

How to measure the Helicity of a Vector Meson?

An Old Physics probe of New Physics

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at **IBS CTPU**

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Plan of the talk

- Measuring the **photon helicity** in $B^\pm \rightarrow K^\pm \pi^\pm \pi^\mp \gamma$ (**LHCb**)
a probe of $F_{\mu\nu} b^\dagger \sigma^{\mu\nu} s_R$ vs $F_{\mu\nu} b^\dagger \sigma^{\mu\nu} s_L$
- Ample data on $\tau^\pm \rightarrow \nu a_1^\pm (\rightarrow \pi^\pm \pi^\pm \pi^\mp)$
with **polarized** a_1^\pm at **BEPC, B-factories, LEP, LHC**.
- Vector meson helicity in the τ rest frame.
- Vector meson helicity in the laboratory frame (**Wigner Rotation**).
- Precision measurement of a_1 **helicity** at the **LHC**
- Summary

References:

arXiv:16**.*****[hep-ph]: KH, H.Ishida, T.Yamada, D.Yang

From the paper

Observation of Photon Polarization in the $b \rightarrow s\gamma$ Transition

by R. Aaij *et al.* (LHCb), PRL 112, 161801 (2014)



Observation of Photon Polarization in the $b \rightarrow s\gamma$ Transition

R. Aaij *et al.**

(LHCb Collaboration)

(Received 27 February 2014; published 22 April 2014)

This Letter presents a study of the flavor-changing neutral current radiative $B^\pm \rightarrow K^\pm \pi^\mp \pi^\pm \gamma$ decays performed using data collected in proton-proton collisions with the LHCb detector at 7 and 8 TeV center-of-mass energies. In this sample, corresponding to an integrated luminosity of 3 fb^{-1} , nearly 14 000 signal events are reconstructed and selected, containing all possible intermediate resonances with a $K^\pm \pi^\mp \pi^\pm$ final state in the $[1.1, 1.9] \text{ GeV}/c^2$ mass range. The distribution of the angle of the photon direction with respect to the plane defined by the final-state hadrons in their rest frame is studied in intervals of $K^\pm \pi^\mp \pi^\pm$ mass and the asymmetry between the number of signal events found on each side of the plane is obtained. The first direct observation of the photon polarization in the $b \rightarrow s\gamma$ transition is reported with a significance of 5.2σ .

implies maximal parity violation up to small corrections of the order m_s/m_b . While the measured inclusive $b \rightarrow s\gamma$ rate [1] agrees with the SM calculations, no direct evidence exists for a nonzero photon polarization in this type of decay. Several extensions of the SM [2], compatible with all current measurements, predict that the photon acquires a significant right-handed component, in particular, due to the exchange of a heavy fermion in the penguin loop [3].

This Letter presents a study of the radiative decay $B^+ \rightarrow K^+\pi^-\pi^+\gamma$, previously observed at the B factories with a measured branching fraction of $(27.6 \pm 2.2) \times 10^{-6}$ [1,4,5]. The inclusion of charge-conjugate processes is implied throughout. Information about the photon polarization is obtained from the angular distribution of the photon direction with respect to the normal to the plane defined by the momenta of the three final-state hadrons in their center-of-mass frame. The shape of this distribution, including the *up-down asymmetry* between the number of events with the photon on either side of the plane, is determined. This investigation is conceptually similar to the historical experiment that discovered parity violation by measuring the up-down asymmetry of the direction of a particle emitted in a weak decay with respect to an axial vector [6]. In $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ decays, the up-down asymmetry is proportional to the photon polarization λ_γ [7,8] and therefore measuring a value different from zero corresponds to demonstrating that the photon is polarized. The

The differential $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ decay rate can be described in terms of θ , defined in the rest frame of the final state hadrons as the angle between the direction opposite to the photon momentum \vec{p}_γ and the normal $\vec{p}_{\pi,\text{slow}} \times \vec{p}_{\pi,\text{fast}}$ to the $K^+\pi^-\pi^+$ plane, where $\vec{p}_{\pi,\text{slow}}$ and $\vec{p}_{\pi,\text{fast}}$ correspond to the momenta of the lower and higher momentum pions, respectively. Following the notation and developments of Ref. [7], the differential decay rate of $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ can be written as a fourth-order polynomial in $\cos\theta$

$$\frac{d\Gamma}{ds ds_{13} ds_{23} d\cos\theta} \propto \sum_{i=0,2,4} a_i(s, s_{13}, s_{23}) \cos^i\theta + \lambda_\gamma \sum_{j=1,3} a_j(s, s_{13}, s_{23}) \cos^j\theta, \quad (1)$$

where $s_{ij} = (p_i + p_j)^2$ and $s = (p_1 + p_2 + p_3)^2$, and p_1 , p_2 , and p_3 are the four-momenta of the π^- , π^+ , and K^+ mesons, respectively. The functions a_k depend on the resonances present in the $K^+\pi^-\pi^+$ mass range of interest and their interferences. The up-down asymmetry is defined as

$$\mathcal{A}_{\text{ud}} \equiv \frac{\int_0^1 d\cos\theta \frac{d\Gamma}{d\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d\Gamma}{d\cos\theta}}{\int_{-1}^1 d\cos\theta \frac{d\Gamma}{d\cos\theta}}, \quad (2)$$

which is proportional to λ_γ .

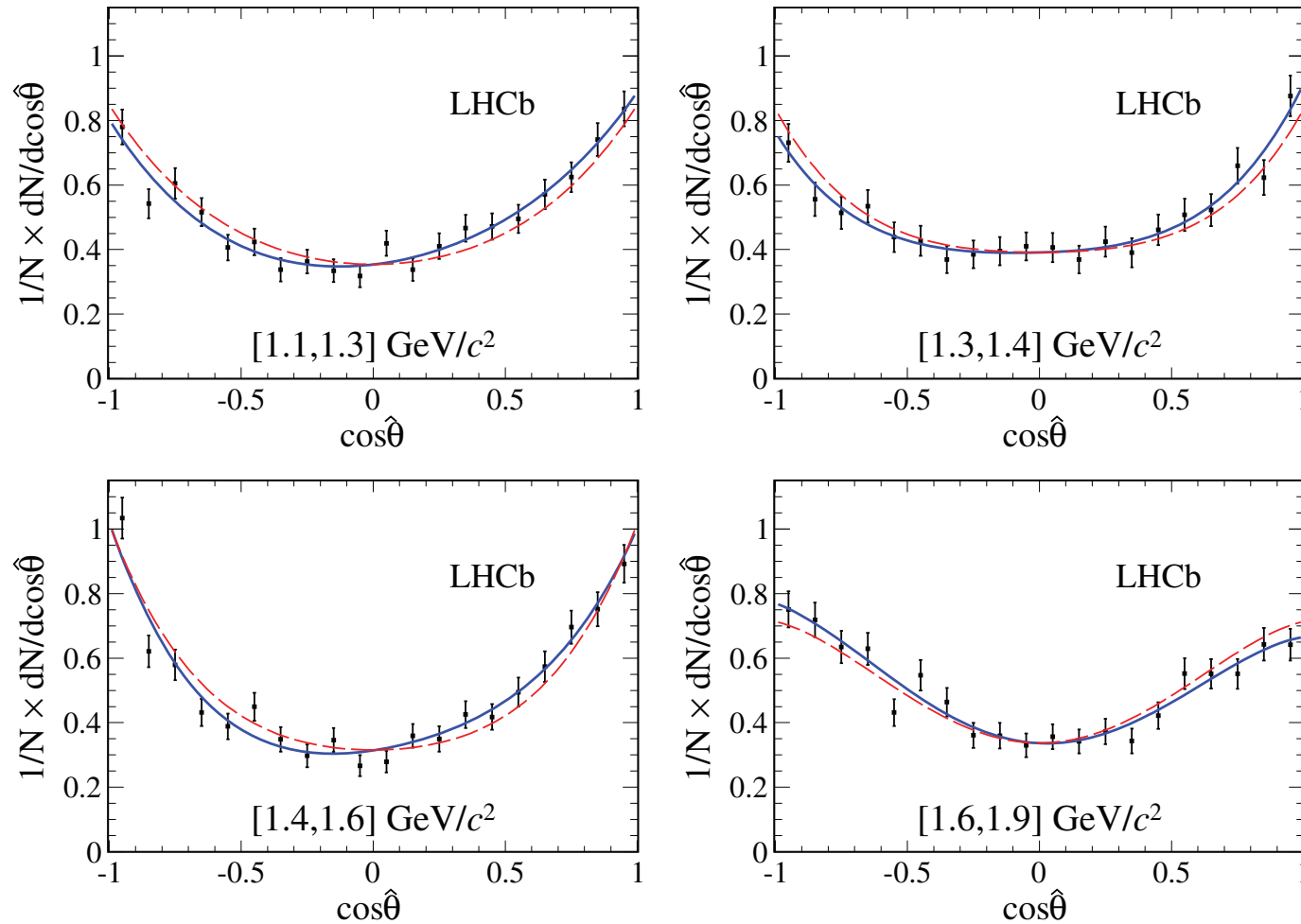


FIG. 3 (color online). Distributions of $\cos \hat{\theta}$ for $B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$ signal in four intervals of $K^+ \pi^- \pi^+$ mass. The solid blue (dashed red) curves are the result of fits allowing all (only even) Legendre components up to the fourth power.

angle, obtained using the normal $\vec{p}_{\pi^-} \times \vec{p}_{\pi^+}$, instead of $\vec{p}_{\pi,\text{slow}} \times \vec{p}_{\pi,\text{fast}}$. The obtained values, along with the relative fit coefficients, are listed in Table II.

To summarize, a study of the inclusive flavor-changing neutral current radiative $B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$ decay, with the $K^+ \pi^- \pi^+$ mass in the $[1.1, 1.9]$ GeV/ c^2 range, is performed on a data sample corresponding to an integrated luminosity of 3 fb^{-1} collected in pp collisions at 7 and 8 TeV center-of-mass energies by the LHCb detector. A total of 13876 ± 153 signal events is observed. The shape of the angular distribution of the photon with respect to the plane defined by the three final-state hadrons in their rest frame is determined in four intervals of interest in the $K^+ \pi^- \pi^+$ mass spectrum. The up-down asymmetry, which is proportional to the photon polarization, is measured for the first

time for each of these $K^+ \pi^- \pi^+$ mass intervals. The first observation of a parity-violating photon polarization different from zero at the 5.2σ significance level in $b \rightarrow s\gamma$ transitions is reported. The shape of the photon angular distribution in each bin, as well as the values for the up-down asymmetry, may be used, if theoretical predictions become available, to determine for the first time a value for the photon polarization, and thus constrain the effects of physics beyond the SM in the $b \rightarrow s\gamma$ sector.

We express our gratitude to our colleagues in the CERN accelerator departments for the excellent performance of the LHC. We thank the technical and administrative staff at the LHCb institutes. We acknowledge support from CERN and from the national agencies: CAPES, CNPq, FAPERJ,

From the paper

Tau Polarization and its Correlations as a Probe of New Physics

by B.Bullock, KH, A.D.Martin, NPB 395, 499 (1993)

We now turn to the vector meson decay modes, $\tau^- \rightarrow v\nu_\tau$ where $v = \rho$ or a_1 . Since the subsequent $\rho \rightarrow 2\pi$ and $a_1 \rightarrow 3\pi$ decay distributions differ according to whether the vector mesons are transversely (T) or longitudinally (L) polarised we study the $\tau \rightarrow v(\text{T})$ and $\tau \rightarrow v(\text{L})$ decay distributions separately. In the τ rest frame, the amplitudes, $\mathcal{M}_{\lambda_\tau\lambda_v}(\hat{\theta})$, describing the $\tau_L^- \rightarrow v\nu_\tau$ decay are of the form

$$\mathcal{M}_{--} = \sqrt{2} \cos \frac{1}{2}\hat{\theta}, \quad \mathcal{M}_{-0} = \frac{m_\tau}{m_v} \sin \frac{1}{2}\hat{\theta}, \quad (2.9)$$

and for $\tau_R^- \rightarrow v\nu_\tau$ are

$$\mathcal{M}_{++} = -\sqrt{2} \sin \frac{1}{2}\hat{\theta}, \quad \mathcal{M}_{+0} = \frac{m_\tau}{m_v} \cos \frac{1}{2}\hat{\theta}, \quad (2.10)$$

where λ_v is the vector meson helicity and where the τ subscripts R, L refer respectively to helicity $\lambda_\tau = +\frac{1}{2}$ and $-\frac{1}{2}$ in the laboratory frame. We note that τ_L^-

Since we do not sum over the polarisation states of the vector meson the boost to the collinear frame is not as straightforward as it was for the single pion or purely leptonic decay mode. We must first perform a Wigner rotation [16] of the vector meson spin quantisation axis,

$$\mathcal{M}'_{\lambda_\tau \lambda'_v} = \sum_{\lambda_v} d_{\lambda'_v \lambda_v}^1(\omega) \mathcal{M}_{\lambda_\tau \lambda_v}, \quad (2.13)$$

which relates the $\tau \rightarrow \nu \nu_\tau$ helicity amplitudes \mathcal{M}' in the collinear frame to the amplitudes \mathcal{M} of (2.9) and (2.10). The angle of rotation is given by

$$\cos \omega = \frac{1 - a^2 + (1 + a^2)\beta \cos \hat{\theta}}{\sqrt{(\cos^2 \hat{\theta} + \gamma^{-2} \sin^2 \hat{\theta})(1 - a^2)^2 + 2(1 - a^4)\beta \cos \hat{\theta} + \beta^2(1 + a^2)^2}}, \quad (2.14)$$

which in the collinear limit ($\beta = 1$) becomes

$$\cos \omega = \frac{1 - a^2 + (1 + a^2) \cos \hat{\theta}}{1 + a^2 + (1 - a^2) \cos \hat{\theta}}, \quad (2.15)$$

where $a = m_\nu/m_\tau$ and $\gamma = (1 - \beta^2)^{-1/2} = E_\tau/m_\tau$ is the boost factor. Using the

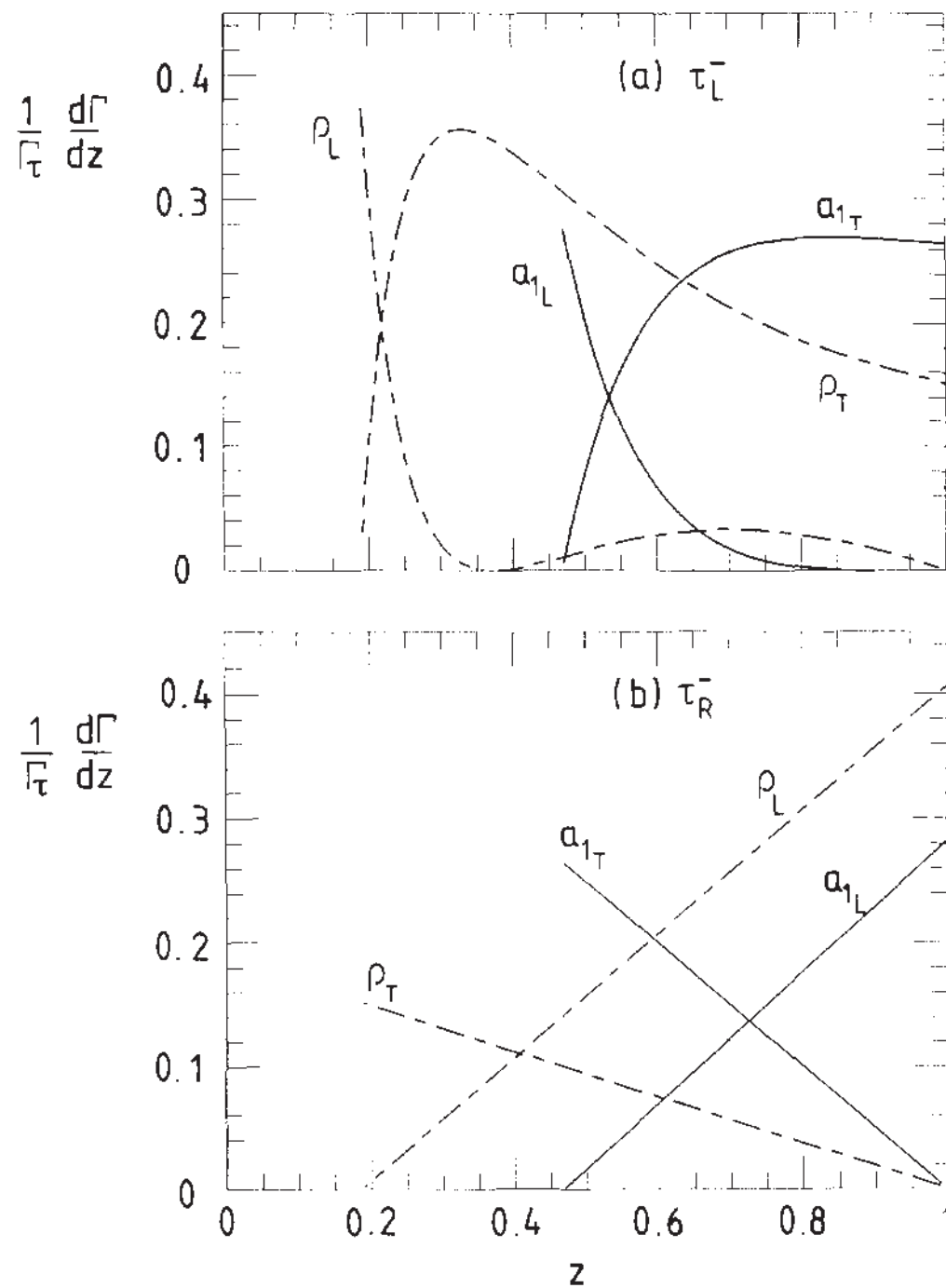
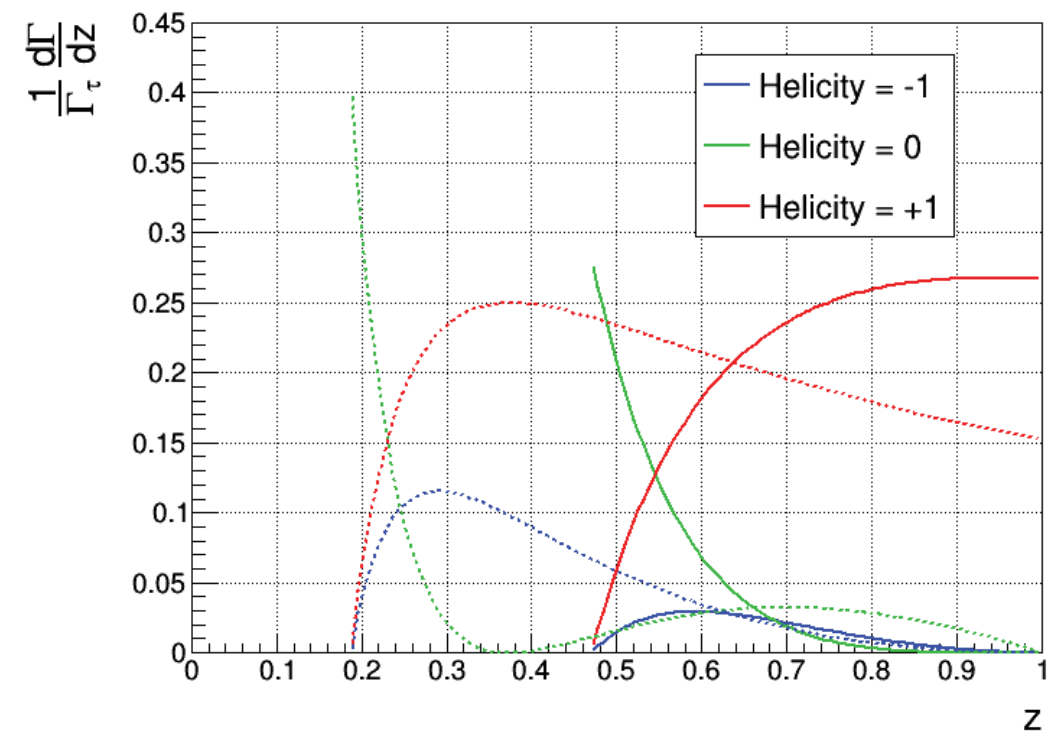
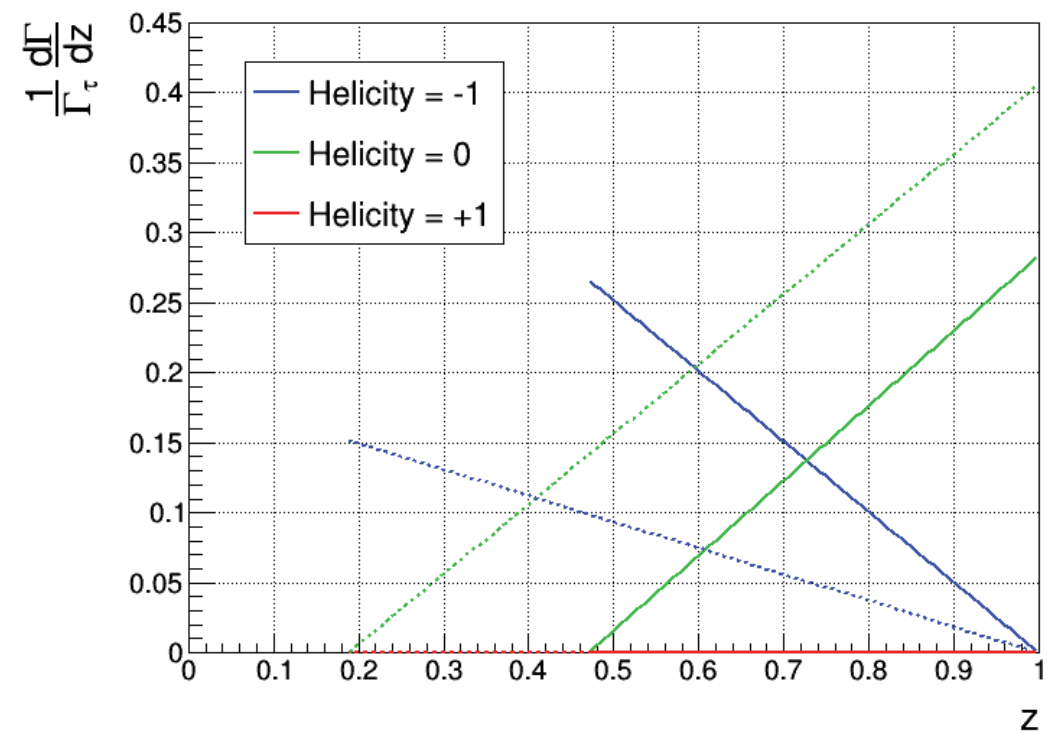


Fig. 1. The fractional energy distributions of polarised ρ and a_1 vector mesons arising from (a) τ_L^- or τ_R^+ decays, (b) τ_R^- or τ_L^+ decays. The energy fraction $z \equiv E_v/E_\tau$, where E_v is the vector meson laboratory energy in the collinear limit ($E_\tau \gg m_\tau$). The masses of the ρ , a_1 mesons are taken to be 0.77

Fractional energy distribution of a_1^+ (solid line) and ρ^+ (dashed line) from τ_R^+ decay



Fractional energy distribution of a_1^+ (solid line) and ρ^+ (dashed line) from τ_L^+ decay



Brief Review of Spin Measurement

- Two methods to measure a particle spin polarization in decays:
 - $\langle \mathbf{s} \cdot \mathbf{p} \rangle \neq 0$ (**Parity-odd**) need Parity-violation (weak decays).
 - $\langle \mathbf{s} \cdot \mathbf{p}_1 \times \mathbf{p}_2 \rangle \neq 0$ (**naive T-odd**) need decays into 3 or more particles, and re-scattering phase.

- Unitarity** of S-matrix:

$$S^\dagger S = (1 + iT)^\dagger (1 + iT) = 1 + i(T - T^\dagger) + T^\dagger T = 1$$

$$-i(T - T^\dagger) = T^\dagger T$$

$$-i \langle f | (T - T^\dagger) | i \rangle = \langle f | T^\dagger T | i \rangle$$

$$-i(T_{fi} - T_{if}^*) = A_{fi} \quad (\text{optical theorem when } |f\rangle = |i\rangle)$$

$$T_{fi} = T_{if}^* + iA_{fi}$$

- Naive T-odd asymmetry** is defined as:

$$|T_{fi}|^2 - |T_{\tilde{f}\tilde{i}}|^2 \quad \text{where } T_{\tilde{f}\tilde{i}} = \langle \tilde{f} | T | \tilde{i} \rangle = \langle f(-\mathbf{p}, -\mathbf{s}) | T | i(-\mathbf{p}, -\mathbf{s}) \rangle$$

- By using the unitarity relation above, we find:

$$\begin{aligned} |T_{fi}|^2 - |T_{\tilde{f}\tilde{i}}|^2 &= |T_{if}|^2 - |T_{\tilde{f}\tilde{i}}|^2 - 2\text{Im}(T_{if}A_{fi}) + |A_{fi}|^2 \\ &= \quad \quad \quad - 2\text{Im}(T_{if}A_{fi}) + |A_{fi}|^2 \quad (\text{if T invariant}). \end{aligned}$$

- Therefore the naive T-odd asymmetry is proportional to the **absorptive amplitude** and its squared, which can be large in strong interactions.

The helicity amplitude of the $a_1^- \rightarrow \pi^- \pi^- \pi^+$ process is

$$\mathcal{M}_\lambda = \epsilon^\mu(\lambda) J_\mu \quad (1)$$

where $\lambda = +, 0, -$ denotes the a_1^- meson helicity, $\epsilon^\mu(\lambda)$ the polarization vector and J_μ the hadronic current, which is given by

$$\begin{aligned} J^\mu(p_1, p_2, p_3) &= \langle \pi_1^-(p_1) \pi_2^-(p_2) \pi_3^+(p_3) | (V^\mu - A^\mu) | 0 \rangle \\ &= \frac{1}{f_\pi} \frac{2\sqrt{2}}{3} [(p_{1\nu} - p_{3\nu}) F_\rho(s_2) + (p_{2\nu} - p_{3\nu}) F_\rho(s_1)] \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) \end{aligned} \quad (2)$$

where f_π is the pion decay constant, p_1, p_2 denote the momenta of the two same-sign pions and p_3 that of the opposite sign pion, and s_1, s_2, Q^μ are defined as $s_1 \equiv (p_2 + p_3)^2$, $s_2 \equiv (p_3 + p_1)^2$, $Q^\mu \equiv p_1^\mu + p_2^\mu + p_3^\mu$. The two same-sign pions π_1^-, π_2^- are distinguished by requiring $Q \cdot p_1 > Q \cdot p_2$. The following form is

$$\begin{aligned}
p_3^\mu &= (E_3, \mathbf{p}_3) = \frac{\sqrt{Q^2}}{2} \begin{pmatrix} x_3 \\ \tilde{x}_3 \cos \phi \cos \Psi \\ \tilde{x}_3 \sin \phi \\ -\tilde{x}_3 \sin \Psi \cos \phi \end{pmatrix}, \\
p_1^\mu &= (E_1, \mathbf{p}_1) = \frac{\sqrt{Q^2}}{2} \begin{pmatrix} x_1 \\ \tilde{x}_1 (\cos \Psi \cos \phi \cos \theta_1 - \cos \Psi \sin \phi \sin \theta_1) \\ \tilde{x}_1 (\sin \phi \cos \theta_1 + \cos \phi \sin \theta_1) \\ \tilde{x}_1 (-\sin \Psi \cos \phi \cos \theta_1 + \sin \Psi \sin \phi \sin \theta_1) \end{pmatrix}, \\
p_2^\mu &= (E_2, \mathbf{p}_2) = \frac{\sqrt{Q^2}}{2} \begin{pmatrix} x_2 \\ \tilde{x}_2 (\cos \Psi \cos \phi \cos \theta_2 + \cos \Psi \sin \phi \sin \theta_2) \\ \tilde{x}_2 (\sin \phi \cos \theta_2 - \cos \phi \sin \theta_2) \\ \tilde{x}_2 (-\sin \Psi \sin \phi \cos \theta_2 - \sin \Psi \cos \phi \sin \theta_2) \end{pmatrix}, \tag{4}
\end{aligned}$$

where $x_k \equiv \frac{2Q \cdot p_k}{Q^2} = 1 - \frac{s_k - m_\pi^2}{Q^2}$ and $\tilde{x}_k \equiv \sqrt{x_k^2 - 4 \frac{m_\pi^2}{Q^2}}$ ($k = 1, 2, 3$) with $s_1 = (p_2 + p_3)^2$, $s_2 = (p_3 + p_1)^2$

and $s_3 = (p_1 + p_2)^2$, and θ_i ($i = 1, 2$) are the angles between \mathbf{p}_i and \mathbf{p}_3 satisfying $\cos \theta_1 = \frac{\tilde{x}_2^2 - \tilde{x}_1^2 - \tilde{x}_3^2}{2\tilde{x}_1\tilde{x}_3}$,

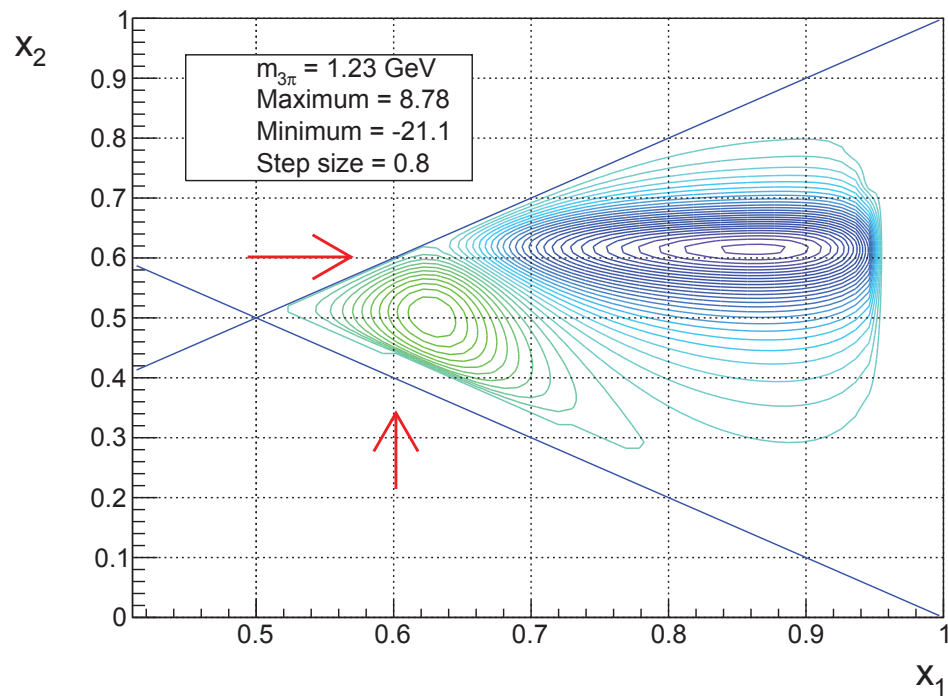
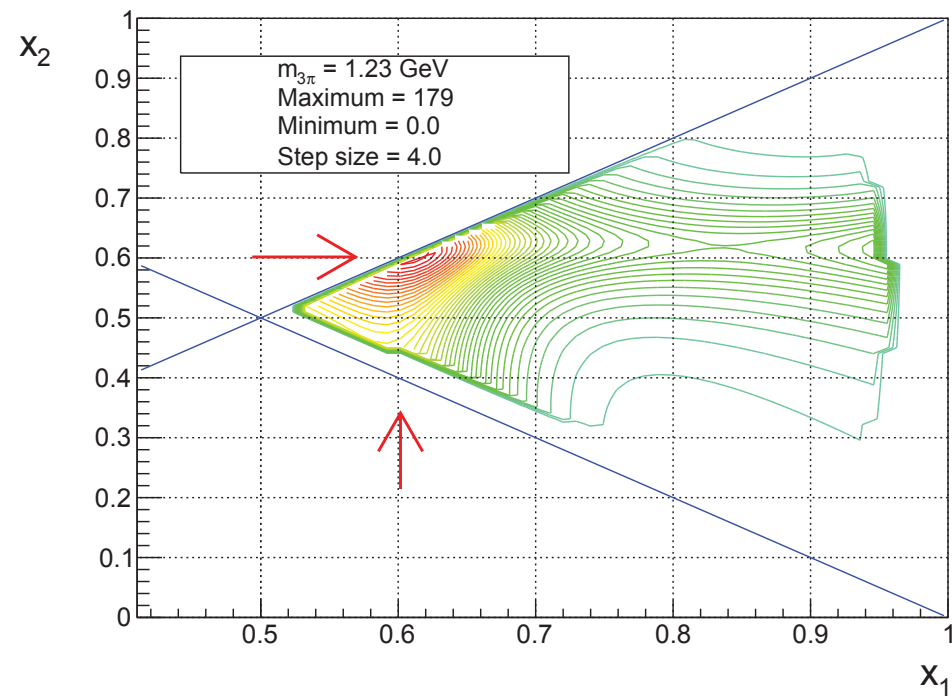
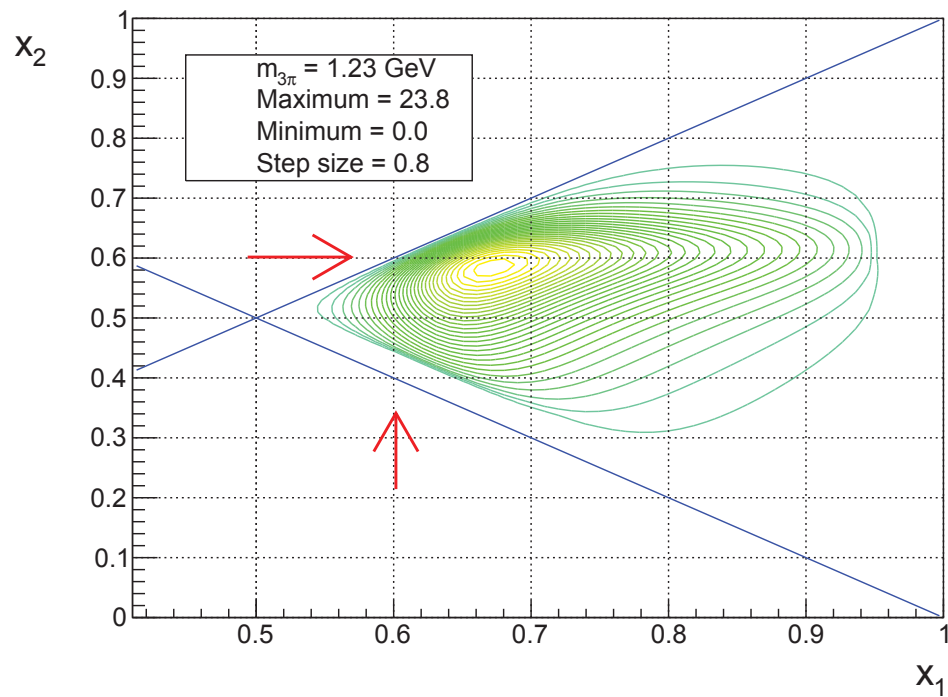
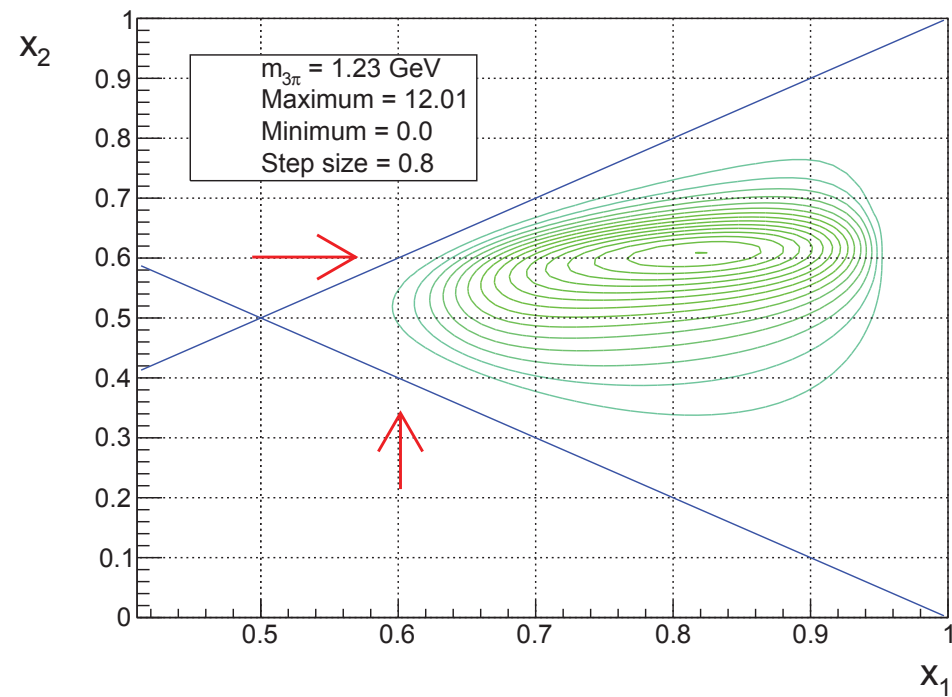
$\cos \theta_2 = \frac{\tilde{x}_1^2 - \tilde{x}_2^2 - \tilde{x}_3^2}{2\tilde{x}_2\tilde{x}_3}$. We then find

$$\begin{aligned}
M_+ &= \epsilon^\mu(+)\mathcal{J}_\mu = \frac{1}{3f_\pi} \sqrt{Q^2} (\cos \Psi [\cos \phi A(x_1, x_2) - \sin \phi B(x_1, x_2)] + i [\sin \phi A(x_1, x_2) + \cos \phi B(x_1, x_2)]), \\
M_0 &= \epsilon^\mu(0)\mathcal{J}_\mu = \frac{\sqrt{2}}{3f_\pi} \sqrt{Q^2} \sin \Psi [\cos \phi A(x_1, x_2) - \sin \phi B(x_1, x_2)], \\
M_- &= \epsilon^\mu(-)\mathcal{J}_\mu = \frac{1}{3f_\pi} \sqrt{Q^2} (-\cos \Psi [\cos \phi A(x_1, x_2) - \sin \phi B(x_1, x_2)] + i [\sin \phi A(x_1, x_2) + \cos \phi B(x_1, x_2)]).
\end{aligned}$$

$$\begin{aligned}
d\Phi_3(a_1 \rightarrow 3\pi) &= d\Phi_2(a_1 \rightarrow \pi^2 + (\pi^1\pi^3)) \frac{ds_2}{2\pi} d\Phi_2((\pi^1\pi^3) \rightarrow \pi^1 + \pi^3) \\
&= \frac{1}{8\pi} \bar{\beta}\left(\frac{m_\pi^2}{Q^2}, \frac{s_2}{Q^2}\right) \frac{d\cos\Psi}{2} \frac{d\phi}{2\pi} \frac{ds_2}{2\pi} \frac{1}{8\pi} \bar{\beta}\left(\frac{m_\pi^2}{Q^2}, \frac{m_\pi^2}{Q^2}\right) \frac{d\cos\hat{\theta}}{2} \frac{d\hat{\phi}}{2\pi} \\
&= \frac{1}{128\pi^3} \frac{1}{Q^2} ds_1 ds_2 \frac{d\cos\Psi}{2} \frac{d\phi}{2\pi},
\end{aligned} \tag{7}$$

we obtain the following differential decay rate for each a_1^- helicity λ :

$$\begin{aligned}
&\frac{d^4\Gamma_{\lambda=+}(a_1^- \rightarrow \pi^- \pi^- \pi^+)}{d\cos\Psi d\phi ds_1 ds_2} \\
&= \frac{1}{512\pi^4} \frac{1}{2\sqrt{Q^2}} \left(\frac{1}{3f_\pi}\right)^2 [|A|^2 + |B|^2 - (1 - \cos^2\Psi) \{ \cos^2\phi |A|^2 + \sin^2\phi |B|^2 - \sin 2\phi \text{Re}(A \cdot B^*) \} \\
&\quad + 2\cos\Psi \text{Im}(A \cdot B^*)], \\
&\frac{d^4\Gamma_{\lambda=0}(a_1^- \rightarrow \pi^- \pi^- \pi^+)}{d\cos\Psi d\phi ds_1 ds_2} \\
&= \frac{1}{512\pi^4} \frac{1}{2\sqrt{Q^2}} \left(\frac{1}{3f_\pi}\right)^2 2(1 - \cos^2\Psi) \{ \cos^2\phi |A|^2 + \sin^2\phi |B|^2 - \sin 2\phi \text{Re}(A \cdot B^*) \}, \\
&\frac{d^4\Gamma_{\lambda=-}(a_1^- \rightarrow \pi^- \pi^- \pi^+)}{d\cos\Psi d\phi ds_1 ds_2} \\
&= \frac{1}{512\pi^4} \frac{1}{2\sqrt{Q^2}} \left(\frac{1}{3f_\pi}\right)^2 [|A|^2 + |B|^2 - (1 - \cos^2\Psi) \{ \cos^2\phi |A|^2 + \sin^2\phi |B|^2 - \sin 2\phi \text{Re}(A \cdot B^*) \} \\
&\quad - 2\cos\Psi \text{Im}(A \cdot B^*)].
\end{aligned} \tag{8}$$

(c) $\text{Re}(AB^*)$ (a) $|A|^2$ (d) $\text{Im}(AB^*)$ (b) $|B|^2$ 

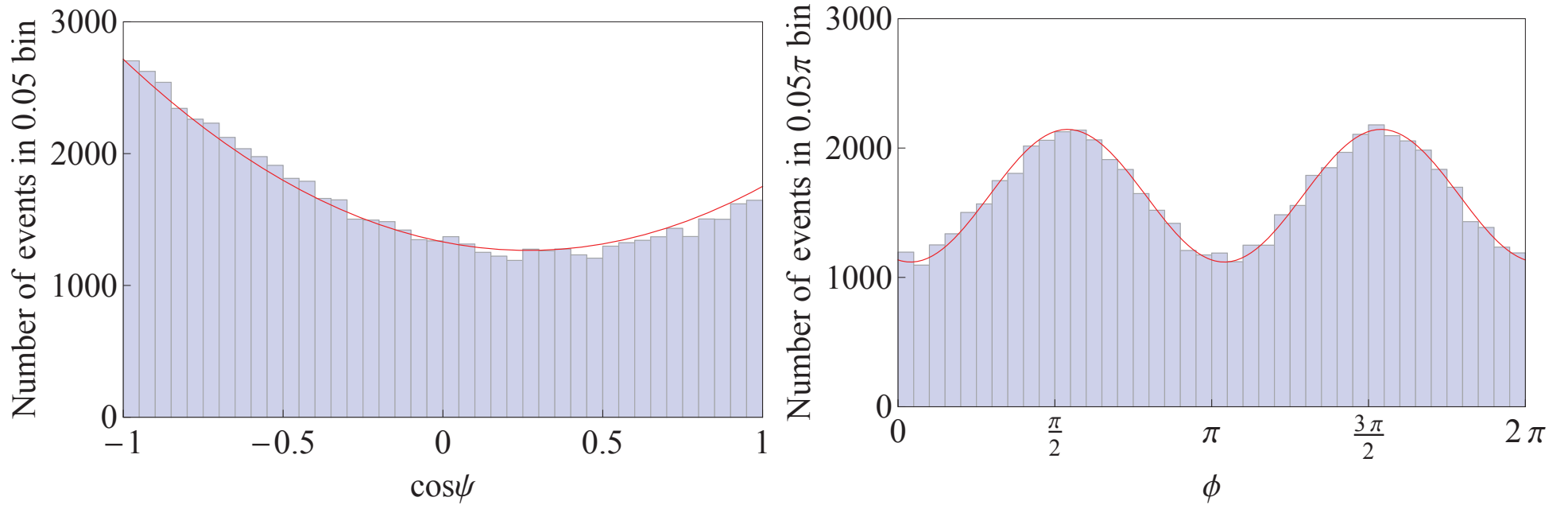


FIG. 4: Left: The distribution of $\cos\Psi$ in the Monte Carlo sample of events, overlaid with the probability distribution function Eq. (10) into which we substitute the estimated values of (\bar{P}_T, \bar{P}_A) and $Q_0^2 = (1.23 \text{ GeV})^2$, integrated over (ϕ, s_1, s_2) . The probability distribution function is normalized by the number of the Monte Carlo events. The histogram is the distribution of Monte Carlo events, and the solid line the probability distribution function.

Right: The same as the left plot, except that the distribution of ϕ is shown, overlaid with the probability distribution function integrated over $(\cos\Psi, s_1, s_2)$.

Summary

- Polarization of a_1 meson in $\tau^\pm \rightarrow \nu(\bar{\nu})a_1^\pm$ decay is precisely known in the SM.
- Helicity of a_1 meson from τ decay in laboratory frame can be calculated by Lorentz transformation (**Wigner Rotation**).
- High degree of a_1 meson polarization is predicted both in $\tau^+\tau^-$ production from virtual γ (**BEPC, B factories**) and from Z (**LEP, LHC**), and also in $W^\pm \rightarrow \tau^\pm\nu(\bar{\nu})$ decays (**LHC**).
- The helicity of a_1 meson can be measured accurately by measuring **Parity-even** and **naive-T-odd** asymmetry proportional to $\mathbf{s}(a_1) \cdot \mathbf{p}_{\pi_1} \times \mathbf{p}_{\pi_2}$ in the a_1 rest frame, i.e., the correlation between the orientation of the decay plane and the a_1 spin polarization along its momentum direction in the laboratory frame.
- Once the technique is established, it can be directly used to measure the photon helicity in rare decays $B^\pm \rightarrow a_1^\pm\gamma$, which constrains the ratio of $F_{\mu\nu}b^\dagger\sigma^{\mu\nu}d_R$ vs $F_{\mu\nu}b^\dagger\sigma^{\mu\nu}d_L$.
- We expect that the lowest lying K^* meson decays $K^{*\pm} \rightarrow K^\pm\pi^\pm\pi^\mp$ have similar matrix elements mediated by two interfering amplitudes, one with $K^{*0} \rightarrow K^+\pi^-$ and the other with $\rho^0 \rightarrow \pi^+\pi^-$. Therefore, by restricting the analysis to the lowest $K\pi\pi$ invariant mass region, one can measure the photon helicity in the $b \rightarrow s\gamma$ transitions.