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Dark Side of the Universe, Daejeon

Two-loop Anomalous Dimensions for Four-Fermi Operators in Supersymmetric Theories

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Based on

arXiv:1703.08329 with J.Hisano, Y. Omura, and T. Sato (Nagoya U.)

Introduction

After Higgs Discovery

- We know all standard model (SM) parameters
- We can prepare theoretical predictions of physics beyond the SM (BSM) more precisely

Intensity Frontier

Flavor Physics
CP violation
etc..

- Indirect measurements
- Approach to high scale physics

Flavor violation: ~$10^{5\text{-}7}$ GeV
Proton Decay: ~10^{16} GeV

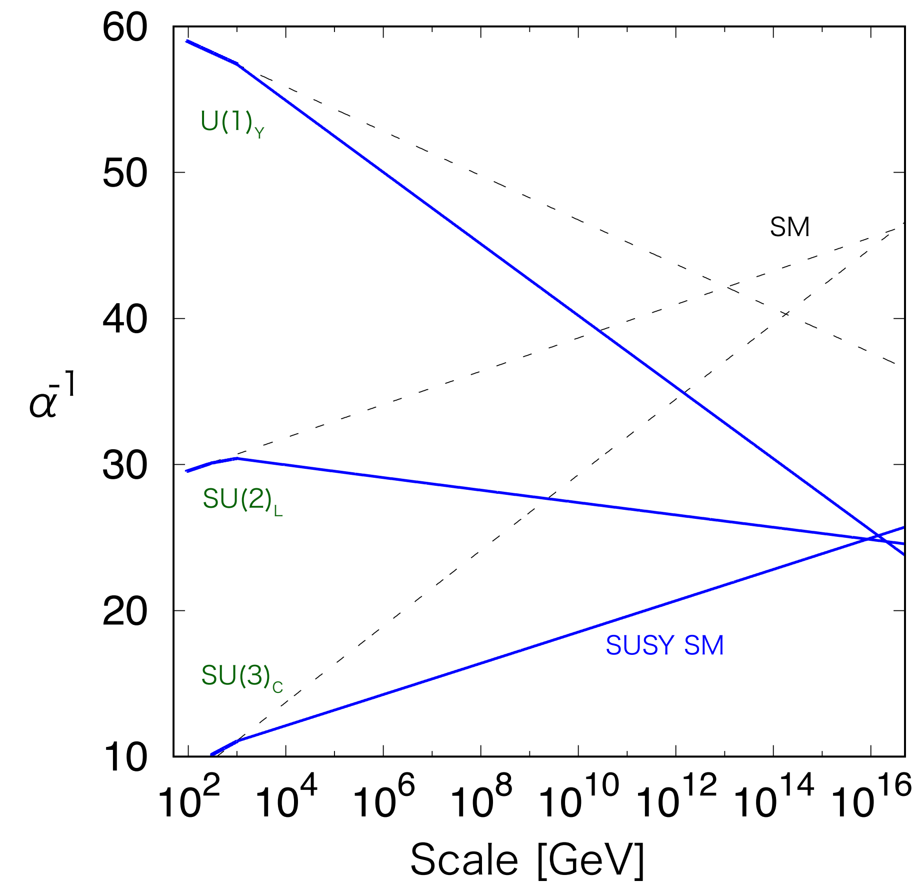
Collider Physics $\longleftrightarrow$ Cosmology

Complementary studies are important

Promising extensions of the SM;

Supersymmetric (SUSY) Theories

- Dark matter candidate
- Consistent with Unification



SUSY GUTs

- Unification of strong and electroweak forces
- Unifications of matters

Nucleon decay in SUSY GUTs

- Various supersymmetric models are proposed
  - High-intensity physics has a potential to distinguish models

Flavour Violating Processes (Meson oscillation etc.)

- Soft breaking parameters
- Flavor violating couplings in U(1)' extended SUSY SMs
- Matter mixings in the context of SUSY GUTs
- etc..

Quantum Corrections in SUSY Limit

Lagrangian

$$\mathcal{L} = \underbrace{\int d^4\theta\ \mathcal{K}(\Phi, \Phi^\dagger)}_{\text{Kähler potential}} + \Big(\underbrace{\int d^2\theta\ W(\Phi)}_{\text{Superpotential}} + \mathrm{h.c.} \Big)$$

- Superpotential part:
  No vertex corrections (non-renormalization theorem)
  Corrections from wavefunction renormalization

- Kähler potential part:
  Affected by not only wavefunction but also vertex corrections
  * more complicated: to calculate quantum corrections

Theoretical Development of Quantum Corrections to Effective Operators

- <u>Nucleon Decay</u>

  - ☑ 2-loop RGEs in SUSY SM [Hisano, Kobayashi, Nagata, Muramatsu '13]
  - ☑ 2-loop RGEs in SM [Daniel, Penarrocha '84]
  - ☑ 2-loop QCD Correction [Arafune, Nihei '94]
  - ☑ 1-loop Finite (threshold) Correction @ SUSY, GUT scales [Hisano, TK, Omura '15] [Bajc, Hisano, TK, Omura '16]

- <u>Flavor Changing Processes</u>

  - ☑ 2-loop QCD Correction [Buras, Misiak, Urban '00]
  - ☑ 1-loop Finite Corrections from QCD [Buras,Girrbach '12], …

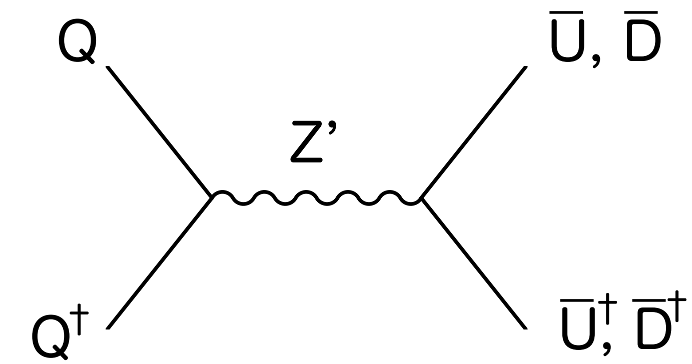
Two-loop Anomalous Dimensions

Four-Fermi operators from Kähler terms

	Q_i	L_i	$\overline{U}_i$	$\overline{D}_i$	$\overline{E}_i$
$SU(3)_C$	$\mathbf{3}$	$\mathbf{1}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_Y$	$1/6$	$-1/2$	$-2/3$	$1/3$	1

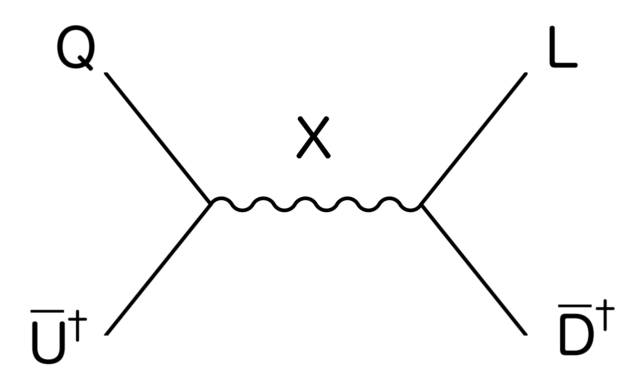
- <u>ΔF=2 flavor-changing processes</u>

$$\int d^4\theta\, \frac{1}{\Lambda^2} Q_2^\dagger \overline{D}_1^\dagger Q_1 \overline{D}_2\,, \qquad \int d^4\theta\, \frac{1}{\Lambda^2} Q_2^\dagger \overline{U}_1^\dagger Q_1 \overline{U}_2$$



- <u>Nucleon decay processes</u>

$$\int d^4\theta\, \frac{1}{\Lambda^2} \overline{U}_1^\dagger \overline{D}_1^\dagger Q_1 L_1\,, \qquad \int d^4\theta\, \frac{1}{\Lambda^2} \overline{E}_1^\dagger \overline{U}_1^\dagger Q_1 Q_1$$



- <u>Operators including SUSY breaking superfields</u>

$$\int d^4\theta\, \frac{1}{\Lambda^2} X^\dagger X Q_i^\dagger Q_j$$

- and so on

Strategy: To obtain RGEs for effective operators

Renormalization scale dependence of Wilson coeff.

$$\mu \frac{d}{d\mu} C(\mu) = \gamma_{\mathcal{O}} C(\mu) \qquad \mathcal{L}_{\text{eff.}} = C(\mu)\mathcal{O}(\mu) + \text{h.c.}$$

Renormalization group eqs. for vertex function with O inserted

$$\left[\mu \frac{\partial}{\partial \mu} + \sum_{\alpha} \beta_{\alpha} \frac{\partial}{\partial g_{\alpha}} - \left(\gamma_{\phi} \phi \frac{\partial}{\partial \phi} + \gamma_{\phi^\dagger} \phi^\dagger \frac{\partial}{\partial \phi^\dagger} \right) + \gamma_{\mathcal{O}} \right] \Gamma_{\mathcal{O}} = 0$$

- (pink) $\log \mu$ derivative of effective Kähler potential
- (blue) β function of gauge couplings
- (green) anomalous dim. (AD) for constituent superfields

$\Rightarrow$ we get AD for Wilson coeff. γ_{O}

<u>Effective Kähler potential</u>

S. Groot Nibbelink, T. S. Nyawelo (2005)

$$\mathcal{K}_{2L} = \frac{1}{2} R^{a\ b}_{\ c\ d} \overline{J}^{c\ d}_{\ a\ b}(M^2) - (GT_I^{(\alpha)}\phi)^c{}_{;b}(\phi^\dagger T_J^{(\alpha)}G)_a{}^{;d} \overline{I}^{a\ b\ IJ}_{\ c\ d}(M^2, M^2, M_V^2)$$
$$- f^{(\alpha)}_{LIN} f^{(\alpha)}_{JKM} \overline{I}^{IJKLMN}(M_V^2, M_V^2, M_V^2)$$

assuming: superpotential W = 0
canonical gauge kinetic function

Dividing tree-level Kähler term into two parts $\quad \mathcal{K} = \sum_\phi \phi_a^\dagger \phi^a + \Delta\mathcal{K}\,,$

<u>Kähler metric</u>

$$G^a{}_b \equiv \mathcal{K}^a{}_b = \frac{\partial^2}{\partial\phi_a^\dagger \partial\phi^b}\mathcal{K}$$

<u>Kähler curvature</u>

$$R^{a\ b}_{\ c\ d} \equiv \mathcal{K}^{ab}_{cd} - \mathcal{K}^{ab}{}_e (G^{-1})^e_f \mathcal{K}_{cd}{}^f$$

Two-loop Anomalous dimensions

J. Hisano, TK, Y. Omura, T. Sato (2017)

Kähler potential including supersymmetric four-Fermi operators

$$\Delta\mathcal{K} = \mathcal{C}(\overline{\lambda}_A^{a_1 a_2}\phi^\dagger_{a_1}\phi^\dagger_{a_2})(\lambda^A_{b_1 b_2}\phi^{b_1}\phi^{b_2}) + \text{h.c.}$$

- Projection λ: composite operator -> irrep part.

Generic form of two-loop ADMs for ΔK

$$\begin{aligned}
&(16\pi^2)^2\gamma^{(2)}_{\mathcal{O}}\mathcal{O} \\
&= 2\left[\sum_\phi (b_\alpha C_\alpha(\phi)\delta_{\alpha,\beta} + 2C_\alpha(\phi)C_\beta(\phi)) - \left(4C^{\text{comp.}}_\alpha - 2\sum_\phi C_\alpha(\phi)\right)C^{\text{comp.}}_\beta\right] g^2_\alpha g^2_\beta\,\mathcal{O} \\
&- 4g^2_\alpha g^2_\beta\left[(\phi^\dagger T^{(\beta)}_J T^{(\alpha)}_I T^{(\alpha)}_I)_a \mathcal{O}^a_b (T^{(\beta)}_J\phi)^b + (\phi^\dagger T^{(\beta)}_J)_a\mathcal{O}^a_b(T^{(\alpha)}_I T^{(\alpha)}_I T^{(\beta)}_J\phi)^b\right] \\
&- 4g^2_\alpha g^2_\beta\left[\sum_\phi \phi^\dagger(T^{(\alpha)}_I T^{(\beta)}_J + T^{(\beta)}_J T^{(\alpha)}_I)\phi\right] \text{tr}\left[T^{(\beta)}_J(\mathcal{O}T^{(\alpha)}_I\phi) + T^{(\alpha)}_I(\mathcal{O}\phi^\dagger T^{(\beta)}_J)\right] \\
&+ 8g^2_\alpha g^2_\beta(\phi^\dagger T^{(\alpha)}_I T^{(\beta)}_J)_a\mathcal{O}^a_b(T^{(\beta)}_J T^{(\alpha)}_I\phi)^b
\end{aligned}$$

terms: induce operator mixing

term: vanishes unless we consider ΔF=1 operators $\mathcal{K} \sim \overline{D}_2^\dagger\overline{D}_1\sum_i Q_i^\dagger Q_i$

Examples: Flavor changing operators

$$\mathcal{O} = \overline{D}_1^\dagger \overline{D}_1^\dagger \overline{D}_2 \overline{D}_2$$

Chiral part of Kähler potential transforms as $\overline{\mathbf{3}} \times \overline{\mathbf{3}} = \overline{\mathbf{6}} + \cancel{\mathbf{3}}$

$$\gamma_{\overline{D}}^{(2)} = \frac{g_3^4}{(16\pi^2)^2}[\gamma_{\overline{D}}^{(2)}]_{33} + \frac{g_3^2 g_Y^2}{(16\pi^2)^2}[\gamma_{\overline{D}}^{(2)}]_{3Y} + \frac{g_Y^4}{(16\pi^2)^2}[\gamma_{\overline{D}}^{(2)}]_{YY} \, ,$$

$$[\gamma_{\overline{D}}^{(2)}]_{33} = \frac{8}{3}(4b_3 + 5) \quad [\gamma_{\overline{D}}^{(2)}]_{YY} = 8q_{\overline{D}}^2(b_Y - 6q_{\overline{D}}^2) \quad [\gamma_{\overline{D}}^{(2)}]_{3Y} = -48q_{\overline{D}}^2$$

$$\mathcal{O} = Q_2^\dagger \overline{D}_1^\dagger Q_1 \overline{D}_2$$

Chiral part of Kähler potential transforms as $\mathbf{3} \times \overline{\mathbf{3}} = \mathbf{8} + \mathbf{1}$

$$\mathcal{O}^{(1)} = 2(Q_2^\dagger T_I^{(\alpha)} \overline{D}_1^\dagger)(\overline{D}_2 T_I^{(\alpha)} Q_1)$$
$$\mathcal{O}^{(2)} = (Q_2^\dagger \overline{D}_1^\dagger)(\overline{D}_2 Q_1)$$

$$\begin{aligned}\gamma_{ij}^{(2)}\mathcal{O}^{(j)} = &\frac{g_3^4}{(16\pi^2)^2}[\gamma_{ij}^{(2)}]_{33}\mathcal{O}^{(j)} + \frac{g_2^4}{(16\pi^2)^2}[\gamma_{ij}^{(2)}]_{22}\mathcal{O}^{(j)} + \frac{g_Y^4}{(16\pi^2)^2}[\gamma_{ij}^{(2)}]_{YY}\mathcal{O}^{(j)} \\ &+ \frac{g_3^2 g_2^2}{(16\pi^2)^2}[\gamma_{ij}^{(2)}]_{32}\mathcal{O}^{(j)} + \frac{g_3^2 g_Y^2}{(16\pi^2)^2}[\gamma_{ij}^{(2)}]_{3Y}\mathcal{O}^{(j)} + \frac{g_2^2 g_Y^2}{(16\pi^2)^2}[\gamma_{ij}^{(2)}]_{2Y}\mathcal{O}^{(j)}\end{aligned}$$

$$[\gamma^{(2)}]_{33} = \begin{pmatrix} \frac{4}{9}(43 + 24b_3) & \frac{160}{27} \\ \frac{20}{3} & \frac{32}{9}(3b_3 + 10) \end{pmatrix}, \quad [\gamma^{(2)}]_{22} = \left(3b_2 + \frac{9}{2}\right)\mathbf{1}\,, \quad [\gamma^{(2)}]_{YY} = \left(\frac{5}{9}b_Y - \frac{7}{54}\right)\mathbf{1}\,,$$
$$[\gamma^{(2)}]_{32} = \begin{pmatrix} 11 & \frac{16}{3} \\ 6 & 24 \end{pmatrix}, \quad [\gamma^{(2)}]_{3Y} = \begin{pmatrix} -\frac{19}{3} & -\frac{32}{81} \\ \frac{2}{9} & \frac{296}{27} \end{pmatrix}, \quad [\gamma^{(2)}]_{2Y} = -\frac{2}{3}\mathbf{1}\,,$$

with 1-loop coefficients of β-fucntion b_N

Summary and Discussion

- ☑ We derived 2-loop ADMs for any four-Fermi operators in supersymmetric theories.
  * Using 2-loop effective Kähler potential & RGEs for operator-inserted vertex functions
  * Only gauge interactions are included.

- ☑ We have not estimated 2-loop ADMs numerically:
  - Model dependences (initial condition, input scale, and so on)

- ☑ Application to operators generating non-holomorphic soft mass terms $\int d^4\theta \, \frac{1}{\Lambda^2} X^\dagger X \Phi^\dagger \Phi$

Remaining issues

- ☐ Model dependent analysis:
  - Flavor violating processes in extra U(1)' scenario (secluded and/or intermediate U(1)' breaking)
  - Flavor violating processes via matter mixings

- ☐ Effects from Yukawa couplings (@ 1-loop)

Backups

Examples: Nucleon Decay operators

$$\mathcal{O}^{(1)} = \overline{U}_1^{\dagger}\overline{D}_1^{\dagger}Q_1L_1\,, \qquad \mathcal{O}^{(2)} = \overline{E}_1^{\dagger}\overline{U}_1^{\dagger}Q_1Q_1$$

$$[\gamma^{(2)}_{\mathcal{O}^{(i)}}] = \frac{g_3^4}{(16\pi^2)^2}[\gamma^{(2)}_{\mathcal{O}^{(i)}}]_{33} + \frac{g_2^4}{(16\pi^2)^2}[\gamma^{(2)}_{\mathcal{O}^{(i)}}]_{22} + \frac{g_Y^4}{(16\pi^2)^2}[\gamma^{(2)}_{\mathcal{O}^{(i)}}]_{YY}$$
$$+ \frac{g_3^2g_2^2}{(16\pi^2)^2}[\gamma^{(2)}_{\mathcal{O}^{(i)}}]_{32} + \frac{g_3^2g_Y^2}{(16\pi^2)^2}[\gamma^{(2)}_{\mathcal{O}^{(i)}}]_{3Y} + \frac{g_2^2g_Y^2}{(16\pi^2)^2}[\gamma^{(2)}_{\mathcal{O}^{(i)}}]_{2Y}$$

$$[\gamma^{(2)}_{\mathcal{O}^{(1)}}]_{33} = [\gamma^{(2)}_{\mathcal{O}^{(2)}}]_{33} = \frac{64}{3} + 8b_3\,,$$
$$[\gamma^{(2)}_{\mathcal{O}^{(1)}}]_{22} = [\gamma^{(2)}_{\mathcal{O}^{(2)}}]_{22} = \frac{9}{2} + 3b_2\,,$$
$$[\gamma^{(2)}_{\mathcal{O}^{(1)}}]_{YY} = \frac{113}{54} + \frac{5}{3}b_Y\,, \qquad [\gamma^{(2)}_{\mathcal{O}^{(2)}}]_{YY} = \frac{91}{18} + 3b_Y\,,$$
$$[\gamma^{(2)}_{\mathcal{O}^{(1)}}]_{32} = 12\,, \qquad [\gamma^{(2)}_{\mathcal{O}^{(2)}}]_{32} = 20\,,$$
$$[\gamma^{(2)}_{\mathcal{O}^{(1)}}]_{2Y} = 2\,, \qquad [\gamma^{(2)}_{\mathcal{O}^{(2)}}]_{2Y} = \frac{2}{3}\,,$$
$$[\gamma^{(2)}_{\mathcal{O}^{(1)}}]_{3Y} = \frac{68}{9}\,, \qquad [\gamma^{(2)}_{\mathcal{O}^{(2)}}]_{3Y} = \frac{76}{9}$$