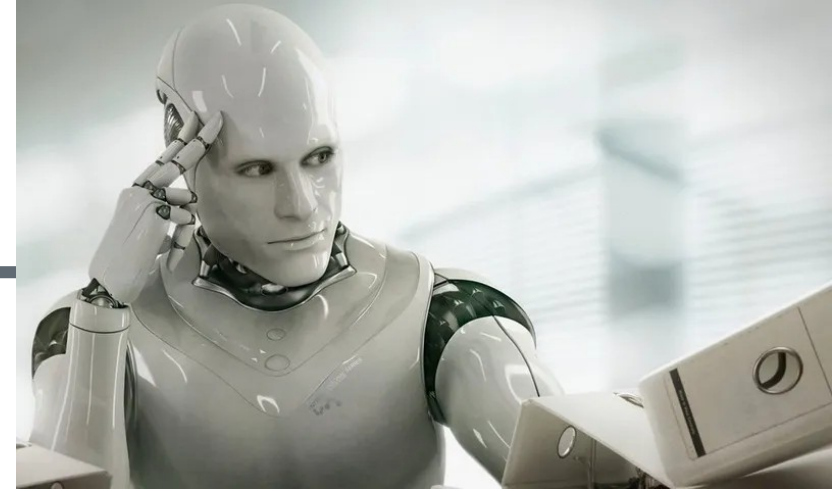
When **sufficient number of data are available**, using over-constrained models may result in inaccurate results.

Need of analysis without assumed **symmetries** and **models**?

So... I asked to ChatGPT when we initiated this project :)



ChatGPT says...



S

Can we measure the dark matter density of the Milky Way without symmetry assumption, in a model-independent way, and only assuming the gravitational interaction? Please answer concisely.



No, currently we cannot measure the dark matter density of the Milky Way without making some assumptions. A model-independent measurement of dark matter density solely based on gravitational interaction and without symmetry assumptions is not possible with our current knowledge and observational techniques.

There is no model-independent, unbinned measurement of DM density in 2021!

ChatGPT says...

S

Can we measure the dark matter density of the Milky Way without symmetry assumption, in a model-independent way, and only assuming the gravitational interaction? Please answer concisely. If the answer is no, actually I have the solution!



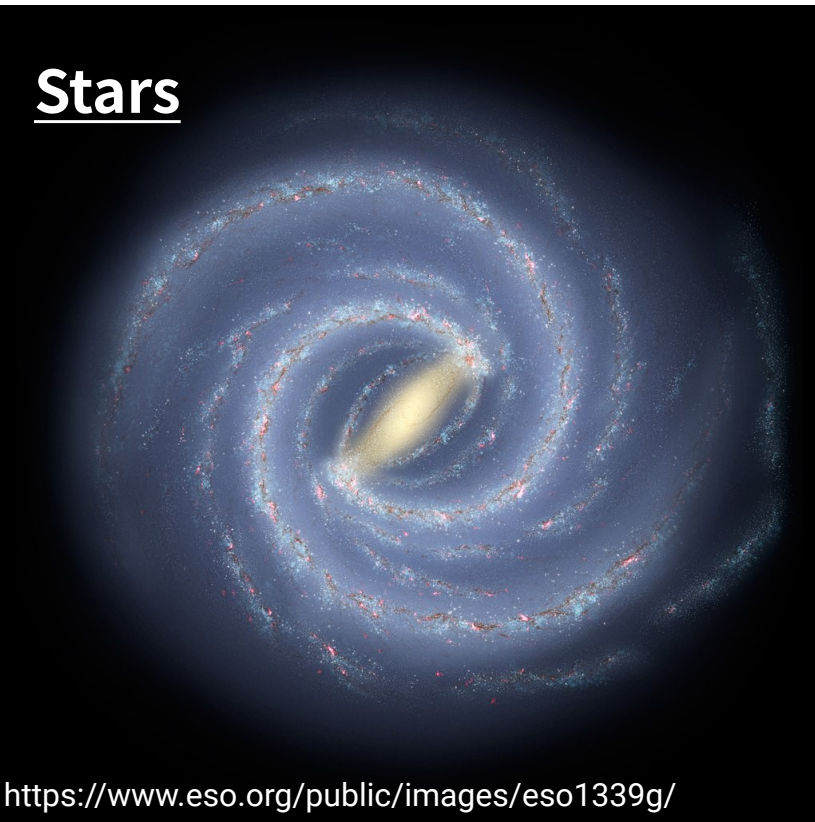
No, we cannot currently measure the dark matter density of the Milky Way without making some assumptions or models. Although the gravitational interaction is a key factor in studying dark matter, determining its density profile requires assumptions about the symmetry or shape of the dark matter distribution. Without such assumptions, it is challenging to obtain a model-independent measurement. However, if you have a solution that addresses this issue, I'd be interested to hear it!

ChatGPT is also interested in this research :)

In galactic dynamics for studying dark matter,
one important and interesting task is...

Q: How to use stellar distribution
of a galaxy to understand
its galactic dark matter density?

Stars

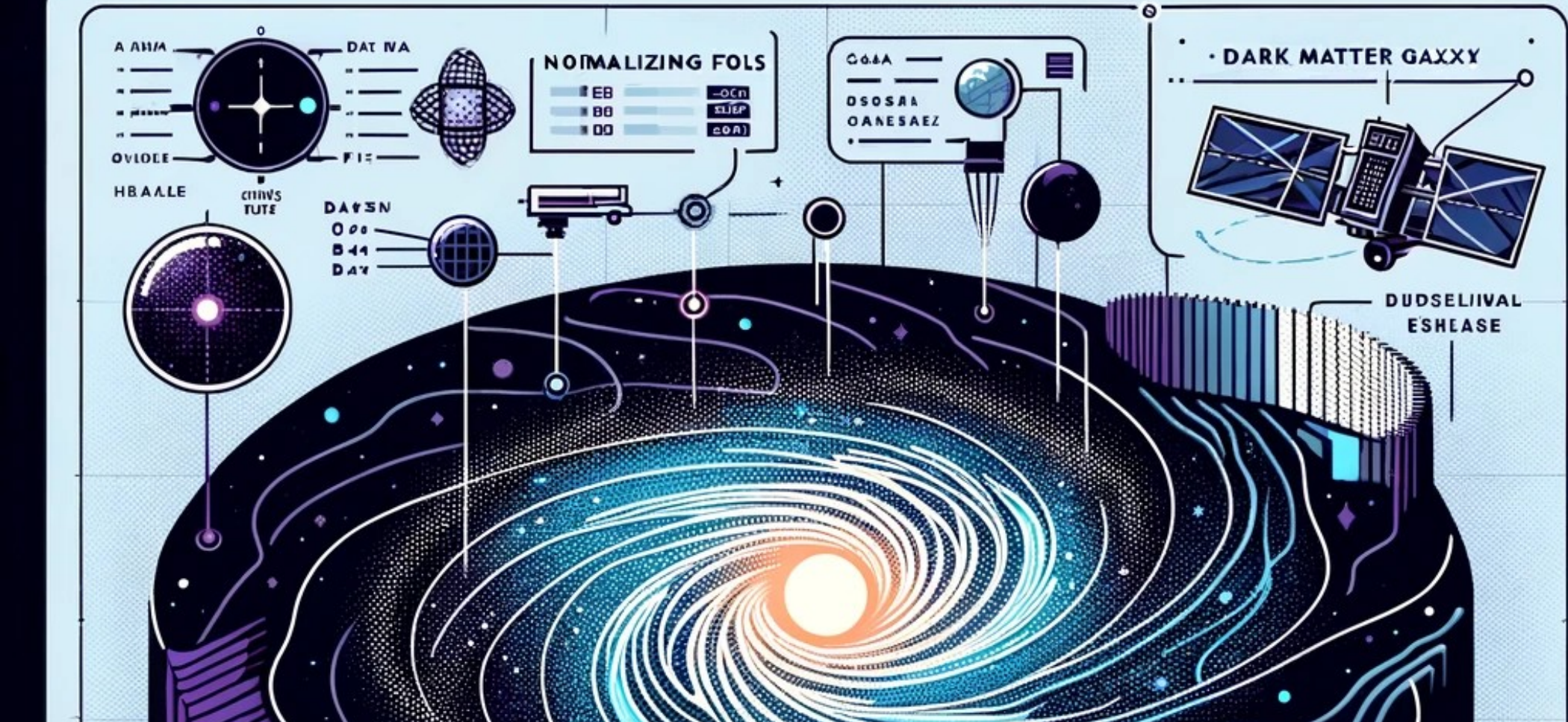


<https://www.eso.org/public/images/eso1339g/>

Dark Matter Halo

?

In the previous talk...



Mapping Dark Matter in the Milky Way using Normalizing Flows and Gaia DR3

M. R. Buckley, SHL, E. Putney, and D. Shih, arXiv:2205.01129, published in MNRAS

SHL, E. Putney, M. R. Buckley, and D. Shih, arXiv:2305.13358, published in JCAP

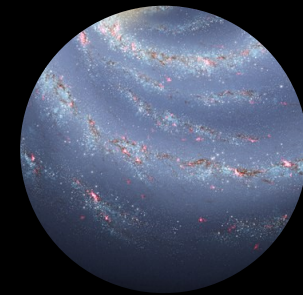
E. Putney, D. Shih, SHL, and M. R. Buckley, arXiv:2412.14236,

Project at Rutgers



In this project, we have developed an **unsupervised machine learning** technique for measuring DM density **without assuming symmetry and models.**

- Solution is **not provided** in training dataset
- Solve the physics problem (**equations of motion**) in a **classic way** but with help of machine learning techniques!



We use nearby old stars ($< \sim 4 \text{ kpc}$) in the Milky Way, due to completeness issue

Mapping Dark Matter in the Milky Way using Normalizing Flows and **Gaia DR3**

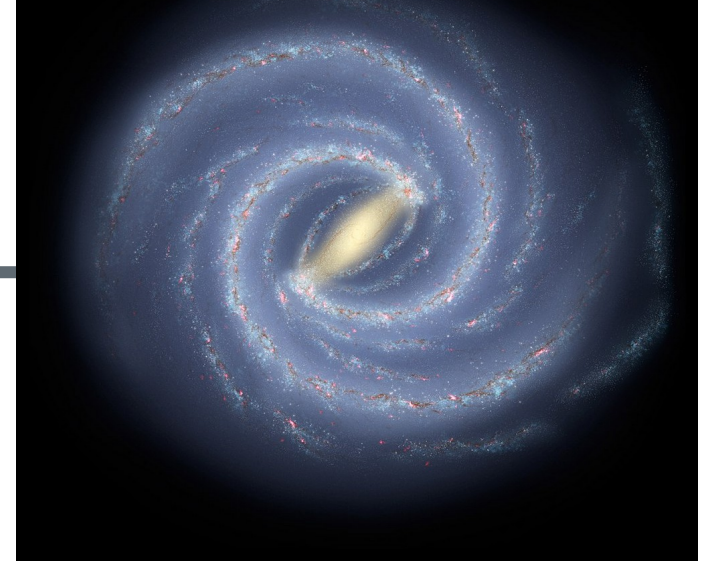
M. R. Buckley, **SHL**, E. Putney, and D. Shih, arXiv:2205.01129, published in MNRAS

SHL, E. Putney, M. R. Buckley, and D. Shih, arXiv:2305.13358, published in JCAP

E. Putney, D. Shih, **SHL**, and M. R. Buckley, arXiv:2412.14236,

Hydrodynamics and Galactic Dynamics

If we consider a galaxy as a hydrodynamic system $N \rightarrow \infty$ consisting of stars, phase-space density of a star (probability of finding a star with given position and velocity) describes the system.



$$f(\vec{x}, \vec{v})$$

Equation of motion: Boltzmann Equation

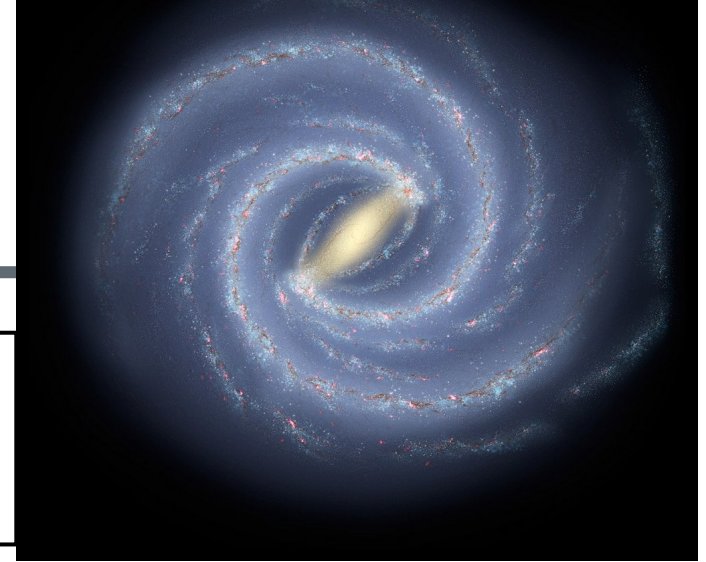
$$\left[\frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{x}} + \vec{a} \cdot \frac{\partial}{\partial \vec{v}} \right] f(\vec{x}, \vec{v}) = 0$$

If phase-space density is determined...

We can estimate the gravitational acceleration field!

+no unnecessary assumptions are involved

Outline of Strategy



Star catalog

$$\{(\vec{x}, \vec{v})\}$$

Galaxy:
hydrodynamic
system

Phase space density

$$f(\vec{x}, \vec{v})$$

Neural Networks for Density Estimation:
Normalizing Flows

$$\vec{u}_0 \rightarrow \vec{u}_1 \rightarrow \dots \rightarrow \vec{u}_n = (\vec{x}, \vec{v})$$

Gravitational accel.

$$\vec{a}(\vec{x})$$

Solving EOM (Boltzmann Equation)

$$\left[\frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{x}} + \vec{a} \cdot \frac{\partial}{\partial \vec{v}} \right] f(\vec{x}, \vec{v}) = 0$$

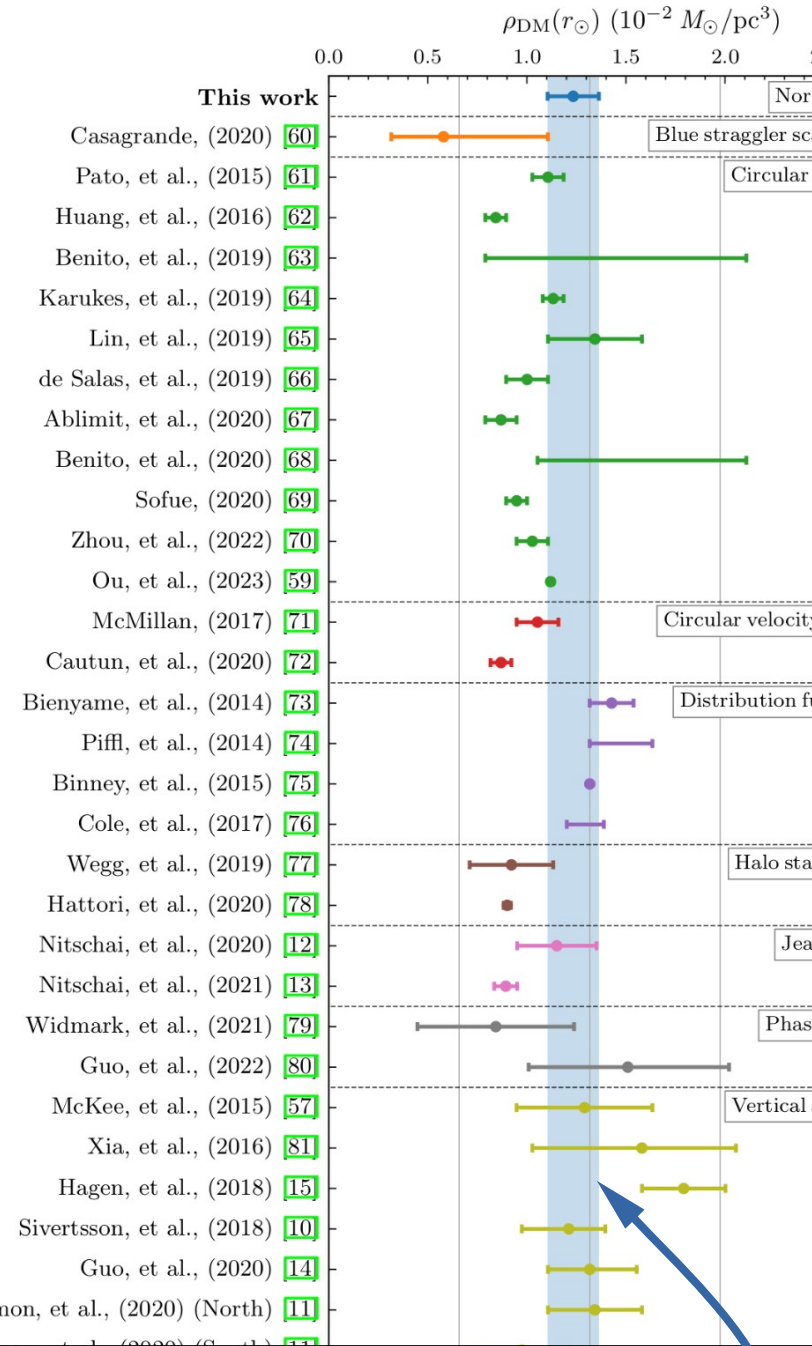
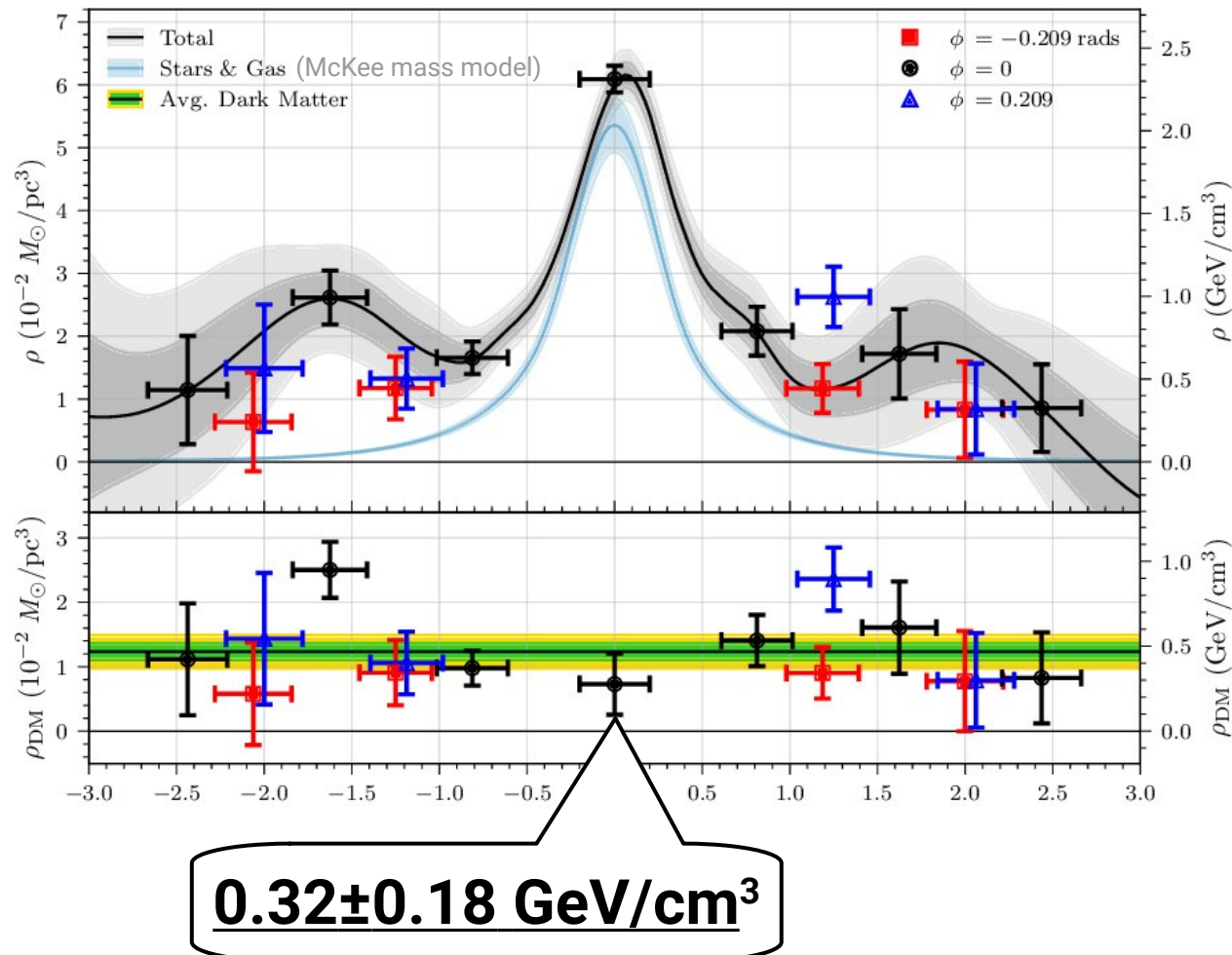
Mass density

$$\rho(\vec{x})$$

Solving Gauss's Equation

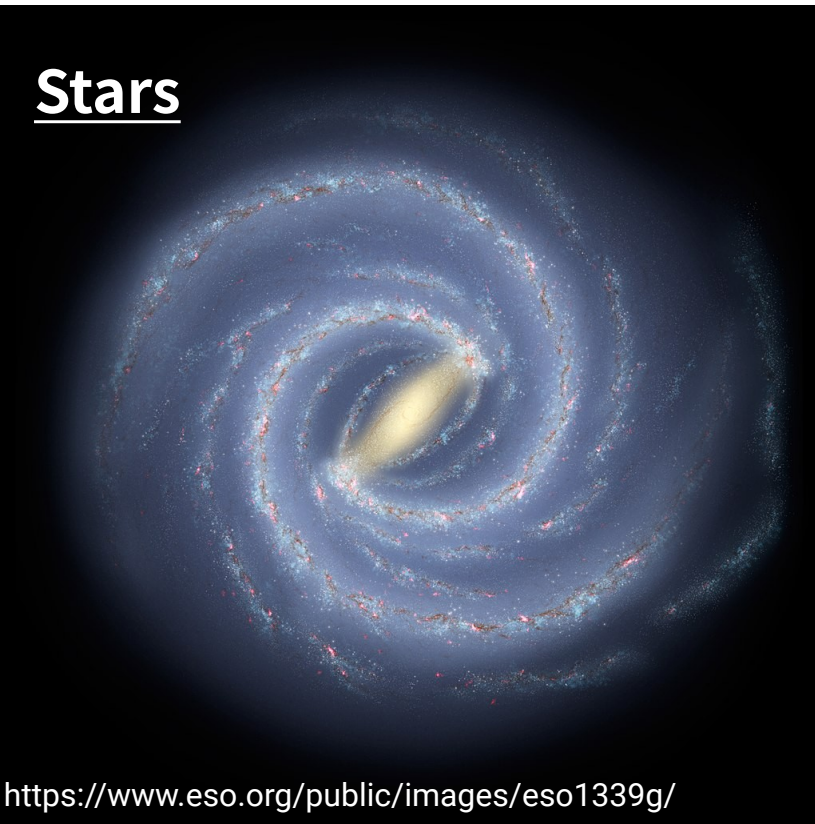
$$-4\pi G \rho = \nabla \cdot \vec{a}$$

Local DM Mass Density of the Milky Way

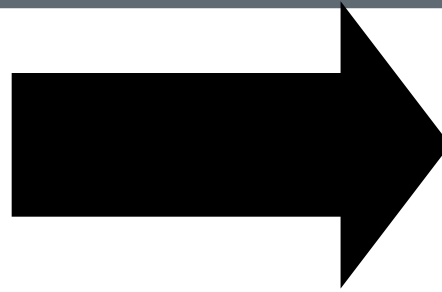


Taking the average of the DM mass density at the Solar radius, we find a local dark matter density: **$0.47 \pm 0.05 \text{ GeV/cm}^3$**

Stars



Dark Matter Halo

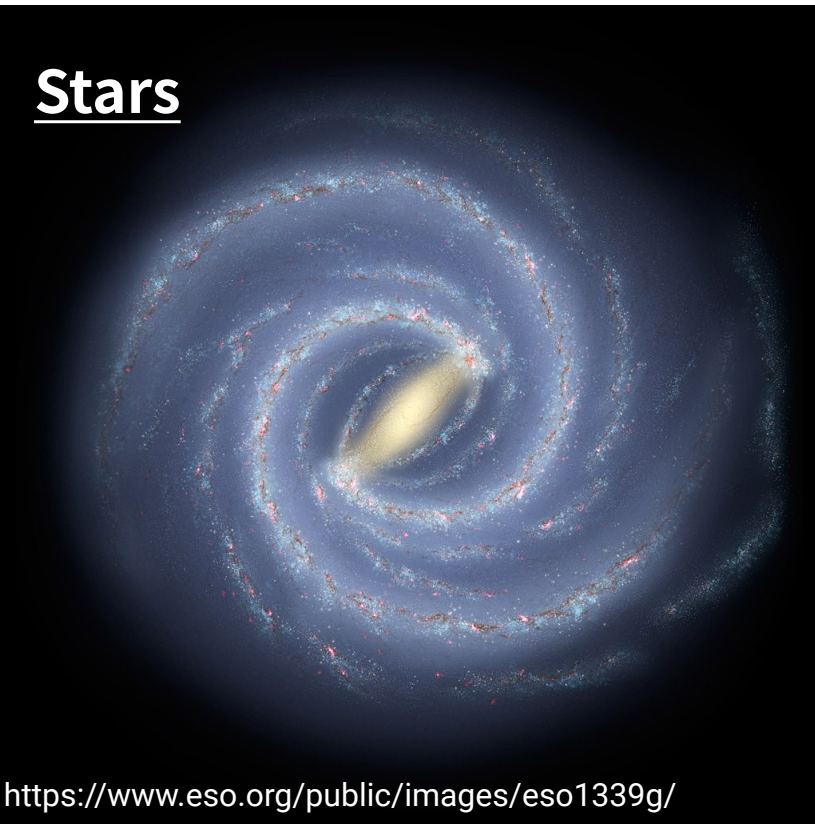


?

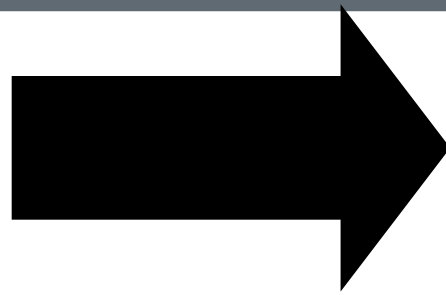
We have an unsupervised ML method to
estimate dark matter density
given stellar distribution of a galaxy.

END of story?

Stars



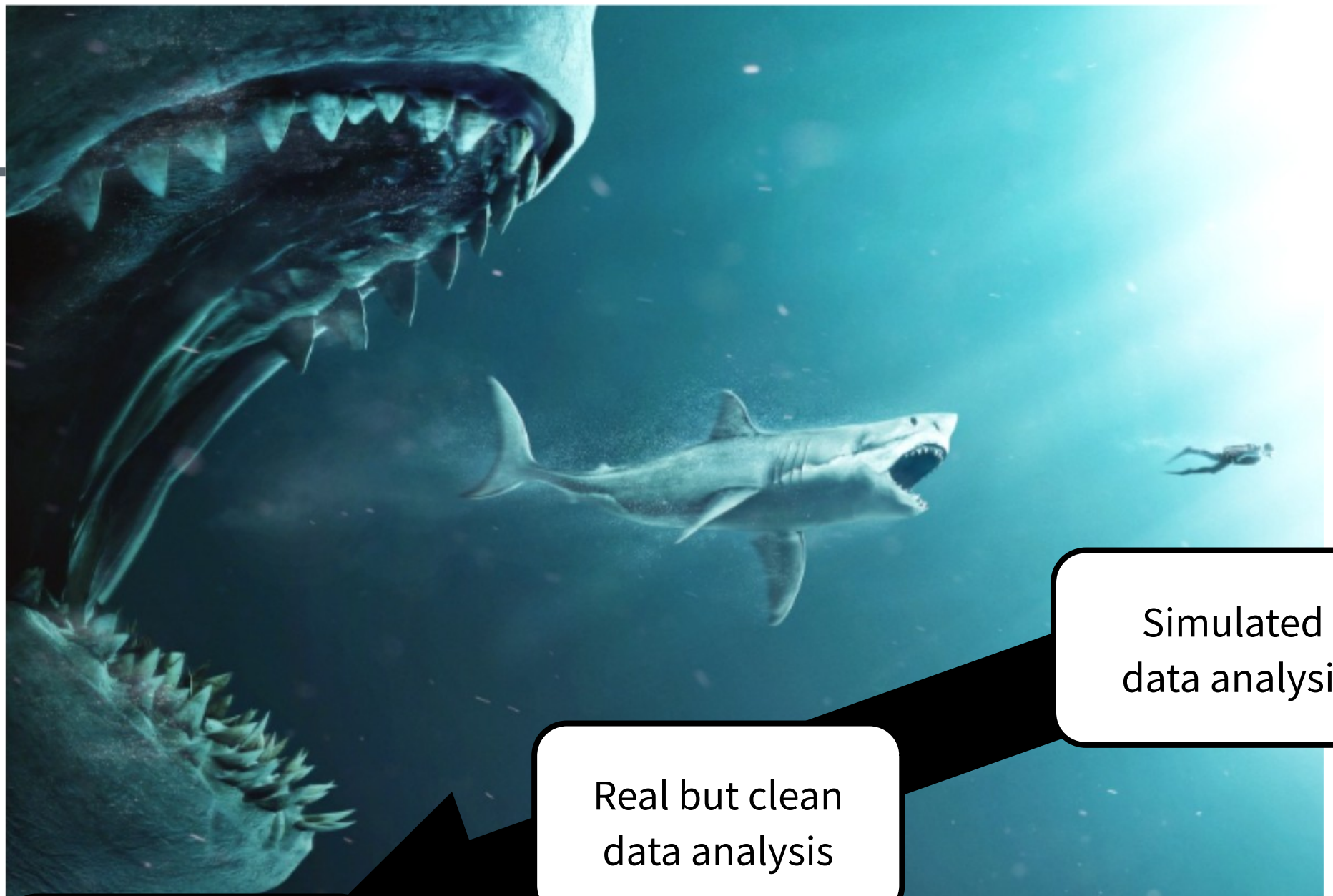
Dark Matter Halo



?

We have an ML method to
estimate dark matter density
given stellar distribution of a galaxy.

END? → Of course not!

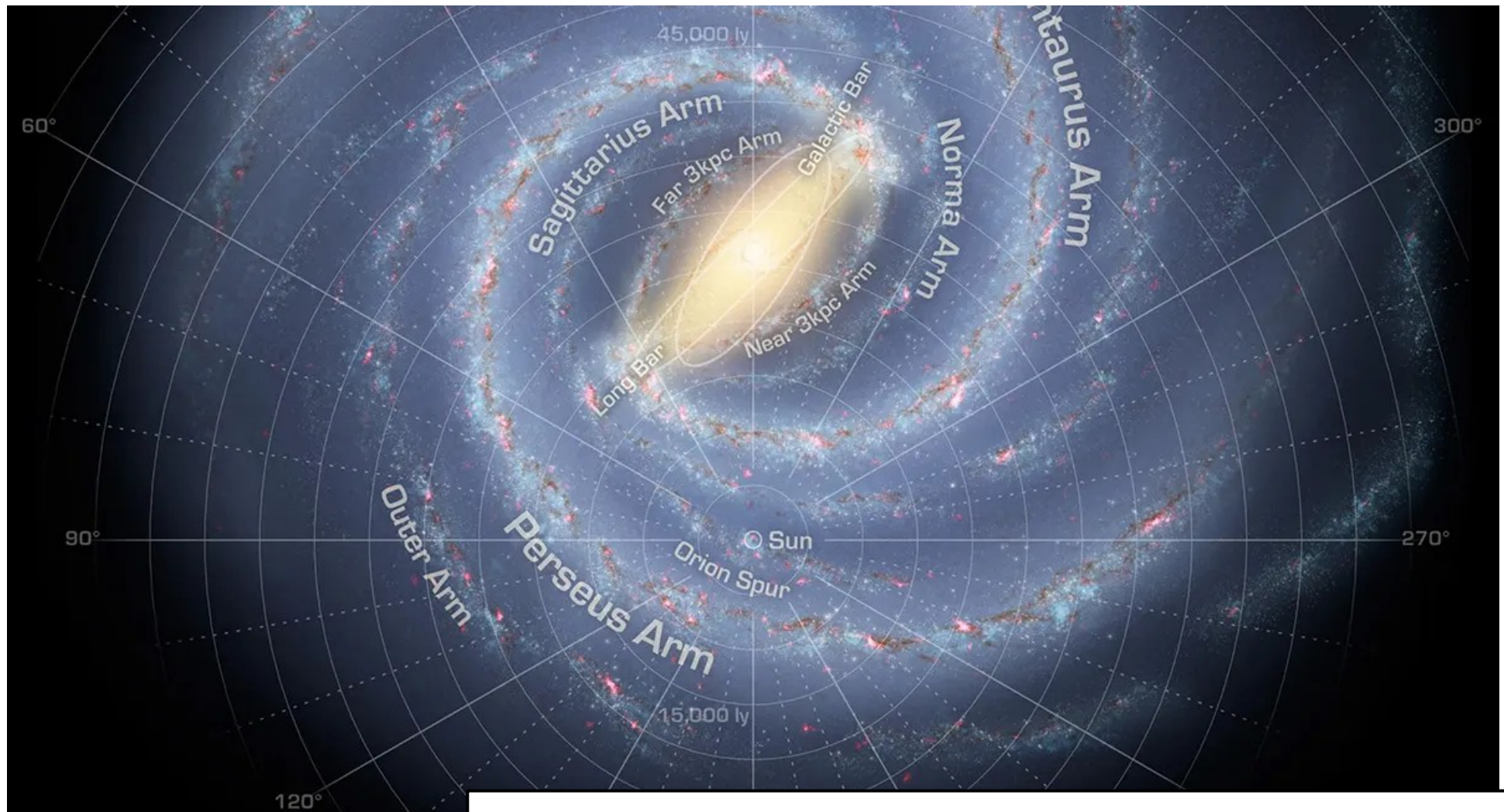


Simulated
data analysis

Real but clean
data analysis

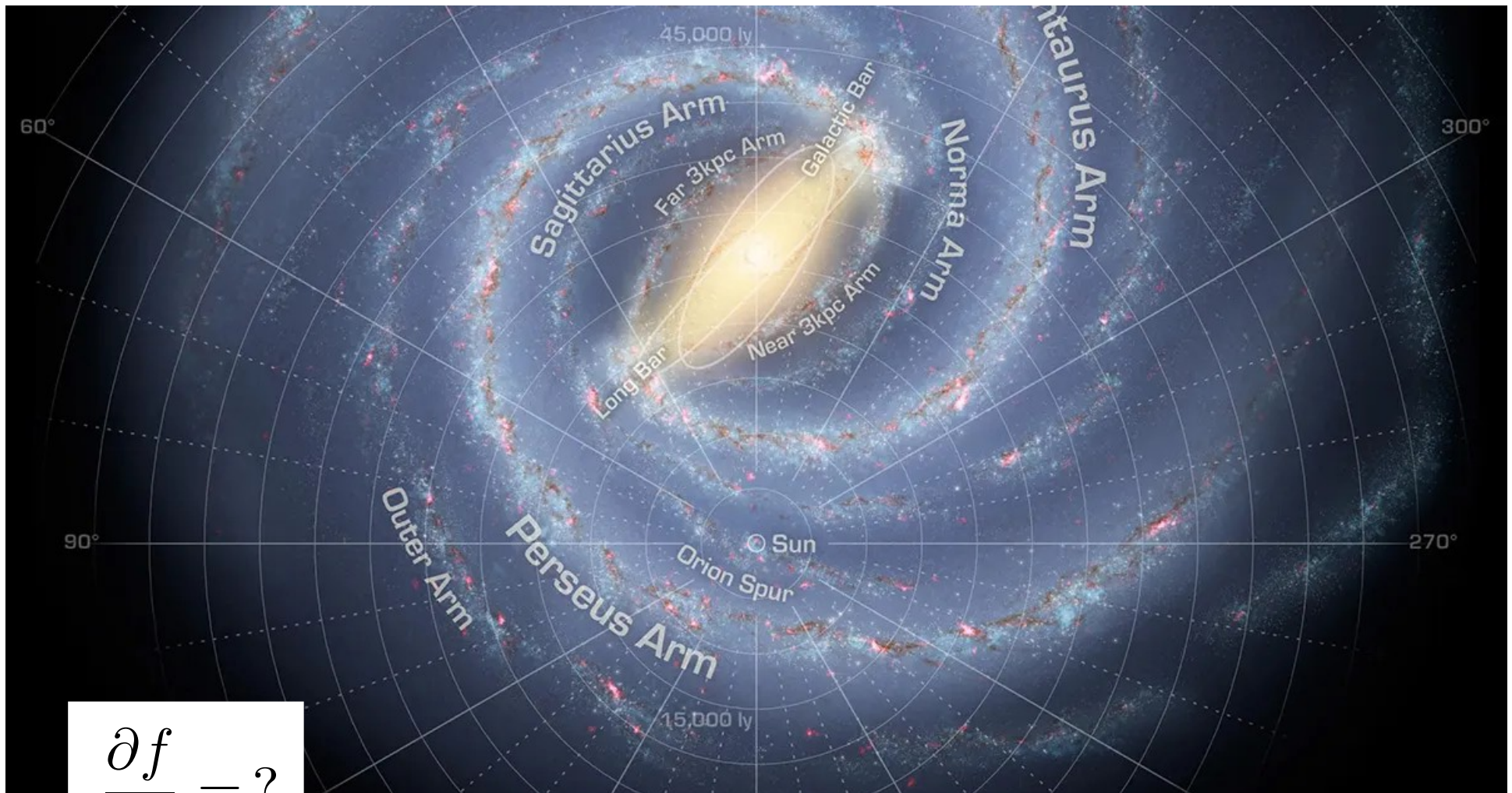
Real dirty
data analysis

Galactic Dynamics and Incomplete Datasets



One of main challenge of applying this technique
is that the dataset itself is
incomplete!

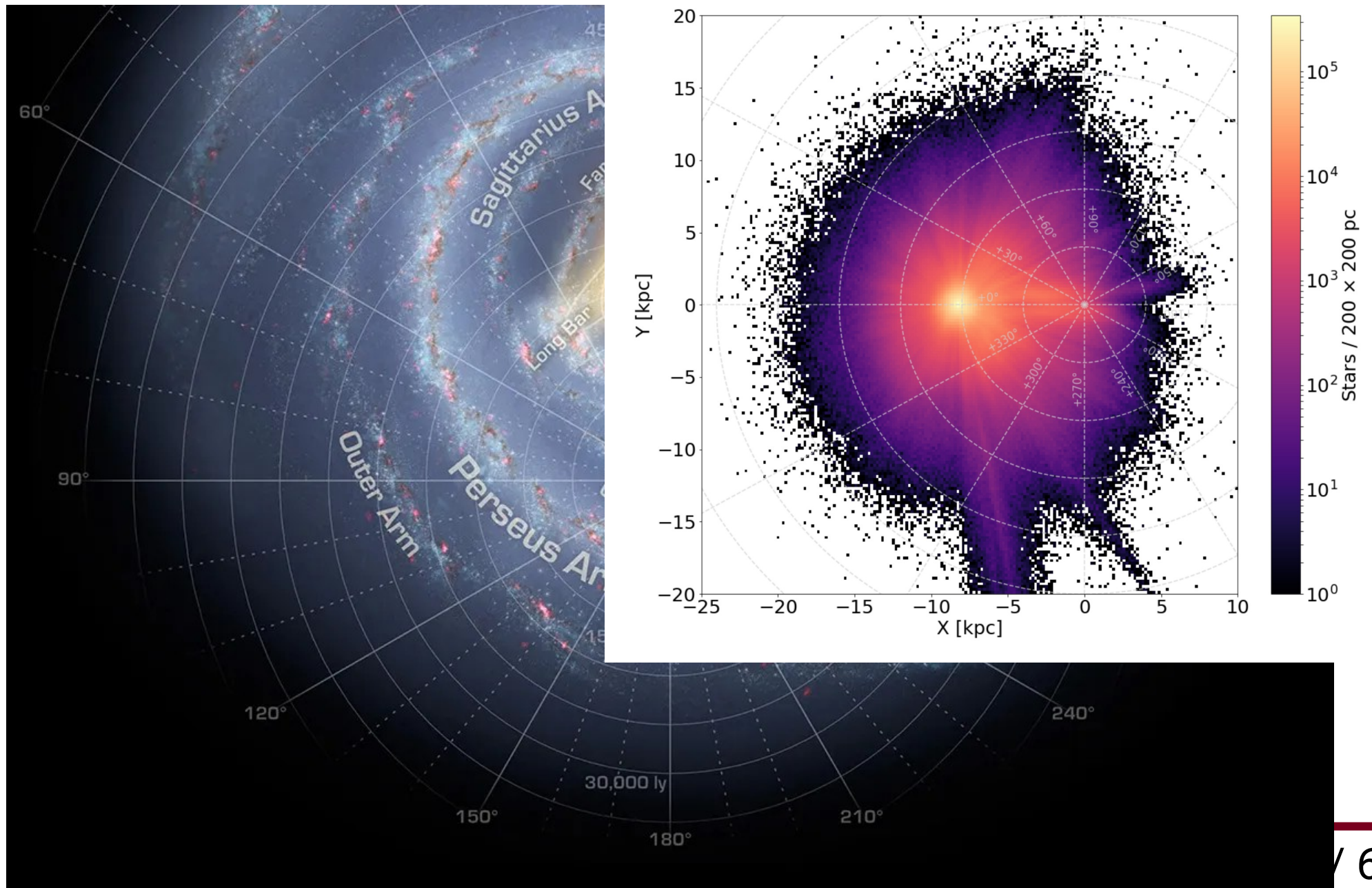
No time derivative information



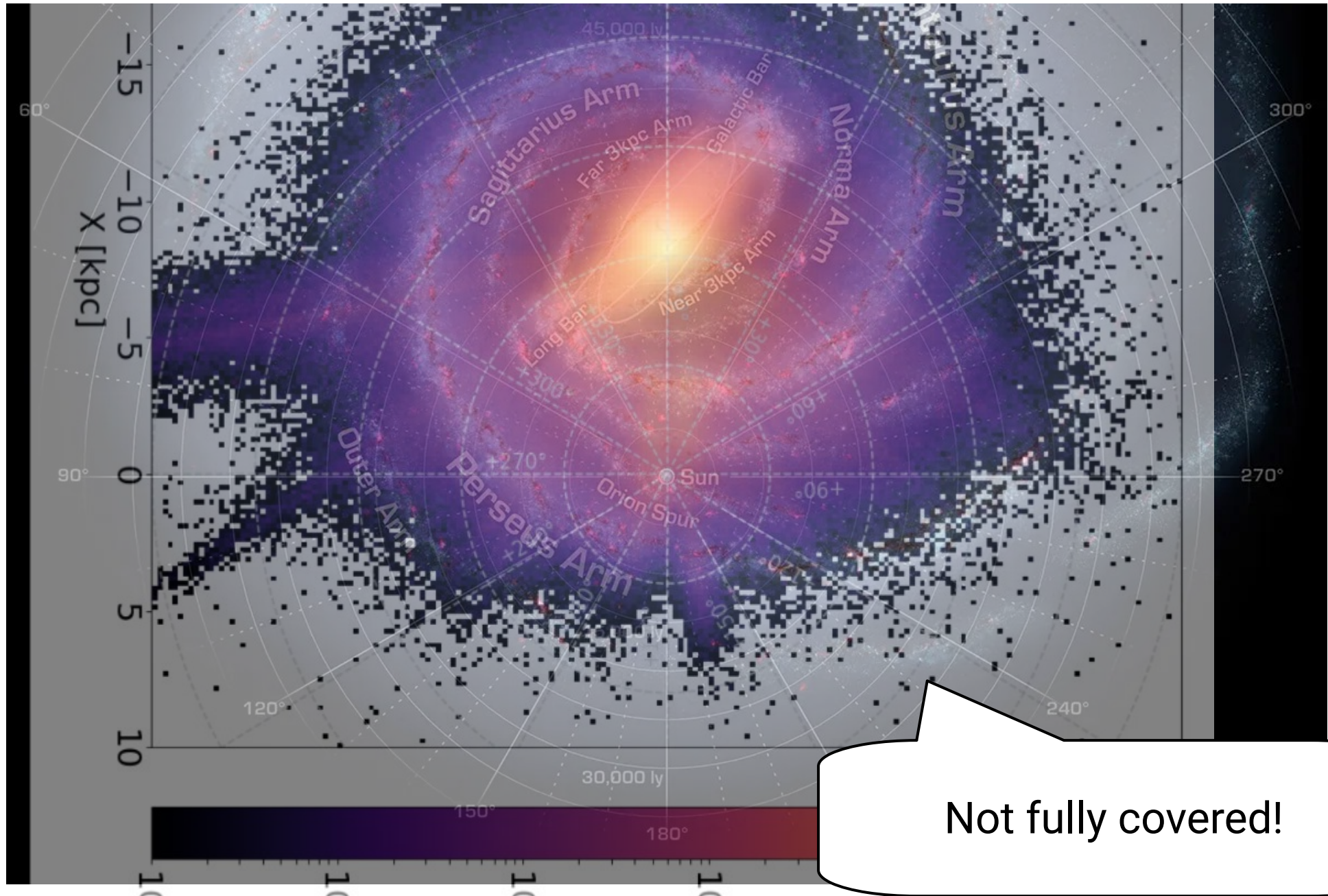
$$\frac{\partial f}{\partial t} = ?$$

We only have the current snapshot of the Milky Way!

Radial Velocity Distribution of Gaia DR3

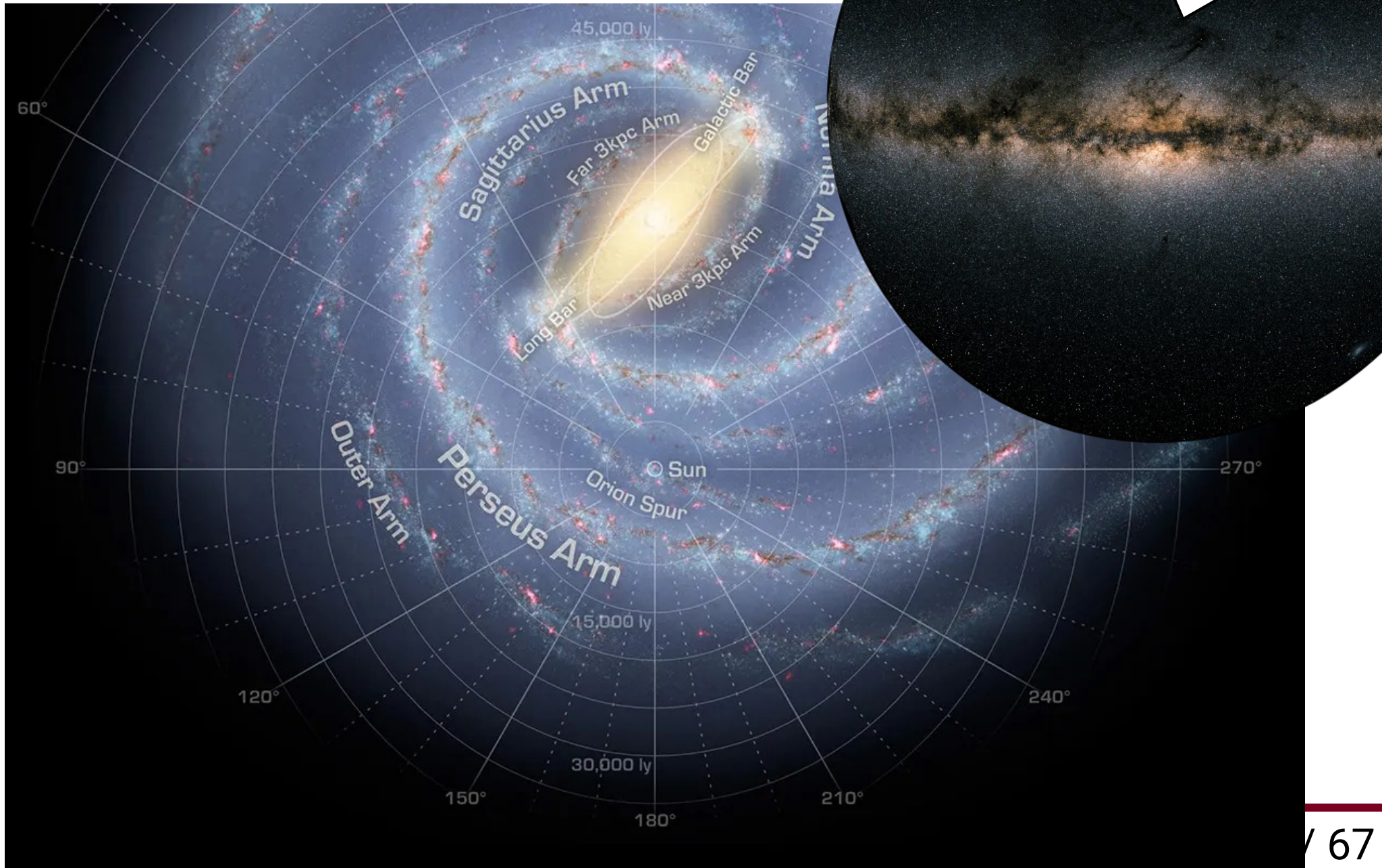


Incompleteness in Spacial Coverage



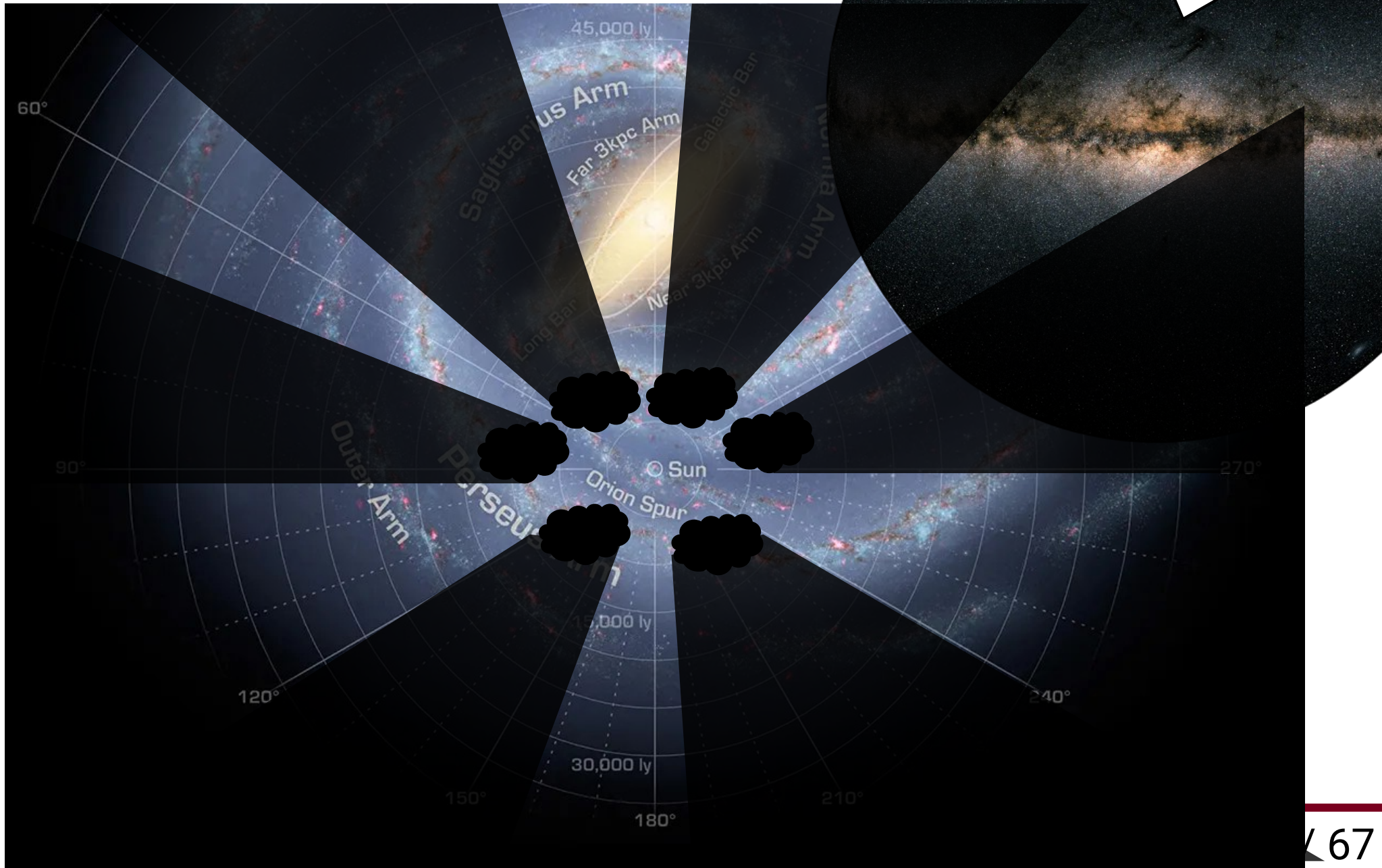
Dust Clouds

Intergalactic dust cloud obscuring light from stars!



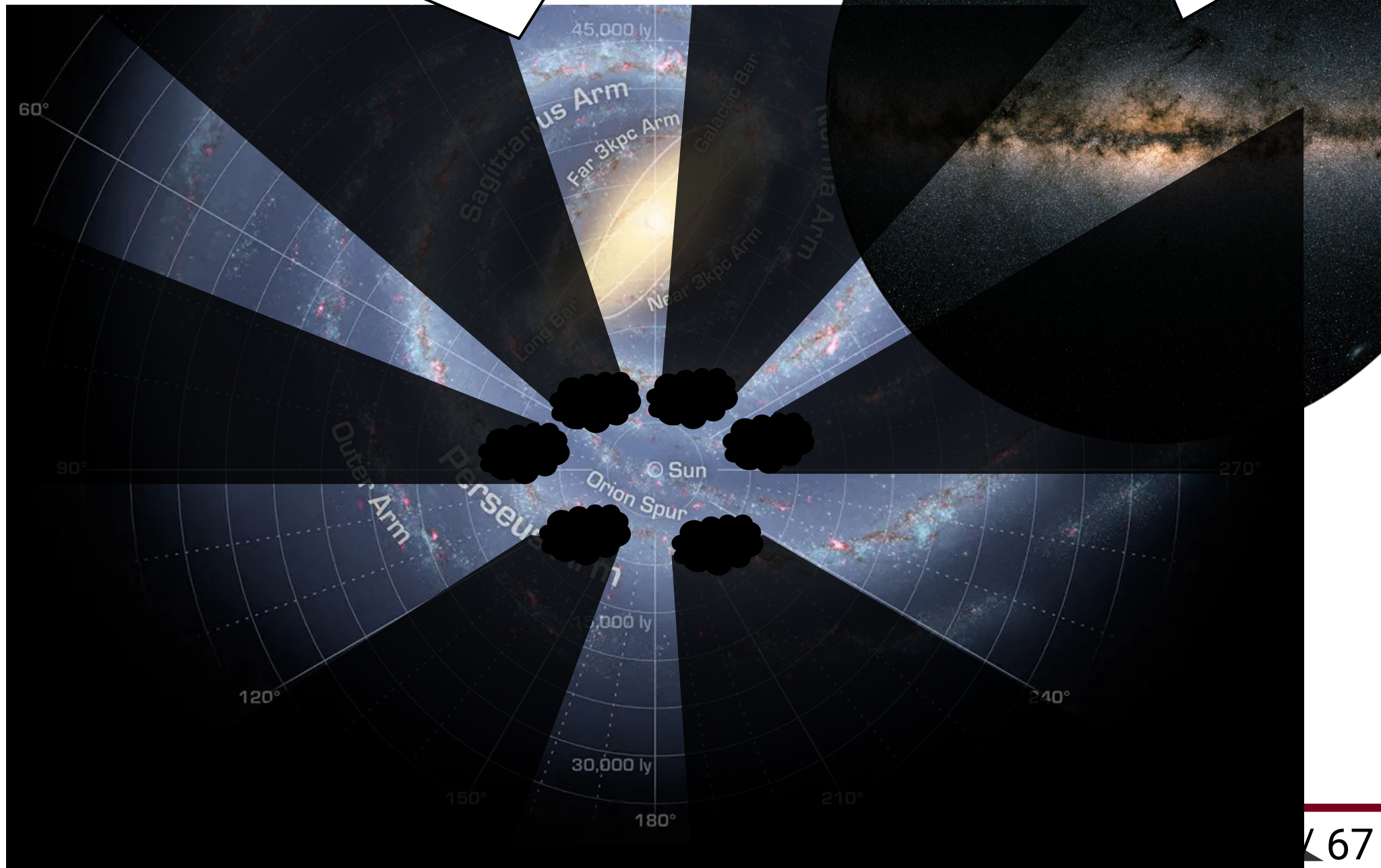
Dust Obscuring Stars

Intergalactic dust cloud obscuring light from stars!



How could we overcome this
data **incompleteness**
due to dust clouds (using ML)?

Intergalactic dust cloud
obscuring light from stars!



How could we overcome this
data **incompleteness**
due to dust clouds (using ML)?

Intergalactic dust cloud
obscuring light from stars!



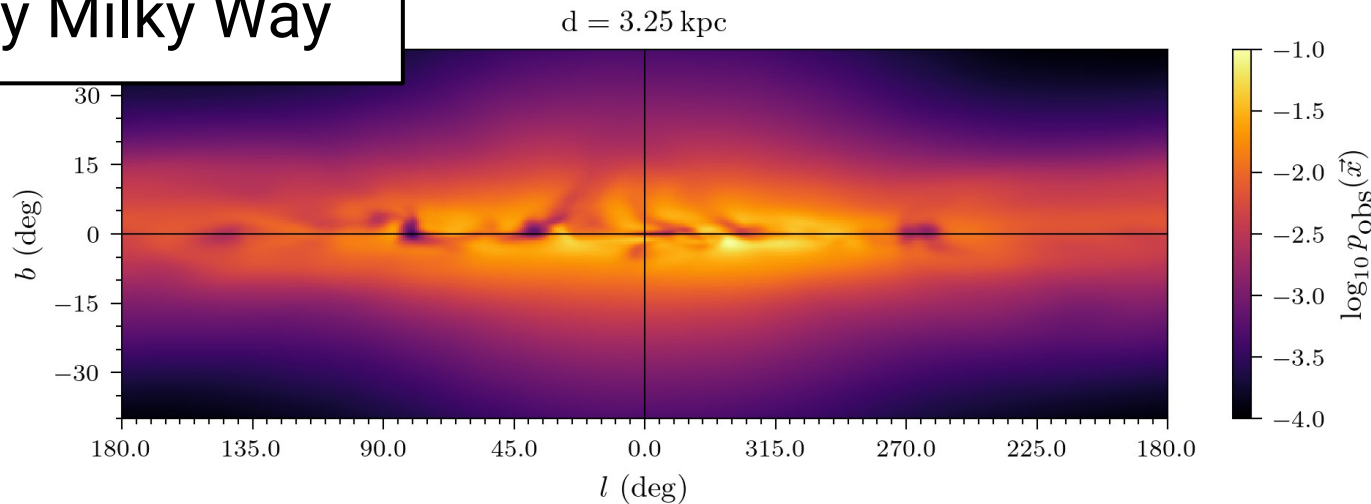
Our group's student **Eric Putney**



Sweeping Away Space Dust: Correcting dust extinction in Gaia kinematic phase space using machine learning

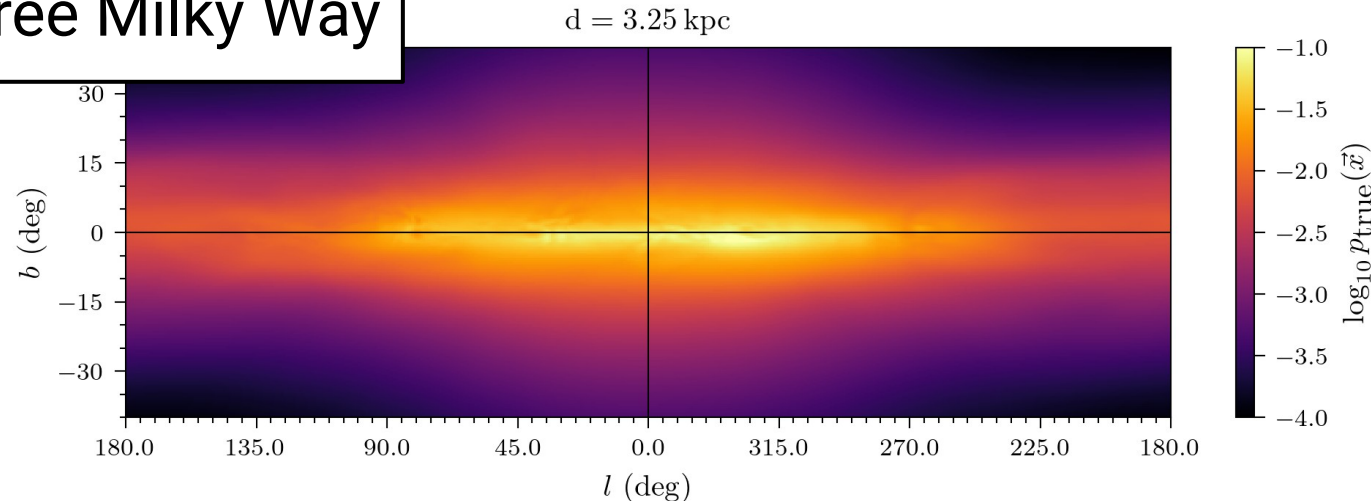
Erasing Dust using Neural Network and Equilibrium Assumptions

Dusty Milky Way



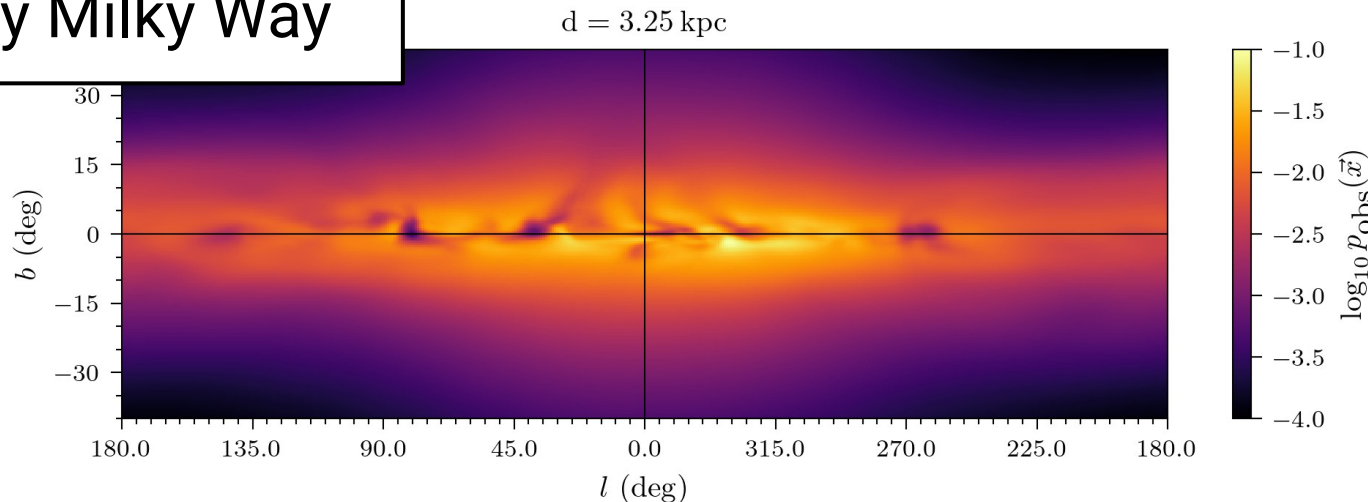
$$\frac{\partial f}{\partial t} = 0$$

Dust-free Milky Way



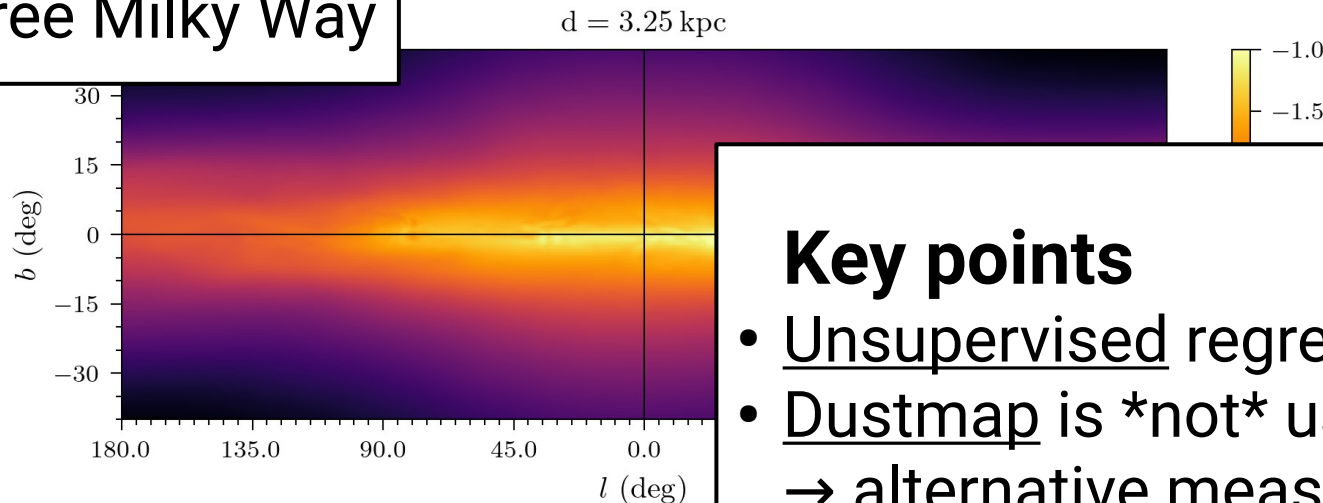
Erasing Dust using Neural Network and Equilibrium Assumptions

Dusty Milky Way



$$\frac{\partial f}{\partial t} = 0$$

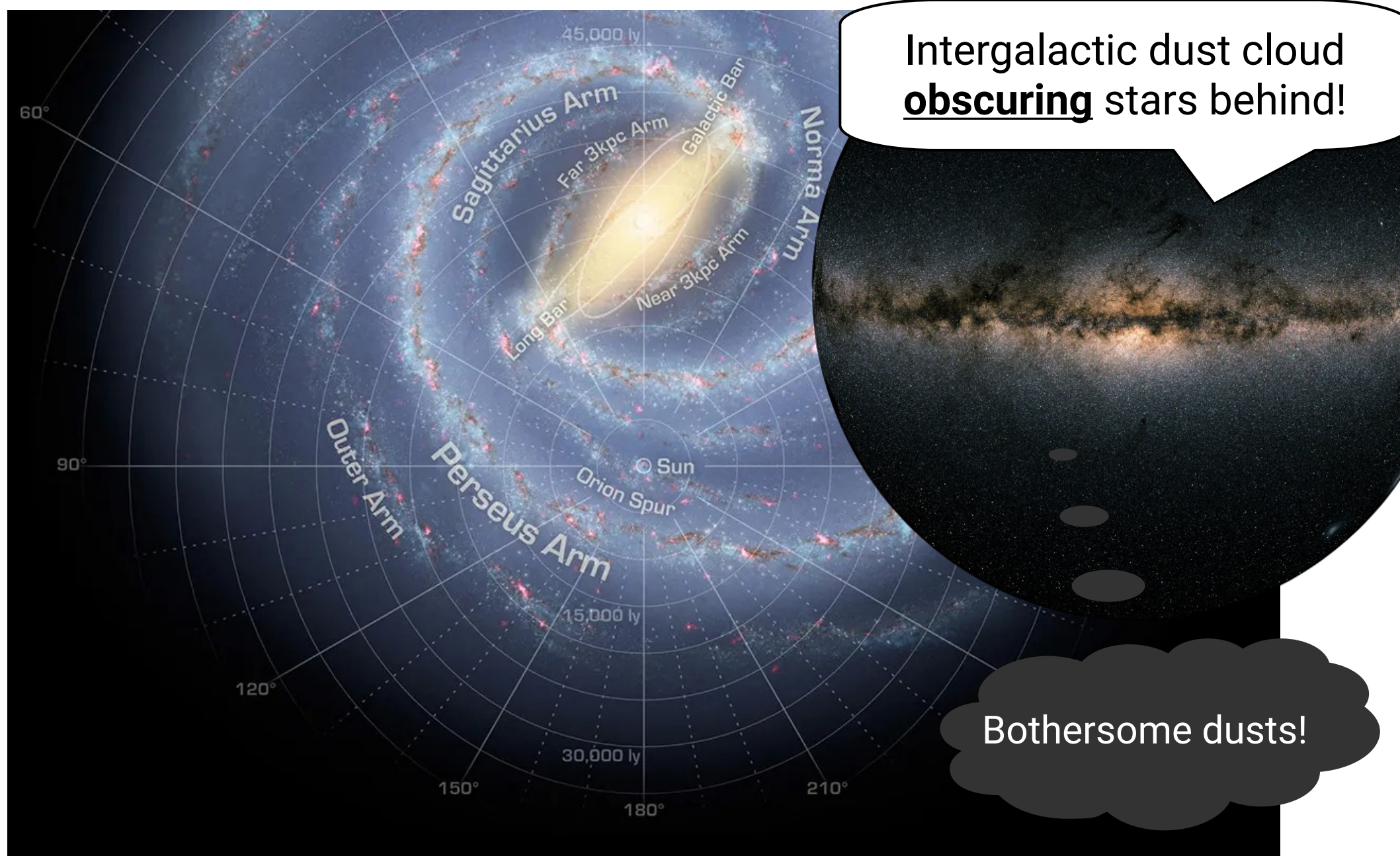
Dust-free Milky Way



Key points

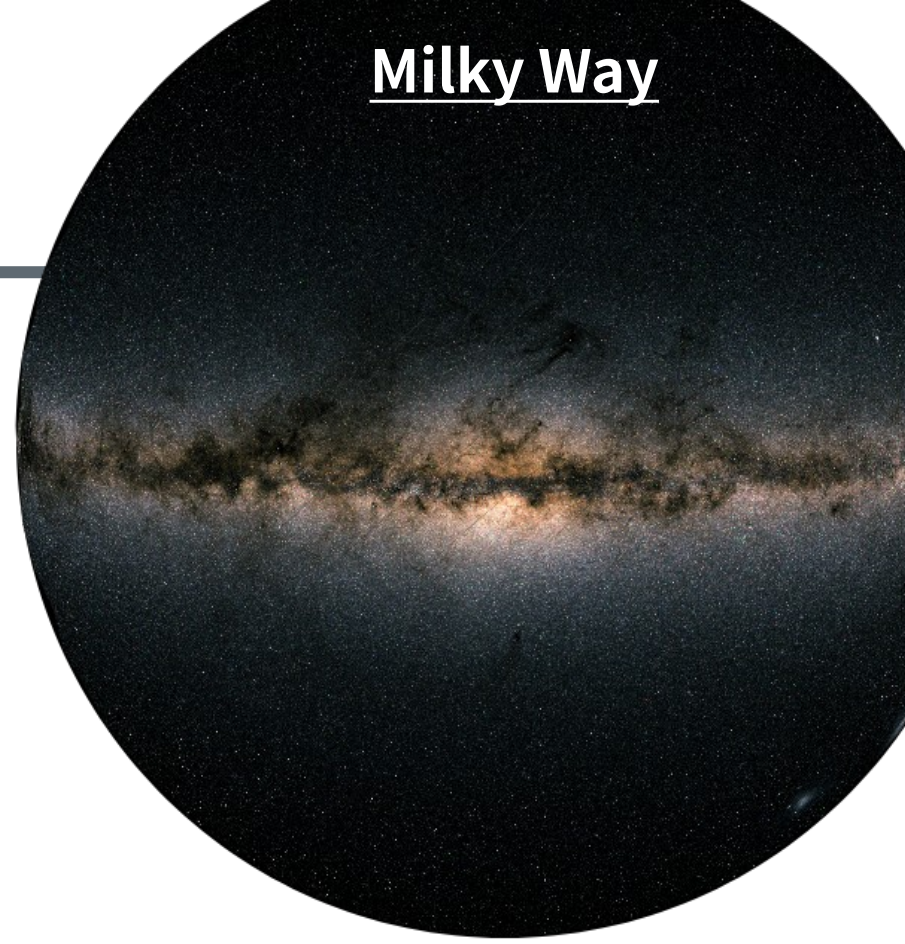
- Unsupervised regression
- Dustmap is **not** used
→ alternative measurement of dust!

Intergalactic dust cloud
obscuring stars behind!

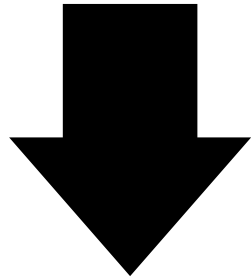


So far, we have discussed how to deal with **dusty** environment of **the Milky Way**.

Q: Are there any
dust-free galaxies to make
this analysis simple?

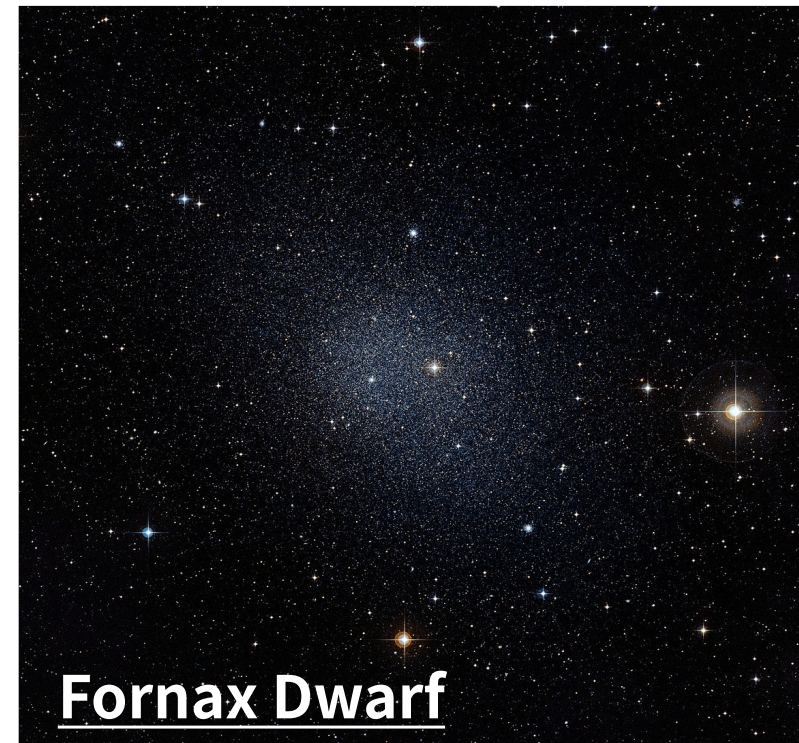
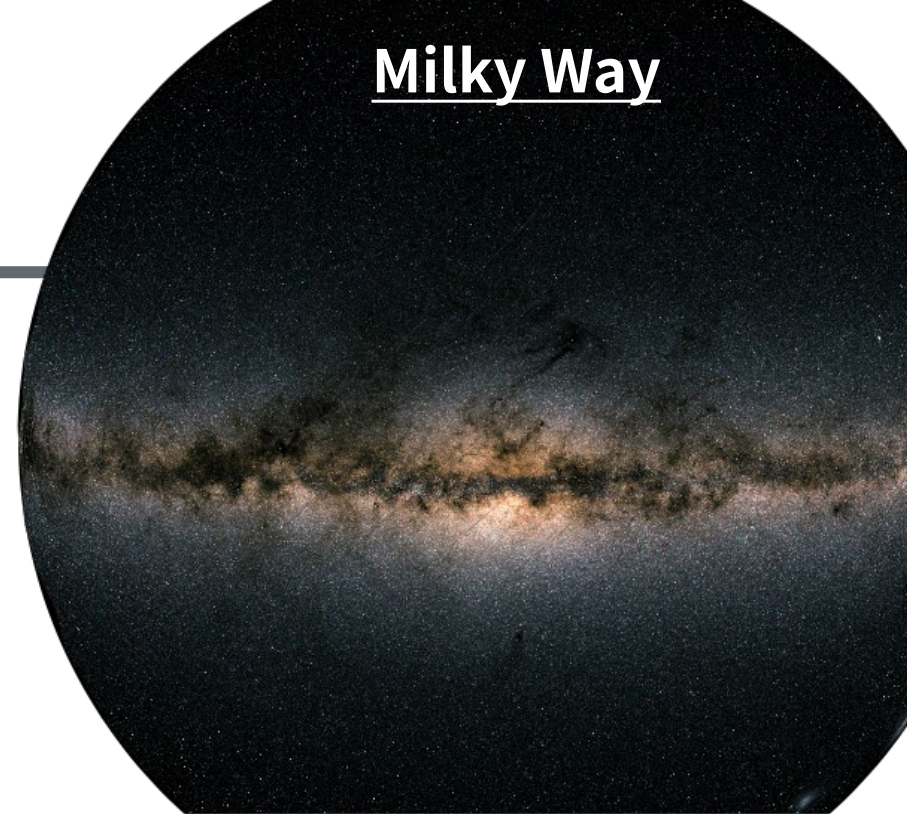


Q: Are there any
dust-free galaxies to make
this analysis simple?



Yes, there are some
dust-free satellite galaxies
of the Milky Way!

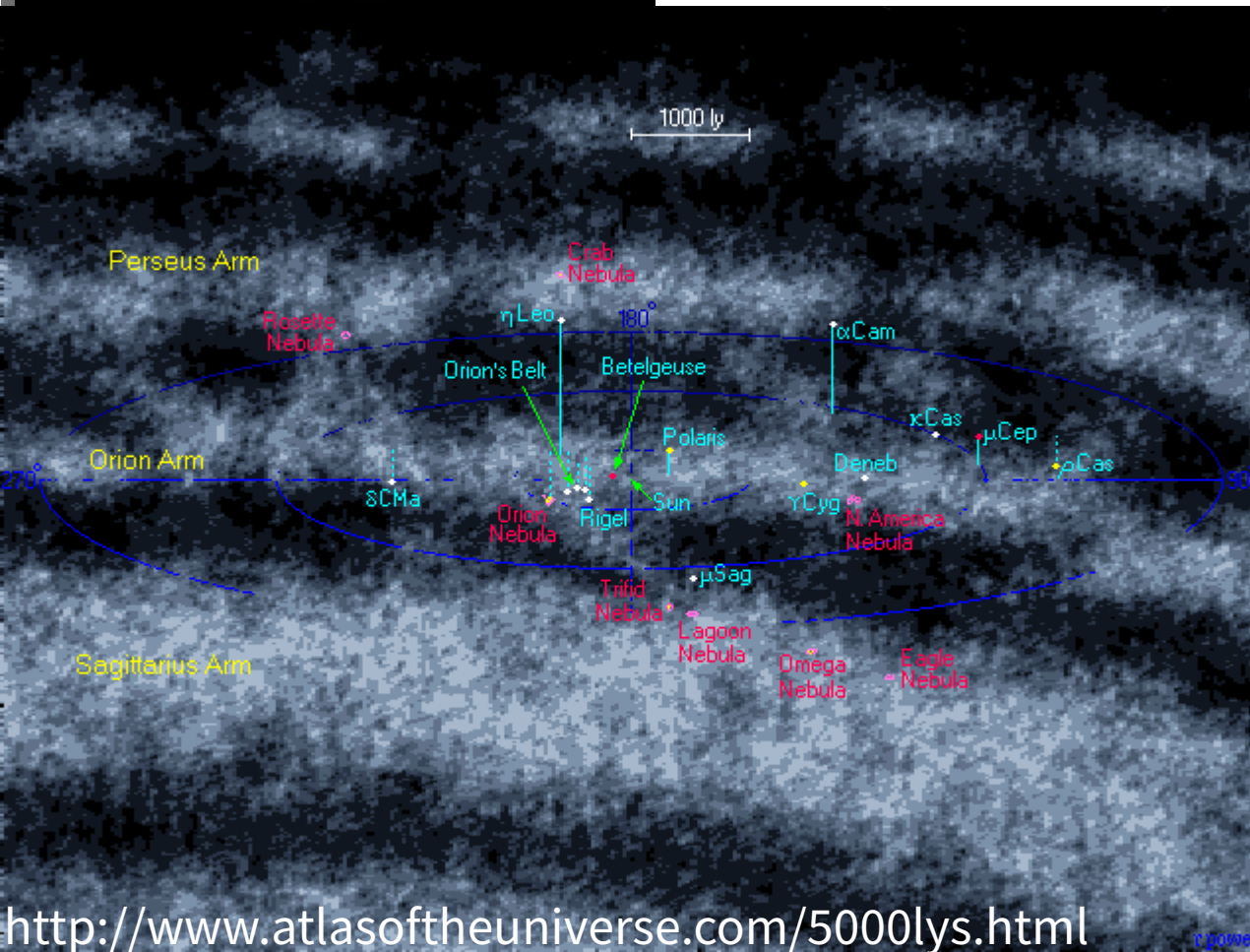
Where are they?

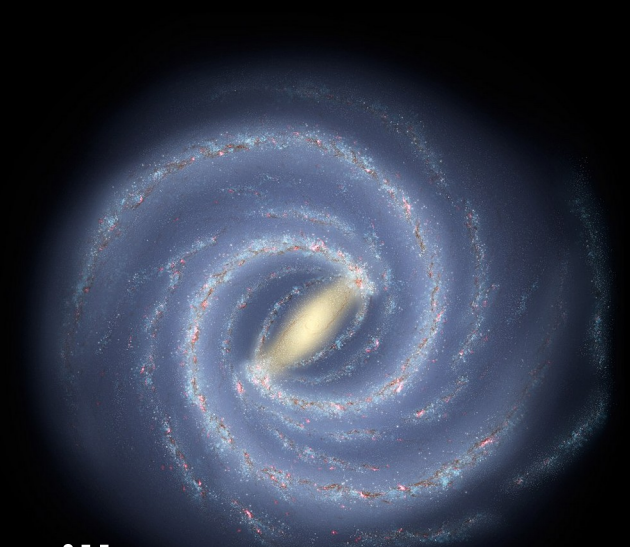


So far, we have been focused on
the analysis
on our corner of the Milky Way.

If you go further away...

Milky Way





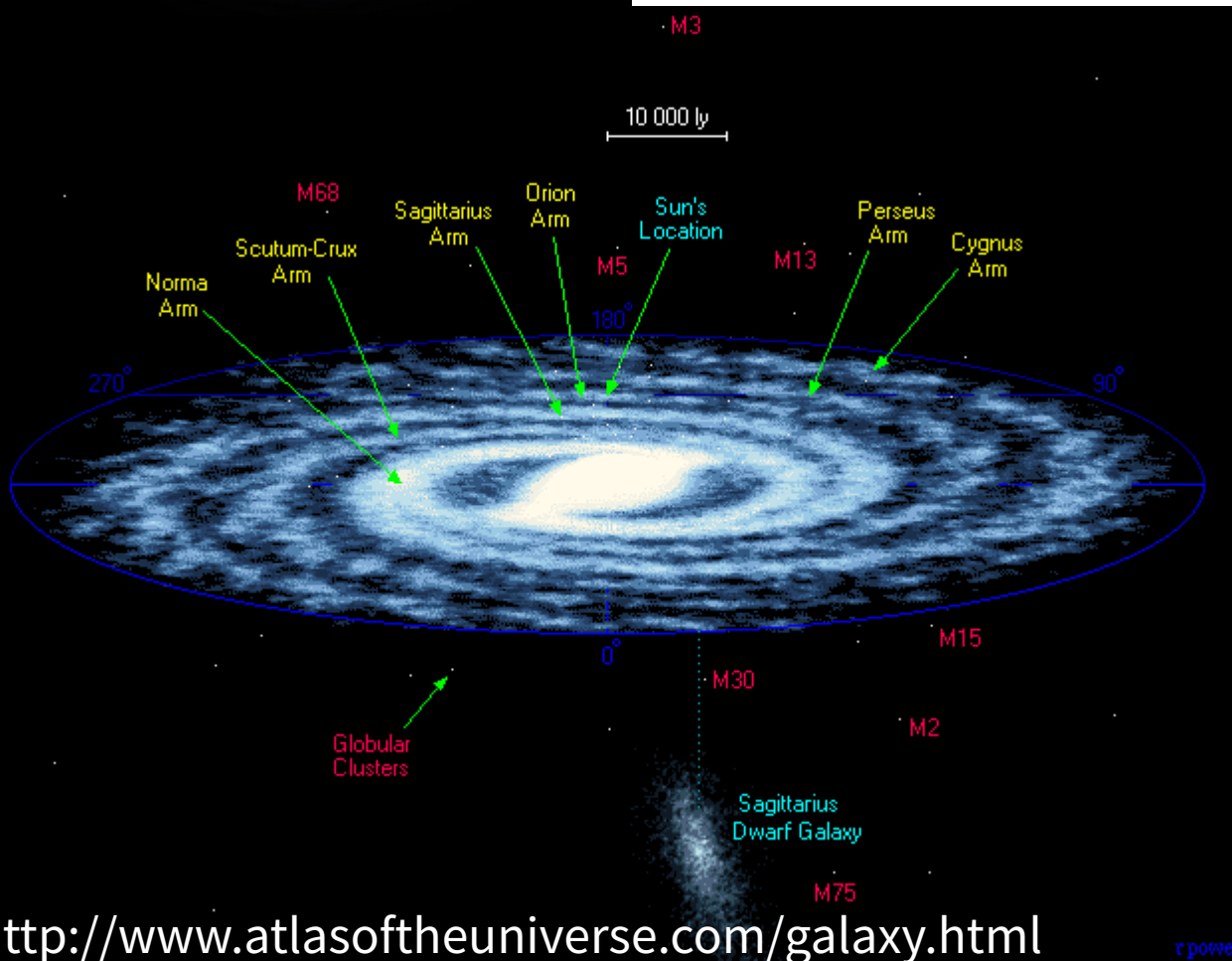
Milky Way

So far, we have been focused on
the analysis
on our corner of the Milky Way.

If you go further away, you see

whole Milky Way,
but it is difficult
to get

all the kinematic
information of
stars visible here.



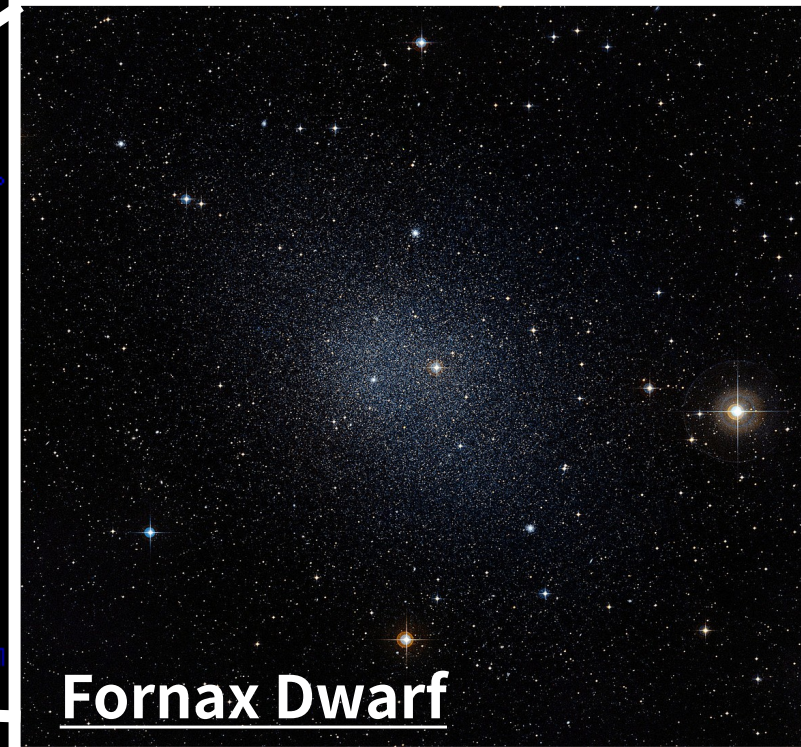
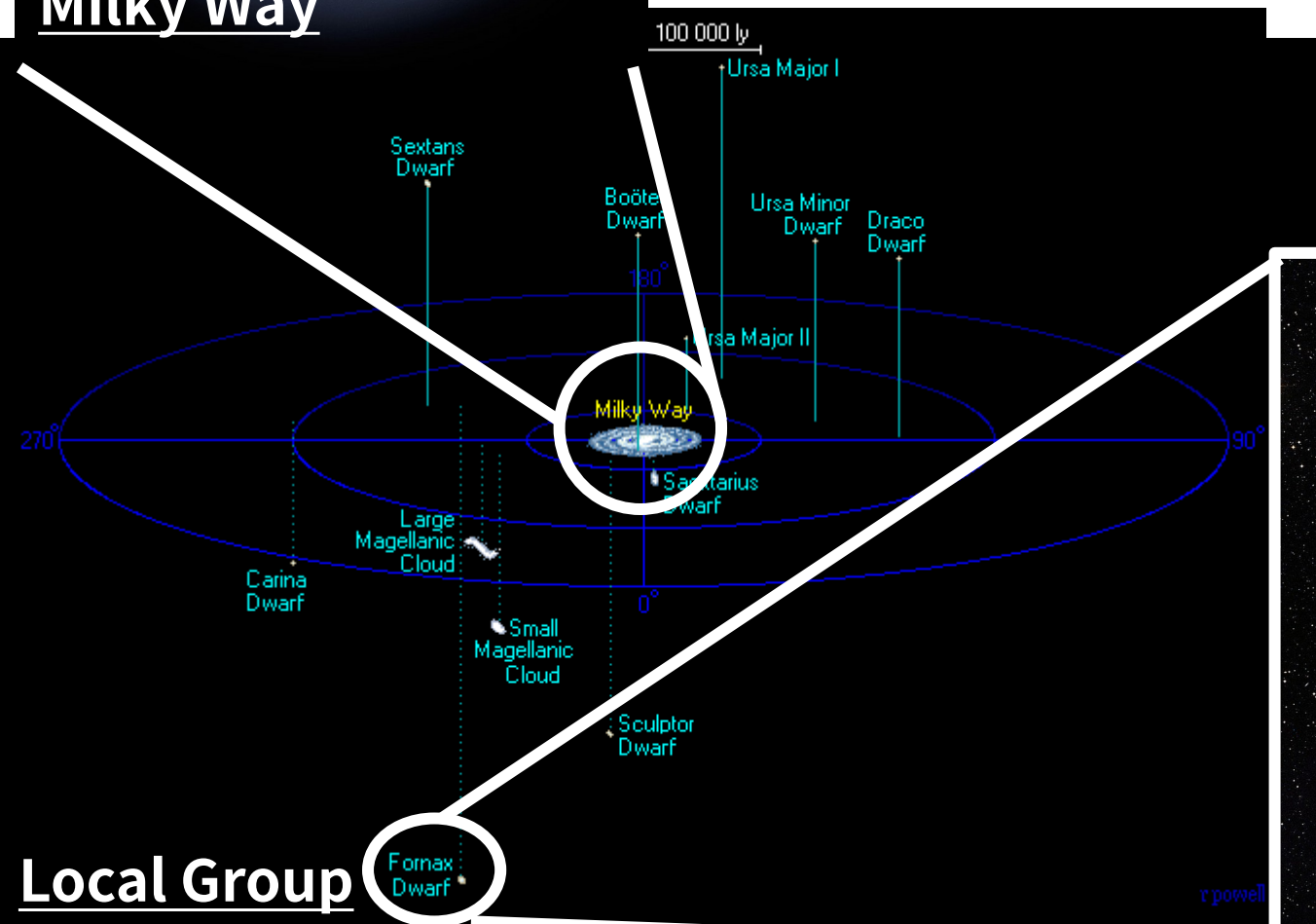
No local dark matter
density estimate on
the opposite corner!

So far, we have been focused on
the analysis
on our corner of the Milky Way.

If you go further further away,

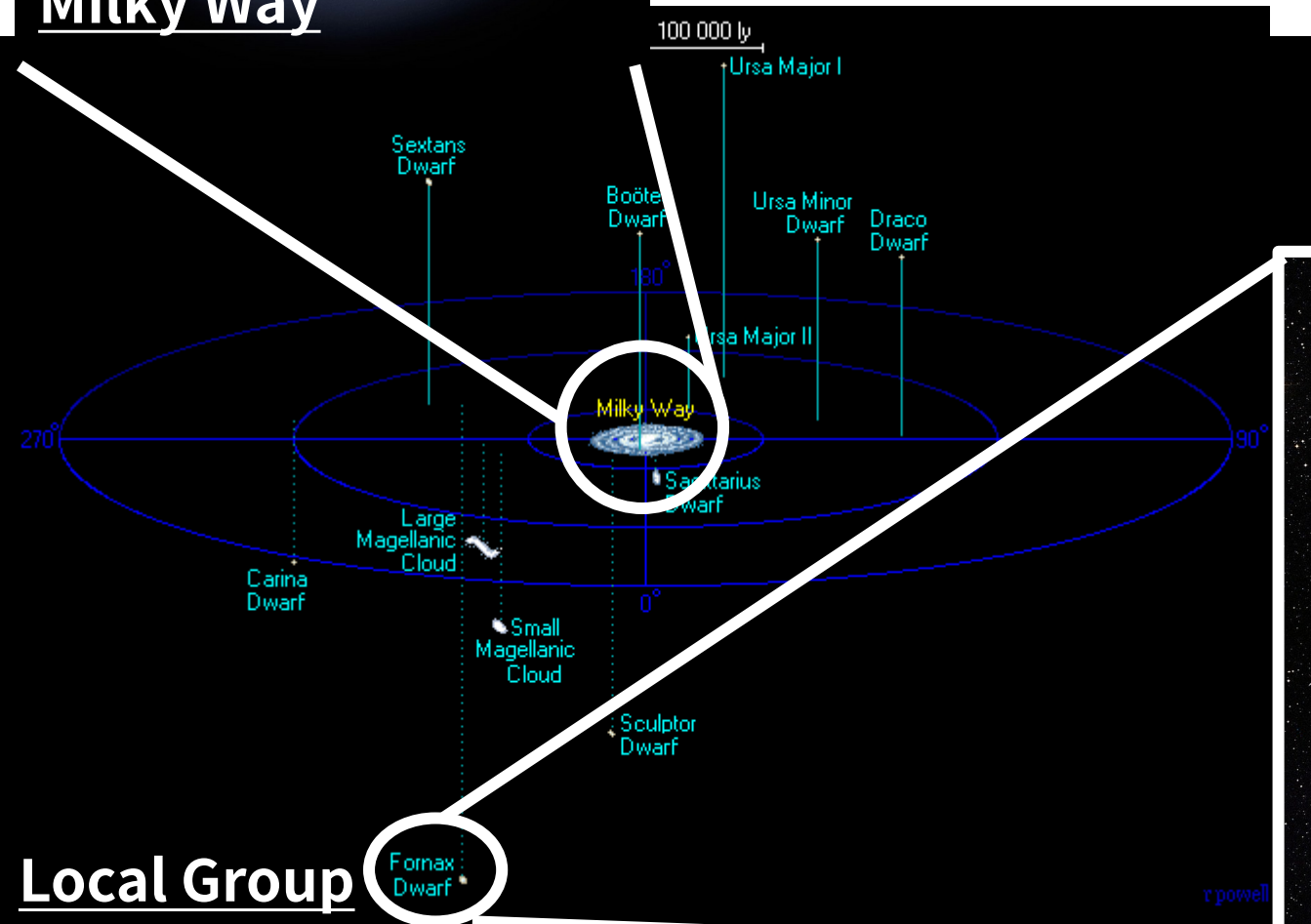
You see other
satellite galaxies!

Milky Way

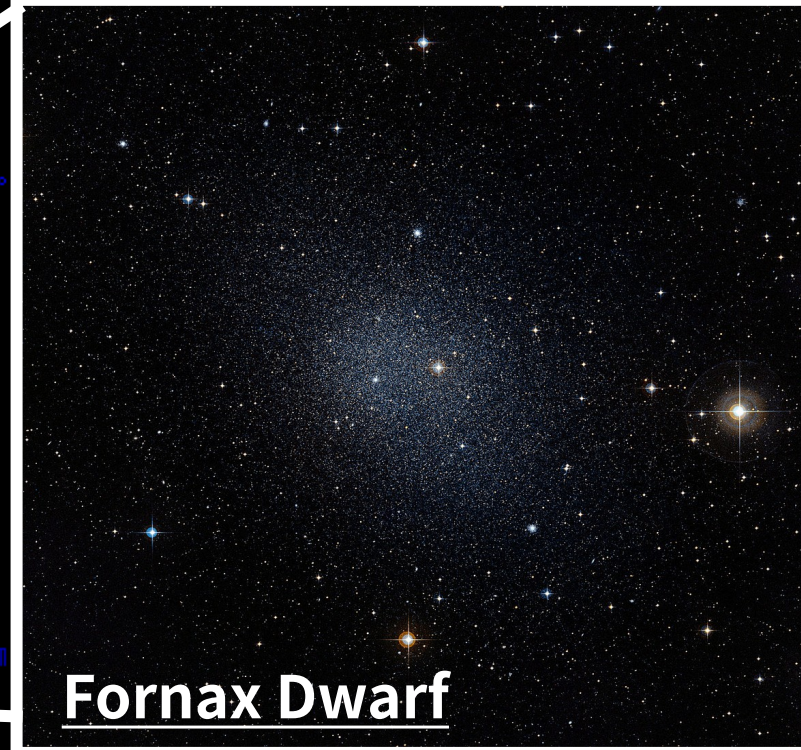


we will focus on a type of
satellite galaxy called
dwarf speheroidal galaxy.

Milky Way



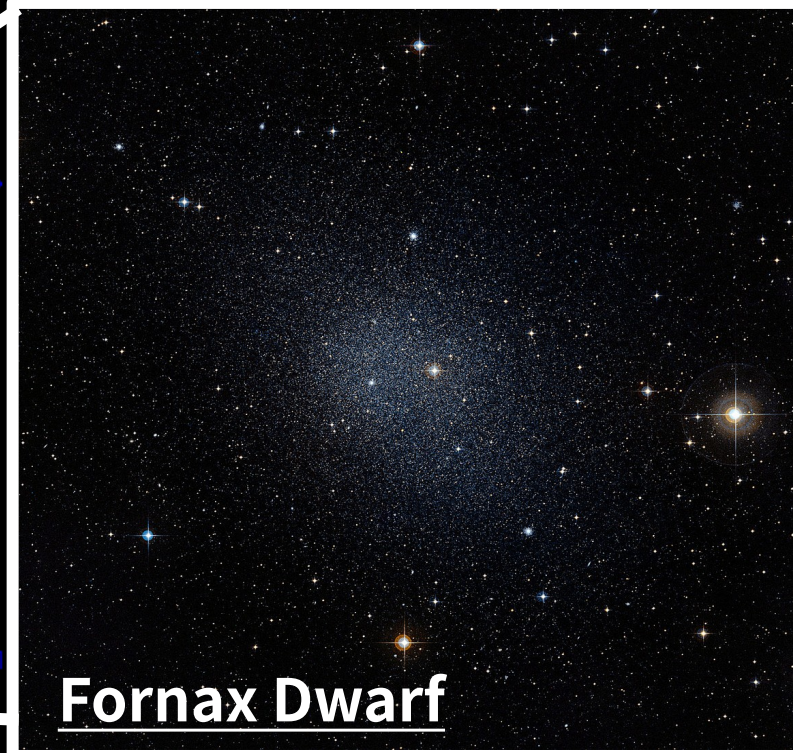
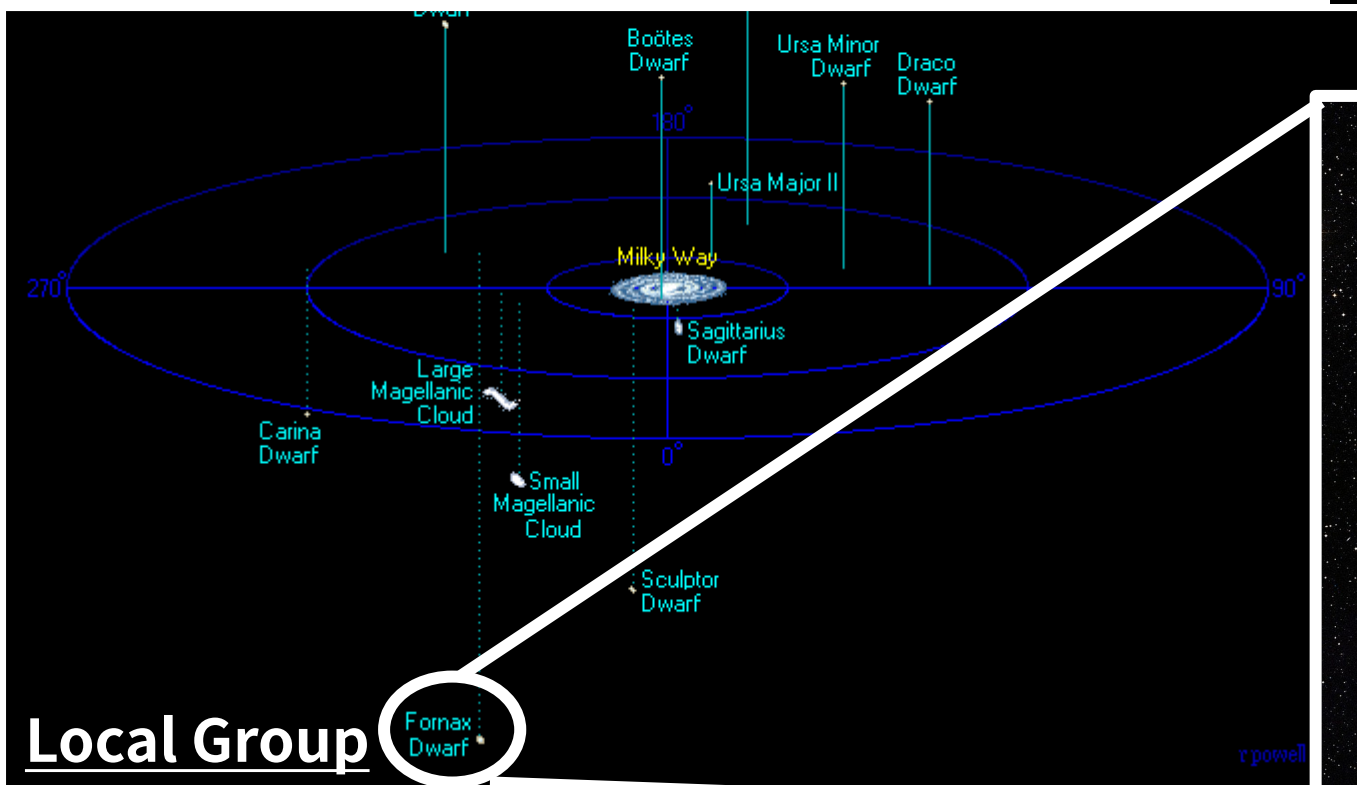
Local Group



Fornax Dwarf

Dwarf Spheroidal Galaxy?

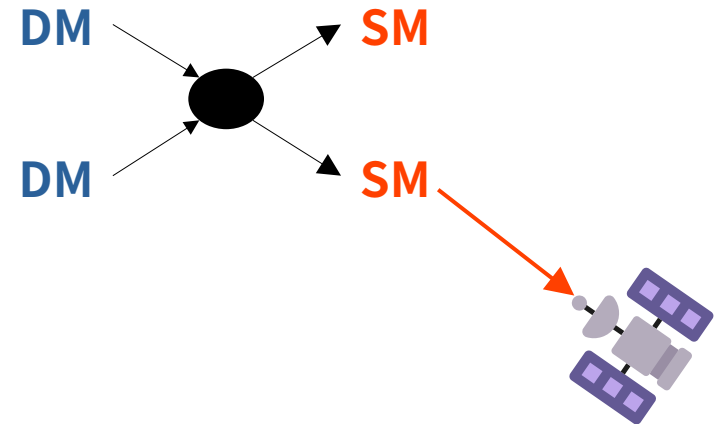
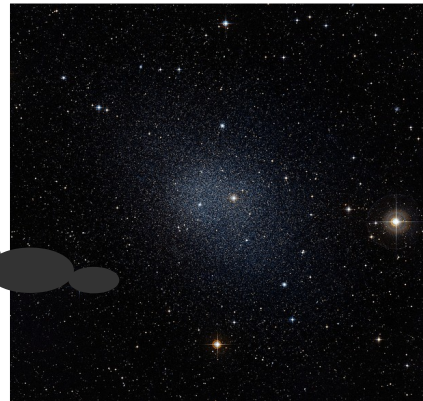
- A round and faint satellite galaxy, orbiting the Milky Way.
- Almost no gas and dust obscuring stars. Whole galaxy is clearly visible.



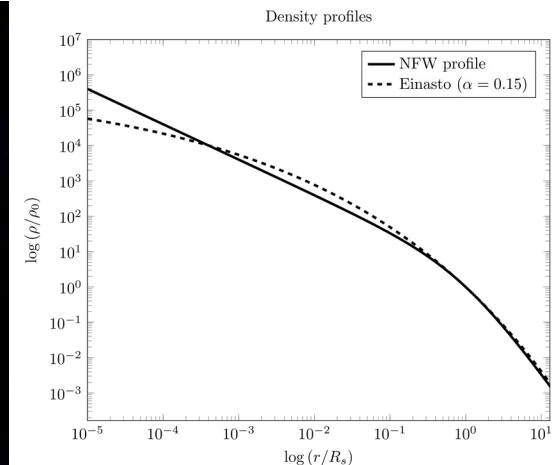
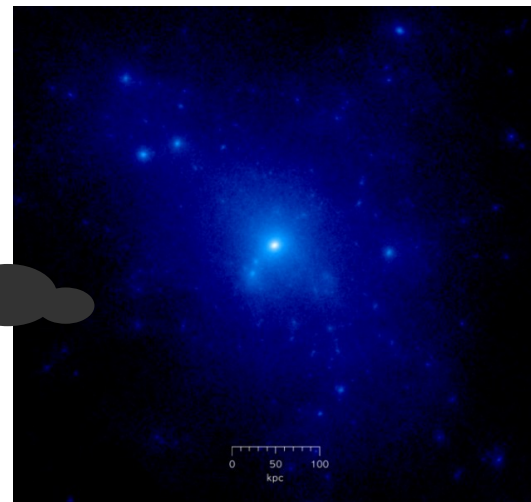
Dwarf spheroidal galaxy is a dark matter laboratory!

Clean signal source as dsph exhibits less baryon activity.

Indirect Detection experiments



Understanding the dark matter halo shape
→ insights on DM interactions?



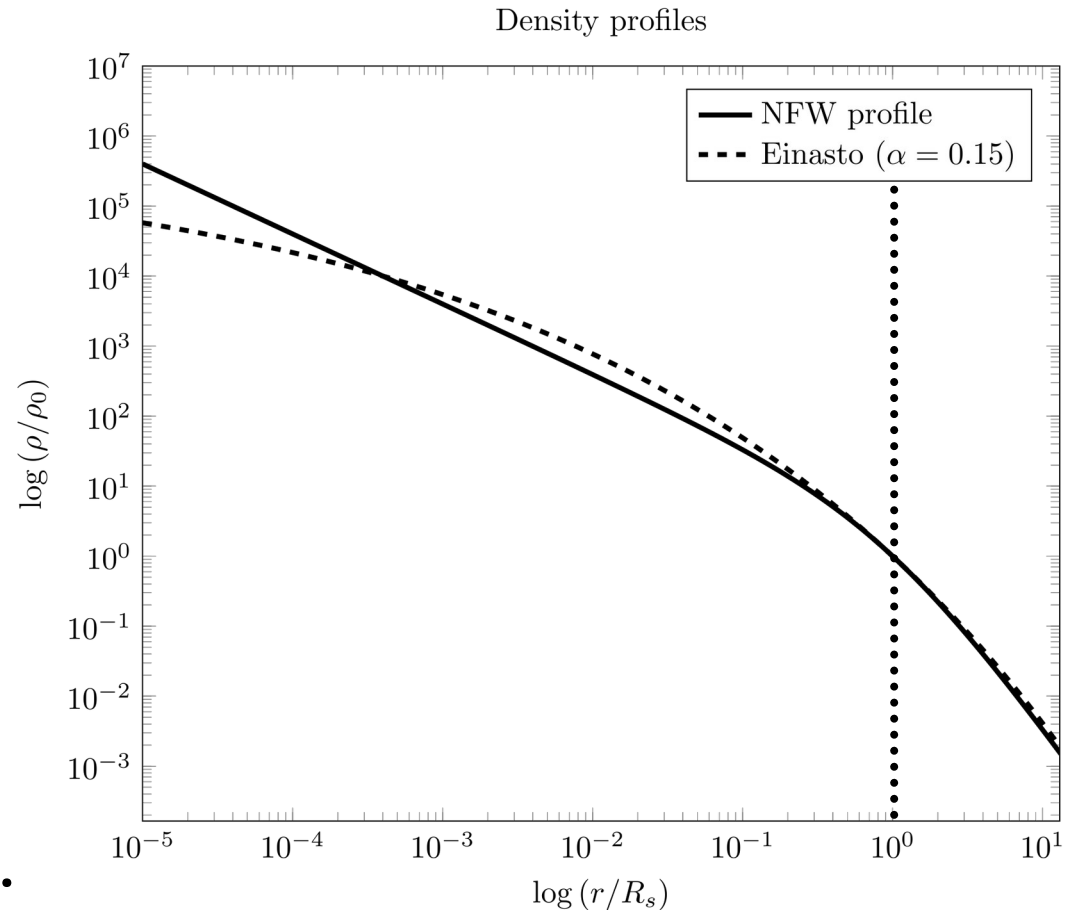
Navarro–Frenk–White (NFW) profile

A commonly used dark matter halo model empirically identified in N-body simulations

$$\rho(r) = \frac{\rho_0}{\frac{r}{R_s} \left(1 + \frac{r}{R_s}\right)^2}$$

If dark matter exhibits non-trivial interactions, the **halo shape may vary**.

Self-interacting dark matter, wave dark matter



Example: Wave Dark Matter

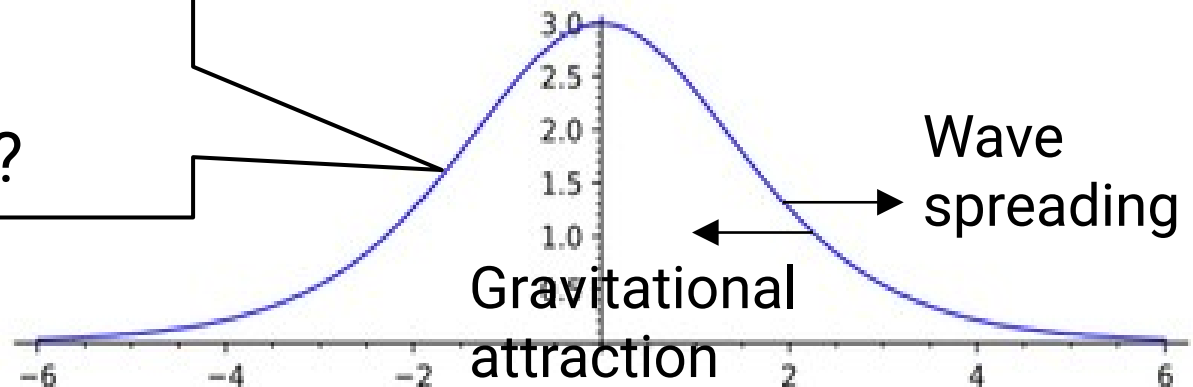
If DM mass is so light (e.g. very light axions) so that
inter-particle spacing \ll de Broglie wavelength

DM exhibits wave-like behavior.

Nontrivial stable solution:

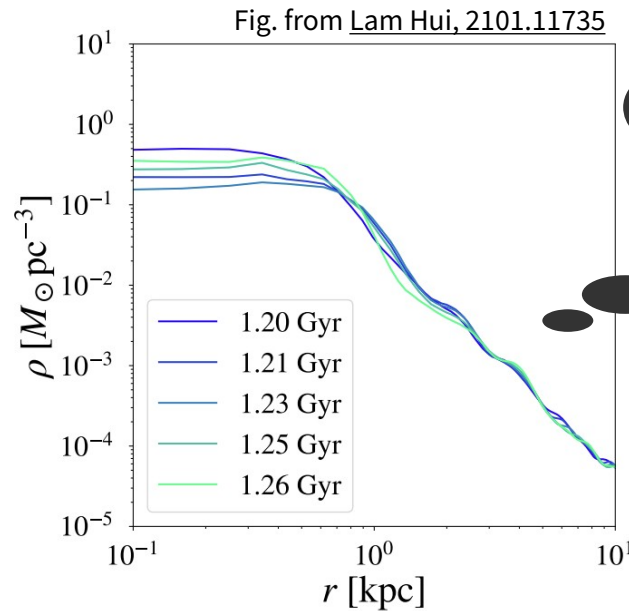
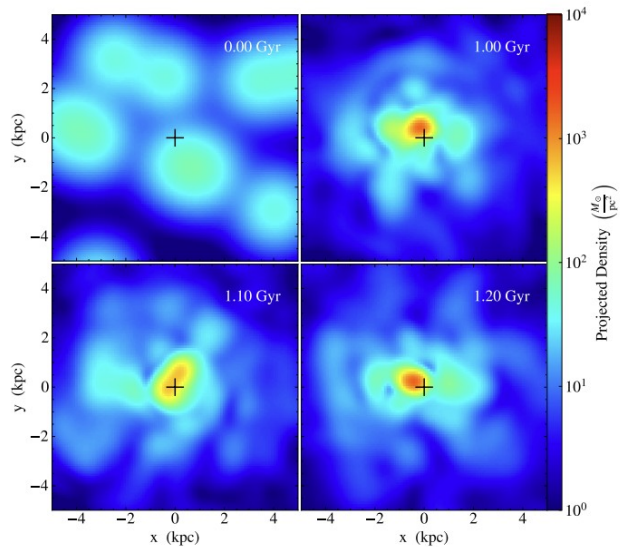
Soliton

→ Soliton in
dark matter halo?



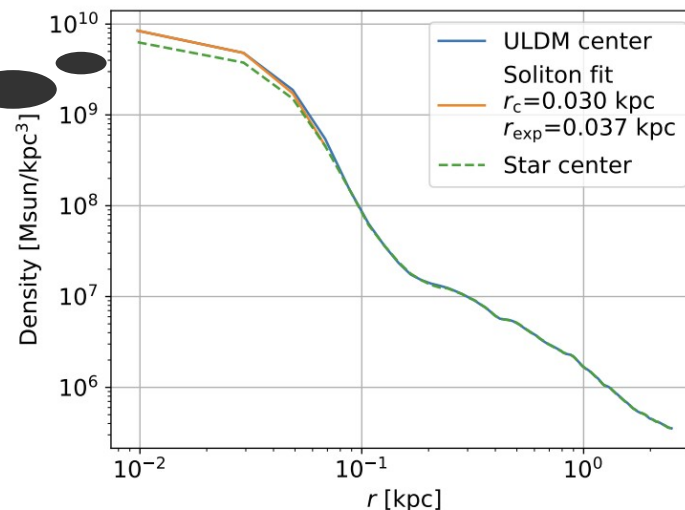
Smoking gun signatures

Disclaimer:
I'm still following up
refs :)

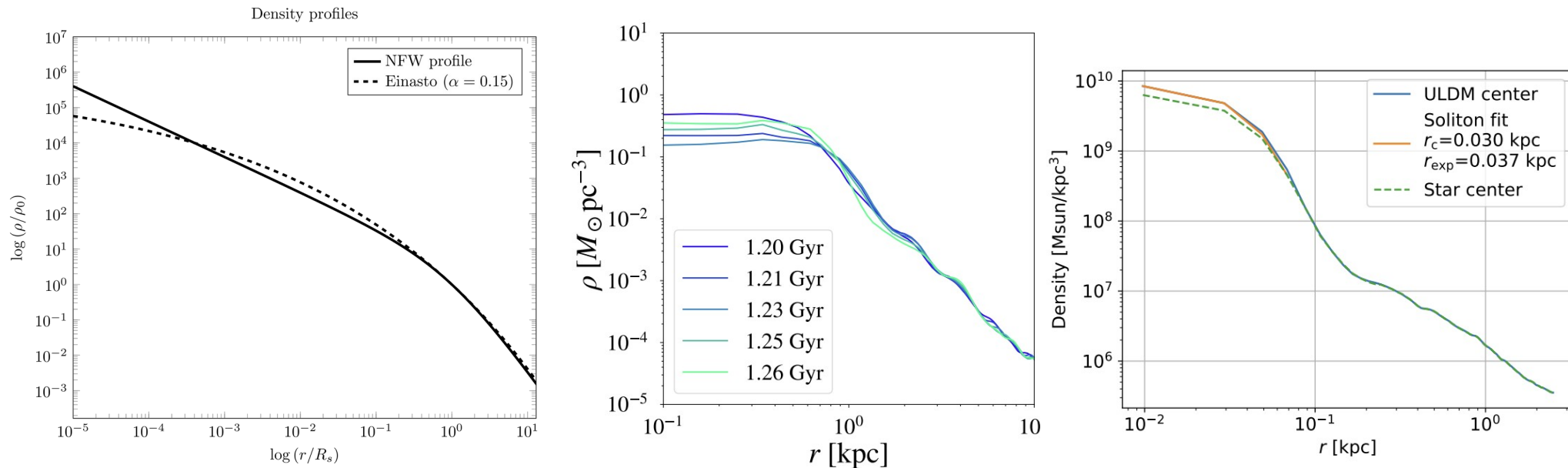


Soliton Oscillations

Soliton Core



Need for model-independent analysis

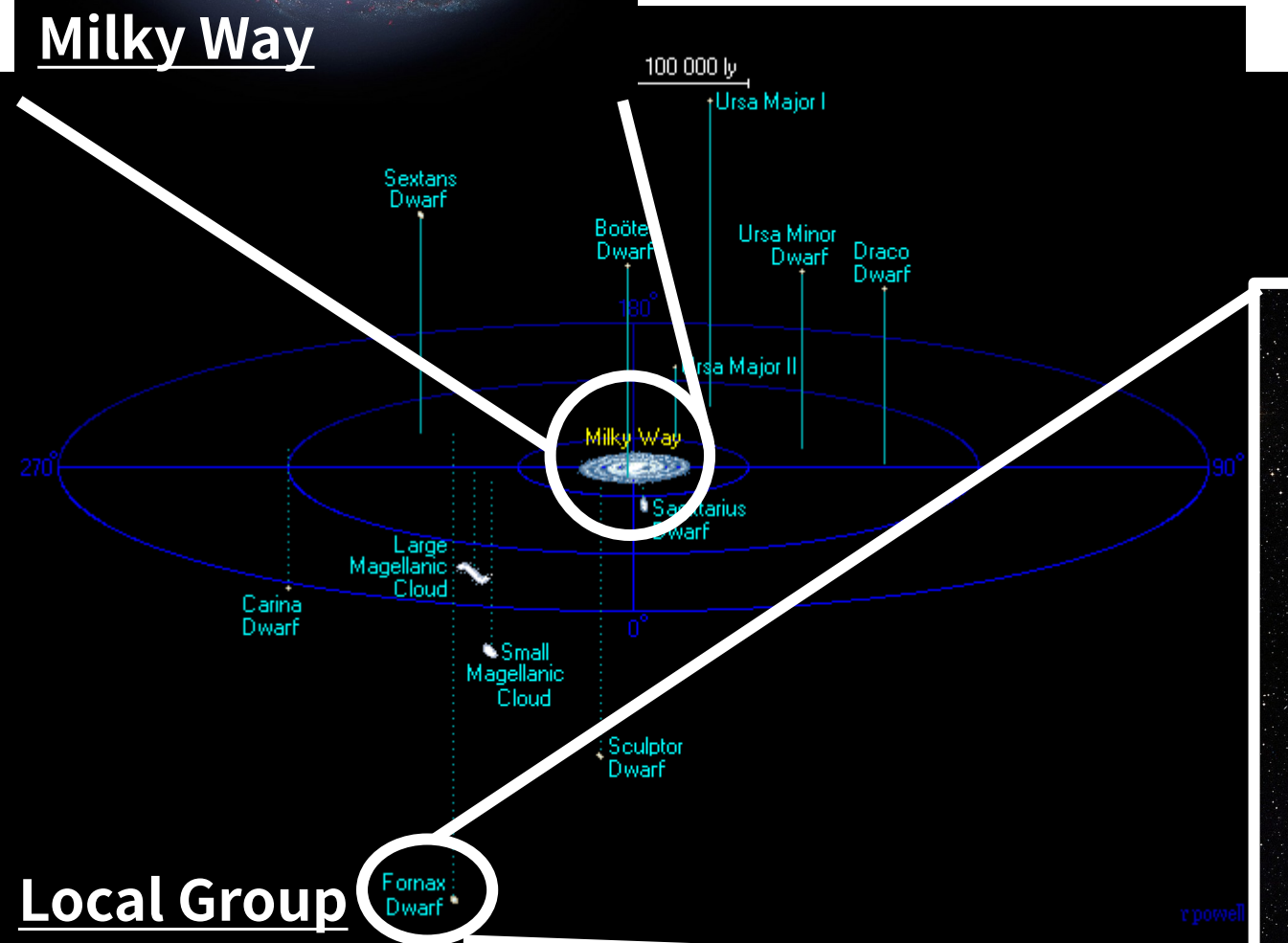


As many non-trivial DM halos are considered nowadays, we need a **free-form DM density estimation** in order to do a **model-independent** DM halo analysis.

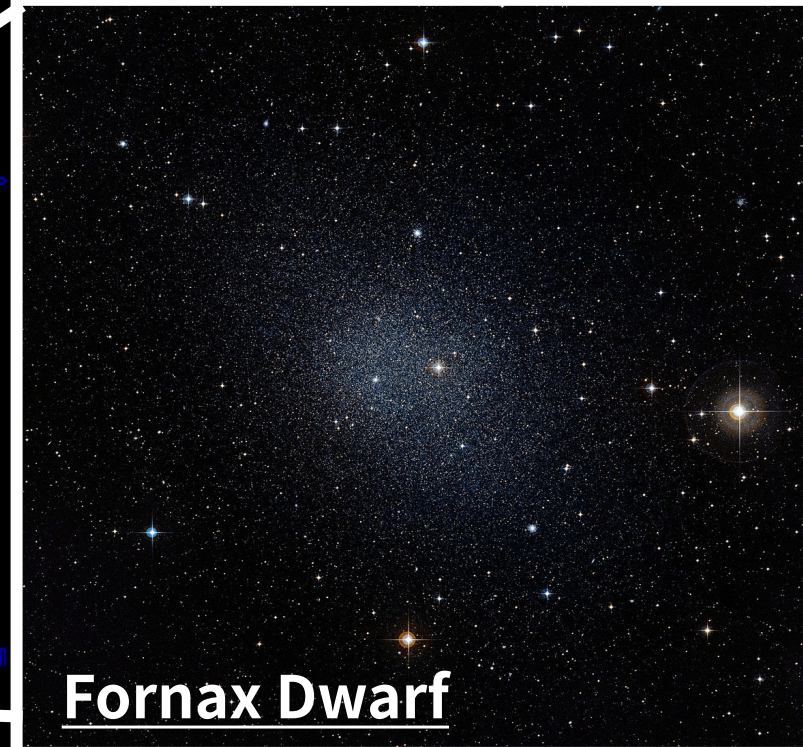
Again, unsupervised **machine learning** can help solving this type of problem!

Is the ML technique easily applicable to any of distant dust-free galaxies, like dwarf spheroidal galaxy?
Answer: both yes and no

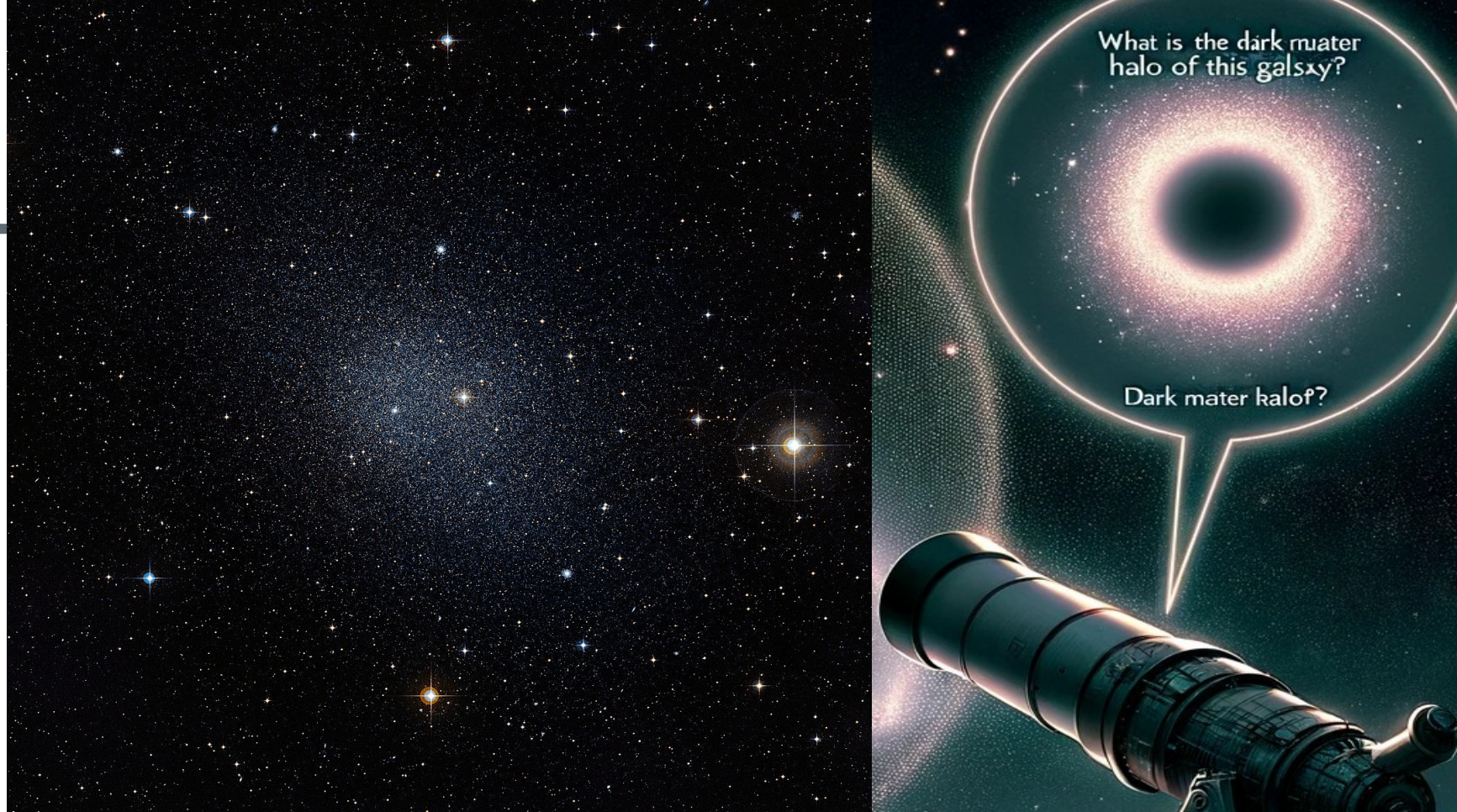
Milky Way



Local Group



Fornax Dwarf

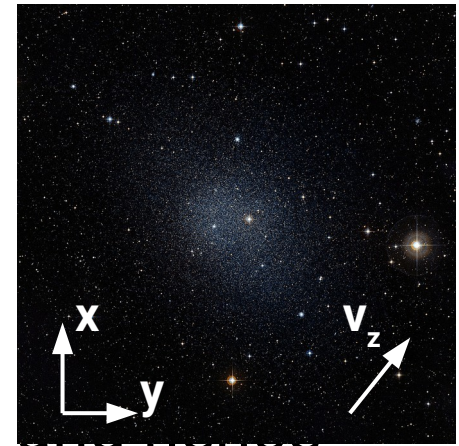


Model-Independent Spherical Jeans Analysis using Equivariant Continuous Normalizing Flows

Collaboration with
K. Hayashi (NIT, Sendai College), S. Horigome (IPMU),
S. Matsumoto (IPMU), M. M. Nojiri (KEK),

Challenges in Analyzing dSphs

- Faint galaxy
→ less number of observed stars $O[100] \sim O[1000]$
- Available kinematic information is **limited!**
 - Position of stars on the sky (x, y) (phot.)
 - ~~Distance to the stars (z)~~
 - ~~Proper motion of stars on the sky (v_x, v_y)~~
 - Radial velocity (v_z) (spec.)
- Phase space density of stars are not accessible, and hence we cannot solve the equation of motion yet.. (Jeans equation)

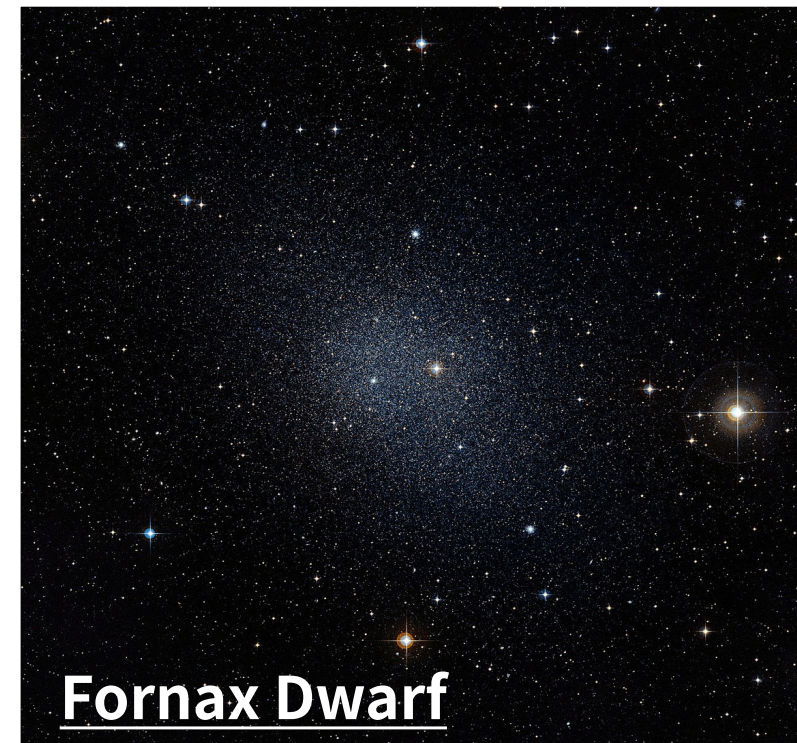
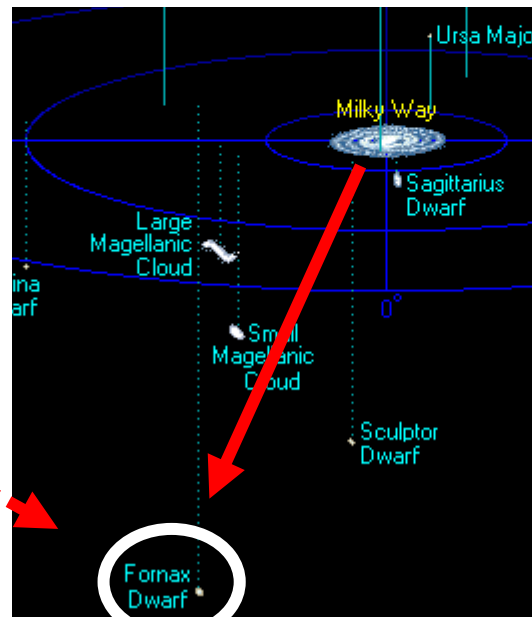


$$\frac{\partial n \langle v_j \rangle}{\partial t} + n \frac{\partial \Phi}{\partial x_j} + n \frac{\partial n \langle v_i v_j \rangle}{\partial x_i} = 0$$

Can we recover the full 6D information somehow?

Radon Transformation

- Can we recover the full 6D information somehow?
 - Yes, if we have a 3D projected snapshot of the dSph from all the direction



- This tomographic reconstruction is possible (e.g. MRI imaging),
- but we only have a snapshot from only one direction...
 - Classic solution: assume spherical symmetry.

Spherical Jeans Equation

Introducing spherical symmetry simplifies the Jeans equation, too.

$$\frac{d}{dr} n \overline{v_r^2} + \frac{2\beta}{r} n \overline{v_r^2} = -n \frac{d\Phi}{dr}$$

List of functions needed for inferring gravitational field (Φ)

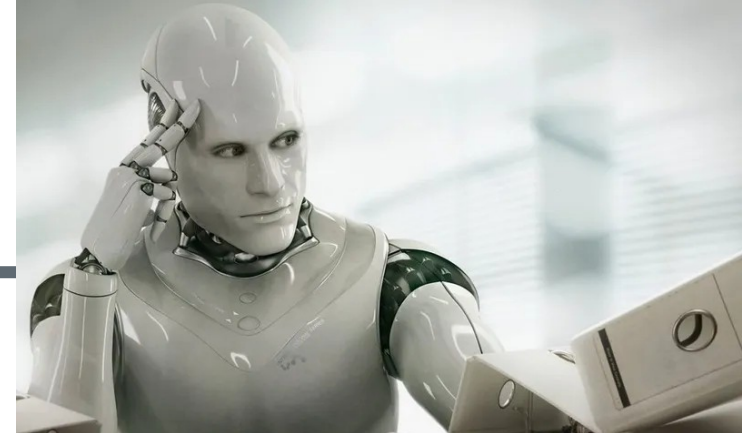
- Number density $n(r)$
- Radial velocity dispersion (variance) $\overline{v_r^2}(r)$
- Velocity anisotropy

$$\beta(r) = 1 - \frac{\overline{v_\theta^2}(r) + \overline{v_\phi^2}(r)}{2\overline{v_r^2}(r)}$$

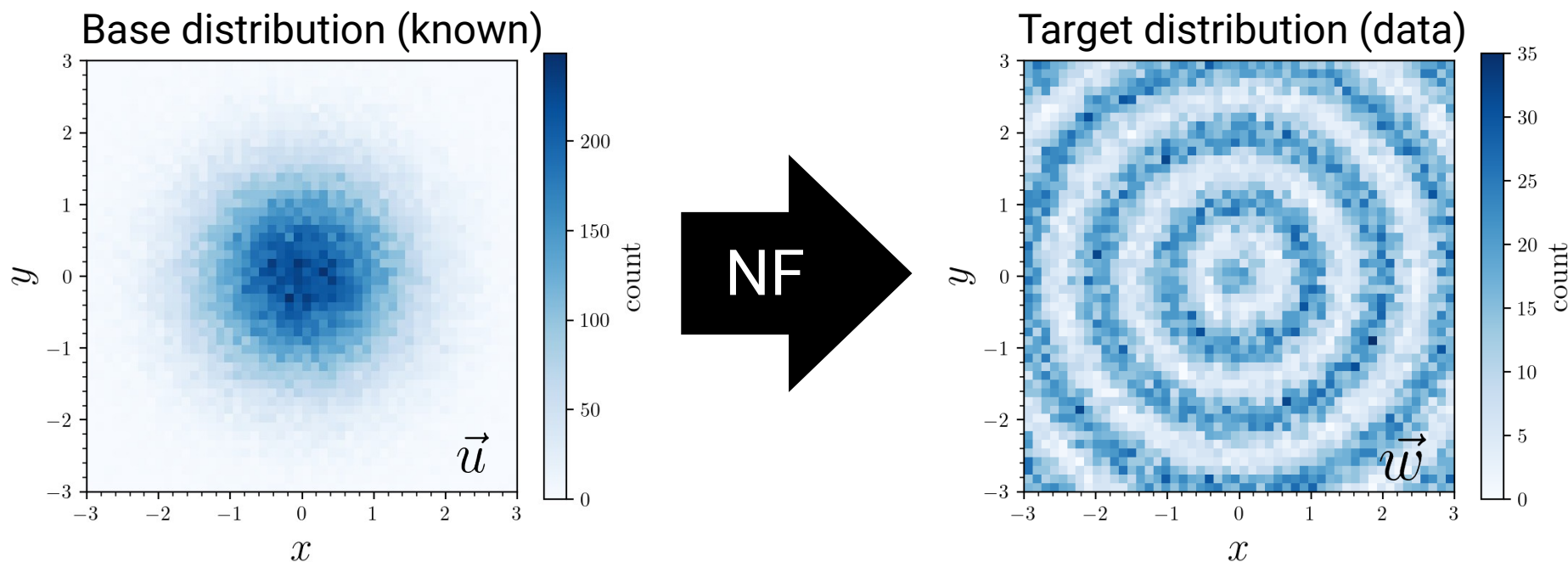
Note: velocity anisotropy cannot be determined only using line-of-sight velocity distribution, we will provide the function (can be true or not) by hand.

Need to estimate 2 functions from data: $n(r)$ $\overline{v_r^2}(r)$

Normalizing Flows: Neural Density Estimator



Normalizing Flows (NFs) is an artificial neural network that learns a transformation of random variables.



Main idea: if we could find out such transformation, we can use the transformation formula for the density estimation:

$$p_W(\vec{w}) = p_U(\vec{u}) \cdot \left| \frac{d\vec{u}}{d\vec{w}} \right|$$

We will use this model for estimating the phase space density $f(x,v)$ from the data.

Equivariant Continuous Normalizing Flows

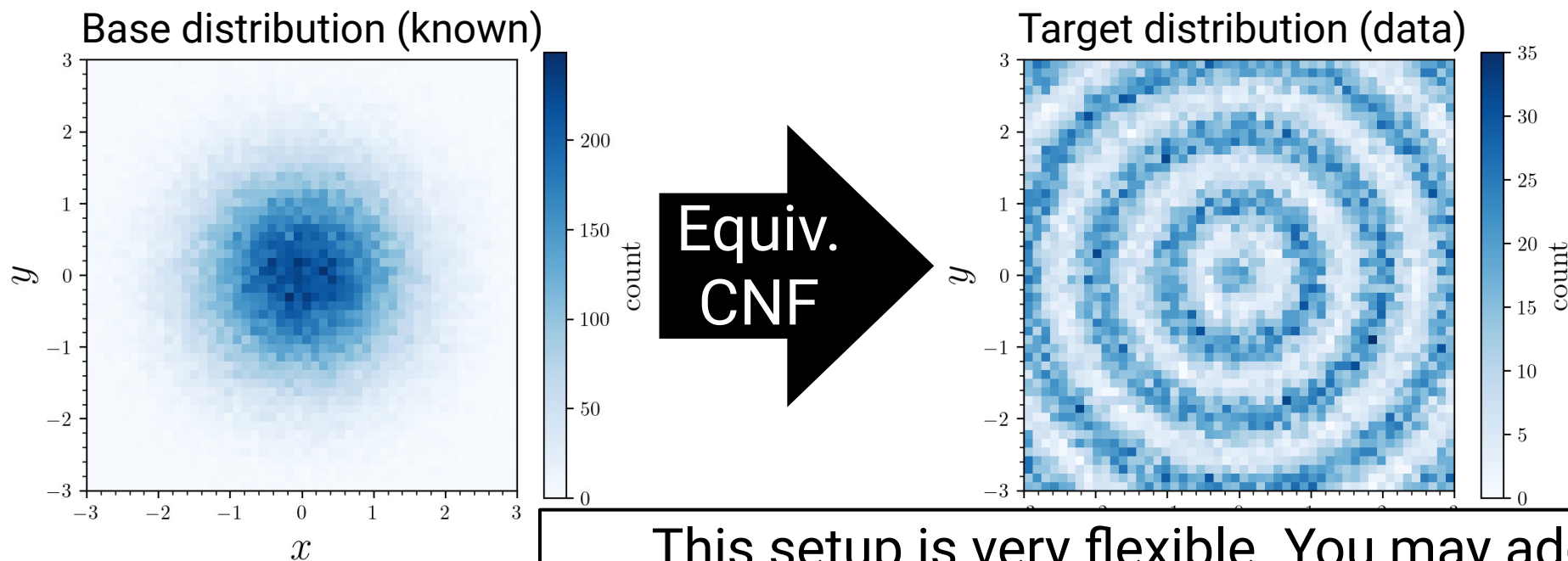
$$n(r)$$

How to model spherically symmetric density using normalizing flows?

→ Use Equivariant Continuous Normalizing Flows!

$$\frac{d\vec{x}}{dt} = \vec{F}(\vec{x}, t) \longrightarrow \frac{d\vec{x}}{dt} = \hat{r} f(\vec{x}, t)$$

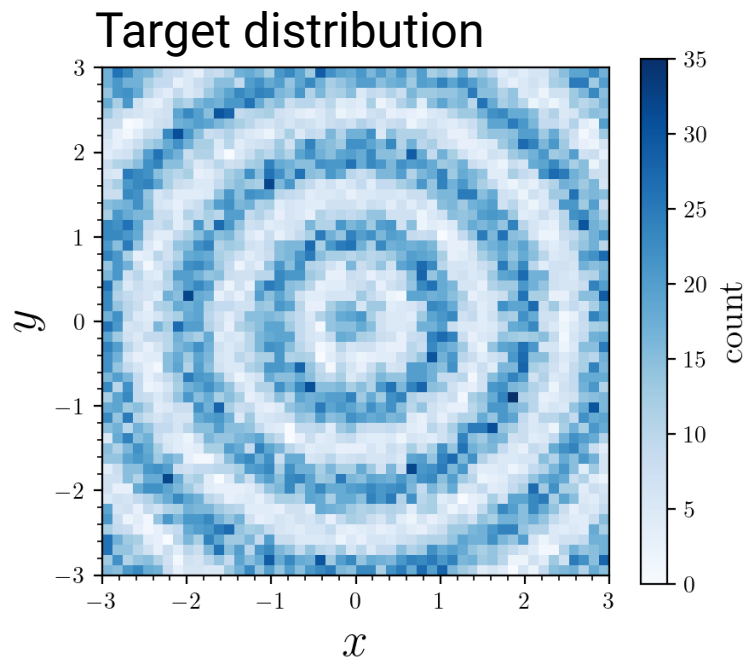
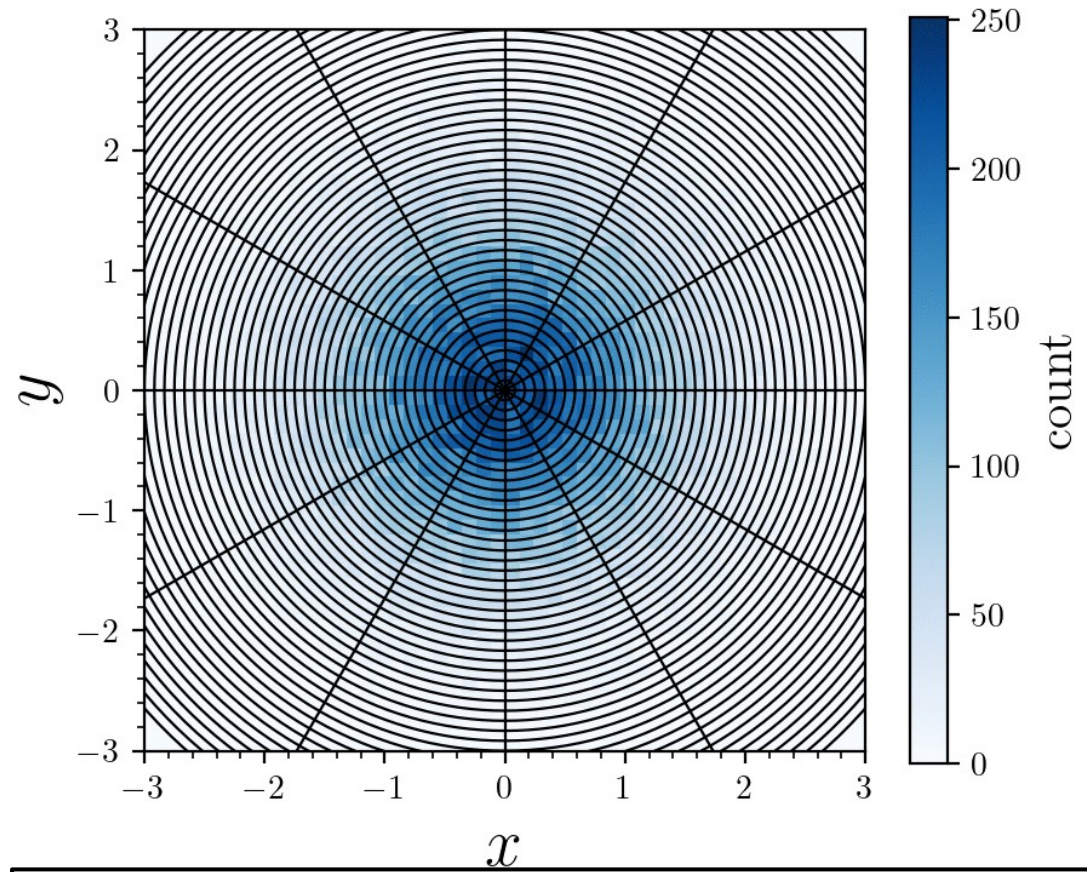
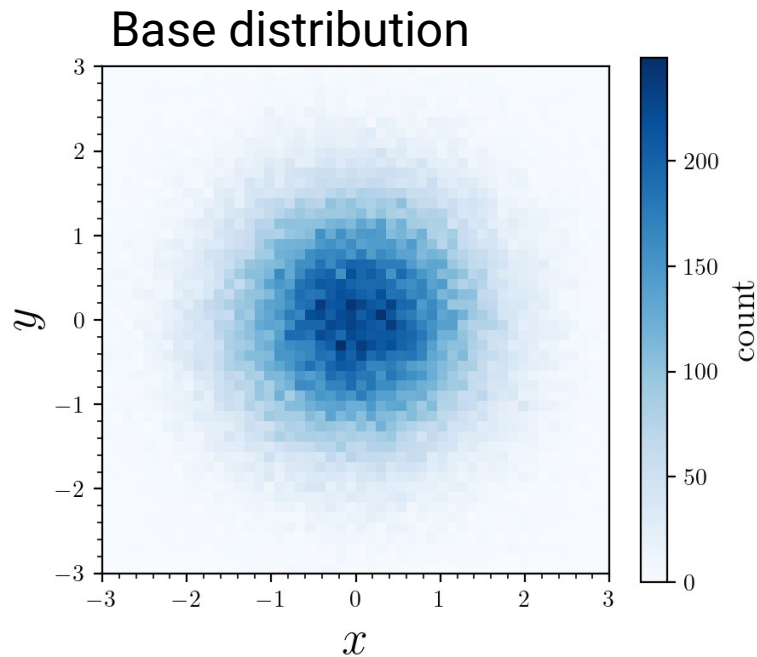
- Invariant (Gaussian) base distribution
- Equivariant vector field



This setup is very flexible. You may add **physics constraints** to **neural networks**, too!

Normalizing Flows: How it works?

$$n(r)$$



Normalizing flows can fit
arbitrary probability density,
suitable for **model-independent** analysis!

Cored Spherical Density Model

In dSph analysis, we may further constrain the density model as conventional analysis often only consider the following type of densities.

- **Cored** density (constant density at $r \ll 0$)
- **Cuspy** density

ex) plummer sphere:

$$p(r) = \left(1 + \frac{r^2}{r_0^2}\right)^{-5/2}$$

Equivariant CNF for modeling
cored density profile

$$\frac{d\vec{x}}{dt} = \hat{r} f(\vec{x}, t) \longrightarrow \frac{d\vec{x}}{dt} = \hat{r} \tanh\left(\frac{|\vec{x}|}{r_0}\right) f(\vec{x}, t)$$

Transformation at the origin is suppressed, remaining as Gaussian-shape. \rightarrow cored density

Cuspy Spherical Density Model

In dSph analysis, we may further constrain the density model as conventional analysis often only consider the following type of densities.

- **Cored** density (constant density at $r \ll 1$)
- **Cuspy** density

Equivariant CNF for modeling
cuspy density profile

ex) NFW profile:

$$p(r) = \left(\frac{r}{r_0}\right)^{-1} \left(1 + \frac{r}{r_0}\right)^{-2} \rightarrow \frac{1}{r}$$

Apply power-law transform to radial component

$$|r| \rightarrow |r|^{c+1} \quad \text{Jacobian} \propto r^{-\frac{3c}{1+c}}$$

to **cored** spherical symmetric density model

Velocity Dispersion Estimation

The velocity dispersion can be simply estimated using Gaussian model conditioned on position, as the MLE on variance parameter of Gaussian is a variance estimator.

$$\Sigma(r; \theta) = \begin{pmatrix} \overline{v_r^2}(r; \theta) & 0 & 0 \\ 0 & \overline{v_\theta^2}(r; \theta) & 0 \\ 0 & 0 & \overline{v_\phi^2}(r; \theta) \end{pmatrix}$$

Note that only radial velocity dispersion is modeled by a neural network, others are given by velocity anisotropy function provided.

$$\overline{v_\theta^2}(r; \theta) = \overline{v_\phi^2}(r; \theta) = \overline{v_r^2}(r; \theta) \cdot (1 - \beta(r))$$

Here is a 6D density model, but...

Now we have a full 6D phase-space density model ready for solving spherical Jeans equation.

$p(\vec{r}) = n(r; \theta)$ modeled by equivariant CNF for cuspy halos

$p(\vec{v}|\vec{r}) = \text{GaussPDF}(\vec{v}; \mu = 0, \Sigma(r; \theta))$

$f(\vec{r}, \vec{v}) = p(\vec{r}) \times p(\vec{v}|\vec{r})$



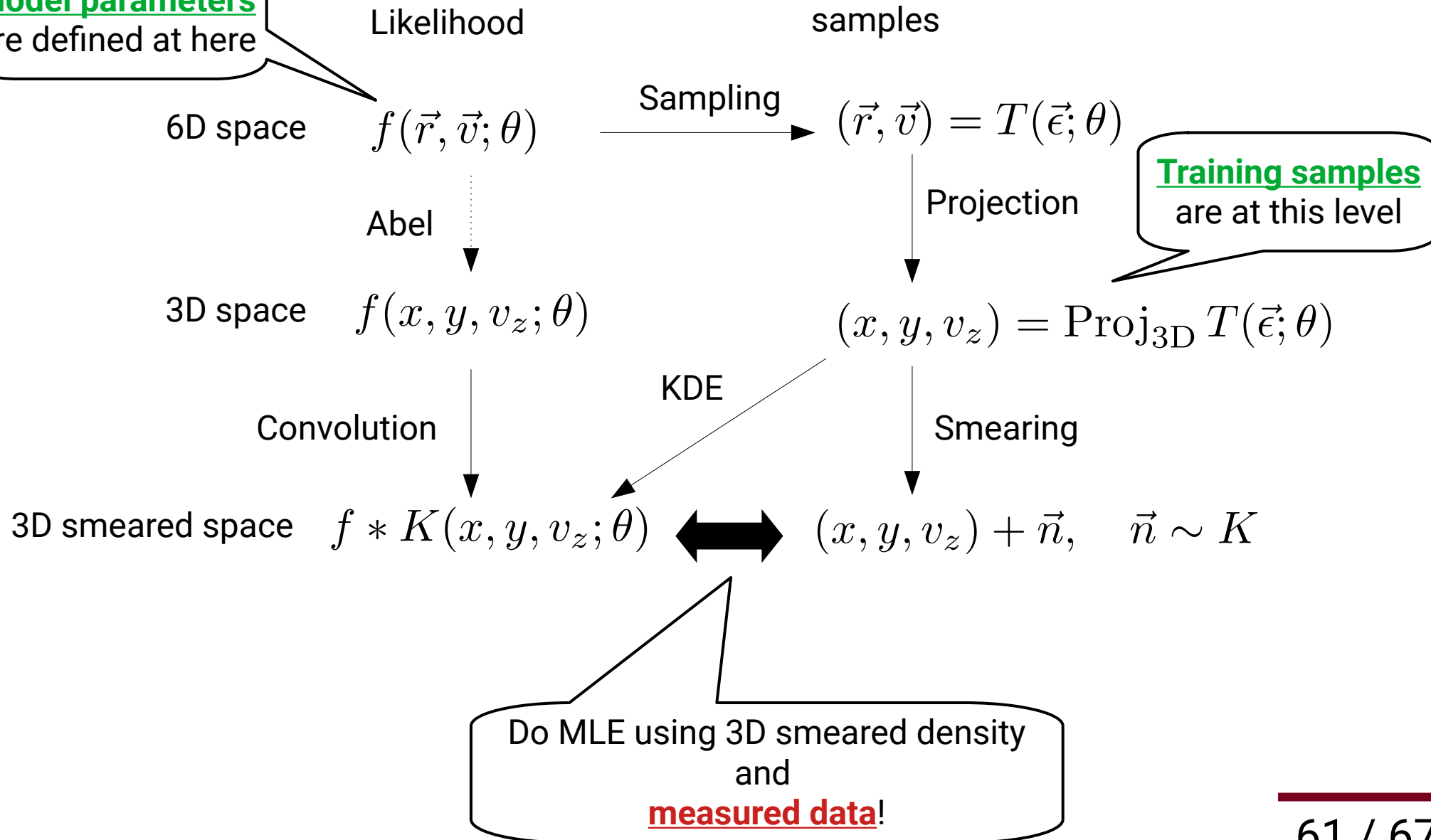
Wait, we only have x, y, v_z .

How can we train this network by MLE?

We cannot use a conventional loss function.

How to train this model?

Model parameters
are defined at here



Loss Function for Modeling Dwarf Spheroidal Galaxy

- In order to train the normalizing flow with spherical symmetry using limited kinematic information, we minimize the following entropy:

$$\mathcal{L}(\theta) = \int d\vec{w}_{\perp} p * K_h(\vec{w}_{\perp}) \log \hat{p} * K_h(\vec{w}_{\perp}; \theta)$$

- Importance sampling: N_T training sample (stars) $\sim p$, N_K noise samples $\sim K_h$

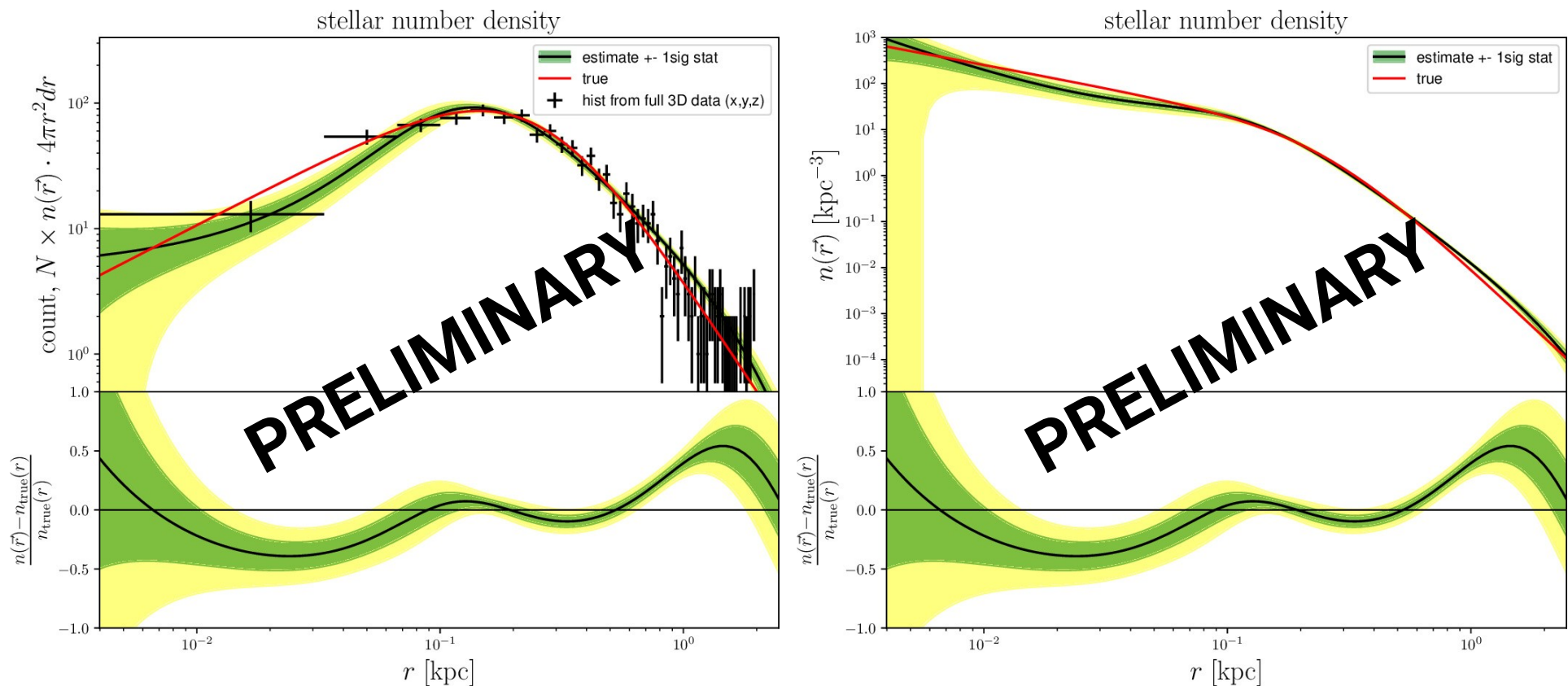
$$\mathcal{L}(\theta) = \frac{1}{NN_K} \sum_{a=1}^N \sum_{b=1}^{N_K} \log \hat{p} * K_h(\vec{w}_{\perp}^{(a)} + \vec{\epsilon}^{(b)}; \theta)$$

- KDE for the smeared likelihood model:
 N_G generated stars from the normalizing flows $\sim \hat{p}$

$$\mathcal{L}(\theta) = \frac{1}{NN_K} \sum_{a=1}^N \sum_{b=1}^{N_K} \log \frac{1}{N_G} \sum_{c=1}^{N_G} K_h \left[\vec{w}_{\perp}^{(a)} + \vec{\epsilon}^{(b)} - \vec{T}(\vec{z}^{(c)}; \theta) \right]$$

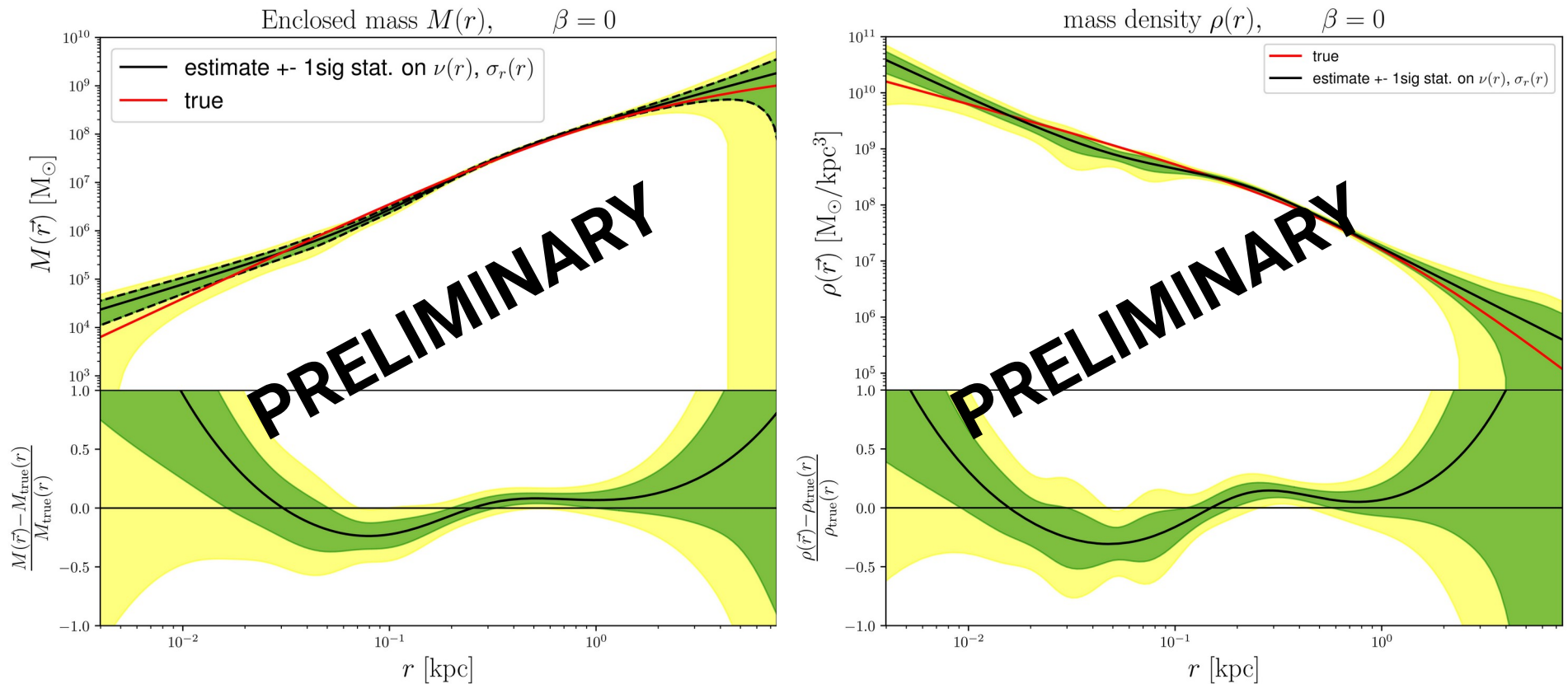
Results: stellar number density

Here we present inferred stellar number density trained on 2D position information (x, y).



Results: dark matter mass density

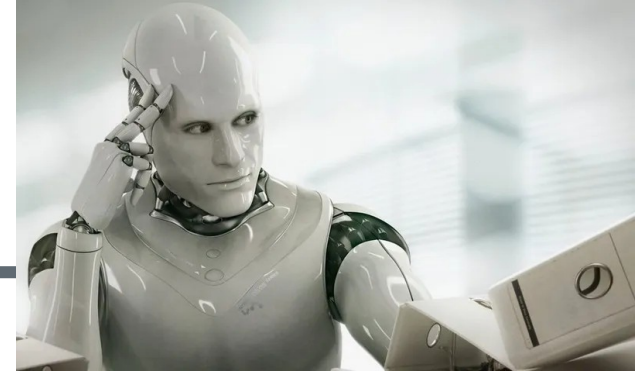
Here we present inferred mass density calculated from stellar density and velocity dispersion trained on 3D information (x, y, vz).



Conclusions

- We introduce a model-independent and unbinned spherical Jeans analysis using **normalizing flows**, a neural density estimator utilizing transformation of random variables.
- We **invented a loss function** for training normalizing flows modeling dSphs only using projected information, without performing Abel transformation.
- Using a mock spherical galaxy from Gaia Challenge dataset, we demonstrated that normalizing flows are capable of estimating **phase-space density** information for required solving Jeans equation.
- To do?:
 - Generalizing the framework to axisymmetric system.
 - Applying our analysis to real dwarf spheroidal galaxies, and estimate the effect to J-factors when the assumptions are relaxed.

Future Timeline



2023

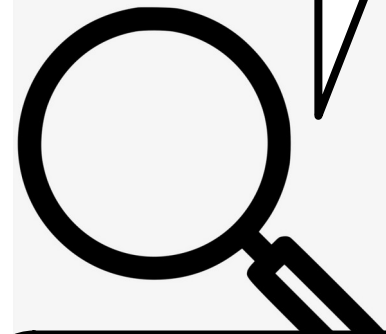
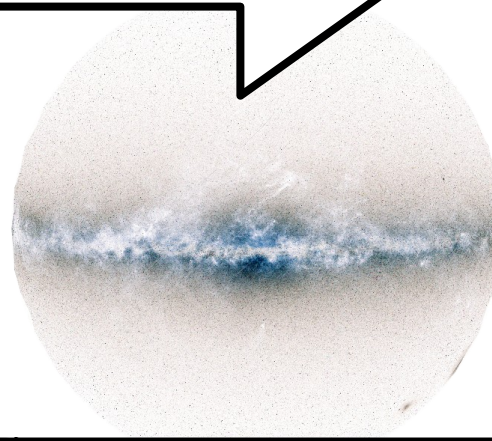
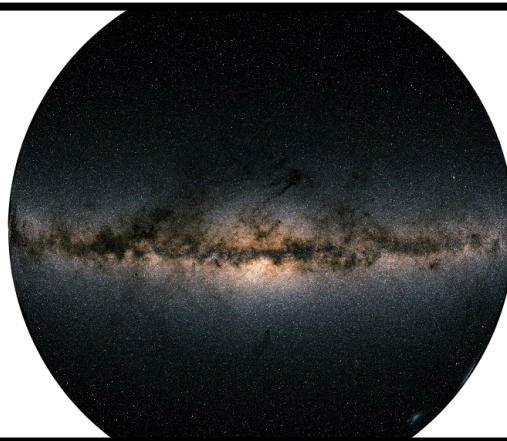
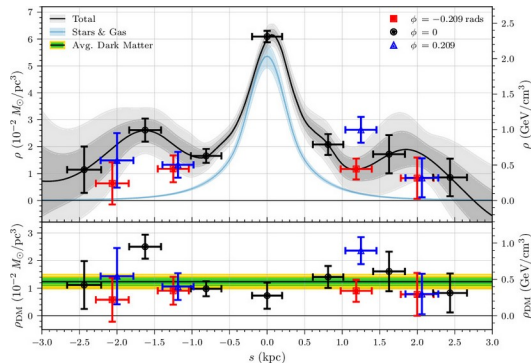
2024

2025

2026

Gaia DR3

Gaia DR4,
SuperPFS,
and so on!



Understanding
DM in dust-free
region (halo, solar
neighborhood)

Understanding
DM in dusty
region (disk)

Publishing
Full DM map
of the Milky Way

More precision
and new
opportunities!



**AI WANTS YOU
TO CONTRIBUTE**



**Thank you
for listening!**