

Features of the integrated perturbation theory for the large-scale structure of the universe

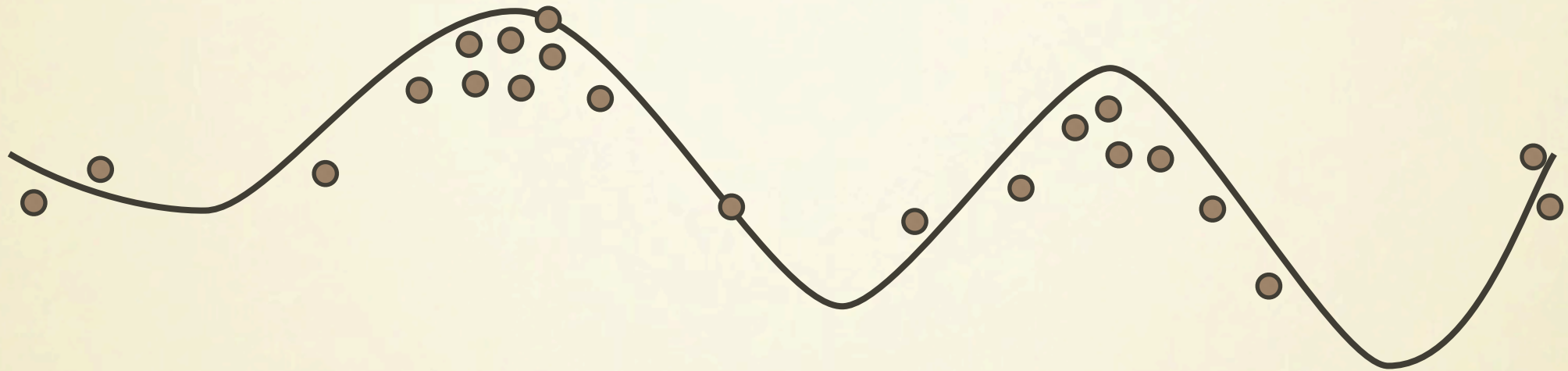
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@Precision Era for Large Scale Structure
CTPU, IBS, Daejeon, Korea
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Integrated perturbation theory (IPT)

- Integration of what?
 - nonlinear perturbation theory
 - general bias (incl. halo bias, nonlocal, etc...)
 - nonlinear redshift-space distortions
 - primordial non-Gaussianity (scale-dep. bias)
- A generalization of nonlinear perturbation theory
 - Many previously known results are reproduced
 - predicts new features on BAO scale in biased power spectrum

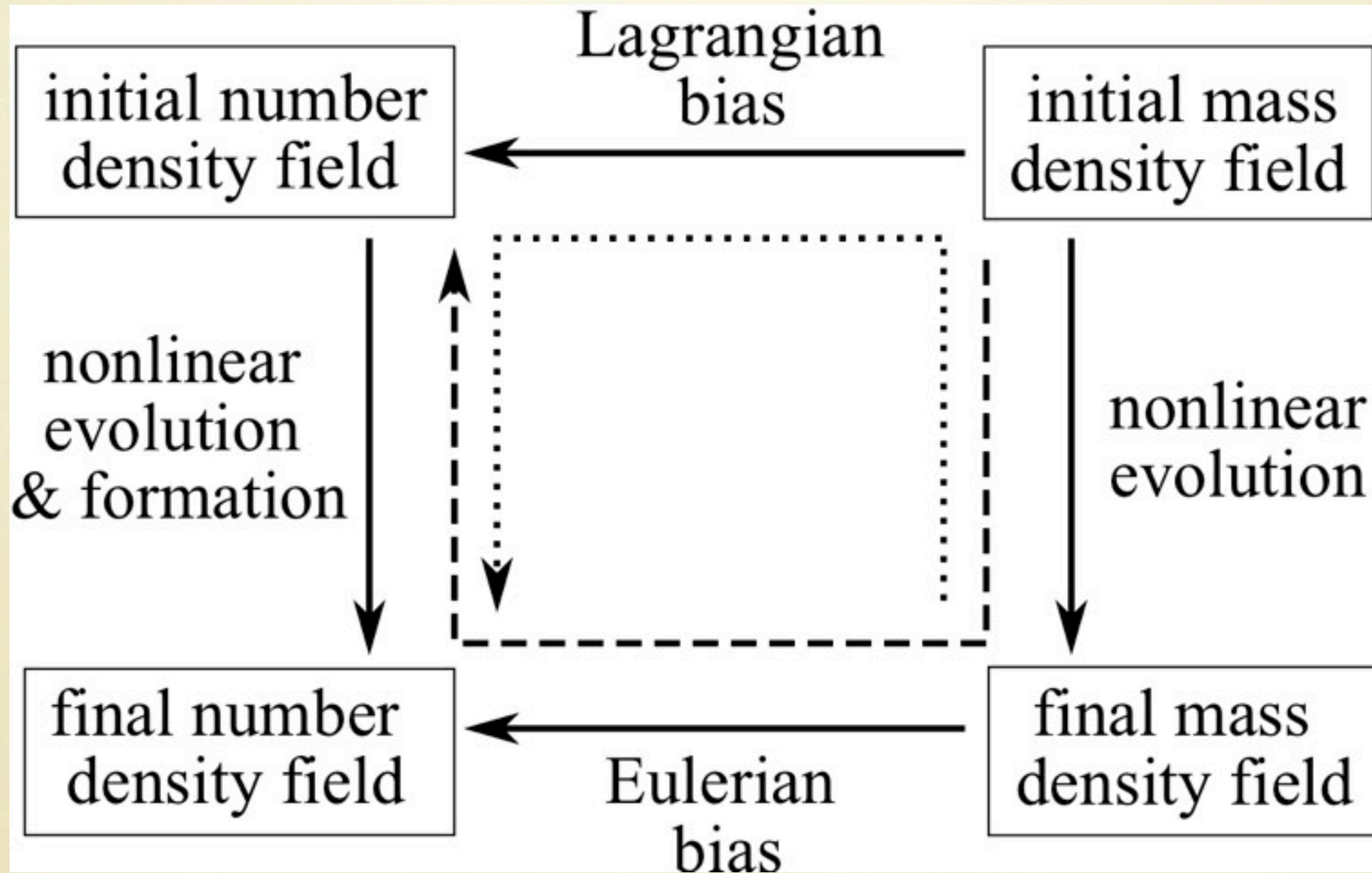
Bias between mass and objects



- Mass density $\neq$ number density (in general)
- Densities of both mass and astronomical objects are determined by initial density field
 - There should be a relation

$$\delta_m(\mathbf{x}) = \frac{\rho_m(\mathbf{x})}{\bar{\rho}_m} - 1 \quad \Leftrightarrow \quad \delta_X(\mathbf{x}) = \frac{\rho_X(\mathbf{x})}{\bar{\rho}_X} - 1$$

Eulerian & Lagrangian bias



Nonlinear dynamics is nonlocal, so is nonlinear bias

Eulerian & Lagrangian bias

- Eulerian and Lagrangian biases are compatible only in a nonlocal formalism
 - The relations can be explicitly derived in perturbation theory:

$$b_1(\mathbf{k}) = b_1^L(\mathbf{k}) + 1,$$

$$b_2(\mathbf{k}_1, \mathbf{k}_2) = b_2^L(\mathbf{k}_1, \mathbf{k}_2) - b_1^L(\mathbf{k}_1 + \mathbf{k}_2)F_2(\mathbf{k}_1, \mathbf{k}_2) \\ + \left(1 + \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_1^2}\right)b_1^L(\mathbf{k}_2) + \left(1 + \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_2^2}\right)b_1^L(\mathbf{k}_1),$$

etc.

Local biases are incompatible

Lagrangian perturbation theory with Lagrangian (nonlocal) bias

- The relation between Eulerian density fluctuations and Lagrangian variables

$$1 + \delta_X(\mathbf{x}) = \int d^3q \left[1 + \delta_X^L(\mathbf{q}) \right] \delta_D^3[\mathbf{x} - \mathbf{q} - \boldsymbol{\Psi}(\mathbf{q})]$$

Eulerian
density field

Biased field in
Lagrangian space

displacement
(& redshift distortions)

- Perturbative expansion in Fourier space

$$\delta_X^L(\mathbf{k}) = \sum_{n=1}^{\infty} \frac{1}{n!} \int \frac{d^3k_1}{(2\pi)^3} \cdots \frac{d^3k_n}{(2\pi)^3} (2\pi)^3 \delta_D^3(\mathbf{k}_{1\dots n} - \mathbf{k}) b_n^L(\mathbf{k}_1, \dots, \mathbf{k}_n) \delta_L(\mathbf{k}_1) \cdots \delta_L(\mathbf{k}_n)$$

Kernel of the Lagrangian bias

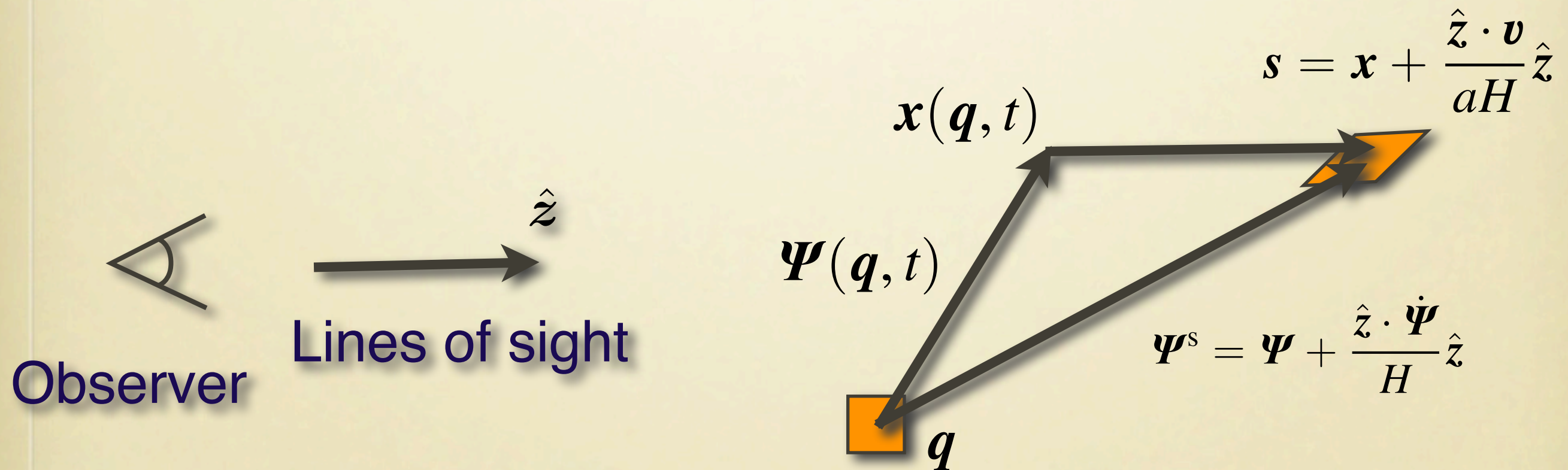
$$\tilde{\boldsymbol{\Psi}}(\mathbf{k}) = \sum_{n=1}^{\infty} \frac{i}{n!} \int \frac{d^3k_1}{(2\pi)^3} \cdots \frac{d^3k_n}{(2\pi)^3} (2\pi)^3 \delta_D^3(\mathbf{k}_{1\dots n} - \mathbf{k}) L_n(\mathbf{k}_1, \dots, \mathbf{k}_n) \delta_L(\mathbf{k}_1) \cdots \delta_L(\mathbf{k}_n)$$

Kernel of the displacement field (& redshift distortions)

$$\mathbf{k}_{1\dots n} \equiv \mathbf{k}_1 + \cdots + \mathbf{k}_n$$

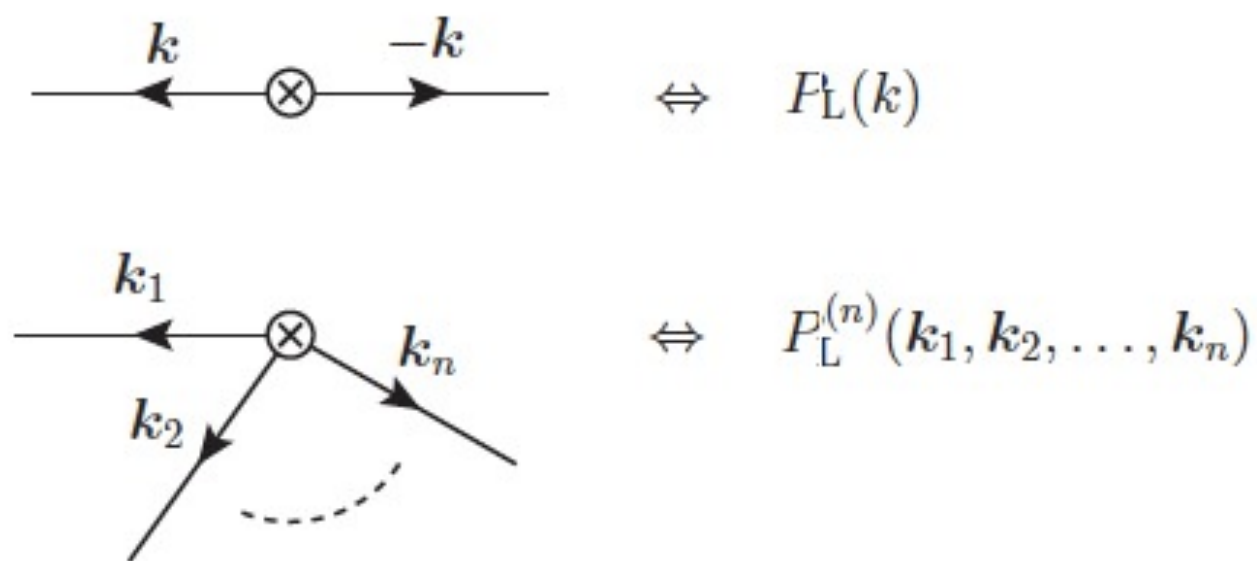
redshift space distortions

- RSD is easier to describe in Lagrangian picture
 - Real space to Redshift space: Exactly linear mapping in Lagrangian
 - c.f.) nonlinear mapping in Eulerian picture
 - Technically just linear transform LPT kernel fns

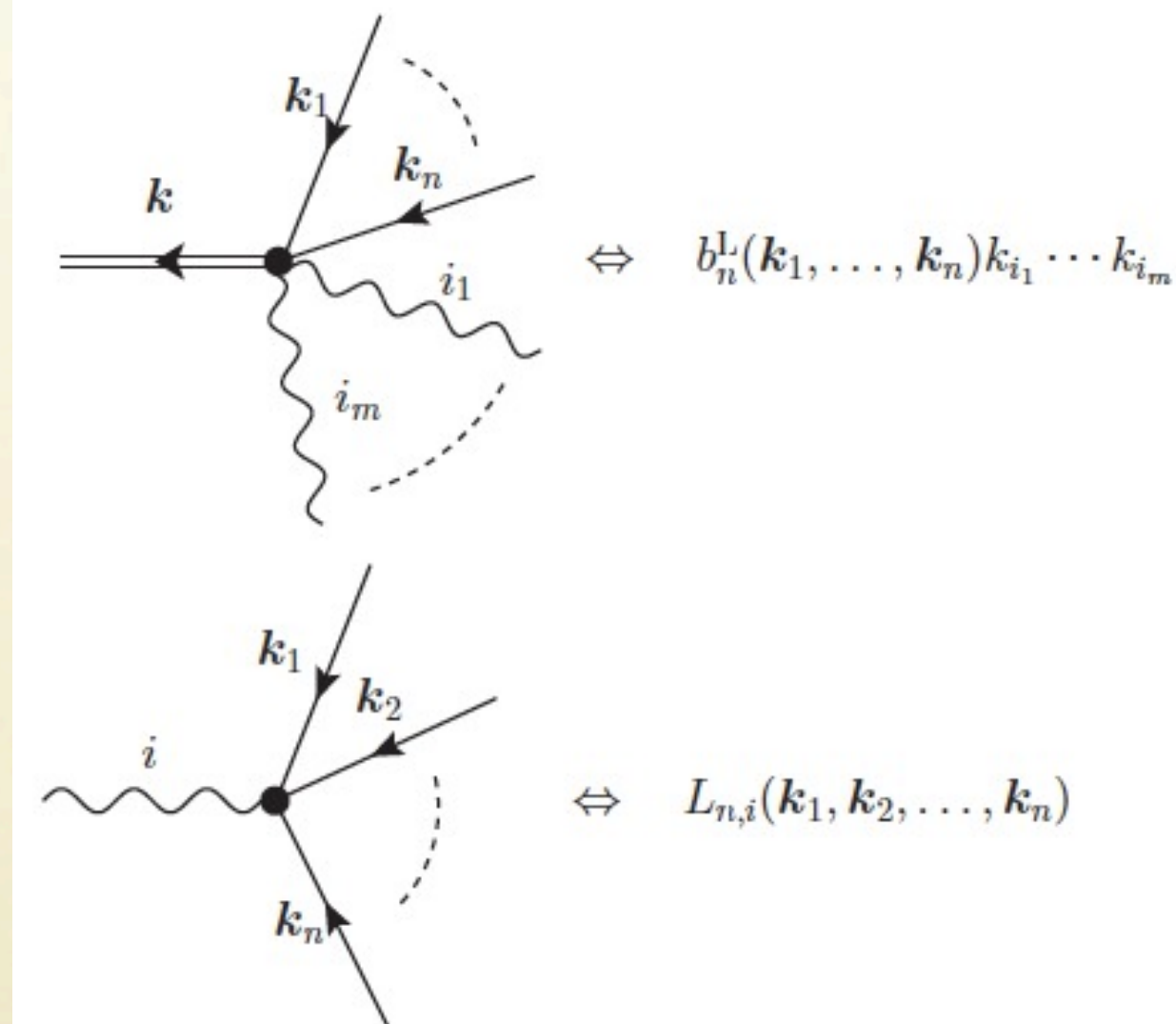


Diagrams in iPT

Primordial spectra



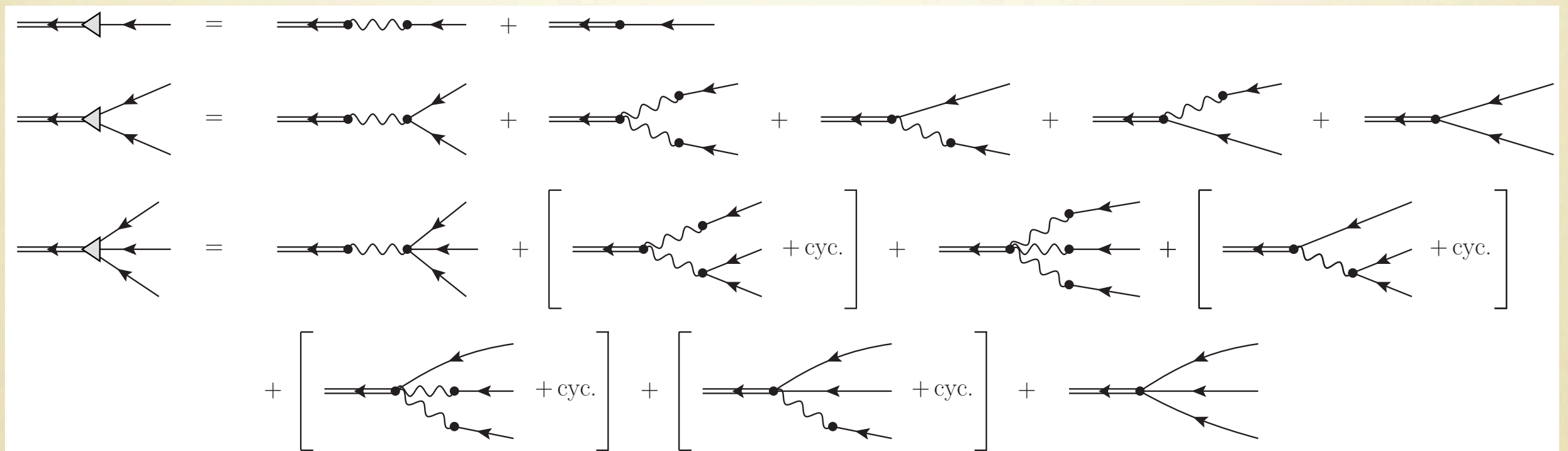
Vertices in Lagrangian PT



can naturally deal with RSD and nG

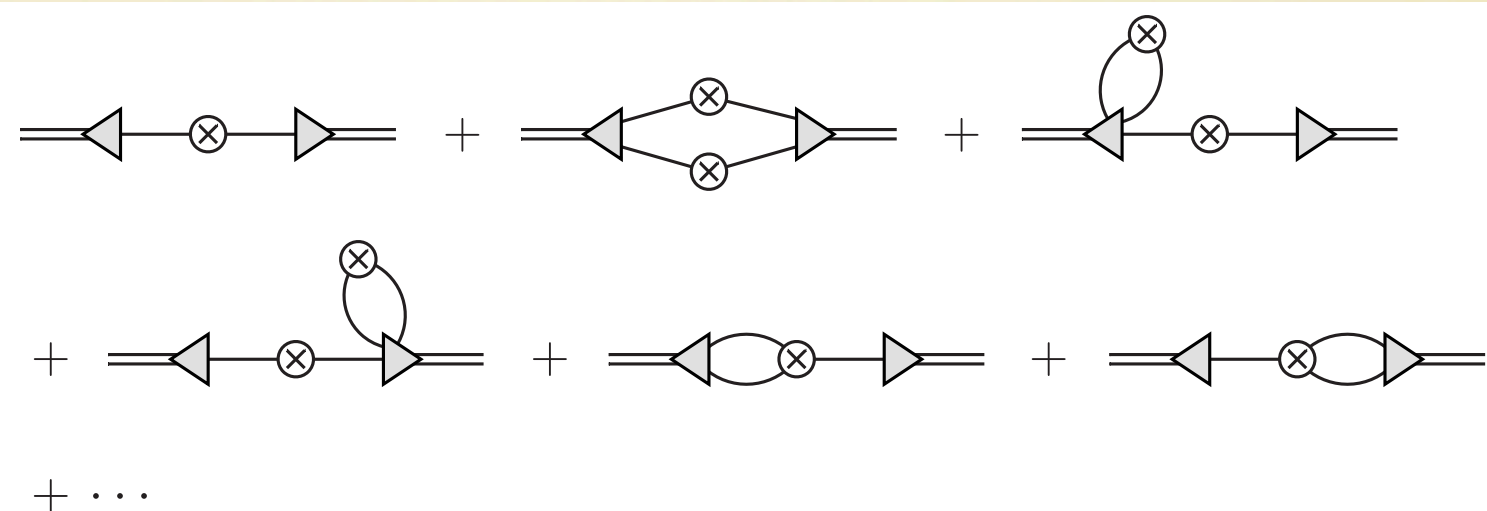
Vertex shrunk

- Shrunk vertices



- Ex.)

$$P_X(k) =$$

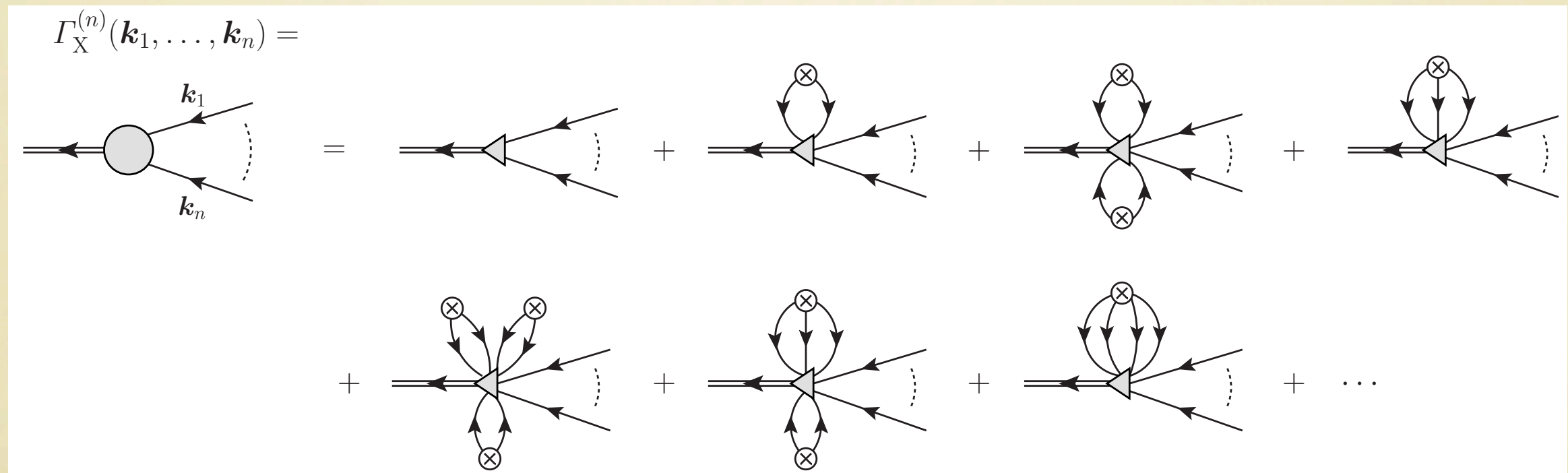


Multi-point propagator

TM (1995); Crocce & Scoccimarro (2006), Bernardeau et al. (2008)

- Density sector of multi-point propagator with nonlocal bias and RSD

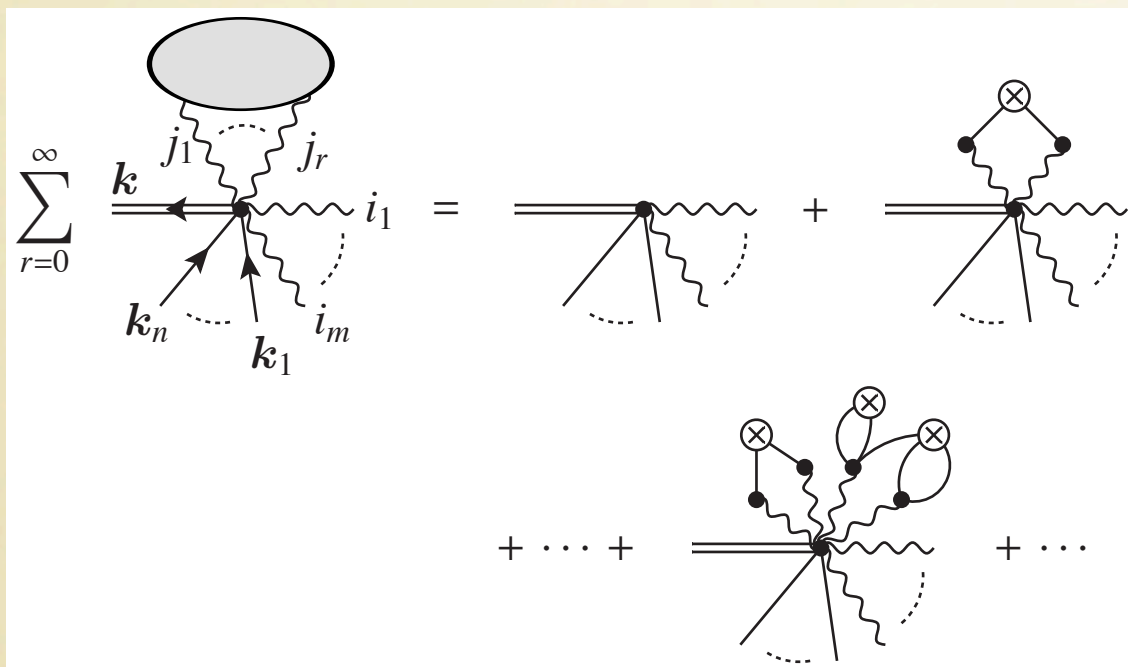
$$\left\langle \frac{\delta^n \delta_X(\mathbf{k})}{\delta \delta_L(\mathbf{k}_1) \cdots \delta \delta_L(\mathbf{k}_n)} \right\rangle = (2\pi)^{3-3n} \delta_D^3(\mathbf{k} - \mathbf{k}_{1\dots n}) \Gamma_X^{(n)}(\mathbf{k}_1, \dots, \mathbf{k}_n)$$



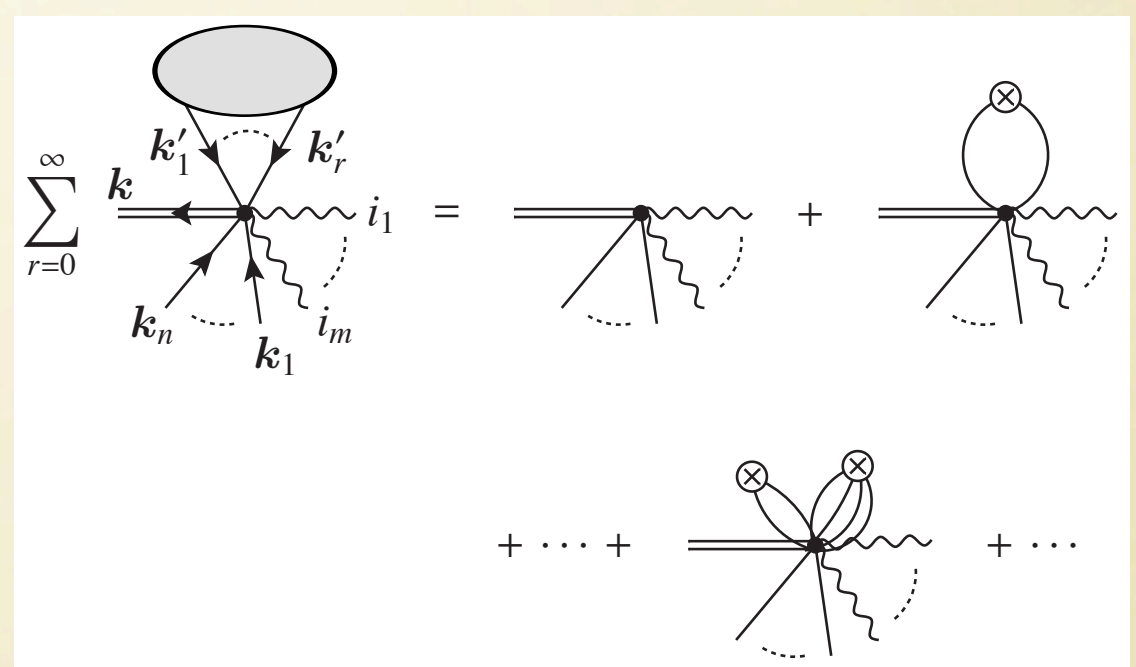
Multi-point propagator

- Full evaluations of MP propagator are difficult
- Partial resummations in the Lagrangian PT

Lagrangian vertex resummation



Lagrangian bias renormalization



$$\begin{aligned} \Pi(\mathbf{k}) &= \langle e^{-i\mathbf{k} \cdot \boldsymbol{\Psi}} \rangle \\ &= \exp \left[\sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \langle (\mathbf{k} \cdot \boldsymbol{\Psi})^n \rangle_c \right] \end{aligned}$$

$$\begin{aligned} b_n^L(\mathbf{k}_1, \dots, \mathbf{k}_n) &= (2\pi)^{3n} \int \frac{d^3 k'}{(2\pi)^3} \frac{\delta^n \delta_X^L(\mathbf{k}')}{\delta \delta_L(\mathbf{k}_1) \cdots \delta \delta_L(\mathbf{k}_n)} \Big|_{\delta_L=0} \\ \Rightarrow c_n^L(\mathbf{k}_1, \dots, \mathbf{k}_n) &= (2\pi)^{3n} \int \frac{d^3 k'}{(2\pi)^3} \left\langle \frac{\delta^n \delta_X^L(\mathbf{k}')}{\delta \delta_L(\mathbf{k}_1) \cdots \delta \delta_L(\mathbf{k}_n)} \right\rangle \end{aligned}$$

Renormalized bias functions

- Introduction of the “renormalized bias functions” is essential in iPT
 - Series of functions to characterize nonlocal bias

$$c_n^L(\mathbf{k}_1, \dots, \mathbf{k}_n) = (2\pi)^{3n} \int \frac{d^3k}{(2\pi)^3} \left\langle \frac{\delta^n \delta_X^L(\mathbf{k})}{\delta\delta_L(\mathbf{k}_1) \cdots \delta\delta_L(\mathbf{k}_n)} \right\rangle,$$

- It can be viewed as a counterpart of multi-point propagator for Lagrangian biasing

$$\left\langle \frac{\delta^n \delta_X^L(\mathbf{k})}{\delta\delta_L(\mathbf{k}_1) \cdots \delta\delta_L(\mathbf{k}_n)} \right\rangle = (2\pi)^{3-3n} \delta_D^3(\mathbf{k}_{1\dots n} - \mathbf{k}) c_n^L(\mathbf{k}_1, \dots, \mathbf{k}_n).$$

(4)

A simple model of renormalized bias functions for halos

- Localization of PS model for the halo number density

$$n(M) = -\frac{2\bar{\rho}_0}{M} \frac{\partial}{\partial M} P(M, \delta_c), \quad \Rightarrow \quad n(\mathbf{x}, M) = -\frac{2\bar{\rho}_0}{M} \frac{\partial}{\partial M} \Theta[\delta_M(\mathbf{x}) - \delta_c],$$

- Applying the above model, we have

$$c_n^L(k_1, \dots, k_n) = \frac{A_n(M)}{\delta_c^n} W(k_1 R) \cdots W(k_n R) + \frac{A_{n-1}(M) \sigma_M^n}{\delta_c^n} \frac{d}{d \ln \sigma_M} \left[\frac{W(k_1 R) \cdots W(k_n R)}{\sigma_M^n} \right],$$

$$A_n(M) \equiv \sum_{j=0}^n \frac{n!}{j!} \delta_c^j b_j^L(M).$$

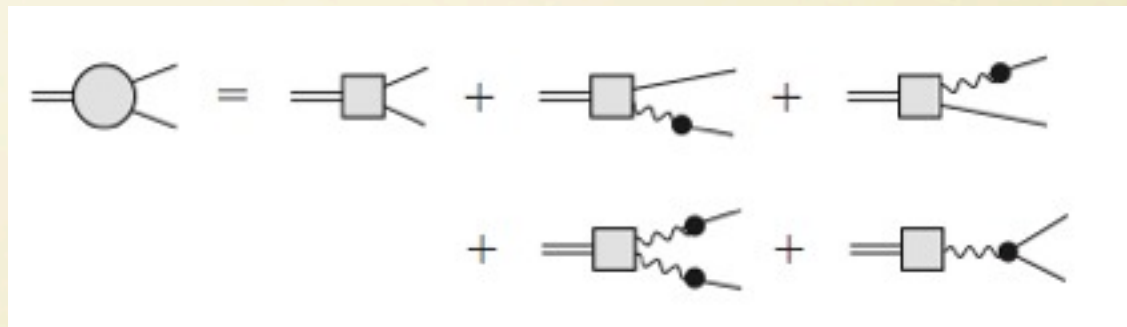
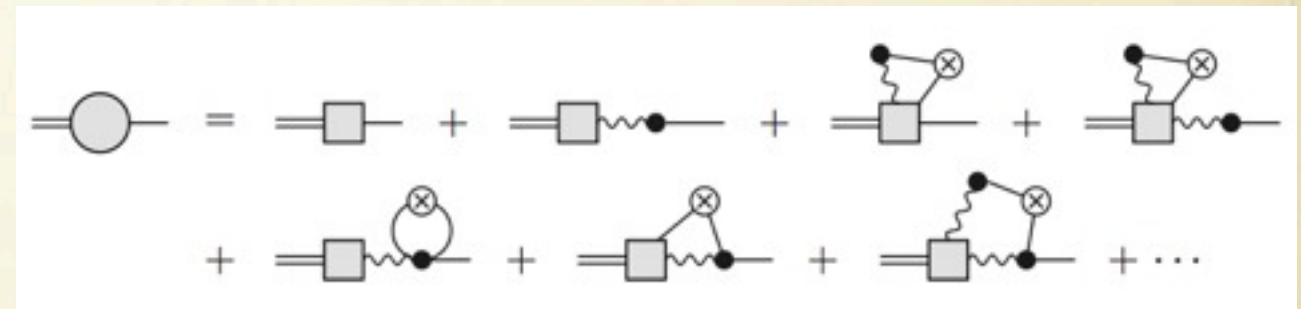
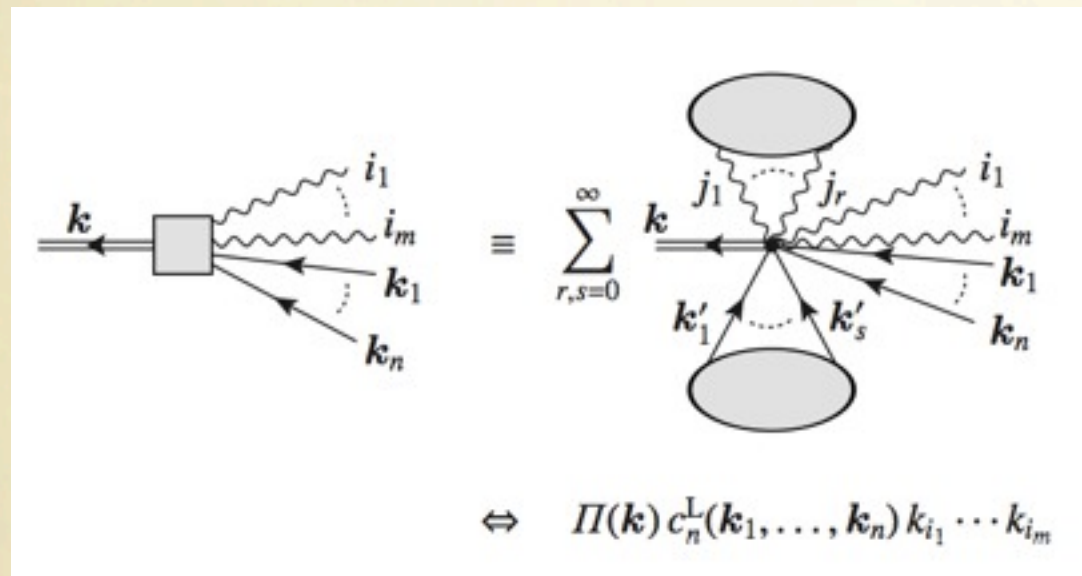
$$A_0(M) = 1,$$

$$A_1(M) = 1 + \delta_c b_1^L(M),$$

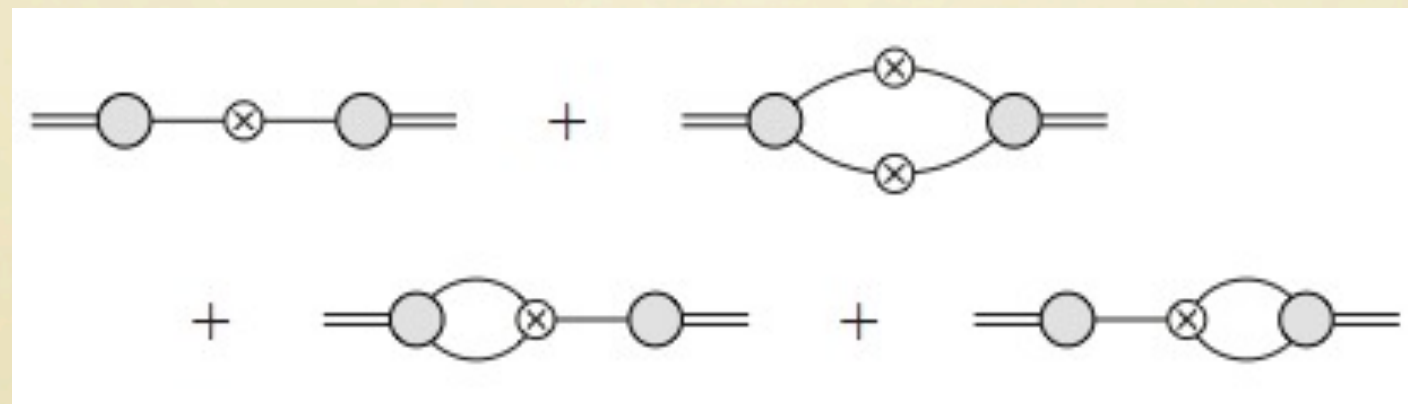
$$A_2(M) = 2 + 2\delta_c b_1^L(M) + \delta_c^2 b_2^L(M).$$

Vertex summations

- Partially renormalized vertex, propagators



- Power spectrum (up to 1-loop, including nG)



Ex.) One-loop formula in real space

$$P_{XY}(k) = \Pi^2(k) \left[\hat{\Gamma}_X^{(1)}(k) \Gamma_Y^{(1)}(k) P_L(k) + \frac{k^3}{8\pi^2} \int_0^\infty dr r^2 \int_{-1}^1 dx \hat{\Gamma}_X^{(2)}(k, r, x) \hat{\Gamma}_Y^{(2)}(k, r, x) P_L(kr) P_L(k \sqrt{1+r^2-2rx}) \right], \quad (1)$$

where indices X and Y are either matter 'm' or halo 'h' in this paper, $P_L(k)$ is the linear matter power spectrum, and vertex factor $\Pi(k)$ is given by

$$\Pi(k) = \exp \left[-\frac{k^2}{12\pi^2} \int dp P_L(p) \right]. \quad (2)$$

The normalized multipoint propagators $\hat{\Gamma}^{(1)}$ and $\hat{\Gamma}^{(2)}$ are given by

$$\hat{\Gamma}_m^{(1)}(k) = 1 + \frac{k^3}{4\pi} \int_0^\infty dr \int_{-1}^1 dx \left[\frac{5}{21} \frac{r^2(1-x^2)^2}{1+r^2-2rx} + \frac{3}{7} \frac{(1-rx)(1-x^2)rx}{1+r^2-2rx} \right] P_L(k), \quad (3)$$

$$\hat{\Gamma}_h^{(1)}(k) = 1 + c_h^{(1)}(k) + \frac{k^3}{4\pi} \int_0^\infty dr \int_{-1}^1 dx \left\{ \frac{5}{21} \frac{r^2(1-x^2)^2}{1+r^2-2rx} + \frac{3}{7} \frac{(1-rx)(1-x^2)}{1+r^2-2rx} [rx + r^2 c_h^{(1)}(kr)] \right\} P_L(k), \quad (4)$$

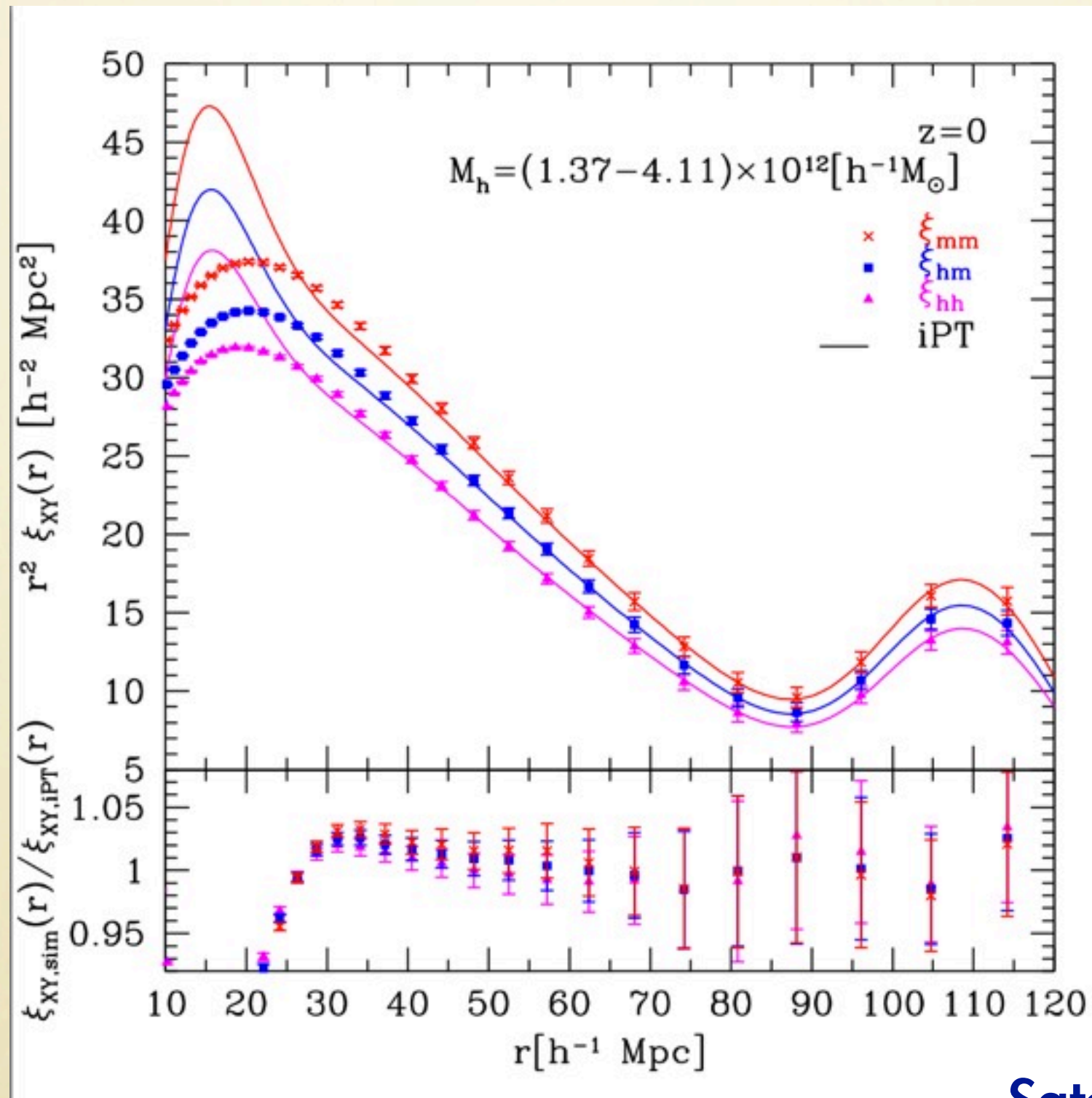
$$\hat{\Gamma}_m^{(2)}(k, r, x) = \frac{x}{r} + \frac{1-rx}{1+r^2-2rx} - \frac{4}{7} \frac{1-x^2}{1+r^2-2rx}, \quad (5)$$

$$\hat{\Gamma}_h^{(2)}(k, r, x) = \frac{x}{r} \left[1 + c_h^{(1)}(k \sqrt{1+r^2-2rx}) \right] + \frac{1-rx}{1+r^2-2rx} \left[1 + c_h^{(1)}(kr) \right] - \frac{4}{7} \frac{1-x^2}{1+r^2-2rx} + c_h^{(2)}(kr, k \sqrt{1+r^2-2rx}), \quad (6)$$

- similar, but much more terms in redshift space

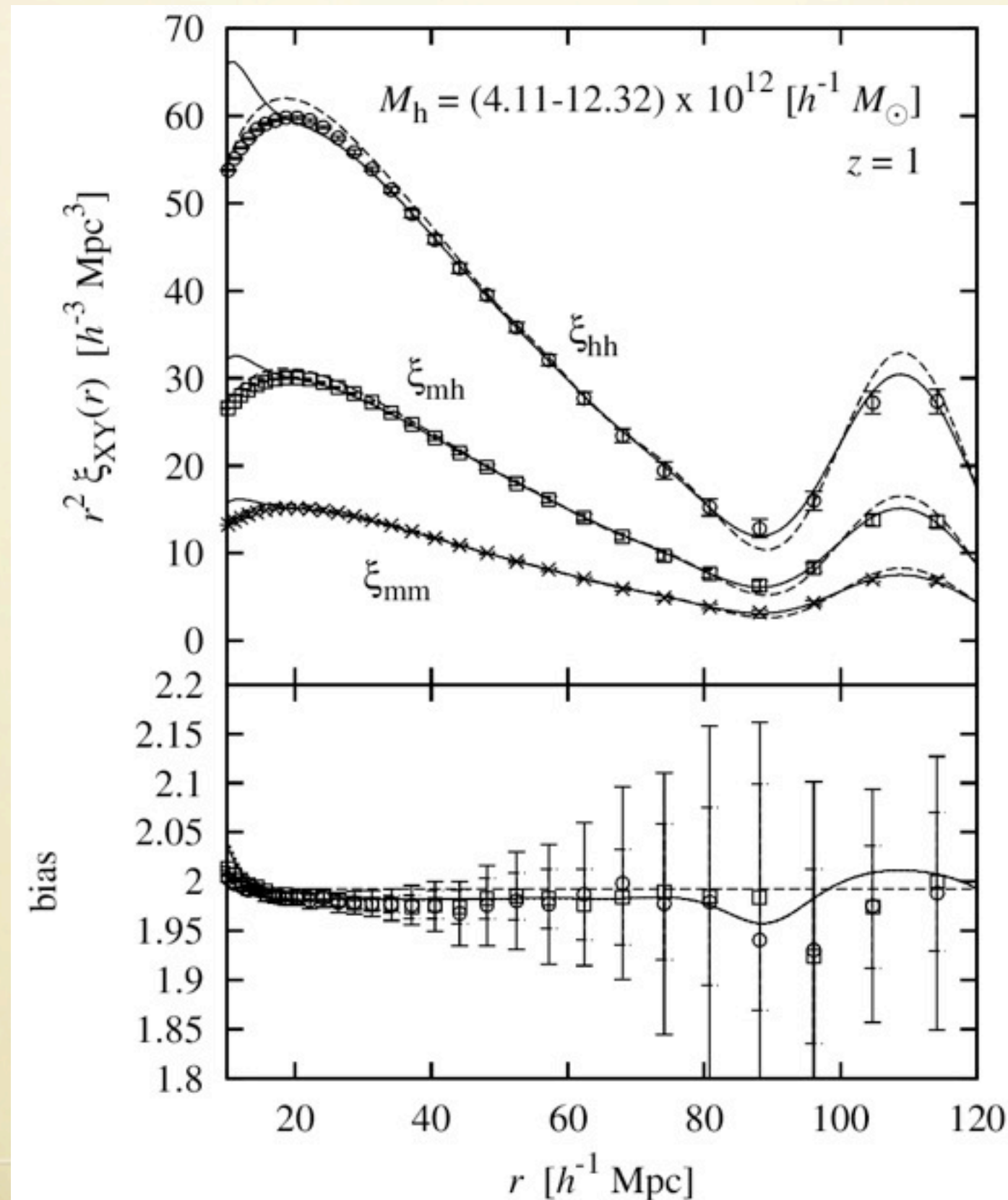
Halo clustering: Comparison with N-body simulations

One-loop



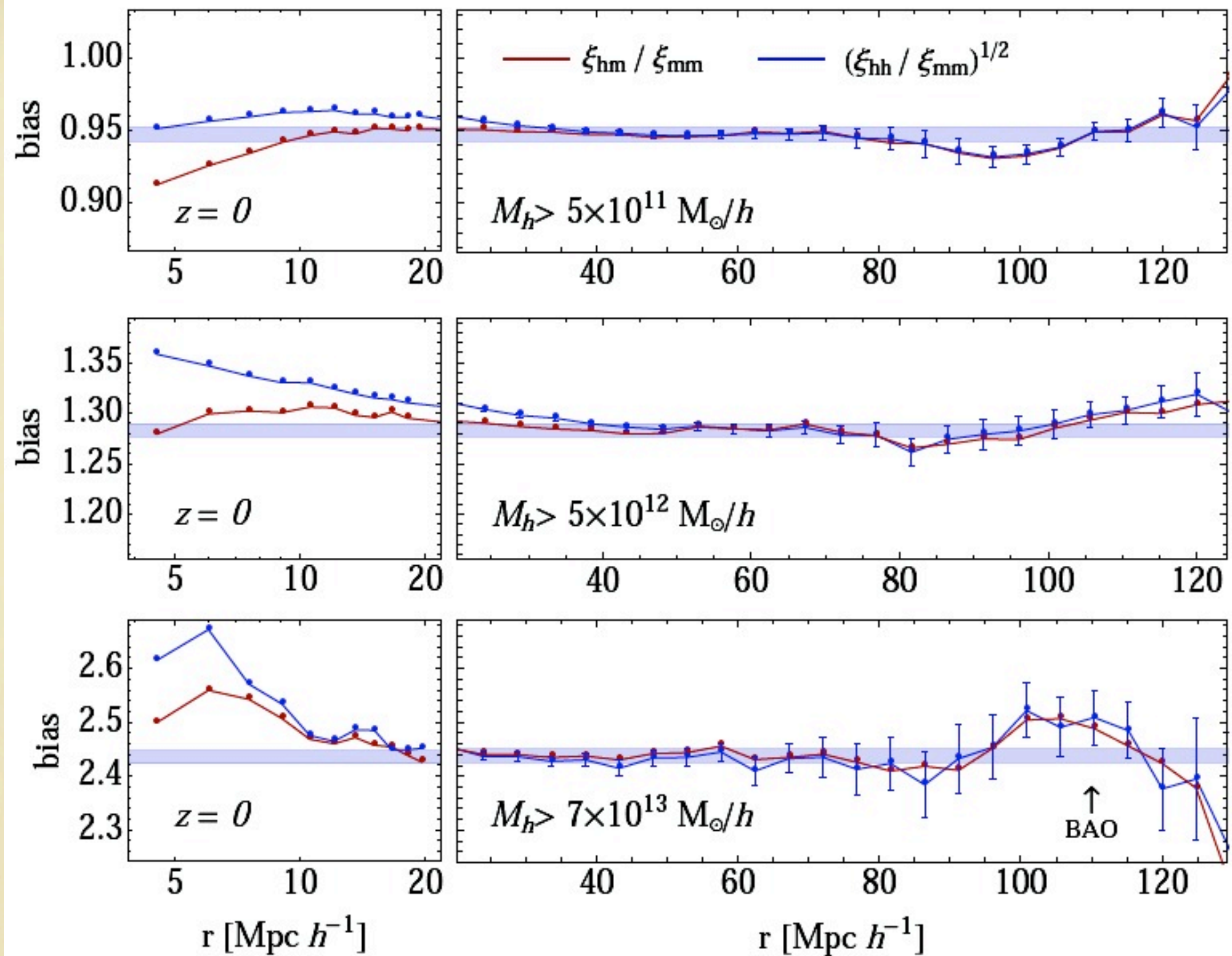
Sato & TM (2013)

Halo clustering: slight scale-dependence of bias around BAO



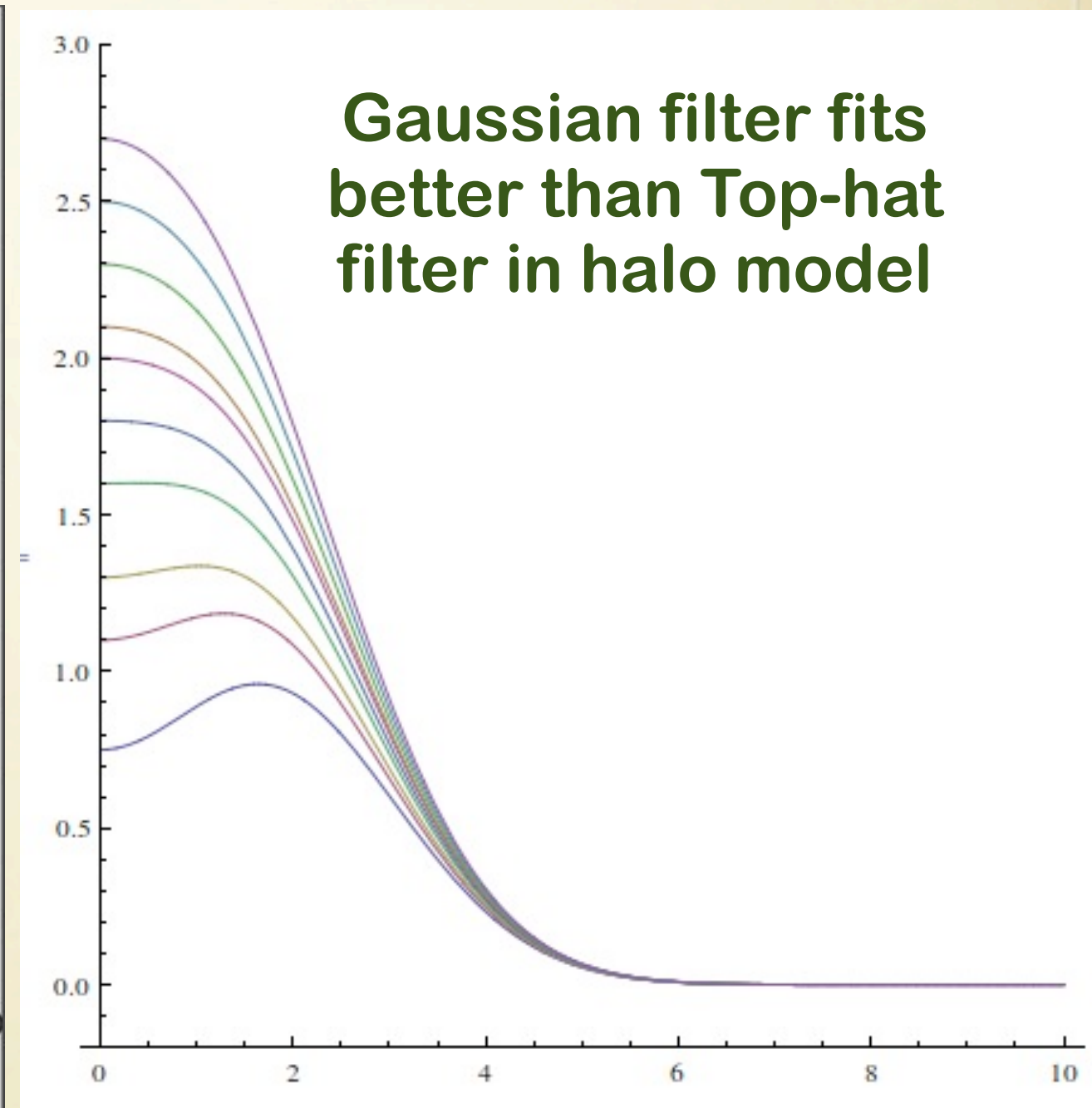
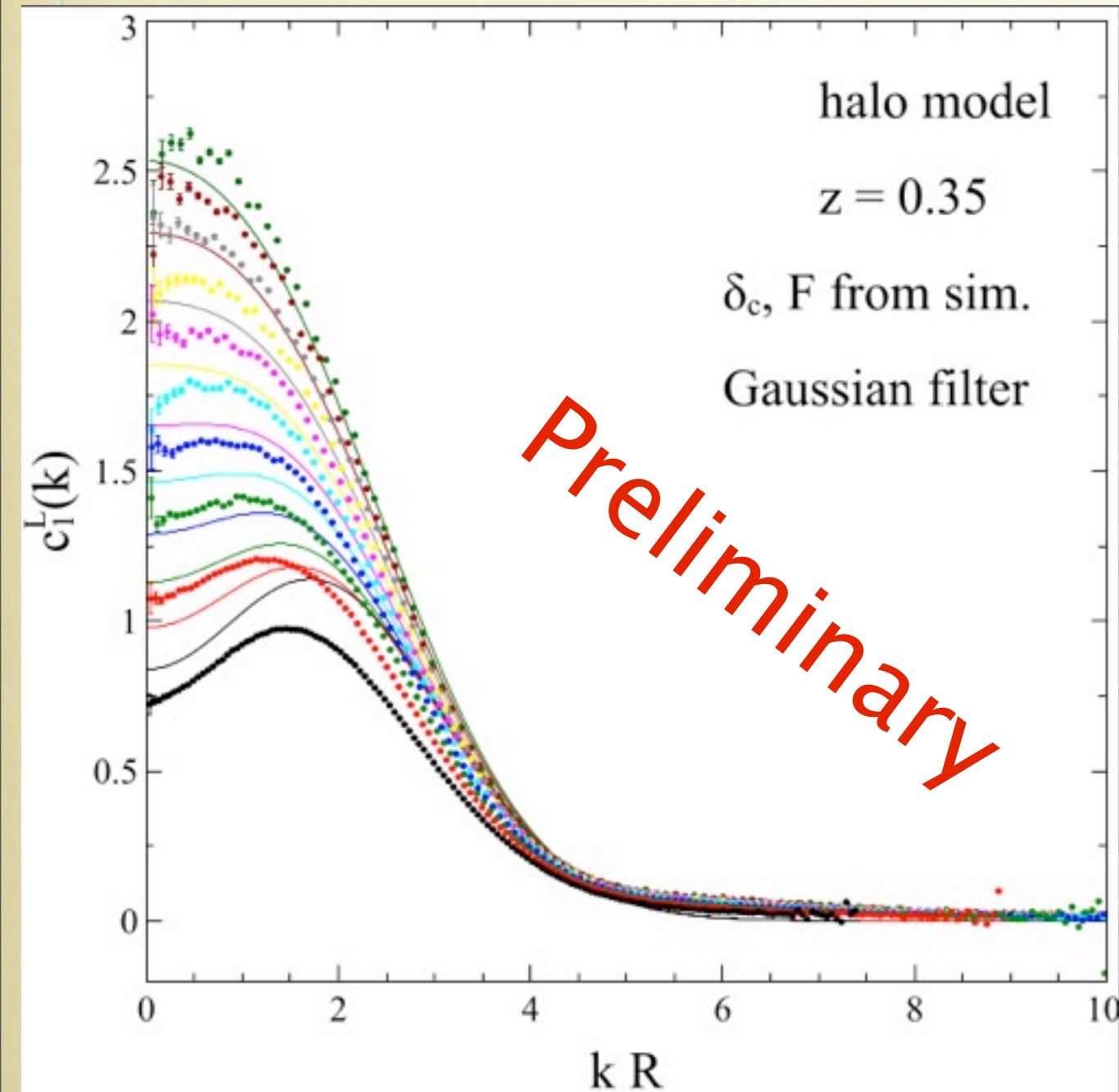
One-loop

TM (2014)



Crocce+ (2013)

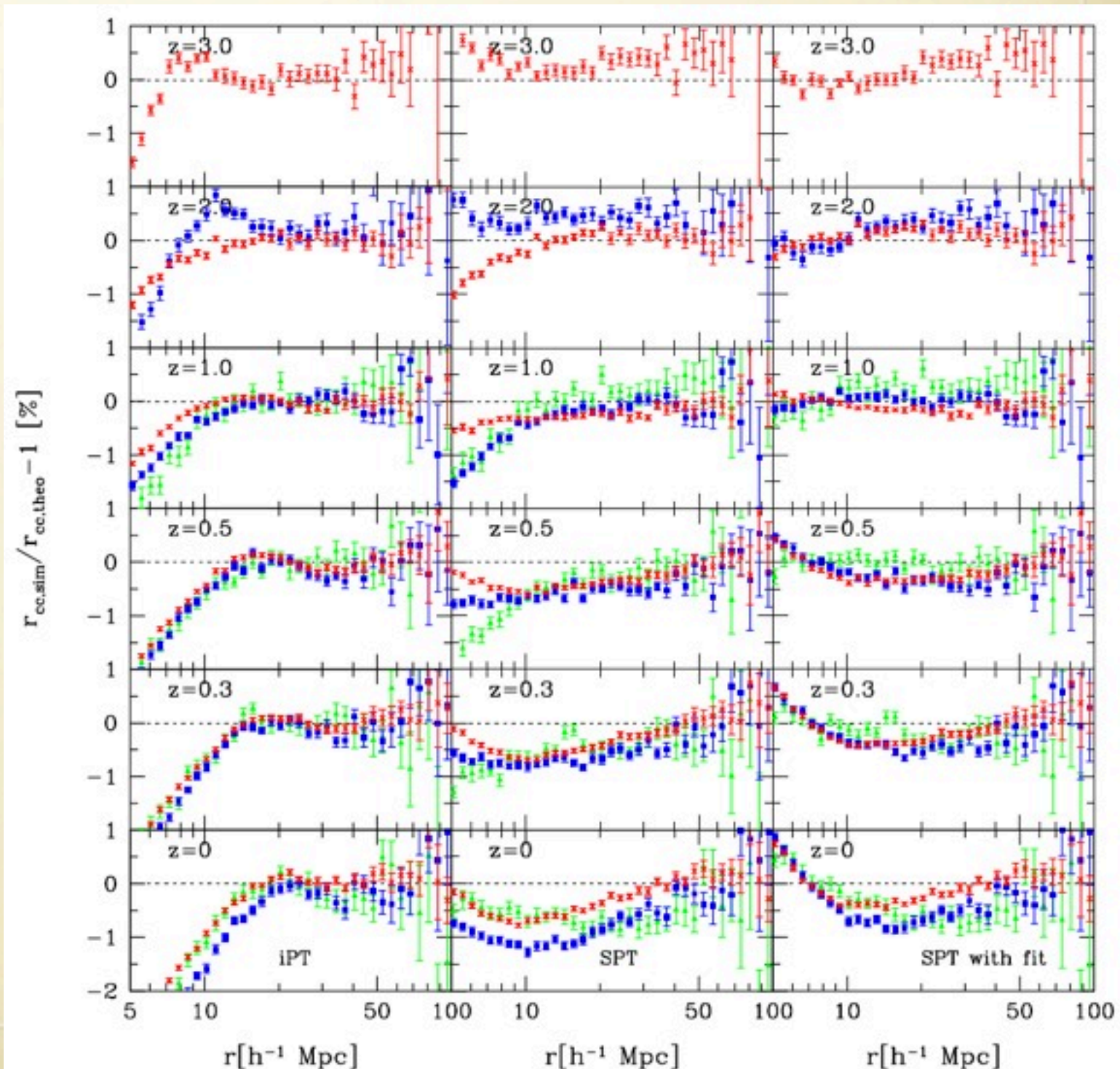
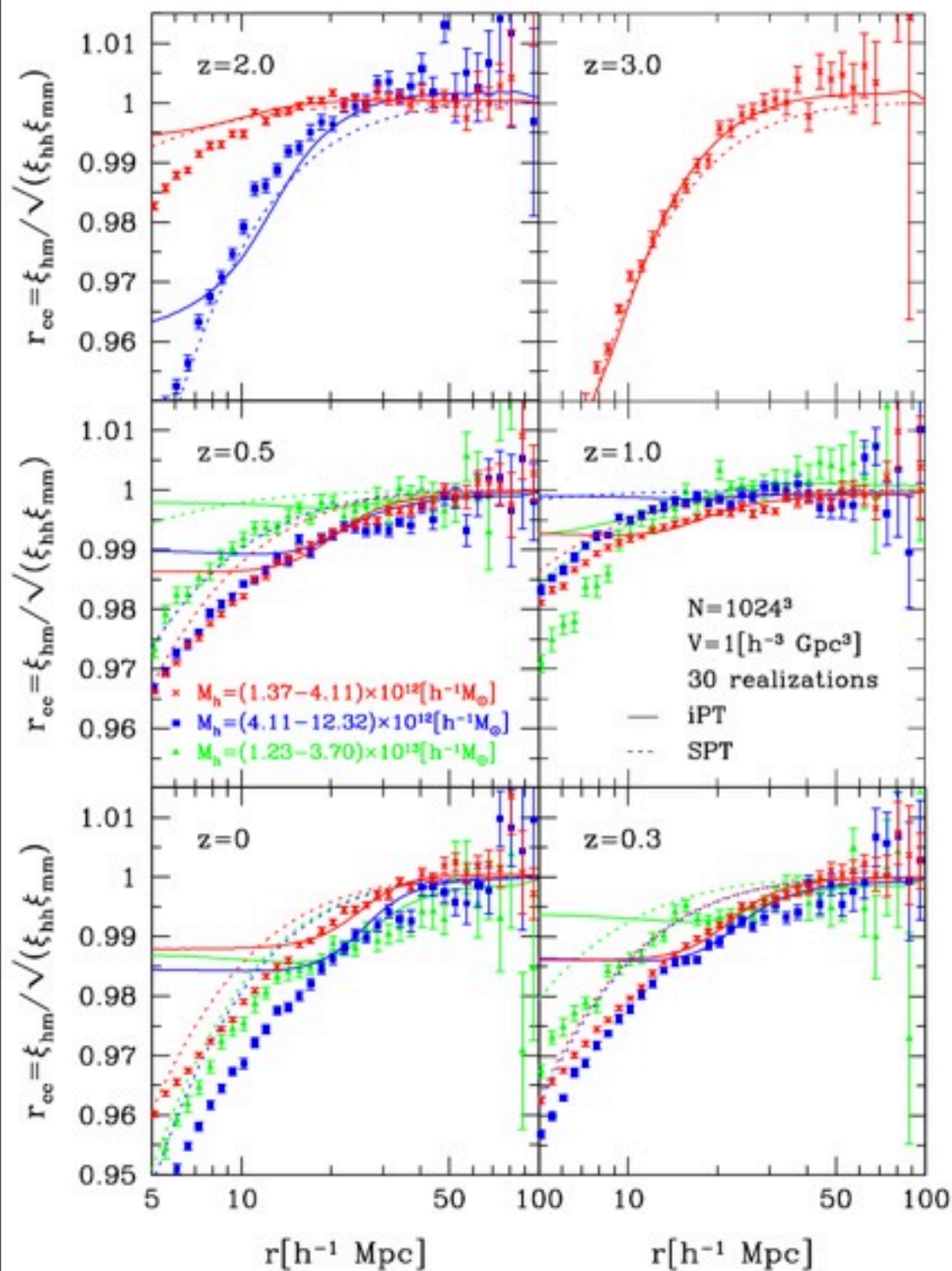
$c_L^L(k)$: numerical vs analytic



Courtesy of T. Nishimichi

Bias stochasticity and iPT

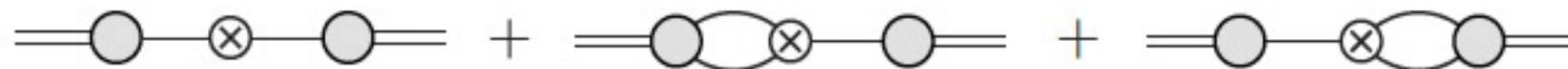
$$r_{cc} = \xi_{hm} / \sqrt{(\xi_{hh} \xi_{mm})}$$



Scale-dependent bias and iPT

- iPT predicts scale-dependent bias in the presence of primordial non-Gaussianity

$$\begin{aligned}
 P_X(k) = & [\Gamma_X^{(1)}(k)]^2 P_L(k) \\
 & + \Gamma_X^{(1)}(k) \int \frac{d^3 k'}{(2\pi)^3} \Gamma_X^{(2)}(k', k - k') B_L(k, k', |k - k'|) \\
 & + \dots,
 \end{aligned}
 \tag{2}$$



General results

- dominant term on large scales:

$$\Delta b(k) \approx \frac{1}{2P_L(k)} \int \frac{d^3 k'}{(2\pi)^3} c_2^L(k', k - k') B_L(k, k', k - k')$$

- scalings do not depend on details of bias

$$\Delta b^{\text{loc.}} \propto k^{-2},$$

$$\Delta b^{\text{eq.}} \propto k^0,$$

$$\Delta b^{\text{fol.}} \propto k^{-1},$$

$$\Delta b^{\text{ort.}} \propto k^{-1}$$

- amplitudes depend on details of bias

Cancellation of highest-order bias parameters in PS mass function

- For the Press-Schechter mass function,

$$b_n^L(M) = \frac{v^{n-1} H_{n+1}(v)}{\delta_c^n}, \quad \Rightarrow \quad A_n(M) = v^n H_n(v) = \frac{\delta_c^{n+1}}{\sigma_M^2} b_{n-1}^L(M),$$

$$c_n^L \sim \frac{\delta_c}{\sigma_M^2} b_{n-1}^L + (\text{nonlocal corrections})$$

- This explains why scale-dependent bias Δb is proportional to b_{-1}^L instead of b_{-2}^L
- Reproduces PBS results in this model with iPT

New general formula

- When the mass function is arbitrary, we have a new formula

$$\Delta b(k) \approx \frac{\sigma_M^2}{2\delta_c^2} \left[\left(2 + 2\delta_c b_1^L + \delta_c^2 b_2^L \right) I(k) + \left(1 + \delta_c b_1^L \right) \frac{dI(k)}{d \ln \sigma_M} \right].$$

- E.g., Sheth-Tormen mass function:

$$\Delta b(k) \approx \left[\frac{q\delta_c b_1^L}{2} + \frac{1}{v^2} \frac{p(qv^2 + 2p + 1)}{1 + (qv^2)^p} \right] I(k) + \left[\frac{q}{2} + \frac{1}{v^2} \frac{p}{1 + (qv^2)^p} \right] \frac{dI(k)}{d \ln \sigma_M}.$$

Scale-dependent bias in redshift space

- Redshift-space distortions are straightforwardly included in iPT

$$\Delta p_0(k) \approx \left(\frac{f}{3} + b_1 \right) Q_2(k), \quad \Delta p_2(k) \approx \frac{2f}{3} Q_2(k),$$

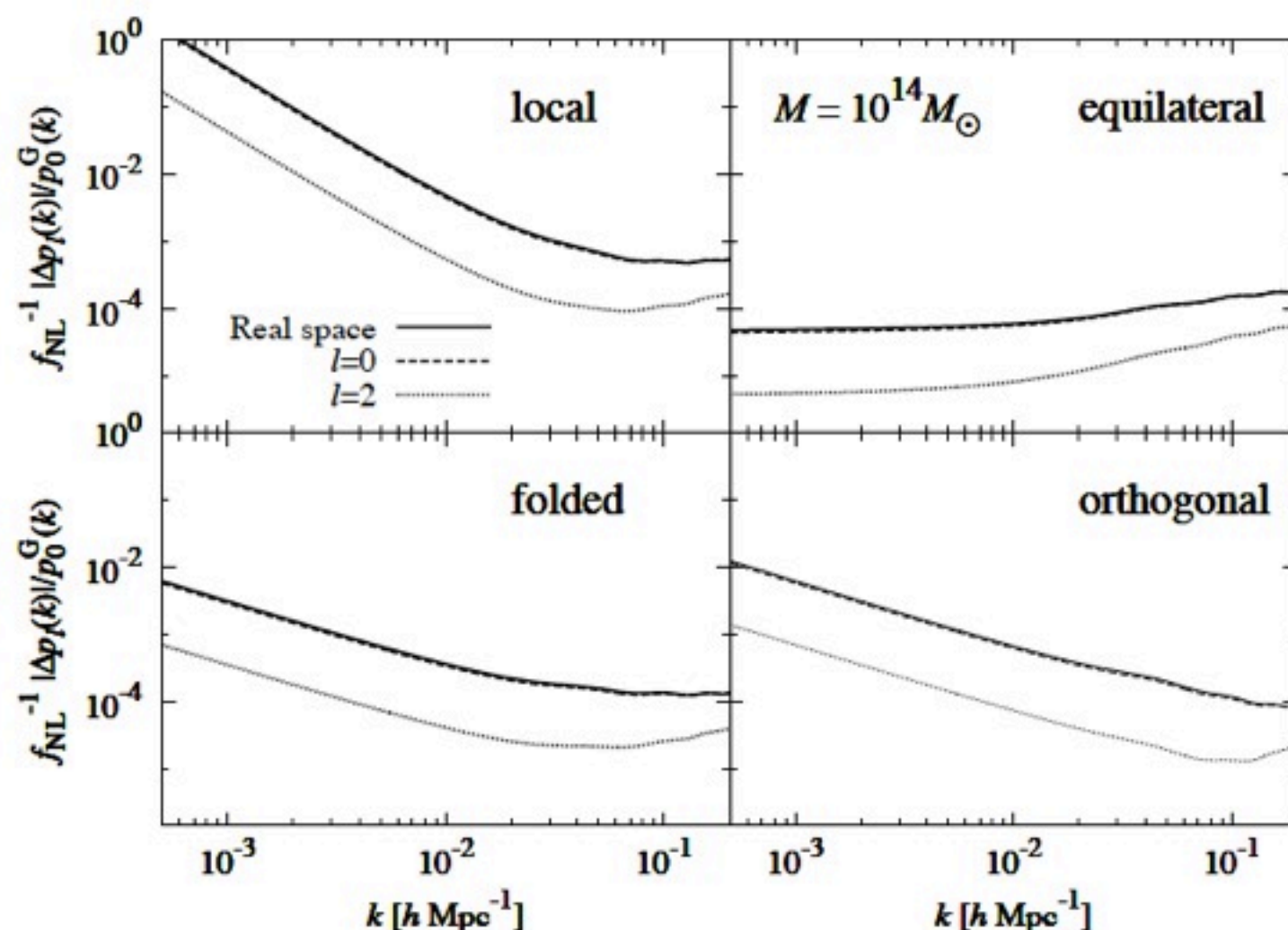
$$\Delta p_4(k), \Delta p_6(k) \ll \Delta p_0(k), \Delta p_2(k),$$

$$\begin{aligned} \Delta p_0(k) = & 2 \left[\frac{f}{3} + \frac{2f^2}{5} + \frac{f^3}{7} + \left(1 + \frac{2f}{3} + \frac{f^2}{5} \right) b_1(k) \right] R_1(k) \\ & - 2 \left[\frac{2f}{21} + \frac{4f^2}{35} + \frac{3f^3}{35} + \left(\frac{2}{7} + \frac{4f}{21} + \frac{2f^2}{15} \right) b_1(k) \right] R_2(k) \\ & + \left[\frac{f}{3} + \frac{f^2}{5} + \left(1 + \frac{f}{3} \right) b_1(k) \right] Q_1(k) \\ & + \left[\frac{f}{3} + b_1(k) \right] Q_2(k), \end{aligned} \quad (147)$$

$$\begin{aligned} \Delta p_2(k) = & 4f \left[\frac{1}{3} + \frac{4f}{7} + \frac{5f^2}{21} + \left(\frac{2}{3} + \frac{2f}{7} \right) b_1(k) \right] R_1(k) \\ & - f \left[\frac{8}{21} + \frac{32f}{49} + \frac{11f^2}{21} + \left(\frac{16}{21} + \frac{13f}{21} \right) b_1(k) \right] R_2(k) \\ & + 2f \left[\frac{1}{3} + \frac{2f}{7} + \frac{1}{3} b_1(k) \right] Q_1(k) + \frac{2f}{3} Q_2(k), \end{aligned} \quad (148)$$

$$\begin{aligned} \Delta p_4(k) = & 16f^2 \left[\frac{2}{35} + \frac{3f}{77} + \frac{1}{35} b_1(k) \right] R_1(k) \\ & - \frac{4f^2}{35} \left[\frac{16}{7} + \frac{26f}{11} + b_1(k) \right] R_2(k) + \frac{8f^2}{35} Q_1(k), \end{aligned} \quad (149)$$

$$\Delta p_6(k) = \frac{32f^3}{231} R_1(k) - \frac{8f^3}{231} R_2(k). \quad (150)$$



Higher-order nonGaussianity

- Higher-order analysis

- (Yokoyama & TM 2012, Yokoyama, Matsubara & Taruya 2013)
- Contributions from gNL (prim. trispec)
- Analysis of bispectrum

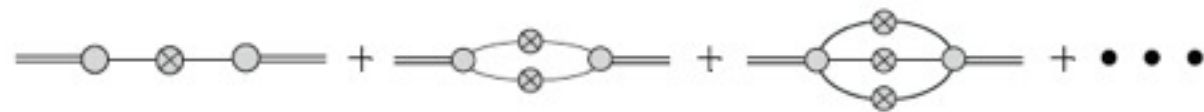


FIG. 5: Pure Gaussian case



FIG. 6: Diagrams linearly proportional to the primordial bispectrum.

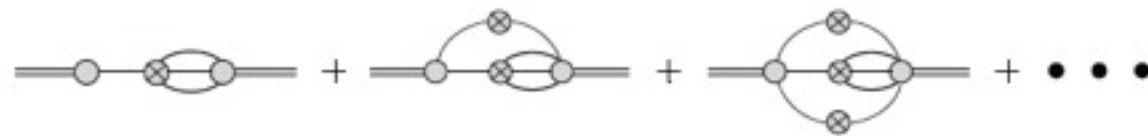
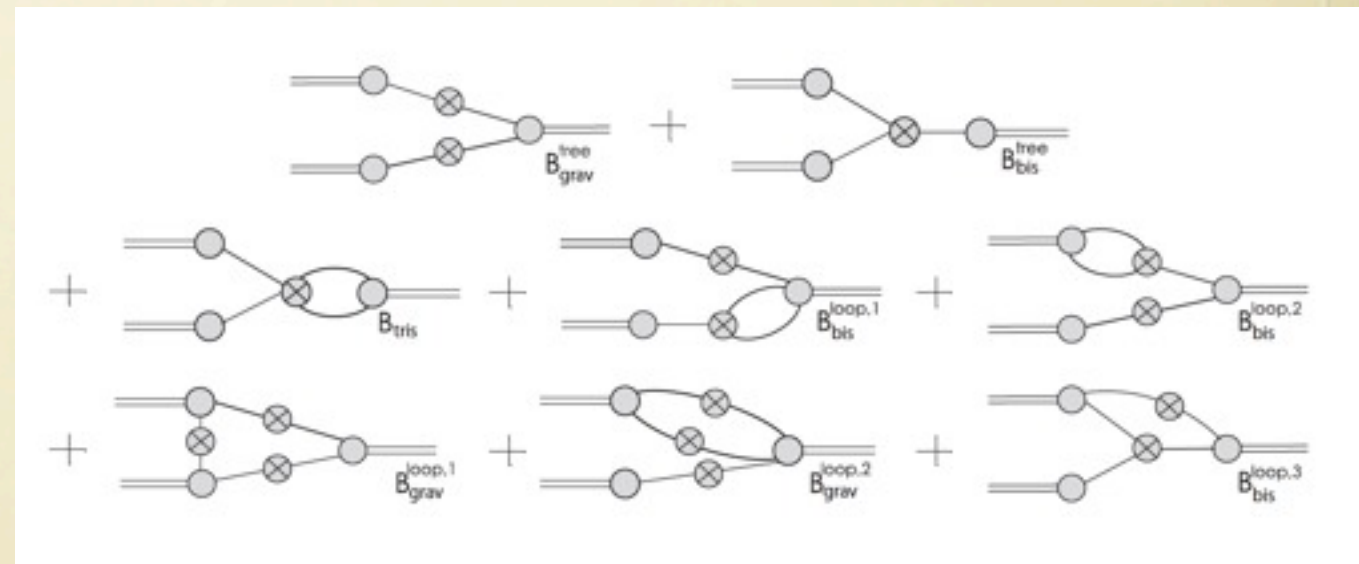
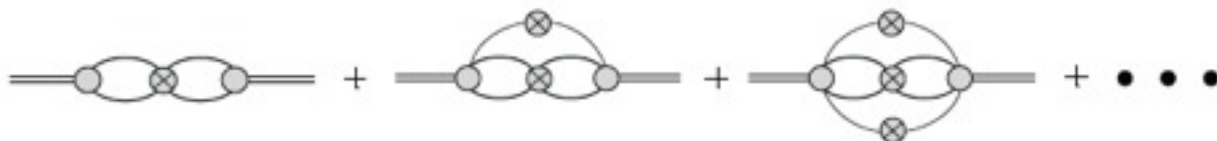
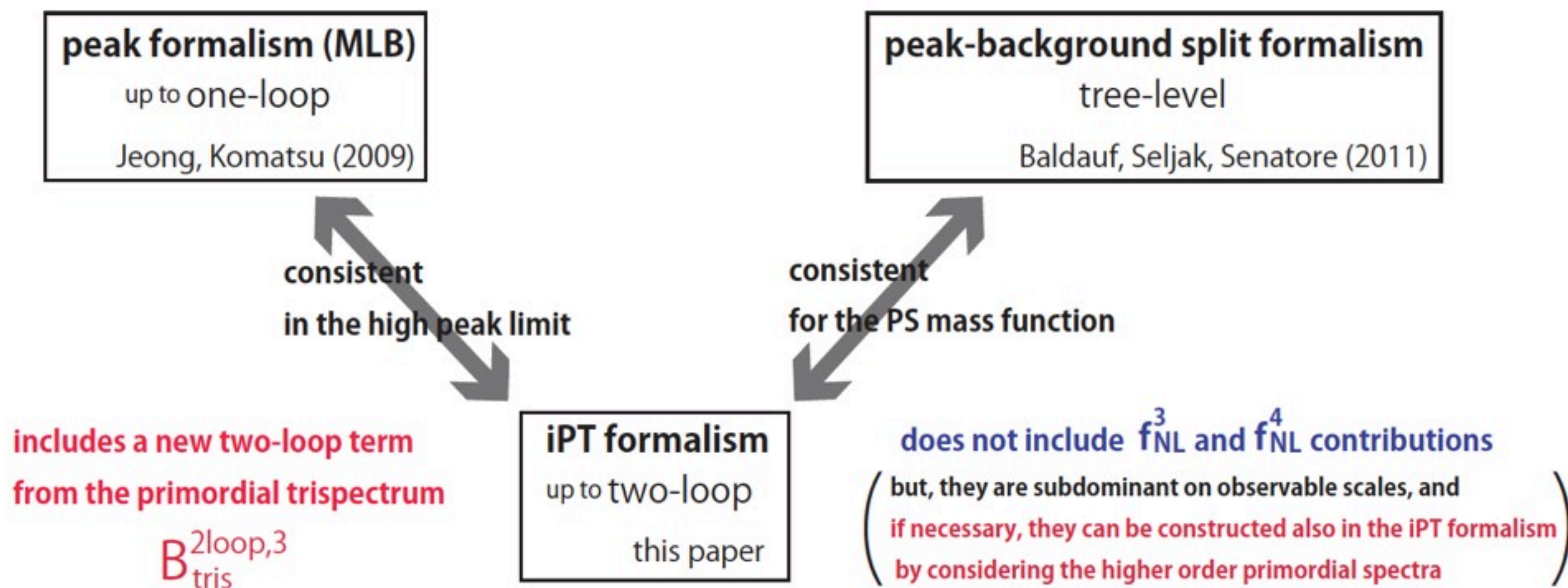


FIG. 7: Diagrams linearly proportional to the primordial trispectrum.

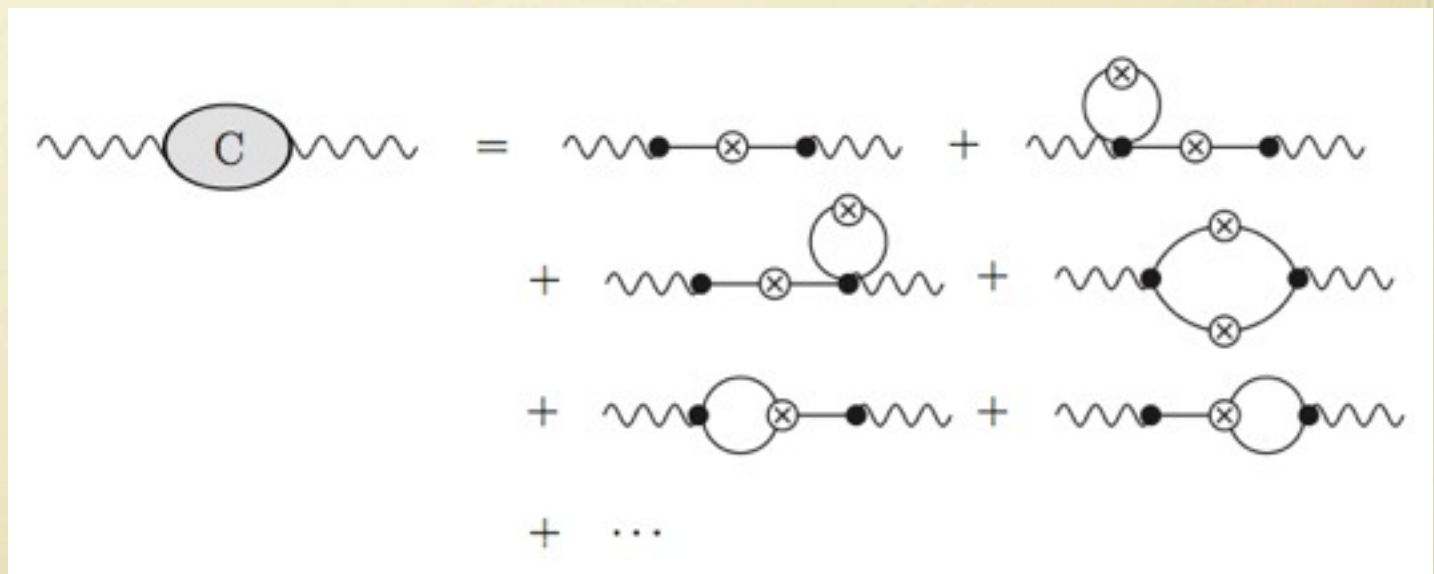
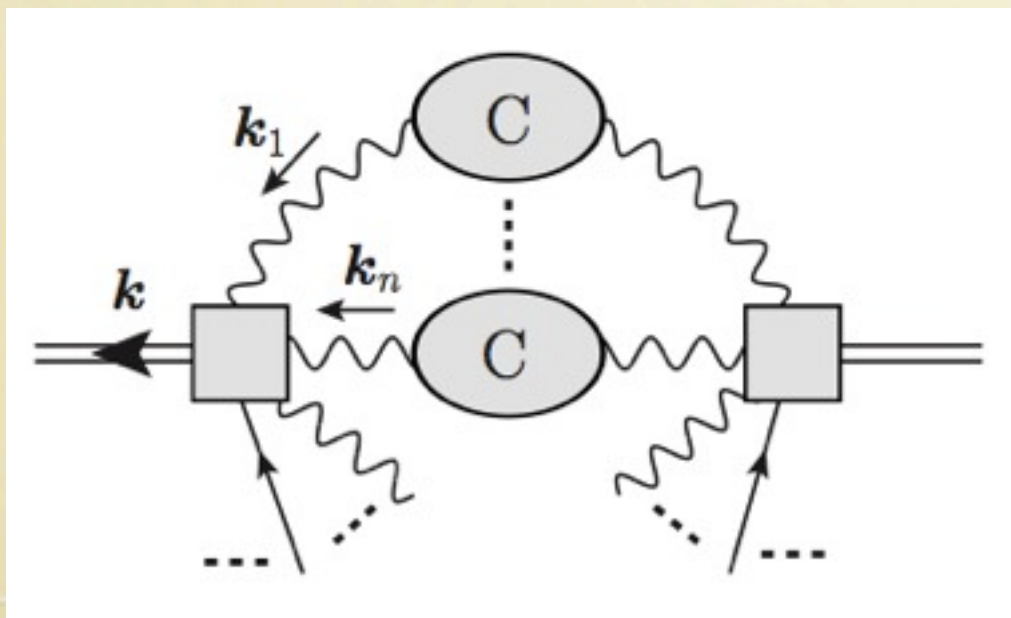
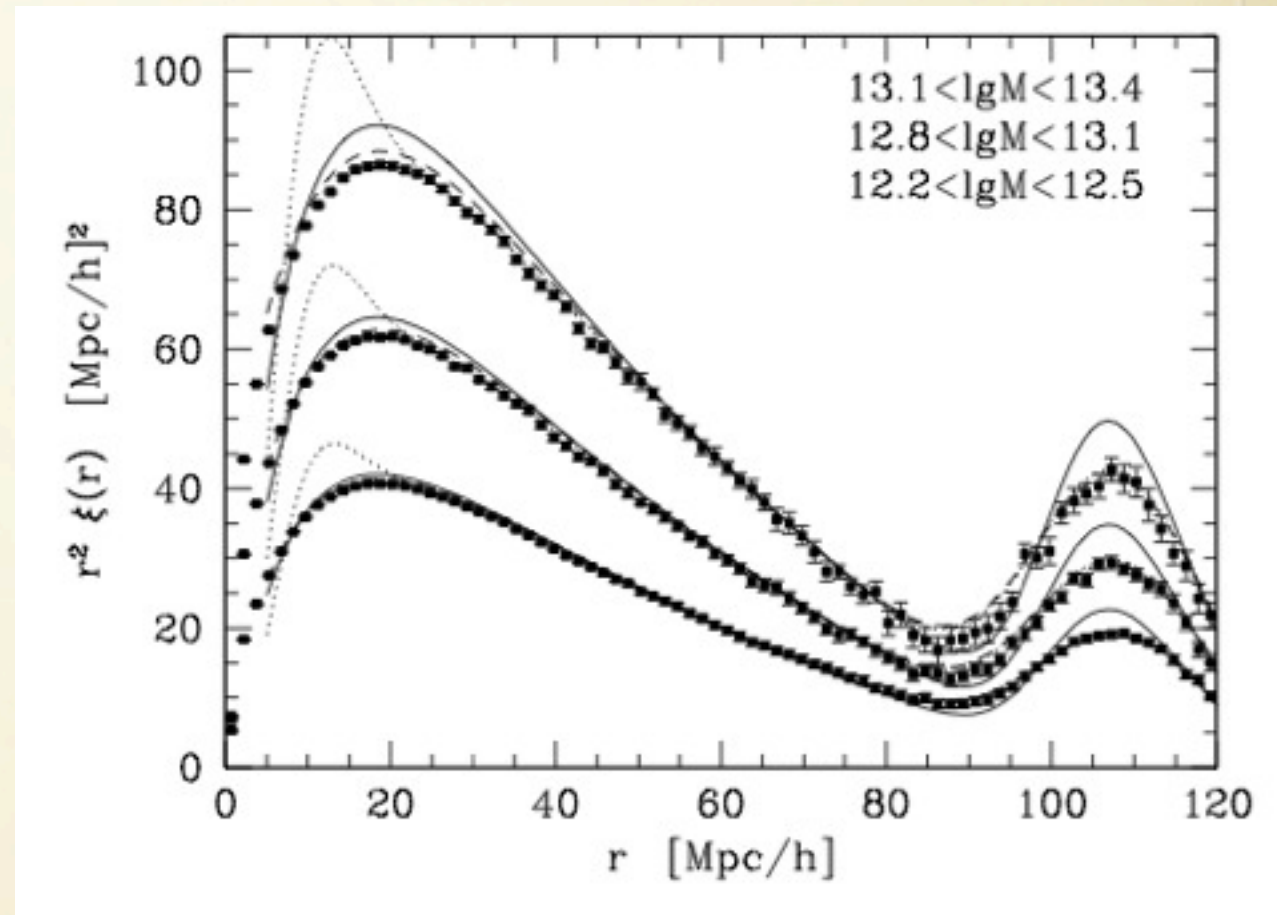


Halo bispectrum on large scales with primordial NG (up to the trispectrum)



Relation to CLPT

- Convolution Lagrangian perturbation theory (Carlson+ 13)
 - additional resummations on top of iPT
 - Diagrammatically, their method is equivalent resumming the following contributions:



Summary

- **Integrated perturbation theory**
 - a consistent formulation of nonlinear perturbation theory, generally including nonlocal bias, RSD and nG
 - vertex resummations and bias renormalizations
- **Scale-dependent bias from iPT**
 - Previous formulas such as those of PBS are re-derived from the new formula by taking appropriate limits
- **and many other applications for observables**
- **Future issue**
 - Improved bias models beyond the simple halo approach
 - Higher-order corrections (2-loop and beyond)

