Center for Astroparticle Physics
GENEVA

UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES

UVB fluctuations with source clustering: an analytic approach

Vincent Desjacques

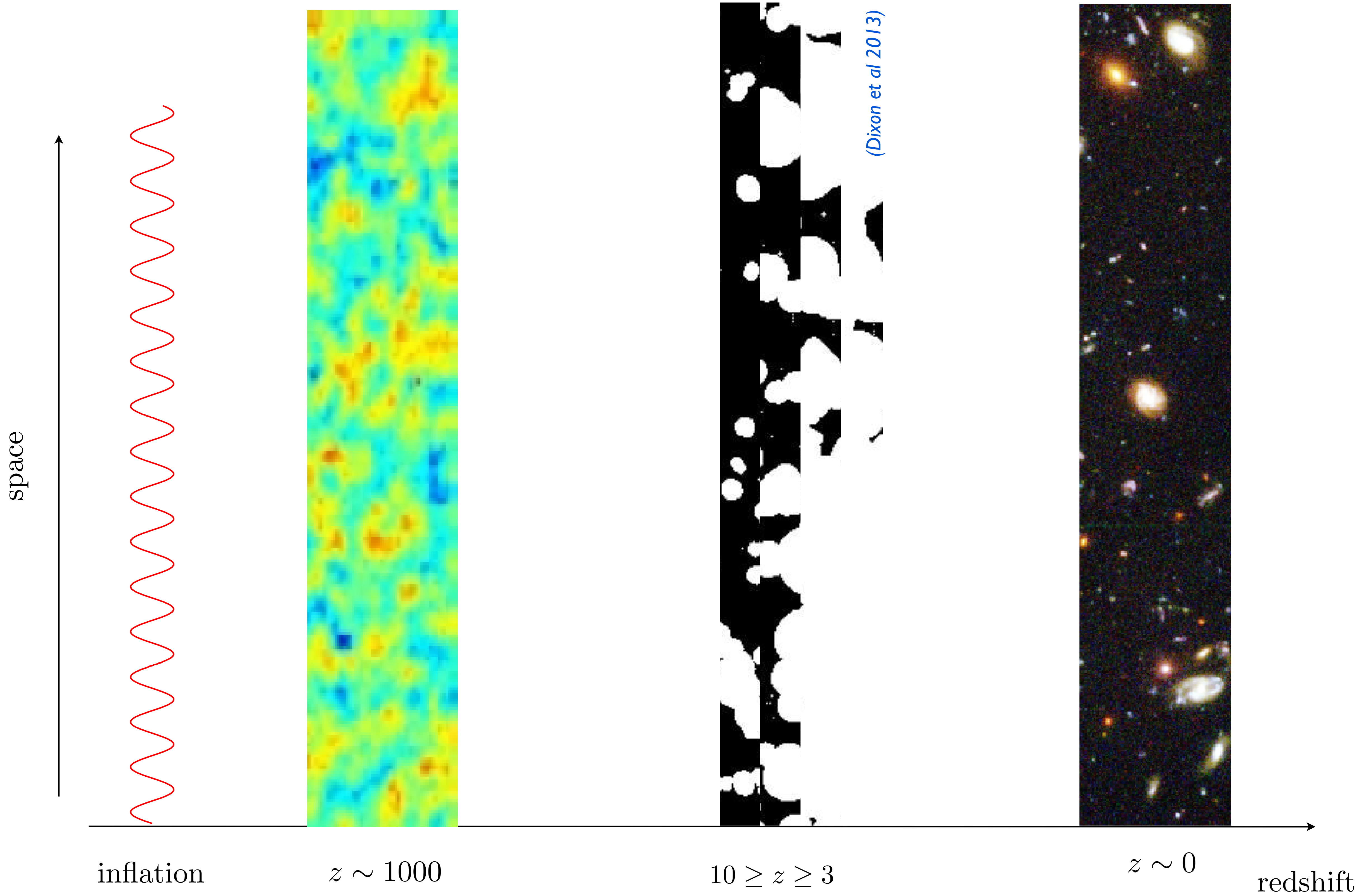
IBS workshop on LSS, Daejeon, 10 July, 2014

Outline

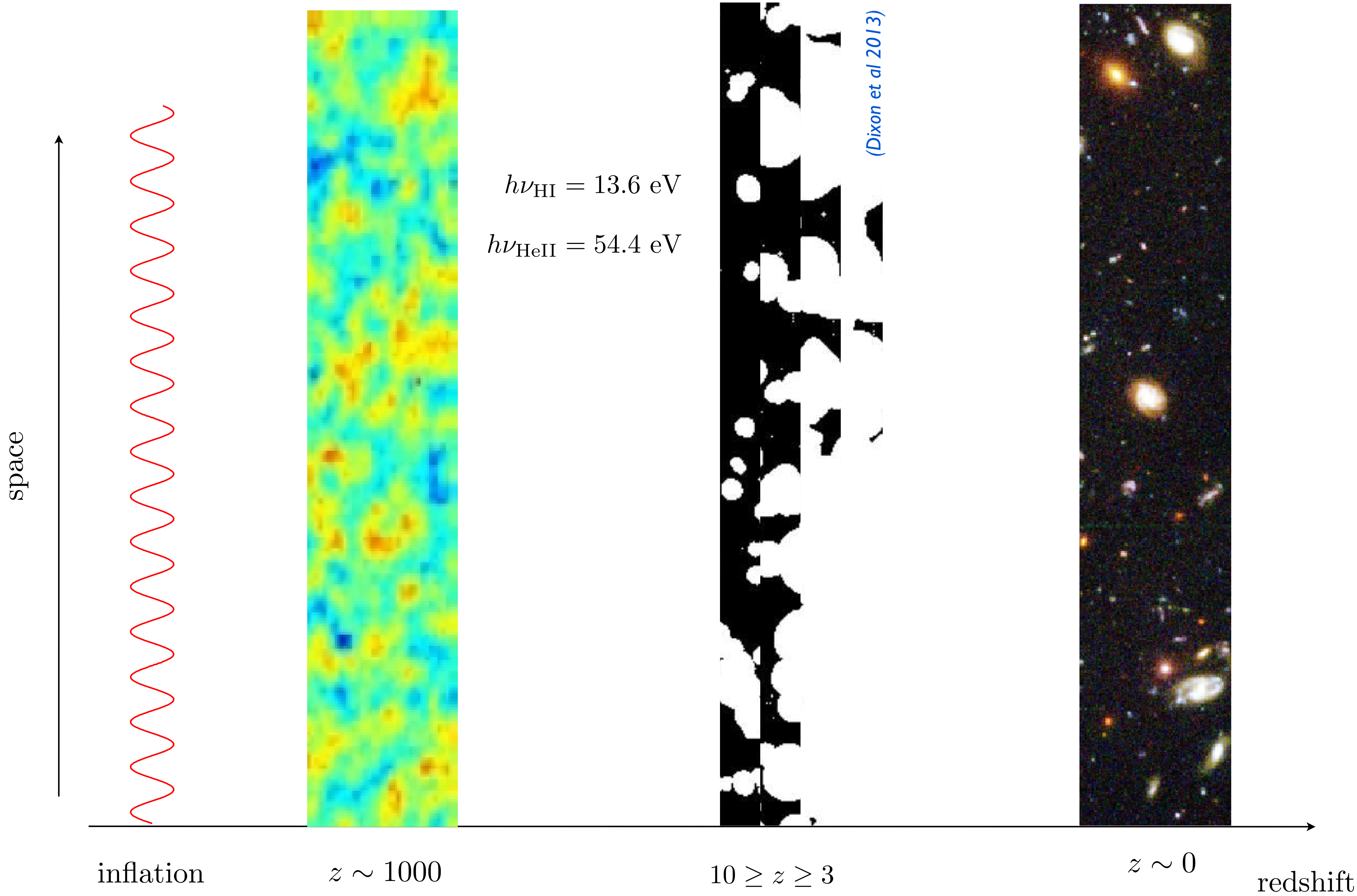
- *Reionization of the Universe*
- *Count-in-cells approach to intensity fluctuations*
- *Application to Helium-II ionizing background*

1406.6379

Reionization



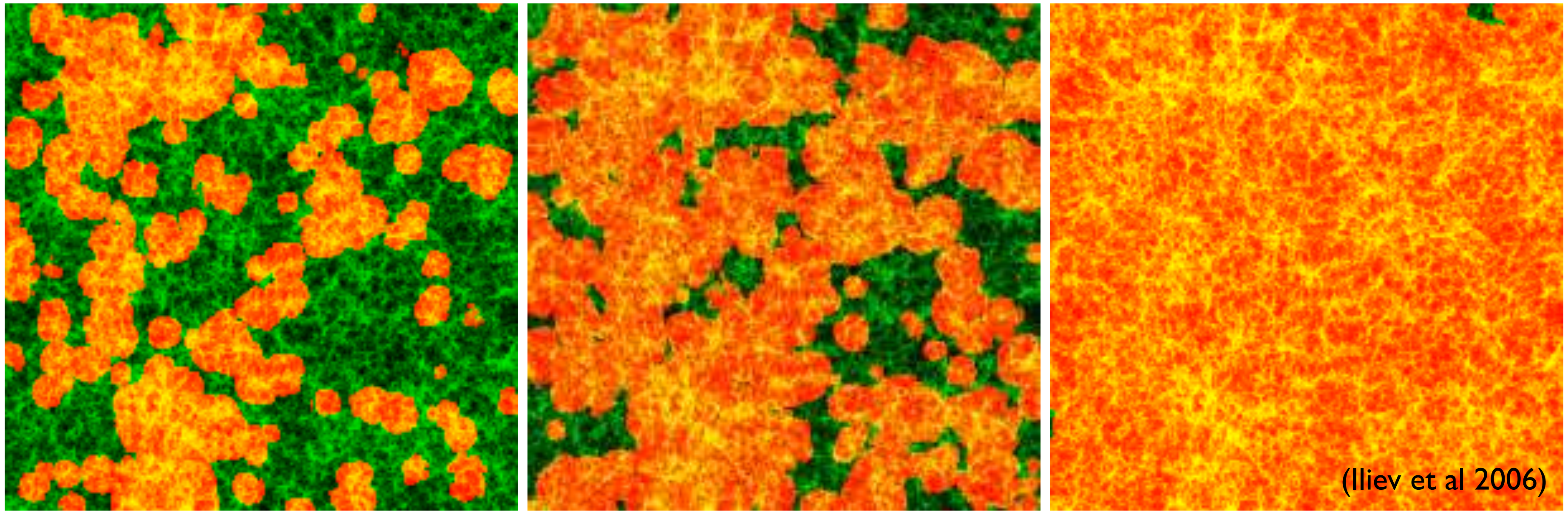
Reionization



Simulating Helium-II reionization

Challenging:

- *Need small scales: IGM physics*
- *Need large scales: quasars (QSOs) are rare*



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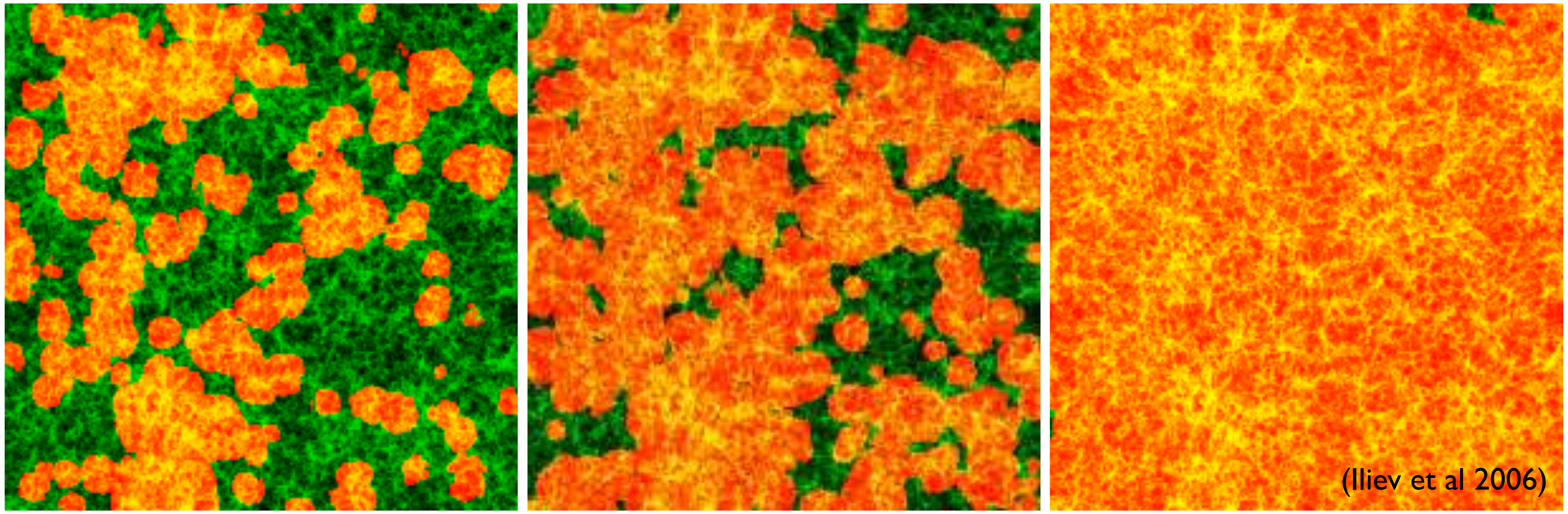

Analytic/semi-numeric methods can provide physical insights into certain aspects of the problem

The importance of source clustering

Is quasar clustering important at the end of Helium-II reionization ?

Scales

- *(Comoving) attenuation length or photon mean free path*

$$r_0 \sim 30 - 50 \text{ Mpc}$$

(Bolton & Haehnelt 2006; Furlanetto & Oh 2008)

- *Quasar (QSO) clustering length*

$$r_\xi \sim 15 - 30 \text{ Mpc}$$

(Shen et al 2007; Francke et al 2008)

- *QSO number density*

$$l = \bar{n}^{-1/3}$$

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Quasar clustering important if:

i) $r_\xi / r_0 \gtrsim 1$

ii) $r_0 / l \gg 1$

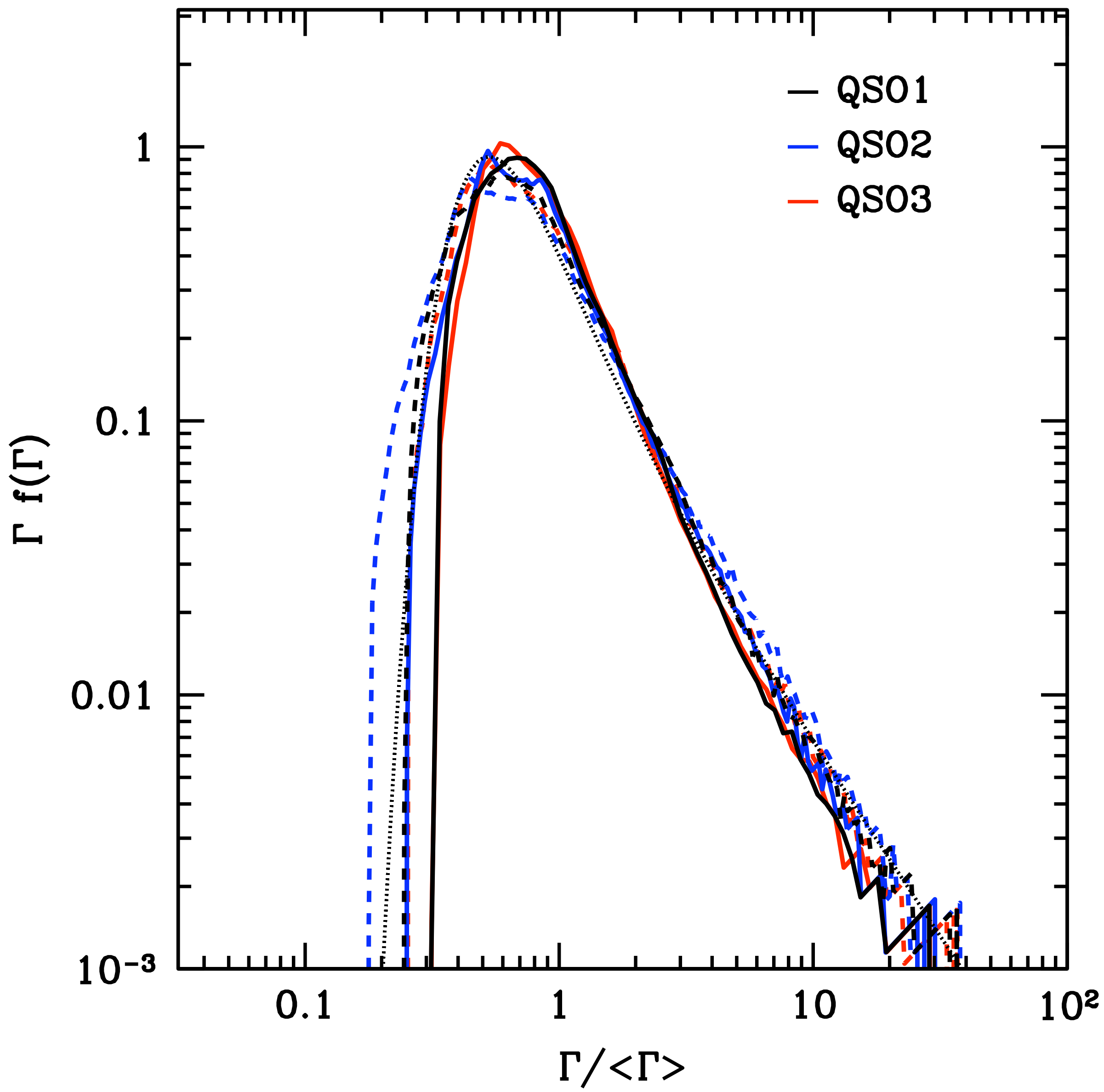
Effect of QSO clustering on intensity distribution

$$\text{Photoionization rate}: \quad \Gamma = \int_{\nu_{\rm HeII}}^{\infty} d\nu \, \frac{J_\nu}{h\nu} \, \sigma_{\rm HeII}(\nu)$$

QSO1 $=$ sub-sample of halos with $M \geq 5 \times 10^{11} M_\odot$

QSO2 $=$ most massive halos

QSO3 $=$ randomly distributed

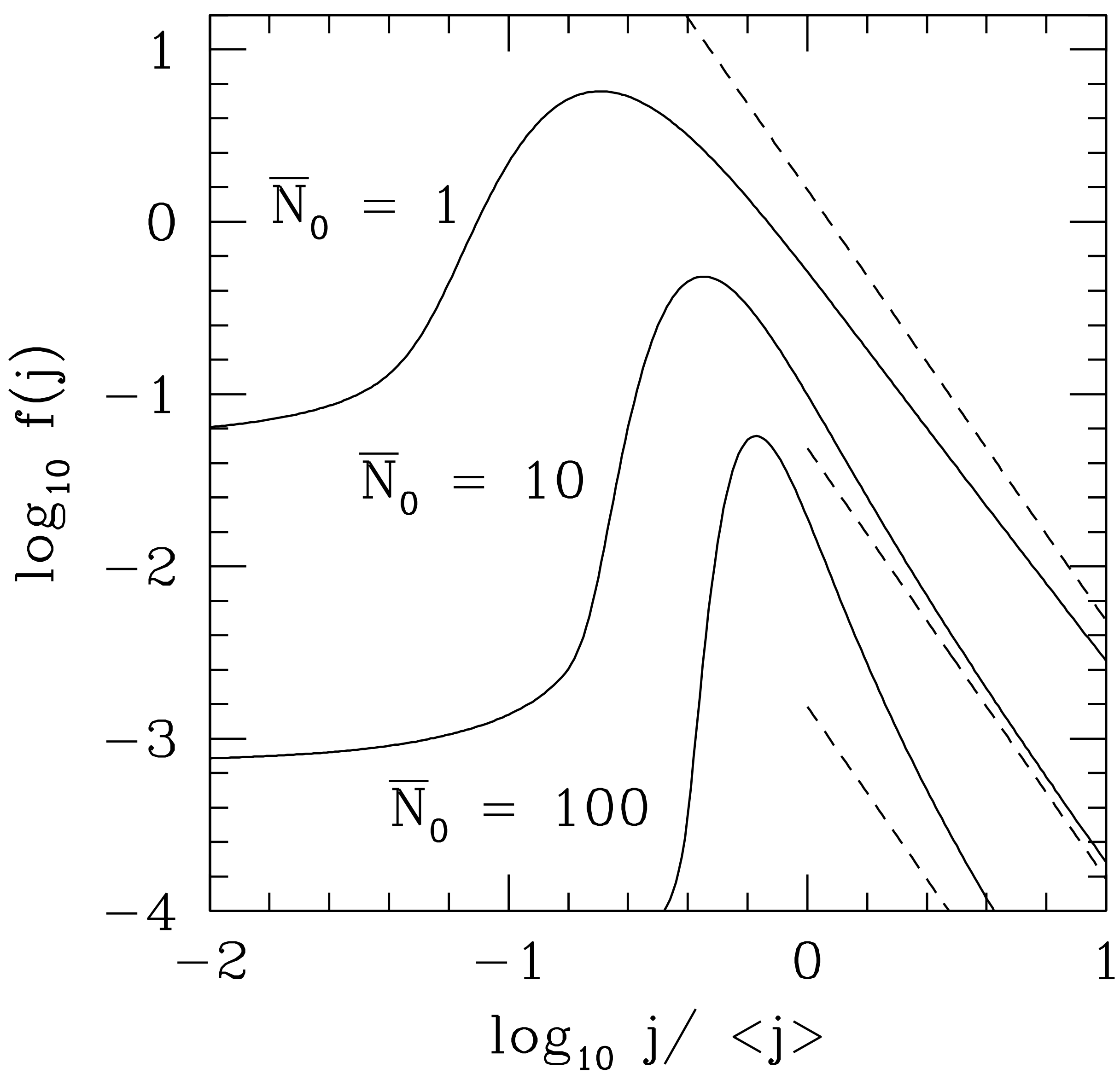


(Dixon, Furlanetto & Mesinger 2013)

Plan

Extend the work of Zuo 1992; Fardall & Shull 1993, Meiksin & White 1993; who worked out P(J) analytically for randomly distributed sources

$$\bar{N}_0 = \frac{4\pi}{3}\bar{n}r_0^3$$



(Meiksin & White 2003)

Count-in-cells formalism (I)

Consider randomly-located cells of volume V.

- *Probability to have an empty cell:*

$$P_0 = P\big(\Phi_0(V)\big) = \exp\big(\mathcal{W}_0(V)\big)$$

- *Conditional void correlation:*

$$\begin{aligned}\mathcal{W}_0(V) &= \sum_{k=1}^{\infty} \frac{(-\bar{n})^k}{k!} \int_V d^3\mathbf{x}_1 \ldots \int_V d^3\mathbf{x}_k\, \xi_k(\mathbf{x}_1, \ldots, \mathbf{x}_k) \\ &= \sum_{k=1}^{\infty} \frac{(-\bar{N})^k}{k!} \bar{\xi}_k(V)\end{aligned}$$

$$\bar{N} = \bar{n} V$$

- *Volume-averaged irreducible correlations:*

$$\bar{\xi}_k(V) \equiv \frac{1}{V^k} \int_V d^3\mathbf{x}_1 \ldots \int_V d^3\mathbf{x}_k\, \xi_k(\mathbf{x}_1, \ldots, \mathbf{x}_k)$$

Fall et al. 1976; White 1979; Peebles 1980; Fry 1985; Balian & Schaefer 1989; Szapudi & Colombi 1996

Count-in-cells formalism (II)

Consider randomly-located cells of volume V.

- *Count-in-cells probabilities*

$$P_N(V) = \frac{(-\bar{n})^N}{N!} \frac{d^N}{d\bar{n}^N} \exp[\mathcal{W}_0(V)]$$

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- *Poisson distribution:* $\bar{\xi}_1(V) \equiv 1, \qquad \mathcal{W}_0(V) = -\bar{N}$

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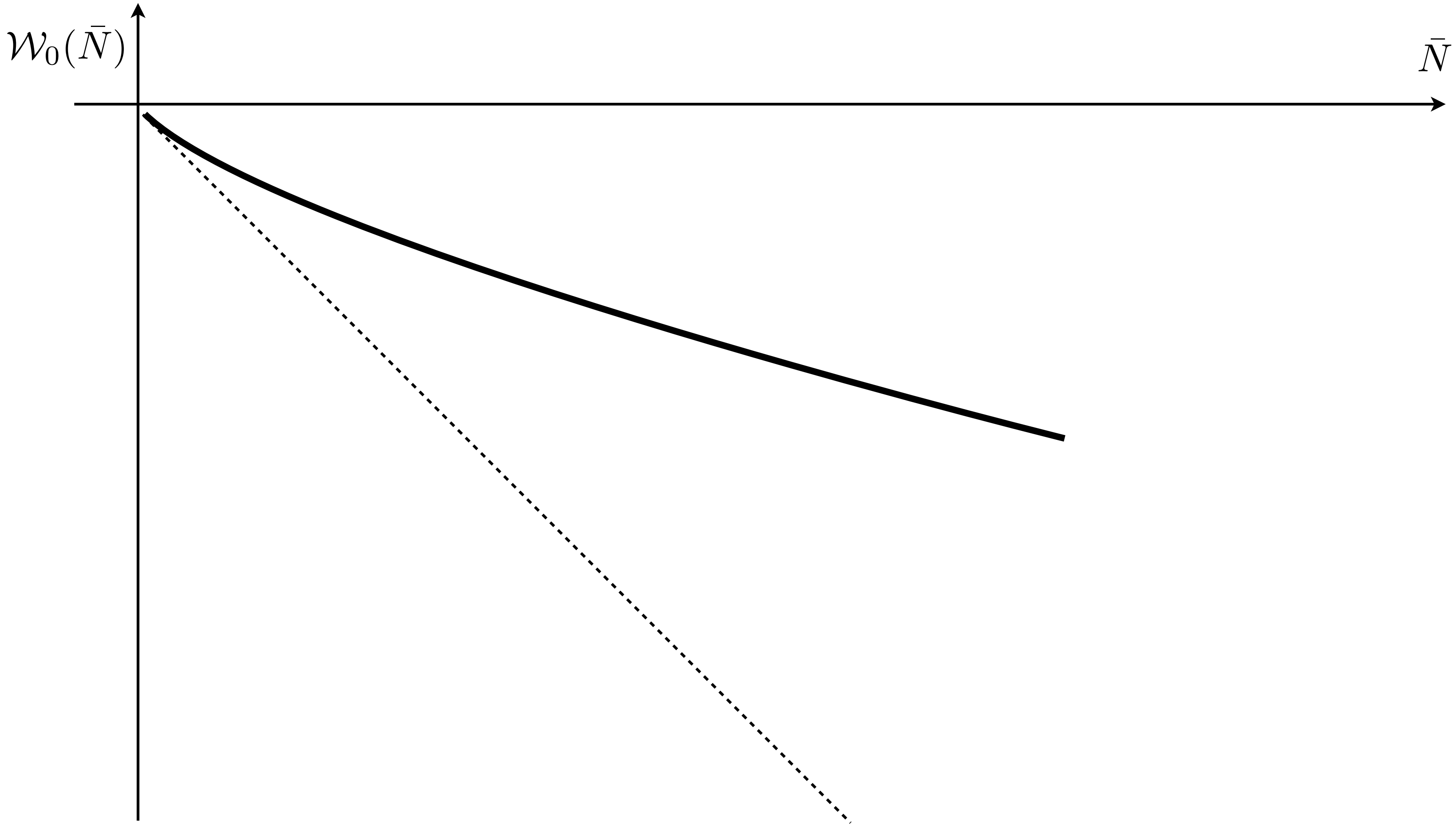

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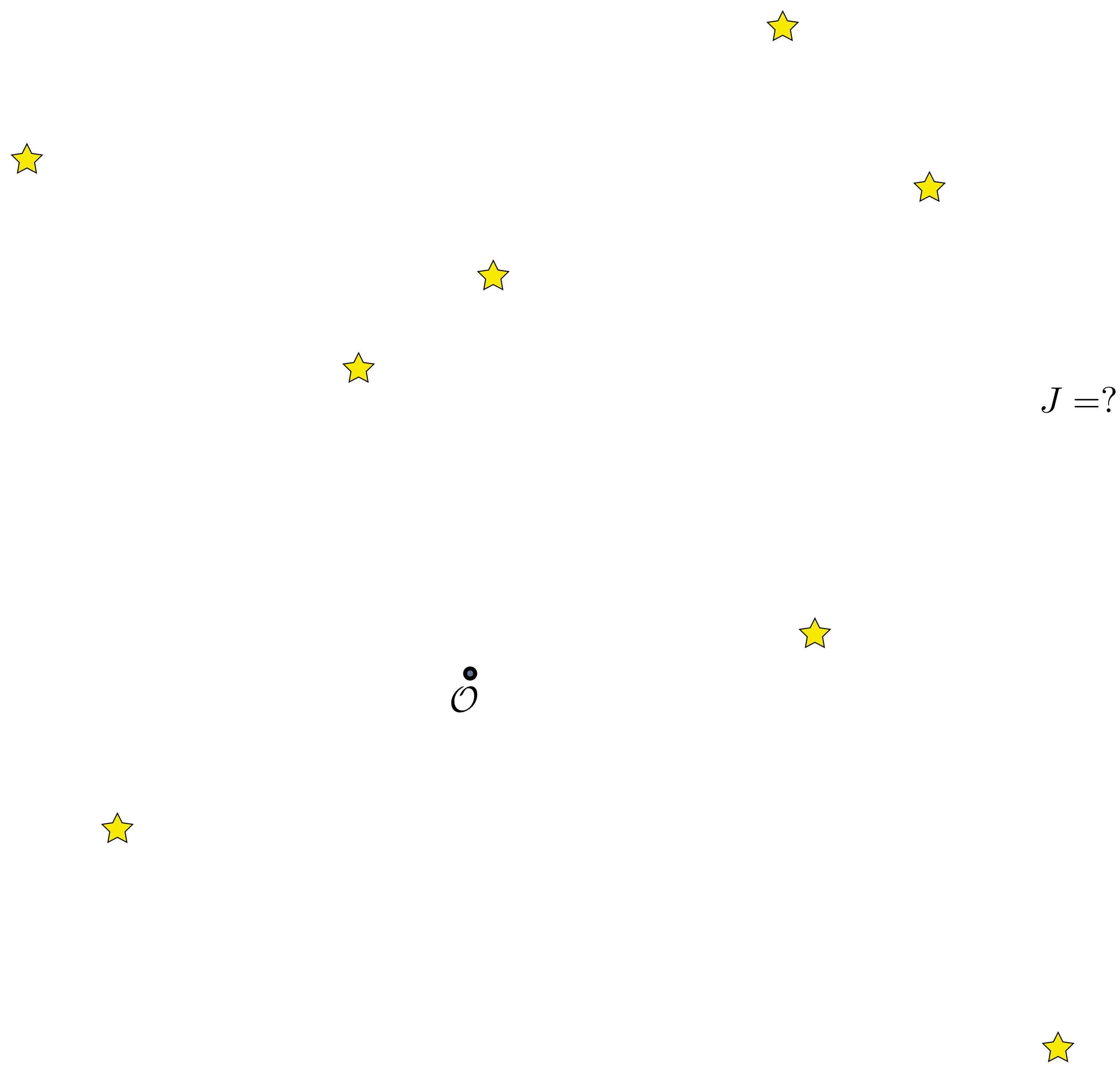
Intensity distribution



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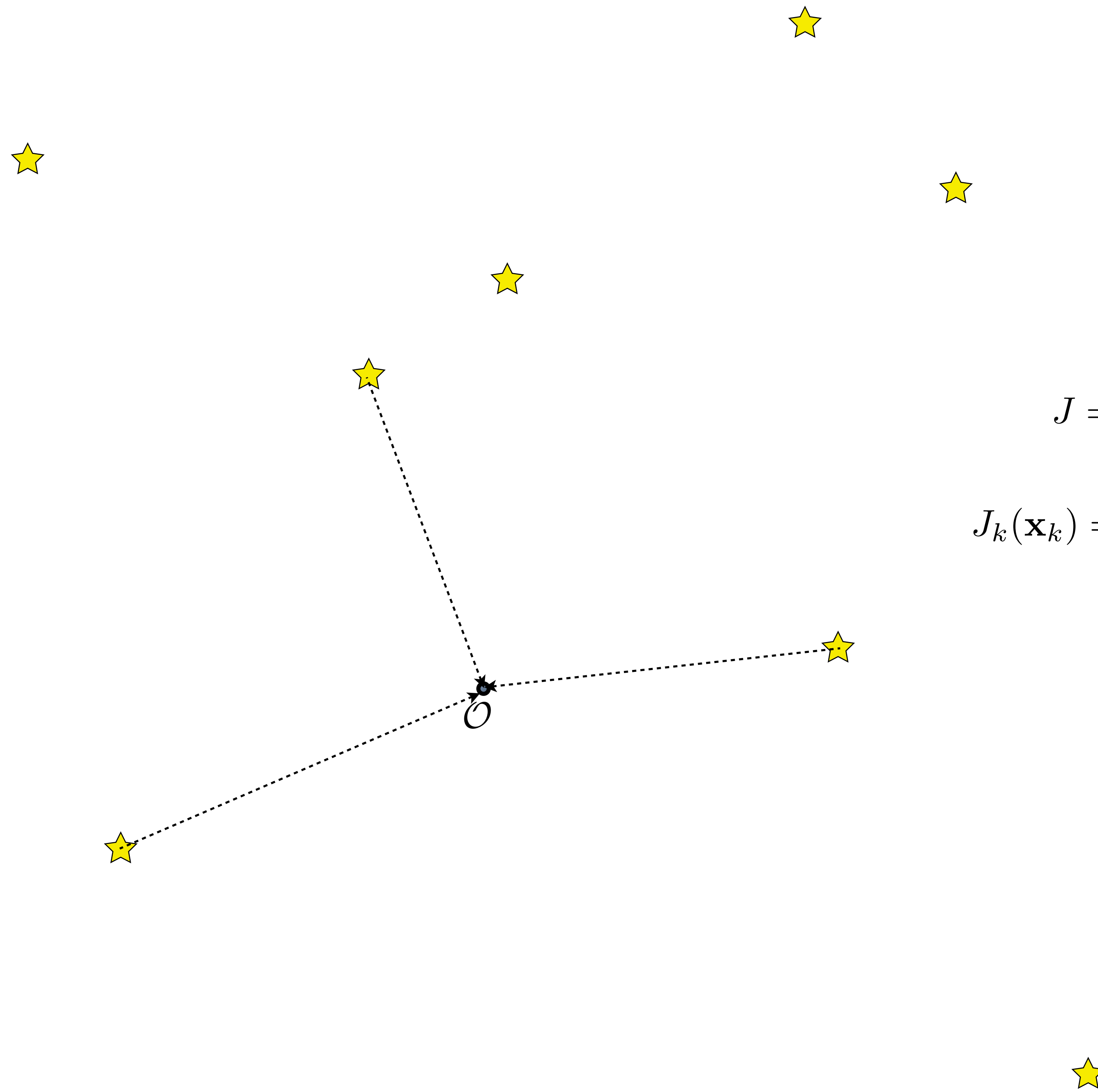
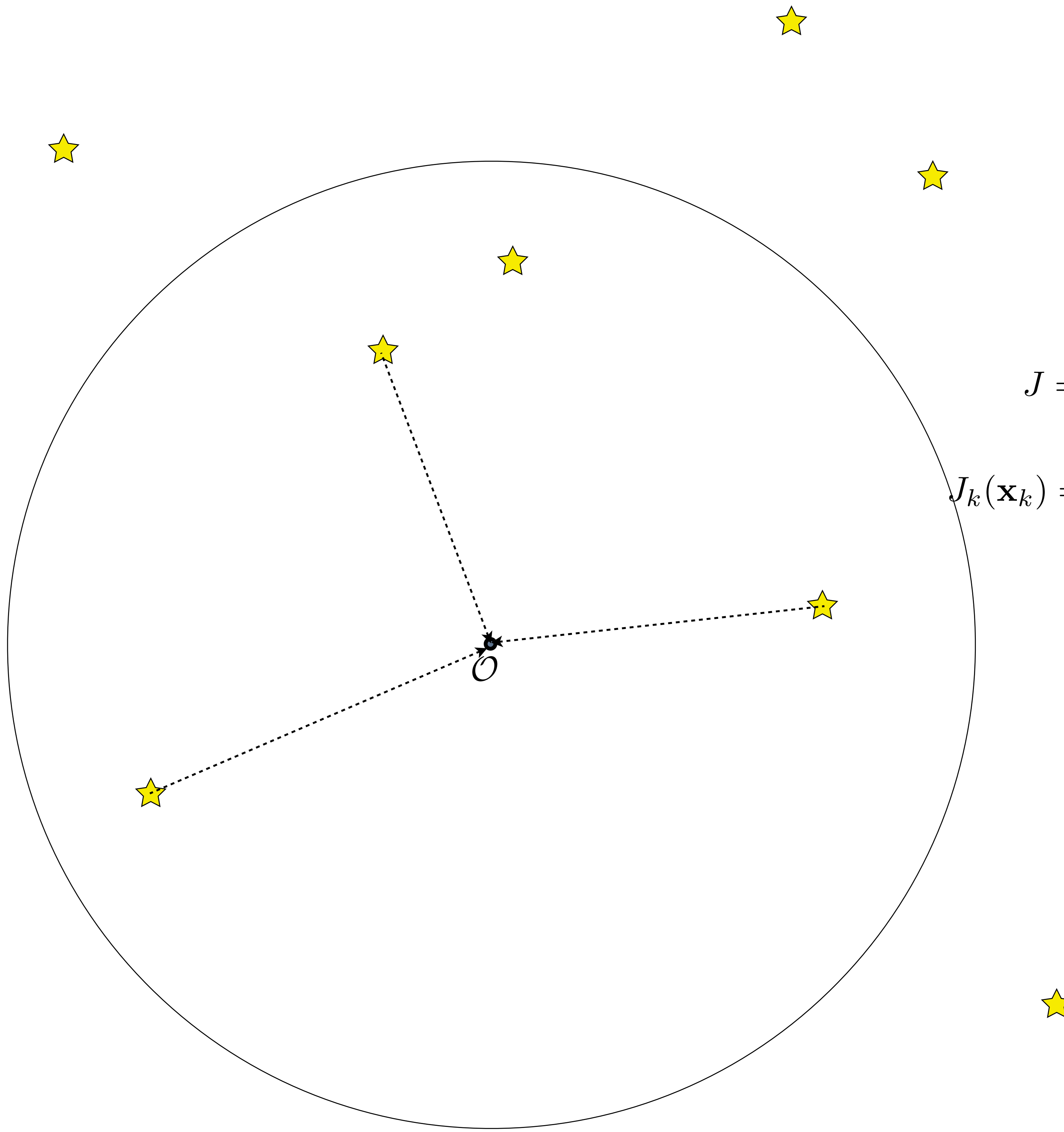


$$J = \sum_k J_k(\mathbf{x}_k)$$

$$J_k(\mathbf{x}_k) = (1+z)^2 \frac{L_k}{(4\pi r_k)^2} e^{-r_k/r_0}$$

Intensity distribution



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Quasar luminosities

- *Standard double power-law form for the bolometric QLF:*

$$\Phi(L,z) = \frac{\Phi_\star(z)/L_\star(z)}{(L/\ L_\star(z))^{\beta_1(z)} + (L/L_\star(z))^{\beta_2(z)}}$$

(Boyle, Shanks & Peterson 1988)

- *Normalized luminosity:*

$$L = \alpha\, L_\star$$

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(Boyle, Shanks & Peterson 1988)

- *Normalized luminosity:*

$$L = \alpha\, L_\star$$

- *At z=3:*

$$\begin{aligned} \Phi_\star &= 2.56 \times 10^{-6}\ \mathrm{Mpc}^{-3} \\ L_\star &= 10^{13.17}\ \mathrm{L}_\odot \\ \beta_1 &= 1.395 \\ \beta_2 &= 3.10 \end{aligned}$$

(Hopkins, Richards & Hernquist 2007)

Count-in-cells with weight

- *Assign a weight to each point:*

$$1 = \sum_{N=0}^{\infty} \frac{1}{N!} \int \ldots \int P\{X_1 \ldots X_N \big| \Phi_0(V)\} \, e^{\mathcal{W}_0(V)}$$

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$$\downarrow$$

$$P_\omega(V) = \sum_{N=0}^{\infty} \frac{1}{N!} \int \dots \int P\{X_1 \dots X_N \big| \Phi_0(V)\} \, \omega(\mathbf{x}_1) \dots \omega(\mathbf{x}_N) \, e^{\mathcal{W}_0(V)}$$

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- *Compact expression:*

$$P_\omega(V) = e^{\mathcal{W}_\omega(V)} - e^{\mathcal{W}_0(V)}$$

$$\mathcal{W}_\omega(V) = \sum_{k=1}^{\infty} \frac{(-\bar{n})^k}{k!} \int_V d^3\mathbf{x}_1 \ldots \int_V d^3\mathbf{x}_k\, \xi_k(\mathbf{x}_1, \ldots, \mathbf{x}_k)\, \big(1 - \omega(\mathbf{x}_1)\big) \ldots \big(1 - \omega(\mathbf{x}_k)\big)$$

Application to UV background

- *Weight is provided by the Quasar contribution to the specific intensity at x=0*

- *Each configuration of N quasars in cells of volume V contributes*

$$\int d\alpha_1 \dots d\alpha_N \, \phi(\alpha_1) \dots \phi(\alpha_N) \times P\{X_1 \dots X_N \Phi_0(V)\} \times \delta_D(J_1 + \cdots + J_N - J)$$

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- *Substitute Laplace/Fourier representation:*

$$\delta_D(J_1 + \cdots + J_N - J) = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} ds \, e^{s(J - J_1 - \cdots - J_N)}$$

$$\omega(\mathbf{x}_k) = \Theta_H(R - |\mathbf{x}_k|) \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha_k \, \phi(\alpha_k) \, e^{-sJ_k(\mathbf{x}_k)}$$

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- *Intensity distribution is*

$$P(J) = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} ds \, e^{sJ + \mathcal{W}_\omega(V)}$$

Hierarchical ansatz

- *Volume-averaged correlation functions are of the form*

$$\bar{\xi}_k = S_k \, \bar{\xi}_2^{k-1}$$

- *Under this assumption, we can recast the conditional void correlation into the form*

$$\begin{aligned}\mathcal{W}_0(V) &= -\bar{N} \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k!} S_k \left(\bar{N}\bar{\xi}_2\right)^{k-1} \\ &\equiv -\bar{N}\chi(\bar{N}\bar{\xi}_2)\end{aligned}$$

$$\text{void scaling function :} \quad \chi = -\frac{\ln\left(P_0\right)}{\bar{N}} = -\frac{\mathcal{W}_0(V)}{\bar{N}}$$

e.g.

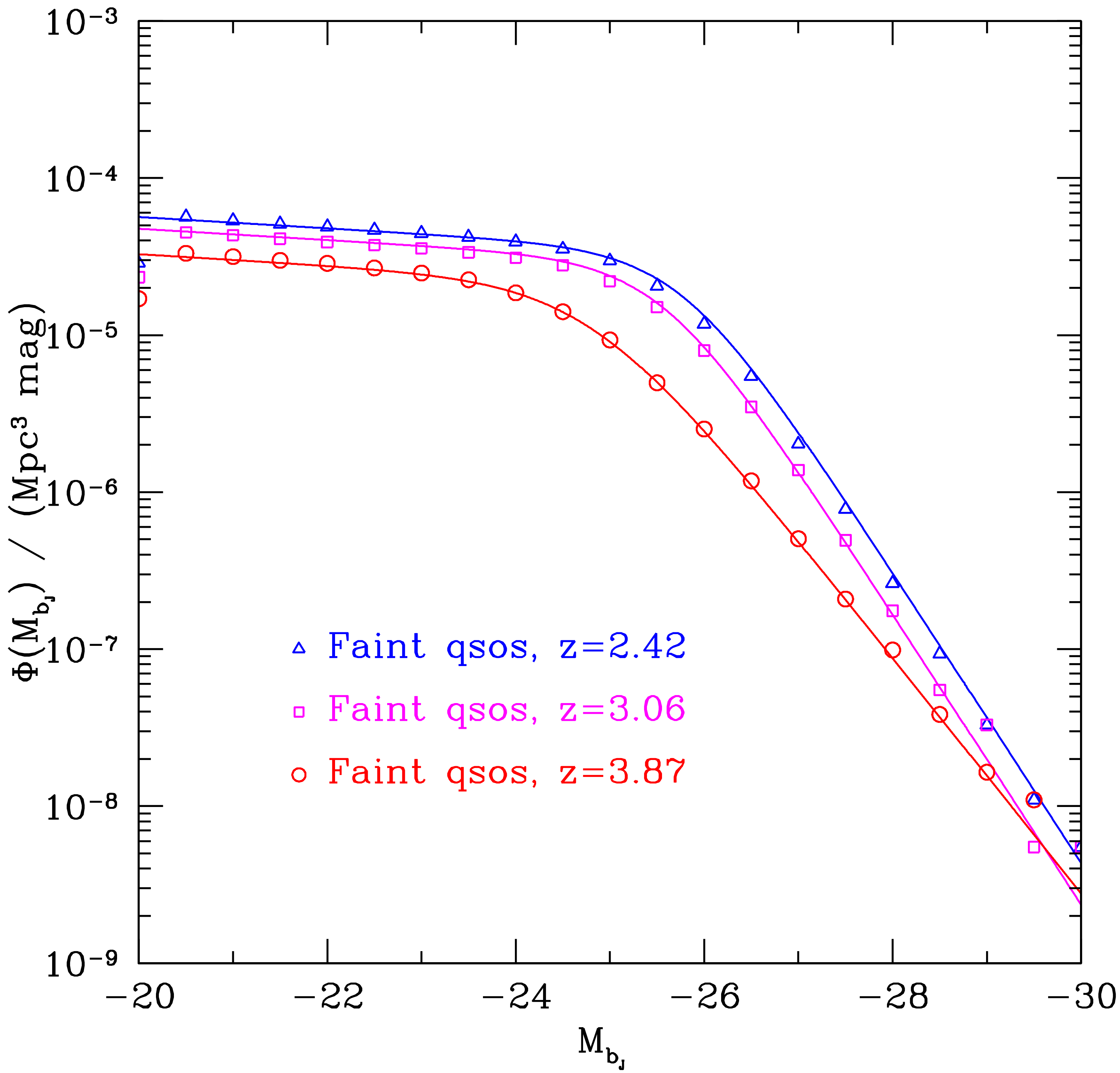
$$\chi(\bar{N}\bar{\xi}_2) = \frac{\ln\left(1+\bar{N}\bar{\xi}_2\right)}{\bar{N}\bar{\xi}_2} \qquad \text{(Negative Binomial)}$$

$$\chi(\bar{N}\bar{\xi}_2) = \frac{1}{1+\frac{1}{2}\bar{N}\bar{\xi}_2} \qquad \text{(Geometric Hierarchical)}$$

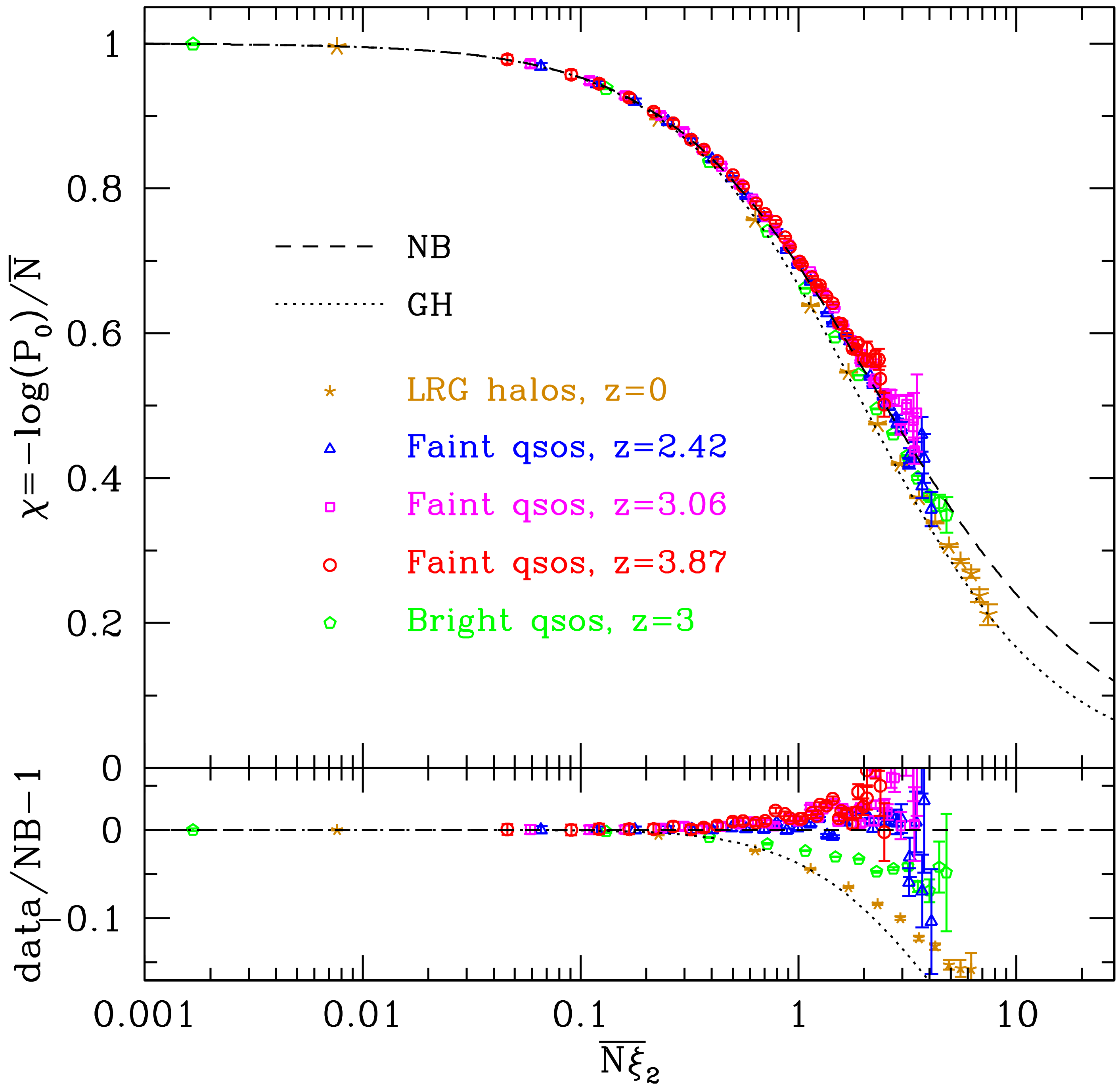
Test with mock quasars

Synthetic QSO catalogues constructed from the Millennium simulation

(Croton 2009)



Quasar void scaling function

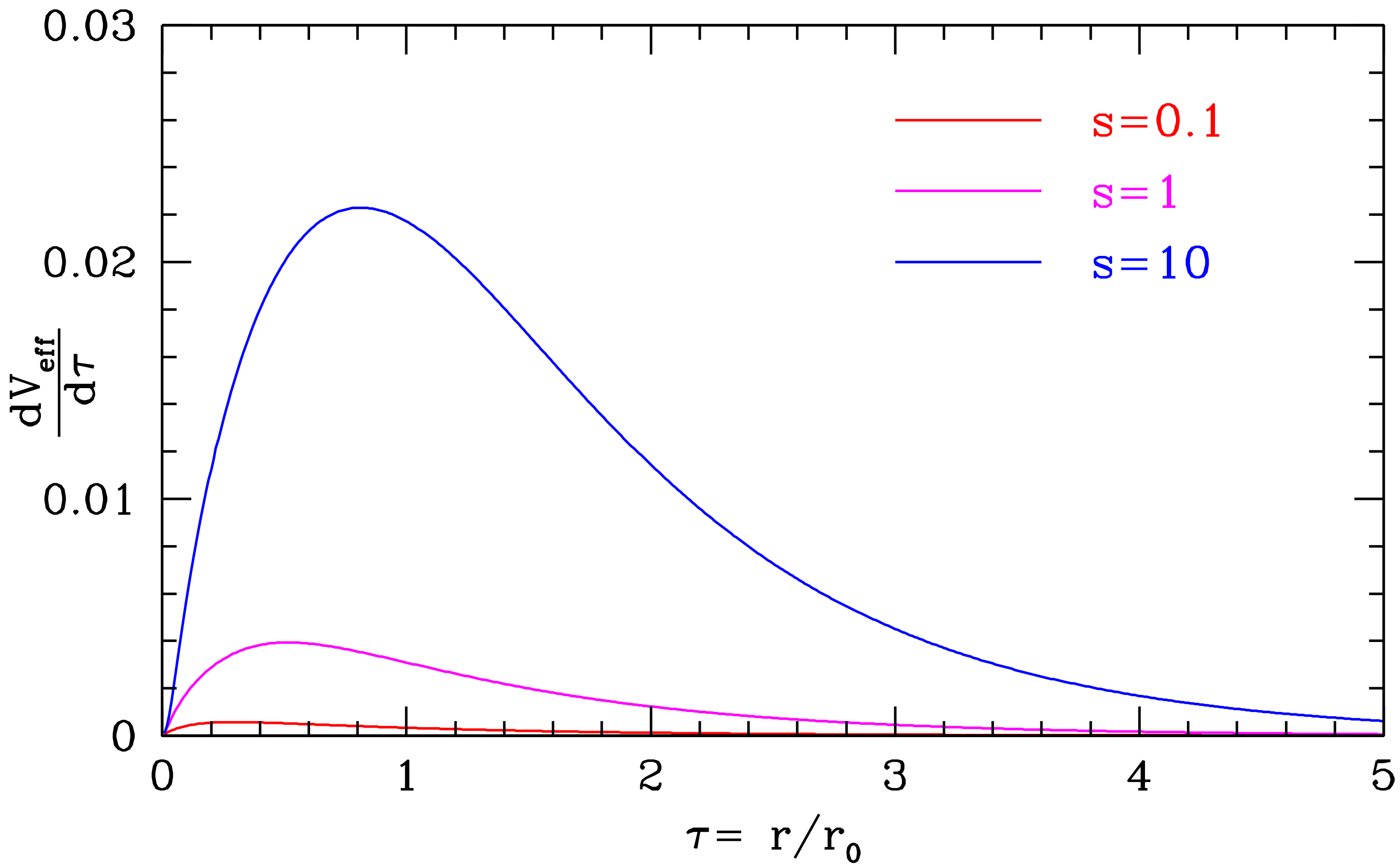


UVB fluctuations in hierarchical models

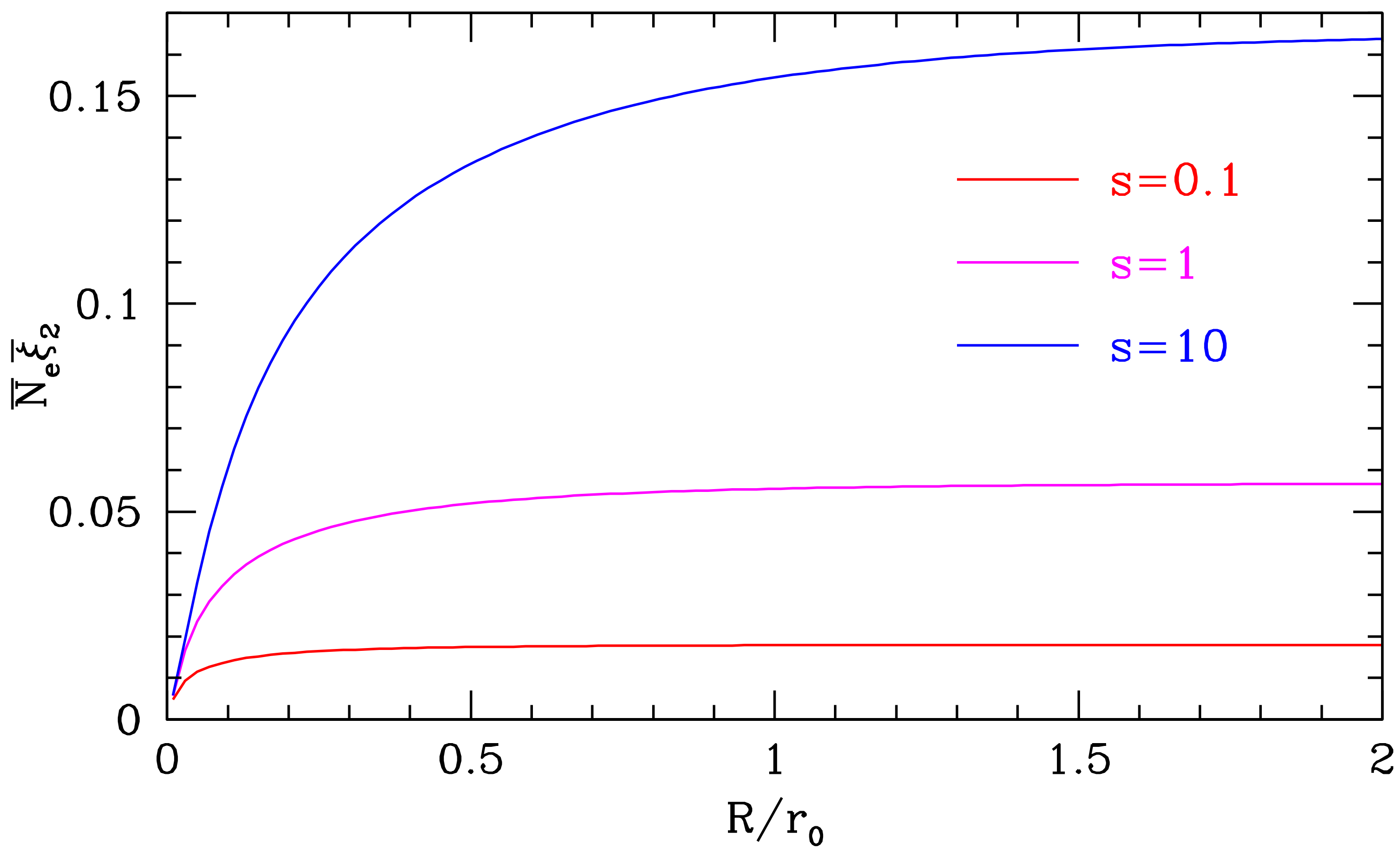
Hierarchical ansatz holds regardless the shape of the window function

$$V_{\rm e}(s,V) \equiv \int d^3\mathbf{x}\,\big(1-\omega(\mathbf{x})\big)\Theta_H(|\mathbf{x}|-R)$$
$$= \int_0^{R/r_0} d\tau\,\frac{dV_e}{d\tau}(s,\tau)$$

$$\tau = r/r_0 = \text{optical depth}$$



Average clustering strength

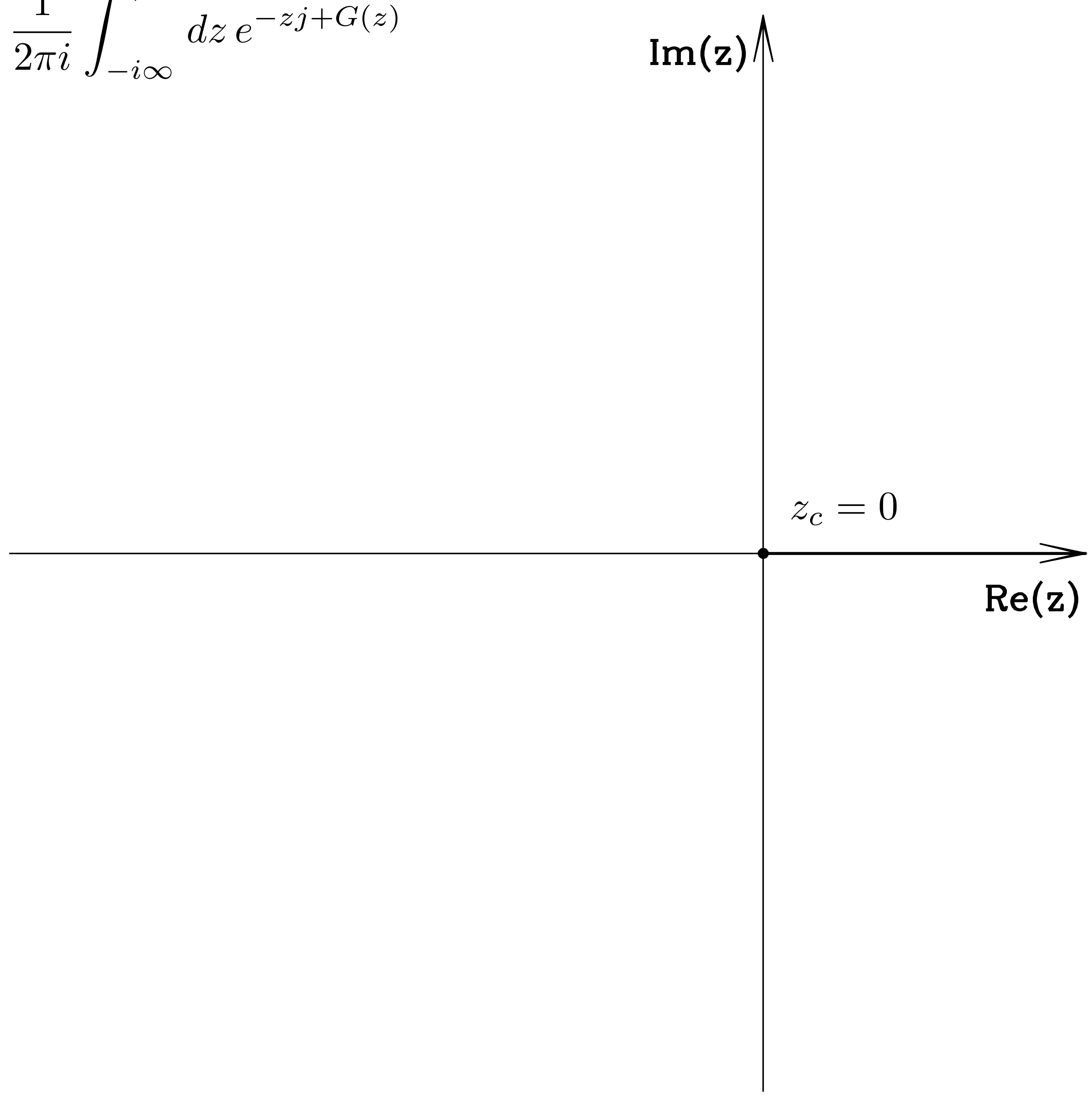


$$s \sim \frac{1}{J} \quad \rightarrow \quad \text{large (small) effects for } J \ll 1 \ \ (J \gg 1)$$

Technicalities

- *Normalized intensity:* $j \equiv J/J_\star, \quad j_\star = L_\star/(4\pi r_0^2)$

$$P(j) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} dz\, e^{-zj+G(z)}$$



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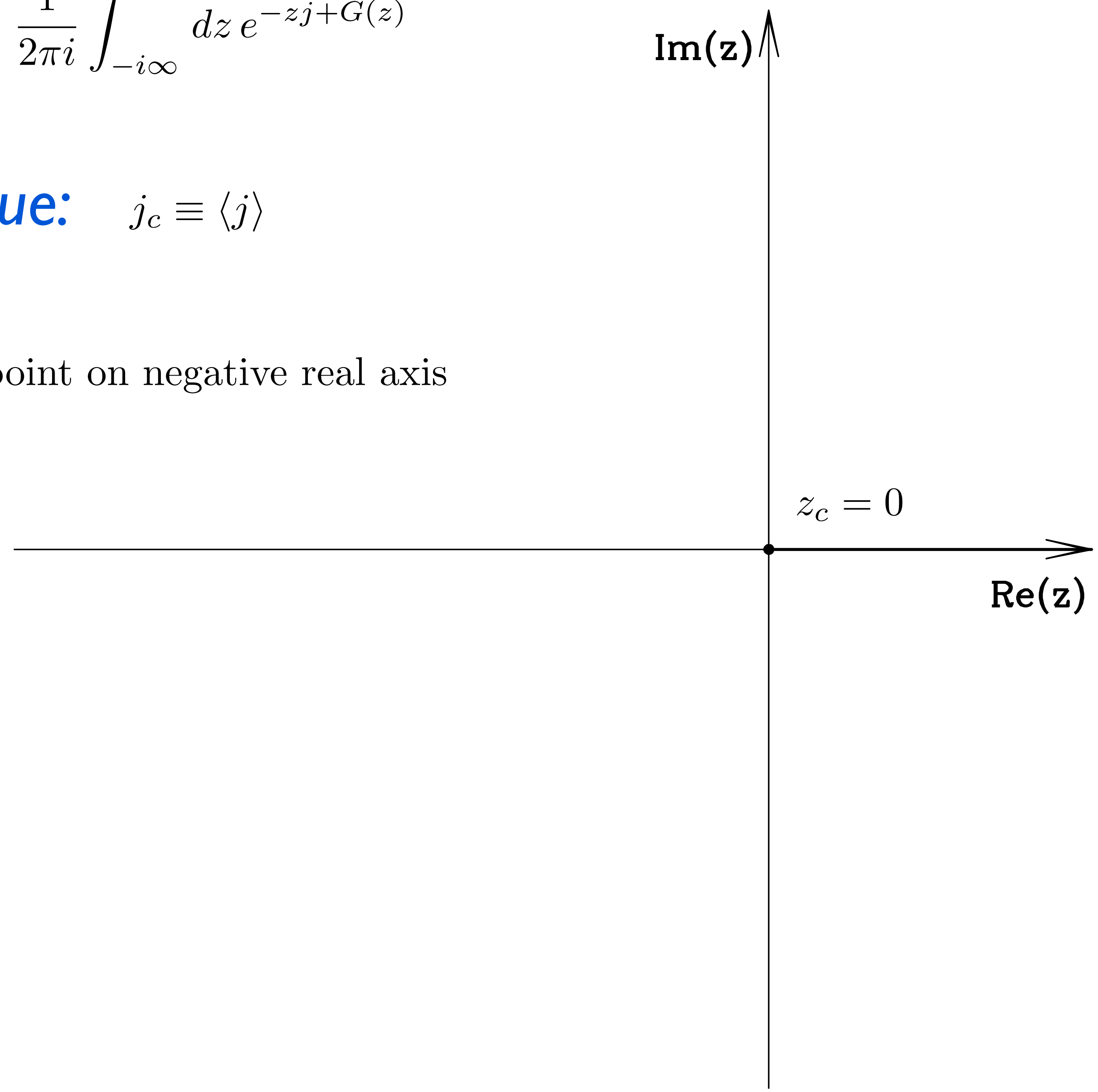
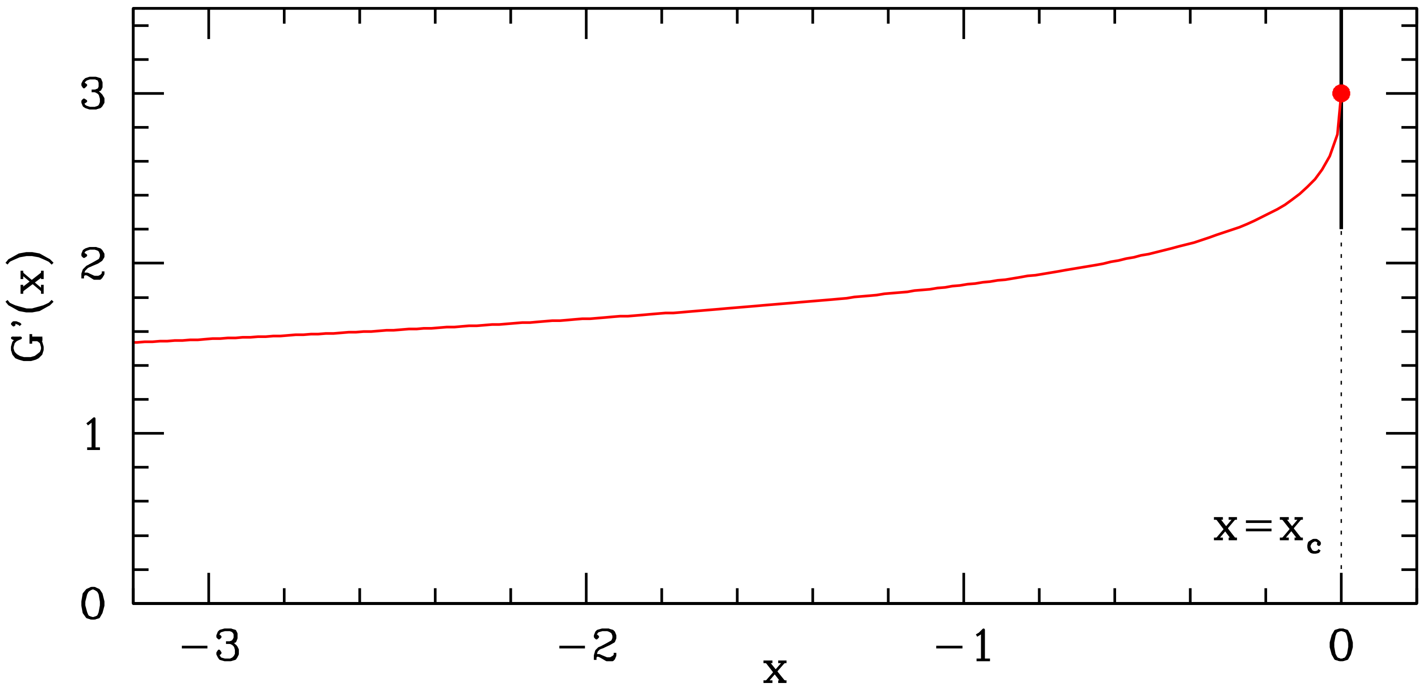
- *There is a critical intensity value:* $j_c \equiv \langle j \rangle$

i) $j \leq j_c \;\Rightarrow\; -zj + G(z)$ admits a saddle point on negative real axis

ii) $j > j_c \;\Rightarrow$ critical point j_c dominates



Legendre transform



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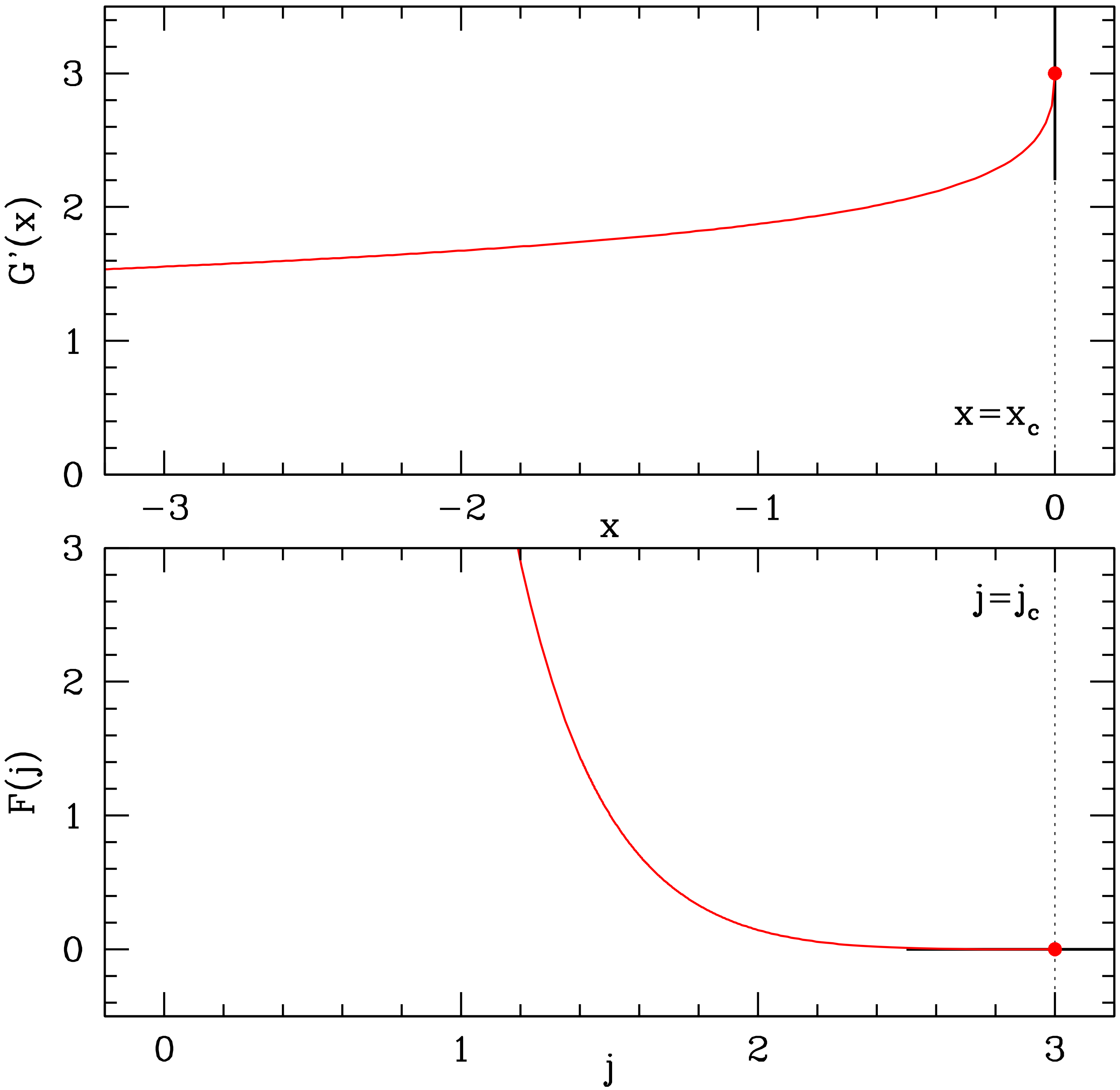
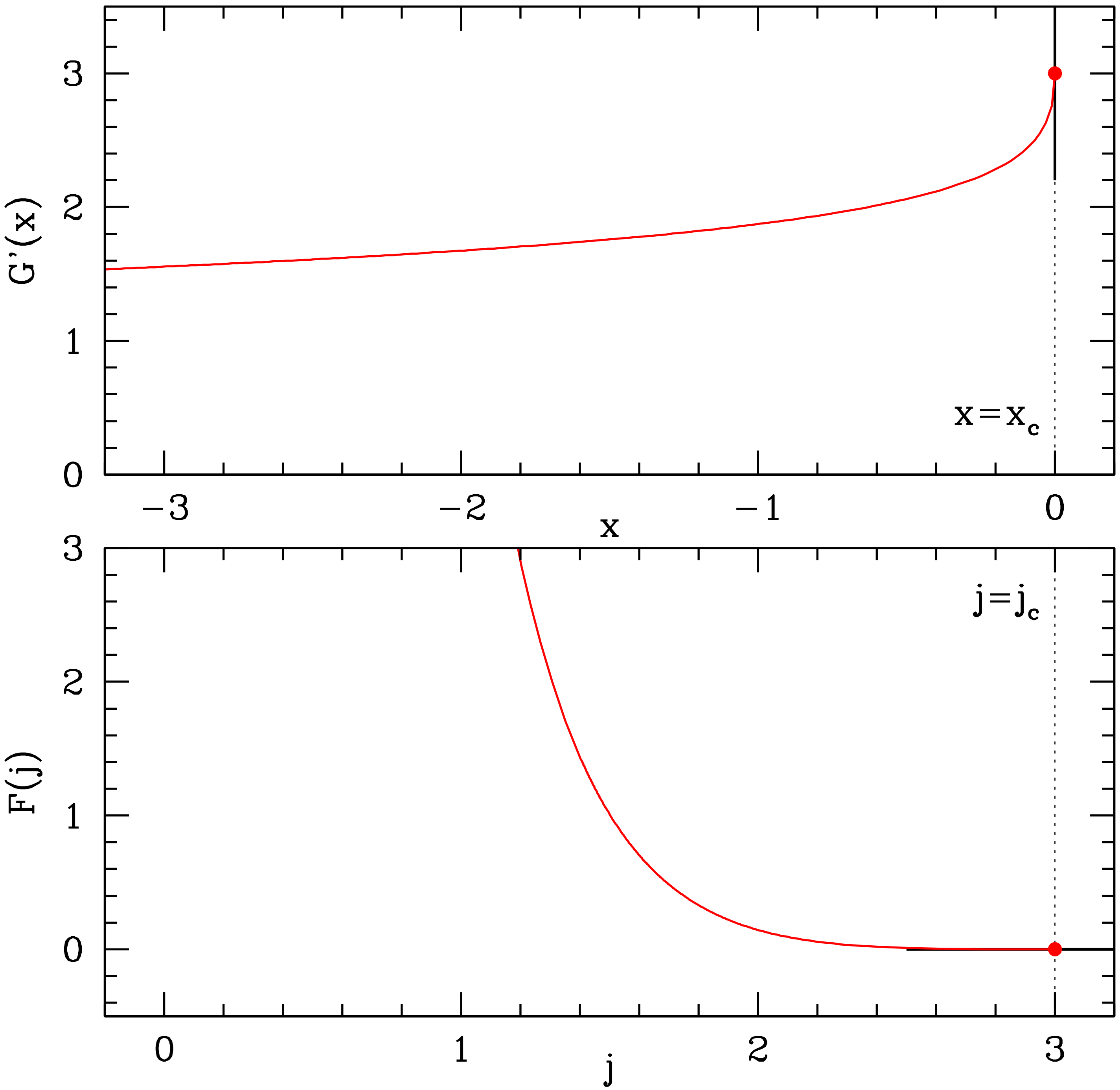


$$G(x) + F(j) = xj$$

Legendre transform



$$G(x) + F(j) = xj$$

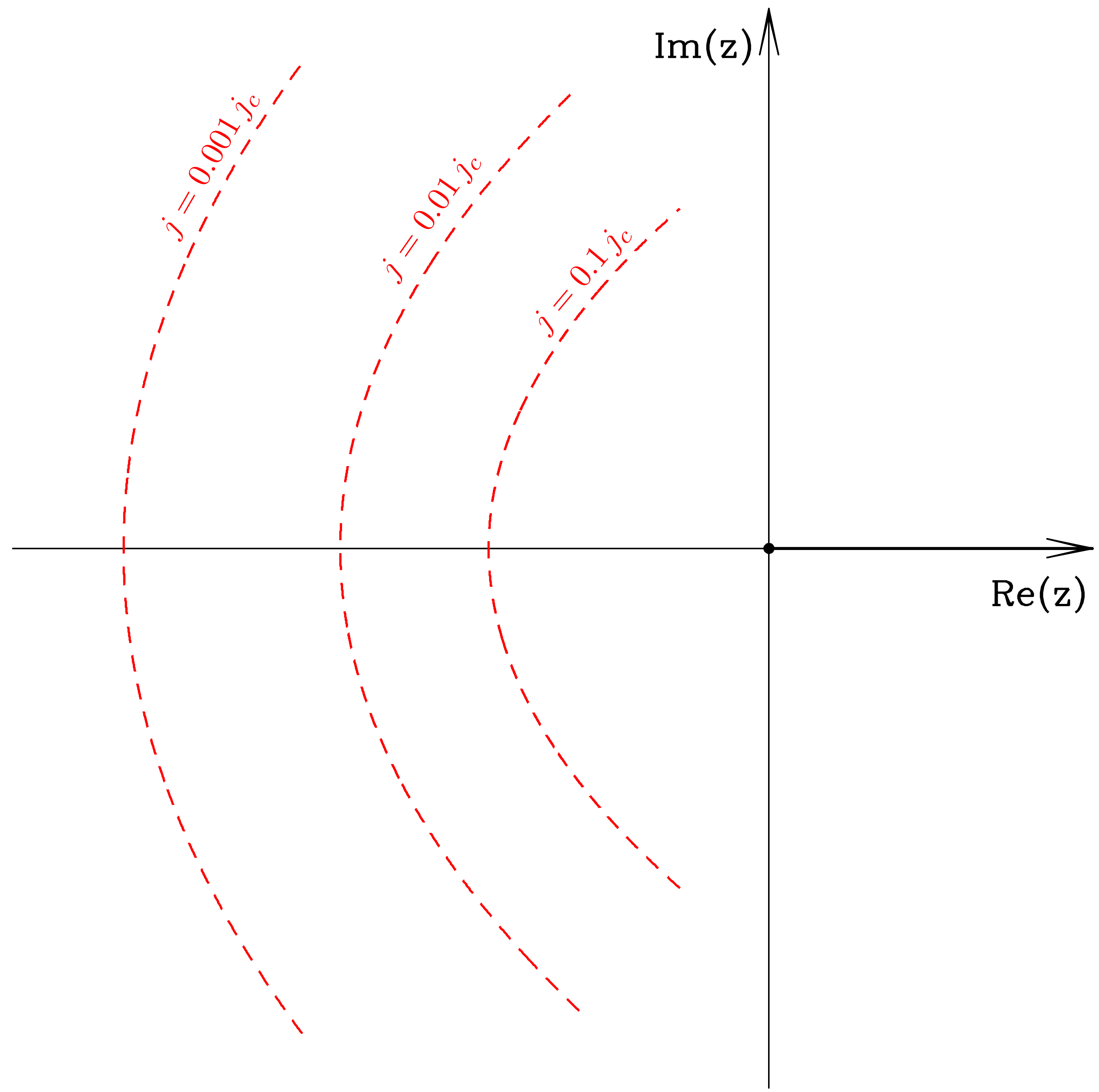
- *Low-intensity tail:*

$$\text{i) } j \le j_c: \quad P(j) \sim \sqrt{F''(j)}\, e^{-F(j)}$$

$$\sim e^{-(\ln j)^m}$$

Saddle point method

Construct paths in the complex plane such that $\delta\big(-zj + G(z)\big) \in \mathbb{R}$



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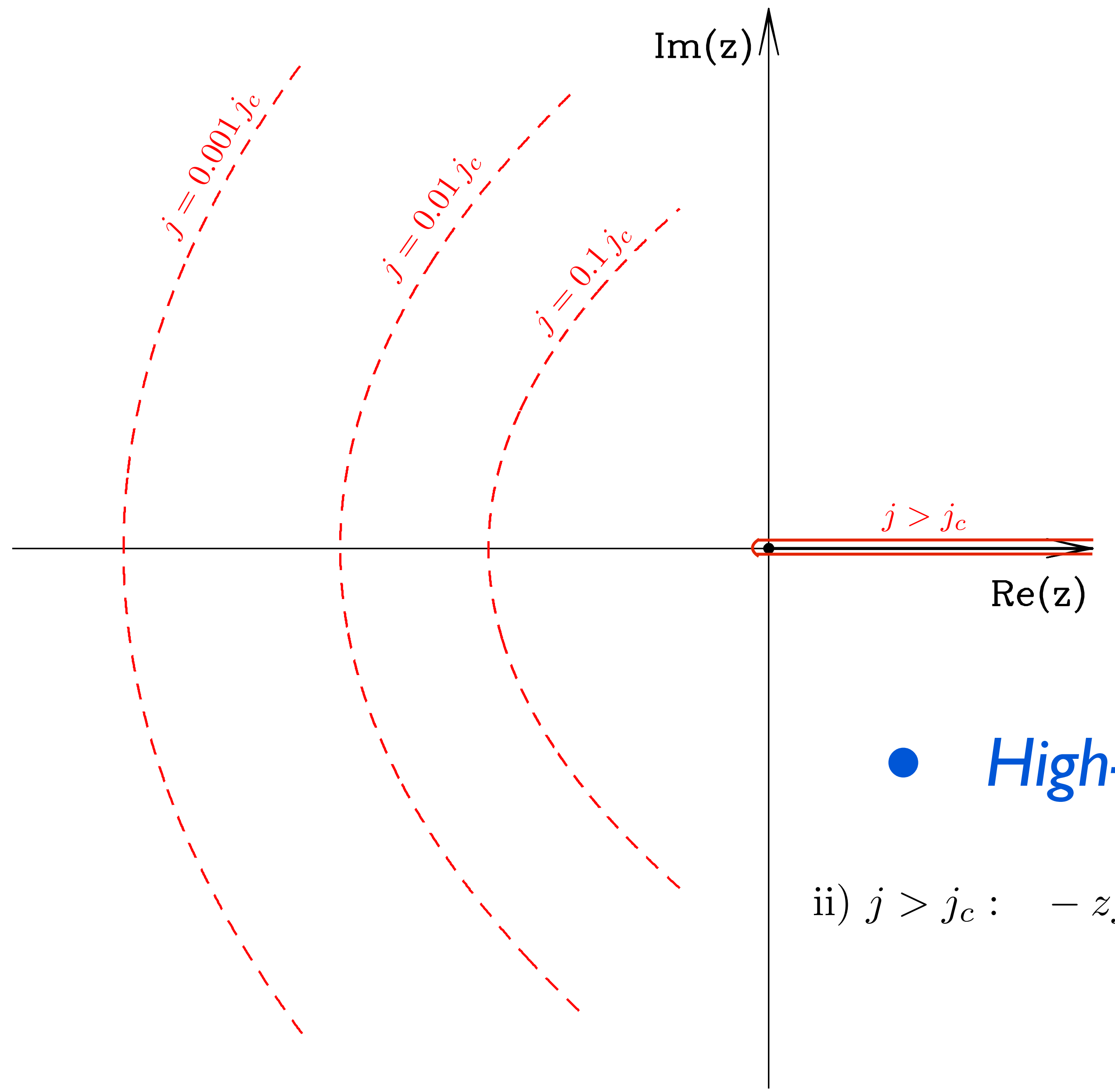


- ***High-intensity tail:***

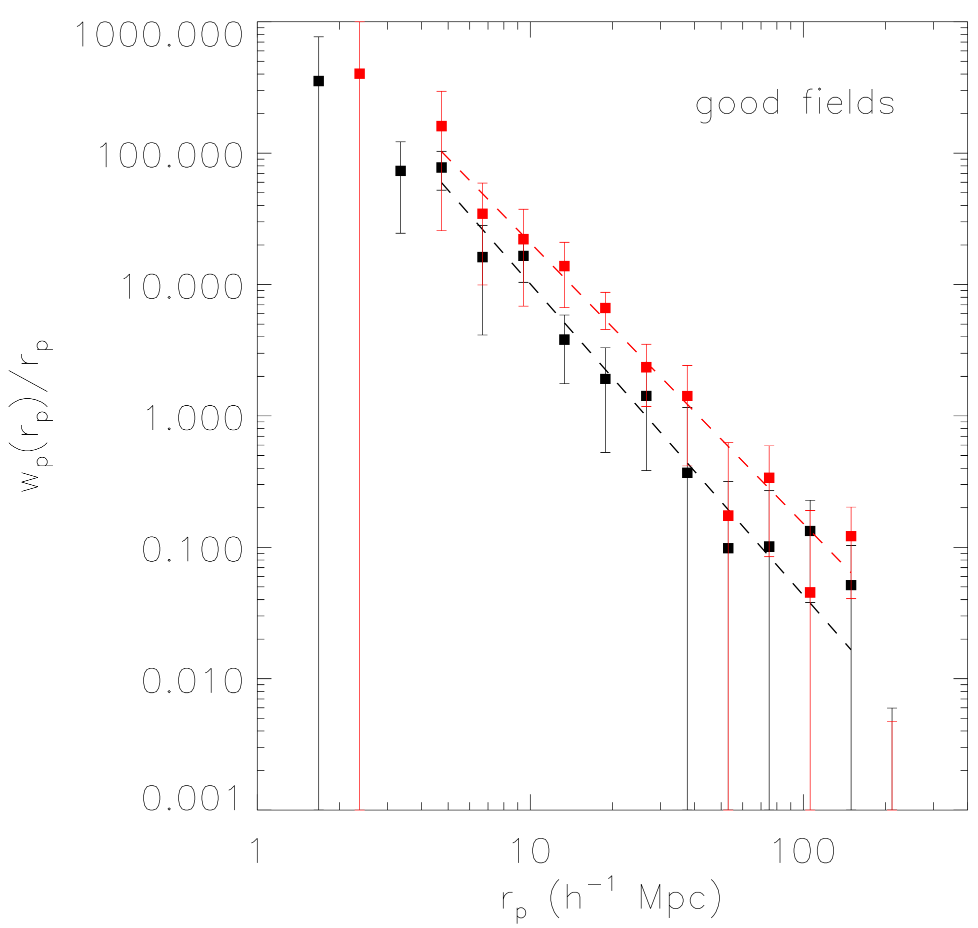
ii) $j > j_c: \quad -zj + G(z) \approx -(j - j_c)z + c_{3/2} z^{3/2}$

$$P(j) \sim j^{-5/2}$$

QSO clustering

- *Power-law form for the QSO correlation function:*

$$\xi_2(r) = \left(\frac{r}{r_\xi}\right)^{-\gamma}$$



(Shen et al 2007)

- *SDSS bright QSOs:*

$$\gamma \approx 2$$

$$r_\xi \sim 24\text{Mpc} \quad (2.9 \leq z \leq 3.5)$$

$$r_\xi \sim 35\text{Mpc} \quad (z \geq 3.5)$$

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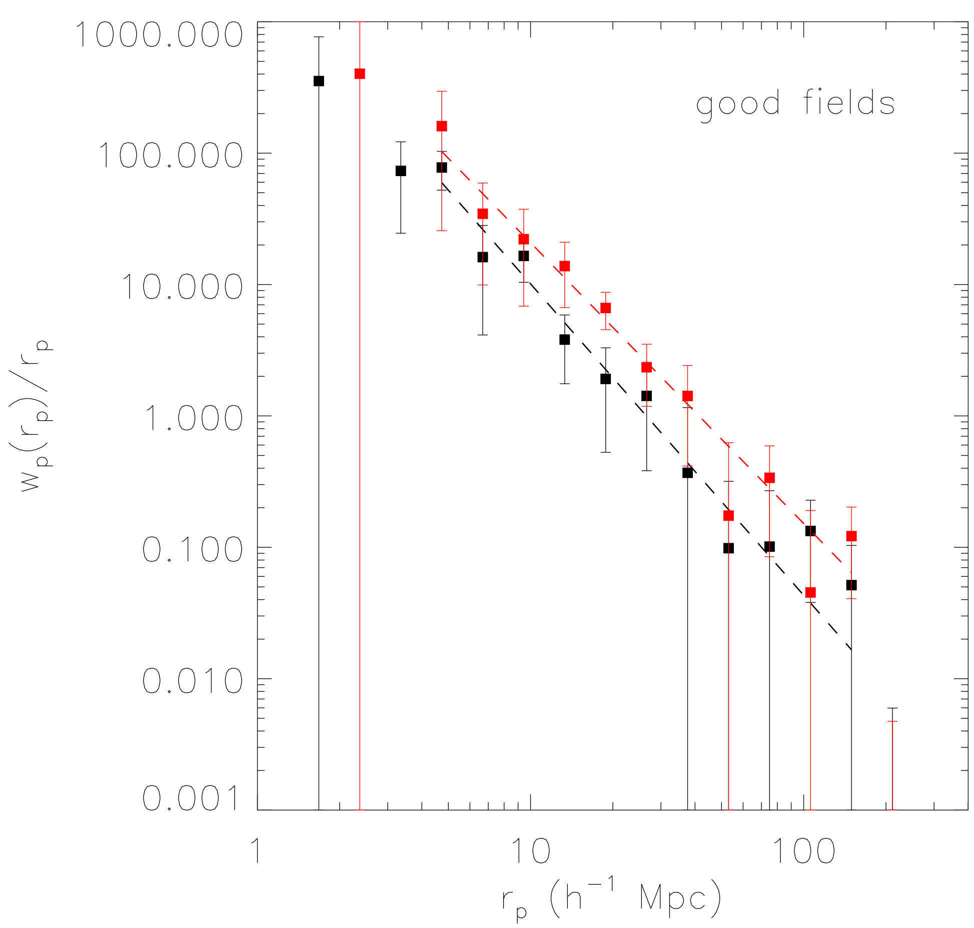


(Shen et al 2007)

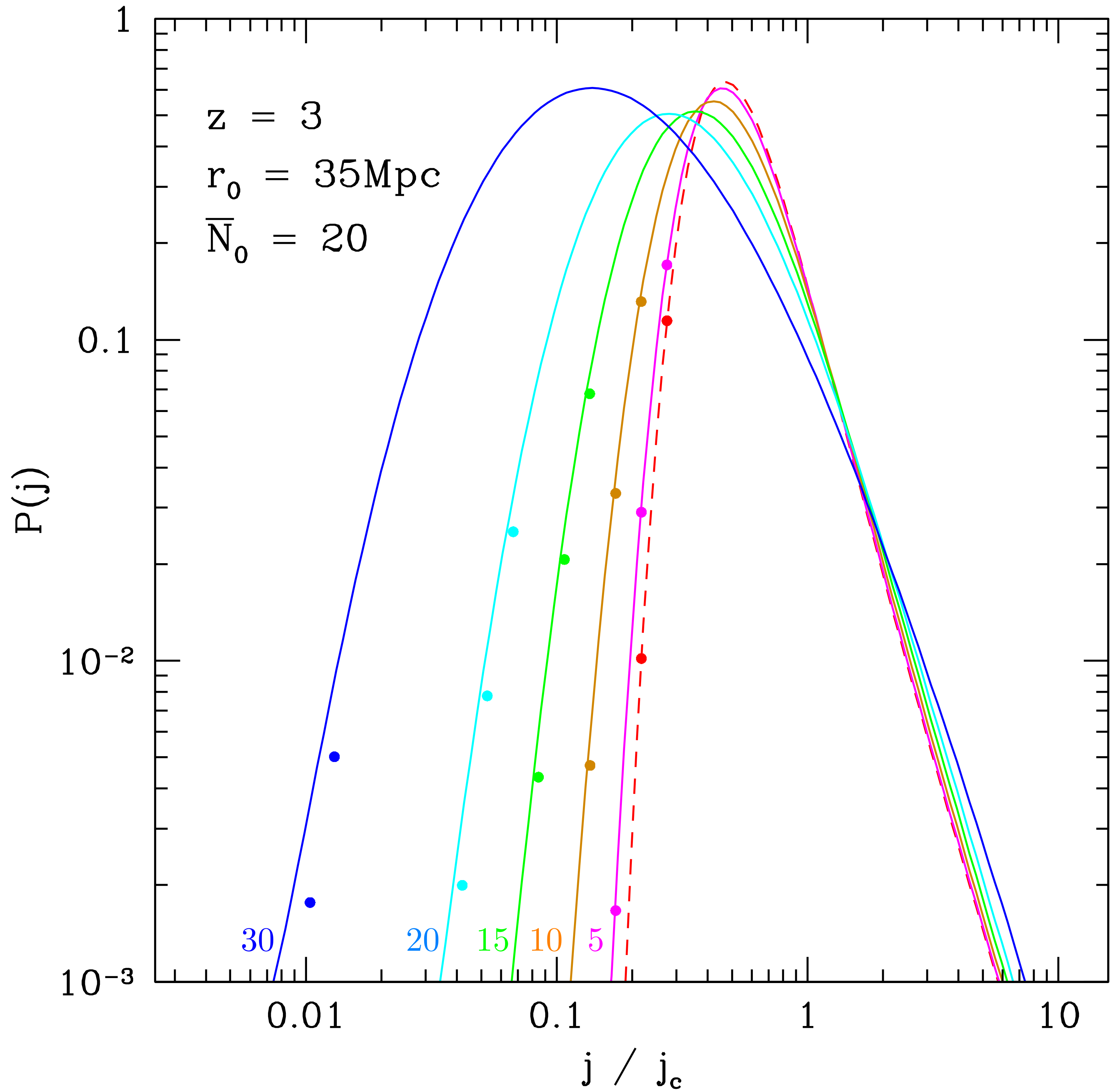
- *Our fiducial choices:*

$$\gamma = 2.1$$

$$r_\xi = 15\text{Mpc}$$

$$\chi = \text{Negative Binomial}$$

Intensity distribution



Varying assumptions

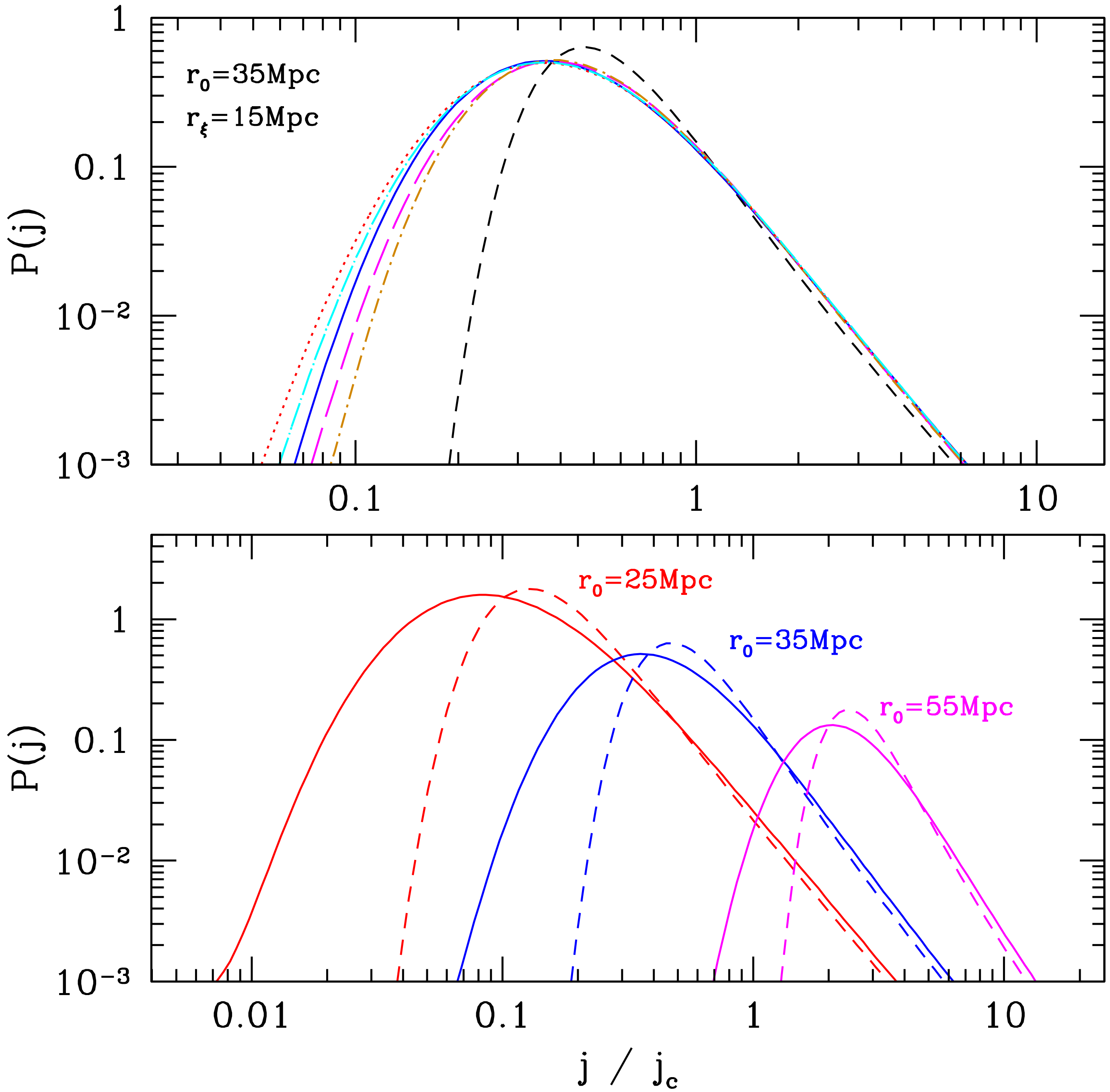


Figure 5. *Top panel* : Effect of changing the behaviour of the quasar correlation function on the distribution $P(j)$. The solid (blue) curve is our fiducial model, the dotted (red) curve was obtained using the GH rather than the NB void scaling function, the long-dashed (magenta) curve has $\xi_2 = 0$ for $r < 1$ Mpc while the dotted-short dashed (orange) assumes $\xi_2 = 0$ outside the range $1 < r < 150$ Mpc. Finally, the dotted-long dashed (cyan) curve assumes a powerlaw slope $\gamma = 1.9$ rather than 2.1. The correlation and attenuation lengths are $r_\xi = 15$ Mpc and $r_0 = 35$ Mpc, respectively. *Bottom panel* : $P(j)$ for 3 different attenuation lengths. Results are shown for randomly distributed (dashed curves) and clustered sources with $r_\xi = 15$ Mpc (solid curves).

Variance of intensity fluctuations

$$\frac{\left.\langle \Delta j^2 \rangle\right|_{\rm clus}}{\left.\langle \Delta j^2 \rangle\right|_{\rm ran}} = \frac{\left.\langle j^2 \rangle - \langle j \rangle^2\right|_{\rm clus}}{\left.\langle j^2 \rangle - \langle j \rangle^2\right|_{\rm ran}}$$

	$r_\xi = 5$	$r_\xi = 10$	$r_\xi = 15$	$r_\xi = 20$	$r_\xi = 30$
$r_0 = 25$	1.02	1.08	1.19	1.35	1.56
$r_0 = 35$	1.03	1.10	1.23	1.41	1.95
$r_0 = 55$	1.05	1.15	1.32	1.56	2.25

Environmental dependence (I)

- *Environmental dependence of conditional void probability:*

$$\mathcal{W}_0(V|\delta) = \sum_{k=0}^{\infty} \frac{(-\bar{n})^k}{k!} \int_V d^3\mathbf{x}_1 \ldots \int_V d^3\mathbf{x}_k \, \xi_k(\mathbf{x}_1, \ldots, \mathbf{x}_k|\delta)$$
$$\equiv \sum_{k=1}^{\infty} \frac{(-\bar{N})^k}{k!} \bar{\xi}_k(V|\delta)$$

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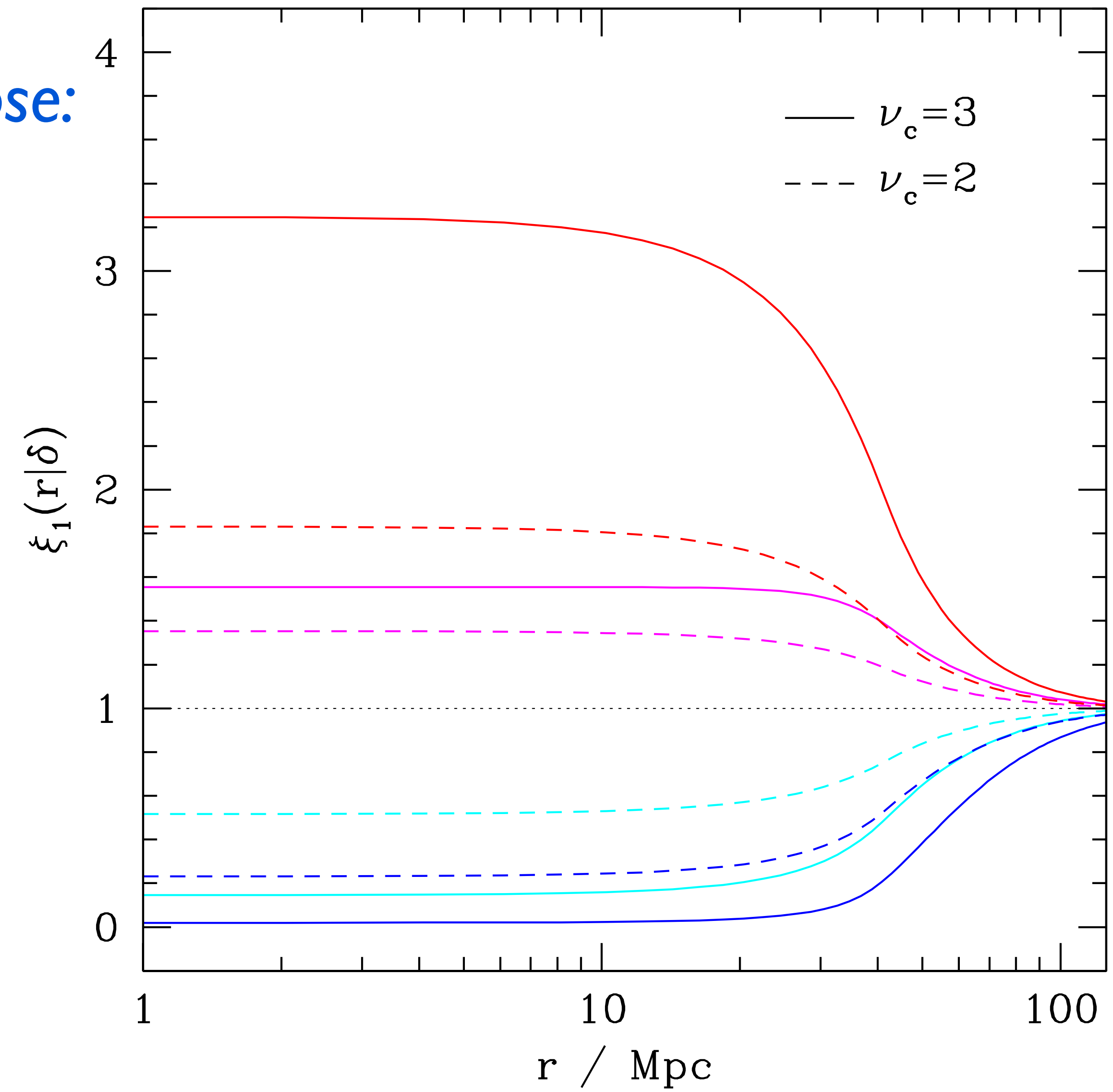
$$\equiv \sum_{k=1}^{\infty} \frac{(-\bar{N})^k}{k!} \bar{\xi}_k(V|\delta)$$

- *Poisson distribution + spherical collapse:*

$$\mathcal{W}_0(V|\delta) = -\bar{N}\bar{\xi}_1(V|\delta)$$



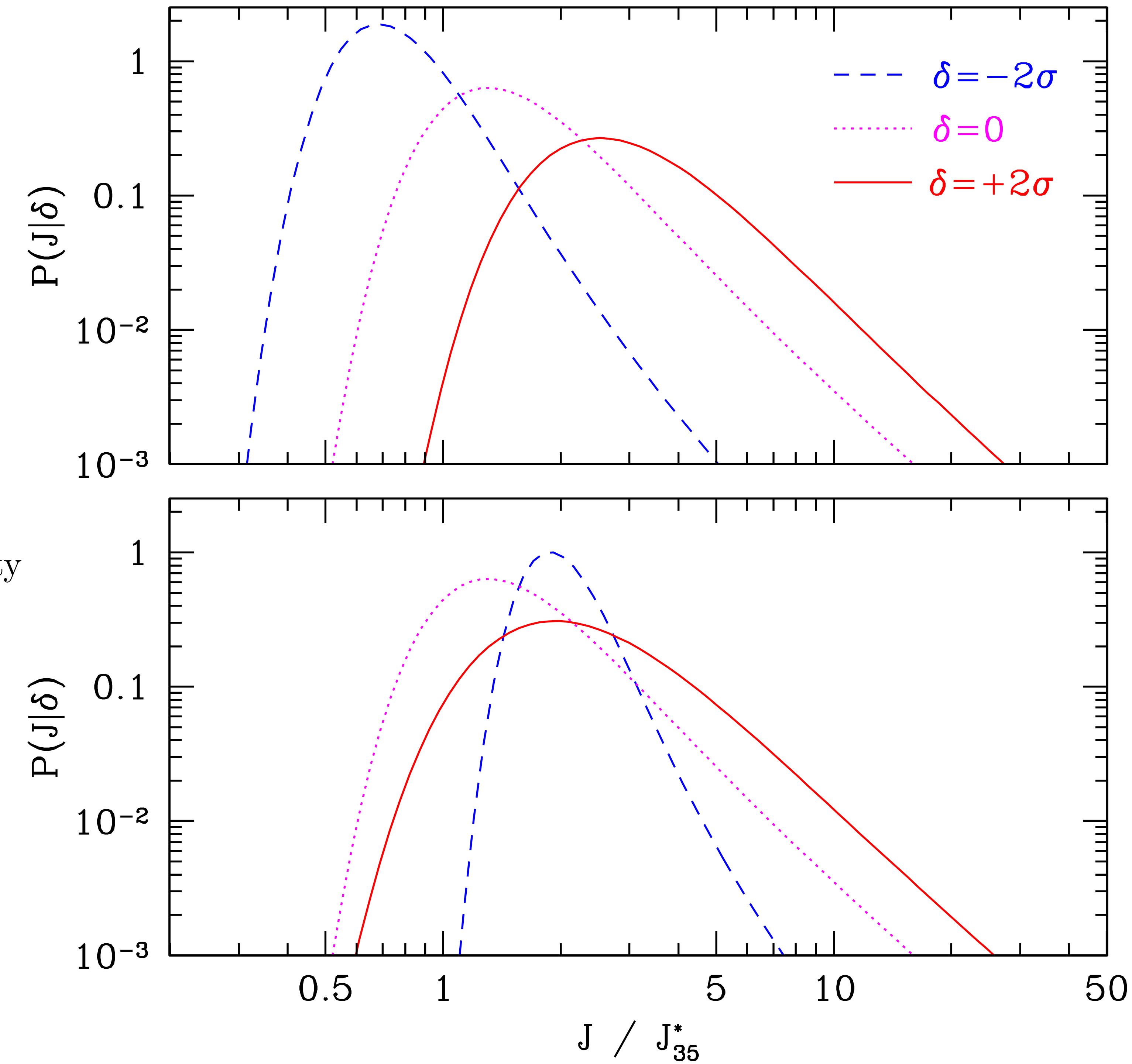
Environmental dependence (II)

constant attenuation length

$r_0 = 35\text{Mpc}$

r_0 depends on environmental density

$29 < r_0 < 57\text{Mpc}$



Summary

- *Count-in-cells approach to ionizing fluctuations with source clustering*

- *Our approach relies on hierarchical ansatz. We have tested it with mock quasar catalogues. QSO void scaling function closely tracks the Negative Binomial model.*

- *Model inputs: observed QLF and quasar 2-point correlation*

- *For a (comoving) attenuation length 25<r0<55Mpc, quasar clustering becomes significant when rx>15-20Mpc*

- *Differences between low- and high-density regions could be a factor of few if quasars are strongly clustered*