Gravitational waves from phase transitions: analytic approach

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Based on arXiv:1605.01403 (PRD95, 024009) & 1707.03111

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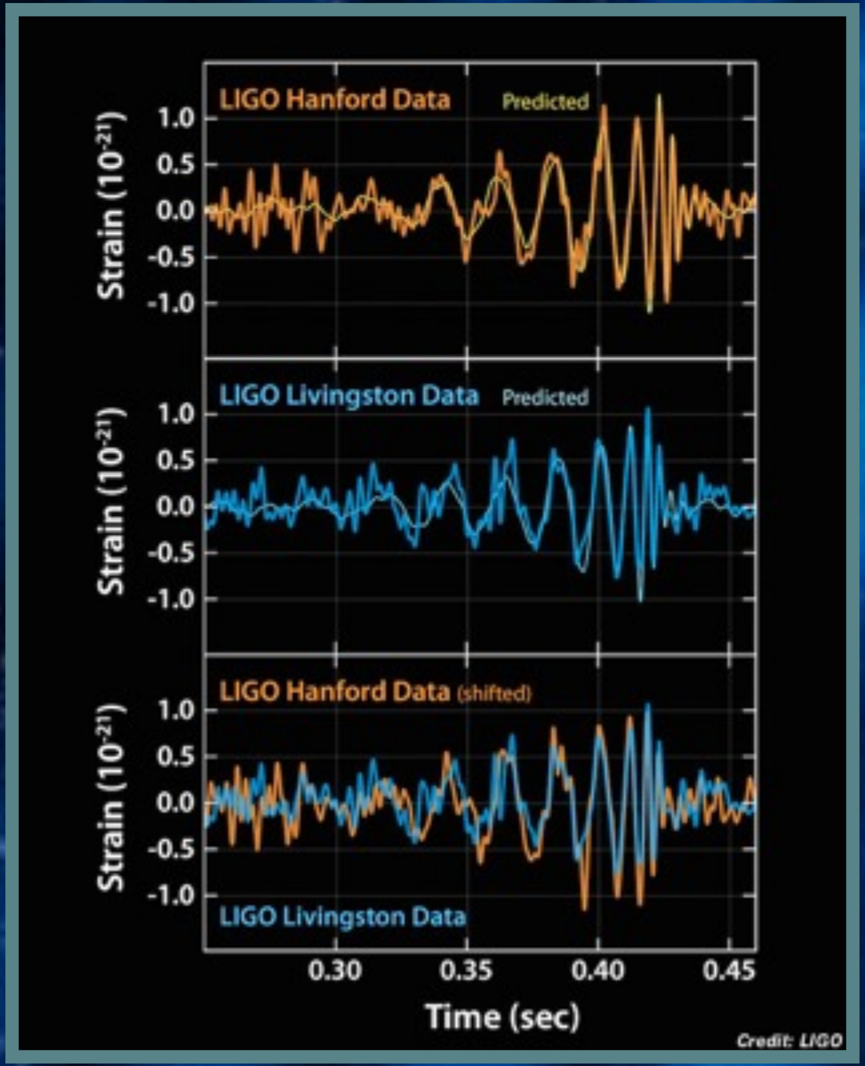
Introduction

ERA OF GRAVITATIONAL WAVES

- Detection of GWs from BH binaries → **GW astronomy** has started



[LIGO]

ERA OF GRAVITATIONAL WAVES

- Detection of GWs from BH binaries → GW astronomy has started

- Next will come GW cosmology with space interferometers

  e.g. LISA, DECIGO, BBO, ...

- First-order phase transitions can be cosmological GW sources

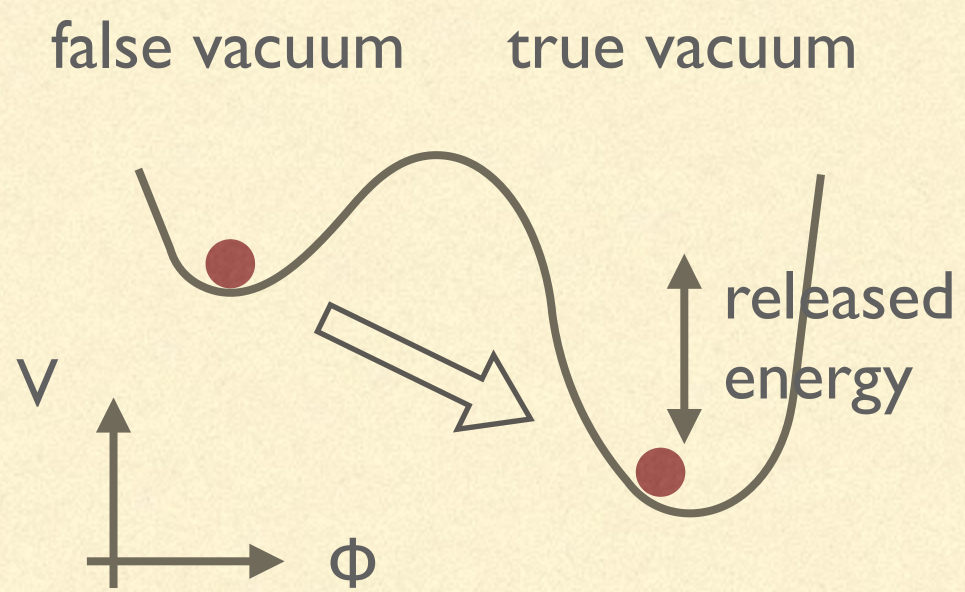
  - Electroweak sym. breaking (w/ extensions)
  - PQ sym. breaking
  - SUSY breaking
  - GUT breaking

  ... etc.

GWS AS A PROBE TO PHASE TRANSITION

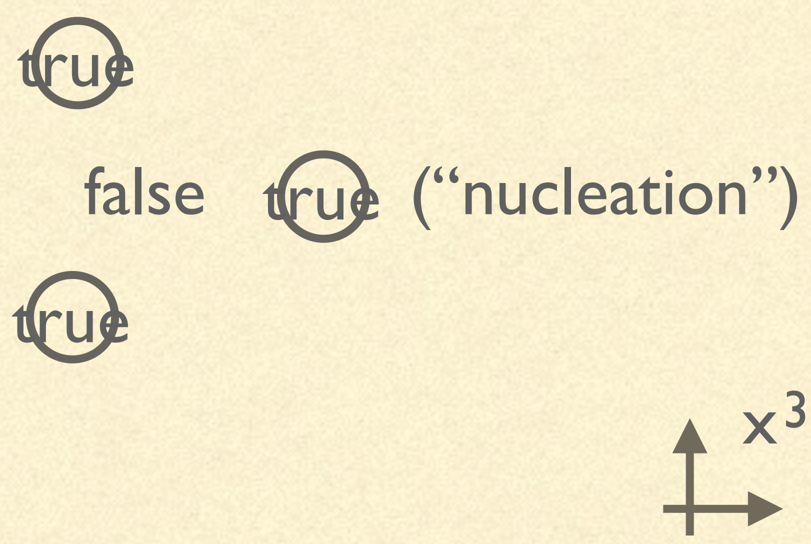
- How thermal first-order phase transition produces GWs

- Field space



Quantum tunneling

- Position space



Bubble formation & GW production

GWS AS A PROBE TO PHASE TRANSITION

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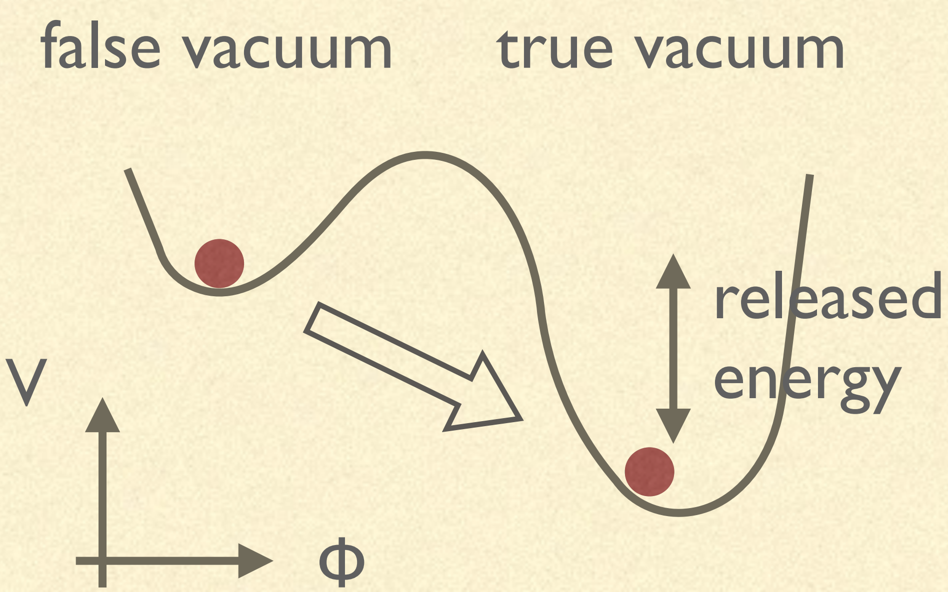

Quantum tunneling

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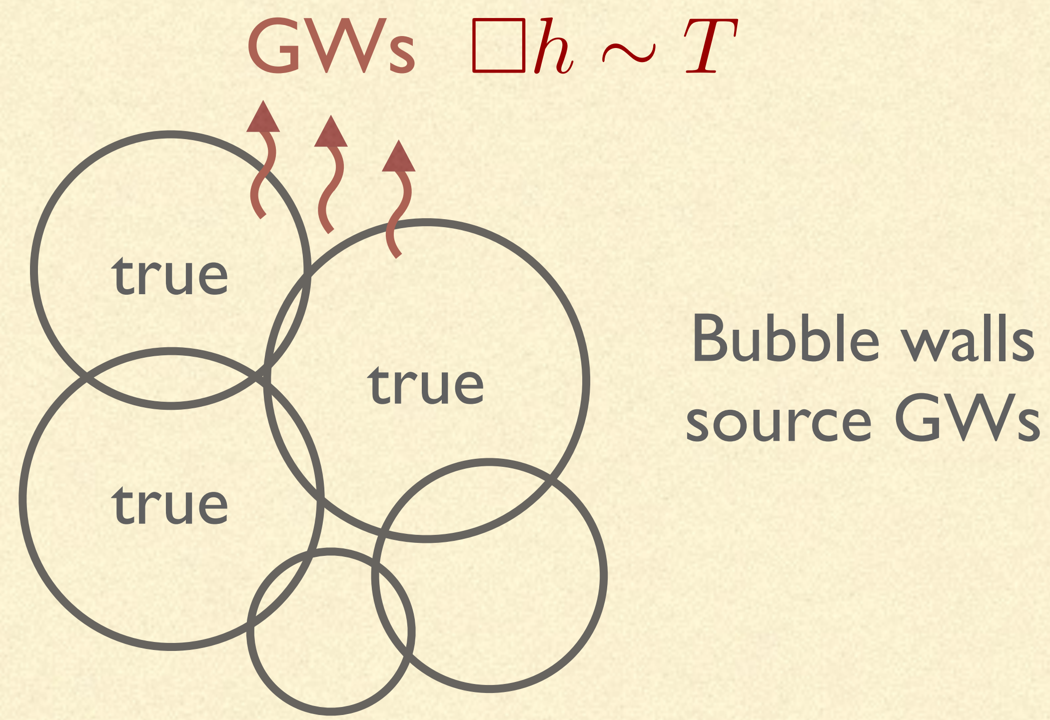


Bubble formation & GW production

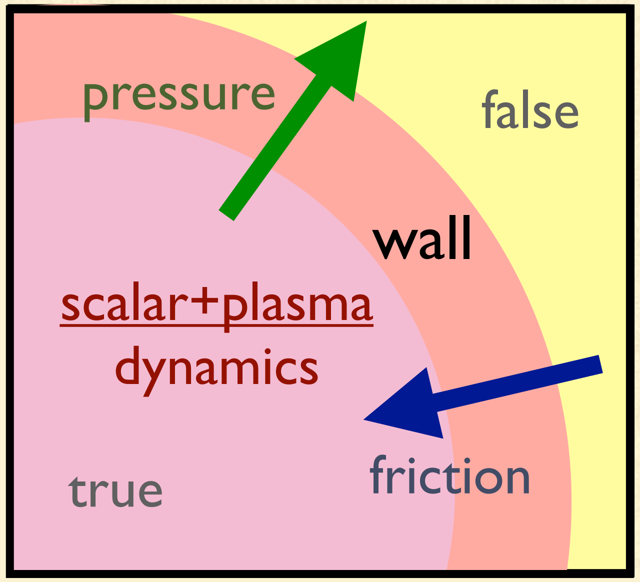
TALK PLAN

0. Introduction ✔
1. GW sourcing in phase transitions
2. Analytic approach
3. Conclusion

1. GW sourcing in phase transitions

CURRENT UNDERSTANDING

- Two main players in the game : **scalar field & plasma**



- Walls (where the scalar field value changes) want to expand ("pressure")
- Walls are pushed back by plasma ("friction")

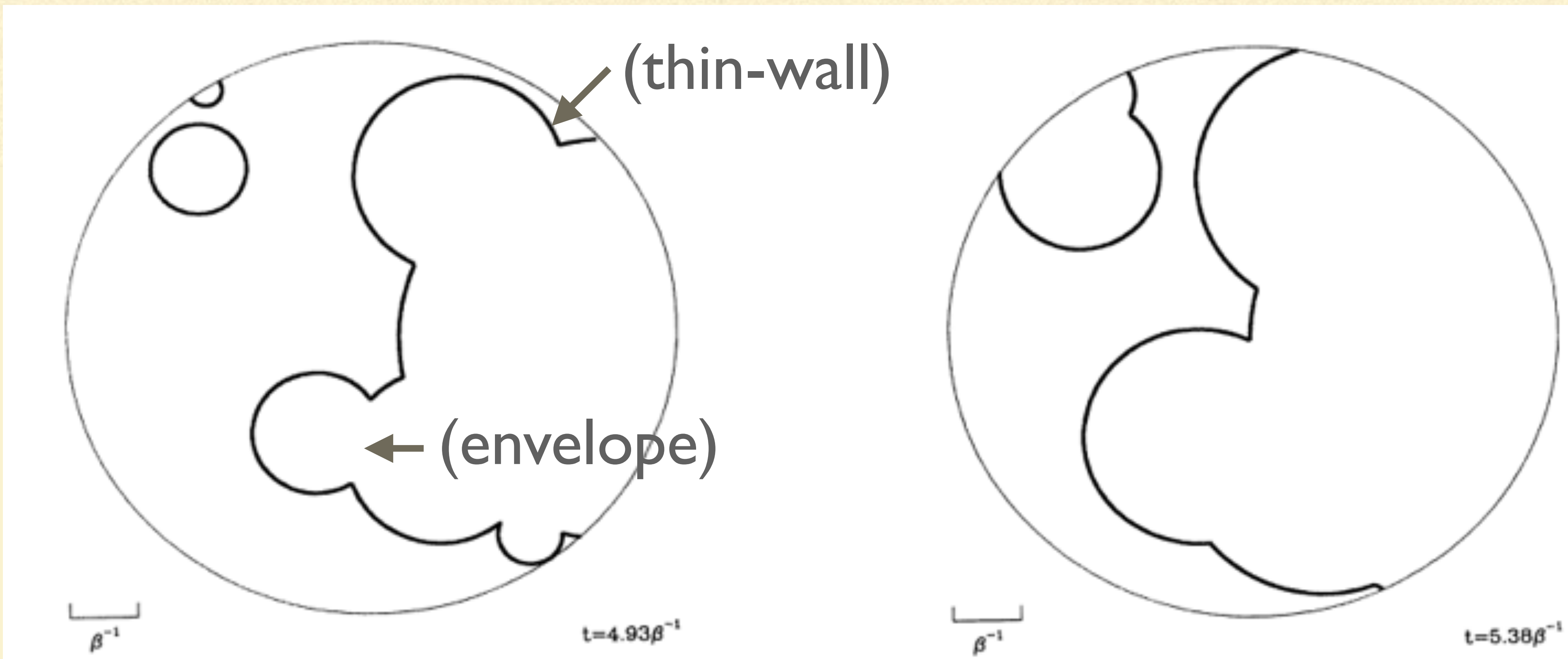
- Three main sources for GWs [Caprini et al.'16]

Bubble collision / Sound wave / Turbulence

(scalar field contribution) (plasma contribution)

CURRENT UNDERSTANDING

- Two



changes)

'friction")

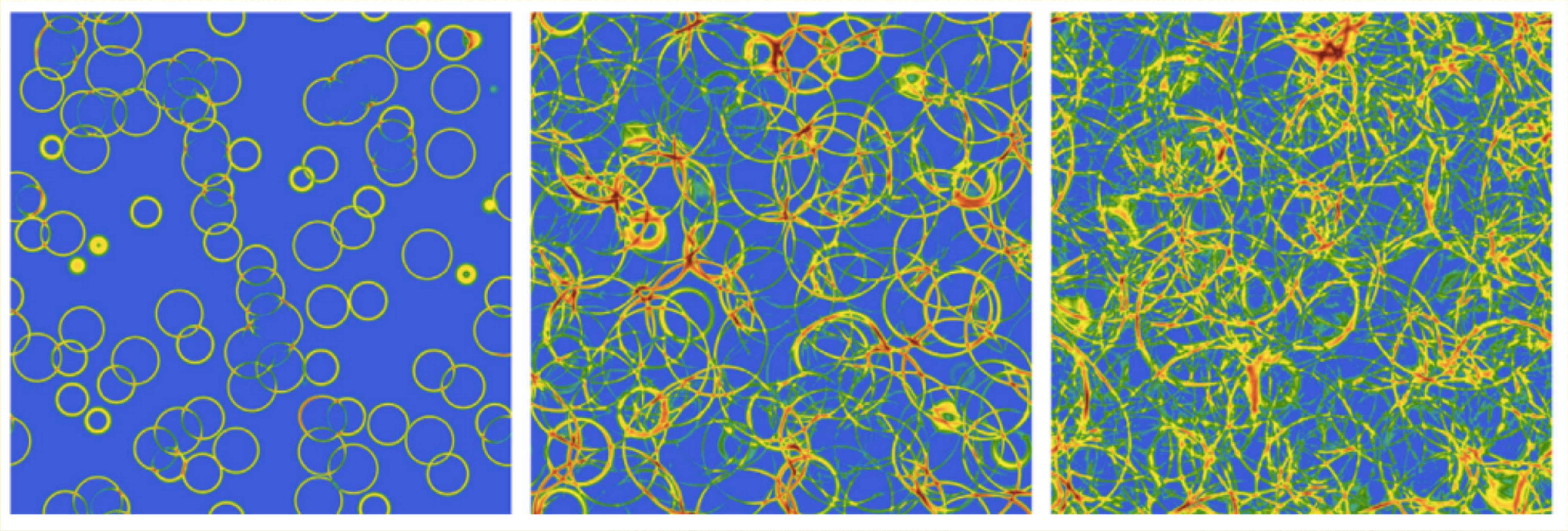
[Kosowski et al. '93]

- Three main sources for GWs [Caprini et al. '16]

Bubble collision / Sound wave / Turbulence

(scalar field contribution) (plasma contribution)

CURRENT UNDERSTANDING



[Hindmarsh et al.'15]

- Three main sources for GWs [Caprini et al.'16]

Bubble collision / Sound wave / Turbulence

(scalar field contribution) (plasma contribution)

NECESSITY OF ANALYTIC APPROACH

- Current understanding mainly comes from developments in

Numerical simulations

- However, we need

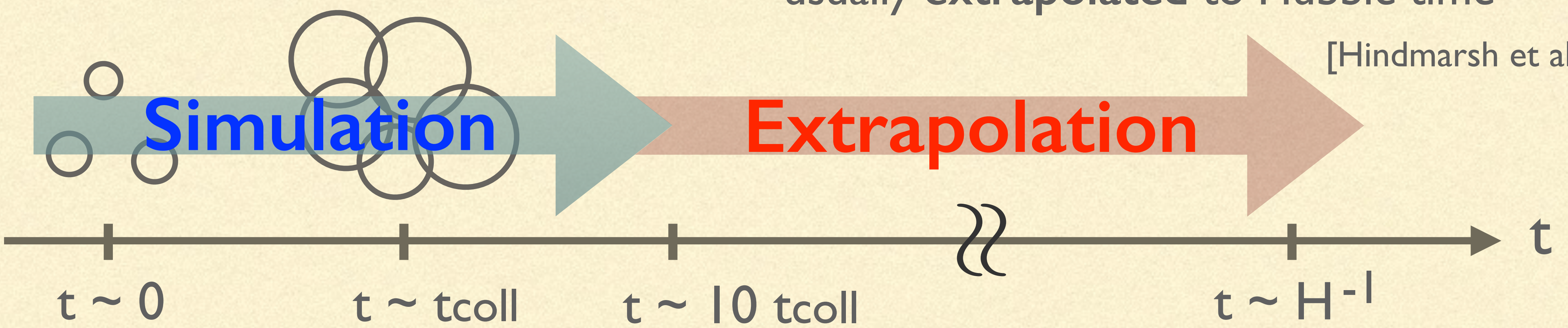
Analytic understanding + Numerical understanding

(Compare the situation w/ e.g. CMB, Lattice QCD etc.)

WHY NUMERICAL SIMULATIONS ARE NOT ENOUGH?

- Numerical simulations for GW production from "sound waves" are usually **extrapolated** to Hubble time

[Hindmarsh et al.'14]



by assuming that GW sources are constantly effective until Hubble time

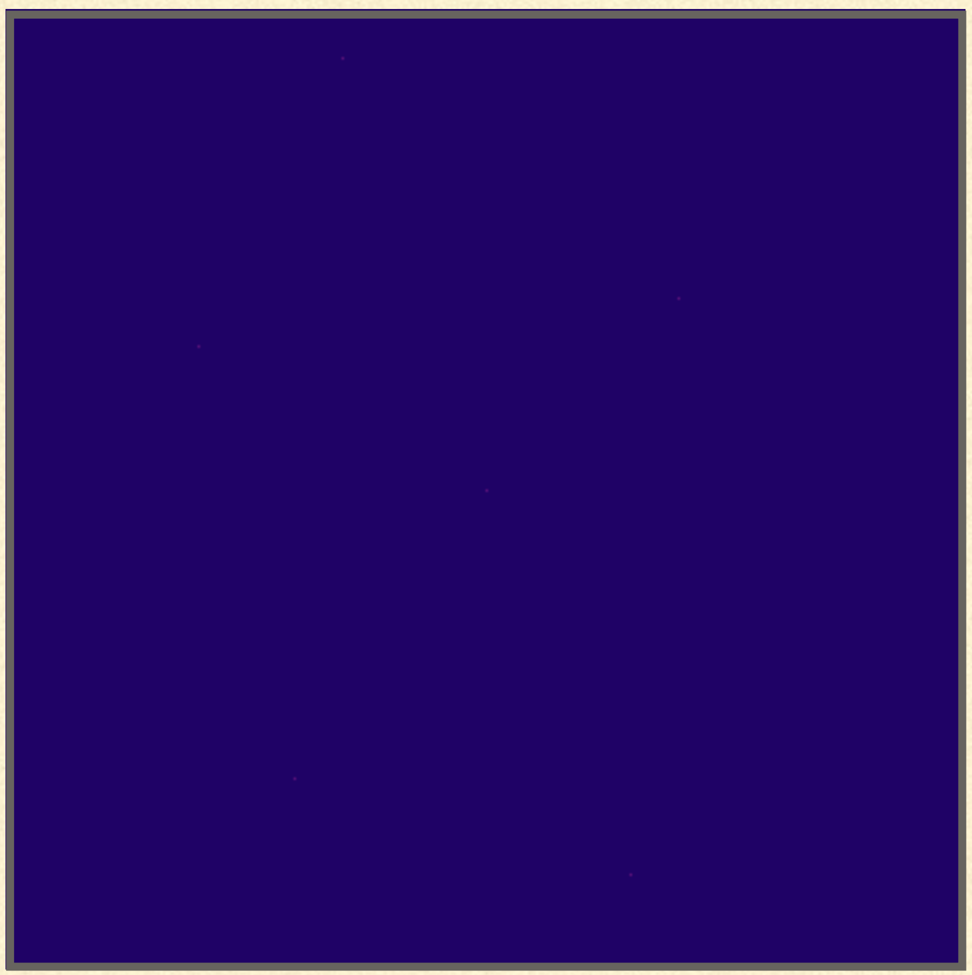
- Many future prospects are derived from these results
- Is this treatment justified?   Alternative way to understand the system?

"Analytic approach"

2. Analytic approach

THE SYSTEM WE WANT TO UNDERSTAND

- Following system will capture the physics



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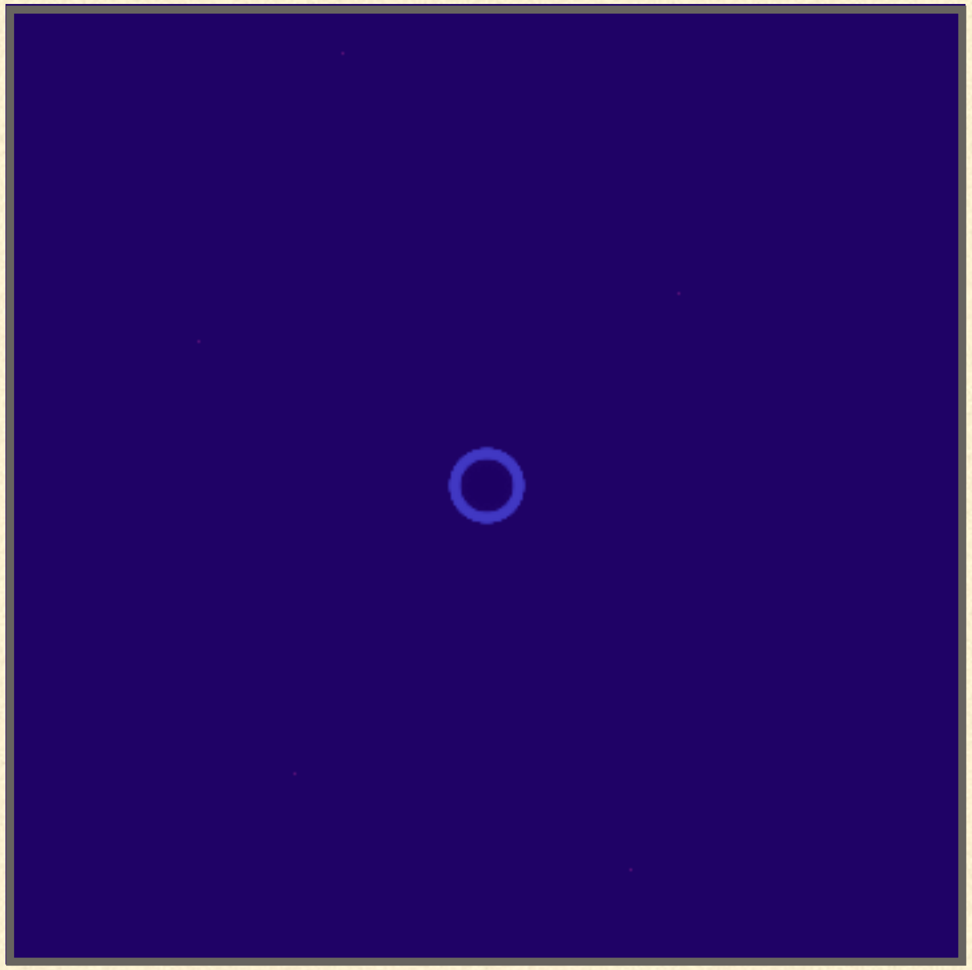

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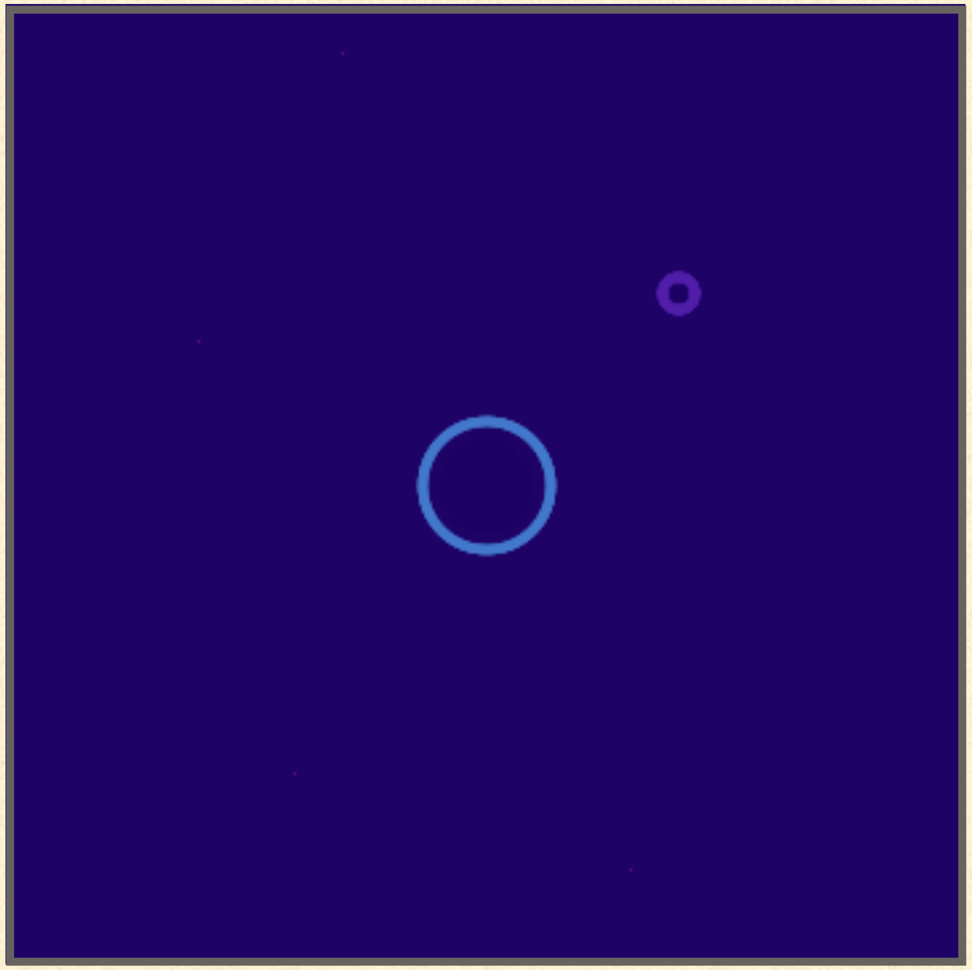

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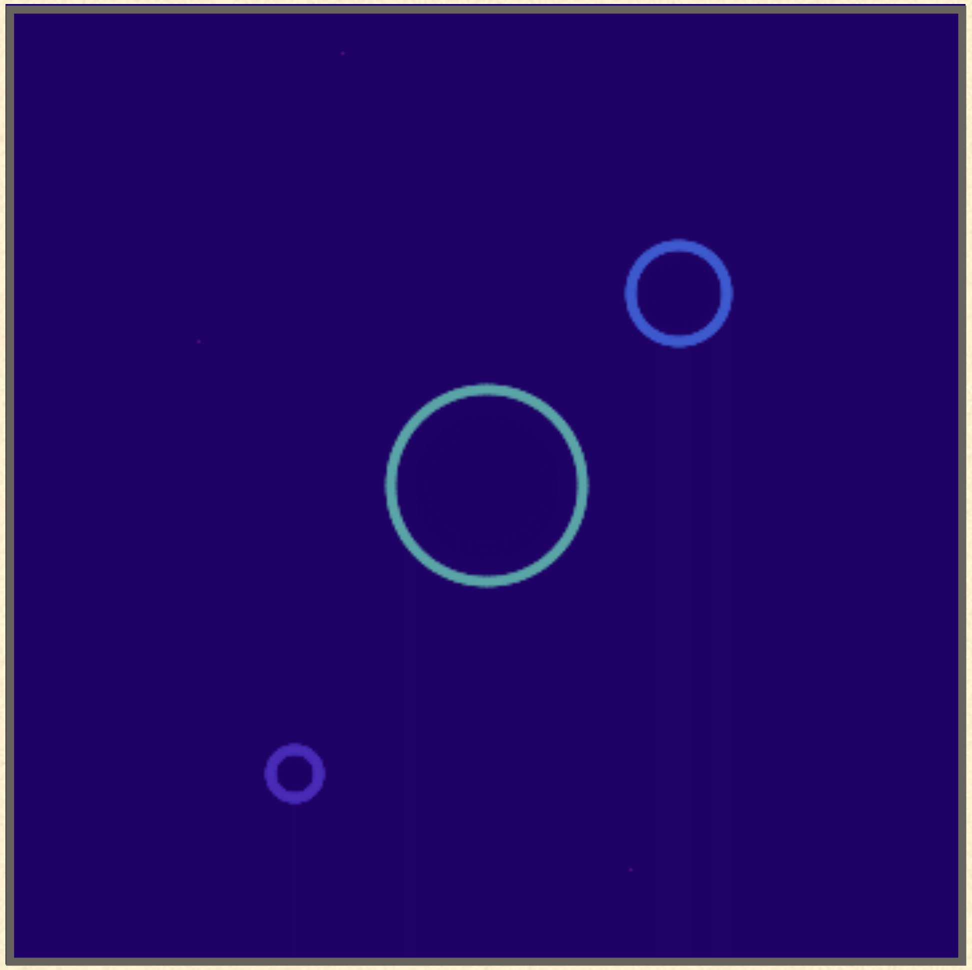

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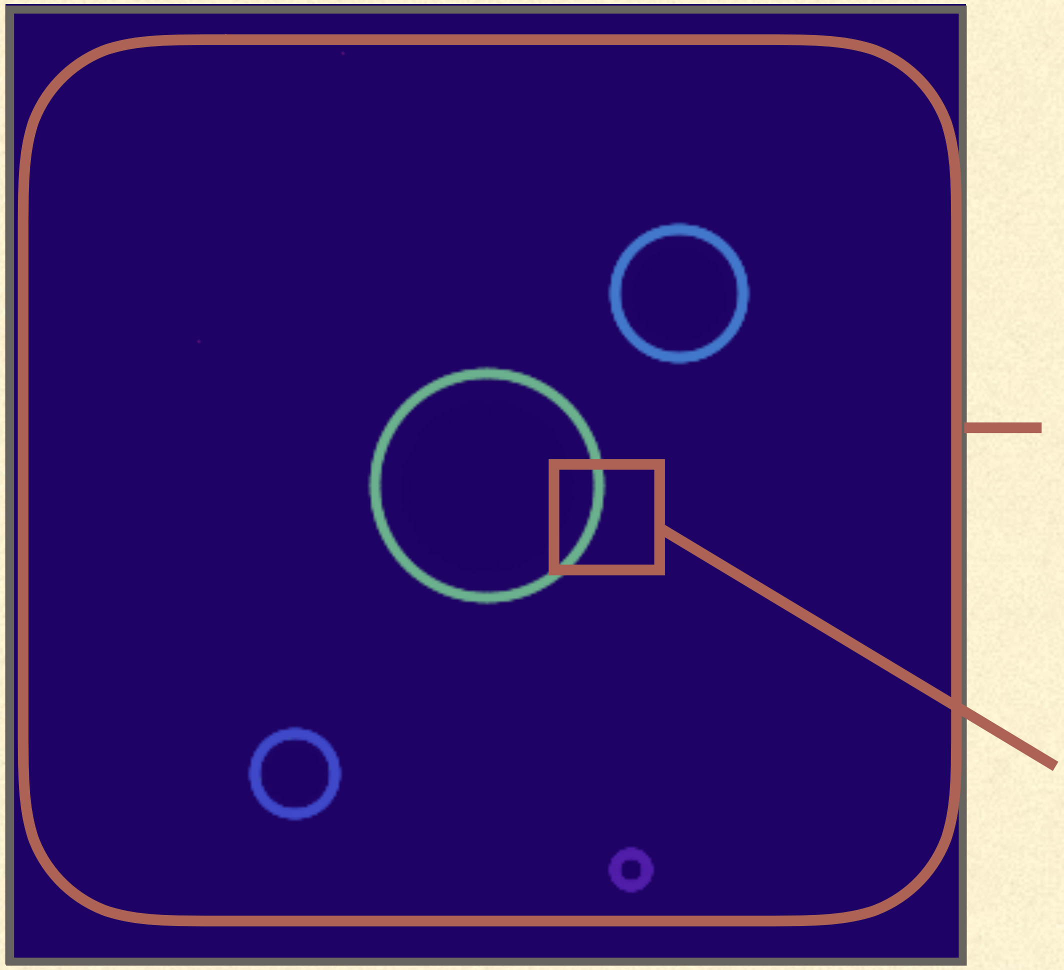

- Cosmic expansion neglected
- Bubbles nucleate with rate Γ
  (Typically $\Gamma \sim e^{\beta t}$ in thermal PTs)
- Walls are approximated to be thin

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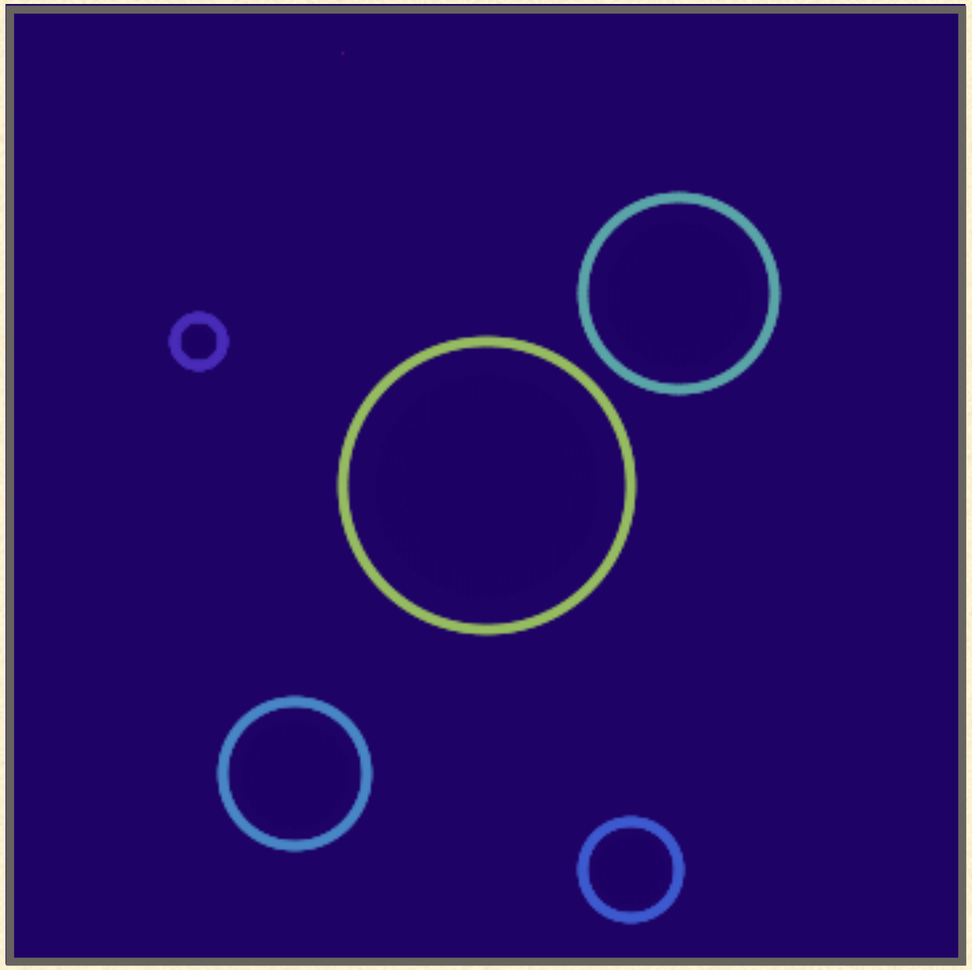

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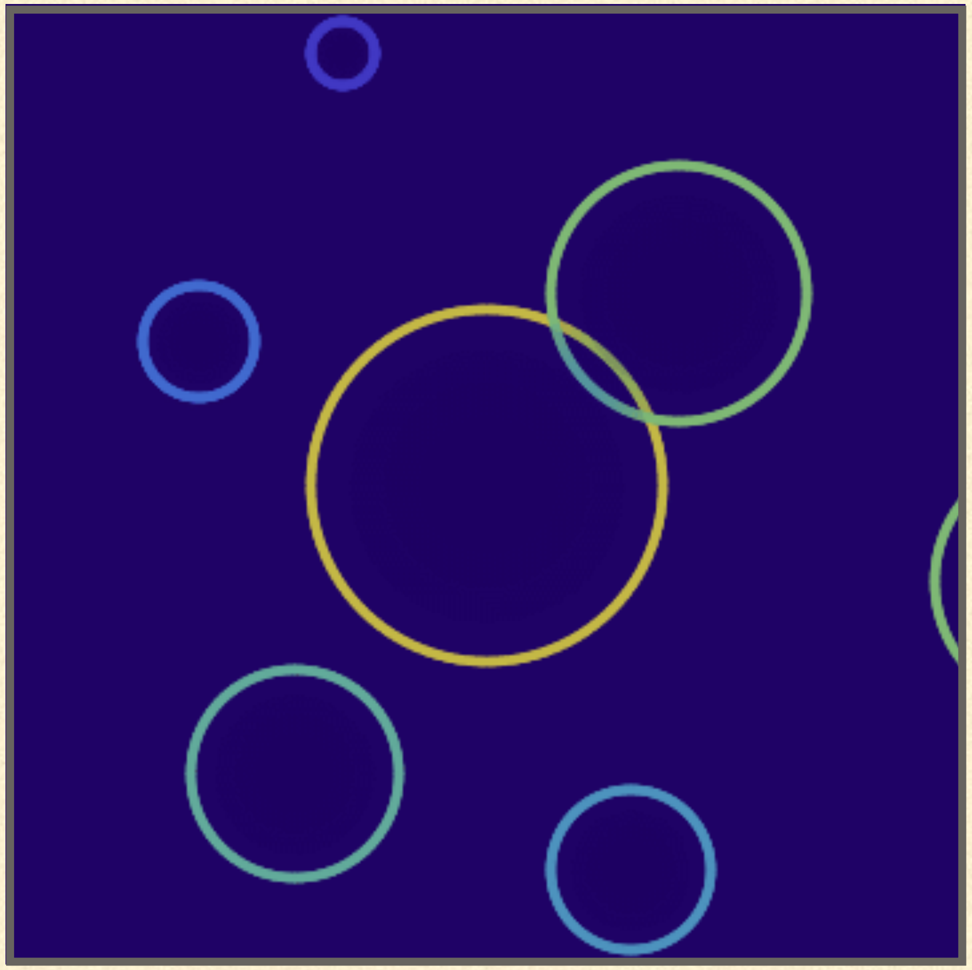

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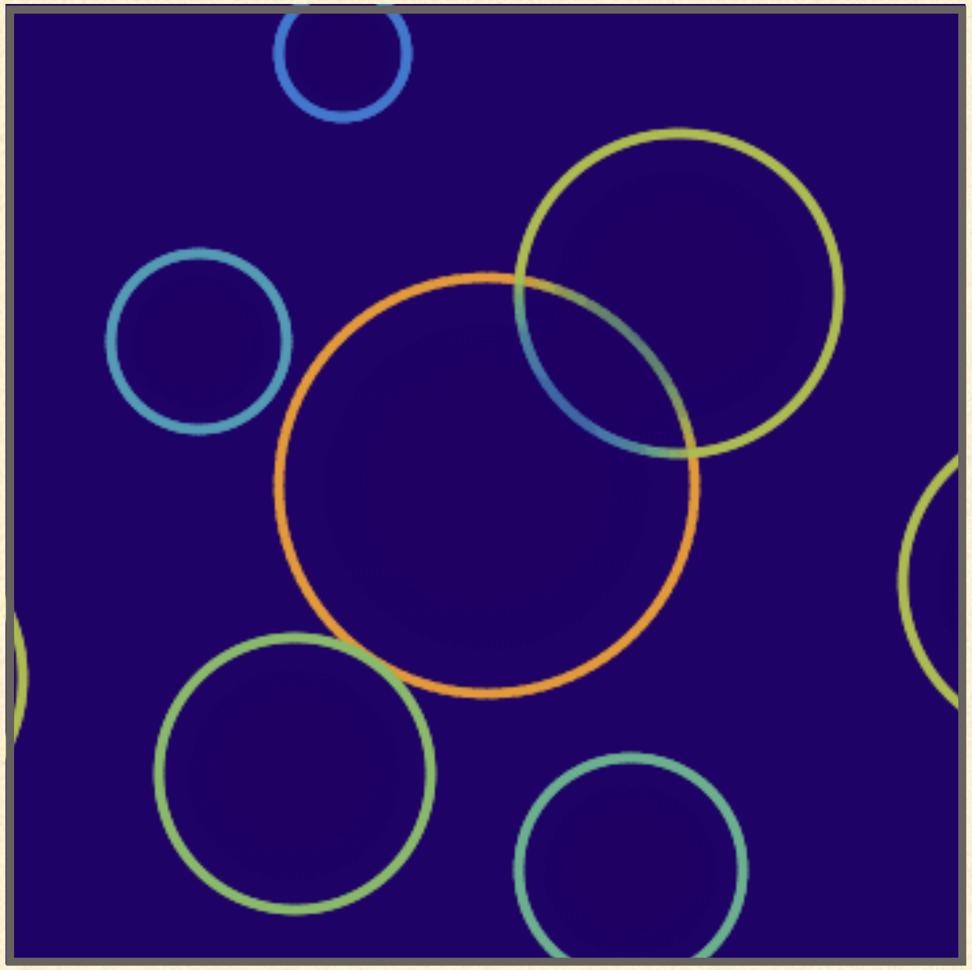

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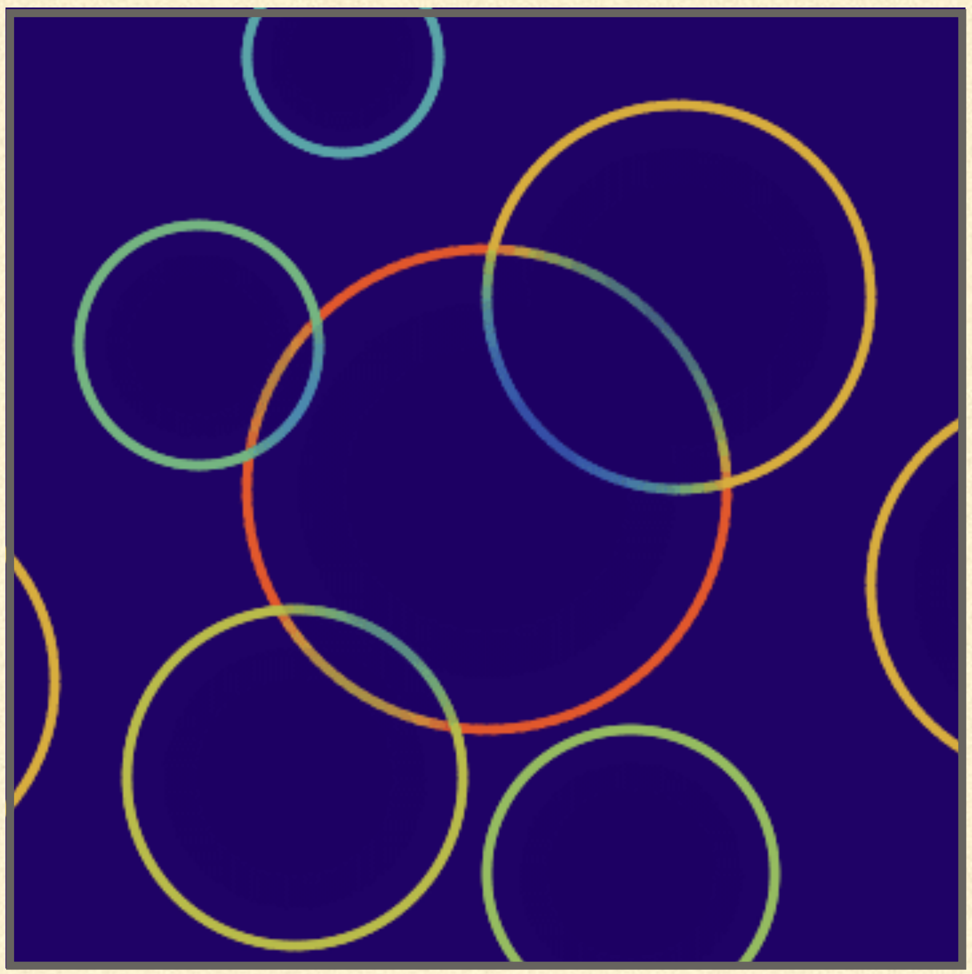

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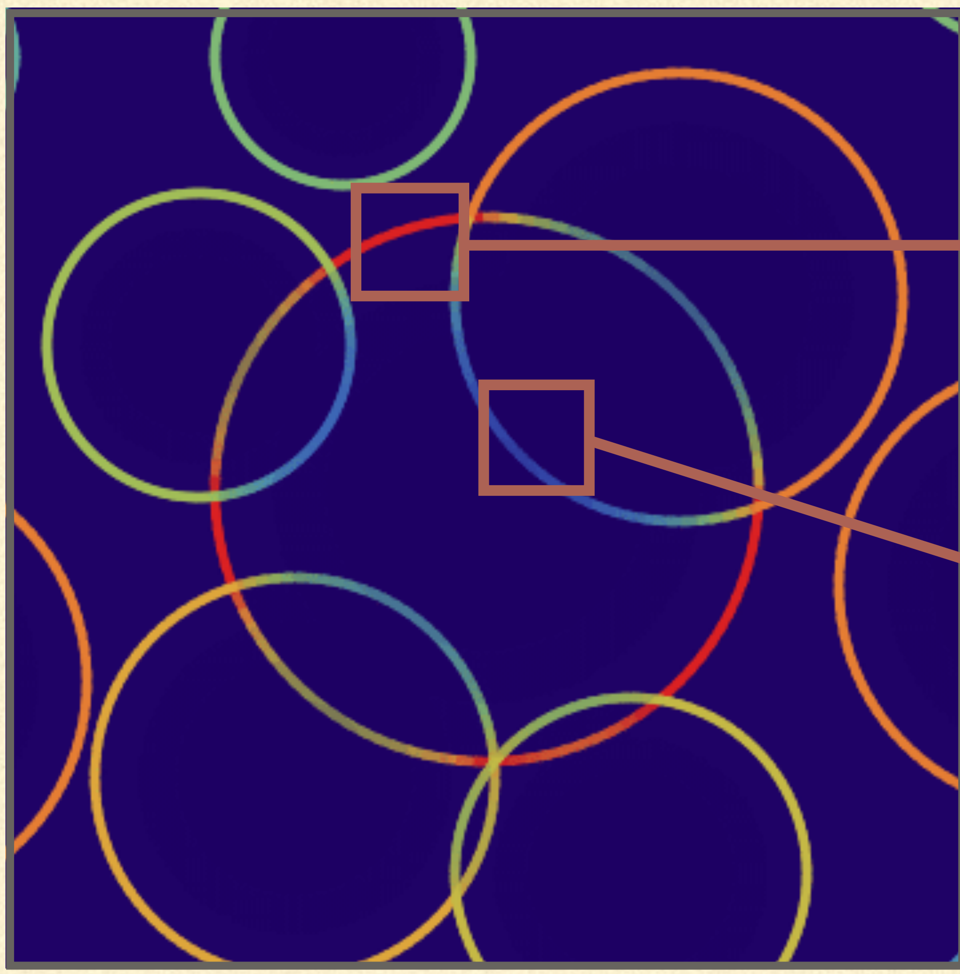

- Walls become more and more energetic
  (typically $T_{ij} \propto$ (bubble radius))
- They lose energy after first collision
  ($T_{ij} \propto$ (bubble radius)$^{-2}$ × (damping func. D))

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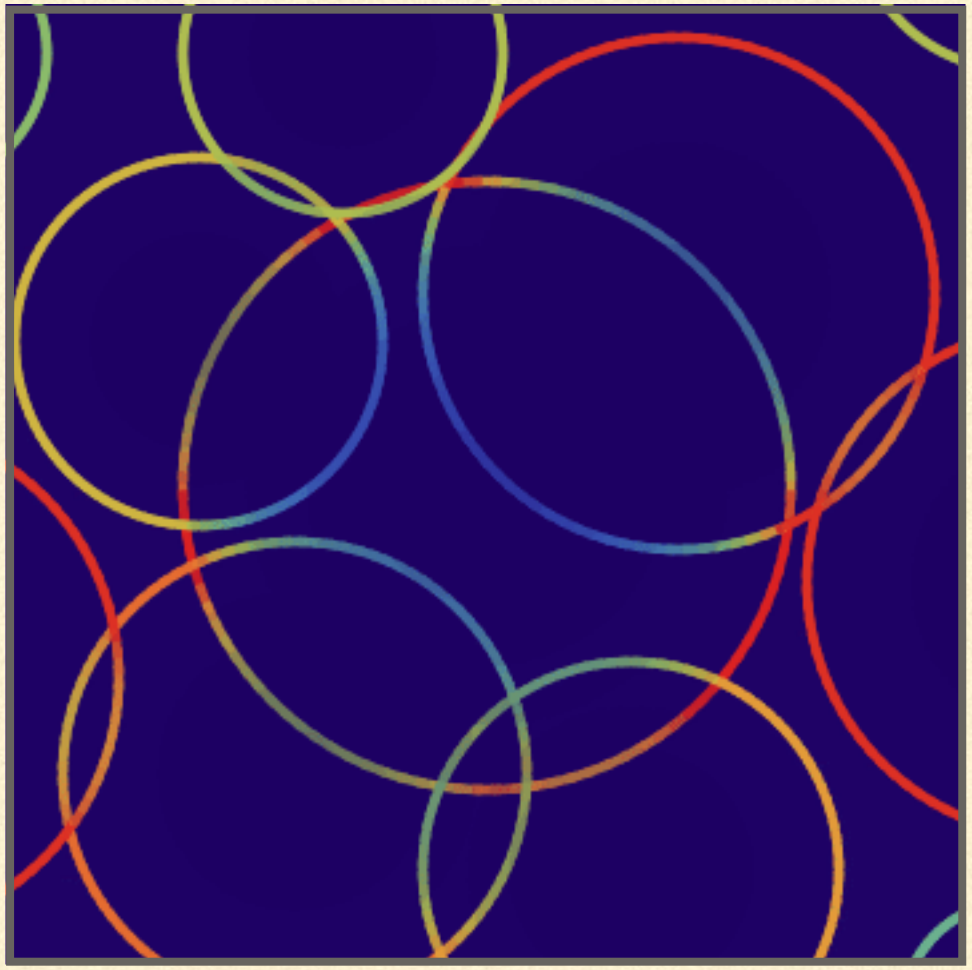

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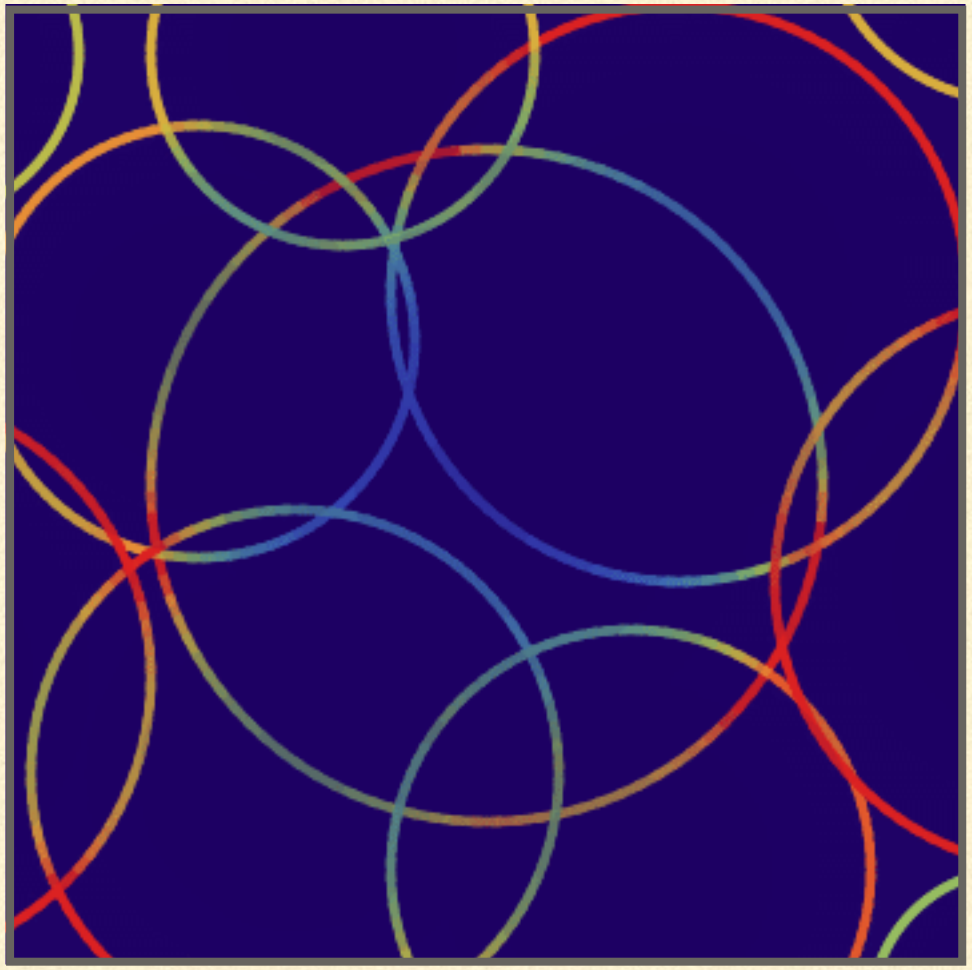

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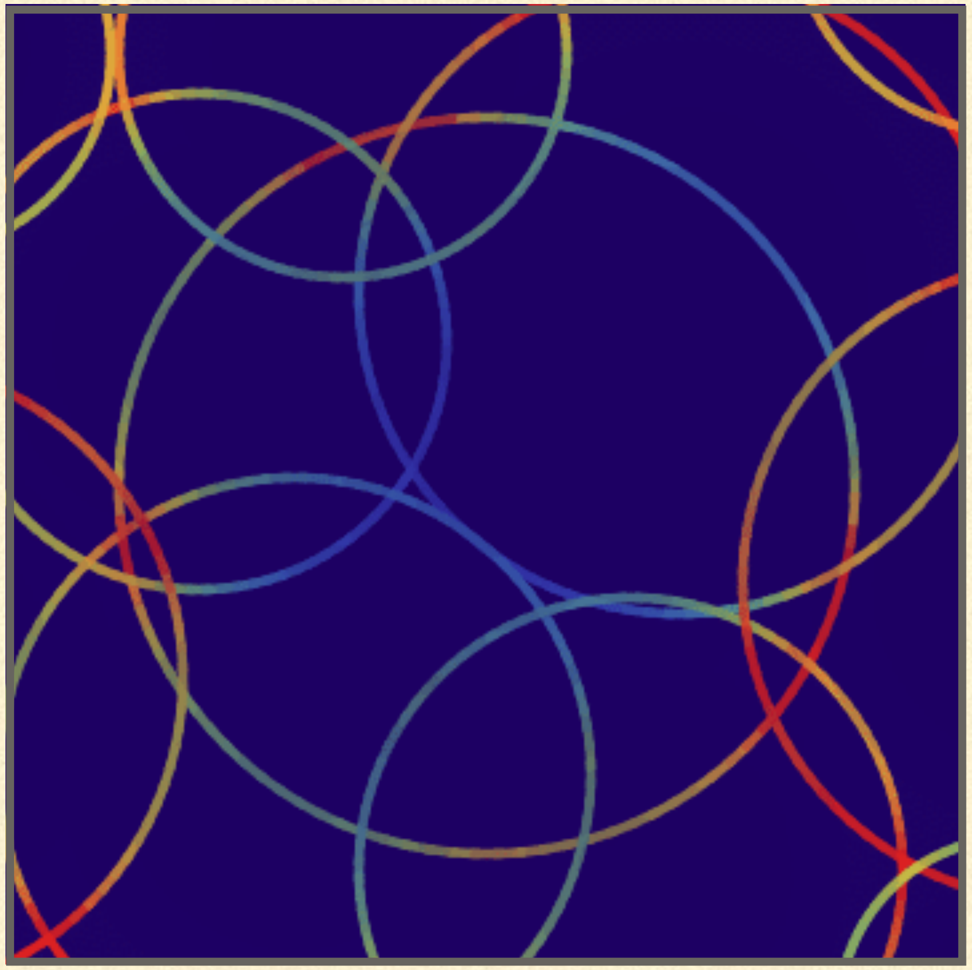

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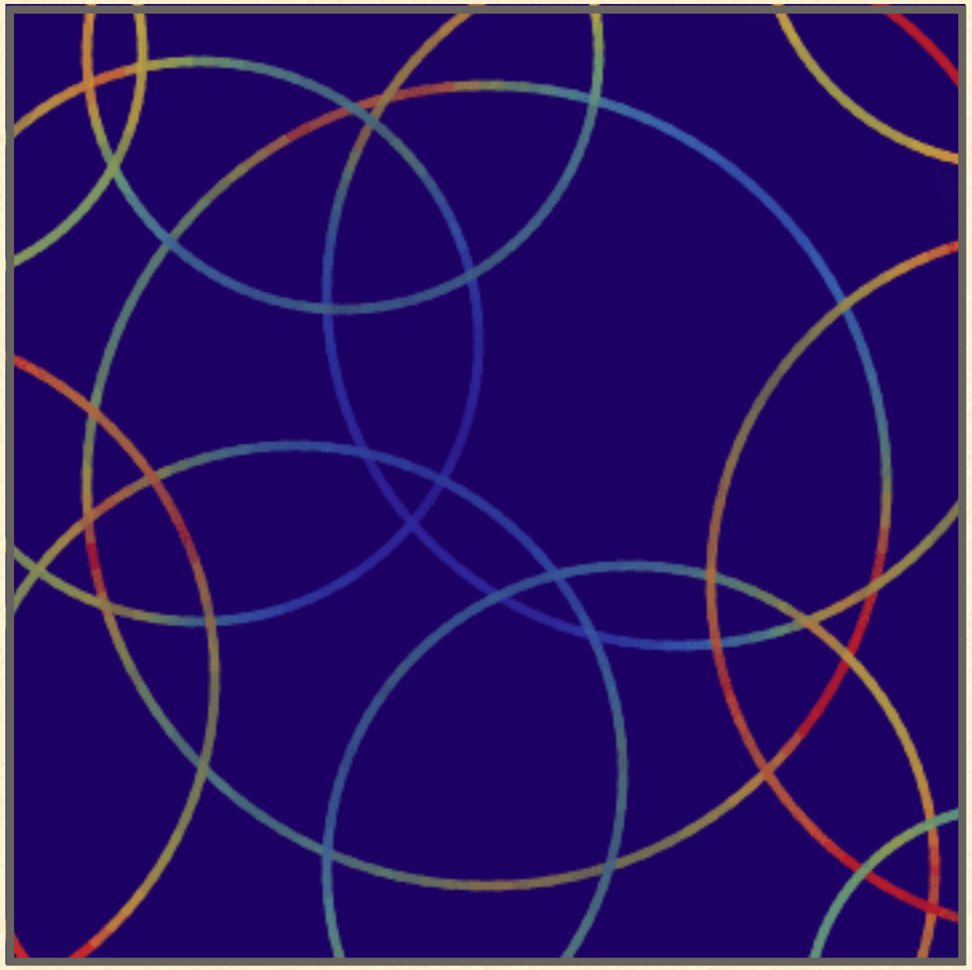

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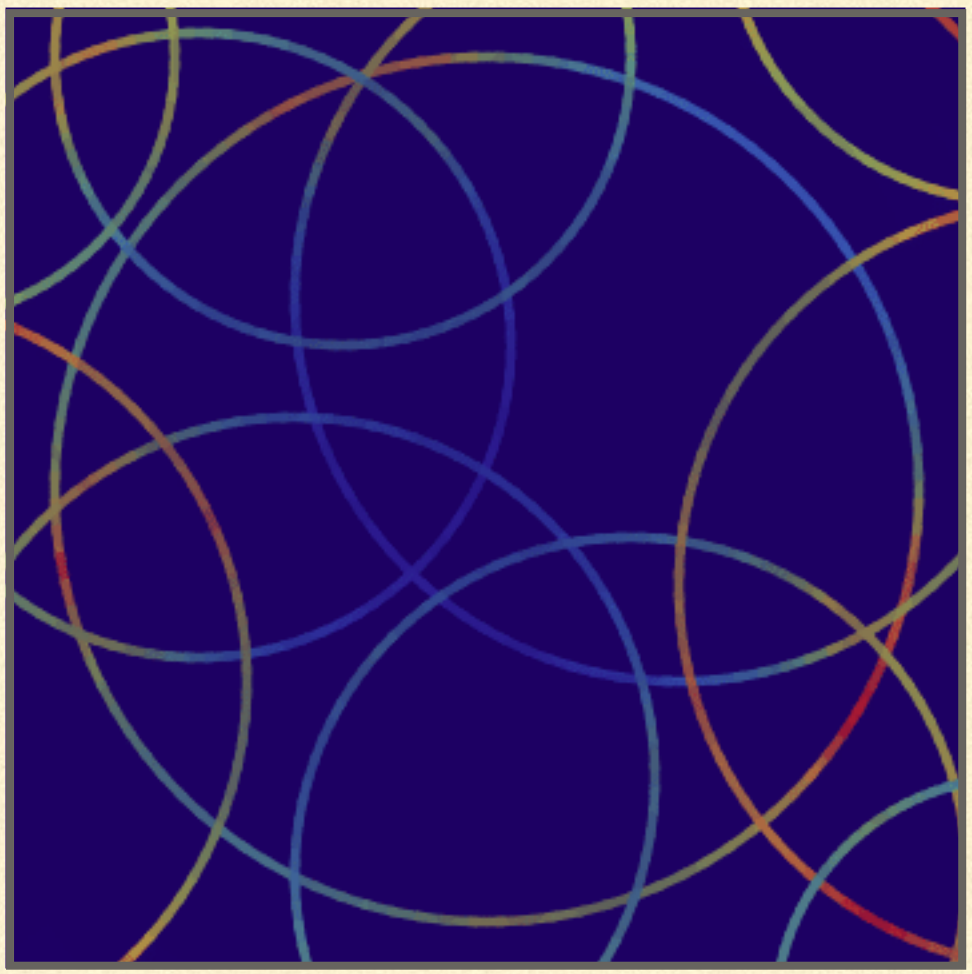

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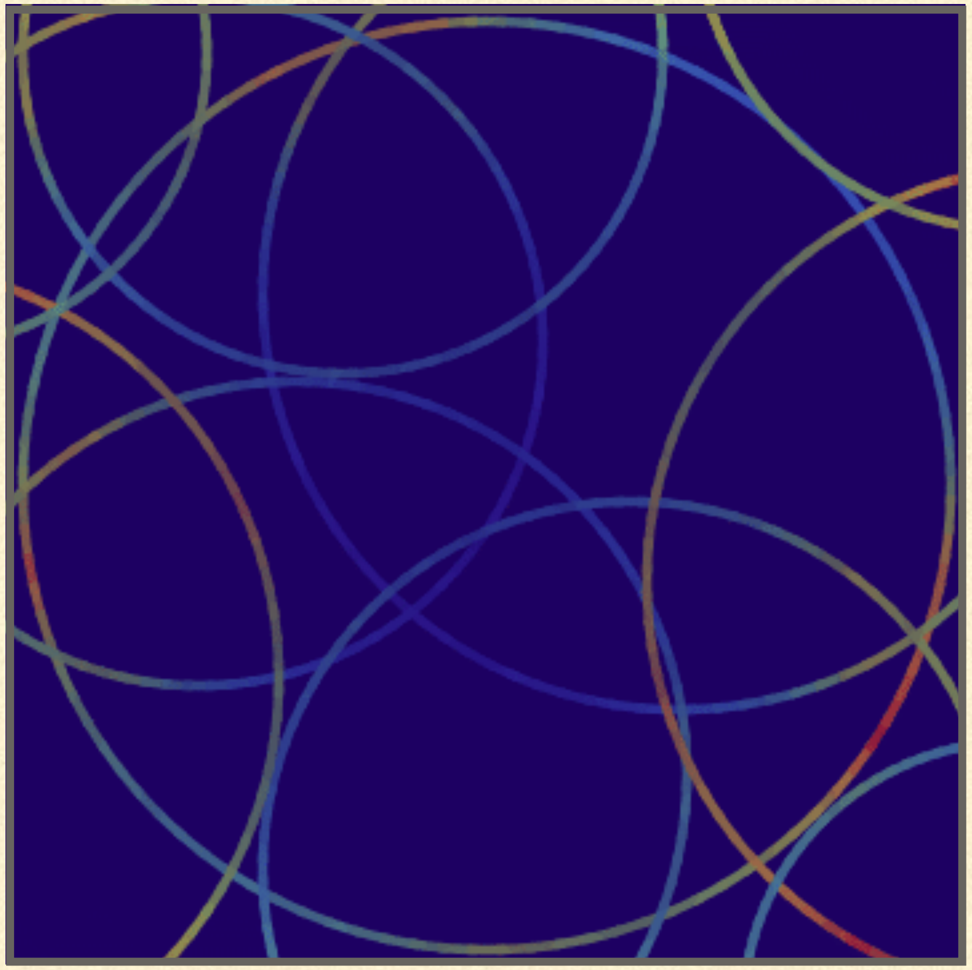

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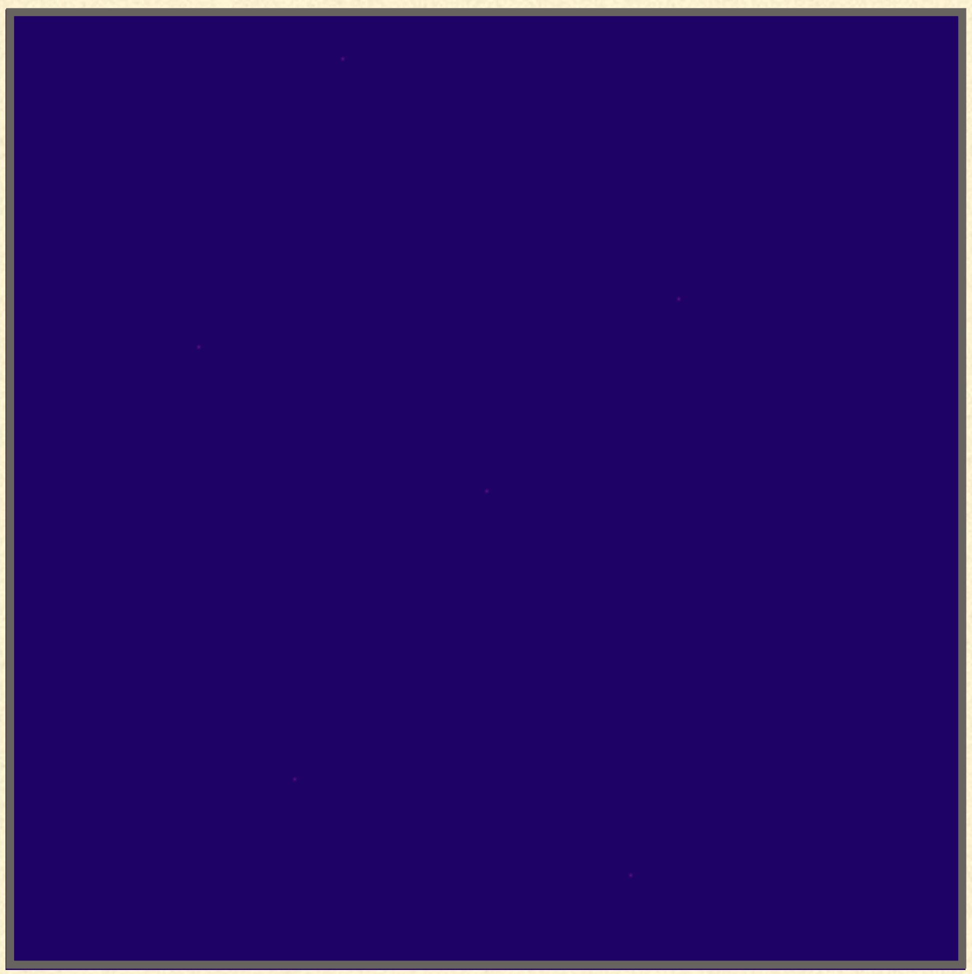

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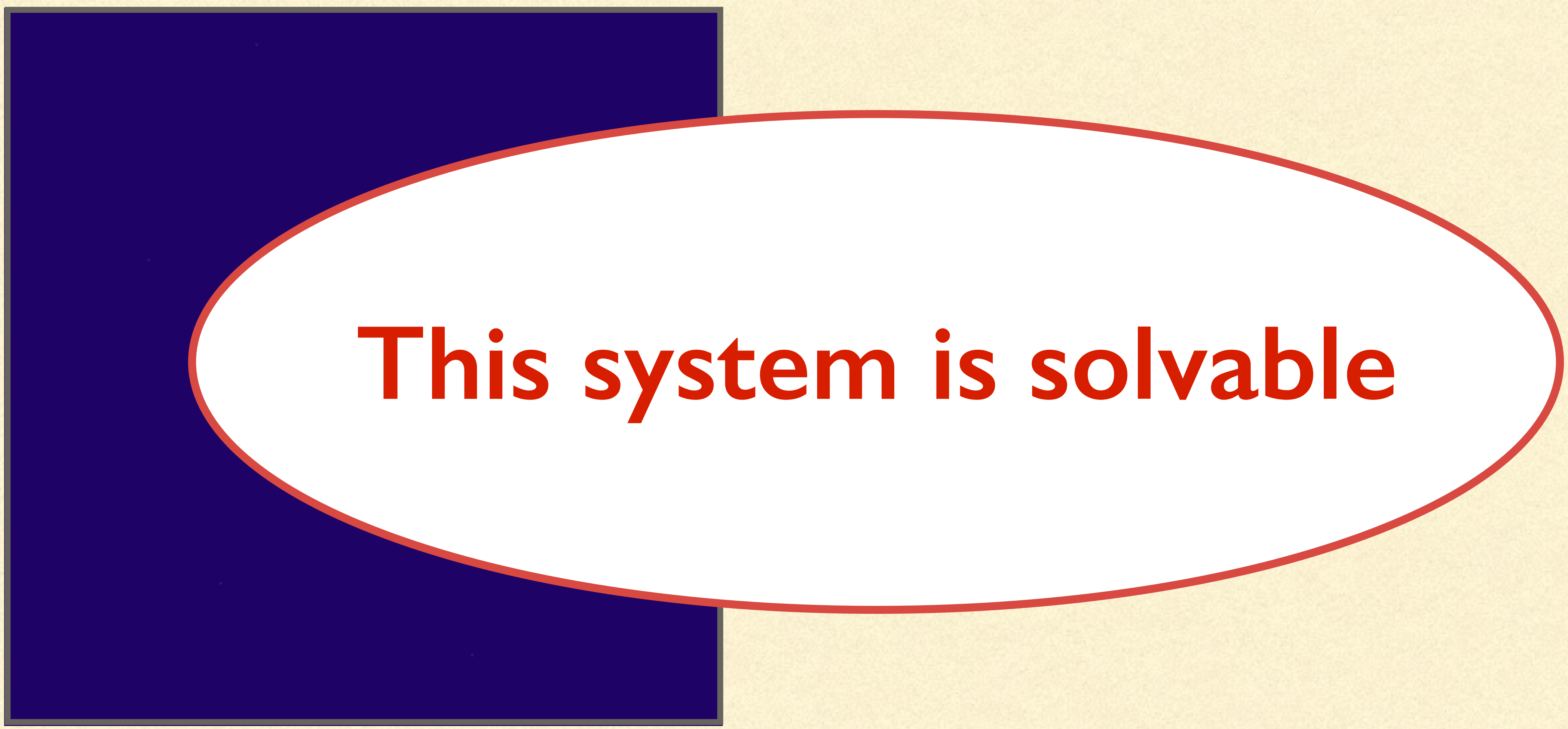


SOLVABLE ?

- Solvable means ...
  - We can write down the resulting GW spectrum **ANALYTICALLY** essentially only by causality arguments
- Full derivation needs an O(1)-hr talk
  - Here we show only the results

FULL EXPRESSIONS

- Full expression reduces to only ~10-dim. integration [Jinno & Takimoto '17]

1. single-bubble + 2. double-bubble

$$\Delta^{(s)} = \int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y \int_{v|t_{x,y}|}^{\infty} dr \int_{-\infty}^{t_{\max}} dt_n \int_{t_n}^{t_x} dt_{xi} \int_{t_n}^{t_y} dt_{yi}$$

$$\frac{k^3}{3} \left[\begin{array}{l} e^{-I(x_i,y_i)}\,\Gamma(t_n)\,\dfrac{r}{r_{xn}^{(s)} r_{yn}^{(s)}} \\ \times \left[j_0(kr)\mathcal{K}_0(n_{xn\times}, n_{yn\times}) + \dfrac{j_1(kr)}{kr}\mathcal{K}_1(n_{xn\times}, n_{yn\times}) + \dfrac{j_2(kr)}{(kr)^2}\mathcal{K}_2(n_{xn\times}, n_{yn\times}) \right] \\ \times\, \partial_{txi}\left[r_B(t_{xi}, t_n)^3 D(t_x, t_{xi}) \right] \partial_{tyi}\left[r_B(t_{yi}, t_n)^3 D(t_y, t_{yi}) \right] \cos(k t_{x,y}) \end{array} \right]$$

FULL EXPRESSIONS

- Full expression reduces to only ~10-dim. integration [Jinno & Takimoto '17]

1. single-bubble

GW spectrum (properly normalized)

General nucleation rate & wall velocity

$$\Delta^{(s)} = \int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y \int_{v|t_{x,y}|}^{\infty} dr \int_{-\infty}^{t_{\max}} dt_n \int_{t_n}^{t_x} dt_{xi} \int_{t_n}^{t_y} dt_{yi}$$
$$\frac{k^3}{3}\left[e^{-I(x_i,y_i)} \Gamma(t_n) \frac{r}{r_{xn}^{(s)} r_{yn}^{(s)}} \times \left[j_0(kr)\mathcal{K}_0(n_{xn\times}, n_{yn\times}) + \frac{j_1(kr)}{kr}\mathcal{K}_1(n_{xn\times}, n_{yn\times}) + \frac{j_2(kr)}{(kr)^2}\mathcal{K}_2(n_{xn\times}, n_{yn\times}) \right] \times \partial_{txi}\left[r_B(t_{xi}, t_n)^3 D(t_x, t_{xi}) \right] \partial_{tyi}\left[r_B(t_{yi}, t_n)^3 D(t_y, t_{yi}) \right] \cos(k t_{x,y}) \right]$$

General "damping" function after wall collision

$T_{ij} \propto$ (bubble radius)$^{-2}$ × D

FULL EXPRESSIONS

- Full expression reduces to only ~10-dim. integration [Jinno & Takimoto '17]

1. single-bubble + 2. double-bubble

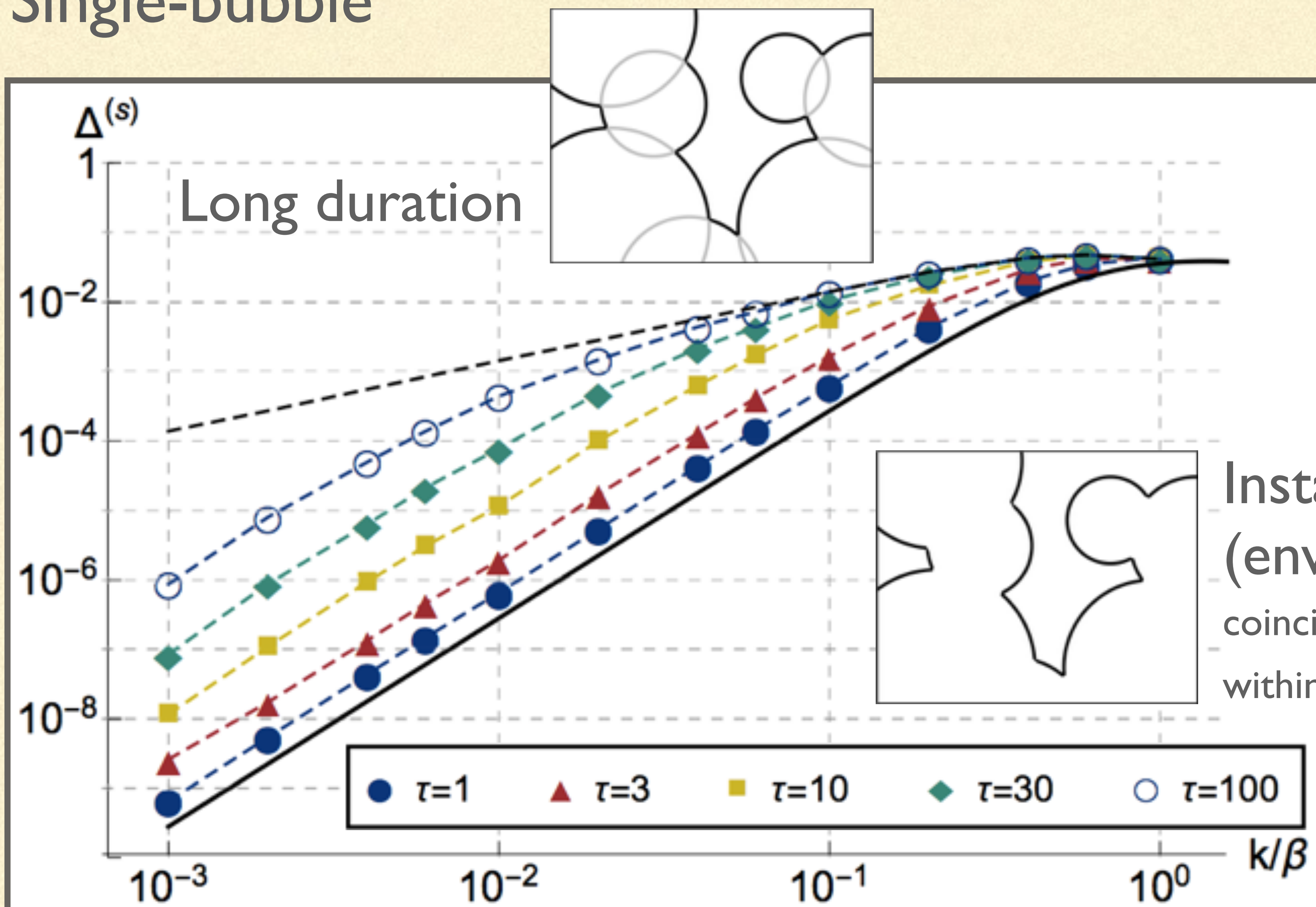
$$\Delta^{(d)} = \int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y$$

$$\int_0^{\infty} dr \int_{-\infty}^{t_x} dt_{xn} \int_{-\infty}^{t_y} dt_{yn} \int_{t_{xn}}^{t_x} dt_{xi} \int_{t_{yn}}^{t_y} dt_{yi} \int_{-1}^{1} dc_{xn} \int_{-1}^{1} dc_{yn} \int_0^{2\pi} d\phi_{xn,yn}$$

$$\frac{k^3}{3} \left[\begin{array}{l} \Theta_{\rm sp}(x_i, y_n)\Theta_{\rm sp}(x_n, y_i) e^{-I(x_i,y_i)} \Gamma(t_{xn})\Gamma(t_{yn}) \\ \times\, r^2 \left[j_0(kr)\mathcal{K}_0(n_{xn}, n_{yn}) + \dfrac{j_1(kr)}{kr}\mathcal{K}_1(n_{xn}, n_{yn}) + \dfrac{j_2(kr)}{(kr)^2}\mathcal{K}_2(n_{xn}, n_{yn}) \right] \\ \times\, \partial_{txi} \left[r_B(t_{xi}, t_{xn})^3 D(t_x, t_{xi}) \right] \partial_{tyi} \left[r_B(t_{yi}, t_{yn})^3 D(t_y, t_{yi}) \right] \cos(k t_{x,y}) \end{array} \right]$$

NUMERICAL RESULT

- Single-bubble



Damping func.

$D \sim e^{-t/\tau}$

after collision

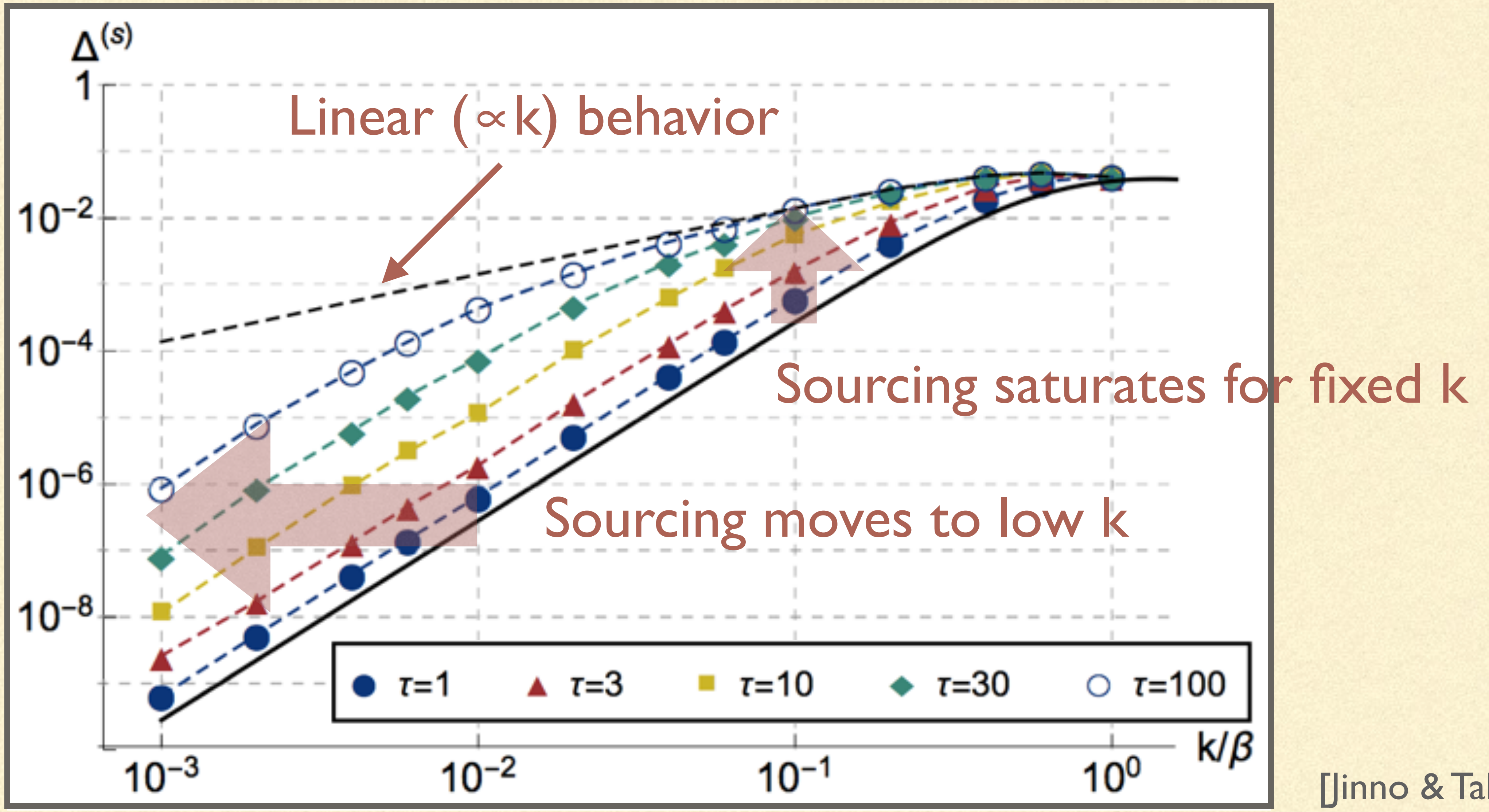
Instant disappearance (envelope) [Jinno&Takimoto '16]

coincide with [Huber&Konstandin '08] within factor 2

[Jinno & Takimoto '17]

NUMERICAL RESULT

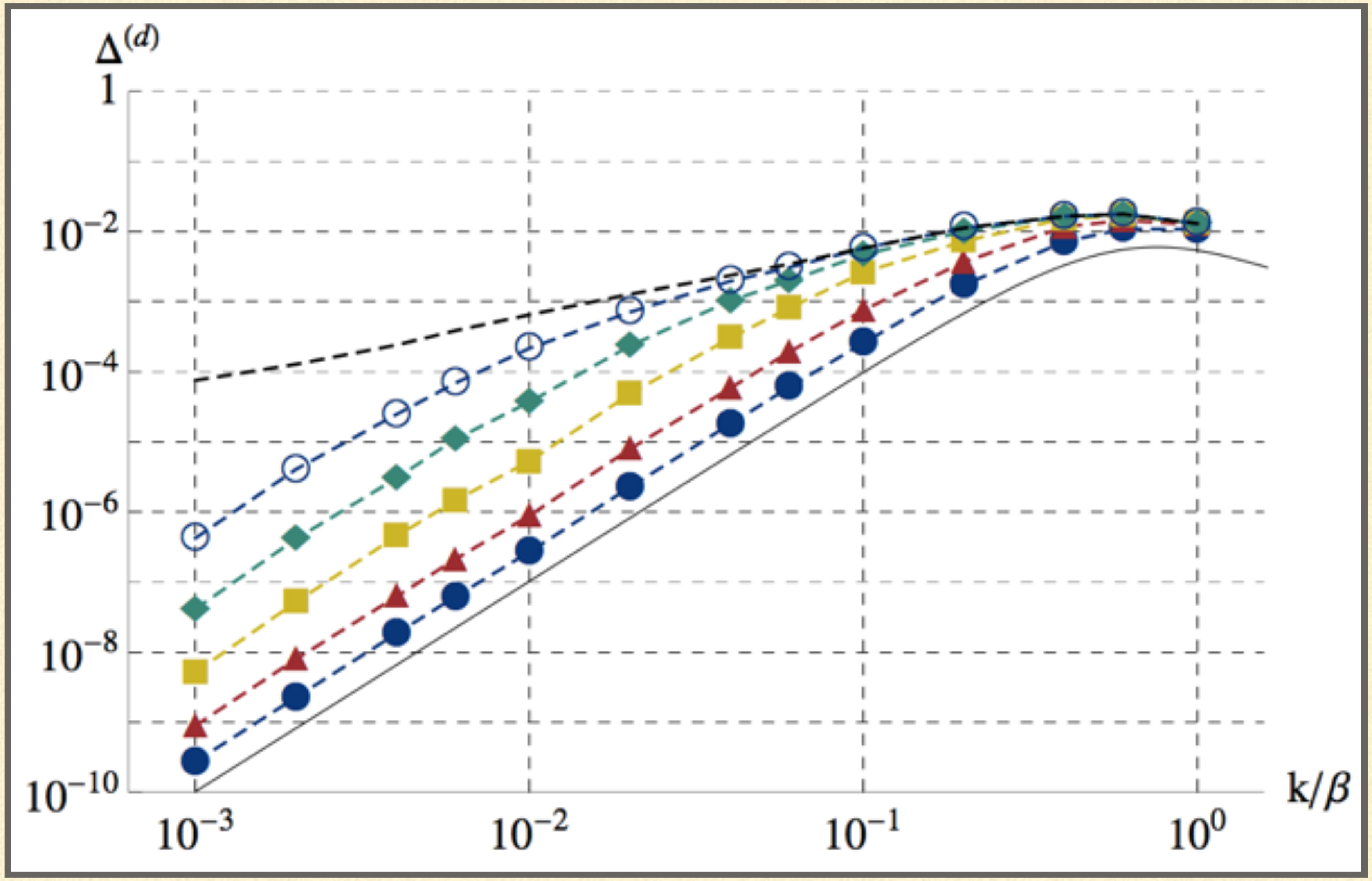
- Single-bubble



[Jinno & Takimoto '17]

NUMERICAL RESULT

- Double-bubble



[Jinno & Takimoto '17]

IMPLICATIONS

- For bubble collision (in this case thin-wall approx. holds well)

  - General expression for the spectrum has been derived

- For sound wave

  - Sourcing SATURATES. Tension with common extrapolation procedure?

SUMMARY & FUTURE PROSPECTS

- GW spectrum w/ thin-wall has been derived **ANALYTICALLY**

  - General nucleation rate & wall velocity & damping of wall energy

- Tension with common understanding on “sound wave”?

  - The origin of this tension must be identified

- Various effects can be implemented

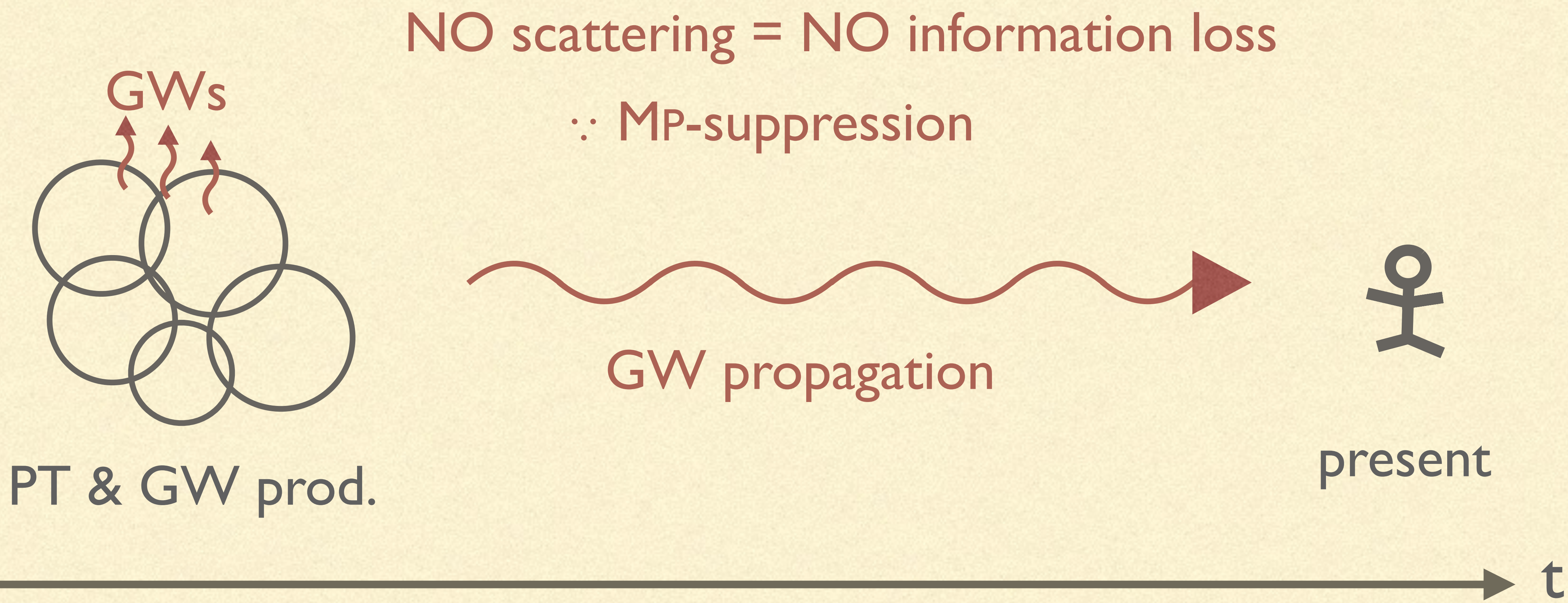
  - Cosmic expansion / Nucl. rate dependence / Wall thickness (w/ truncation)

- Will deepen our understanding on GW sourcing

Back up

GWS AS A PROBE TO PHASE TRANSITION

- How thermal first-order phase transition produces GWs



GW SPECTRUM IS 2-POINT ENSEMBLE AVE.

- Stochastic GWs is essentially $\langle T_{ij}(t_x, \mathbf{x}) T_{kl}(t_y, \mathbf{y}) \rangle_{\rm ens}$ [Caprini et al. '08]

  - Derivation : 1. GW EOM $\quad \Box h \sim T \quad \rightarrow \quad h \sim \int^t dt' \ \mathrm{Green}(t, t') T(t')$

    2. Then $\Omega_{\rm GW} \sim \left\langle \dot{h}^2 \right\rangle_{\rm ens} \sim \int \int \langle TT \rangle_{\rm ens}$

- So, everything is done if we obatain $\langle T(x) T(y) \rangle_{\rm ens}$

  - This is just an expectation value

    1. Fix spacetime points x & y

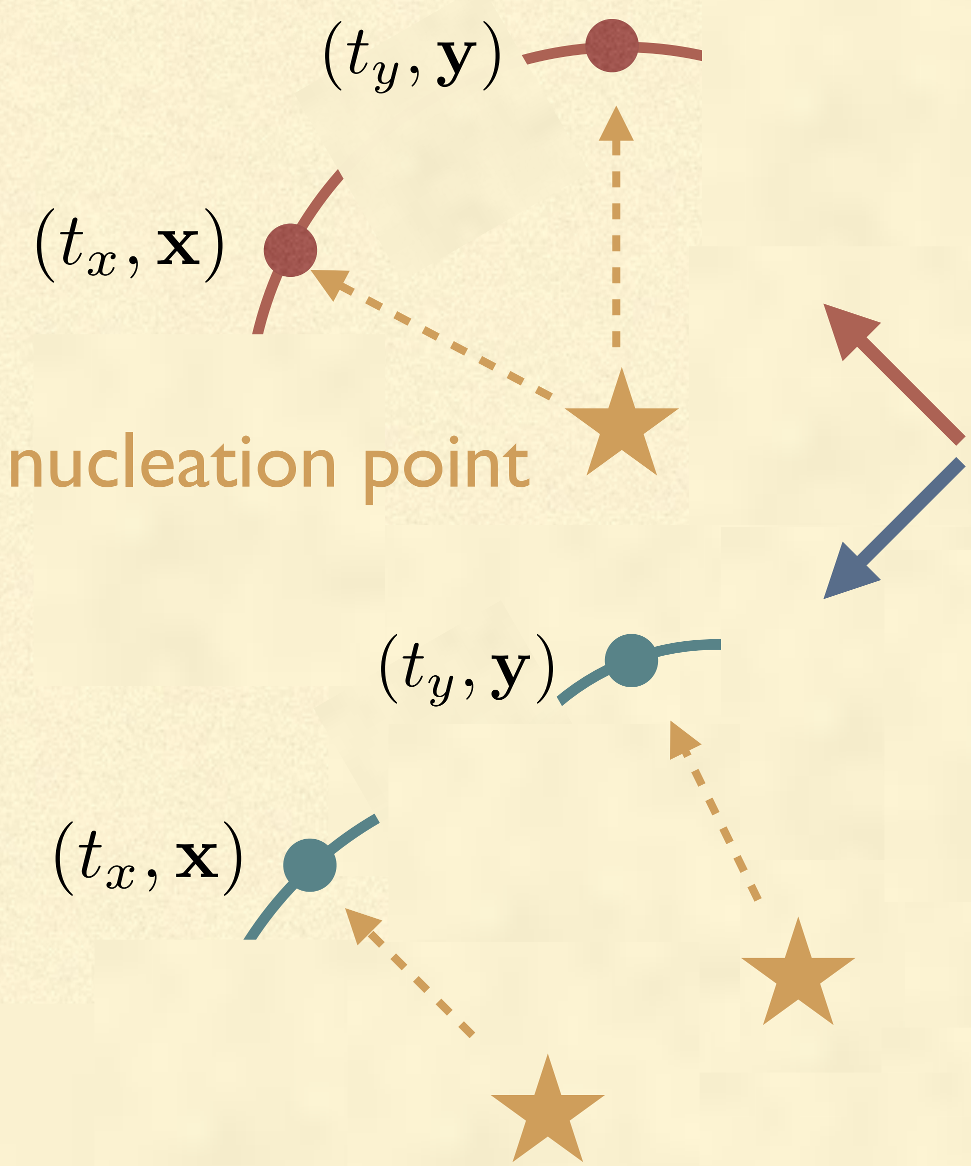
    2. Sum up $\left\{ \begin{array}{c} \text{(prob. for } T(x)T(y) \neq 0) \\ \times \\ \text{(value of } T(x)T(y)\text{)} \end{array} \right.$

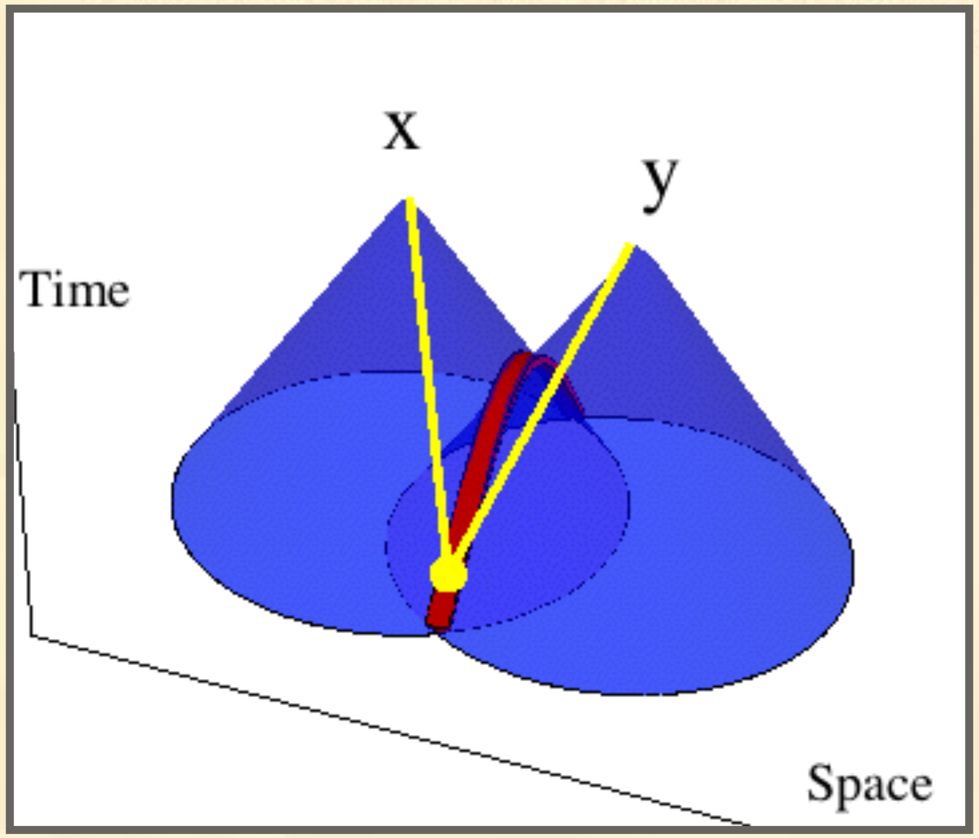
ONLY TWO CASES

- Following two exhaust $T(x)T(y) \neq 0$ possibilities [Jinno & Takimoto '16]

1. single-bubble

2. double-bubble

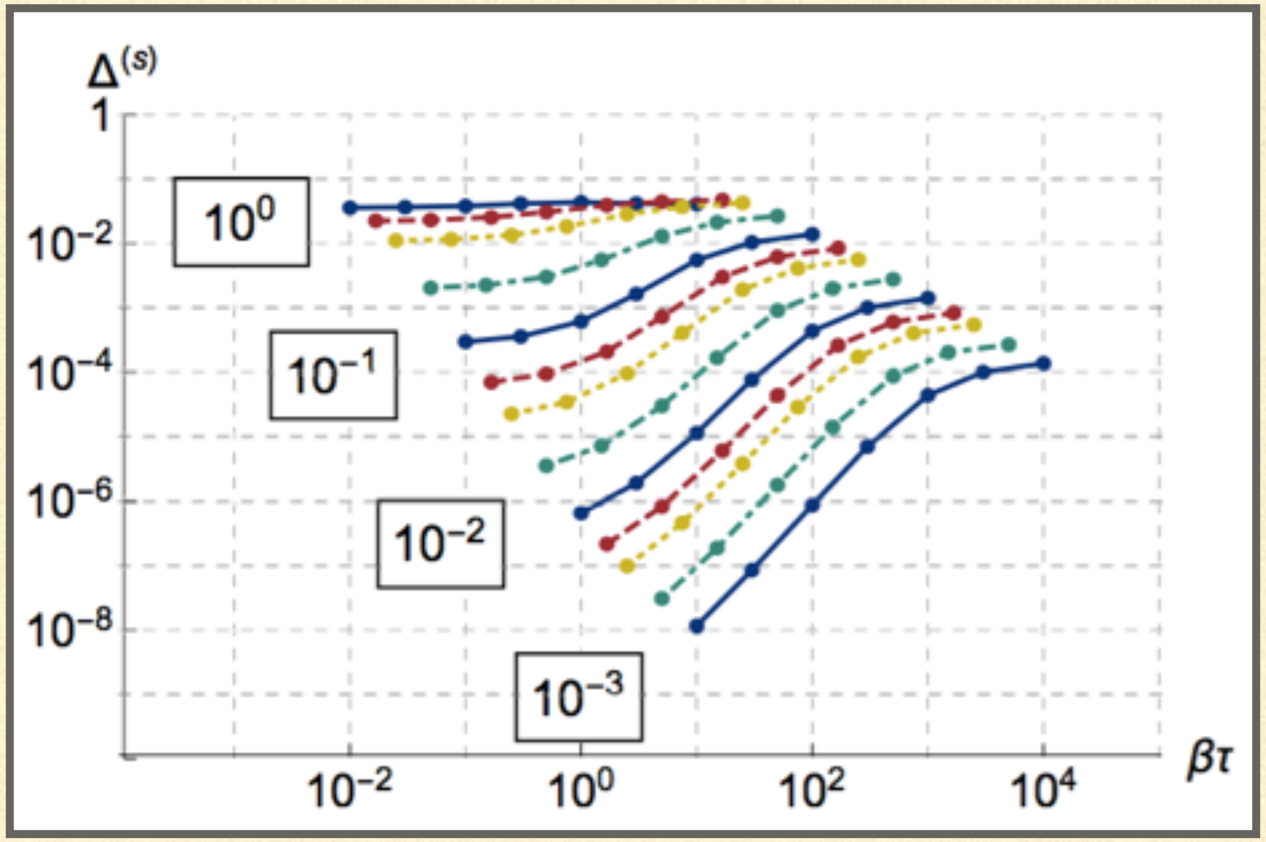




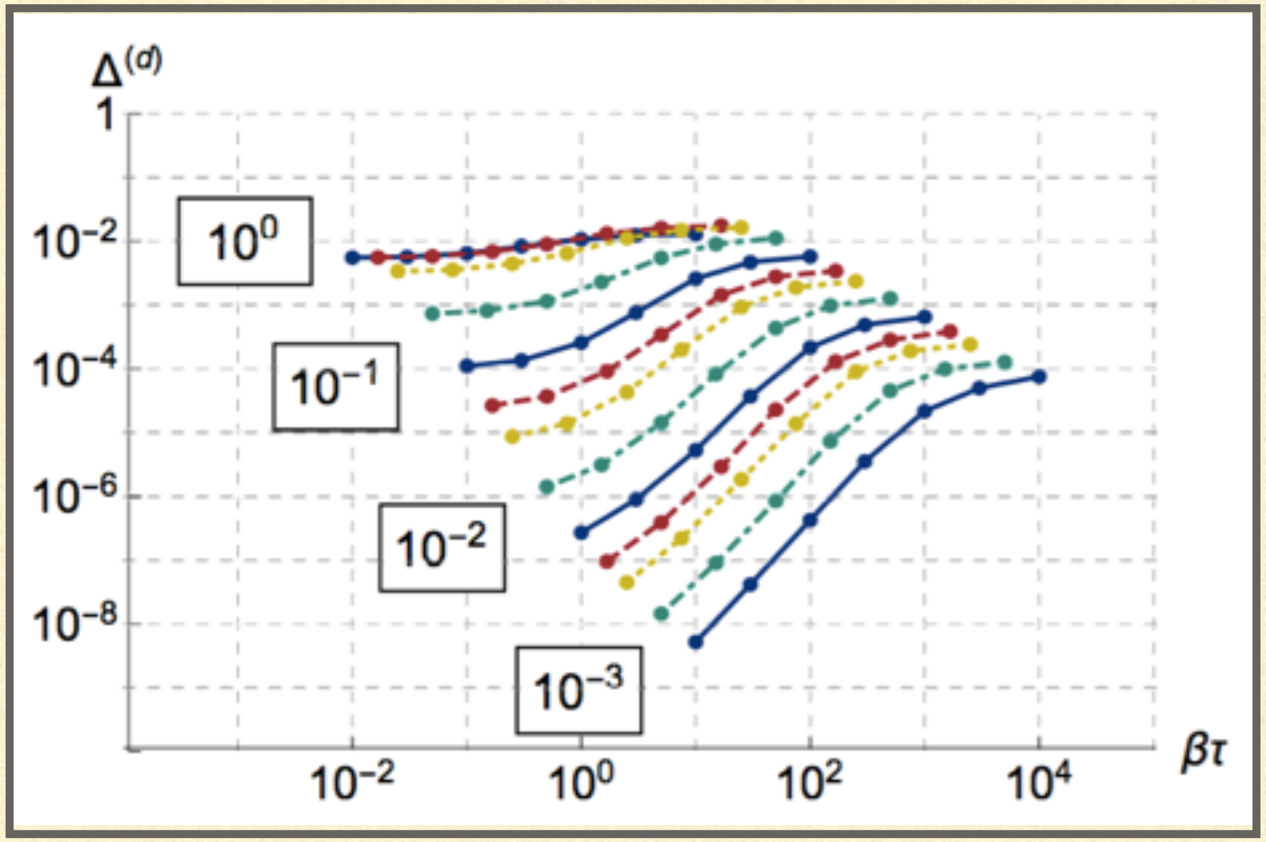
NUMERICAL RESULT

- GW sourcing as a function of time

- Single



- Double



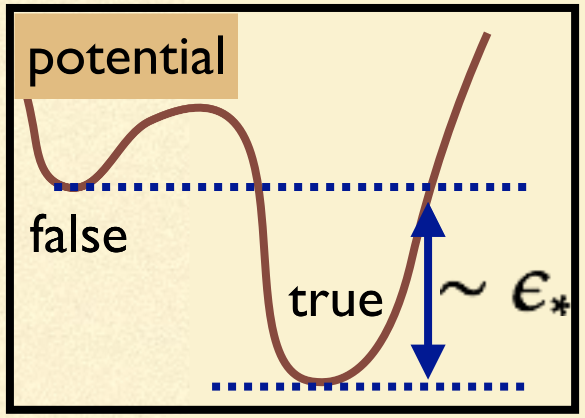
CLASSIFICATION OF WALL DYNAMICS

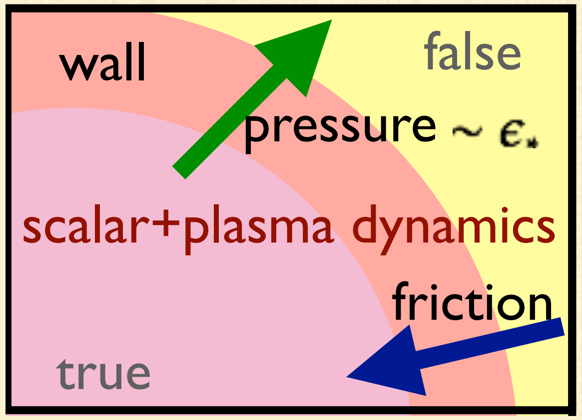
- How good are thin-wall & envelope approximations ?

  - Roughly speaking,

$$\alpha \equiv \epsilon_* / \rho_{\rm radiation}$$

determines bubble-wall behavior



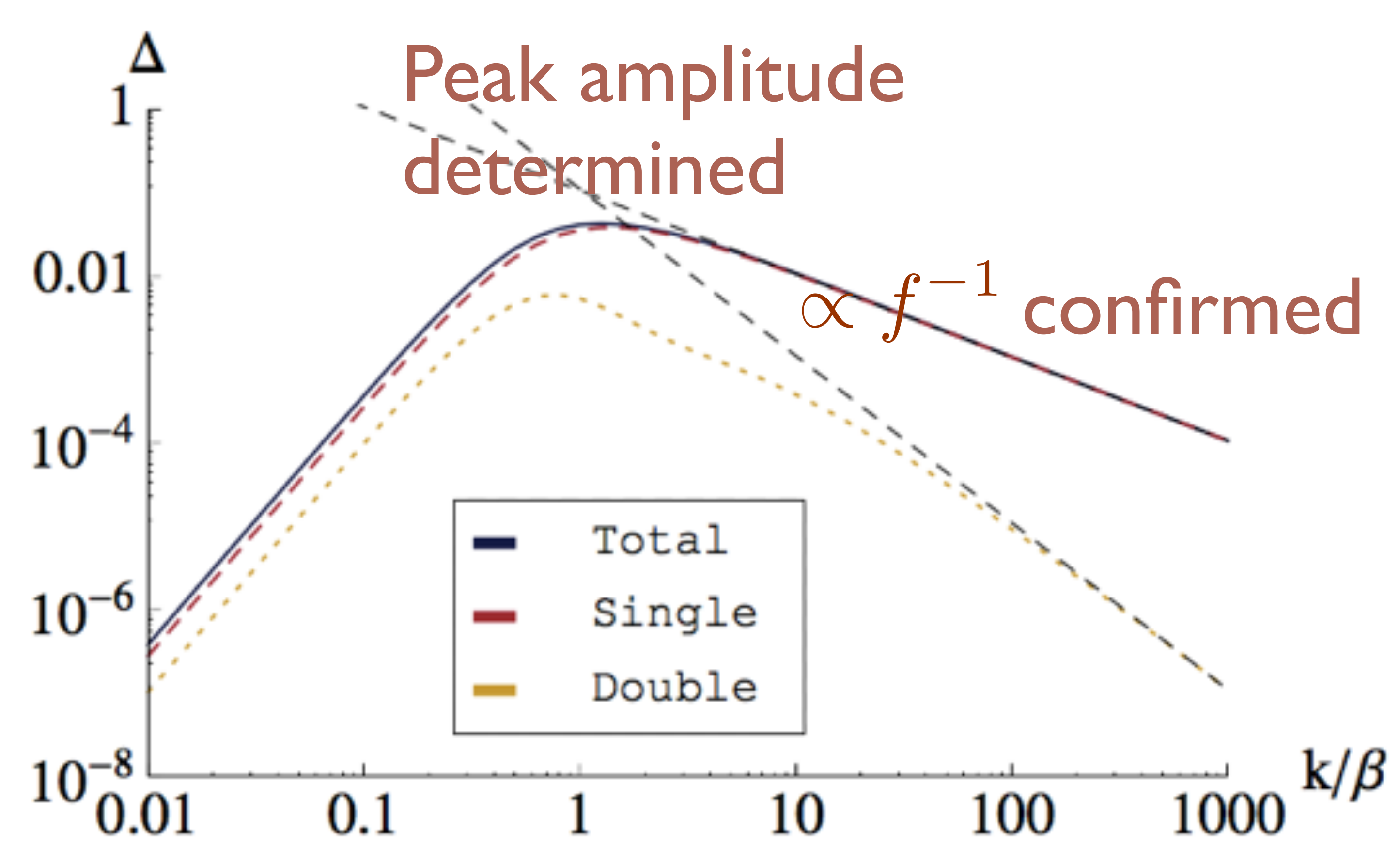


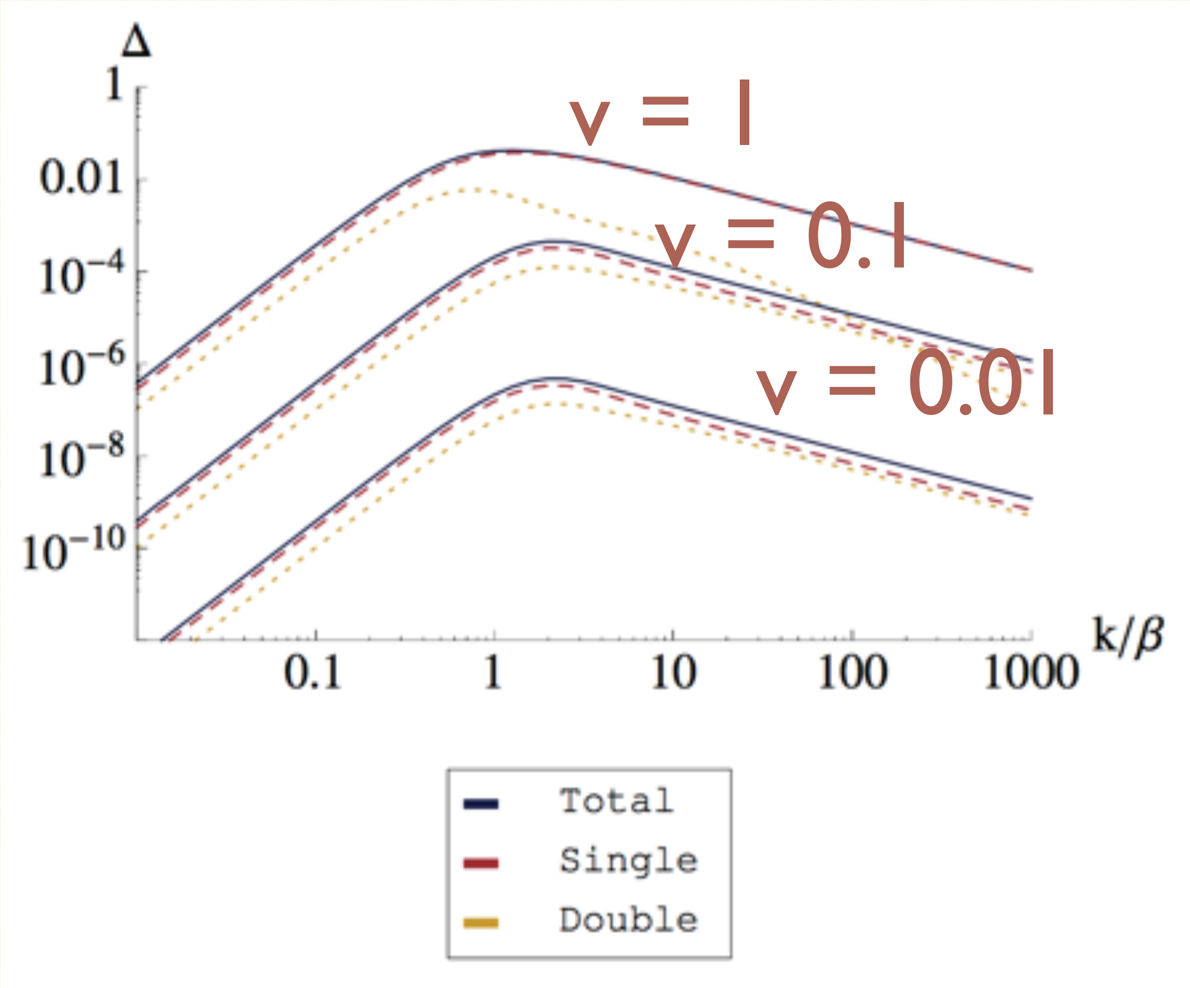
	Wall velocity approaches	Energy dominated by
(R) Runaway case $\alpha \gtrsim O(1)$	speed of light (c)	scalar motion (wall itself)
(T) Terminal velocity case $\alpha \lesssim O(1)$	terminal velocity (< c)	plasma around walls

GW SPECTRUM WITH ENVELOPE

- Result $\left(\Omega_{\rm GW}(k) \equiv \frac{1}{\rho_{\rm tot}} \frac{d\rho_{\rm GW}}{d\ln k} \right.$ & $\left. \Delta(k/\beta) \equiv \frac{3}{8\pi G} \frac{\beta^2 \rho_{\rm tot}}{\kappa^2 \epsilon_*^2} \Omega_{\rm GW}(k) \right)$

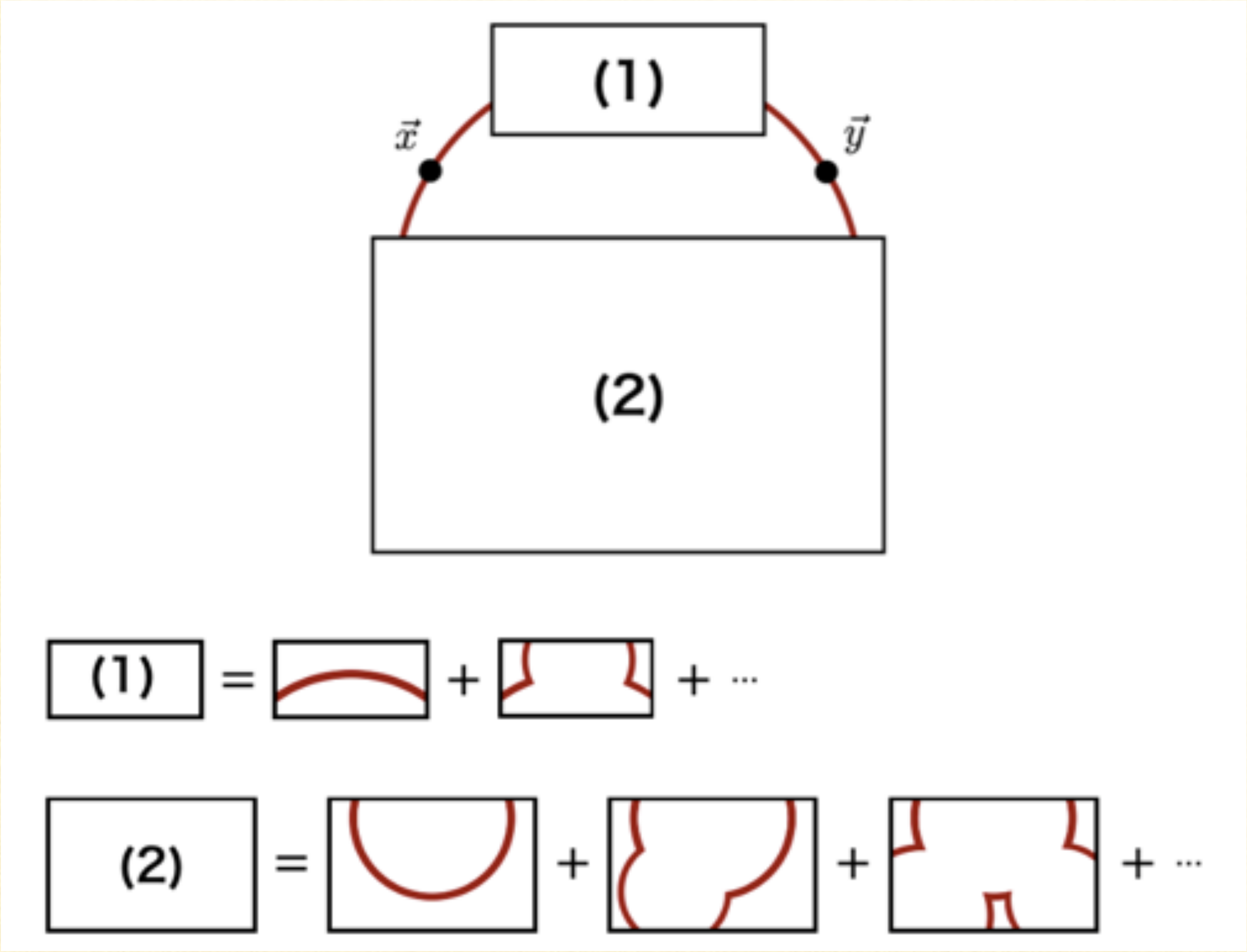
  - consistent with numerical simulation within factor ~2





WHY SINGLE-BUBBLE MATTERS

- Illustration with envelope



- Two bubble-wall fragments must remain uncollided until they reach x and y

- Other parts of the bubble might have collided already

- In this sense, breaking of spherical sym. is automatically taken into account