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# The Universe after G-inflation

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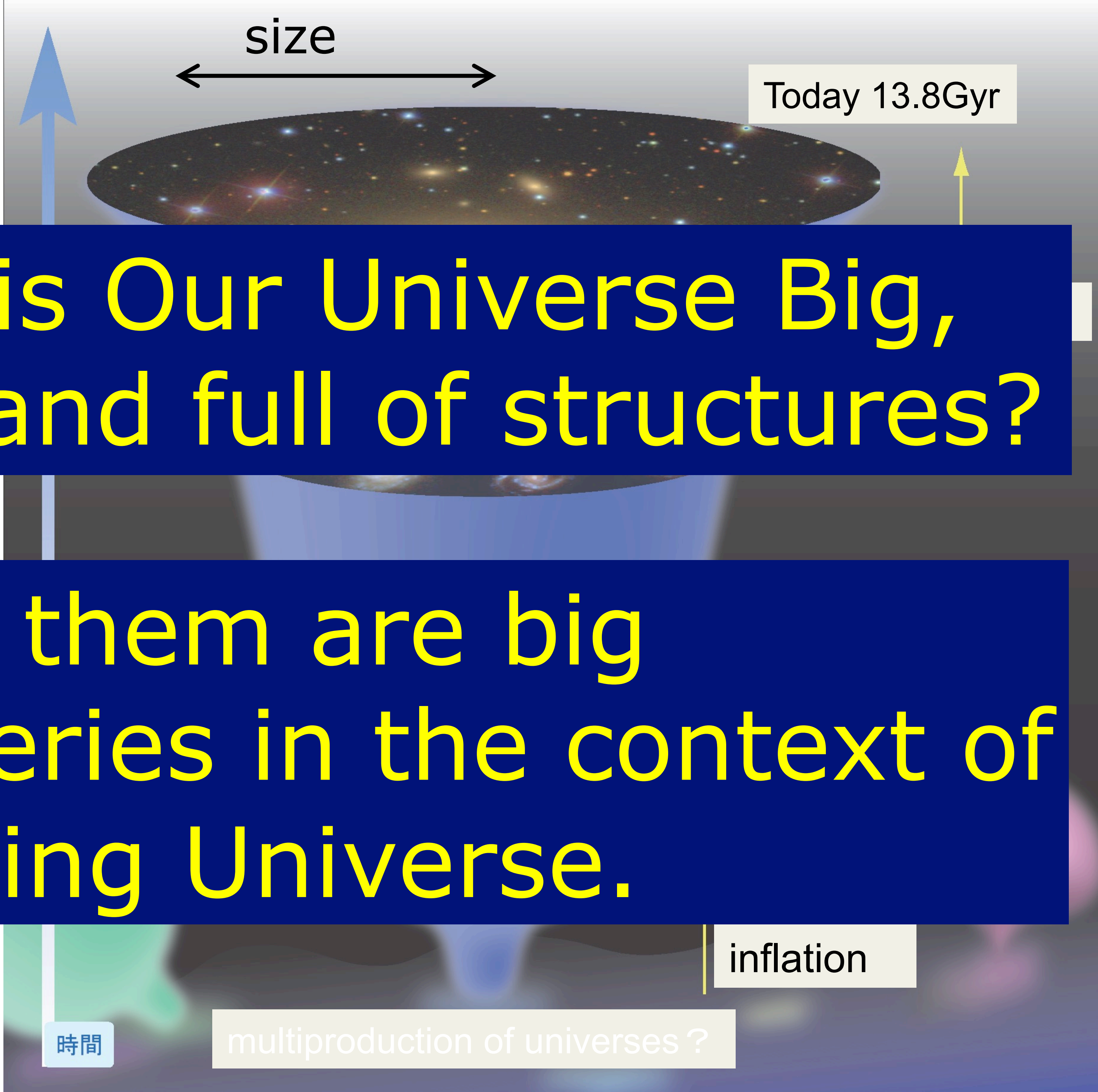
G=Galileon



# Why is Our Universe Big, Old, and full of structures?

All of them are big mysteries in the context of evolving Universe.

# Rapid Accelerated Expansion or INFLATION in the early Universe can solve

## The Horizon Problem

Why is our Universe Big?

## The Flatness Problem

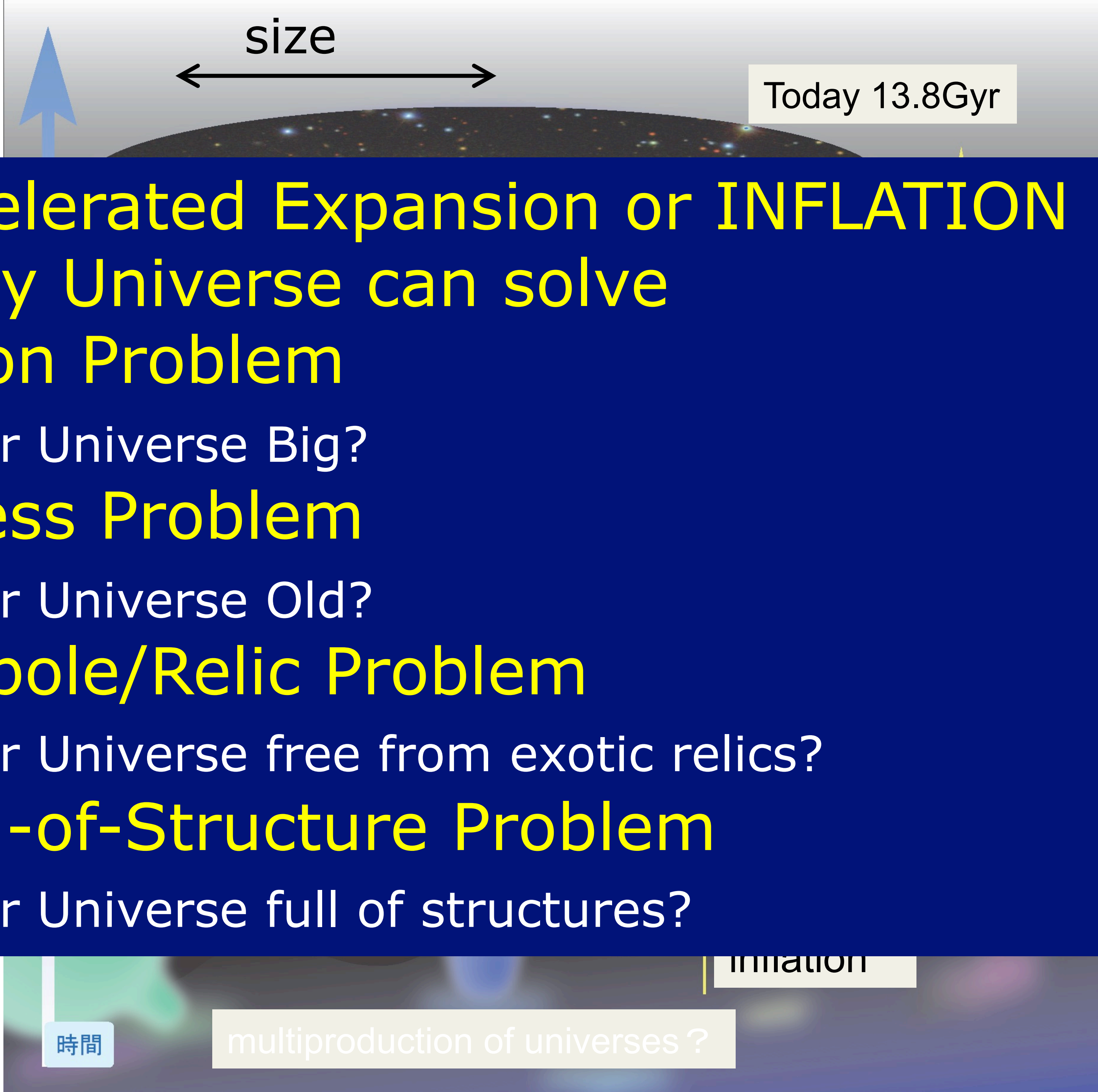
Why is our Universe Old?

## The Monopole/Relic Problem

Why is our Universe free from exotic relics?

## The Origin-of-Structure Problem

Why is our Universe full of structures?



# Generalized G-inflation

The most general single scalar action with second order field equations

Kobayashi, Yamaguchi, JY (2011)

$$S = \sum_{i=2}^{5} \int d^4x \ \sqrt{-g}\mathcal{L}_i$$

$$X = -\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \qquad G_{iX} = \frac{\partial G_i}{\partial X}$$

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X), \\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi, \\
\mathcal{L}_4 &= G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\
\mathcal{L}_5 &= G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right],
\end{aligned}$$

During inflation the Universe must have been very dark, being dominated by the inflaton's energy density…

# The Universe after inflation

## Reheating

or entropy production
to realize Big Bang

- ★ In some models, inflation may end abruptly without being followed by its coherent field oscillation and reheating proceeds through gravitational particle creation.

- ★ Such inflation models include k-inflation and G-inflation models driven by kinetic energy of a scalar field with non-canonical kinetic terms.

# The original kinetically driven G-inflation

$$\mathcal{L} = K(\phi, X) - G_3(\phi, X)\Box\phi \quad X = -\frac{1}{2}(\partial\phi)^2 \quad G_4 = \frac{M_G^2}{2}$$

Einstein gravity

$M_G$ : Reduced Planck Mass

★The case with full shift symmetry

$$K(X) = -X + \frac{X^2}{2M'^3\mu} \quad G_3 = \frac{X}{M'^3} \Rightarrow X = M'^3\mu, \quad H_{\rm inf}^2 = \frac{M'^3\mu}{6M_G^2}.$$

De Sitter inflation (never ends)

★Inflation can be terminated by flipping the sign here.

$$K(X) = -X + \frac{X^2}{2M'^3\mu} \Rightarrow K(X) = -\boxed{A(\phi)}X + \frac{X^2}{2M'^3\mu}$$

A simple choice: $A(\phi) \equiv \tanh\left[\beta\left(\phi_f - \phi\right)/M_G\right]$ with $\beta = O(1)$.

Numerical solutions indicate $\phi$ stalls within one e-fold after crossing $\phi_f$ and all higher derivative terms become negligible.

This function breaks the shift symmetry (severely) only around $\phi \approx \phi_f$ .

Despite its appearance, the linear perturbation $\delta\phi(x,t)$ around the background $\phi(t)$ , $\phi_{total}(x,t)=\phi(t)+\delta\phi(x,t)$ is stable.

The action for the curvature perturbation $\varsigma_\phi = -H\dfrac{\delta\phi}{\dot\phi}$

$$S^{(2)} = \frac{1}{2}\int d\tau d^3xz^2\left[\mathcal{G}\varsigma_\phi'^2 - \mathcal{F}(\nabla\varsigma_\phi)^2\right]$$

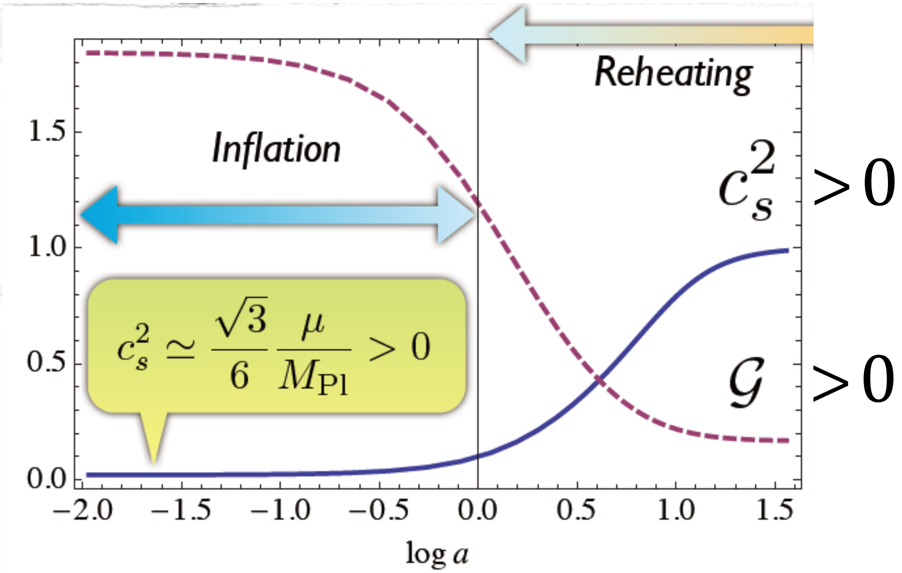
$$\mathcal{F} = K_X + 2G_X(\ddot\phi + 2H\dot\phi) - 2\frac{G_X^2}{M_G^2}X^2 + 2G_{XX}X\ddot\phi - 2(G_\phi - XG_{\phi X}),$$

$$\mathcal{G} = K_X + 2XK_{XX} + 6G_XH\dot\phi + 6\frac{G_X^2}{M_G^2}X^2 - 2(G_\phi - XG_{\phi X}) + 6G_{XX}HX\dot\phi.$$

Perturbation is not a ghost if $\mathcal{G} > 0$

$$z = \frac{a\dot\phi}{H - G_X\dot\phi^3/2M_G^2},$$

There is no gradient instability if $c_s^2 = \dfrac{\mathcal{F}}{\mathcal{G}} > 0$



Both are satisfied throughout.

# Unique signature of G-inflation

In G-inflation, the null energy condition may be violated,

$$2M_G^2\dot{H} = -(\rho + p) = -(2K_X X + 3G_{3X}H\dot{\phi}^3 - 4G_{3\phi}X - 2G_{3X}\ddot{\phi}X) > 0.$$

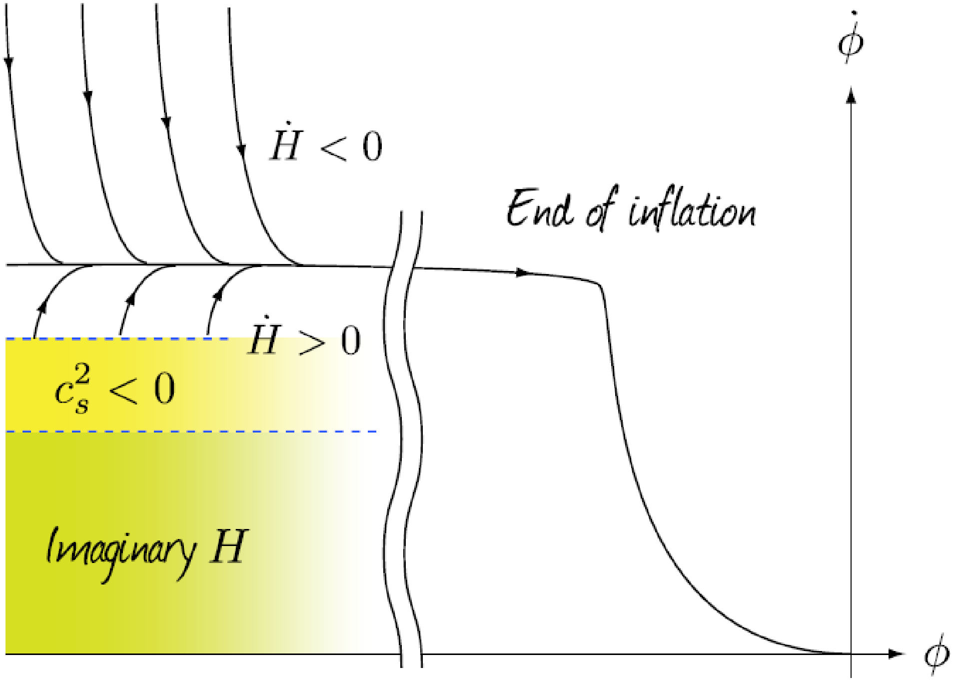
It can be violated without instabilities, keeping $c_s^2 > 0$.

The tensor spectral index can be positive,

$$n_T = -2\varepsilon = 2\frac{\dot{H}}{H^2} > 0.$$

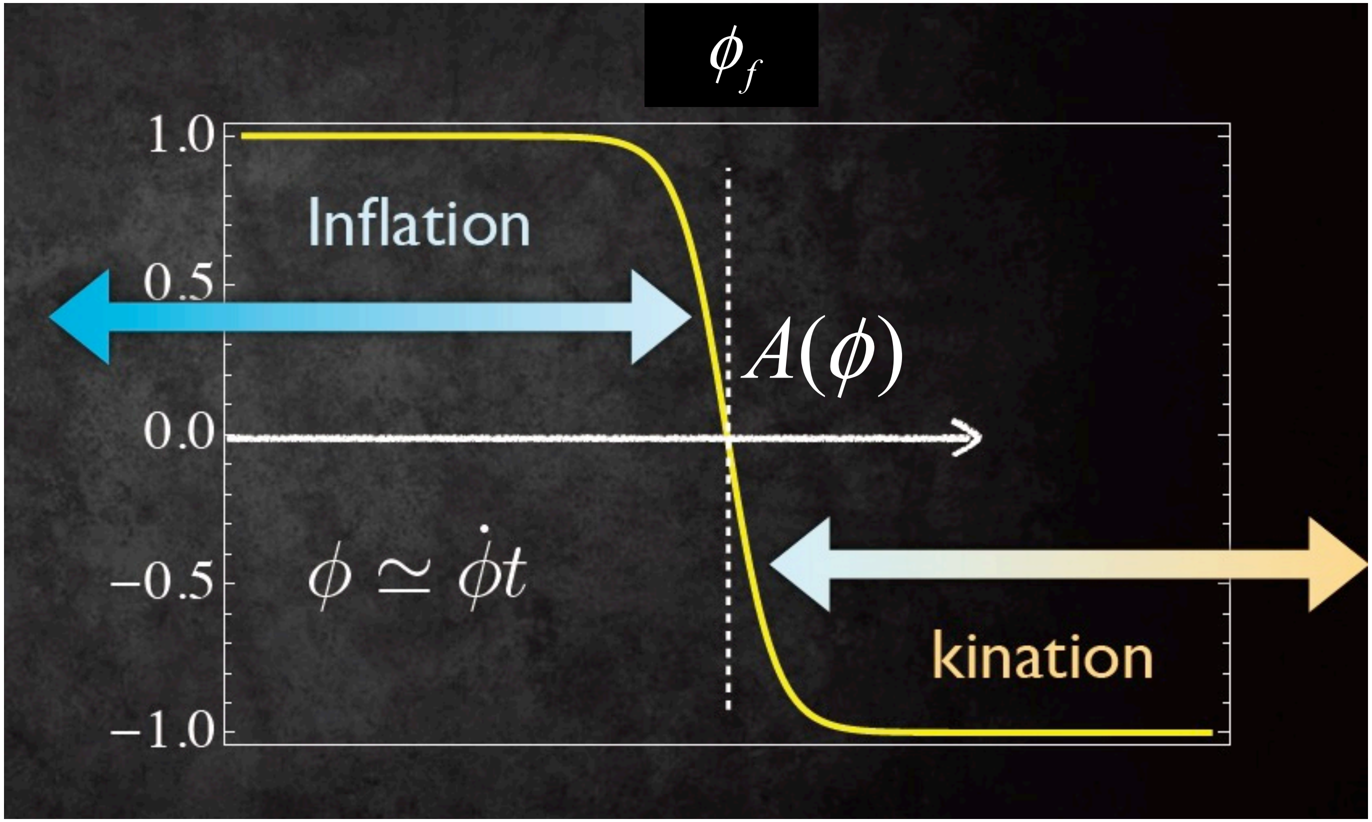
BLUE tensor spectrum!!

(This is not the case with k-inflation.)



Short wave tensor fluctuations may have a larger amplitude at formation.

# The Universe after G-inflation



- ★After $A(\phi)$ has flipped its sign the Universe is soon dominated by the kinetic energy of the inflaton field now with the canonical kinetic term.

$$K(X) \cong X \qquad p = \rho_{tot} = \frac{\dot{\phi}^2}{2} \propto a^{-6}(t) \Rightarrow a(t) \propto t^{\frac{1}{3}}$$

★ **Gravitational particle production** takes place due to this rapid change of the expansion law.

Vacuum state in de Sitter space is different from that in the power-law expanding Universe.

$$a(t) \propto e^{H_{\inf} t} \xrightarrow{\Delta t} a(t) \propto t^{\frac{1}{3}}$$

$$\langle 0_{dS} | a^{\dagger}_{powerlaw\boldsymbol{k}} a_{powerlaw\boldsymbol{k}} | 0_{dS} \rangle \neq 0$$

★ This process can be calculated using the Bogolubov coefficients $\beta_k$.
Consider production of a massless boson $\chi$, whose mode function reads

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \frac{k^2}{a^2}\chi_k = 0 \xrightarrow[X_k \,\equiv\, a^{-1}\chi_k]{dt \,=\, a(t)d\tau \ \ \text{conformal time}} X_k'' + \left(k^2 \boxed{-\frac{a''}{a}}\right)X_k = 0$$

$$\equiv -V(\tau)$$

★ Bogolubov coefficient

$$\beta_k = \frac{i}{2k}\int_{-\infty}^{\infty} e^{-2ik\tau} V(\tau) d\tau \Rightarrow \langle 0_{dS} | a^{\dagger}_{powerlaw\boldsymbol{k}} a_{powerlaw\boldsymbol{k}} | 0_{dS} \rangle = |\beta_k|^2$$

★ Energy density

$$\rho_r(\tau) = \frac{1}{2\pi^2 a^4(\tau)} \int_0^{\infty} |\beta_k|^2 k^3 dk \cong \frac{9H_{\inf}^4}{32\pi^2 a^4} \ln\left(\frac{1}{H_{\inf}\Delta t}\right)$$

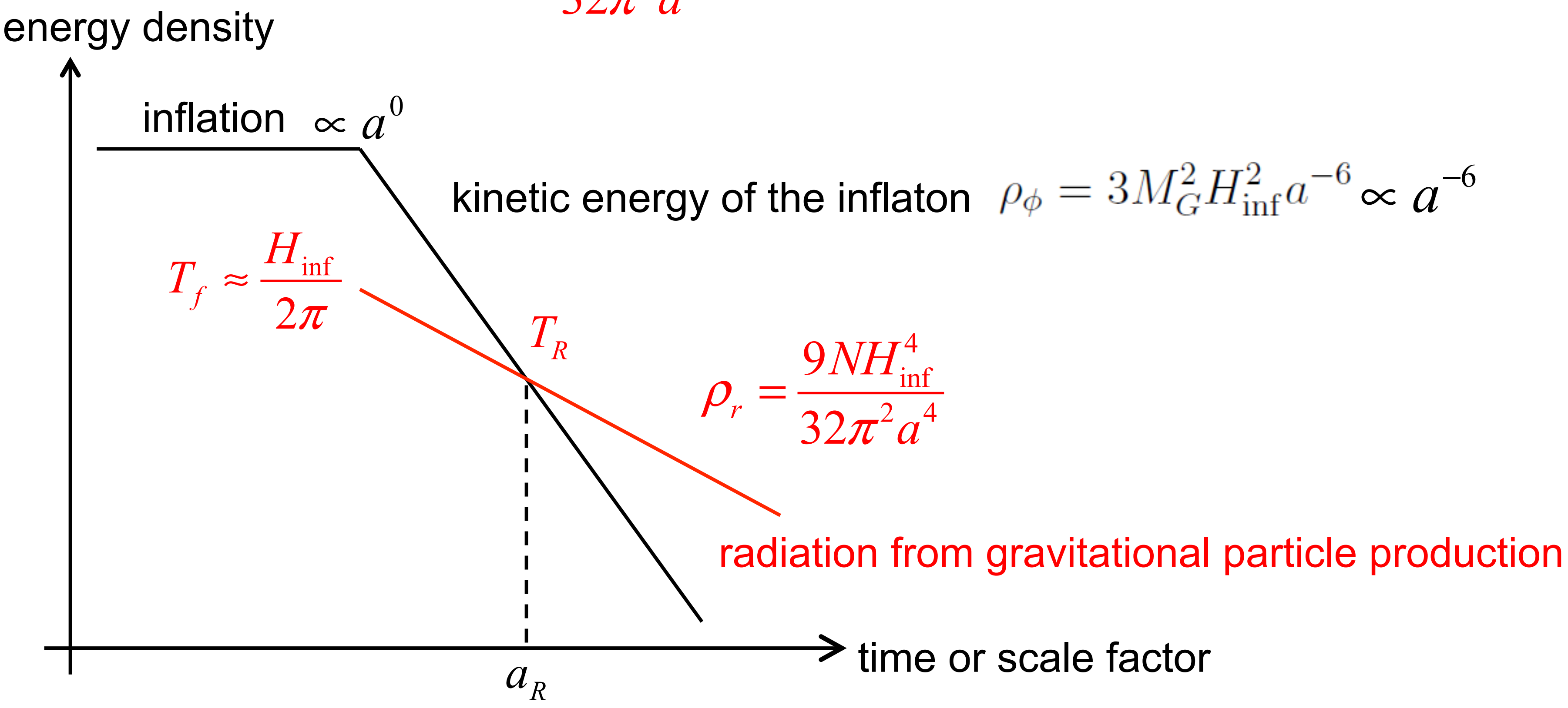
setting $a = 1$ at the end of inflation.

(Ford 1987, Kunimitsu & JY 2014)

- Gravitational particle production produces radiation energy of order of the Hawking temperature of the de Sitter space $T \approx \frac{H_{\inf}}{2\pi}$ at the end of inflation.

- Suppose there are effectively $N$ modes and neglect the logarithmic factor.

$$\rho_r = \frac{9NH_{\inf}^4}{32\pi^2 a^4} \ll \rho_{tot} = 3M_G^2 H_{\inf}^2 \quad \text{initially.}$$



$$\rho_\phi = \rho_r \quad \text{at} \quad a_R^2 = \frac{32\pi^2 M_G^2}{3NH_{\inf}^2} \quad \text{which defines the reheating time.}$$

- ★ The reheating temperature at the onset of the radiation domination is

$$T_R = \frac{3N^{\frac{3}{4}}}{(32\pi^2)^{\frac{3}{4}}} \left(\frac{30}{\pi^2 g_*}\right)^{\frac{1}{4}} \frac{H_{\rm inf}^2}{M_G} \simeq 3.9 \times 10^6 N^{\frac{3}{4}} \left(\frac{g_*}{106.75}\right)^{-\frac{1}{4}} \left(\frac{r}{0.01}\right) \text{GeV}.$$

Here $r = 0.01 \left(\frac{H_{\rm inf}}{2.4 \times 10^{13}\text{GeV}}\right)^2$ is the tensor-scalar ratio.

- ★ The maximum temperature after inflation can be much higher (if thermalized sufficiently rapidly) :

$$T_f \approx \frac{H_{\rm inf}}{2\pi} = 4\times 10^{12} \left(\frac{r}{0.01}\right)^{\frac{1}{2}} \text{GeV}$$

- ★ If we consider the case the Universe is thermalized through 2 body scattering,

from $\Gamma_2 = \langle n\sigma v\rangle \approx \frac{N' T^3}{\pi^2} \frac{\alpha^2}{T^2} > H = H_{\rm inf} a^{-3}$, we find the thermalization temperature

$$T_{th} \approx 8\times 10^{10} \left(\frac{r}{0.01}\right)^{\frac{1}{2}} \left(\frac{N'}{10}\right)^{\frac{1}{2}} \left(\frac{\alpha}{0.05}\right) \text{GeV}$$

$\alpha$ : coupling contant

$N'$ : number of modes

# Reheating through direct interaction

(Bazrafshan Moghaddam, Brandenberger, & JY 2017)

★ More efficient reheating is possible if χ is directly coupled with the inflaton.

$$\mathcal{L}_{int} = \frac{1}{2M^2}(\partial\phi)^2\chi^2 = -\frac{1}{2M^2}\dot{\phi}^2\chi^2$$

which preserves the shift symmetry of $\phi$.

★ Mode function satisfies

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + M^{-2}\dot{\phi}^2\right)\chi_k = 0$$

$$X_k \equiv a^{-1}\chi_k$$

$$dt = a(t)d\tau$$

$$X_k'' + \left(k^2 + \underbrace{\boxed{M^{-2}\dot{\phi}^2a^2 - \frac{a''}{a}}}_{\equiv -V(\tau)}\right)X_k = 0$$

This problem can also be solved using the Bogolubov coefficients.

$$\rho_r(\tau) = \frac{1}{2\pi^2 a^4(\tau)} \int_0^\infty |\beta_k|^2 k^3 dk \qquad \beta_k = \frac{i}{2k} \int_{-\infty}^{\infty} e^{-2ik\tau} V(\tau) d\tau$$

$$V(\tau) = \left( 12\frac{M_G^2}{M^2} - 2 \right) \frac{1}{\tau^2}$$

During inflation

$$a(\tau_f) = -\frac{1}{H_{\inf}\tau_f} = 1$$

$$V(\tau) = \left( 3\frac{M_G^2}{M^2} - \frac{1}{4} \right) \frac{1}{\left(\tau + \frac{3}{2H_{\inf}}\right)^2}$$

Kination regime after inflation

direct interaction

gravitational particle production

★The final radiation energy density is given by

$$\rho_r(\tau) = \frac{-1}{32\pi^2 a^4} \int_{-\infty}^{\tau} d\tau_1 d\tau_2 \ln(\mu\,|(\tau_1 - \tau_2)|) V'(\tau_1) V'(\tau_2)$$

$$\rho_r(\tau) = \frac{-1}{32\pi^2 a^4}\int_{-\infty}^{\tau} d\tau_1 d\tau_2 \ln(\mu\,|(\tau_1-\tau_2)|)V'(\tau_1)V'(\tau_2)$$

$$\simeq \frac{H_{\text{inf}}^4}{128\pi^2 a^4}\left[\underbrace{5\frac{M_G^4}{M^4}}_{\text{direct interaction}} + \underbrace{12\ln\left(\frac{1}{H_{\text{inf}}\Delta t}\right)}_{\text{gravitational particle production}}\right]$$

★ If $M \leq M_G$ the direct interaction is more important than gravitational particle production and realizes a higher reheating temperature.

- ★ The reheating temperature when direct interaction is dominant.

$$T_R = \frac{5H_{\inf}^2 M_G^2}{32\pi^2 (3g_*)^{1/4} M^3} = 1.2\times 10^{13}\left(\frac{g_*}{106.75}\right)^{-\frac{1}{4}}\left(\frac{r}{0.01}\right)\left(\frac{M}{10^{16}\,\mathrm{GeV}}\right)^{-3}\mathrm{GeV}$$

- ★ The maximum possible temperature after inflation is enhanced as

$$T_f \approx \frac{H_{\inf}}{2\pi}\frac{2M_G}{3M} = 6\times 10^{14}\left(\frac{r}{0.01}\right)^{\frac{1}{2}}\left(\frac{M}{10^{16}\,\mathrm{GeV}}\right)^{-1}\mathrm{GeV}$$

- ★ The thermalization temperature by 2 body scattering is given by

$$T_{th} = 2\times 10^{14}\left(\frac{r}{0.01}\right)^{\frac{1}{2}}\left(\frac{N'}{10}\right)^{\frac{1}{2}}\left(\frac{\alpha}{0.05}\right)\left(\frac{M}{10^{16}\,\mathrm{GeV}}\right)^{-\frac{3}{2}}\mathrm{GeV}$$

Higher thermalization and reheating temperatures are possible.

# Conclusion

The Universe after G-inflation (as well as k-inflation) is dominated by the kinetic energy density of the inflaton field which evolves as $\dot{\phi}=\dot{\phi}_f a^{-3}=\sqrt{6}M_G H_{\inf} a^{-3}$

The reheating proceeds through gravitational particle production and/or direct interaction through derivative coupling preserving the shift symmetry of the theory.

One can consider spontaneous baryogenesis or asymmetric dark matter genesis by introducing a derivative coupling between the inflaton and their respective currents.

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