Flavor Origin of Dark Matter and Lepton Mixing

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Based on: Phys.Rev.D 93(2016)115041
& JHEP 1705 (2017) 068

DSU 2017, IBS *13.07.2017*

Why we are interested in flavor symmetries?

SM flavor problem:
- Why there are three families?
- Fermion mass hierarchy
- Different quark and lepton mixing

Flavor Symmetries :

→ Models based on **discrete groups** : $A_4, S_3, S_4, \Delta(27)$ etc

↓ Models based on **continuous groups**: $G_F = U(1), SU(2)$ etc

Continuous flavor symmetry

Froggatt-Nielsen like models

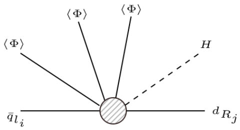
Froggatt, Nielsen '79
Leurer, Seiberg, Nir '92

- SM fermions are charged under G_F
- Flavon fields (Φ) are introduced (charged under G_F)
- Forbids Yukawa couplings at the normalizable level
- Yukawa terms are allowed as higher dimensional operator:

$$\mathcal{L} \in \bar{q_{L_i}} d_{R_j} H \left(\frac{\Phi}{\Lambda}\right)^{n_{ij}} \Rightarrow m^d_{ij} = \left(\frac{\langle\Phi\rangle}{\Lambda}\right) \frac{v}{\sqrt{2}}$$

Λ : Cut-off scale introduced → can be interpreted as the mass scale of the new degrees of freedom

↓ might be related to new physics (neutrino mass, DM, baryogenesis etc.)



• $\langle\Phi\rangle < \Lambda \Rightarrow \epsilon = \frac{\langle\Phi\rangle}{\Lambda}$: small parameter, $n_{ij} \rightarrow$ dictated by symmetry
• quark mass hierarchy and mixing can be explained

Discrete flavor symmetry

The A_4 is considered to be a favored symmetry in the neutrino sector Ma, Rajasekharan '01
Babu, Ma, Valle '03
Altarelli, Feruglio '05

A_4 is the minimal group which contains 3 dim. representation

- **A_4** → Even Permutation of 4 objects; Group of order 12; invariant group of a tetrahedron
- **minimal group** → It has $1, 1', 1'', 3$
- **3 dim.** → Can accommodate three flavors of leptons $\begin{pmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{pmatrix} \sim 3;\ e_R \sim 1, \mu_R \sim 1'', \tau_R \sim 1'$

- Product rule: $3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3_A \oplus 3_S$
- $1 \otimes 1 = 1,\ 1' \otimes 1' = 1'', 1' \otimes 1'' = 1$
  $1'' \otimes 1'' = 1'$

Altarelli-Feruglio (AF) Model

- SM singlet scalars (flavons) introduced for neutrino sector: $\phi_S \sim 3,\ \xi \sim 1,\ \phi_T \sim 3$

$$\langle\phi_S\rangle = v_S\,(1,1,1)^T\ ,\ \langle\xi\rangle = v_\xi,\ \langle\phi_T\rangle = v_T\,(1,0,0)^T$$

- Neutrino mass follows from: $\dfrac{\ell_i H \ell_j H}{\Lambda}\left(y_1\dfrac{\phi_S}{\Lambda} + y_2\dfrac{\xi}{\Lambda}\right)$

- Light neutrino mass matrix

$$(m_\nu)_0 = \begin{pmatrix} a - 2b/3 & b/3 & b/3 \\ b/3 & -2b/3 & a + b/3 \\ b/3 & a + b/3 & -2b/3 \end{pmatrix},\quad \begin{array}{l} a = y_1(v^2/\Lambda)\epsilon \\ b = y_2(v^2/\Lambda)\epsilon \end{array},\ \epsilon = v_\xi/\Lambda = v_S/\Lambda$$

- Charge lepton sector becomes diagonal

- Resulting lepton mixing matrix : $U_{TB} = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

TBM Predictions: $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{23} = 1/2$ **and** $\boxed{\sin\theta_{13} = 0}$

Consistent with experimental finding: $\theta_{12} \sim 35^\circ$, $\theta_{23} \sim 45^\circ$ however $\boldsymbol{\theta_{13} = 8^\circ - 9^\circ}$ from results of Double CHOOZ, Daya-Bay, Reno, T2K.

Q1: θ_{13} has to be generated

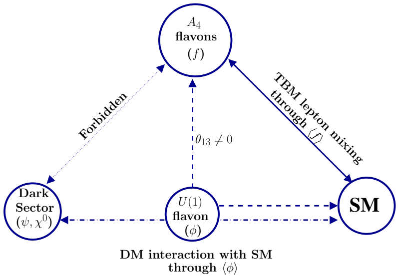
Q2: Can we extend the flavor symmetry in the Dark sector ?

Dark Sector

We consider the dark sector consists of two vector-like fermions: one $SU(2)_L$ doublet $\psi = \begin{pmatrix} \psi^0 \\ \psi^- \end{pmatrix} \equiv (1, 2, -1)$ and one SM singlet $\chi^0 \equiv (1, 1, 0)$

- **Motivation :**
  (i) We presume a replication of the SM Yukawa like interaction to be present in the Dark sector also;
  (ii) Phenomenological reason: $(\overline{\chi^0}\chi^0 H^\dagger H)/\Lambda$ [over abundant DM]
  ψ : gauge interaction [under abundant DM]
- **DM Candidate:** The neutral components mix to give rise a fermionic DM, ψ_1.

Role of flavor symmetry: A4 is relevant in explaining the flavor mixing
G_F is controlling the Yukawa interaction



We consider (i) an extra $U(1)$ symmetry under which SM fields are uncharged; and (ii) a flavon ϕ charged under $U(1)$.

Our Construction

- ψ, χ^0 being **vector-like** fermions:

$$-\mathcal{L}_{\text{Yuk}} \supset M_\psi \overline{\psi}\psi + M_\chi \overline{\chi^0}\chi^0 + \left[\left(\frac{\phi}{\Lambda}\right)^n \overline{\psi}\widetilde{H}\chi^0 + \text{h.c.}\right] \text{ with } Y = \epsilon^n = \left(\frac{\langle\phi\rangle}{\Lambda}\right)^n$$

Neutrino sector: AF model + correction from $\phi(1)$ and $\eta(1')$ —— U(1) ——→ **DM Sector**

↓ $IHIH\phi\eta \implies$ nonzero θ_{13} $(\propto \epsilon)$

↓ $Y = \epsilon^n = \left(\frac{\langle\phi\rangle}{\Lambda}\right)^n$

- For simplicity we consider vevs of all flavons to be same
- Stability of the DM : ensured by $U(1) \Rightarrow \mathcal{Z}_2$
- Interaction controlled by ϵ and n
- ϵ : fixed from generation of θ_{13}
- Use that information + constriants on Y from {relic density + direct search}

$\Downarrow$

Estimate n

Structure of the Model:

Field	e_R	μ_R	τ_R	ℓ	H	ψ	χ^o	ϕ_S	ϕ_T	ξ	η	ϕ
$SU(2)_L$	1	1	1	2	2	2	1	1	1	1	1	1
A_4	1	$1''$	$1'$	3	1	1	1	3	3	1	$1'$	1
Z_3	ω	ω	ω	ω	1	1	1	ω	1	ω	ω	1
Z_2	-1	-1	-1	1	1	-1	-1	1	-1	1	1	1
$U(1)$	0	0	0	0	0	q_1	q_2	0	0	0	$-x$	x

- charged lepton (diagonal): $\frac{y_e}{\Lambda}(\bar{\ell}\phi_T)He_R + \frac{y_\mu}{\Lambda}(\bar{\ell}\phi_T)' H\mu_R + \frac{y_\tau}{\Lambda}(\bar{\ell}\phi_T)'' H\tau_R$ with $\langle\phi_T\rangle = v_T\,(1,1,1)^T$.

$$\textbf{Neutrino Sector}: \quad -\mathcal{L}_{\nu_0} - \delta\mathcal{L}_\nu = (\ell H\ell H)(y_1\xi - y_2\phi_S)/\Lambda^2 + y_3\frac{(\ell H\ell H)\phi\eta}{\Lambda^3}$$

$$(m_\nu)_0 + \delta m_\nu = \begin{pmatrix} a-2b/3 & b/3 & b/3 \\ b/3 & -2b/3 & a+b/3 \\ b/3 & a+b/3 & -2b/3 \end{pmatrix} + \begin{pmatrix} 0 & 0 & d \\ 0 & d & 0 \\ d & 0 & 0 \end{pmatrix}$$

Diagonalizing matrix

$$\begin{aligned} U_{PMNS} \approx U_\nu &= U_{TB} U_1 U_m \\ &= \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \cos\theta_\nu & 0 & \sin\theta_\nu e^{-i\psi} \\ 0 & 1 & 0 \\ -\sin\theta_\nu e^{i\psi} & 0 & \cos\theta_\nu \end{pmatrix} U_m \end{aligned}$$

Constraining ϵ from neutrino oscillation data:

- Obtaining θ_{13} (with $\delta = 0$ or π; $|y_{1,3}| = y$ and $|y_2| = k$):

$$\tan 2\theta_\nu = \frac{\sqrt{3}\epsilon}{\epsilon - 2} \Rightarrow \sin\theta_{13} = \sqrt{\frac{2}{3}}\,|\sin\theta_\nu|$$

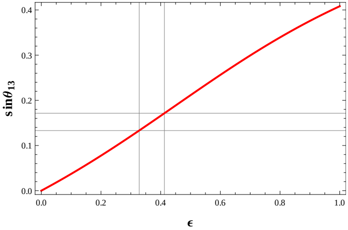
- Mass eigenvalues:

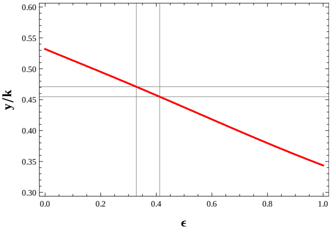
$$m_{1,3} = \alpha\frac{y}{k}\left|\sqrt{1-\epsilon+\epsilon^2} + k/y\right|,$$
$$m_2 = \alpha\frac{y}{k}(1+\epsilon)\,;\, \alpha = \frac{k}{\Lambda}v^2\epsilon$$

- Solar to atmospheric mass-squared difference:

$$r = \frac{\Delta m^2_\odot}{|\Delta m^2_{atm}|} = \frac{m_2^2 - m_1^2}{|m_3^2 - m_1^2|} = 0.03$$

- Constrain ϵ with $\sin\theta_{13} = 0.1330 - 0.1715$ (3σ) Forero, Tortola, Valle 2014





$\epsilon = 0.328 - 0.4125$, Σm_i=(0.102 - 0.106) eV, $|m_{ee}|$=(0.00764-0.00848) eV

Analysis: Dark Sector

• Dark matter: After EW symmetry breaking, the mass matrix M for vector-like fermions:

$$\mathcal{M} = \begin{pmatrix} M_\chi & m_D \\ m_D & M_\psi \end{pmatrix} \text{ in } (\chi^0 \ \psi^0) \text{ basis}, \quad m_D = \epsilon^n v = Yv.$$

$$\begin{aligned} \psi_1 &= \cos\theta_d \chi^0 + \sin\theta_d \psi^0 \\ \psi_2 &= \cos\theta_d \psi^0 - \sin\theta_d \chi^0 \end{aligned} \qquad \tan 2\theta_d = \frac{2m_D}{(M_\psi - M_\chi)}$$

We work in small mixing regime:

$$\sin\theta_d \simeq \frac{Yv}{\Delta M} \text{ with } \Delta M = M_2 - M_1$$

• Charged fermion $\psi^+(\psi^-)$ masses:

$$M^+(M^-) = M_1 \sin^2\theta_d + M_2 \cos^2\theta_d$$

In the limit $\theta_d \to 0$, $M^\pm = M_2 = M_\psi$

Gauge interactions:

$$\frac{g}{\sqrt{2}}\overline{\psi_0}\gamma^\mu W_\mu^+\psi^- + \text{h.c.} \to \frac{g\sin\theta_d}{\sqrt{2}}\overline{\psi_1}\gamma^\mu W_\mu^+\psi^- + \frac{g\cos\theta_d}{\sqrt{2}}\overline{\psi_2}\gamma^\mu W_\mu^+\psi^- + \text{h.c.}$$

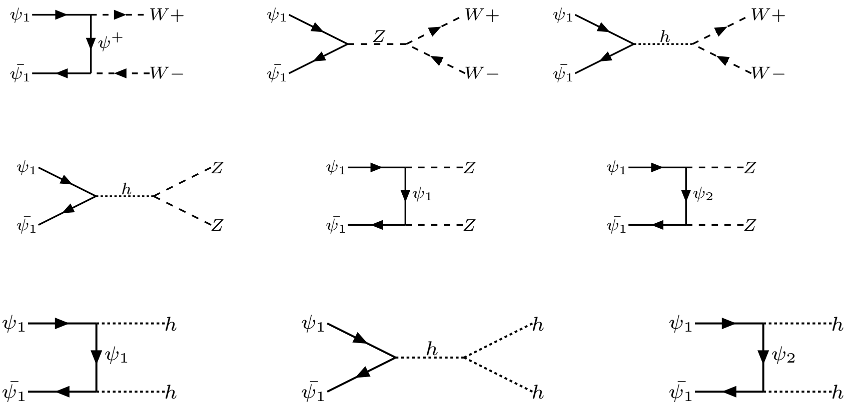
$$\frac{g}{2\cos\theta_w}\overline{\psi_0}\gamma^\mu Z_\mu\psi_0 \to \frac{g}{2\cos\theta_w}\Big(\sin^2\theta_d\overline{\psi_1}\gamma^\mu Z_\mu\psi_1 + \frac{\sin 2\theta_d}{2}(\overline{\psi_1}\gamma^\mu Z_\mu\psi_2 + \overline{\psi_2}\gamma^\mu Z_\mu\psi_1) + \cos^2\theta_d\overline{\psi_2}\gamma^\mu Z_\mu\psi_2\Big)$$

Dominant annihilations contributing to relic density:

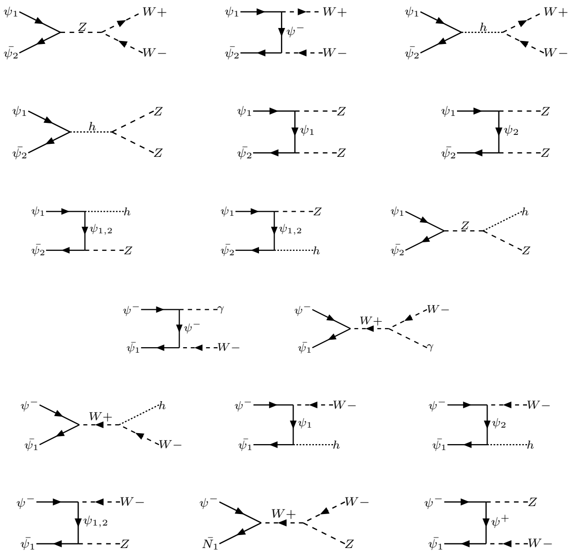
(i) $\overline{\psi_1}\psi_1 \to W^+W^-, ZZ$ through $SU(2)_L$ gauge coupling

(ii) $\overline{\psi_1}\psi_1 \to hh$ through Yukawa coupling $\propto Y$

Dominant Annihilation processes to Higgs and Gauge boson final states



Co-Annihilations



Relic abundance of dark matter:

$$\Omega_{\psi_1} h^2 = \frac{1.09 \times 10^9 \ \mathrm{GeV}^{-1}}{g_\star^{1/2} M_{PL}} \frac{1}{J(x_f)}; \quad J(x_f) = \int_{x_f}^{\infty} \frac{\langle \sigma |v| \rangle_{eff}}{x^2} \ dx$$

$$\langle \sigma |v| \rangle_{eff} = \frac{g_1^2}{g_{eff}^2} \langle \sigma_{ann} |v| \rangle_{eff} + 2 \sum_{i,j} \frac{g_{x_i} g_{x_j}}{g_{eff}^2} \langle \sigma_{co-ann} |v| \rangle_{eff} \left(1 + \frac{\Delta M}{M_1}\right) e^{\left(\frac{-x \Delta M}{M_1}\right)}$$

$$x_{i,j} = \psi_1, \psi_2, \psi^\pm$$

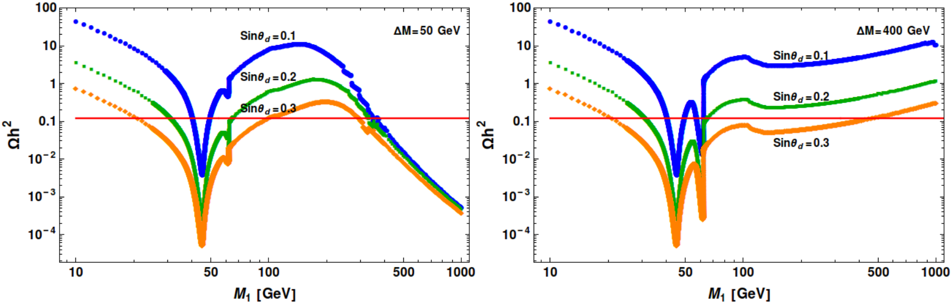
3 parameters of the model: $\sin\theta_d, M_1$ **and** ΔM $\left(\text{with } \sin\theta_d \simeq \frac{Y_V}{\Delta M}\right)$

Cosmological observation:

$$0.094 \leq \Omega_{\rm DM} h^2 \leq 0.130$$

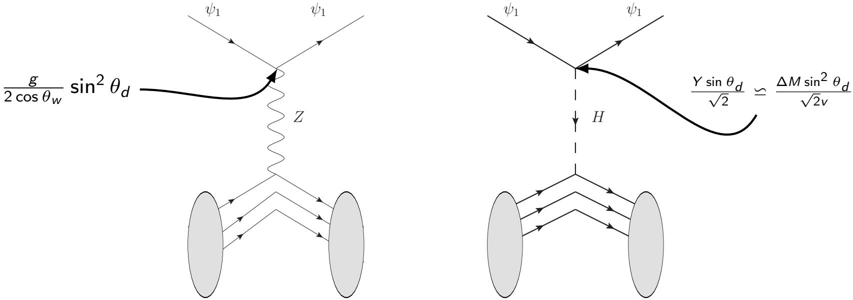
WMAP Collaboration, 1212.5226

Relic density vs. DM mass



- for smaller $\sin\theta_d$, relic density can easily be satisfied.
- co-Annihilations plays significant role.
- resonances appear for $M_1 \sim m_Z/2$ and $m_h/2$.

Direct Detection : $\psi_1 + n \to \psi_1 + n$

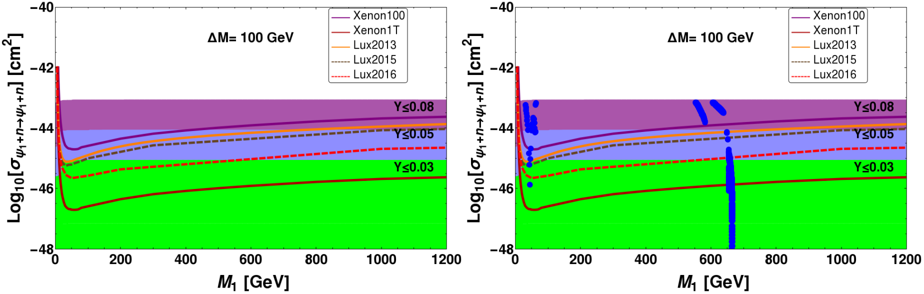


Why θ_d is taken to be small?

(i) Easier to satisfy the relic density constraint as seen from relic density plot

(ii) θ_d is mainly restricted from direct detection

Direct Detection plots:



ΔM is fixed at 100 GeV; Y=0.03 satisfies the LUX 2016 limit for $M_1 \gtrsim$ 600 GeV;

Other constraints on Model Parameters:

Invisible decay of Z : $\Gamma_Z(\text{invisible}) = 499 \pm 1.5$ MeV

If Z is allowed to decay to $\psi_1\psi_1$, $\psi_2\psi_2$ [*i.e.* for $M_{1,2} \lesssim 45$ GeV] then $\sin\theta_d \lesssim 10^{-3}$.

Invisible Higgs decay: If $M_{1,2} \lesssim 63$ GeV

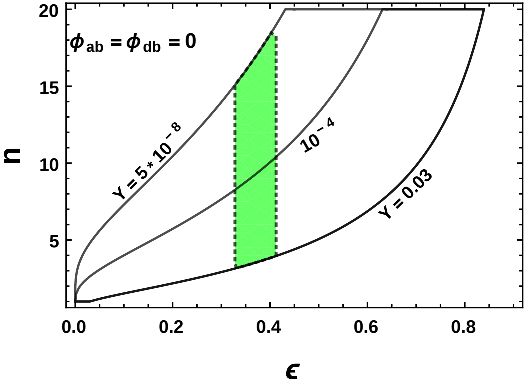
$\sin\theta_d$ is strongly restricted

We consider $M_{1,2} > 63$ GeV

Correlation between dark and neutrino sector:

ϵ : [0.328 − 0.4125] for θ_{13} to be in 3σ range

Y : [10^{-7} − 0.03] from direct search constraint and relic density satisfaction



$\delta = 0,\ n \geq 2$

Correlation between dark and neutrino sector: in presence of nonzero Dirac CP phase

$$\tan 2\theta_\nu = \frac{\sqrt{3}\epsilon \cos\phi_{db}}{(\epsilon\cos\phi_{db} - 2\cos\phi_{ab})\cos\varphi}; \ \sin\theta_{13} = \sqrt{\frac{2}{3}}\,|\sin\theta_\nu|\,,$$

$$\tan\varphi = \tan\delta = \frac{y}{k}\frac{\sin(\phi_{db}-\phi_{ab})}{\cos\phi_{db}}; \quad \delta = \arg[(U_1)_{13}]; \quad \phi_{ab} = \phi_a - \phi_b \quad \phi_{db} = \phi_d - \phi_b$$

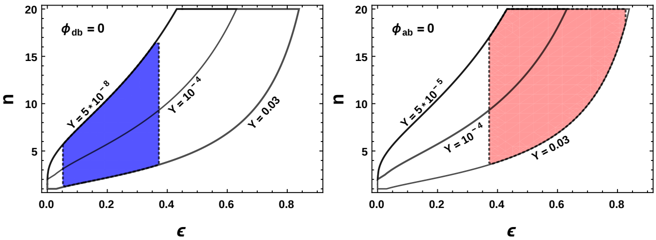


Figure: n vs ϵ to generate different values of $Y = \epsilon^n$ for $\delta \neq 0$

$\delta \neq 0$, limit of n changes

Conclusions:

- Flavor symmetric extension of SM: $A_4 \times U(1)$.
- Dark sector consists of two vector-like fermions.
- SM Yukawa like interaction is replicated in dark sector in presence of a $U(1)$ flavon.
- DM stability is ensured as a result of a remnant $\mathcal{Z}_2$ after $U(1)$ breaking.
- Leptonic mixing θ_{13} restricts the strength of the Yukawa interaction of the DM with SM Higgs.
- Simultaneous satisfaction of θ_{13} and DM relic density accompanied by the direct search limits restricts the $U(1)$ charge of the flavon.

Thank You