Constraints to dark energy using PADE parameterizations

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Jul, 2017

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Outline

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PADE rational approximation

PADE rational approximation

PADE approximations in Mathematics

$$f(x) = \frac{a_0 + a_1 x + a_2 x^2 + ... + a_n x^n}{b_0 + b_1 x + b_2 x^2 + ... + b_n x^m} \ , \tag{1}$$

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PADE rational approximation

PADE approximations in Mathematics

$$f(x) = \frac{a_0 + a_1 x + a_2 x^2 + ... + a_n x^n}{b_0 + b_1 x + b_2 x^2 + ... + b_n x^m} \ , \qquad (1)$$

PADE(I)

$$w_{\rm de}(a) = \frac{w_0 + w_1(1-a)}{1 + w_2(1-a)} \ . \qquad (2)$$

w_0, w_1 and w_2 are the free parameters of the model.

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Simplified PADE(I)

$$w_{\rm de}(a) = \frac{w_0}{1 + w_2(1-a)} \ . \quad (3)$$

w_0 and w_2 are the free parameters of the model.

PADE(II)

$$w_{\rm de}(a) = \frac{w_0 + w_1 \ln a}{1 + w_2 \ln a} \ , \quad (4)$$

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Hubble flow in PADE cosmologies

- Freidmann Equation

$$H^2 = \frac{8\pi G}{3}(\rho_r + \rho_m + \rho_{de}) \ , \tag{5}$$

- Continuity Equations

$$\dot{\rho_r} + 4H\rho_r = 0 \ , \tag{6}$$

$$\dot{\rho_m} + 3H\rho_m = 0 \ , \tag{7}$$

$$\dot{\rho_{de}} + 3H(1 + w_{de})\rho_{de} = 0 \ , \tag{8}$$

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$$\rho_{\rm de}^{\rm (PADEI)} = \rho_{\rm de}^{(0)} a^{-3\left(\frac{1+w_0+w_1+w_2}{1+w_2}\right)} [1+w_2(1-a)]^{-3\left(\frac{w_1-w_0w_2}{w_2(1+w_2)}\right)} , \quad (9)$$

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$$\rho_{\rm de}^{\rm (padeII)} = \rho_{\rm de}^{0} a^{-3\left(\frac{w_1+w_2}{w_2}\right)} (1+w_2\ln a)^{3\left(\frac{w_1-w_0w_2}{w_2{}^2}\right)} . \quad (11)$$

- Energy equations for DE in CPL parameterization

$$\rho_{\rm de}^{\rm CPL} = \rho_{\rm de}^{(0)} a^{-3(1+w_0+w_1)} \exp\{-3w_1(1-a)\} \quad (12)$$

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- Hubble parameter in PADE parameterization

$$E^2_{\rm PADEI} = \Omega_{\rm r0}a^{-4} + \Omega_{\rm m0}a^{-3} + (1-[\Omega_{\rm r0}+\Omega_{\rm m0}]) \times a^{-3(\frac{1+w_0+w_1+w_2}{1+w_2})} \times (1+w_2-aw_2)^{-3(\frac{w_1-w_0w_2}{w_2(1+w_2)})} , \quad (13)$$

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- In this framework, the joint likelihood function is the product of the individual likelihoods:

$$\mathcal{L}_{\rm tot}(\mathbf{p}) = \mathcal{L}_{\rm sn} \times \mathcal{L}_{\rm bao} \times \mathcal{L}_{\rm cmb} \times \mathcal{L}_{\rm h} \times \mathcal{L}_{\rm bbn} \,, \tag{17}$$

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- Total chi-square $\chi^2_{\rm tot}$ is given by:

$$\chi^2_{\rm tot}(\mathbf{p}) = \chi^2_{\rm sn} + \chi^2_{\rm bao} + \chi^2_{\rm cmb} + \chi^2_{\rm h} + \chi^2_{\rm bbn} \ . \tag{18}$$

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Free parameters of PADE models

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- $\mathbf{p} = \{\Omega_{\rm DM0}, \Omega_{\rm b0}, h, w_0, w_1\}$ in the case of CPL parameterisation.
- $\mathbf{p} = \{\Omega_{\rm DM0}, \Omega_{\rm b0}, h\}$ in the case of concordance ΛCDM model.

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Results of our analysis

Results of our analysis

Table: The statistical results for the various DE parameterisations used in the analysis. These results are based on the expansion data. The concordance ΛCDM model is shown for comparison.

Model	PADE I	simp. PADE I	PADE II	CPL	ΛCDM
k	6	5	6	5	3
$\chi^2_{\rm min}$	567.6	567.7	567.9	567.6	574.4
AIC	579.6	577.7	579.9	577.6	580.4

- The AIC is given by

$$\mathrm{AIC} = -2\ln\mathcal{L}_{\max} + 2k \;, \tag{19}$$

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- We find $\Delta\mathrm{AIC} = AIC - AIC_{\Lambda} < 4$ hence, the DE parameterisations explored in this study are consistent with the expansion data as equally as standard model.

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Table: A summary of the best-fit parameters for the various DE parameterisations using the background data.

Model	PADE I	simplified PADE I	PADE II	CPL	ΛCDM
$\Omega_m^{(0)}$	0.286 ± 0.010	0.270 ± 0.010	0.2864 ± 0.0096	0.2896 ± 0.0090	0.2891 ± 0.0090
h	0.682 ± 0.012	0.682 ± 0.012	0.686 ± 0.013	0.682 ± 0.012	0.6837 ± 0.0084
w_0	-0.825 ± 0.091	-0.845 ± 0.039	-0.889 ± 0.080	-0.80 ± 0.11	–
w_1	$-0.09^{+0.39}_{-0.32}$	–	$0.37^{+0.29}_{-0.23}$	$-0.51^{+0.48}_{-0.38}$	–
w_2	$-0.683^{+0.040}_{-0.034}$	-0.387 ± 0.034	$-0.353^{+0.038}_{-0.034}$	–	–
$w_{de}(z=0)$	-0.825	-0.845	-0.889	-0.80	-1.0
$\Omega_{de}(z=0)$	0.714	0.730	0.7136	0.7104	0.7109

- All PADE parameterizations remains in quintessence regime with $-1 < w_{de} < -1/3$ and can not cross the phantom line.

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Linear growth of perturbations

Basic equations in linear regime

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Basic equations in linear regime

- The basic equations that govern the evolution of non-relativistic matter and DE perturbations in linear regime are given by [10]

$$\dot{\delta_{\rm m}} + \frac{\theta_{\rm m}}{a} = 0\ , \tag{20}$$

$$\dot{\delta_{\rm de}} + (1+w_{\rm de})\frac{\theta_{\rm de}}{a} + 3H(c_{\rm eff}^2 - w_{\rm de})\delta_{\rm de} = 0\ , \tag{21}$$

$$\dot{\theta_{\rm m}} + H\theta_{\rm m} - \frac{k^2\phi}{a} = 0\ , \tag{22}$$

$$\dot{\theta_{\rm de}} + H\theta_{\rm de} - \frac{k^2 c_{\rm eff}^2\theta_{\rm de}}{(1+w_{\rm de})a} - \frac{k^2\phi}{a} = 0\ , \tag{23}$$

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- The Poisson equation also reeds

$$-\frac{k^2}{a^2}\phi = \frac{3}{2}H^2[\Omega_{\rm m}\delta_{\rm m} + (1+3c_{\rm eff}^2)\Omega_{\rm de}\delta_{\rm de}] \,, \tag{24}$$

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Linear growth of perturbations

Two types of DE models

- In homogeneous DE models $c_{eff} = 1$ and $\delta_{de} = 0$, Equations for perturbations are simplified to those in standard cosmology.
- In clustered DE models $c_{eff} = 0$ and δ_{de} can grow due to gravitational instability in similar way to dark matter fluctuations.

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Initial conditions

- Initial conditions for solving the equations of perturbations are [11, 12, 13]

$$\begin{aligned} \delta'_{\rm mi} &= \frac{\delta_{\rm mi}}{a_{\rm i}} \ , \\ \delta_{\rm dei} &= \frac{1 + w_{\rm dei}}{1 - 3w_{\rm dei}}\delta_{\rm mi} \ , \\ \delta'_{\rm dei} &= \frac{4w'_{\rm dei}}{(1 - 3w_{\rm dei})^2}\delta_{\rm mi} + \frac{1 + w_{\rm dei}}{1 - 3w_{\rm dei}}\delta'_{\rm mi} \ , \end{aligned} \tag{25}$$

where we fix $a_{\rm i} = 10^{-4}$ and $\delta_{\rm mi} = 1.5 \times 10^{-5}$.

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$$\delta_{\rm dei} = \frac{1+w_{\rm dei}}{1-3w_{\rm dei}}\delta_{\rm mi} \ ,$$
$$\delta'_{\rm dei} = \frac{4w'_{\rm dei}}{(1-3w_{\rm dei})^2}\delta_{\rm mi} + \frac{1+w_{\rm dei}}{1-3w_{\rm dei}}\delta'_{\rm mi} \ , \tag{25}$$

where we fix $a_{\rm i} = 10^{-4}$ and $\delta_{\rm mi} = 1.5 \times 10^{-5}$.

- From the above equations, we first calculate $\delta_m(z)$ and $\delta_{de}(z)$ and then the observable quantity $f(z)\sigma_8(z)$ for different PADE parameterizations considered in this analysis.

Joint Likelihood analysis in background and perturbation levels

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Joint Likelihood analysis in background and perturbation levels

Growth data used in our analysis

- We use the latest observational data for the growth rate $f(z)\sigma_8(z)$ from [14]

z	$f\sigma_8$	Survey
0.02	0.36 ± 0.04	
0.067	0.423 ± 0.055	6dF Galaxy Survey
0.15	0.49 ± 0.15	SDSS DR7 MGS
0.17	0.51 ± 0.06	2dF Galaxy Redshift Survey
0.22	0.42 ± 0.07	WiggleZ Dark Energy Survey
0.25	0.351 ± 0.058	SDSS II LRG
0.3	0.407 ± 0.055	SDSS I/II LRG + SDSS III BOSS CMASS
0.32	0.394 ± 0.062	SDSS III BOSS DR12 LOWZ
0.35	0.440 ± 0.05	SDSS DR5 LRG
0.37	0.460 ± 0.038	SDSS II LRG
0.38	0.430 ± 0.054	SDSS III BOSS DR12
0.4	0.419 ± 0.041	SDSS I/II LRG + SDSS III BOSS CMASS
0.41	0.45 ± 0.04	WiggleZ Dark Energy Survey
0.44	0.413 ± 0.080	WiggleZ Dark Energy Survey + Alcock-Paczynski distortion
0.5	0.427 ± 0.043	SDSS I/II LRG + SDSS III BOSS CMASS
0.51	0.452 ± 0.057	SDSS III BOSS DR12
0.57	0.444 ± 0.038	SDSS III BOSS DR12 CMASS
0.59	0.488 ± 0.06	SDSS III BOSS DR12 CMASS
0.60	0.43 ± 0.04	WiggleZ Dark Energy Survey
0.6	0.433 ± 0.067	SDSS I/II LRG + SDSS III BOSS CMASS
0.60	0.390 ± 0.063	WiggleZ Dark Energy Survey + Alcock-Paczynski distortion
0.61	0.457 ± 0.052	SDSS III BOSS DR12
0.73	0.437 ± 0.072	WiggleZ Dark Energy Survey + Alcock-Paczynski distortion
0.77	0.490 ± 0.18	VIMOS-VLT Deep Survey
0.78	0.38 ± 0.04	WiggleZ Dark Energy Survey
0.8	0.470 ± 0.08	VIMOS Public Extragalactic Redshift Survey
1.36	0.482 ± 0.116	FastSound

Joint Likelihood analysis in background and perturbation levels

Joint Likelihood analysis in background and perturbation levels

- The total likelihood function using the combined cosmological data in background level and growth data in perturbation level is given by

$$\mathcal{L}_{\rm tot}(\mathbf{p}) = \mathcal{L}_{\rm sn} \times \mathcal{L}_{\rm bao} \times \mathcal{L}_{\rm cmb} \times \mathcal{L}_{\rm h} \times \mathcal{L}_{\rm bbn} \times \mathcal{L}_{\rm gr} \ , \tag{26}$$

hence

$$\chi^2_{\rm tot}(\mathbf{p}) = \chi^2_{\rm sn} + \chi^2_{\rm bao} + \chi^2_{\rm cmb} + \chi^2_{\rm h} + \chi^2_{\rm bbn} + \chi^2_{\rm gr} \ , \tag{27}$$

where the statistical vector $\mathbf{p}$ contains an additional free parameter, namely $\sigma_8 \equiv \sigma_8(z=0)$

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Results overall statistical analysis

Table: The statistical results for homogeneous (clustered) DE parameterisations used in the analysis. These results are based on the expansion+growth rate data. The concordance ΛCDM model is shown for comparison.

Model	PADE I	simplified PADE I	PADE II	CPL	ΛCDM
k	7	6	7	6	4
$\chi^2_{\rm min}$	576.4(576.5)	576.4(576.7)	576.9(577.1)	576.5(576.7)	582.6
AIC	590.4(590.5)	588.4(588.7)	590.9(591.1)	588.5(588.7)	590.6

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AIC	590.4(590.5)	588.4(588.7)	590.9(591.1)	588.5(588.7)	590.6

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Results overall statistical analysis

Table: A summary of the best-fit parameters for homogeneous (clustered) DE parameterisations using the background+growth rate data.

Model	PADE I	simplified PADE I	PADE II	CPL	ΛCDM
$\Omega_{\rm m}^{(0)}$	0.288 ± 0.010 (0.288 ± 0.010)	0.288 ± 0.010 (0.2888 ± 0.0099)	0.2721 ± 0.0097 (0.2723 ± 0.0098)	0.2875 ± 0.0095 (0.2882 ± 0.0093)	0.2902 ± 0.00
h	0.681 ± 0.012 (0.681 ± 0.012)	0.680 ± 0.012 (0.679 ± 0.012)	0.684 ± 0.012 (0.683 ± 0.012)	0.681 ± 0.011 (0.680 ± 0.011)	0.6833 ± 0.00
w_0	-0.856 ± 0.088 $(-0.874^{+0.086}_{-0.097})$	-0.839 ± 0.038 (-0.836 ± 0.037)	-0.893 ± 0.075 (-0.896 ± 0.078)	$-0.81^{+0.10}_{-0.12}$ (-0.81 ± 0.10)	–
w_1	$0.07^{+0.37}_{-0.29}$ $(0.14^{+0.38}_{-0.29})$	–	$0.41^{+0.26}_{-0.22}$ $(0.43^{+0.27}_{-0.22})$	$-0.41^{+0.46}_{-0.37}$ $(-0.39^{+0.42}_{-0.36})$	–
w_2	$-0.694^{+0.040}_{-0.036}$ $(-0.699^{+0.042}_{-0.038})$	-0.388 ± 0.034 (-0.388 ± 0.035)	$-0.357^{+0.039}_{-0.033}$ $(-0.358^{+0.038}_{-0.034})$	–	–
σ_8	0.751 ± 0.015 (0.755 ± 0.016)	0.751 ± 0.015 (0.758 ± 0.015)	0.771 ± 0.015 (0.771 ± 0.016)	0.751 ± 0.015 (0.756 ± 0.015)	0.744 ± 0.01
$w_{\rm de}(z=0)$	$-0.856(-0.874)$	$-0.839(-0.836)$	$-0.893(-0.896)$	$-0.81(-0.81)$	-1.0
$\Omega_{\rm de}(z=0)$	$0.712(0.712)$	$0.712(0.7112)$	$0.7279(0.7277)$	$0.7125(0.7118)$	0.7098

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Results overall statistical analysis

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w_1	$0.07^{+0.37}_{-0.29}$ $(0.14^{+0.38}_{-0.29})$	–	$0.41^{+0.26}_{-0.22}$ $(0.43^{+0.27}_{-0.22})$	$-0.41^{+0.46}_{-0.37}$ $(-0.39^{+0.42}_{-0.36})$	–
w_2	$-0.694^{+0.040}_{-0.036}$ $(-0.699^{+0.042}_{-0.038})$	-0.388 ± 0.034 (-0.388 ± 0.035)	$-0.357^{+0.039}_{-0.033}$ $(-0.358^{+0.038}_{-0.034})$	–	–
σ_8	0.751 ± 0.015 (0.755 ± 0.016)	0.751 ± 0.015 (0.758 ± 0.015)	0.771 ± 0.015 (0.771 ± 0.016)	0.751 ± 0.015 (0.756 ± 0.015)	0.744 ± 0.01
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Likelihood contours

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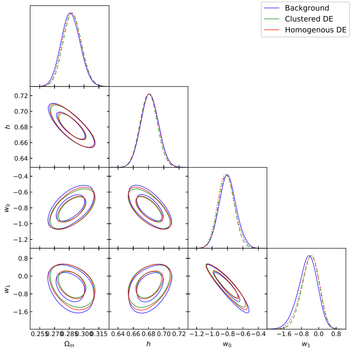
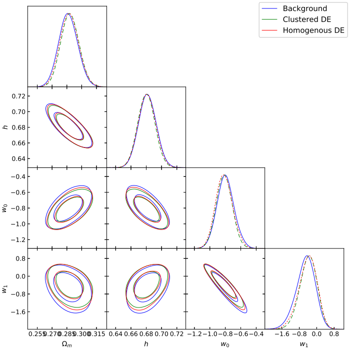
Likelihood contours

CPL

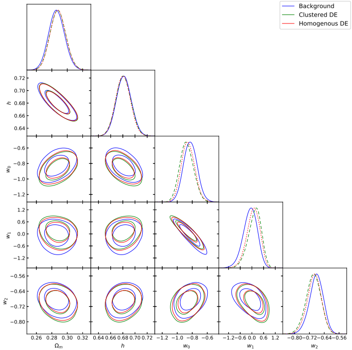


Likelihood contours

CPL



PADE I



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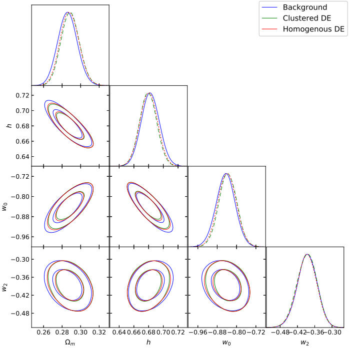
Likelihood contours

Likelihood contours

Simplified PADE I



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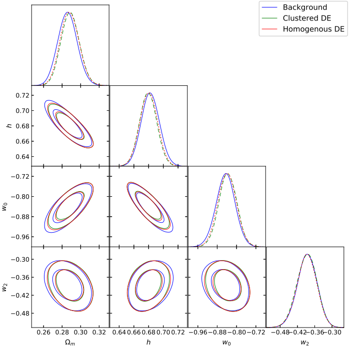
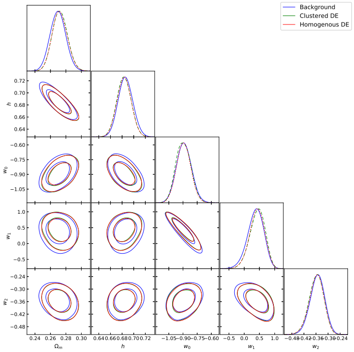
Likelihood contours

Simplified PADE I



PADE II



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Likelihood contours

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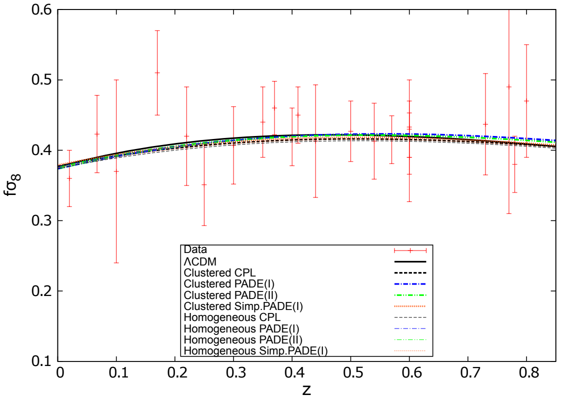
Likelihood contours

The predicted growth rate $f(z)\sigma_8(z)$ for different cosmological models in this analysis.



All PADE models are well fitted to growth rate data as equally as ΛCDM case.

Conclusion

In the context of MCMC analysis, we showed that all PADE parameterizations are well fitted to geometrical data in background level as equally as CPL and ΛCDM model.

Combining the geometrical data with growth rate data in perturbation level, we showed that all PADE parameterizations are well fitted to the full set of observatioal data similar to that of the CPL and ΛCDM models.

The current data of growth rate are not sufficient to discriminate between homogeneous and clustered DE models.

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