Toward Accurate Modeling of the Nonlinear Galaxy Bispectrum

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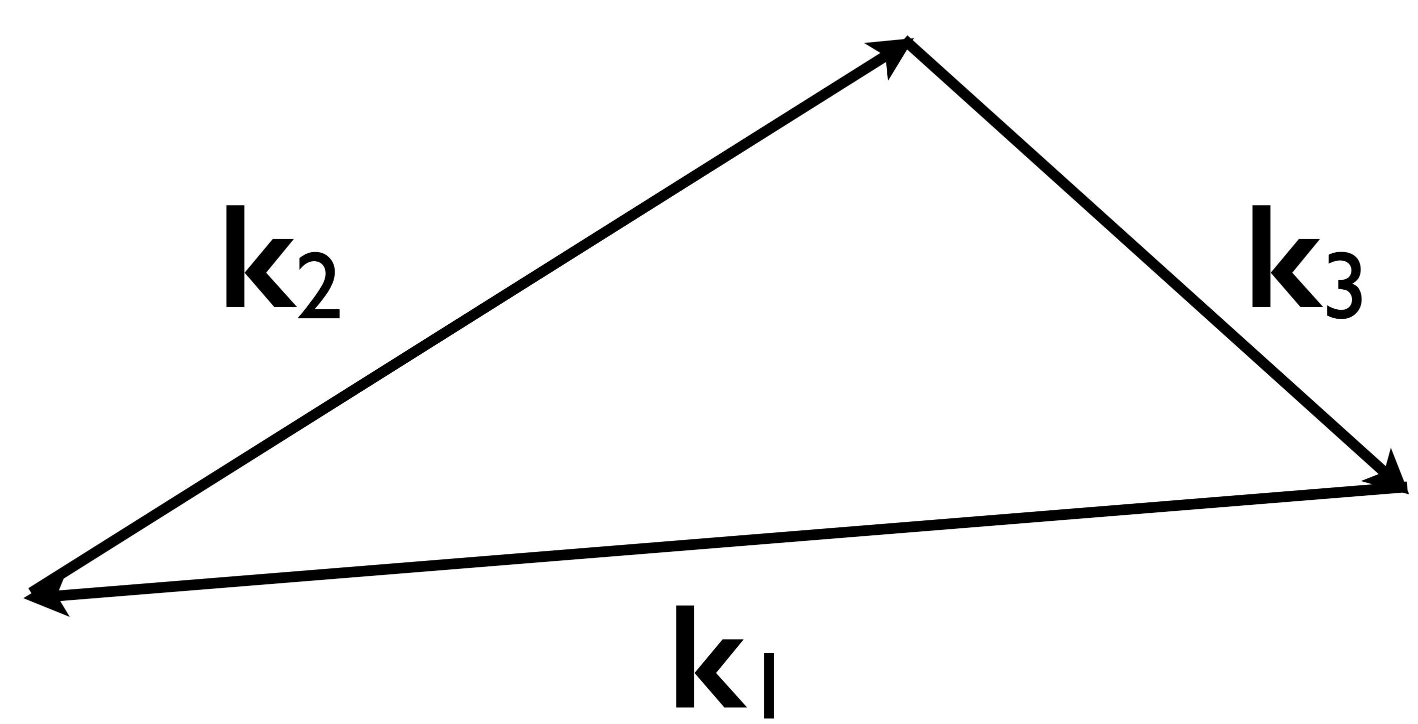
8 July 2014, Precision Era for Large Scale Structure @ IBS, Daejeon, Korea

What is the galaxy bispectrum?

- The galaxy bispectrum is the Fourier transform of the galaxy three-point correlation function:

$$\langle \delta_g(\mathbf{k}_1)\delta_g(\mathbf{k}_2)\delta_g(\mathbf{k}_3)\rangle = (2\pi)^3 B_g(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)\delta^D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$



$$\delta_g(\mathbf{k}) = \int \mathrm{d}^3\mathbf{x} \left(\frac{n(\mathbf{x}) - \bar{n}}{\bar{n}}\right) e^{-i\mathbf{k}\cdot\mathbf{x}}$$

density contrast

Why studying bispectrum?

- The galaxy bispectrum is a treasure island yet to be explored!
- By exploiting the galaxy bispectrum, we can learn about
  (Scoccimarro+ 1998, Scoccimarro+ 1999, Sefusatti+ 2006, Sefusatti & Komatsu 2007, Jeong & Komatsu 2009, Baldauf+ 2011, Greig+ 2013)
  - Physics of inflation: non-Gaussianity
  - Non-linear structure formation
  - Astrophysics
    - Formation, evolution, radiative transfer of galaxies

Challenges to exploit bispectrum

- For any analysis, we need : χ^2=(theory-data)^2/Covariance
  - [1] Theory (non-linearities, non-Gaussianities, bias)
  - [2] Data (window function, optimal weighting)
  - [3] Covariance matrix (6-point correlation function)
- For bispectrum, all three are ***NOT YET*** very well understood to the level that we can confidently apply them for the real analysis

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Visualizing the bispectrum

- With statistical isotropy, the bispectrum only depends on the three sides of triangles (convention $k_1 \geq k_2 \geq k_3$):

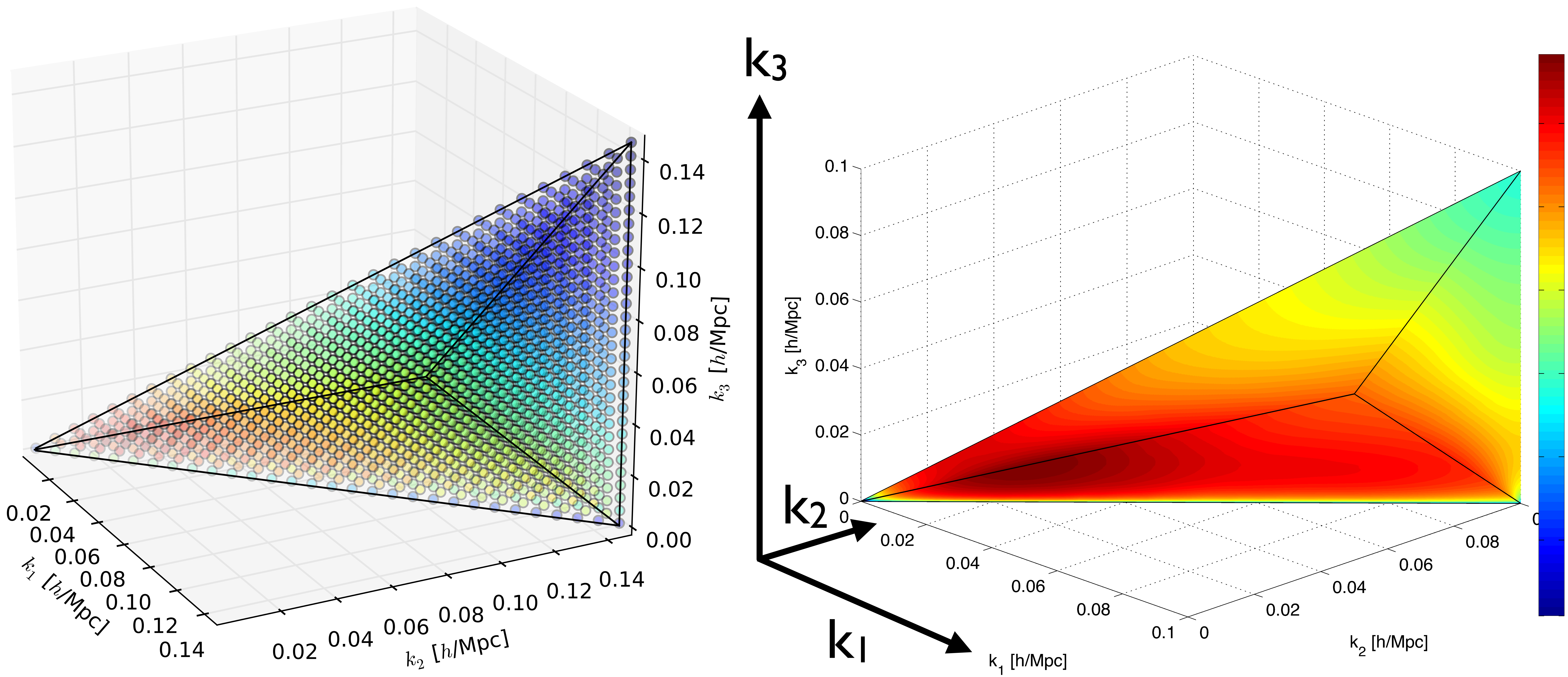
$$\langle \delta_m(\mathbf{k}_1)\delta_m(\mathbf{k}_2)\delta_m(\mathbf{k}_3)\rangle = (2\pi)^3 B_m(k_1, k_2, k_3)\delta^D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$

- Therefore, it may be natural to show bispectrum in 3D.
- Example: the leading order theory prediction

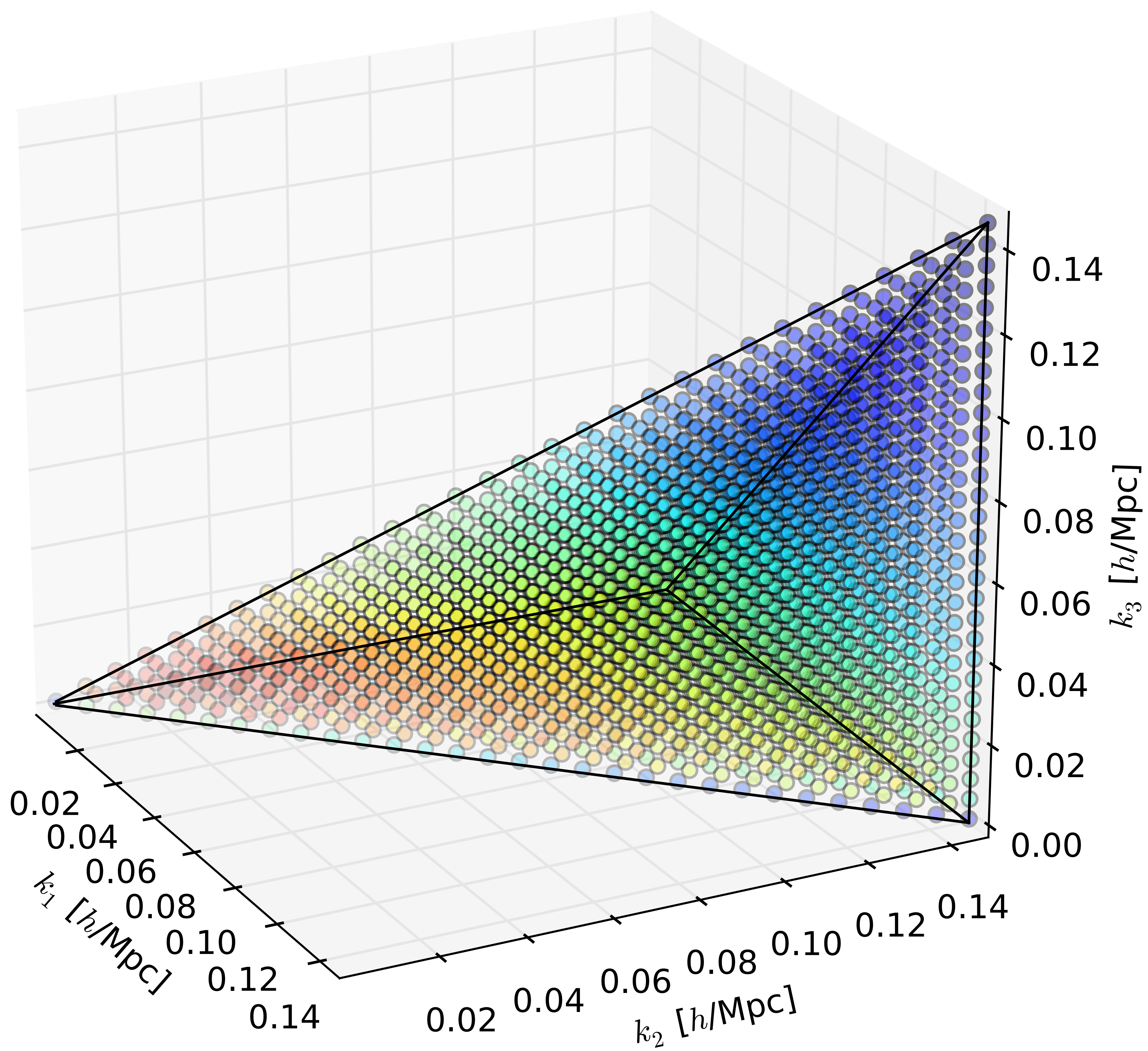
$$B_m(k_1, k_2, k_3) = \frac{3k_3^2(k_1^2+k_2^2) - 5(k_1^2-k_2^2)^2 + 2k_3^2}{14k_1^2k_2^2} P_L(k_1)P_L(k_2) + (2 \text{ cyclic})$$

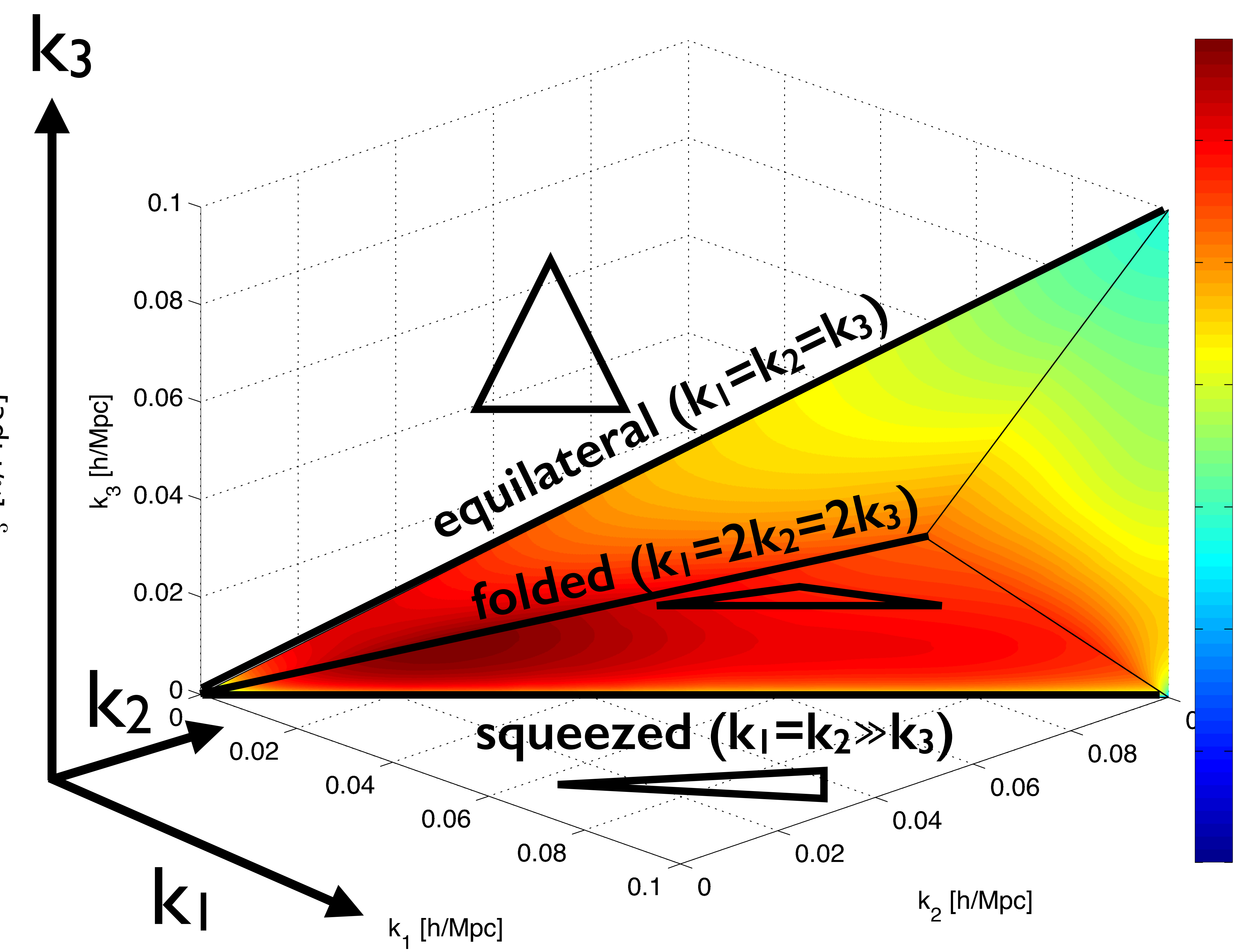
$=2F_2(\mathbf{k}_1,\mathbf{k}_2)$

Bispectrum in 3D (k_1,k_2,k_3)

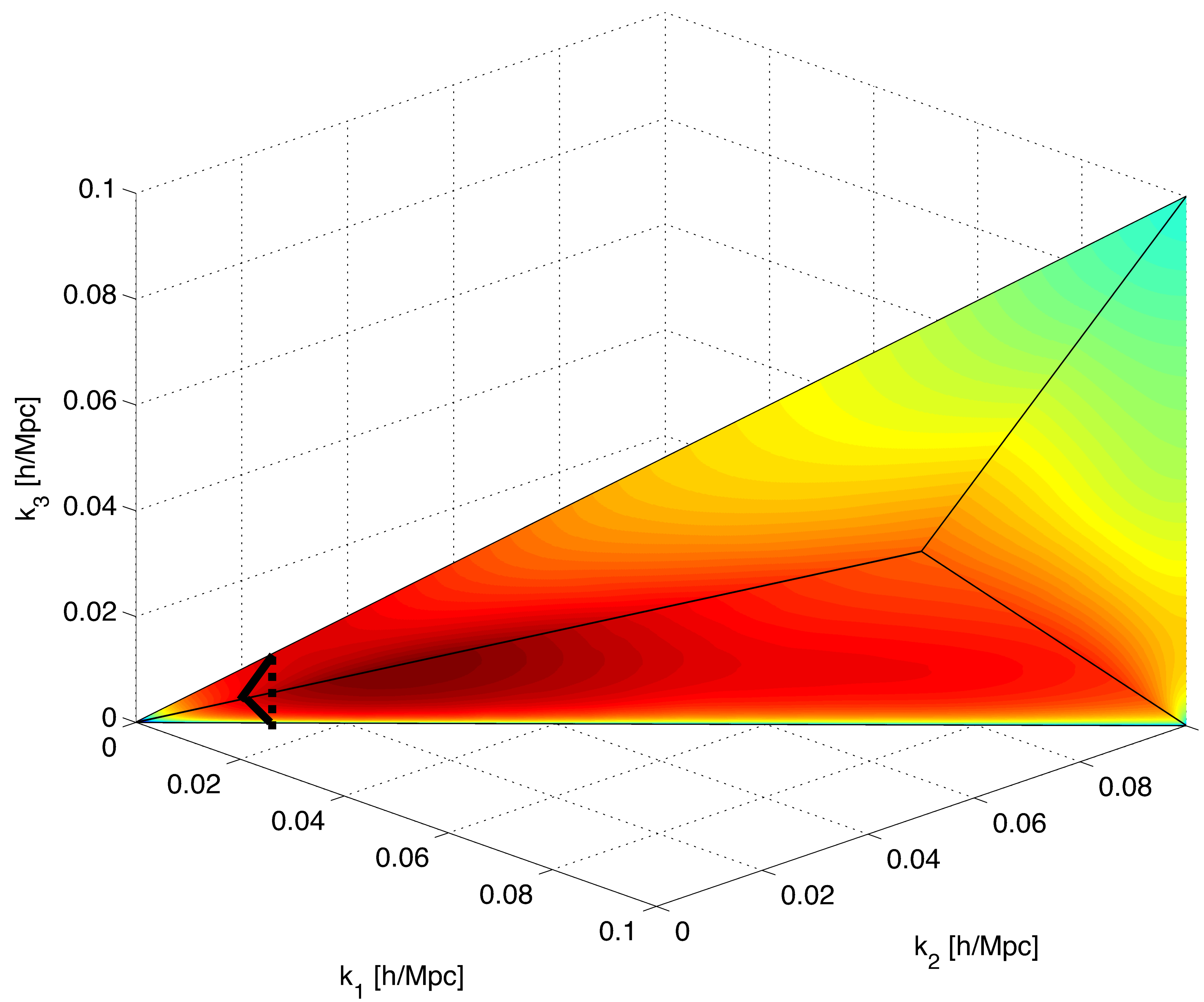


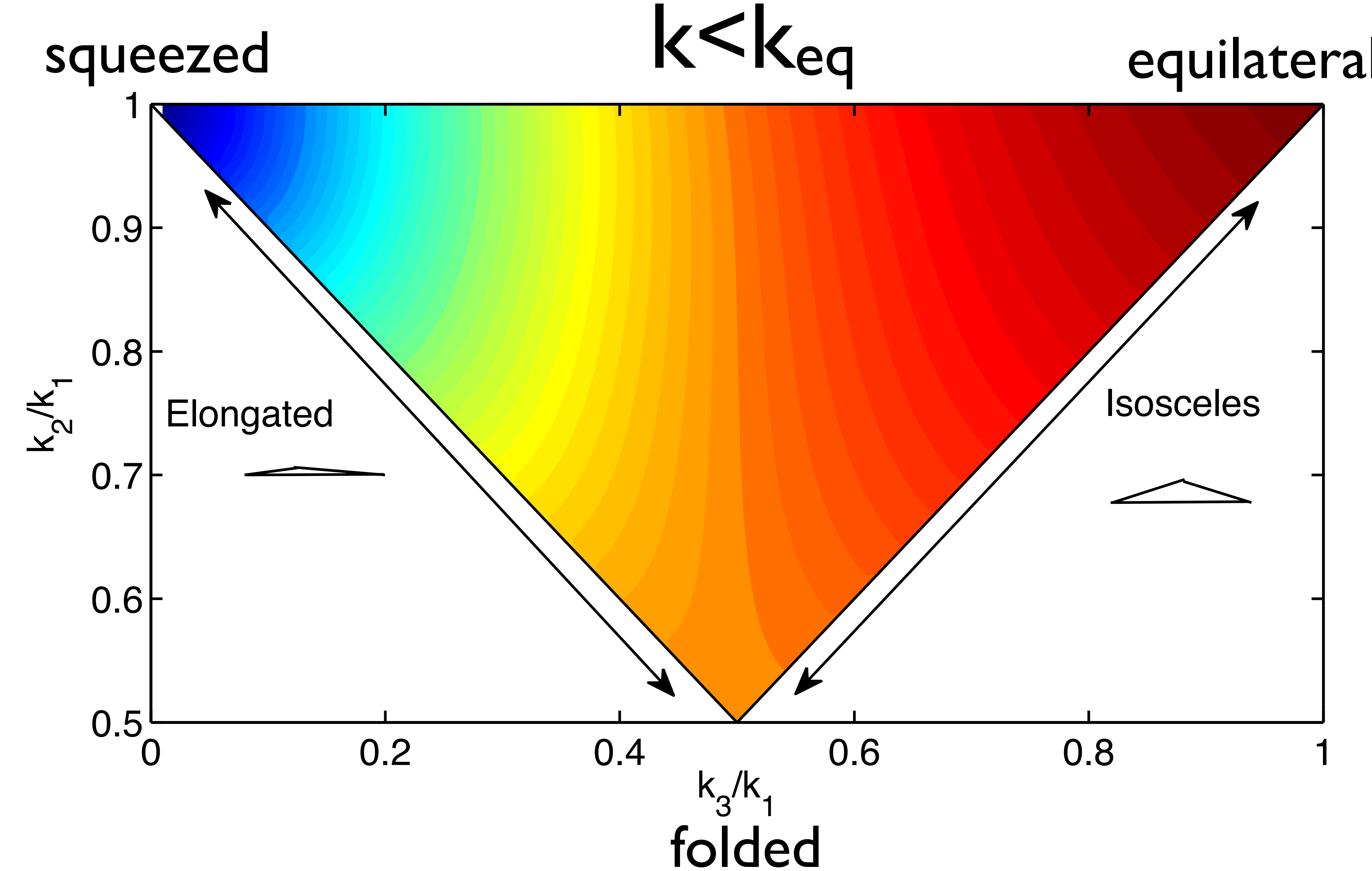
Bispectrum in 3D (k_1,k_2,k_3)





Bispectrum in 2D (k_1 fixed; k_2, k_3)

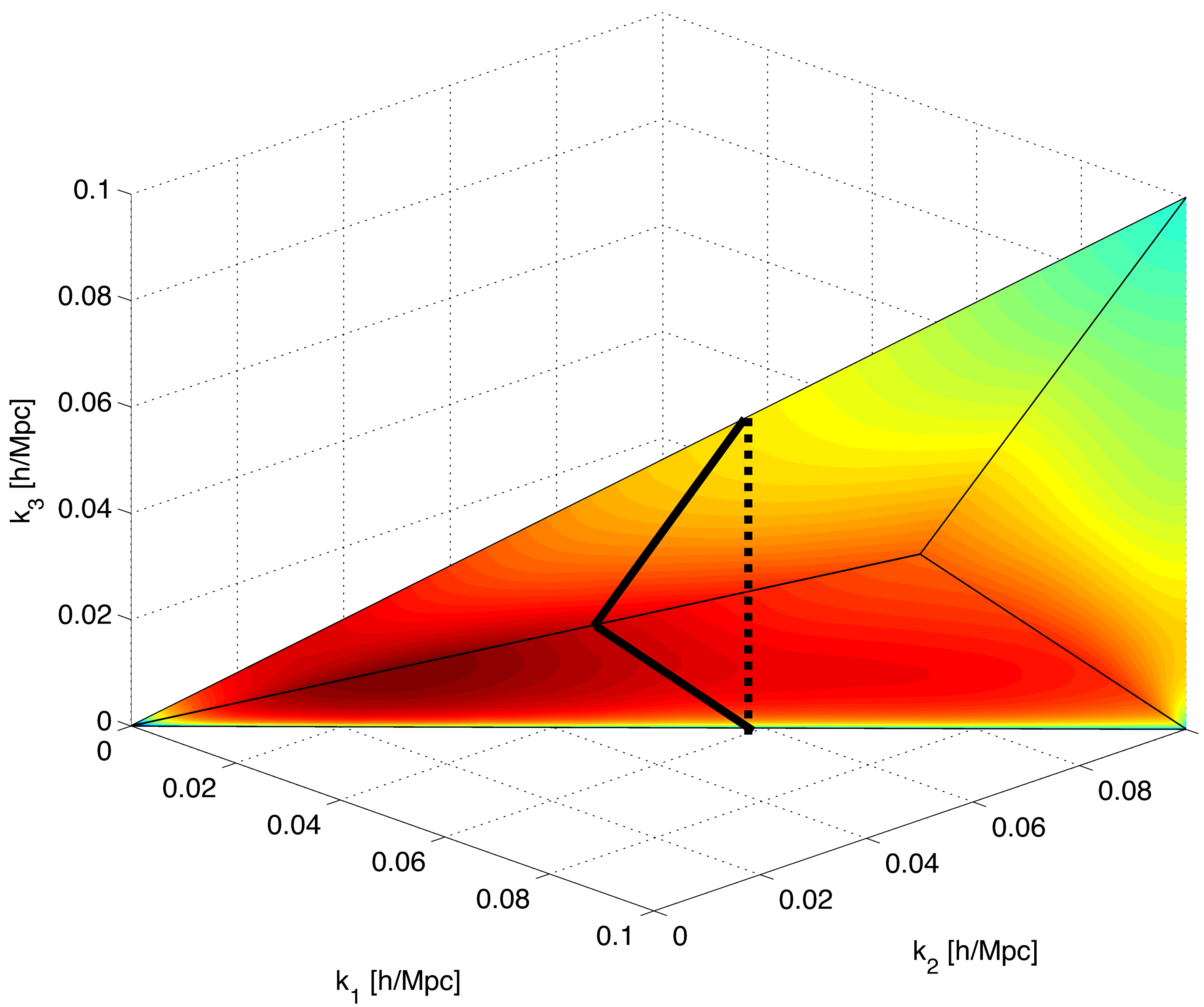




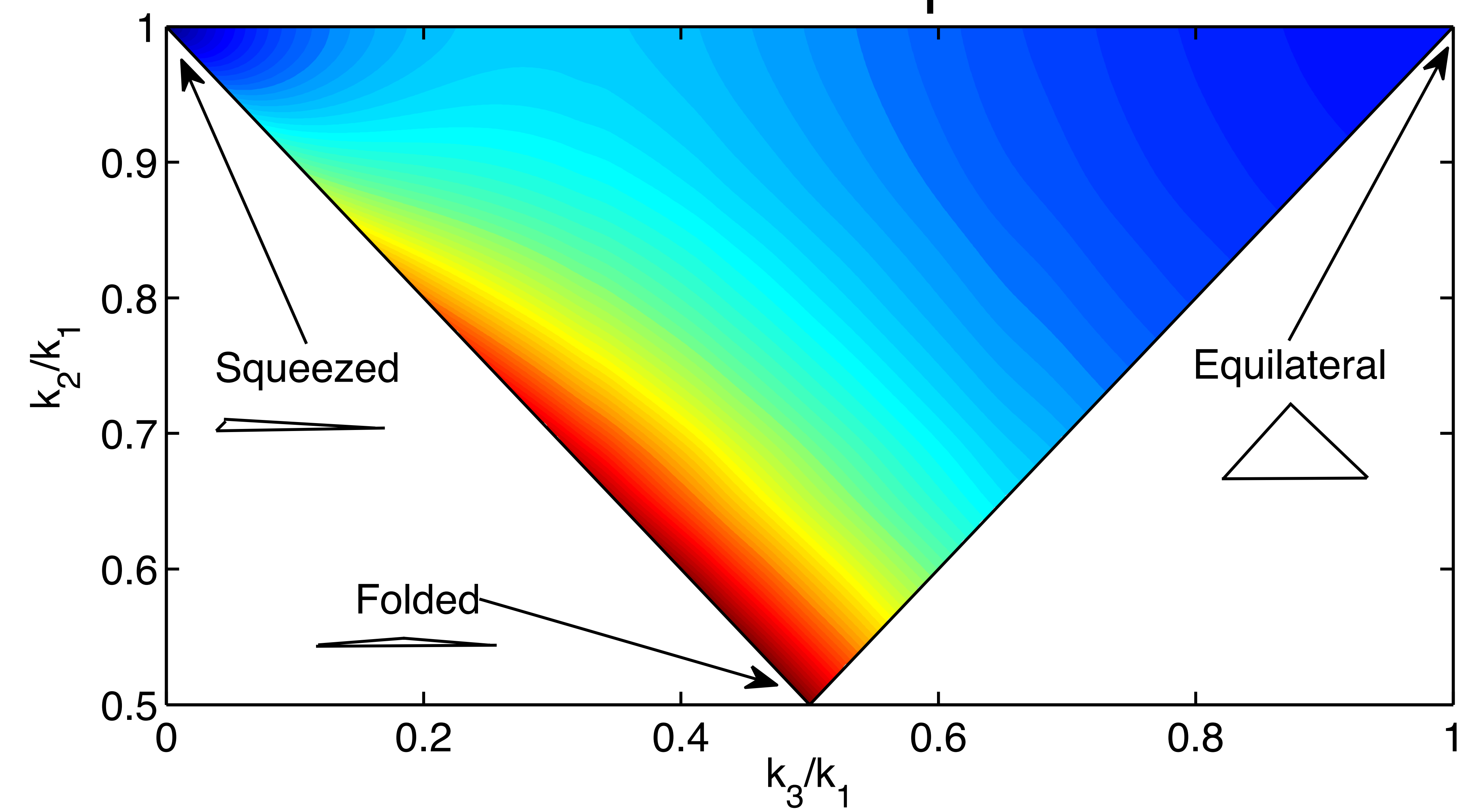
The slope of power spectrum

$d\ln P/d\ln k > 0$

Bispectrum in 2D (k_1 fixed; k_2, k_3)



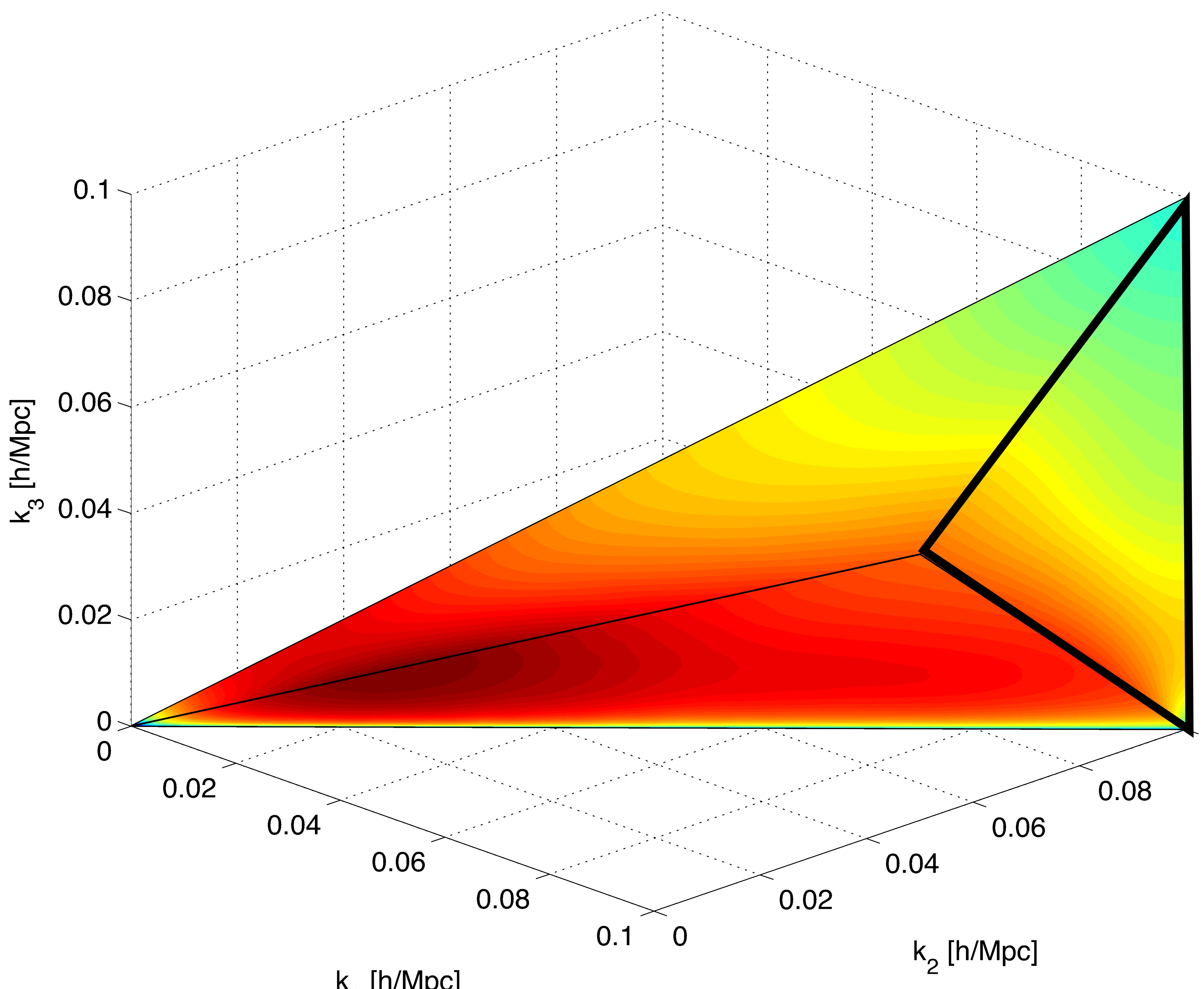
$k \gtrsim k_{eq}$

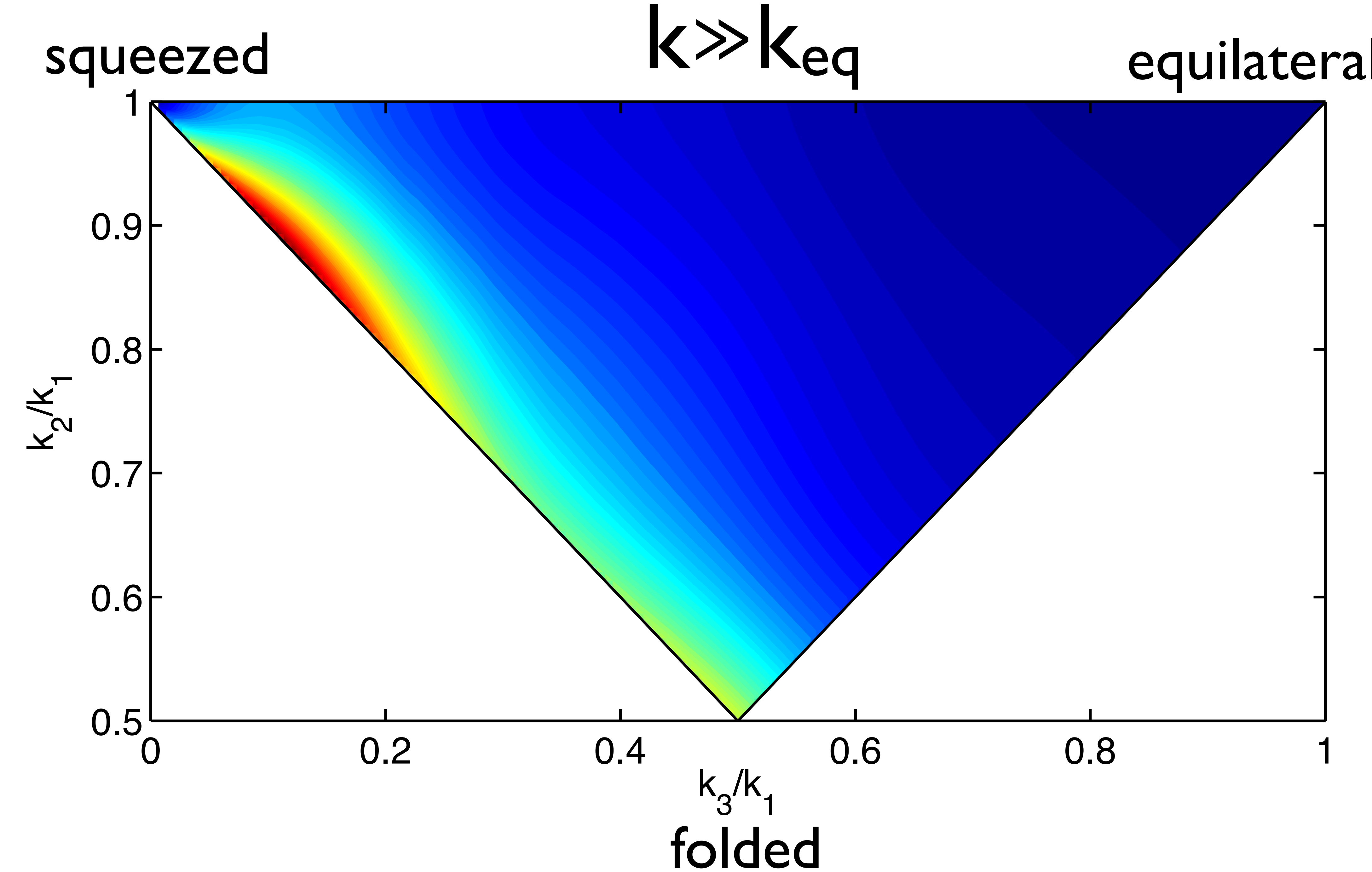


The slope of power spectrum

$d\ln P/d\ln k \lesssim 0$

Bispectrum in 2D (k_1 fixed; k_2, k_3)

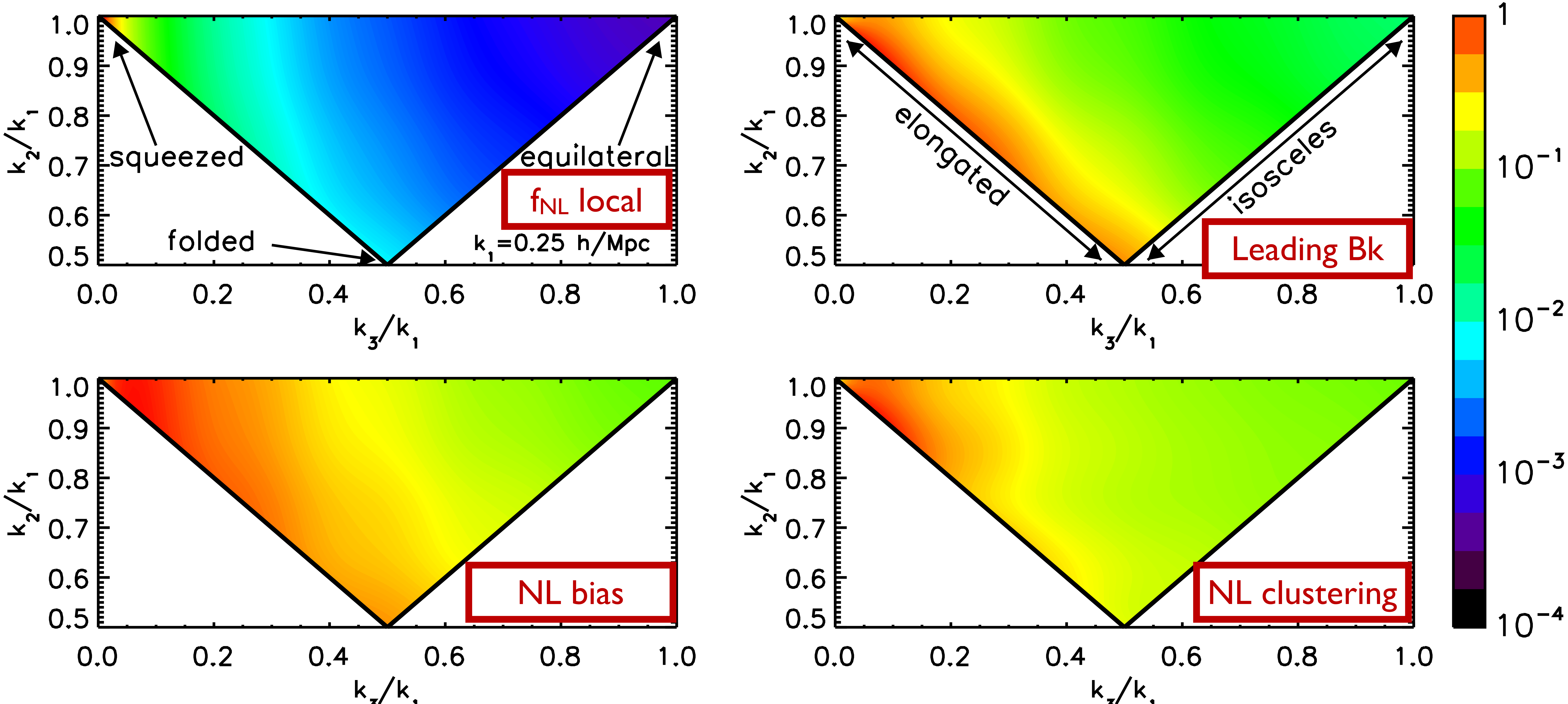




The slope of power spectrum

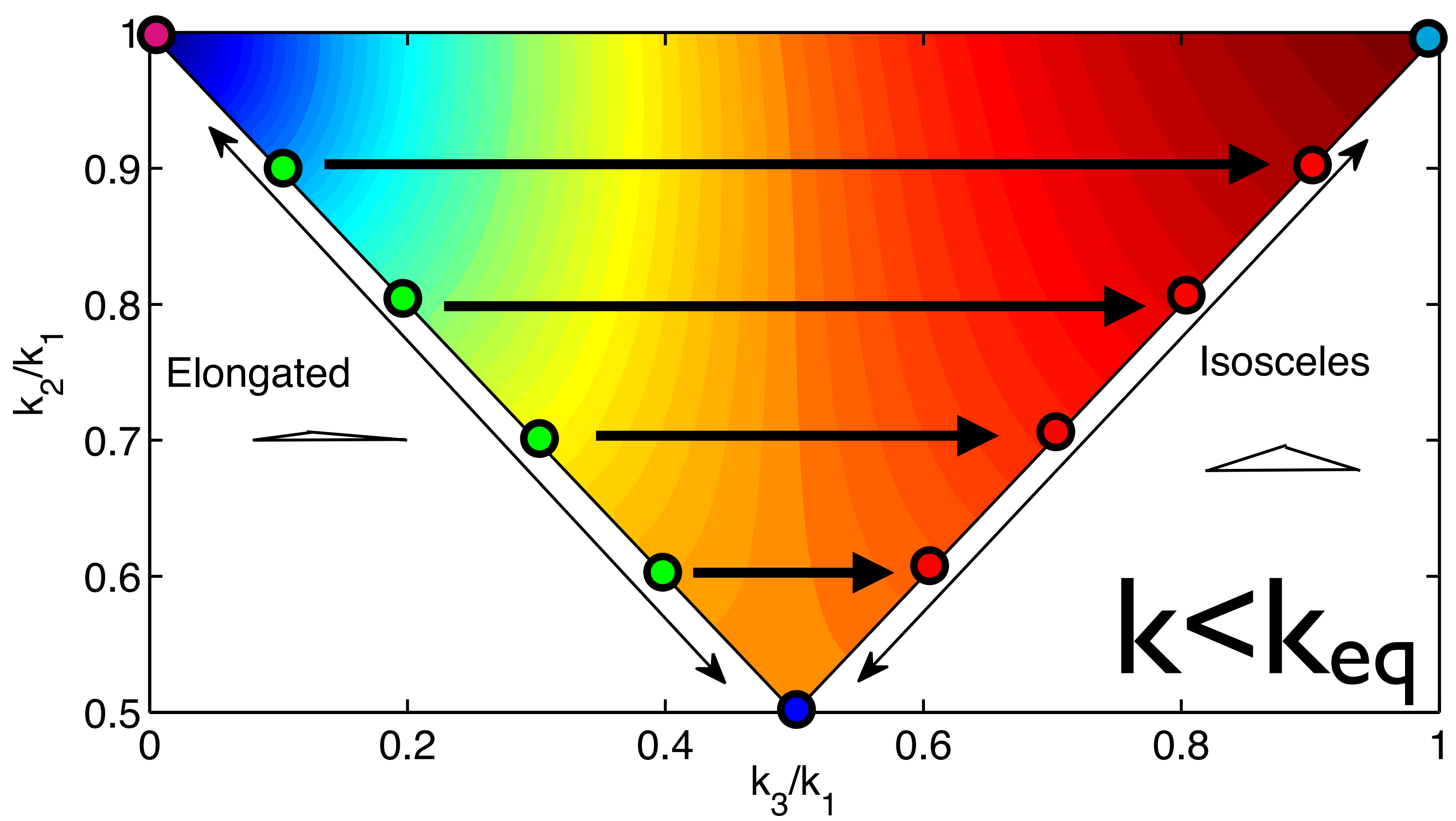
$d\ln P/d\ln k < 0$

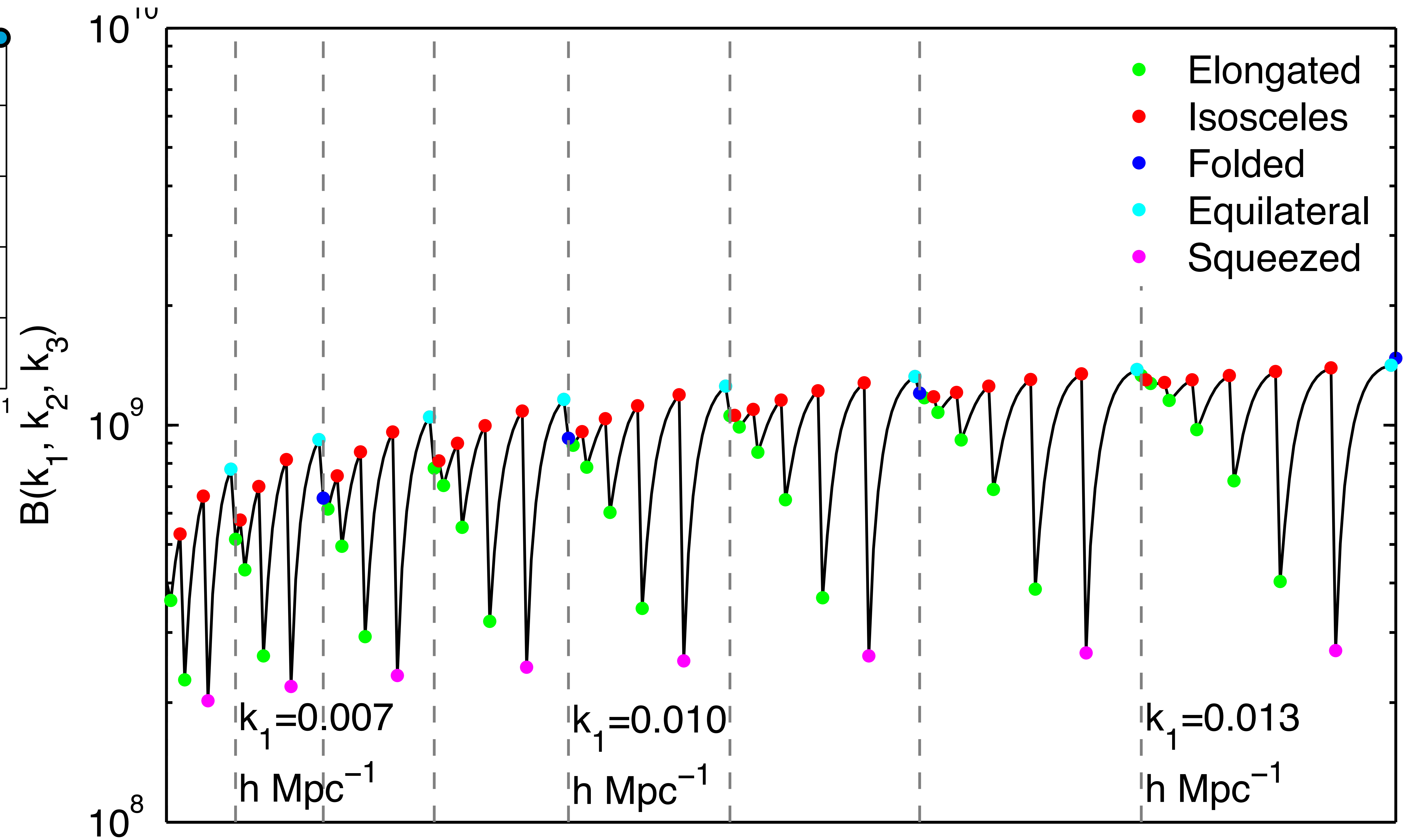
Shape of each component



Flattened Bispectrum $(k_1, k_2, k_3)[n]$

row-major, ascending order





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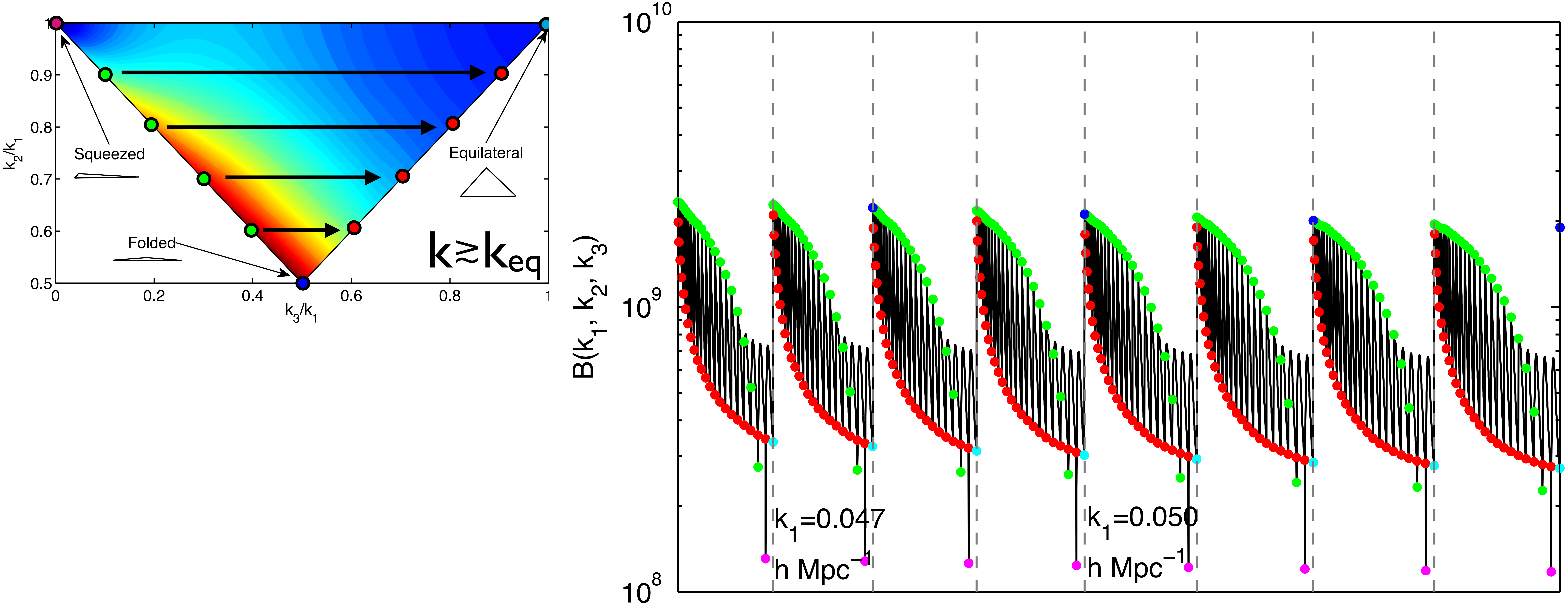

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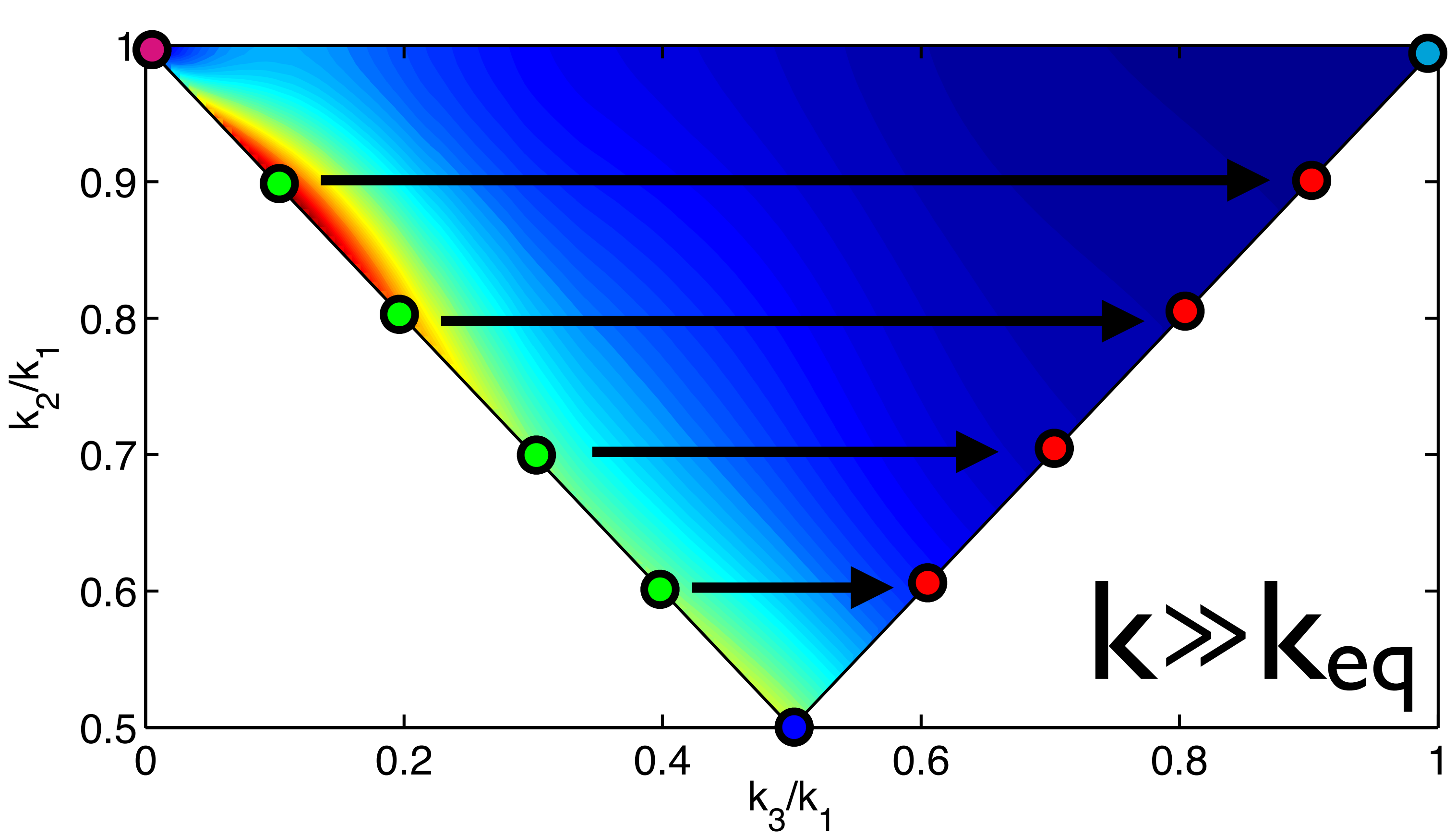
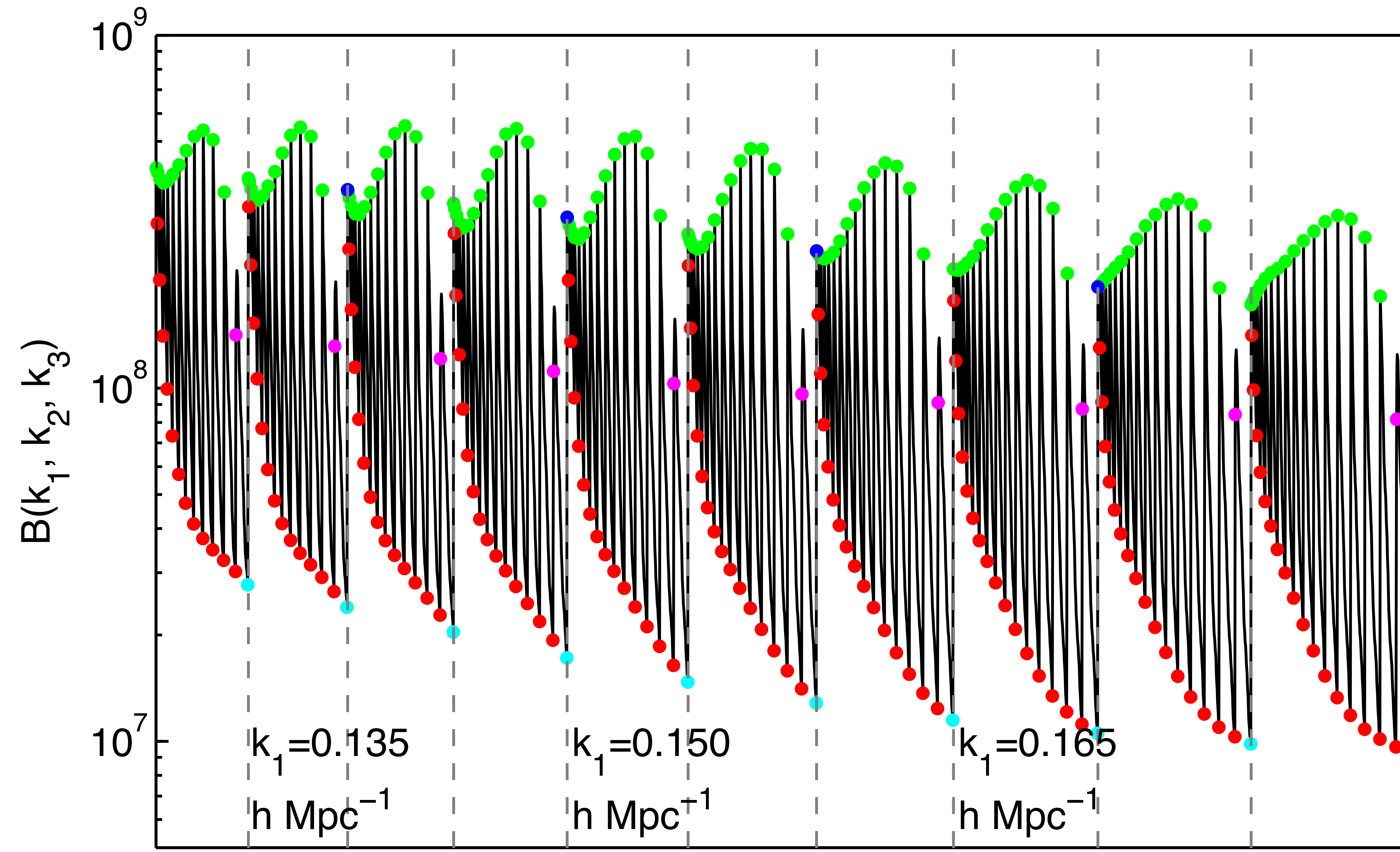




Modeling non-linearities

- On large scales:
  'tree-level' bispectrum from second order density contrast
  $B^{(0)} = <\delta_2\delta_1\delta_1>$; here, δ_n = n-th order density contrast

- Smaller scales:
  'one-loop' bispectrum $B^{(1)}=<\delta_4\delta_1\delta_1>+<\delta_3\delta_2\delta_1>+<\delta_2\delta_2\delta_2>$

- Even smaller scales (highly non-linear):
  There are empirical fitting formulas from simulations.

‘Large’, ‘Smaller,’ ‘even smaller’?

- are not a particularly accurate scientific term…
- Need N-body simulations to draw the line between scales
- Q: How reliable are N-body simulations?

Transient from initial conditions

- To correctly capture non-linearities at all subsequent redshifts, initial condition of N-body simulation must be at the fastest growing modes at all order.
- We don't know how to generate that initial condition. In fact, that is one of the reasons that we run simulations.
- Instead, we use the Lagrangian perturbation theory prescription: linear order (Zeldovich approximation) or second order (2LPT)

Scoccimarro (1998), Crocce+ (2006)

Decaying modes

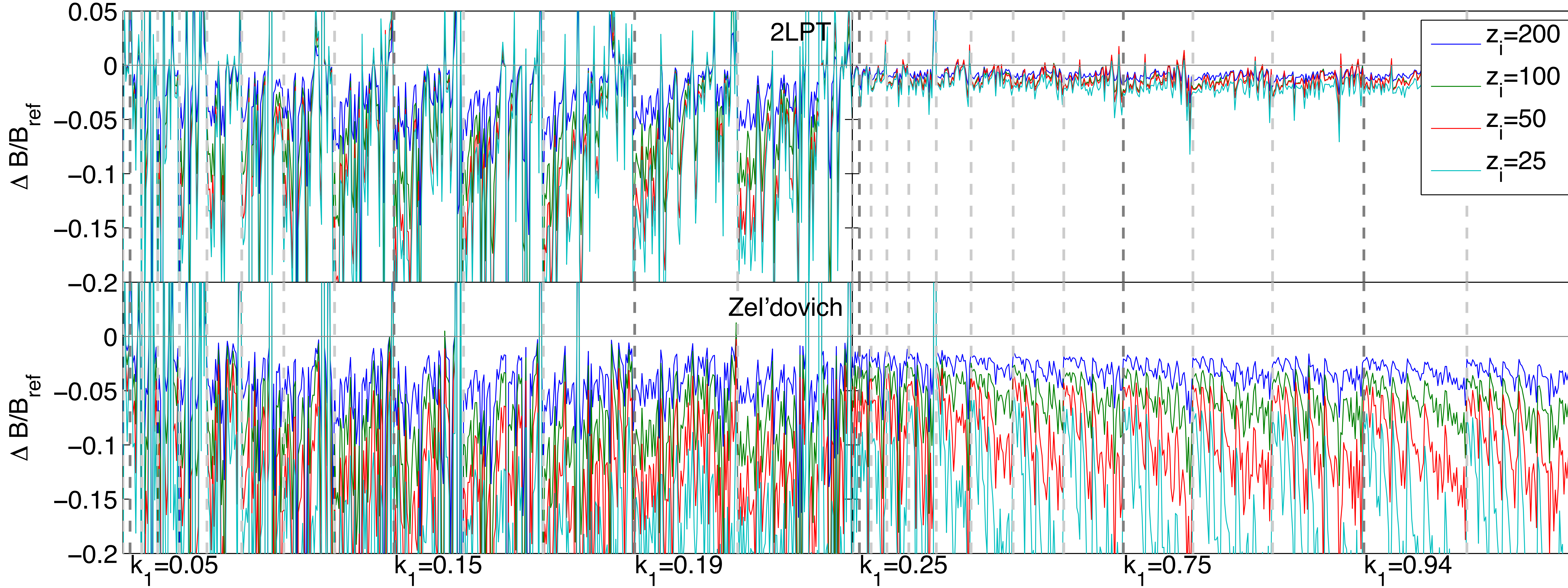
- Initial condition from Zeldovich approximation correctly captures fastest growing mode in the linear order, but generates decaying mode from the second order
- Initial condition from 2LPT approximation correctly captures fastest growing mode in the second order, but generates decaying mode from the third order
- Leading order ('tree-level') bispectrum $B^{(0)} = \langle\delta_2\delta_1\delta_1\rangle$
- How large is the effect?

McCullagh, DJ, Szalay (coming soon)

Transient at z=6

$$\frac{Bk(2LPT/Zeldovich, z_{init})}{Bk(2LPT, z_{init}=400)}$$

(1000 Mpc/h)³ (200 Mpc/h)³



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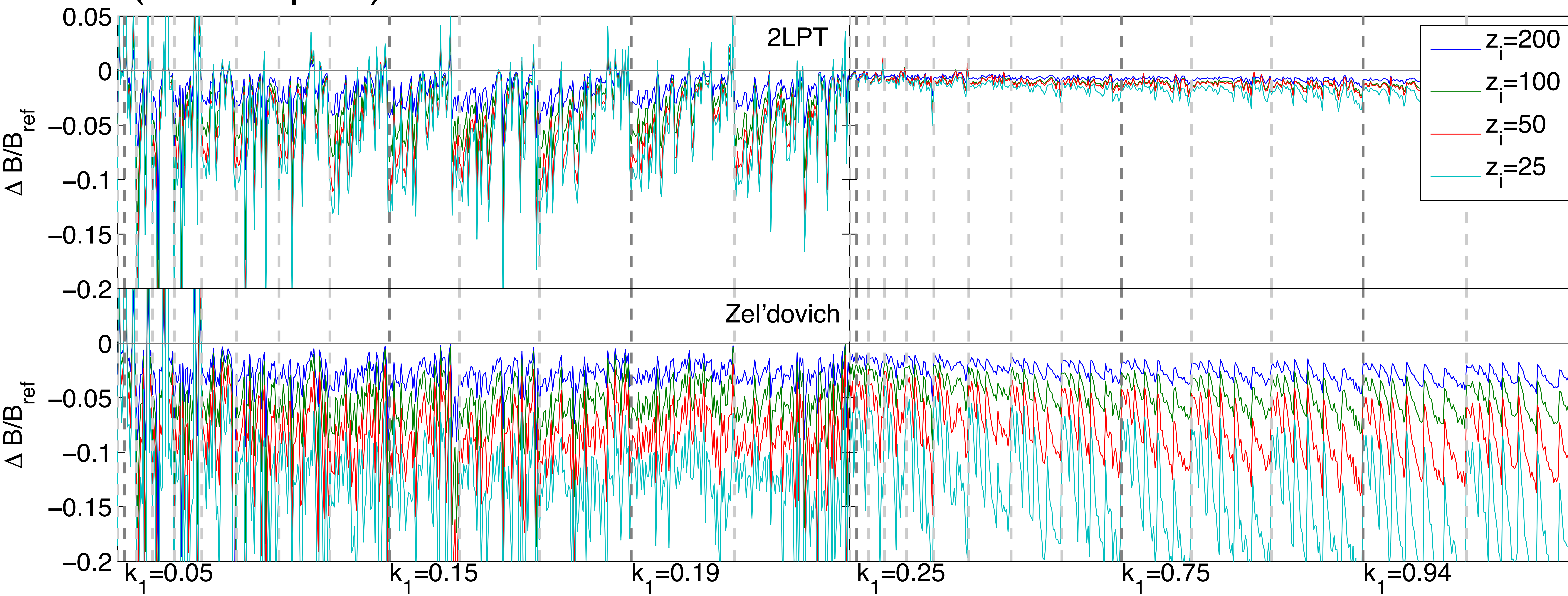
Transient at z=3

$$\frac{Bk(2LPT/Zeldovich, z_{init})}{Bk(2LPT, z_{init}=400)}$$

(1000 Mpc/h)^3 (200 Mpc/h)^3



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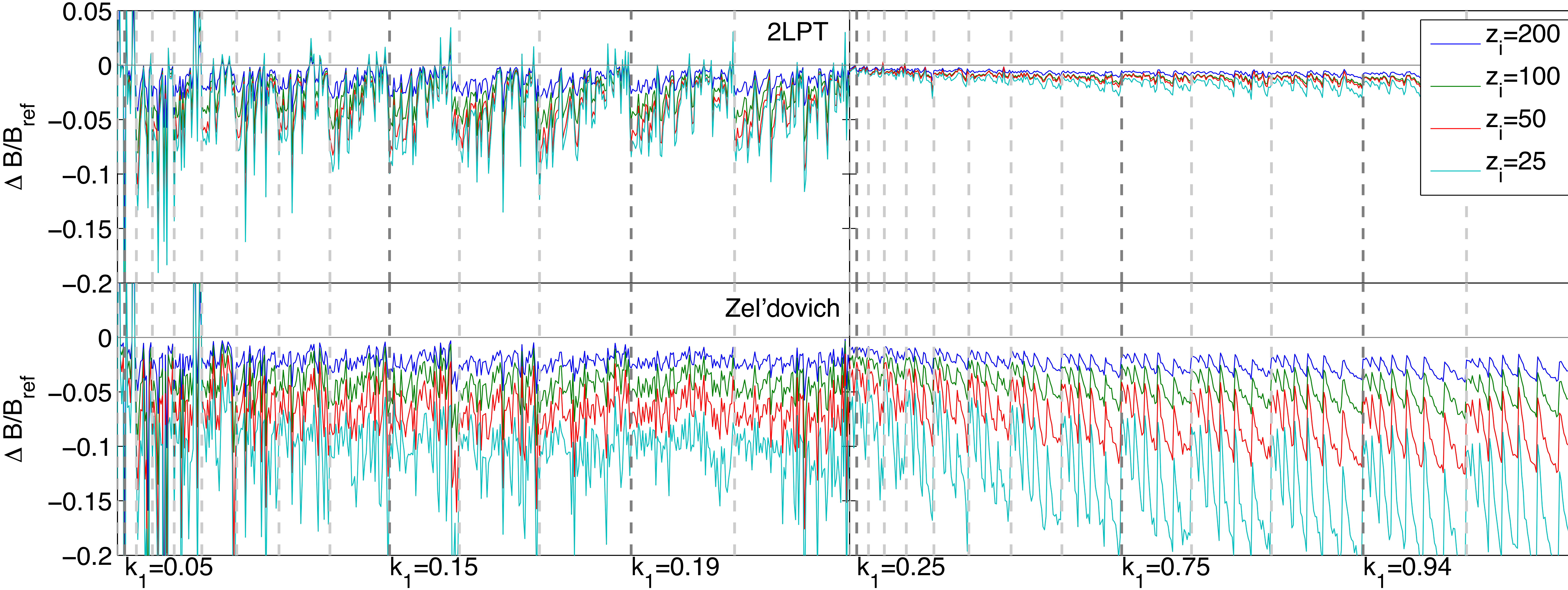
Transient at z=2

$$\frac{Bk(2LPT/Zeldovich, z_{init})}{Bk(2LPT, z_{init}=400)}$$

(1000 Mpc/h)³ (200 Mpc/h)³

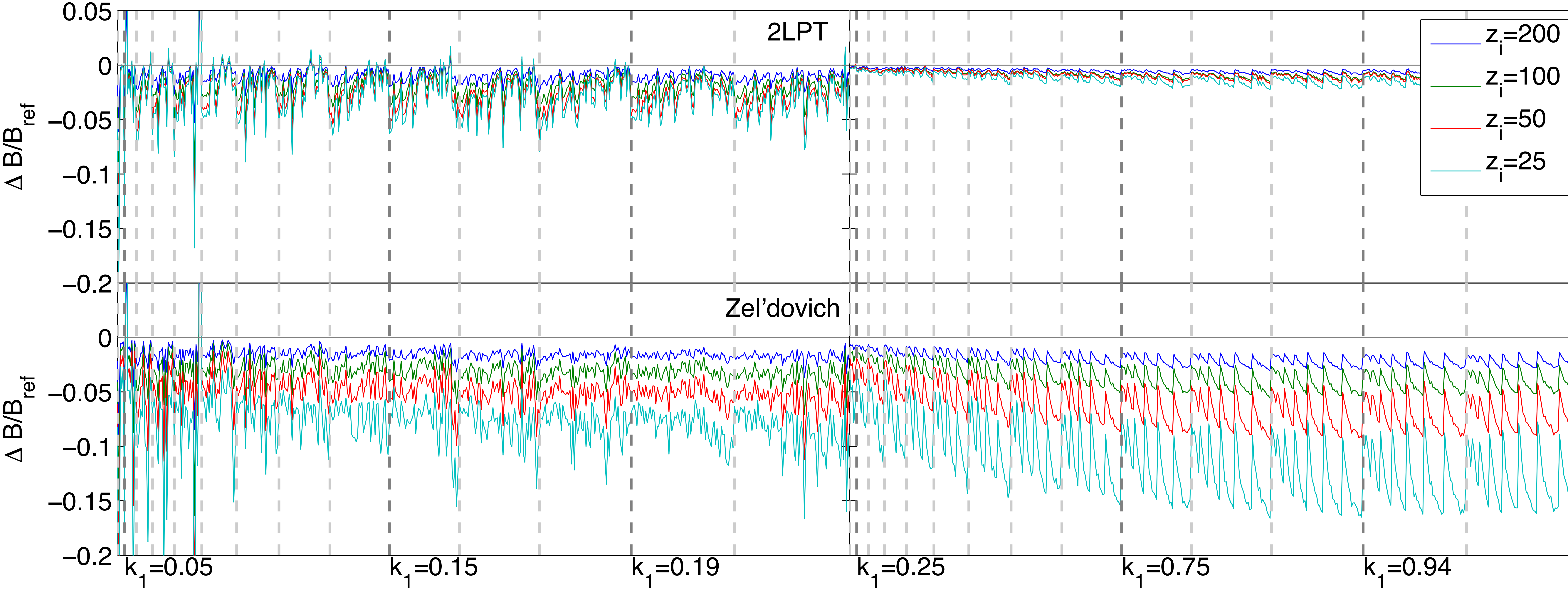


McCullagh, DJ, Szalay (coming soon)

Transient at z=1

$$\frac{\text{Bk(2LPT/Zeldovich}, z_{init})}{\text{Bk(2LPT}, z_{init}=400)}$$

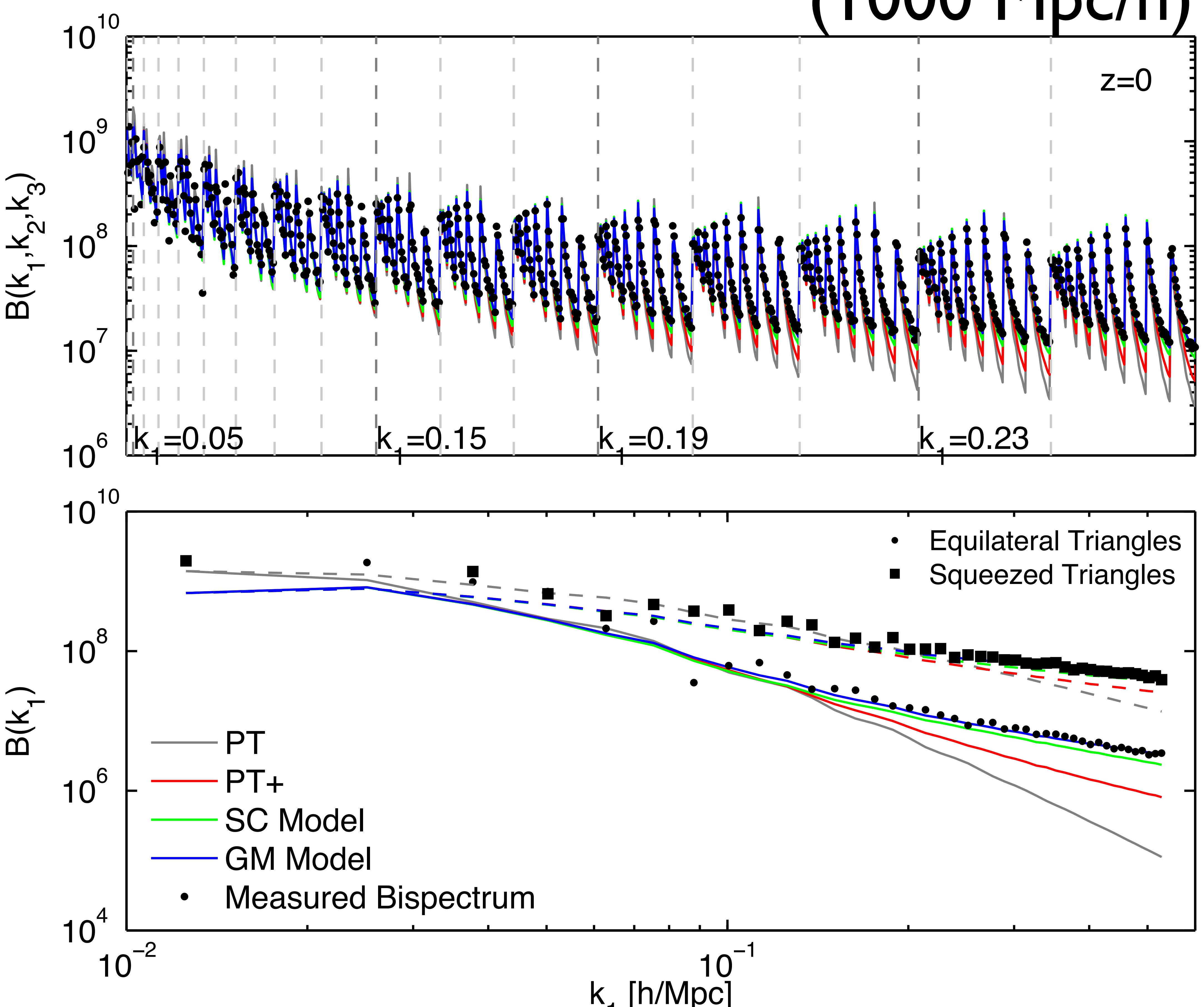
(1000 Mpc/h)^3 (200 Mpc/h)^3



McCullagh, DJ, Szalay (coming soon)

Non-linear bispectrum (z=0)

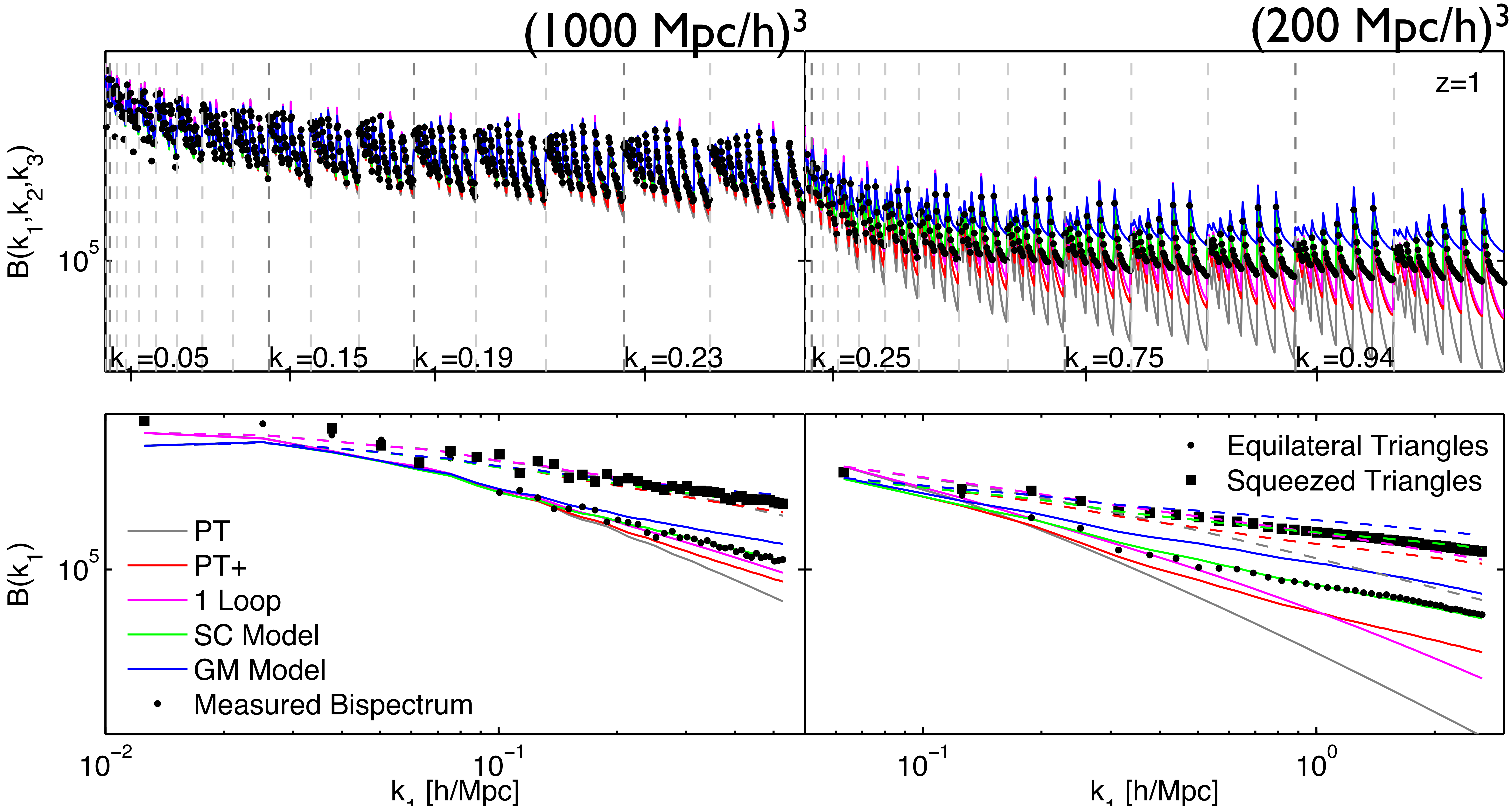
(1000 Mpc/h)^3



- Simulation from 2LPT, z_{ini}=400
- scale & configuration dep.
- Deviation from the tree-level (PT) is more apparent for equilateral-like configurations!
- Two fitting formulas
  SC (Scoccimarro & Couchman 2001)
  GM (Gil-Marin+ 2012)

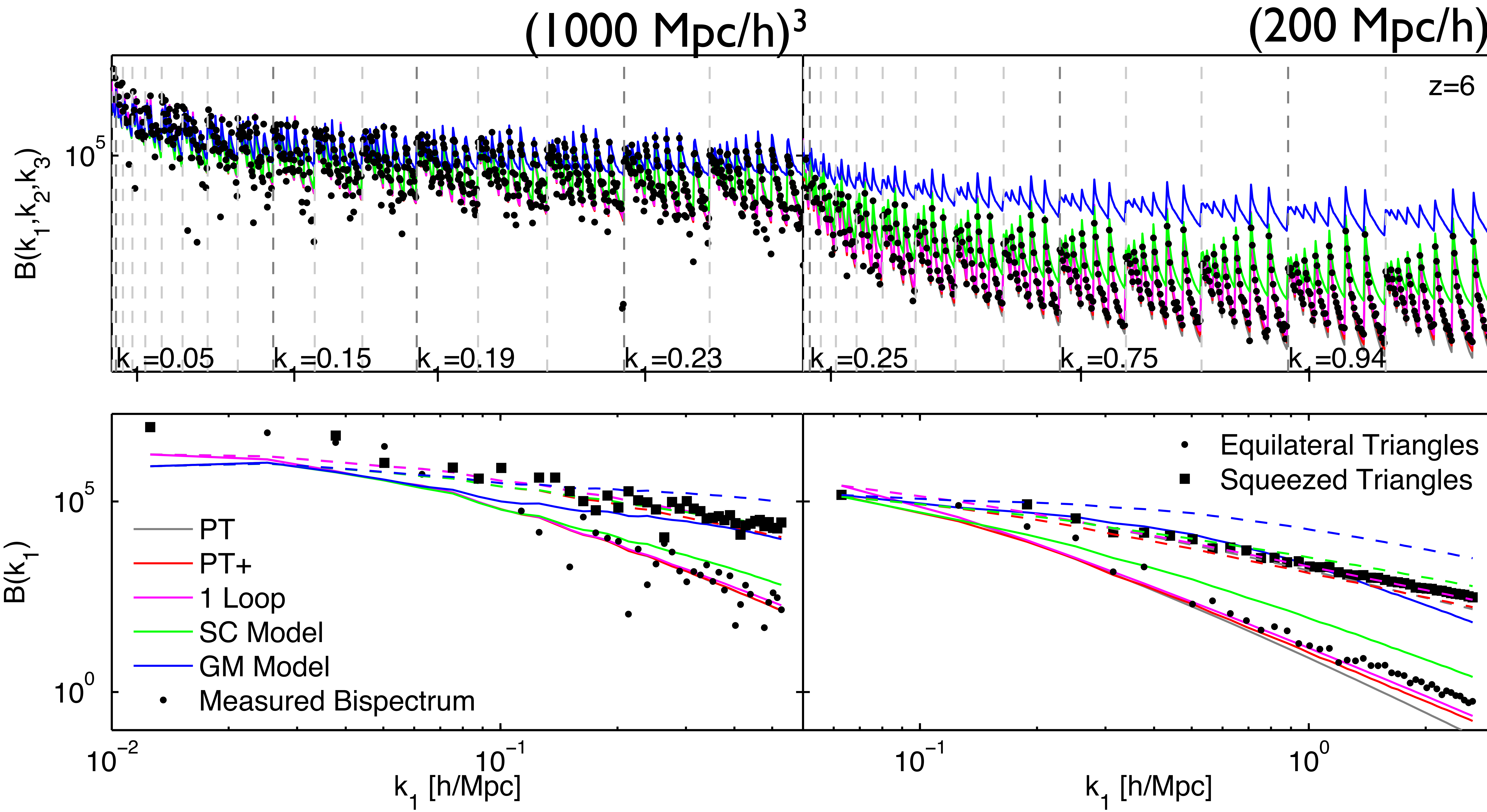
McCullagh, DJ, Szalay (coming soon)

Non-linear bispectrum (z=1)



McCullagh, DJ, Szalay (coming soon)

Non-linear bispectrum (z=6)



McCullagh, DJ, Szalay (coming soon)

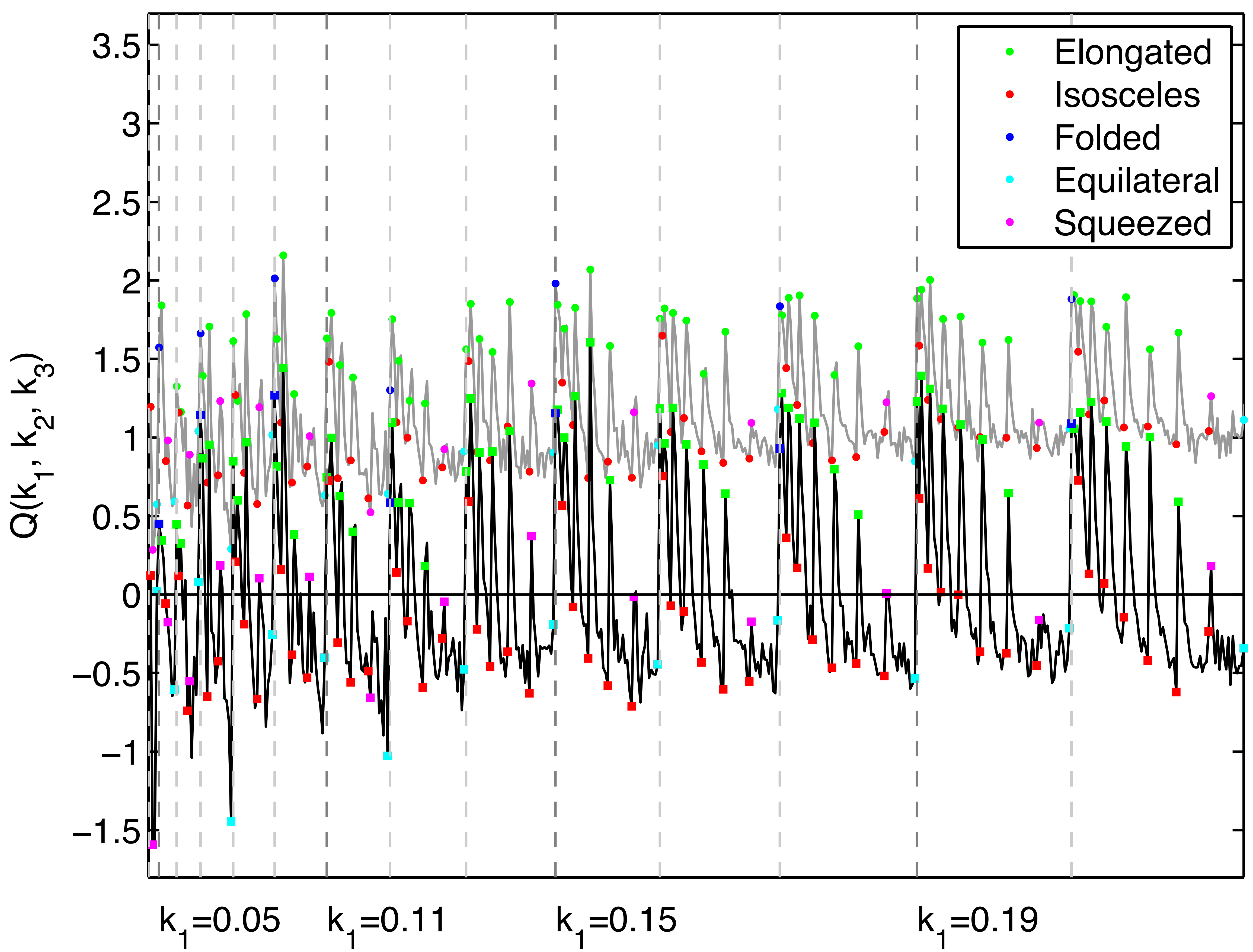
Log-normal density field

- Cosmological density field is close to log-normal: (Neyrinck+ 2009)
  - 1pt pdf indicates the log-normality
  - log-transformed density field has lower covariance and better fidelity to the linear theory
- If EXACTLY log-normal, we are done modeling the bispectrum!

$$B(k_1, k_2, k_3) = P(k_1)P(k_2) + (2 \text{ cyclic}) + \int \frac{d^3q}{(2\pi)^3} P(q)P(|\mathbf{k}_1 + \mathbf{q}|)P(|\mathbf{k}_2 - \mathbf{k}|)$$

McCullagh, DJ, Szalay (coming soon)

(reduced) Bispectrum of log-density



- reduced bispectrum $Q_{123} = B_{123}/(P_1P_2 + \text{cyclic})$
- Skewness ~ $\sum Q_{123}$ = small
- IF log-normal, Q must be 0!
- Log-normality **CANNOT** be extended beyond the 1pt PDF!

Conclusion

- To recover correct non-linear bispectrum with <1% transient at z<6, 2LPT initial condition generator with z_{init}>100 is necessary!
- Non-linearities in bispectrum are stronger on smaller scales and equilateral-like triangles. One-loop bispectrum works well on 'smaller scales', and SC and GM fitting formula works well on highly non-linear scales at z=0 (caution: NOT at high-z).
- Log-normality cannot be extended beyond 1pt PDF.