Probing the extra dimension with dark energy

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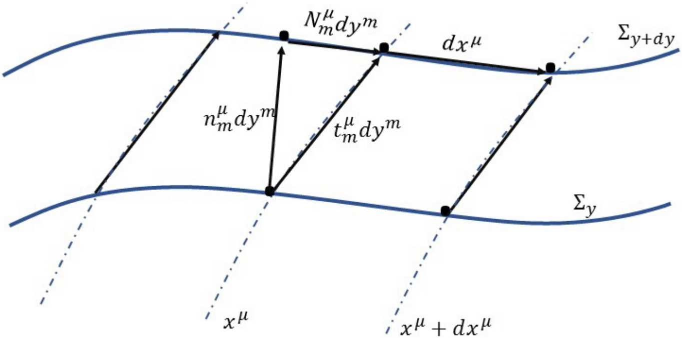
Based on: T. Moon and P. Oh, 1705.00866;
S. Kouwn, C. Park and P. Oh, in preparation

Intodaction

- Cosmic spacetime anisotropy
- Horava-Lifshitz Gravity
  - space and time are not treated on an equal footing
  - general covariance is broken down to foliation preserving diffeomorphism
  - $x^{\mu} \longrightarrow x'^{\mu}(x) \Rightarrow \ \vec{x} \longrightarrow \vec{x}'(\vec{x}, t), t \longrightarrow t'(t)$
  - $t \longrightarrow b^3 t, x \longrightarrow bx$
  - perturbatively renormalizable and free of ghosts
- Extra dimensions
- Cosmological test of the parameter z

- ADM Decomposition



$$\gamma_{mn} = n^{\mu}_{m} n_{n\mu}$$

$$\boxed{ds^2 = g_{\mu\nu}(dx^{\mu} + N^{\mu}_{m}dy^{m})(dx^{\nu} + N^{\nu}_{n}dy^{n}) + \gamma_{mn}dy^{m}dy^{n}}$$

- Foliation Preserving Diff. ;

$$x^\mu \to x'^\mu \equiv x'^\mu(x,y),\ y^m \to y'^m \equiv y'^m(y)$$

$$x^{\mu\prime} = x^\mu + \xi^\mu(x,y),\ \ y^{m\prime} = y^m + \xi^m(y); \qquad \partial_\nu \xi^m = 0$$

$$\begin{aligned} \delta g_{\mu\nu} &= \mathcal{L}_\xi(x) g_{\mu\nu} + \xi^m(y)\partial_m g_{\mu\nu}, \\ \delta N^\mu_m &= \mathcal{L}_\xi(x) N^\mu_m + \mathcal{L}_\xi(y) N^\mu_m + \partial_m \xi^\mu \\ \delta \gamma_{mn} &= \mathcal{L}_\xi(y)\gamma_{mn} + \xi^\mu \partial_\mu \gamma_{mn} \end{aligned}$$

- Finite transformations;

$$\begin{aligned} g'_{\mu\nu}(x',y') &= \left(\frac{\partial x^\rho}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu}\right) g_{\rho\sigma}(x,y) \\ N'^{\ \mu}_m(x',y') &= \left(\frac{\partial y^n}{\partial y'^m}\right)\left[\frac{\partial x'^\mu}{\partial x^\nu} N^\nu_n(x,y) - \frac{\partial x'^\mu}{\partial y^n}\right] \\ \gamma'_{mn}(x',y') &= \left(\frac{\partial y^p}{\partial y'^m}\frac{\partial y^q}{\partial y'^n}\right)\gamma_{pq}(x,y) \end{aligned}$$

- Covariant Quantities

$$\mathcal{D}_m g_{\mu\nu} = \partial_m g_{\mu\nu} - N_m^\rho \partial_\rho g_{\mu\nu} - \partial_\mu N_m^\rho g_{\rho\nu} - \partial_\nu N_m^\rho g_{\mu\rho}.$$

$$\mathcal{F}_{mn}^\mu = \partial_m N_n^\mu - \partial_n N_m^\mu - N_m^\rho \partial_\rho N_n^\mu + N_n^\rho \partial_\rho N_m^\mu$$

○ Under foliation preserving diffeomorphism

$$\begin{aligned}
(\mathcal{D}_m g_{\mu\nu})' &= \left(\frac{\partial y^n}{\partial y'^m}\right)\left(\frac{\partial x^\rho}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu}\right)\mathcal{D}_n g_{\rho\sigma} \\
(\mathcal{F}_{mn}^\mu)' &= \left(\frac{\partial y^p}{\partial y'^m}\frac{\partial y^q}{\partial y'^n}\right)\left(\frac{\partial x'^\mu}{\partial x^\nu}\right)\mathcal{F}_{pq}^\nu \\
(\partial_\mu \gamma_{mn})' &= \left(\frac{\partial x^\nu}{\partial x'^\mu}\right)\left(\frac{\partial y^p}{\partial y'^m}\frac{\partial y^q}{\partial y'^n}\right)\partial_\nu \gamma_{pq}
\end{aligned}$$

4+D FPDG Action

Horizontal vector fields $\hat{\partial}_m = \partial_m - N_m^\mu \partial_\mu$;

$$\hat{\partial}'_m = \frac{\partial y^n}{\partial y'^m}\hat{\partial}_n$$

$$\begin{aligned} S = \int d^{4+D}x\sqrt{-g}\sqrt{-\gamma}\Big[& R^{(4)} + \alpha\hat{R}^{(D)} - \tfrac{\lambda}{4}\gamma^{mn}\gamma^{pq}g_{\mu\nu}F^\mu_{mp}F^\nu_{nq} \\ & + \tfrac{\delta}{4}\gamma^{mn}g^{\mu\nu}g^{\alpha\beta}(\mathcal{D}_m g_{\mu\alpha}\mathcal{D}_n g_{\nu\beta} - \beta\mathcal{D}_m g_{\mu\nu}\mathcal{D}_n g_{\alpha\beta}) \\ & + \tfrac{\epsilon}{4}g^{\mu\nu}\gamma^{mn}\gamma^{pq}(\partial_\mu\gamma_{mp}\partial_\nu\gamma_{nq} - \eta\partial_\mu\gamma_{mn}\partial_\nu\gamma_{pq})\Big] + S_m, \end{aligned}$$

$$R^{(4)} = g^{\mu\nu}R_{\mu\nu}, \quad \hat{R}^{(D)} = \gamma^{mn}\hat{R}_{mn}(\hat{\Gamma}, \hat{\partial}\hat{\Gamma}),$$

- Einstein-Hilbert action is not conformally invariant. A scalar field ϕ is introduced;

$$S = \int d^n x \sqrt{-g} \left[\phi^2 R + \frac{4(n-1)}{n-2} \nabla_\gamma \phi \nabla^\gamma \phi - \lambda \phi^{\frac{2n}{n-2}} \right]$$

Invariance under $g_{\mu\nu} \rightarrow e^{2\omega(x)} g_{\mu\nu}, \; \phi \rightarrow e^{\frac{2-n}{2}\omega(x)} \phi$

- Isotropic conformal transformation $(n = 4 + D)$

$$g_{\mu\nu} \rightarrow e^{2\omega(x,y)} g_{\mu\nu}, \;\; N^\mu_m \rightarrow N^\mu_m, \;\; \gamma_{mn} \rightarrow e^{2\omega(x,y)} \gamma_{mn}$$

- Anisotropic conformal transformation

$$\begin{aligned} g_{\mu\nu} &\rightarrow e^{2\omega(x,y)} g_{\mu\nu}, \;\; N^\mu_m \rightarrow N^\mu_m, \;\; \gamma_{mn} \rightarrow e^{2z\omega(x,y)} \gamma_{mn} \\ \phi &\rightarrow e^{-v(z)\omega(x)} \phi \end{aligned}$$

- Critical Exponent

$$y^m \rightarrow b^z y^m, \;\; x^\mu \rightarrow b x^\mu, \;\; N^\mu_m \rightarrow b^{1-z} N^\mu_m, \;\; \varphi \rightarrow b^{-v(z)} \varphi,$$

4+D FPDCG Action

$$
\begin{aligned}
S_{(C)} = \int d^{4+D}x\sqrt{-g^{(4)}}\sqrt{\gamma}M_*^{2+D}\Bigg[\phi^{\frac{2+Dz}{v}}\left(R^{(4)} + A_1\frac{\nabla_\mu\nabla^\mu\phi}{\phi} + A_2\frac{\nabla_\mu\phi\nabla^\mu\phi}{\phi^2}\right) \\
- V_0\phi^{2N} + \alpha_1\phi^{\frac{4+(D-2)z}{v}}\left(\hat{R}^{(D)} + B_1\frac{\hat{\nabla}_m\hat{\nabla}^m\phi}{\phi} + B_2\frac{\hat{\nabla}_m\phi\hat{\nabla}^m\phi}{\phi^2}\right) \\
- \alpha_2\phi^{\frac{6+(D-4)z}{v}}\frac{1}{4}\gamma^{mn}\gamma^{pq}g_{\mu\nu}F^\mu_{mp}F^\nu_{nq} \\
+ \alpha_3\phi^{\frac{4+(D-2)z}{v}}\Bigg\{\frac{1}{4}\gamma^{mn}g^{\mu\nu}g^{\alpha\beta}(\mathcal{D}_m g_{\mu\alpha}\mathcal{D}_n g_{\nu\beta} - \alpha_4\mathcal{D}_m g_{\mu\nu}\mathcal{D}_n g_{\alpha\beta}) \\
+ C_1\gamma^{mn}g^{\mu\nu}\mathcal{D}_m g_{\mu\nu}\frac{\hat{\nabla}_n\phi}{\phi} + C_2\gamma^{mn}\frac{\hat{\nabla}_m\phi\hat{\nabla}_n\phi}{\phi^2}\Bigg\} \\
- \alpha_5\phi^{\frac{2+Dz}{v}}\Bigg\{\frac{1}{4}g^{\mu\nu}\gamma^{mn}\gamma^{pq}(\nabla_\mu\gamma_{mp}\nabla_\nu\gamma_{nq} - \alpha_6\nabla_\mu\gamma_{mn}\nabla_\nu\gamma_{pq}) \\
+ D_1 g^{\mu\nu}\gamma^{mn}\nabla_\mu\gamma_{mn}\frac{\nabla_\nu\phi}{\phi} + D_2 g^{\mu\nu}\frac{\nabla_\mu\phi\nabla_\nu\phi}{\phi^2}\Bigg\}\Bigg]
\end{aligned}
$$

The 5D z-Weyl gravity

$$S = \int dy d^4x \sqrt{-g} N M_*^3 \Bigg[\varphi^2 \left(R - \tfrac{12}{z+2} \tfrac{\nabla_\mu \nabla^\mu \varphi}{\varphi} + \tfrac{12z}{(z+2)^2} \tfrac{\nabla_\mu \varphi \nabla^\mu \varphi}{\varphi^2} \right)$$

$$+ \beta_1 \varphi^{-\frac{2(z-4)}{z+2}} \left\{ B_{\mu\nu} B^{\mu\nu} - \lambda B^2 \right\} + \beta_2 \varphi^2 A_\mu A^\mu - V(\varphi) \Bigg].$$

$$B_{\mu\nu} = K_{\mu\nu} + \frac{2}{(z+2)N\varphi} g_{\mu\nu} (\partial_y \varphi - \nabla_\rho \varphi N^\rho), \; B \equiv g^{\mu\nu} B_{\mu\nu},$$

$$A_\mu = \frac{\partial_\mu N}{N} + \frac{2z}{z+2} \frac{\partial_\mu \varphi}{\varphi}$$

$$V = V_0 \varphi^{\frac{2(z+4)}{z+2}}$$

- $N^\mu = 0$
- Ground state: $g_{MN}(x,y) = \begin{pmatrix} g_{\mu\nu}(x) & 0 \\ 0 & N(x) \end{pmatrix}$
- $N = 1$

Cosmological test of anisotropic conformal symmetry

$$S = \int dy d^4x \sqrt{-g} M_*^3 \left[\varphi^2 \left(R - \frac{12}{z+2} \frac{\nabla_\mu \nabla^\mu \varphi}{\varphi} + \frac{12z}{(z+2)^2} \frac{\nabla_\mu \varphi \nabla^\mu \varphi}{\varphi^2} \right) \right.$$

$$\left. + \beta_2 \frac{4z^2}{(z+2)^2} \nabla_\mu \varphi \nabla^\mu \varphi - V_0 \varphi^{\frac{2(z+4)}{z+2}} \right].$$

$$\Downarrow \quad \begin{cases} \mathrm{M}_*^3 L \equiv \gamma_1 M_p^2/2, \ M_*^3 L V_0 \equiv \gamma_2 M_p^4 \\ \omega \equiv \frac{-4(z+1)(\beta_2 z+6)}{(z+2)^2} \end{cases}$$

$$S = \int d^4x \sqrt{-g} \left[\frac{\gamma_1 M_p^2}{2} \varphi^2 R - \frac{\gamma_1 M_p^2 \omega}{2} \nabla_\mu \varphi \nabla^\mu \varphi - \gamma_2 M_p^4 \varphi^{\frac{2z+8}{z+2}} \right]$$

$$\Downarrow \quad \varphi \to \tilde{\varphi} = \sqrt{\gamma_1} \varphi$$

$$\boxed{\mathrm{S} = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} \tilde{\varphi}^2 R - \frac{M_p^2 \omega}{2} \nabla_\mu \tilde{\varphi} \nabla^\mu \tilde{\varphi} - \gamma_2 \gamma_1^{-\frac{z+4}{z+2}} M_p^4 \tilde{\varphi}^{\frac{2z+8}{z+2}} \right]}$$

Evolution equations

$$3H^2 = \frac{\omega}{2}\frac{\dot{\varphi}^2}{\varphi^2} - 6H\frac{\dot{\varphi}}{\varphi} + \frac{V}{M_p^2\varphi^2} + \frac{\rho_{r,0}}{M_p^2\varphi^2 a^4} + \frac{\rho_{m,0}}{M_p^2\varphi^2 a^3}\,,$$

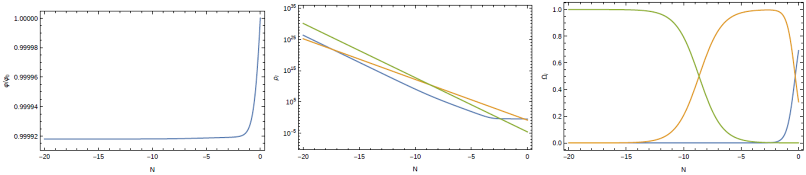
$$-3H^2 - 2\dot{H} = \frac{2\ddot{\varphi}}{\varphi} + 4H\frac{\dot{\varphi}}{\varphi} + \left(2 + \frac{\omega}{2}\right)\frac{\dot{\varphi}^2}{\varphi^2} - \frac{V}{M_p^2\varphi^2} + \frac{\rho_{r,0}}{3M_p^2\varphi^2 a^4}$$

$$\ddot{\varphi} + 3H\dot{\varphi} - \frac{6}{\omega}(2H^2 + \dot{H})\varphi + \frac{V_\varphi}{\omega M_p^2} = 0$$

$$\rho_{DE} = \frac{\varphi_0^2}{\varphi^2}\left(\frac{\omega M_p^2}{2}\dot{\varphi}^2 - 6M_p^2 H\varphi\dot{\varphi} + V\right) + \left(\frac{\varphi_0^2}{\varphi^2} - 1\right)\frac{\rho_{r,0}}{a^4} + \left(\frac{\varphi_0^2}{\varphi^2} - 1\right)\frac{\rho_{m,0}}{a^3}$$

$$p_{DE} = \frac{\varphi_0^2}{\varphi^2}\left[2M_p^2\varphi\ddot{\varphi} + 4M_p^2 H\varphi\dot{\varphi} + M_p^2\left(2 + \frac{\omega}{2}\right)\dot{\varphi}^2 - V\right] + \frac{1}{3}\left(\frac{\varphi_0^2}{\varphi^2} - 1\right)\frac{\rho_{r,0}}{a^4}$$

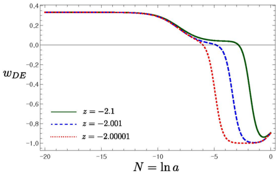
Numerical Analysis and Data Comparison

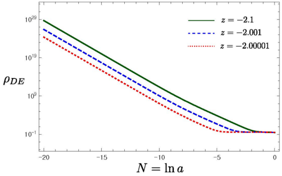


	5D Brans-Dicke Model		ΛCDM Model	
	$H(z)$ + SN + BAO + WMAP9	$H(z)$ + SN + BAO + PLANCK	$H(z)$ + SN + BAO +WMAP9	$H(z)$ + SN + BAO +PLANCK
H_0	$67.94^{+0.89}_{-0.62}$	$67.56^{+0.49}_{-0.67}$	$69.57^{+0.83}_{-0.80}$	$69.32^{+0.67}_{-0.69}$
$\Omega_m h^2$	$0.1384^{+0.0023}_{-0.0027}$	$0.1429^{+0.0015}_{-0.0017}$	$0.1410^{+0.0022}_{-0.0024}$	$0.1446^{+0.0014}_{-0.0015}$
$\Omega_b h^2$	$0.0243^{+0.0004}_{-0.0005}$	$0.0241^{+0.0003}_{-0.0003}$	$0.0239^{+0.0004}_{-0.0004}$	$0.0239^{+0.0003}_{-0.0003}$
z	> -2.0596 (95% CL)	> -2.0374 (95% CL)	-	-
$\hat{V}_0$	$0.3008^{+0.0126}_{-0.0085}$	$0.2905^{+0.0084}_{-0.0081}$	-	-
$\Omega_\Lambda h^2$	-	-	$0.3433^{+0.0120}_{-0.0118}$	$0.3355^{+0.0105}_{-0.0105}$
$\chi^2_{\rm min}$	599.999	607.286	588.366	590.724
χ^2_ν	0.97879	0.99068	0.95825	0.96209
$\lvert\gamma^{\rm PPN} - 1\rvert$	$< 5.7 \times 10^{-4}$ (95% CL)	$< 1.9 \times 10^{-4}$ (95% CL)	-	-
$\delta G/G$	$< 1.5 \times 10^{-2}$ (95% CL)	$< 7.8 \times 10^{-3}$ (95% CL)	-	-
$\dot{G}/G$ $[10^{-13}{\rm yr}^{-1}]$	> -1.23 (95% CL)	> -0.73 (95% CL)	-	-

Discussions

- $z = -2$ is the Einstein limit $+ \Lambda$?





- Mass Hierarchy?;

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \nabla_\mu s \nabla_\nu s - \alpha^2 \phi_0^{\frac{8+2z}{2+z}} \left[\left(\frac{s}{\phi_0} \right)^2 - w^2 \right]^2$$

$$\Longrightarrow \quad m_h \sim \phi_0^{\frac{4}{2+z}} M_{pl}$$

- Matter coupling and KK spectrum around $z = -2$
- Beyond SM?