ibS CTPU-CGA

Survival of the Fittest: Testing Superradiance Termination with Simulated Binary Black Hole Statistics

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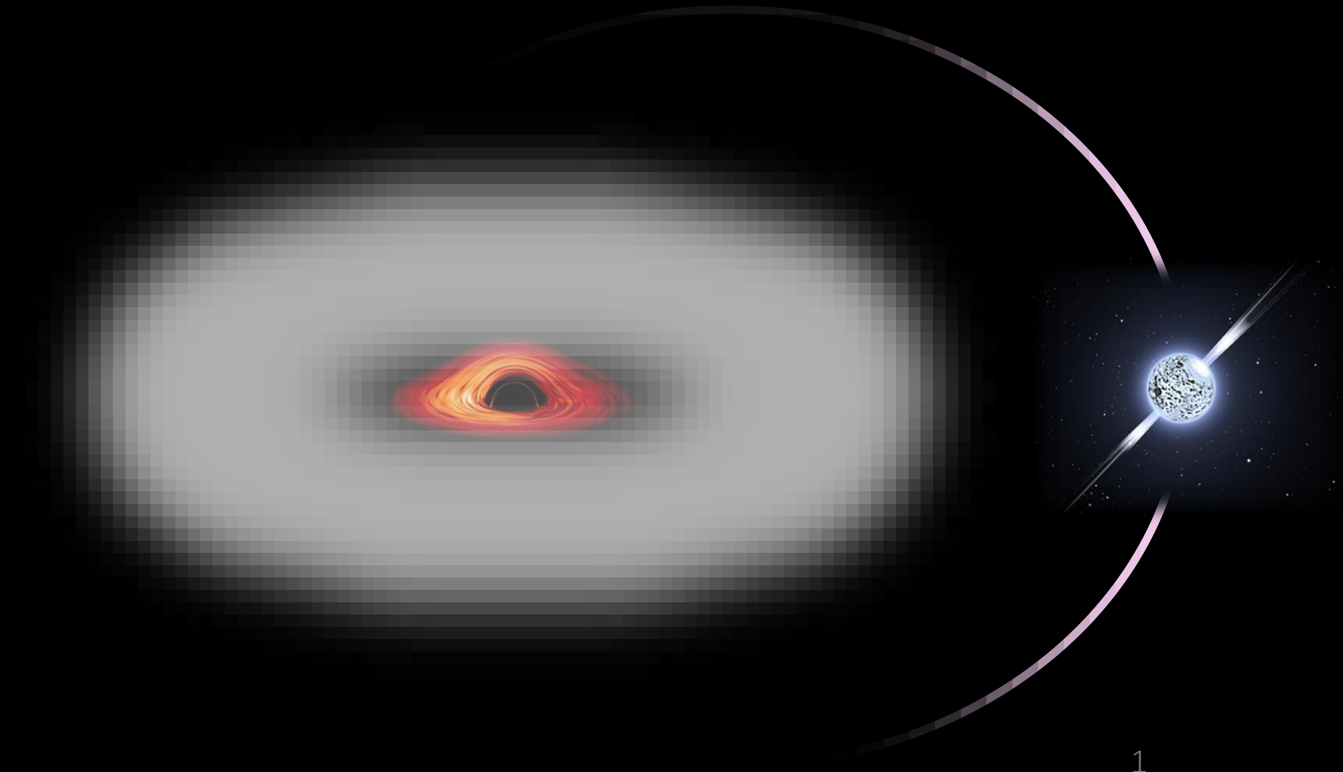
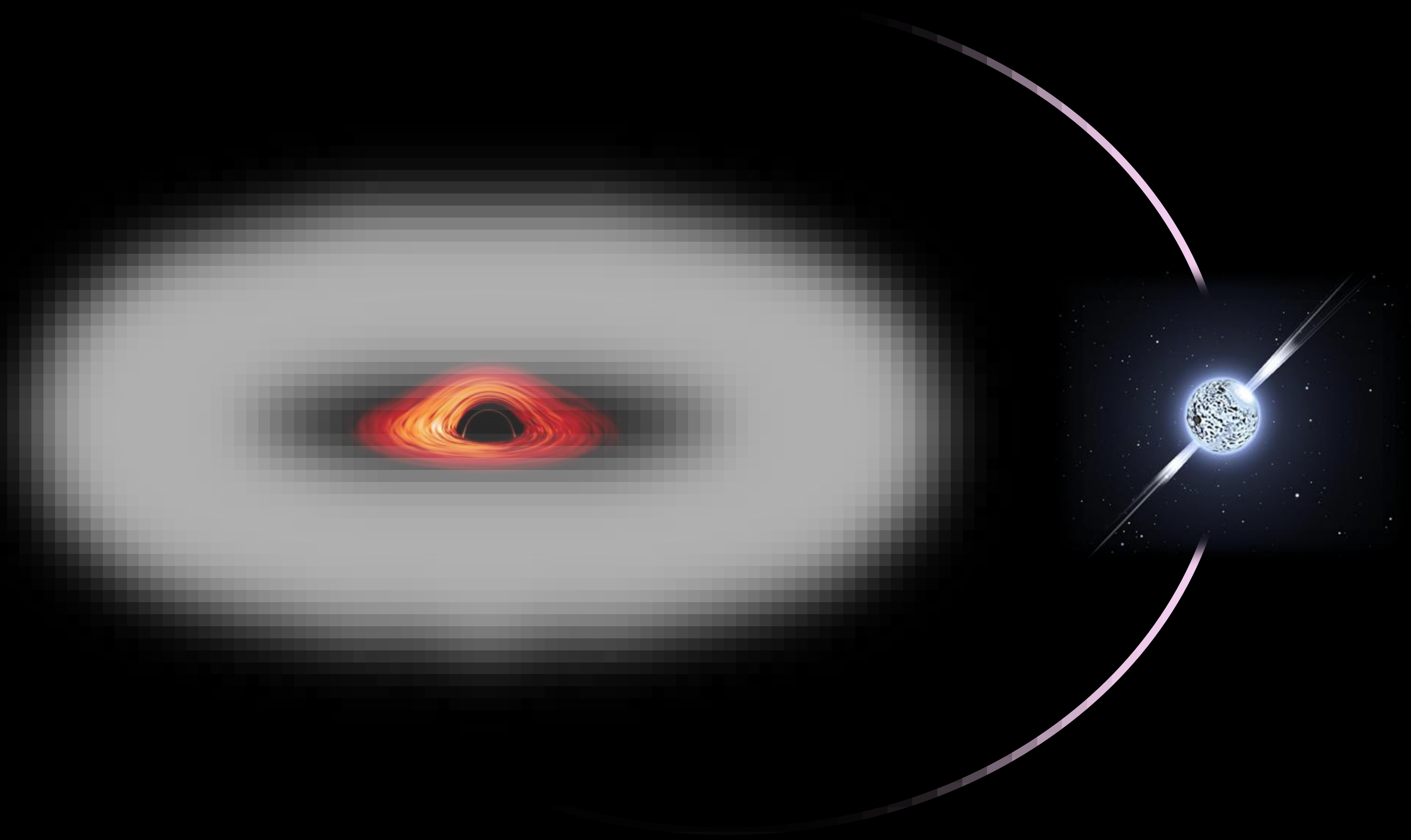
Based on:
PhysRevD.106.043002
PhysRevD.109.024059
ApJ 981 165

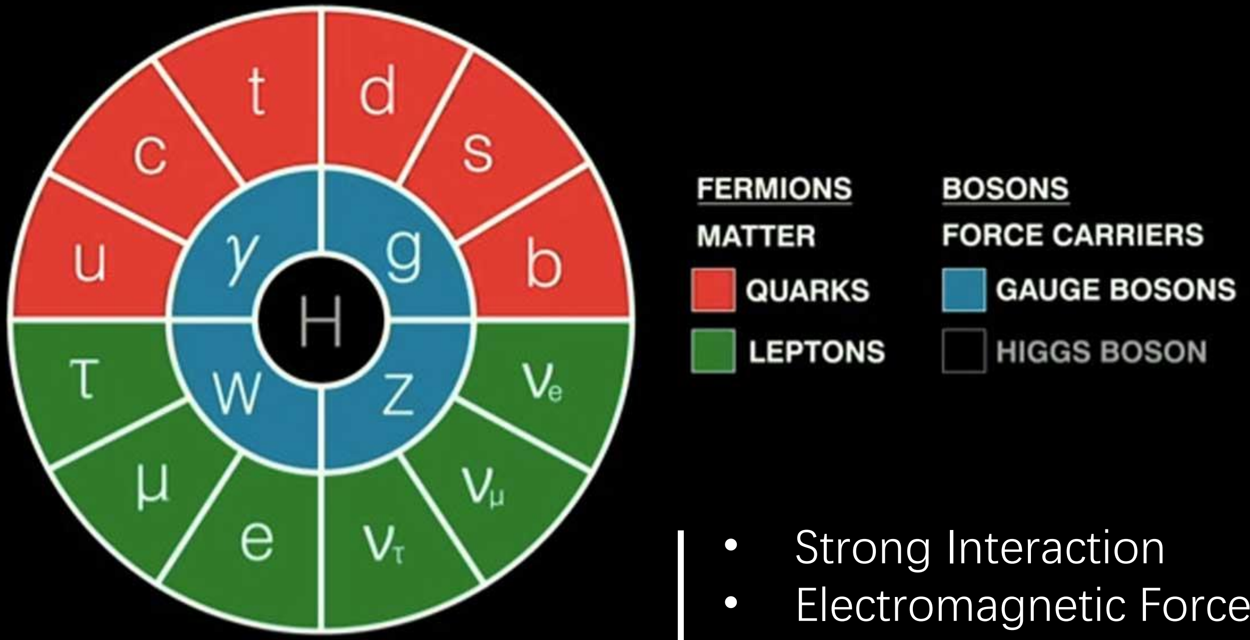


Outline

- Black Hole Superradiance and Gravitational Atom
- Off Resonance Termination of Superradiance
- Statistical Test on Superradiant Termination
- Summary



Standard Model?



- Strong Interaction
- Electromagnetic Force
- Weak Interaction
- (Excluding Gravity)

[https://factmyth.com/the-standard-model-of-particle-physics-explained/]

Unsolved mysteries
Beyond Standard Model (BSM)

- Galaxy rotation curves
- Galaxy clusters
- Gravitational lensing
- …

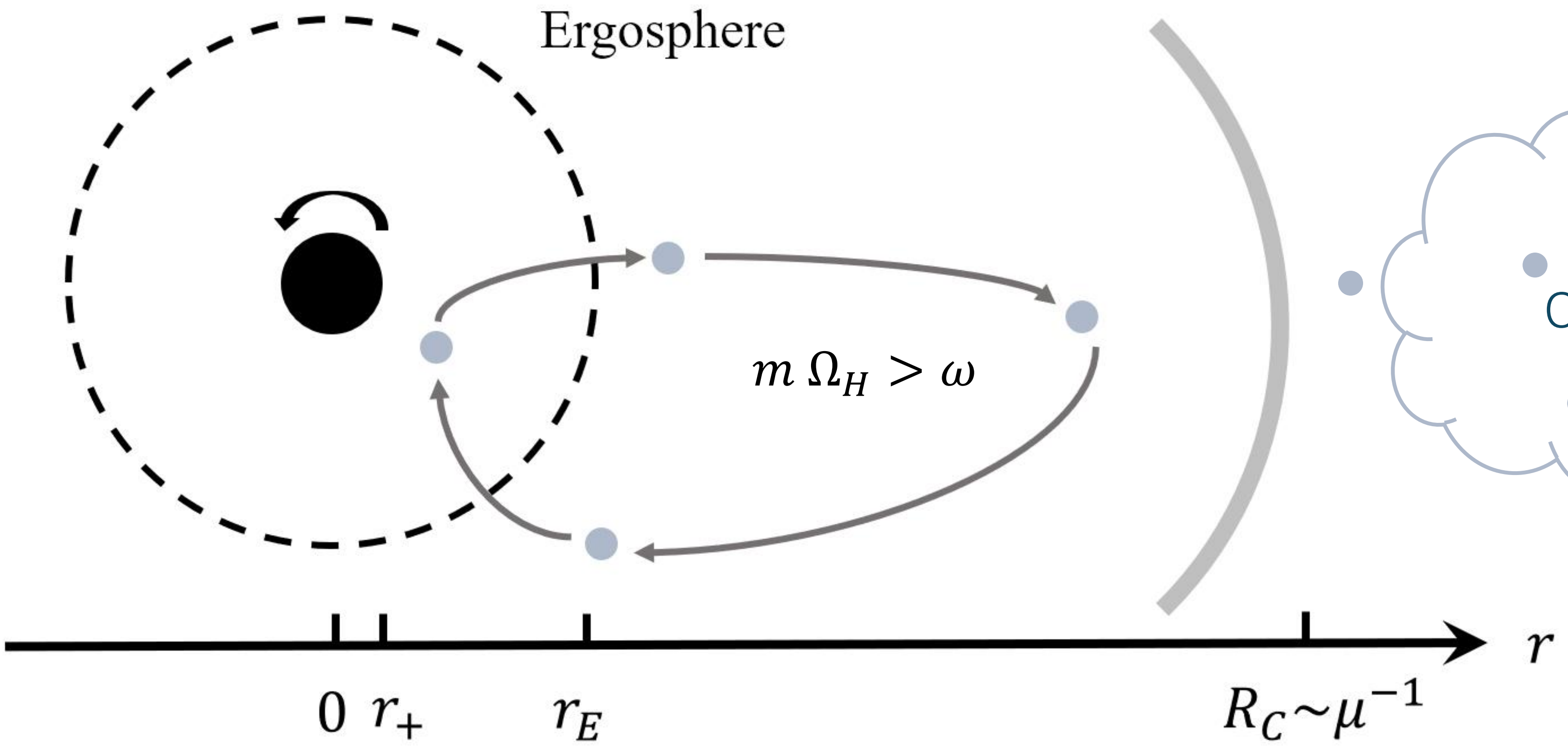
Dark Matter!

Colliders?

🤔 BH is a good tool to test BSM particle

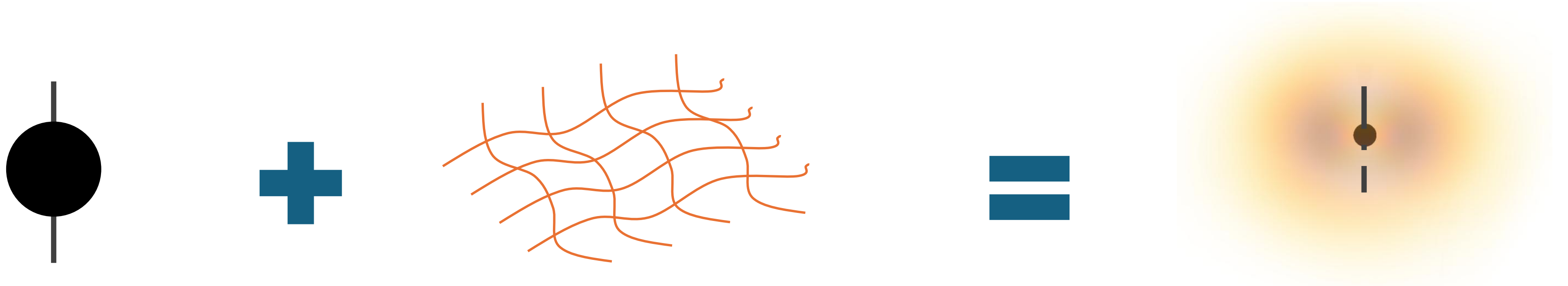
Superradiant Instability

[Press & Teukolsky, 1972]
[Damour et al. 1976]



- Superradiant instability ⇨ Kerr BH grows a ultralight bosonic cloud

BH as a GA



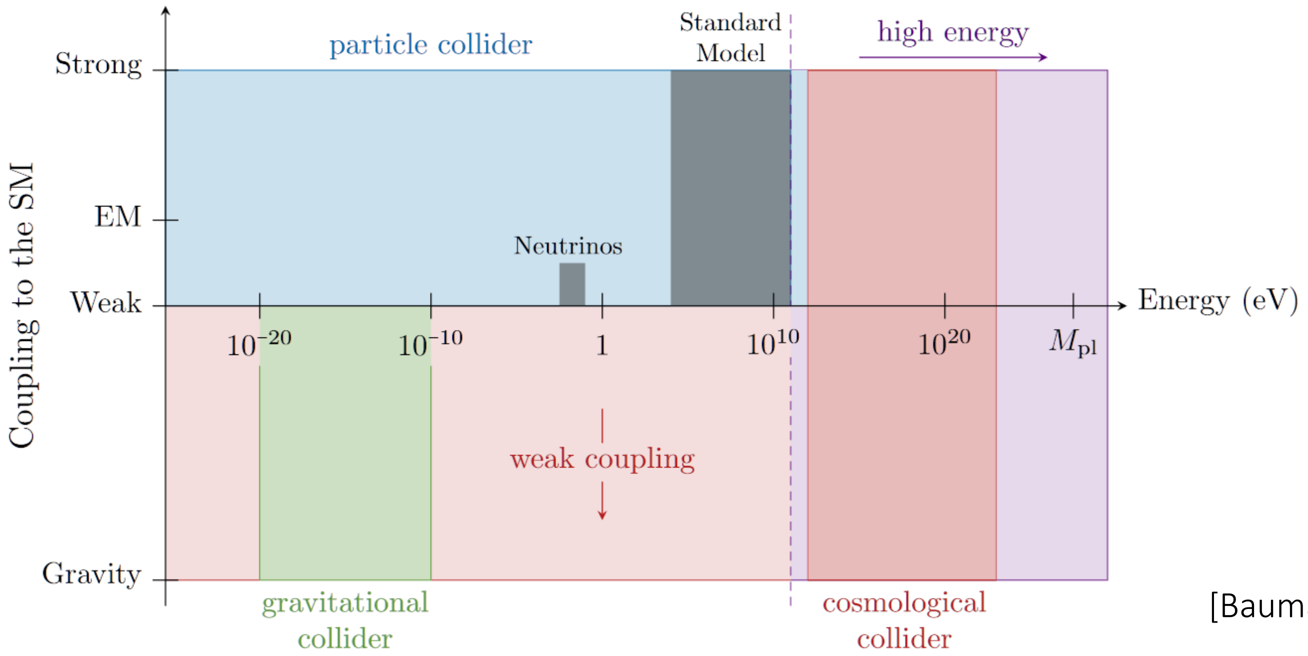
Kerr BH of mass M | Bosonic field of mass μ | Gravitational Atom

$$\mu^{-1} > GM$$

Compton wavelength | Black hole radius

BH

Ke



[Baumann et al, 2019]

$$\mu^{-1} > GM$$

Compton wavelength

Black hole radius

$$M_B \sim [10 M_\odot, 10^{10} M_\odot]$$

BH as a GA

KG in Kerr:

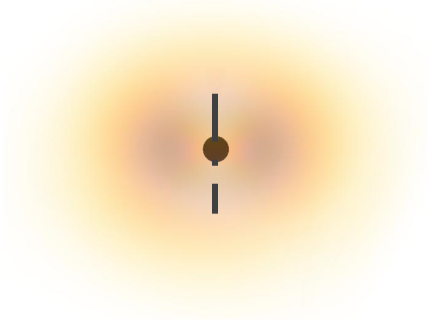
$$\left(g^{\alpha\beta}\,\nabla_\alpha\,\nabla_\beta - \mu^2\right)\Phi = 0$$

Gravitational fine structure constant

$$\alpha \equiv GM\mu \sim \frac{R_S}{\lambda_C} < 1$$

Factoring out the rest mass

$$\Phi \equiv \frac{1}{\sqrt{2\mu}} e^{-i\mu t}\psi + \text{c.c.}$$



Gravitational Atom

Hydrogen-like Schrodinger eq

$$i\partial_t \psi(t,\mathbf{r}) = H_0 \psi(t,\mathbf{r})\ ,\ H_0 \equiv -\frac{1}{2\mu}\partial_{\mathbf{r}}^2 + V(r)$$

with $V(r) = -\frac{\alpha}{r} + \mathcal{O}(\alpha^2)$

[Press & Teukolsky, 1972]
[Damour et al., 1976]
[Detweiler, 1980]
[Baumann et al, 2019, 2020]

GA in a nutshell

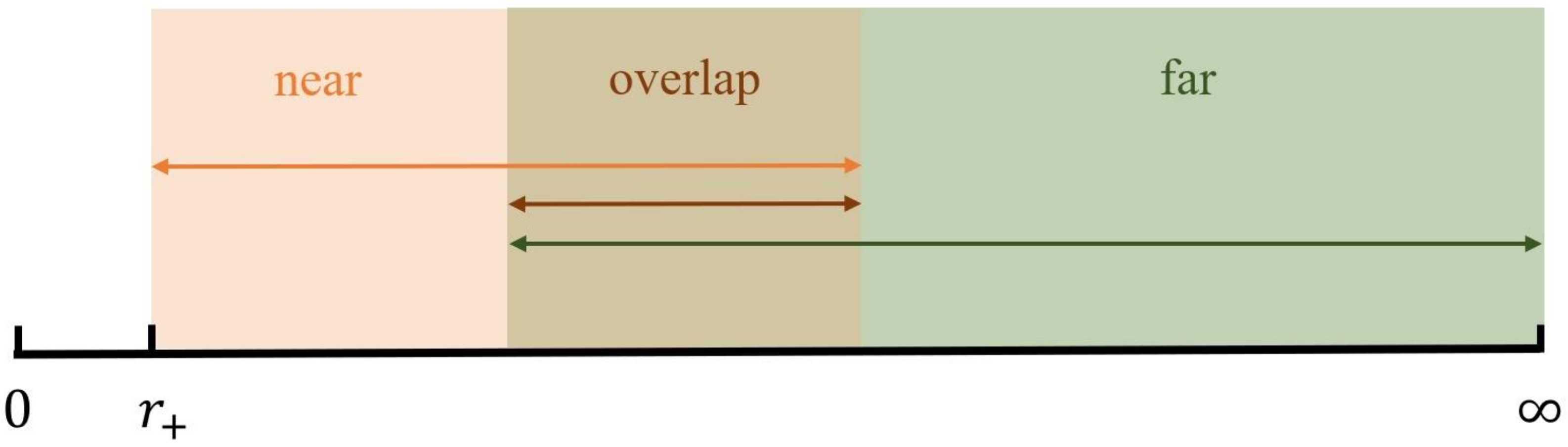
[Press & Teukolsky, 1972]
[Damour et al., 1976]
[Detweiler, 1980]
[Baumann et al, 2019, 2020]

- With decaying B.C. at infinity
- With in-going B.C. at the BH outer horizon

Solutions:

$|\psi_{nlm}\rangle$ with $\omega_{nlm} = E_{nlm} + i\Gamma_{nlm}$ ← By in-going B.C.

Determined by near far formalism :



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- With decaying B.C. at infinity
- With in-going B.C. at the BH outer horizon

Solutions:

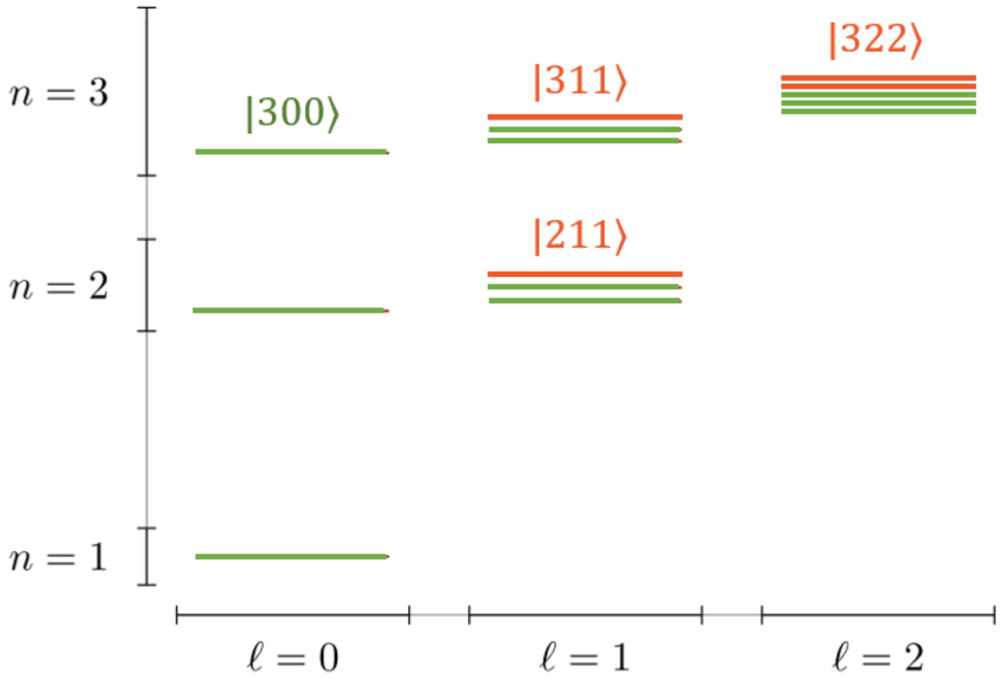
$|\psi_{nlm}\rangle$ with $\omega_{nlm} = E_{nlm} + i\Gamma_{nlm}$ ← By in-going B.C.

$$E_{nlm} = \mu\left(1 - \frac{\alpha^2}{2n^2} + \alpha^4 A(n,l) + \alpha^5 \tilde{a} m\, B(n,l) + \cdots\right)$$

Rest mass   Bohr   Fine   Hyperfine

$$\Gamma_{nlm} \propto (m\Omega_H - \mu)\alpha^{4l+5} \begin{cases} > 0 & \text{Superradiance} \\ < 0 & \text{Absorption} \end{cases}$$

$$\psi_{nlm} \sim e^{-i\omega_{nlm}t} \sim e^{\Gamma_{nlm}t}$$



GA in a nutshell

[Press & Teukolsky, 1972]
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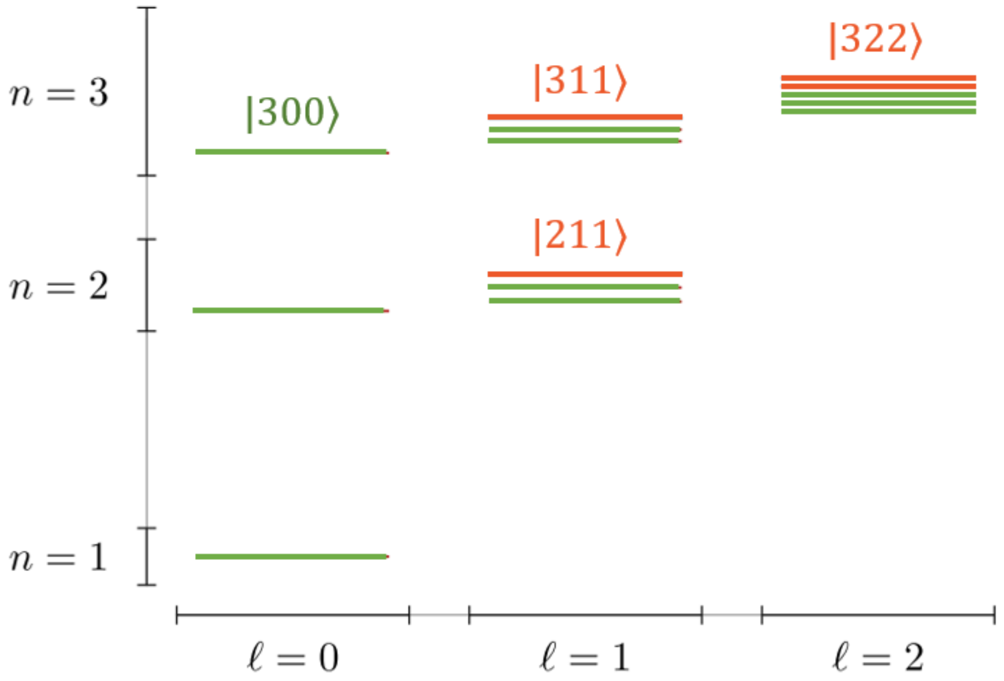
[Baumann et al, 2019, 2020]

$$\Gamma_{nlm} \propto (m\Omega_H - \mu)\alpha^{4l+5}$$ with $$\alpha \equiv GM\mu \sim \frac{R_S}{\lambda_C} < 1$$

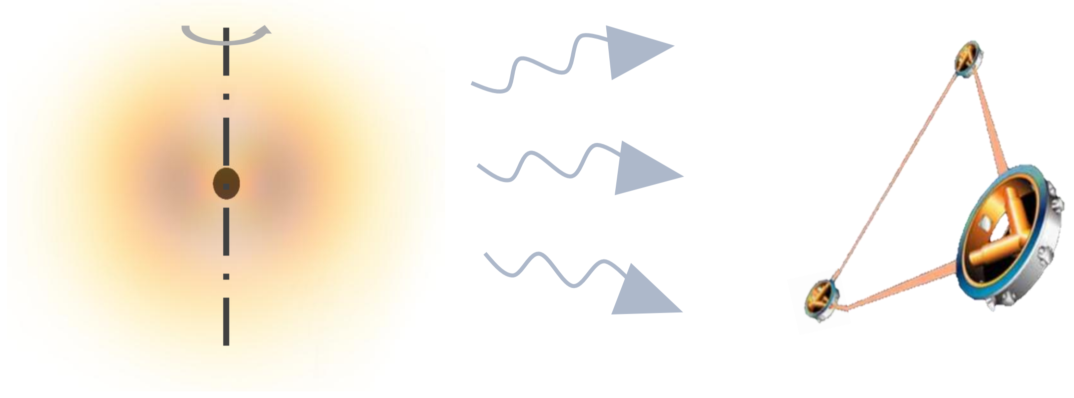
- From $m\Omega_H - \mu$
  - Cloud saturated when spin is small enough.
  - $m \leq 0$ mode always absorptive.
  - Lower m modes need higher spin.
- From α^{4l+5} :
  - The lower l modes grow much quicker.
  - $|211\rangle$ mode will dominant first if spin is enough.



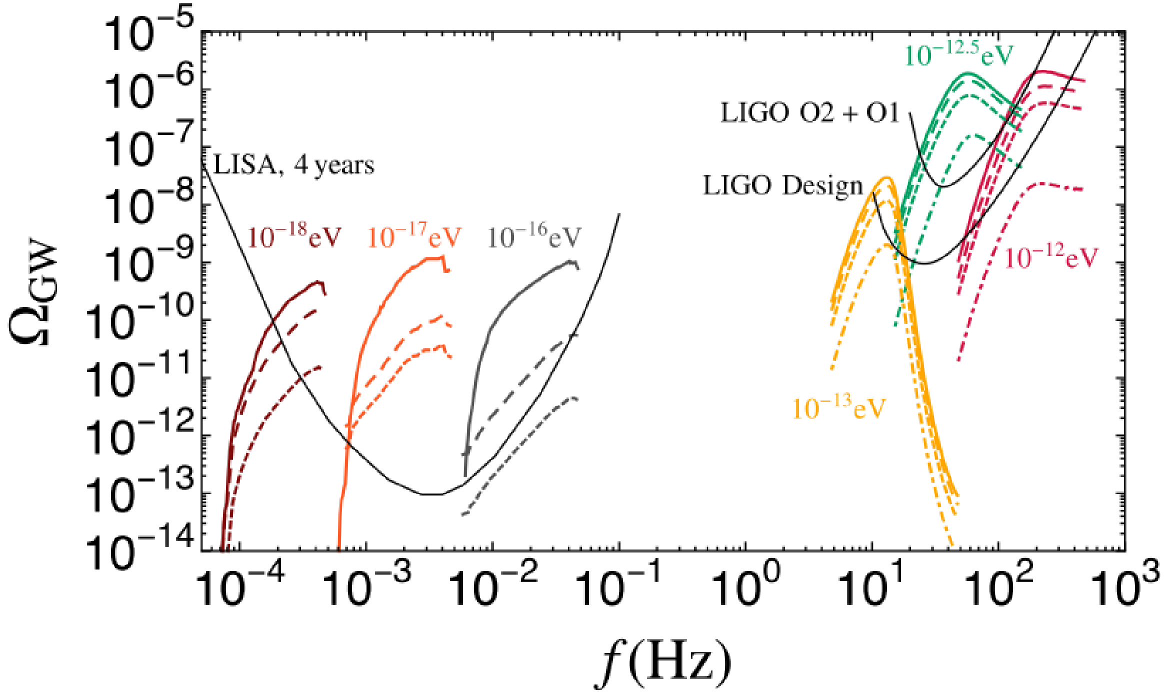
🤔 What phenomena does Gravitational Atom have?

GA phenomenology in isolation

- Near-monochromatic GW



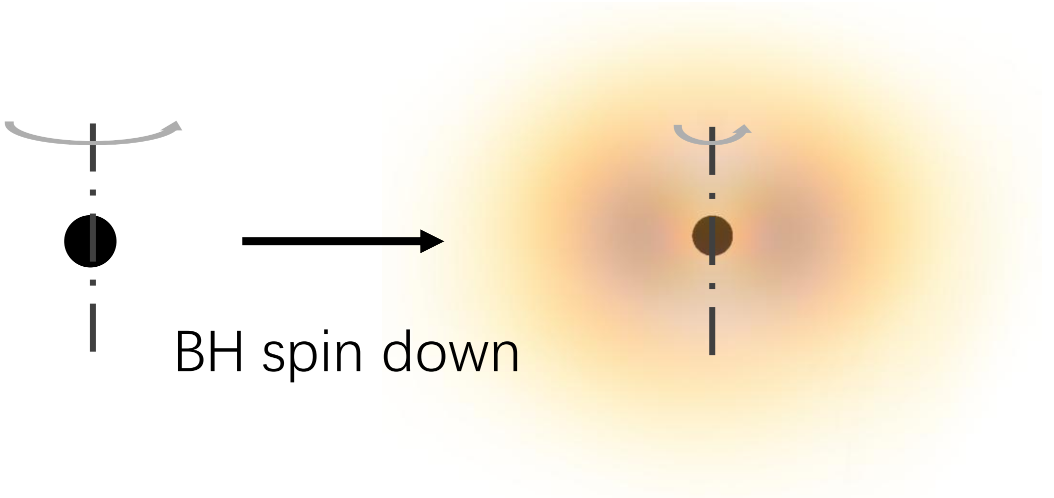
$$f_{\rm GW} \sim \omega_R/\pi \sim 5\,{\rm kHz}\left(\frac{\mu\hbar}{10^{-11}{\rm eV}}\right)$$

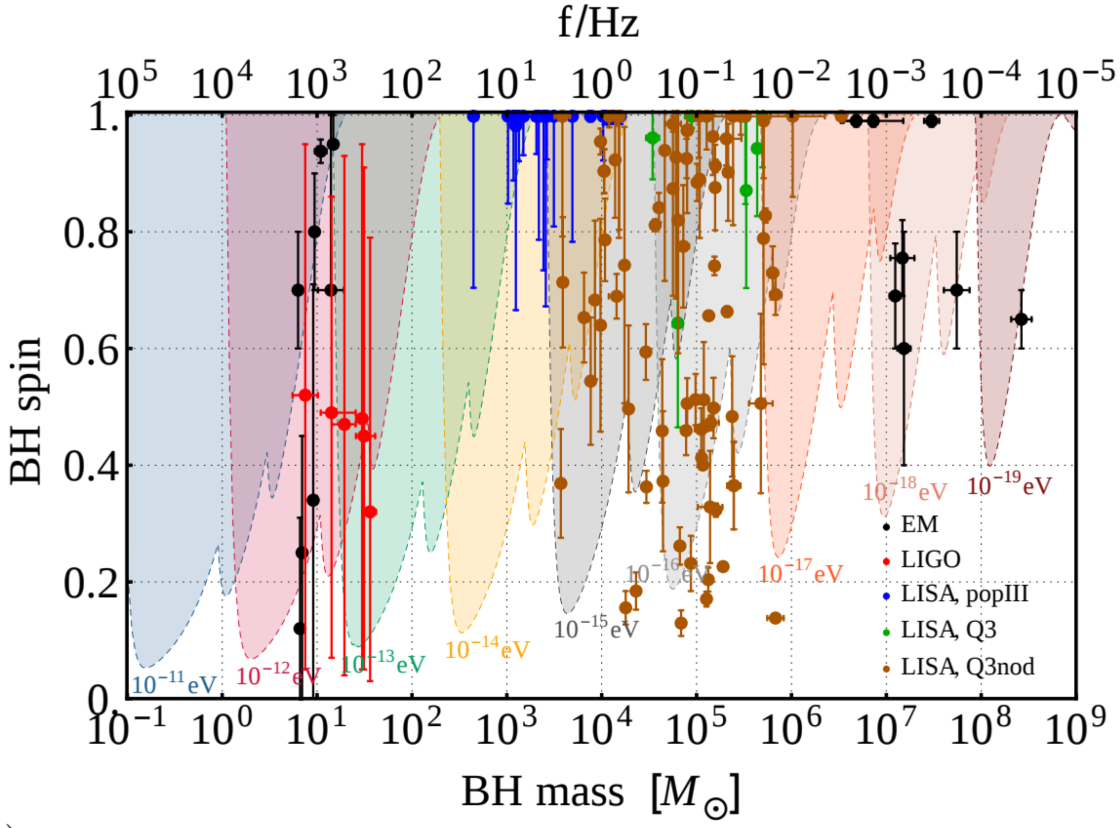


[Brito et al., 2017]

GA phenomenology in isolation

- Spin cutoff by superradiance





$$m\Omega_H \downarrow > \omega \sim \mu \quad \Rightarrow \quad \frac{a}{M} = \frac{4m(M\omega)}{m^2 + 4(M\omega)^2} = \frac{4\alpha}{m} + \mathcal{O}(\alpha^3)$$

Spin at saturation

[Brito et al., 2017]

GA phenomenology in binaries

[Baumann et al, 2019]
[Baumann et al, 2020]
[Baumann et al, 2022]

- Atomic resonant transitions

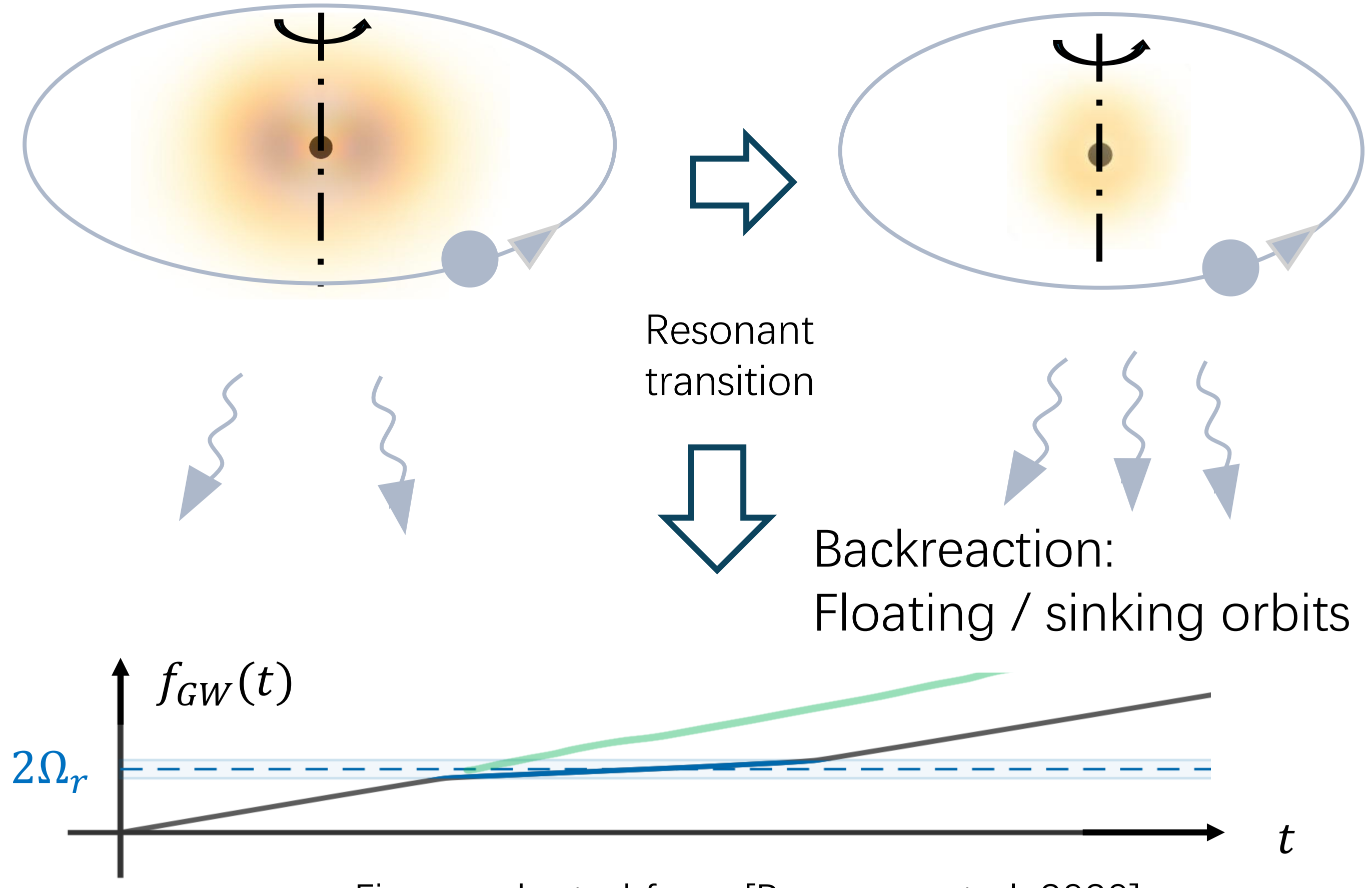
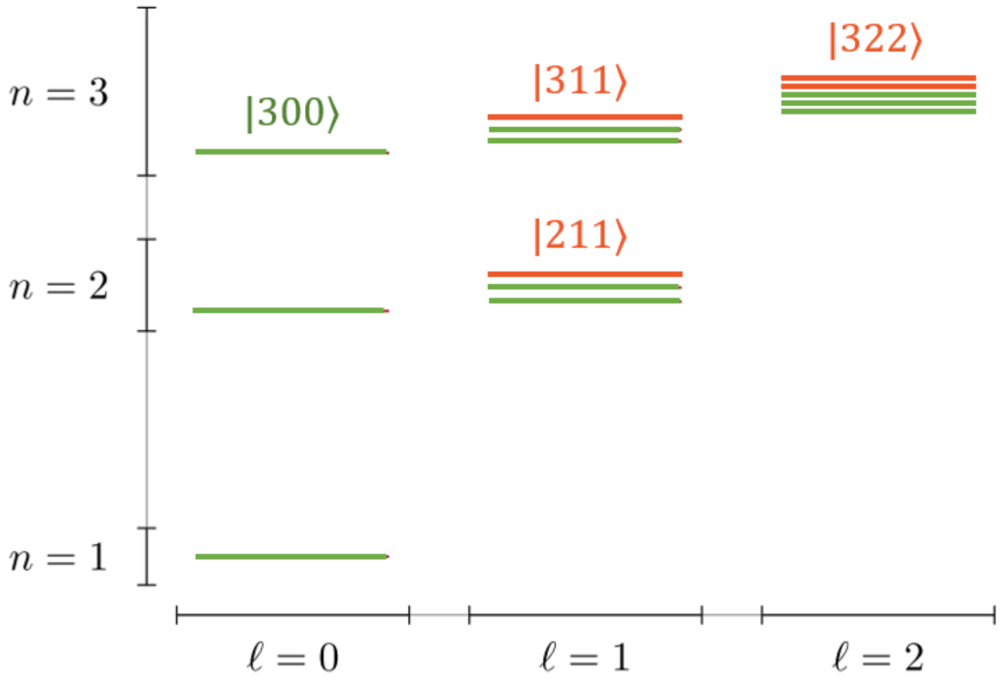


Figure adapted from [Baumann et al, 2020]



🤔 A hidden assumption: there ***is*** a boson cloud

Effect of a binary companion: State mixture

GA in isolation

$|\psi_{322}\rangle$

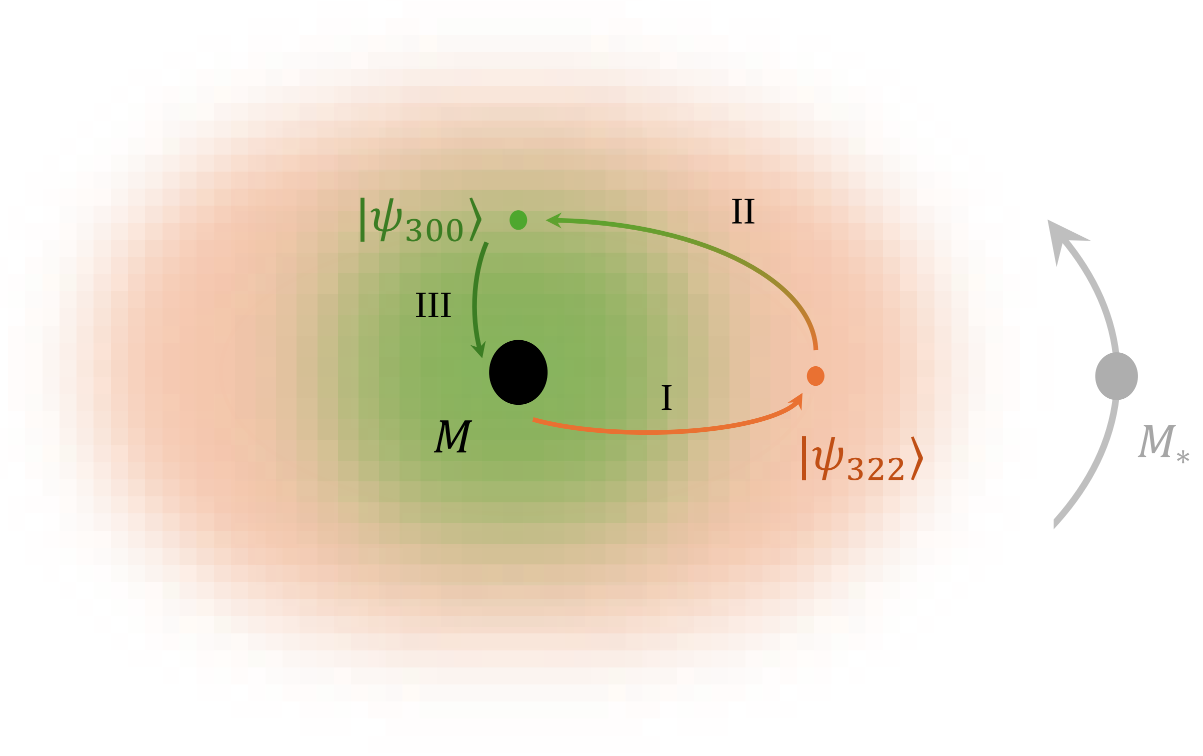
$$\Gamma_{322} \simeq 3 \times 10^{-13}\ \mathrm{s}^{-1} \left(\frac{\alpha}{0.1}\right)^{13} \left(\frac{M}{10M_\odot}\right)^{-1}$$

$|\psi_{300}\rangle$

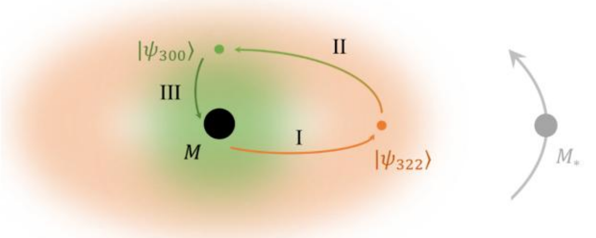
$$\Gamma_{300} \simeq -3 \times 10^{-3}\ \mathrm{s}^{-1} \left(\frac{\alpha}{0.1}\right)^{5} \left(\frac{M}{10M_\odot}\right)^{-1}$$

GA in a binary

[Tong, Wang & **Zhu**, 2022]



Superradiance Termination (ST)



[Tong, Wang & **Zhu**, 2022]

- GA isolated: $i\partial_t \psi(t,\mathbf{r}) = H_0 \psi(t,\mathbf{r})\ ,\ H_0 \equiv -\frac{1}{2\mu}\partial_{\mathbf{r}}^2 + V(r)$
- GA in a binary: $i\partial_t \psi(t,r) = H\psi(t,r)$, with $H = H_0 + V_*(t)$

$$V_* = -\alpha q \sum_{l_* \geqslant 2} \sum_{|m_*| \leqslant l_*} \mathcal{E}_{l_* m_*}(\iota_*, \varphi_*)\, Y_{l_* m_*}(\theta, \phi) \times \left(\frac{r^{l_*}}{R_*^{l_*+1}} \Theta(R_* - r) + \frac{R_*^{l_*}}{r_*^{l_*+1}} \Theta(r - R_*) \right)$$

- E.g., consider a two-state subspace $\{|1\rangle, |2\rangle\}$

  $|1\rangle$ is superradiant with $\Gamma_1 > 0$, (e.g., $|322\rangle$)
  $|2\rangle$ is absorptive with $\Gamma_2 < 0$, (e.g., $|300\rangle$)

Tidal perturbation

"Free"cloud

$$H = \begin{pmatrix} E_1 + i\Gamma_1 & 0 \\ 0 & E_2 + i\Gamma_2 \end{pmatrix} + \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} = \begin{pmatrix} \bar{E}_1 + i\Gamma_1 & \eta^* \\ \eta & \bar{E}_2 + i\Gamma_2 \end{pmatrix}$$

$$V_{ij} \equiv \langle i|V_*|j\rangle$$

$$\bar{E}_i \equiv E_i + V_{ii}$$

$$\eta \equiv V_{21}$$

Superradiance termination

[Tong, Wang & **Zhu**, 2022]

- Schrodinger eq: $i\partial_t|\psi\rangle = \begin{pmatrix} \bar{E}_1 + i\Gamma_1 & \eta^* \\ \eta & \bar{E}_2 + i\Gamma_2 \end{pmatrix} |\psi\rangle$

Eigenfrequencies: $\lambda_\pm \simeq \begin{cases} \bar{E}_1 + \frac{|\eta|^2}{\bar{E}_1 - \bar{E}_2} + i\left[\Gamma_1 - \frac{\Gamma_1 - \Gamma_2}{(\bar{E}_1 - \bar{E}_2)^2}|\eta|^2\right], + \\ \bar{E}_2 + \frac{|\eta|^2}{\bar{E}_2 - \bar{E}_1} + i\left[\Gamma_2 - \frac{\Gamma_2 - \Gamma_1}{(\bar{E}_1 - \bar{E}_2)^2}|\eta|^2\right], - \end{cases}$

$\Delta\Gamma_1$: Correction to the superradiance rate

- Significant correction if the binary is close:

At a binary separation $R_* = 10^5 M$

$$\Gamma_{322} \simeq 3 \times 10^{-13}\ \mathrm{s}^{-1} \left(\frac{\alpha}{0.1}\right)^{13} \left(\frac{M}{10M_\odot}\right)^{-1}$$

$$\Delta\Gamma_{322} \simeq -7 \times 10^3 \frac{q^2}{\alpha^{10}} \frac{M^5}{R_*^6}$$

$$\simeq -0.6 \times 10^{-13}\ \mathrm{s}^{-1} \left(\frac{\alpha}{0.1}\right)^{-10} \left(\frac{q}{0.2}\right)^2 \left(\frac{M}{10M_\odot}\right)^{-1}$$

20% reduction!

Superradiance termination

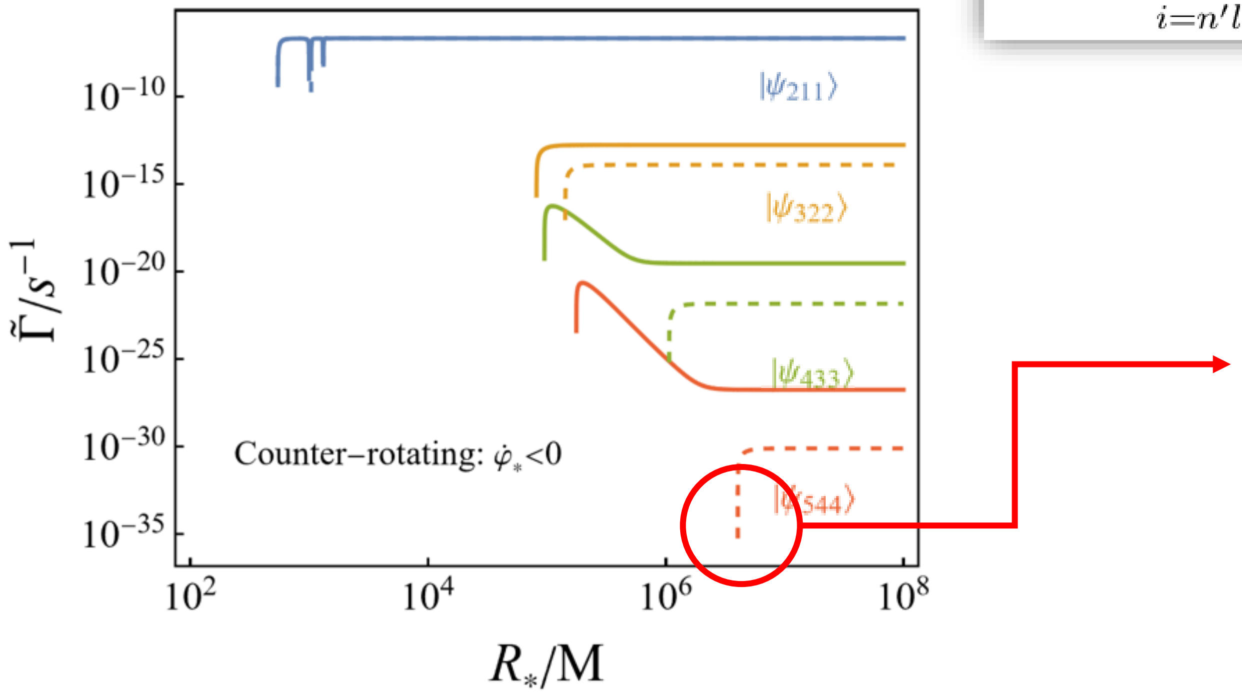
[Tong, Wang & **Zhu**, 2022]

- More generally, considering multiple states and rotation effects

Corrected superradiance rate $\tilde{\Gamma}_1 = \Gamma_1 + \Delta\Gamma_1$

$$\Delta\Gamma_1 \simeq \sum_{i=n'l'm'} \frac{\Gamma_1 - \Gamma_i}{[\bar{E}_1 - \bar{E}_i - (m_1 - m_i)\dot{\phi}_*(R_*)]^2} |\eta_{1i}(R_*)|^2$$

with $\eta_{ij} \equiv V_{ij} = \langle i|V_*|j\rangle$



$\tilde{\Gamma}$ drops to 0 at a finite binary separation, ***terminating*** superradiance

Mass ratio: $q = \frac{M_*}{M}$

Fine structure const: $\alpha \equiv GM\mu$

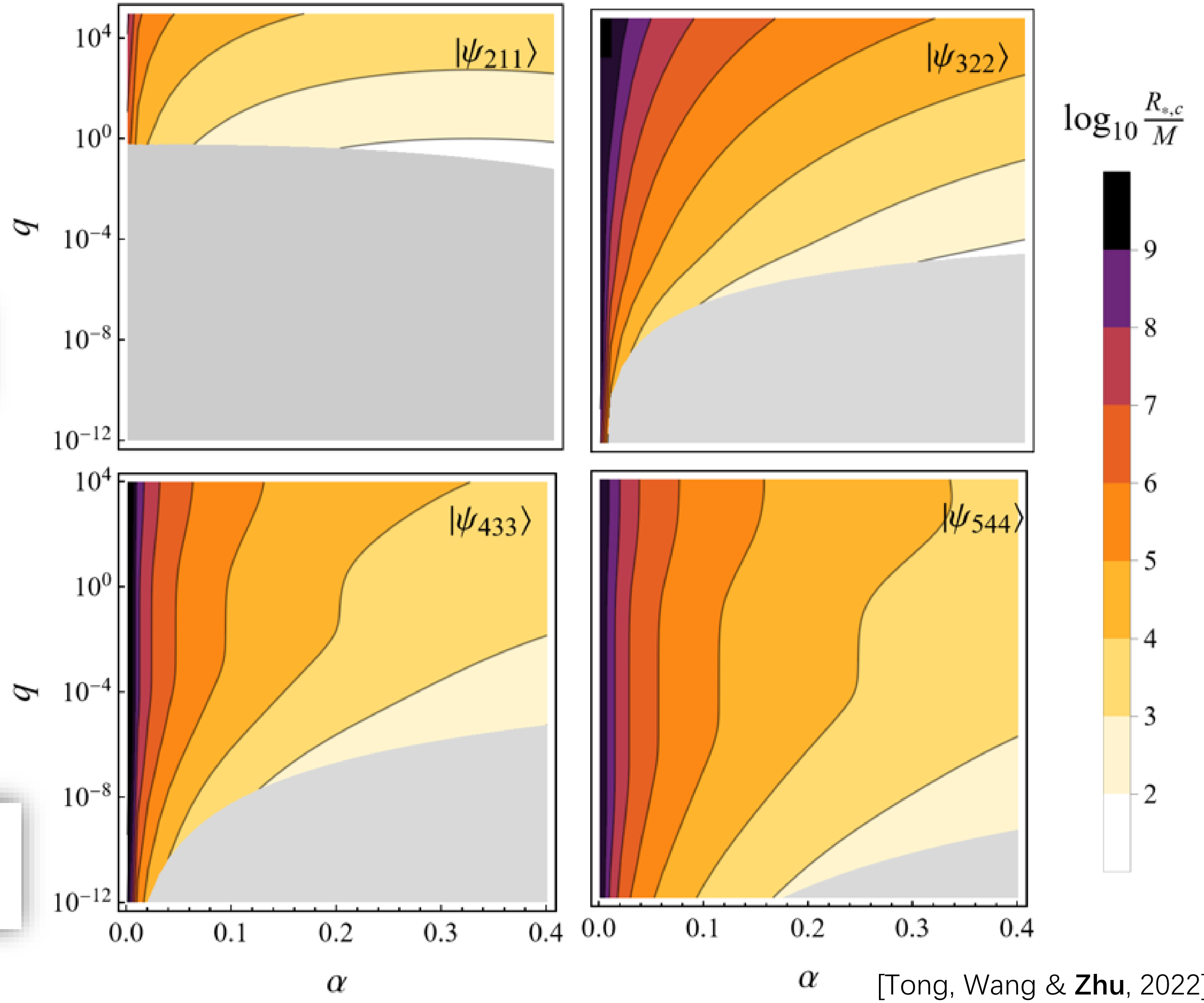
A critical distance

- The critical distance $R_{*,c}$ of $|\psi_{nlm}\rangle$ is defined as

$$\tilde{\Gamma}_{nlm}(R_{*,c}) = \Gamma_{nlm} + \Delta\Gamma_{nlm}(R_{*,c}) \equiv 0$$

- $R_{*,c}(nlm)$ is the distance below which ***no superradiance*** can happen

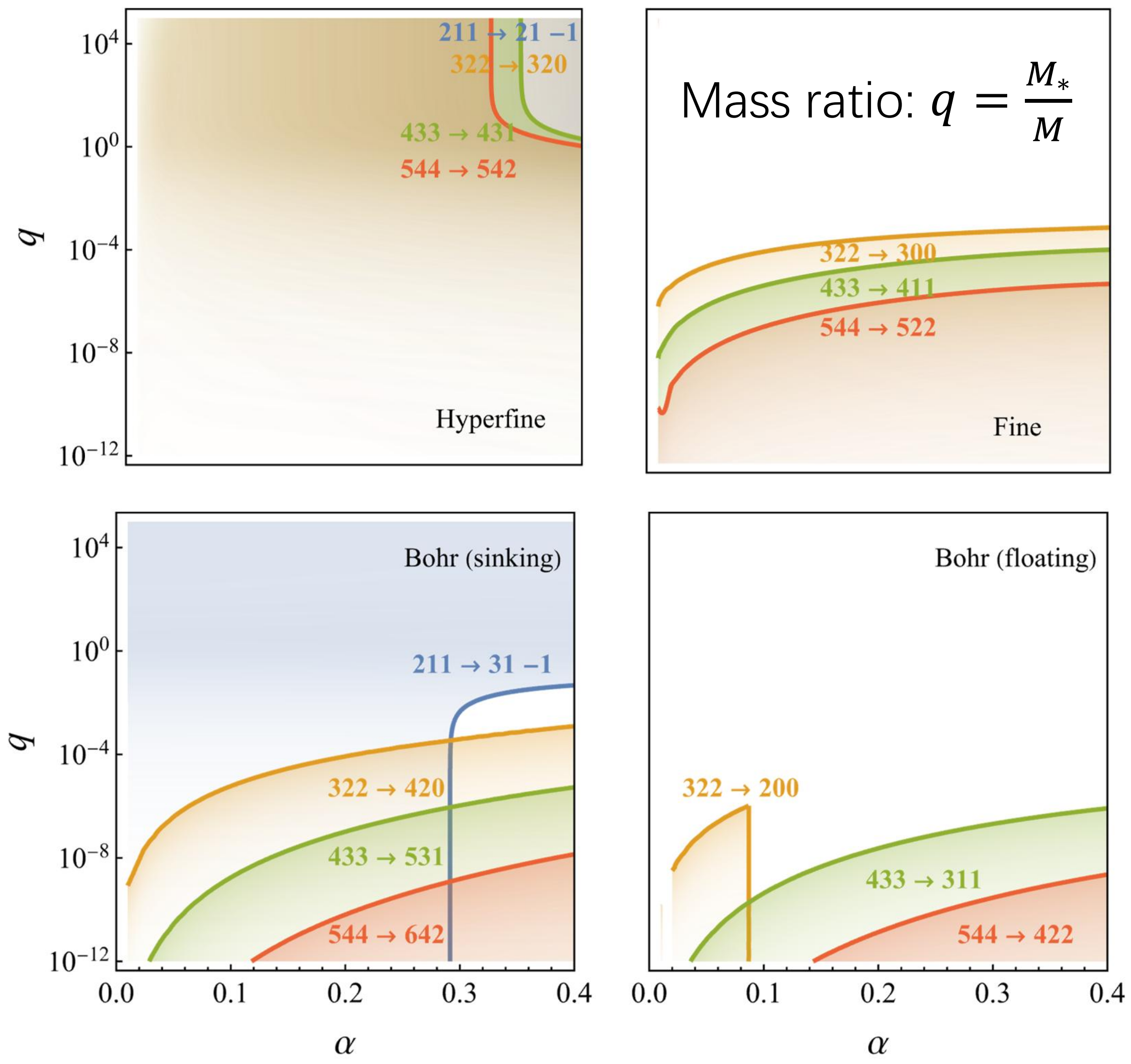
$$R_{*,c}(322) \simeq 10^6 \text{ km} \left(\frac{\alpha}{0.1}\right)^{-23/6} \left(\frac{q}{0.2}\right)^{1/3} \frac{M}{10M_\odot}$$

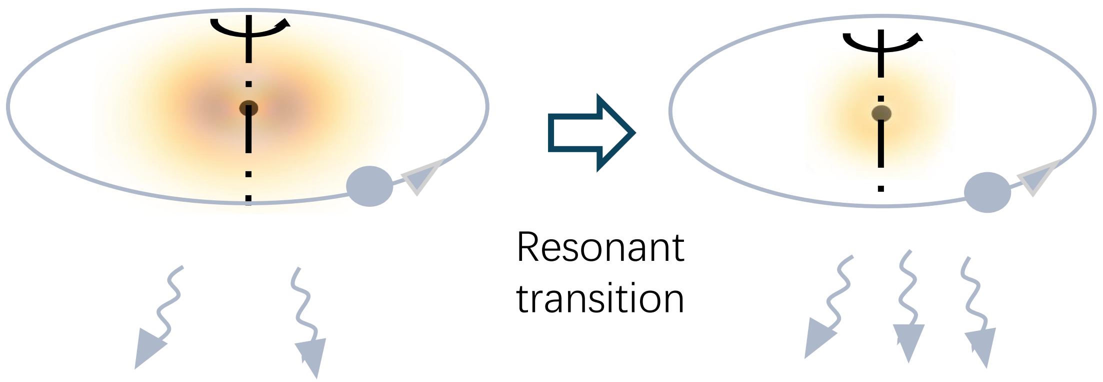


[Tong, Wang & **Zhu**, 2022]

- Superradiance is terminated below a critical binary distance
- What are the phenomenological consequences?

Consequences of ST: Impact on resonance





- Viable GCP transition requires

$$R_{*,r}(nlm \to n'l'm') > R_{*,c}(nlm)$$

[Tong, Wang & **Zhu**, 2022]

ST backreaction: Orbital flow of EMRIs ($q \ll 1$)

[Fan, Tong, Wang & **Zhu**, 2023]

- General binary orbits: $\{p(t), e(t), \iota(t)\} \cup \{S_c(t)\}$

Cloud angular momentum

Inclination angle

Eccentricity

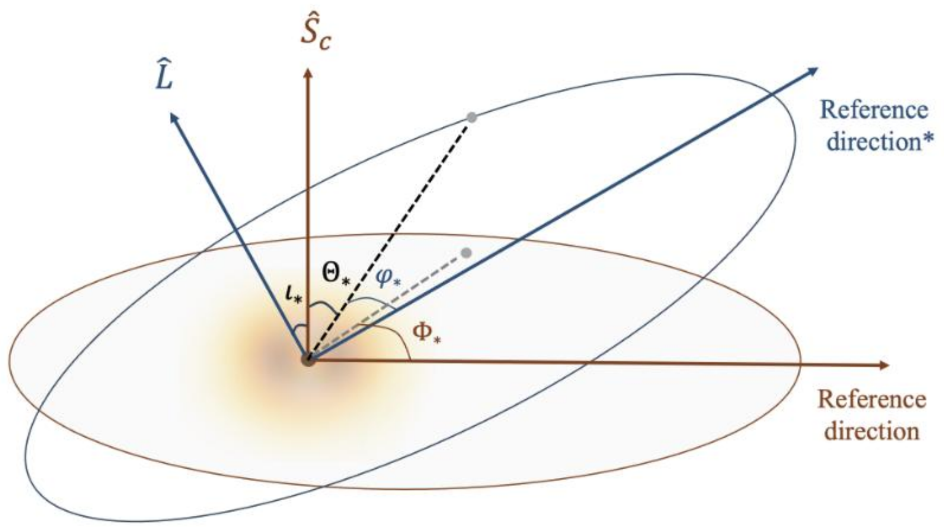
Semi-latus rectum

$$\frac{\mathrm{d}}{\mathrm{d}t}[L(t)\,\cos\iota_*(t)] = \tau_\mathrm{c} + \tau_\mathrm{bGW}\,\cos\iota_*(t)\ ,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}[L(t)\,\sin\iota_*(t)] = \tau_\mathrm{bGW}\,\sin\iota_*(t)\ .$$

$$\frac{\mathrm{d}E(t)}{\mathrm{d}t} = P_\mathrm{c} + P_\mathrm{bGW}\ .$$

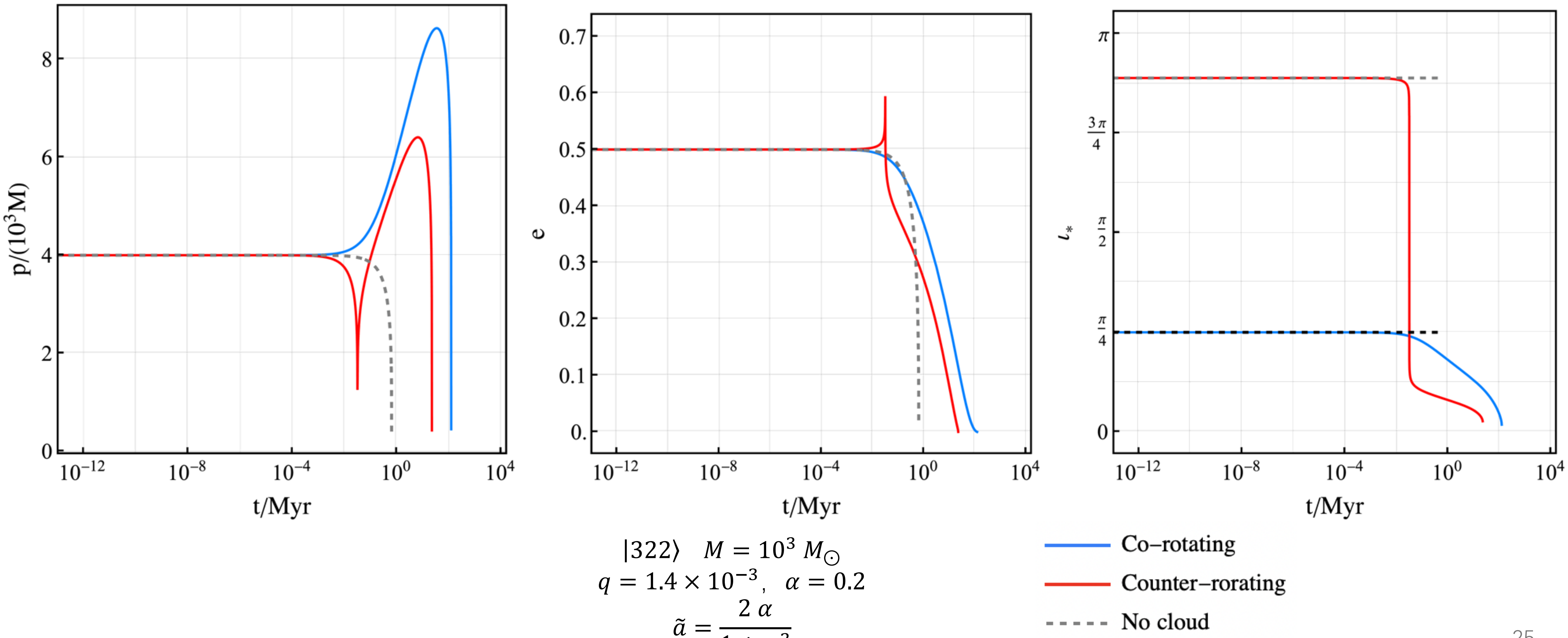
$$\frac{\mathrm{d}S_\mathrm{c}(t)}{\mathrm{d}t} = \left(\frac{\mathrm{d}S_\mathrm{c}(t)}{\mathrm{d}t}\right)_\mathrm{ST} + \left(\frac{\mathrm{d}S_\mathrm{c}(t)}{\mathrm{d}t}\right)_\mathrm{cGW}$$



ST backreaction: Orbital flow of EMRIs

[Fan, Tong, Wang & **Zhu**, 2023]

- Orbital evolution



$|322\rangle \quad M = 10^3\, M_\odot$

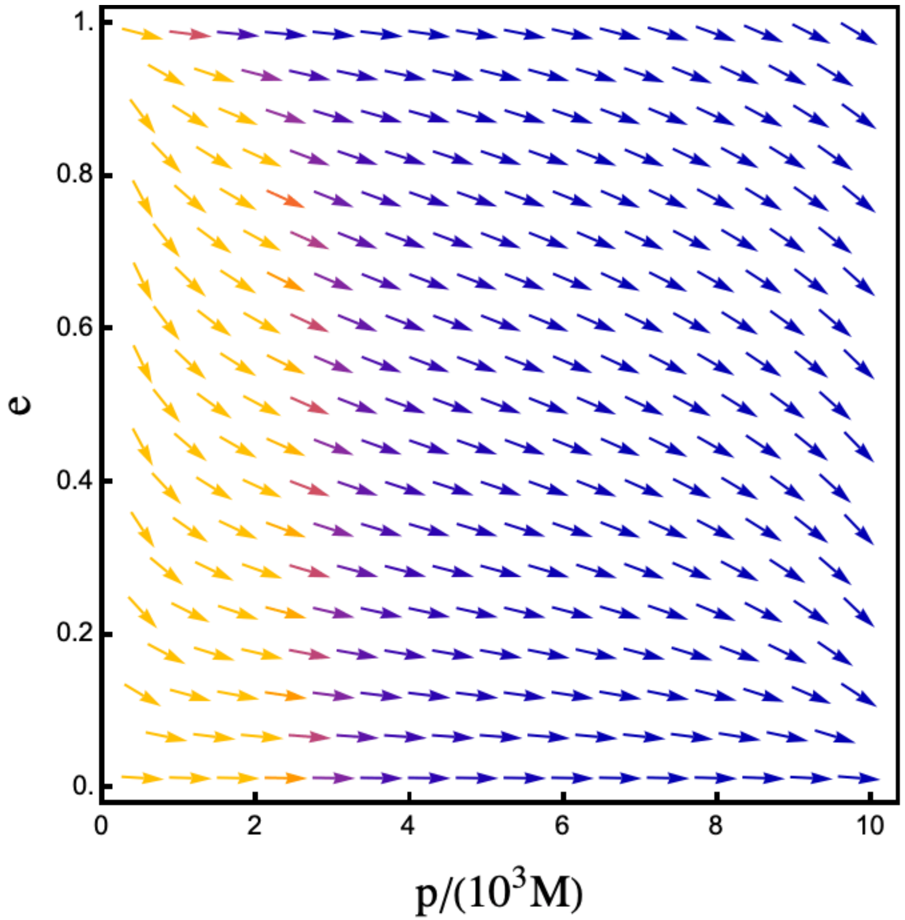
$q = 1.4 \times 10^{-3}, \quad \alpha = 0.2$

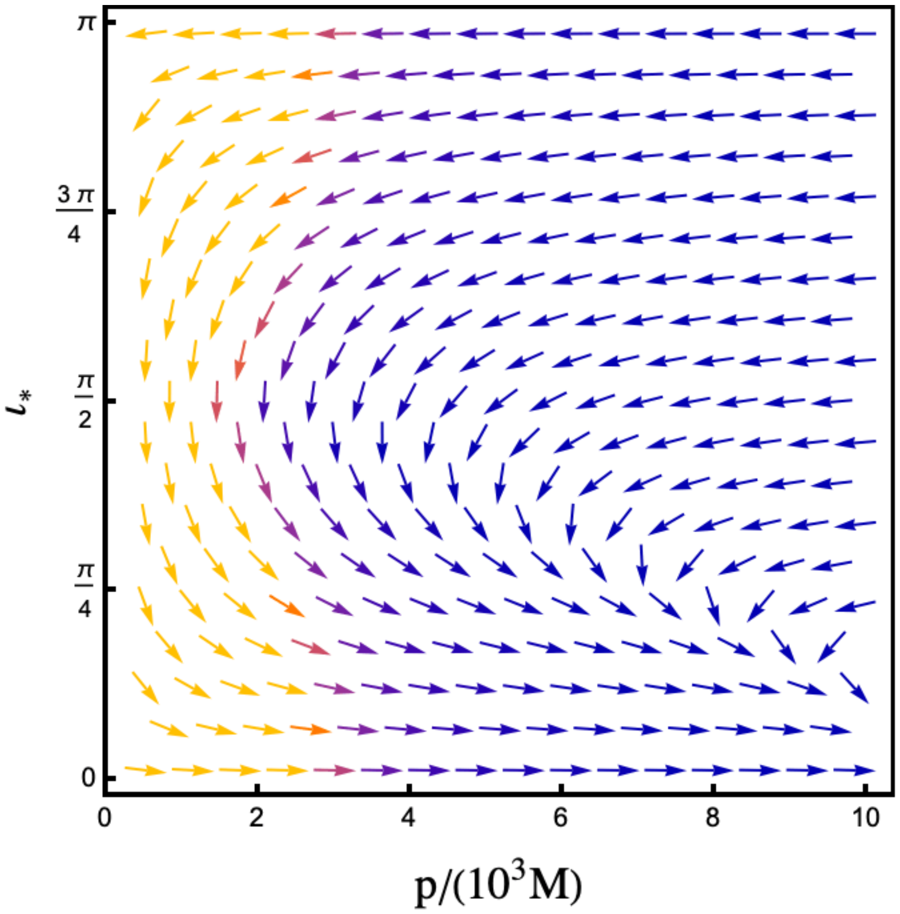
$$\tilde{a} = \frac{2\,\alpha}{1+\alpha^2}$$

ST backreaction: Orbital flow of EMRIs

[Fan, Tong, Wang & **Zhu**, 2023]

- Flow of orbital parameters





$|322\rangle$

$M = 10^3\ M_\odot$

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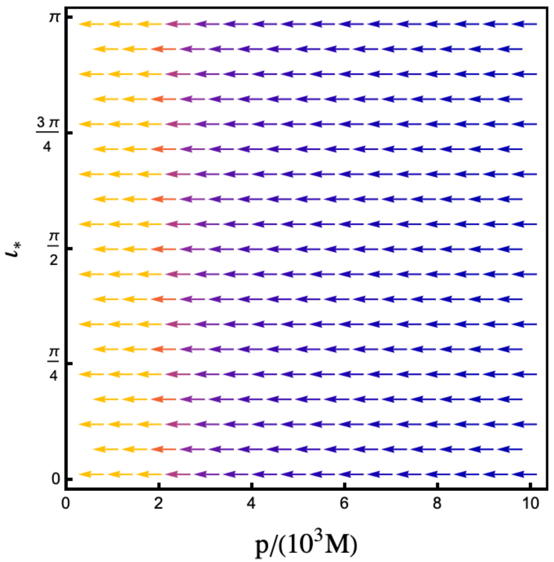
$\alpha = 0.2$

$$\tilde{a} = \frac{2\,\alpha}{1+\alpha^2}$$



How significantly does ST influence the BBH systems in reality from a statistical standpoint?

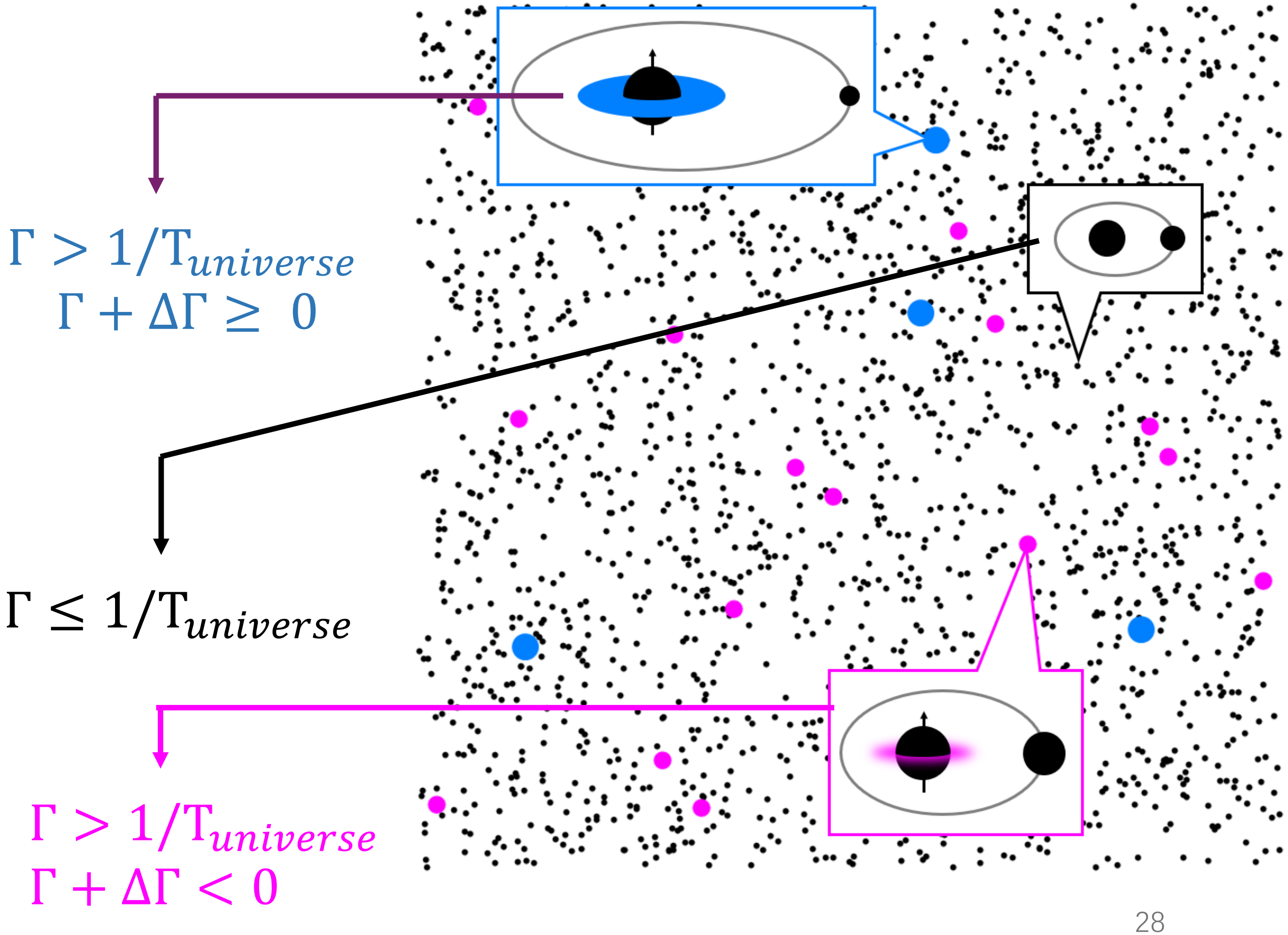
[**Zhu**, Tong, Manzoni & Ma, 2024]

ST Statistic: Survival Rate of GA

- Stellar Evolution for N-body (SEVN) for 8×10^6 BBH systems

- List of
  - Mass M_B and M_*
  - Semi-major axis a
  - Eccentricity e

- Consider
  - Inclination $\iota = 0$
  - BH spin $\tilde{a}$

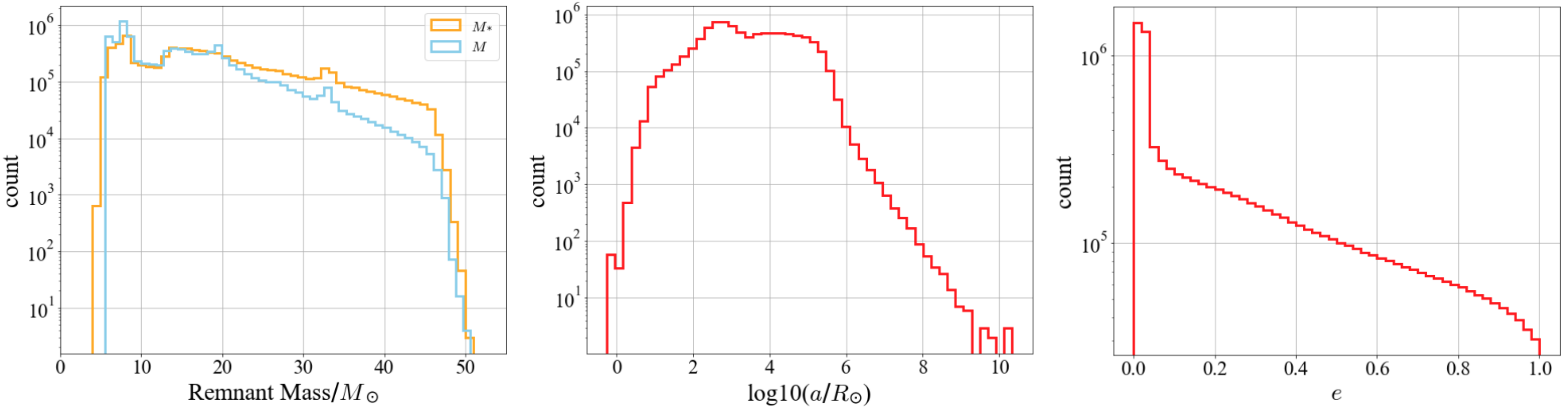


[**Zhu**, Tong, Manzoni & Ma, 2024]

ST Statistic: Survival Rate of GA

- Stellar Evolution for N-body (SEVN) for 8×10^6 BBH systems

- Initial Condition : Mass distribution, orbital period distribution, metallicity of stellar binary systems in the milky way, with uniform spin distribution.
- **Output :**



ST Statistic: Survival Rate of GA

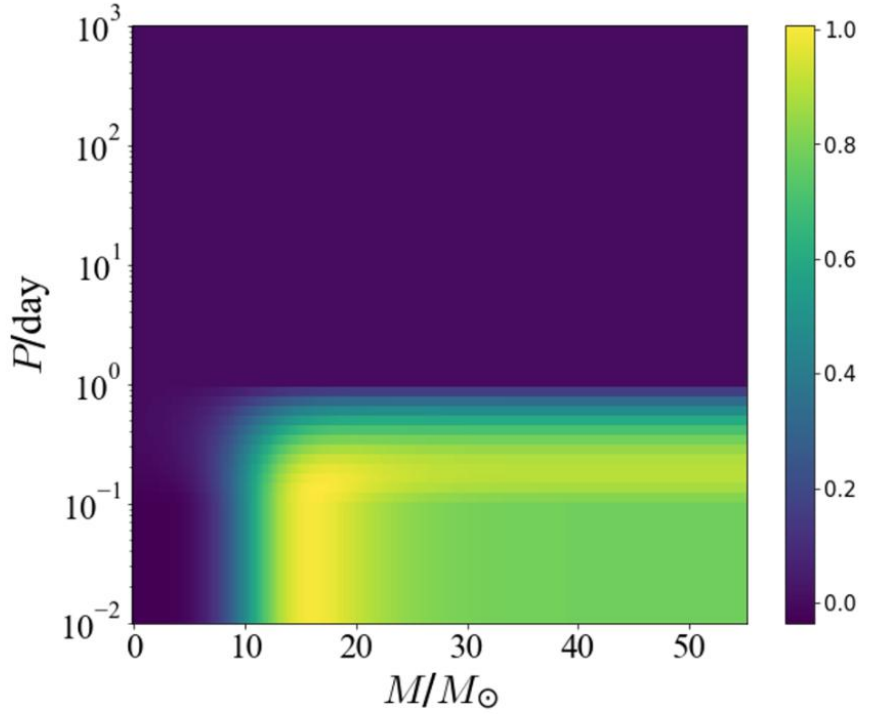
[**Zhu**, Tong, Manzoni & Ma, 2024]

- Stellar Evolution for N-body (SEVN) for 8×10^6 BBH systems

- Initial Condition : Mass distribution, orbital period distribution, metallicity of stellar binary systems in the milky way, with uniform spin distribution.

- Spin Model of BBH : **Wolf-Rayet (WR) Spin Model**
  - Primary BH has small spin.
  - Secondary BH has spin :

$$\tilde{a} = \begin{cases} f^{\alpha} \log_{10}^2\left(\frac{P}{\text{day}}\right) + f^{\beta} \log_{10}\left(\frac{P}{\text{day}}\right) & , \ 0.1 \le \frac{P}{\text{day}} \le 1 \\ 0 & , \ \frac{P}{\text{day}} > 1 \\ \tilde{a}|_{P=0.1\,\text{day}} & , \ \frac{P}{\text{day}} < 0.1 \end{cases}$$

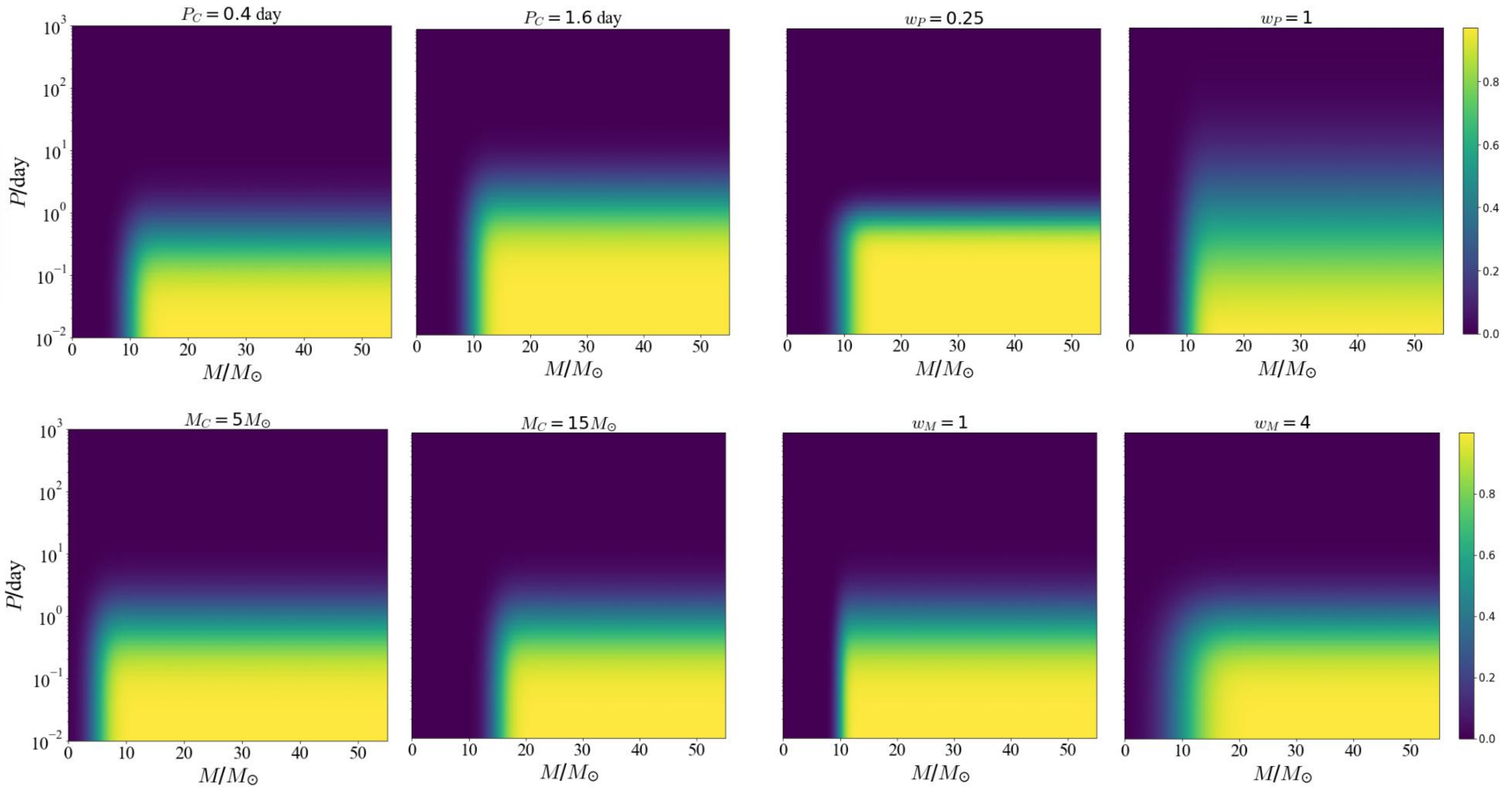


ST Statistic: Survival Rate of GA

[**Zhu**, Tong, Manzoni & Ma, 2024]

- Spin Model of BBH : **Plateau (PT) Spin Model**

$$\tilde{a} = \frac{1}{4}\mathrm{erfc}\left(\frac{\ln P/P_C}{\sqrt{2}w_P}\right)\mathrm{erfc}\left(\frac{M_C - M}{\sqrt{2}w_M}\right)$$

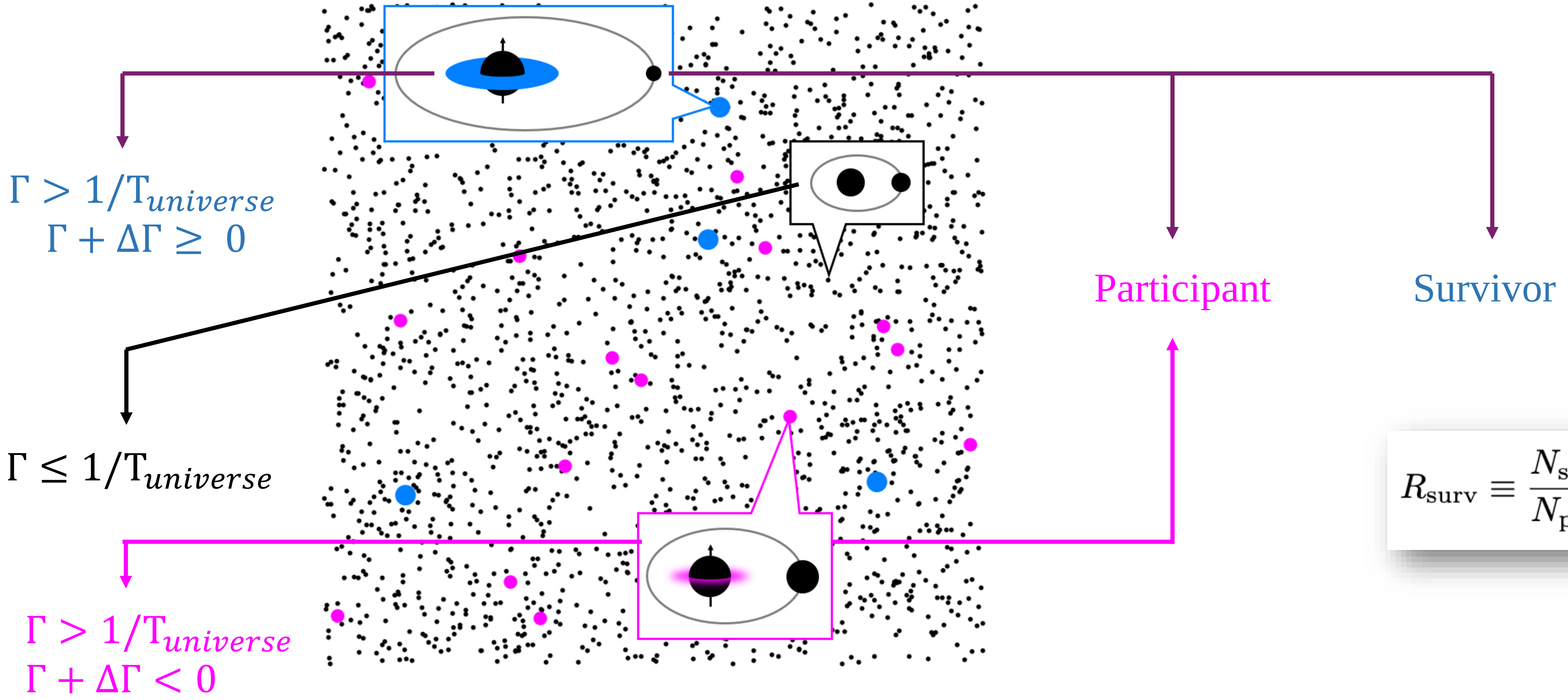


- P_C : Cut off of orbital period
- w_P : Width parameter of period
- M_C : Cut off of parameter of BH mass
- w_M : Width parameter of BH mass

[**Zhu**, Tong, Manzoni & Ma, 2024]

ST Statistic: Survival Rate of GA

- Stellar Evolution for N-body (SEVN) for 8×10^6 BBH systems

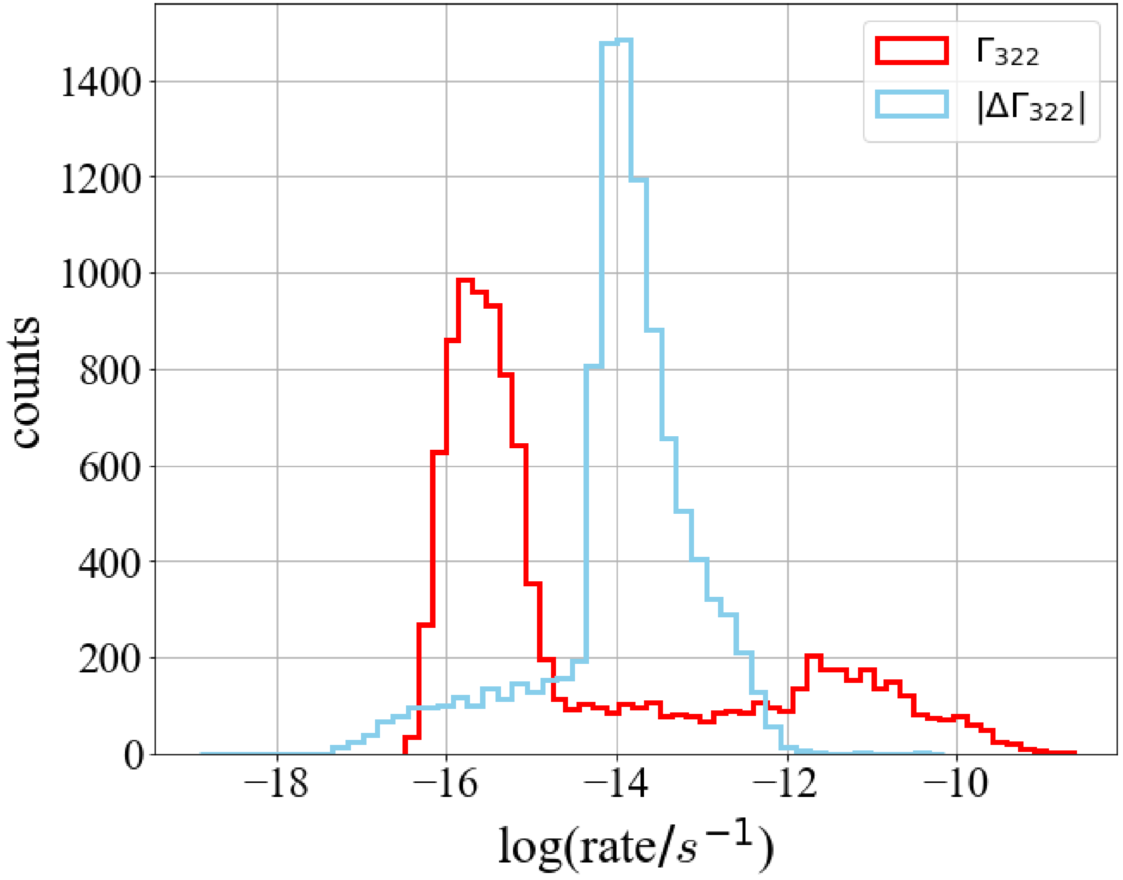


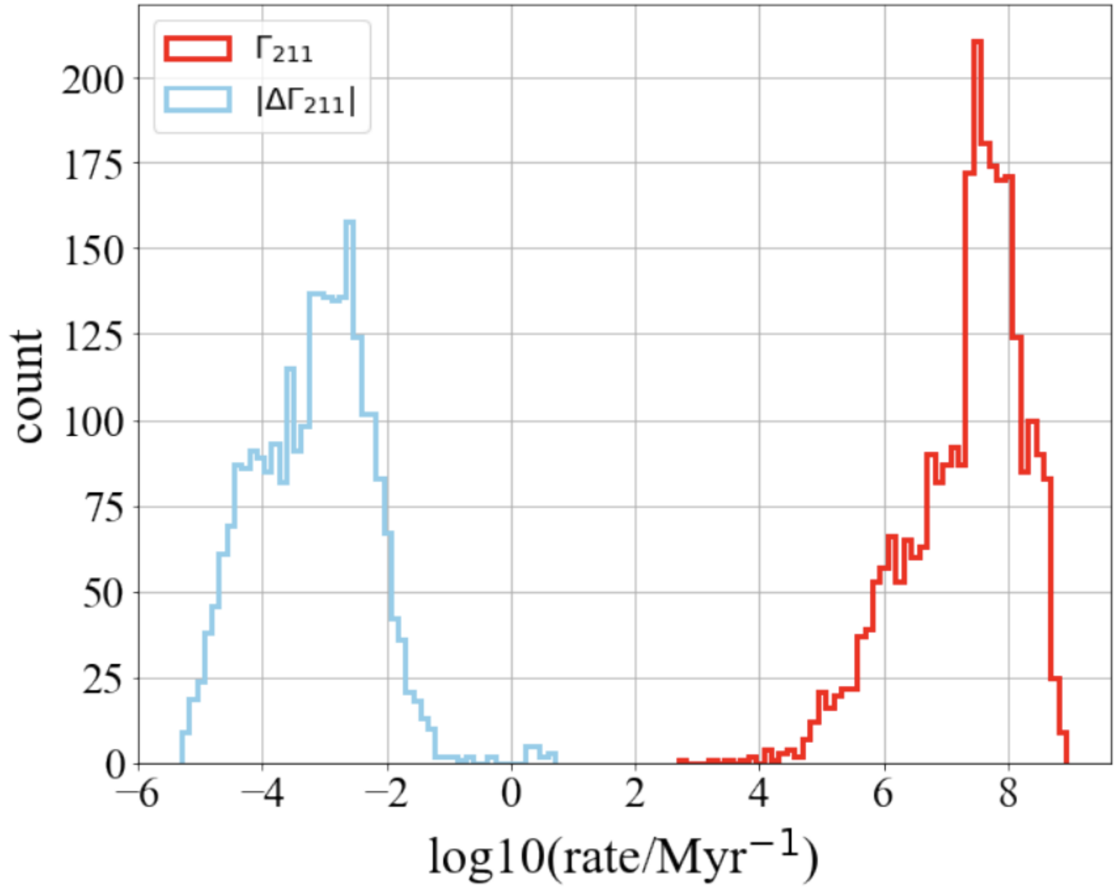
$$R_{\rm surv} \equiv \frac{N_{\rm surv}}{N_{\rm part}}$$

ST Statistic: Survival Rate of GA

[**Zhu**, Tong, Manzoni & Ma, 2024]

- Stellar Evolution for N-body (SEVN) for 8×10^6 BBH systems with **WR spin model**

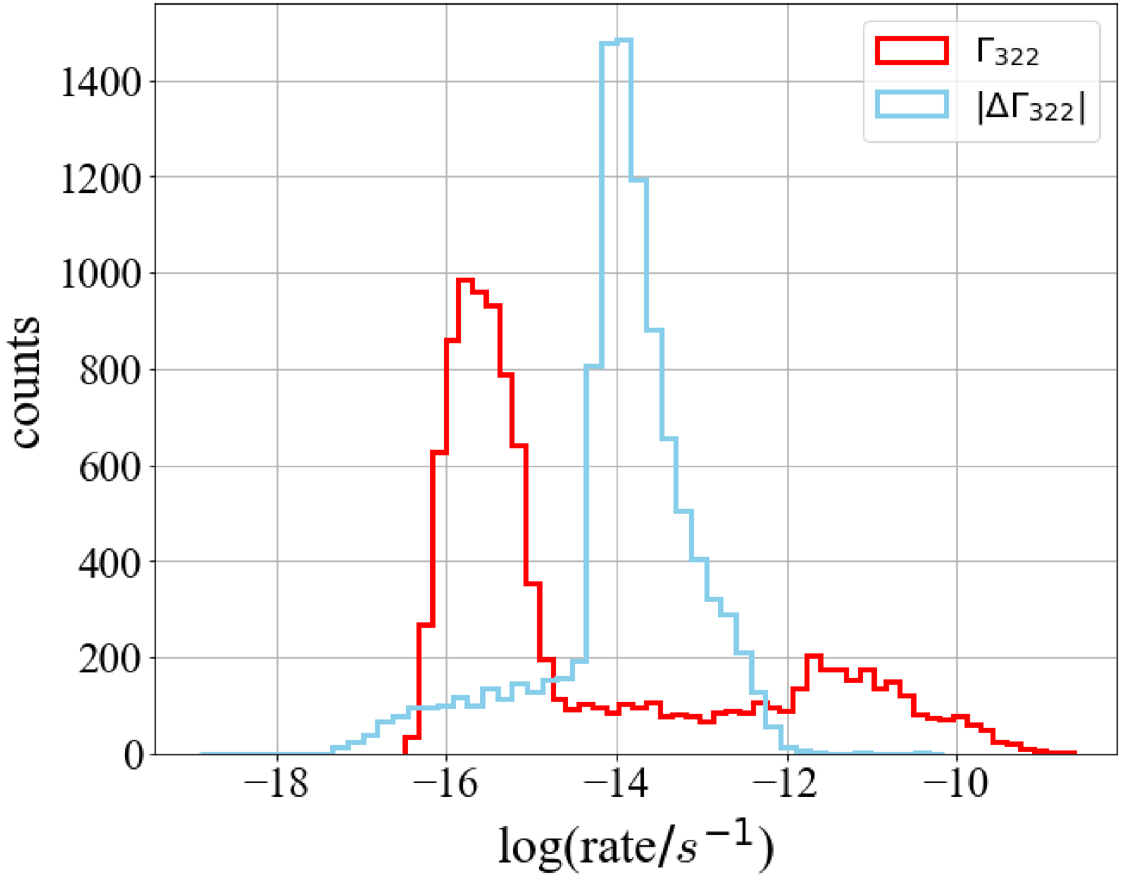


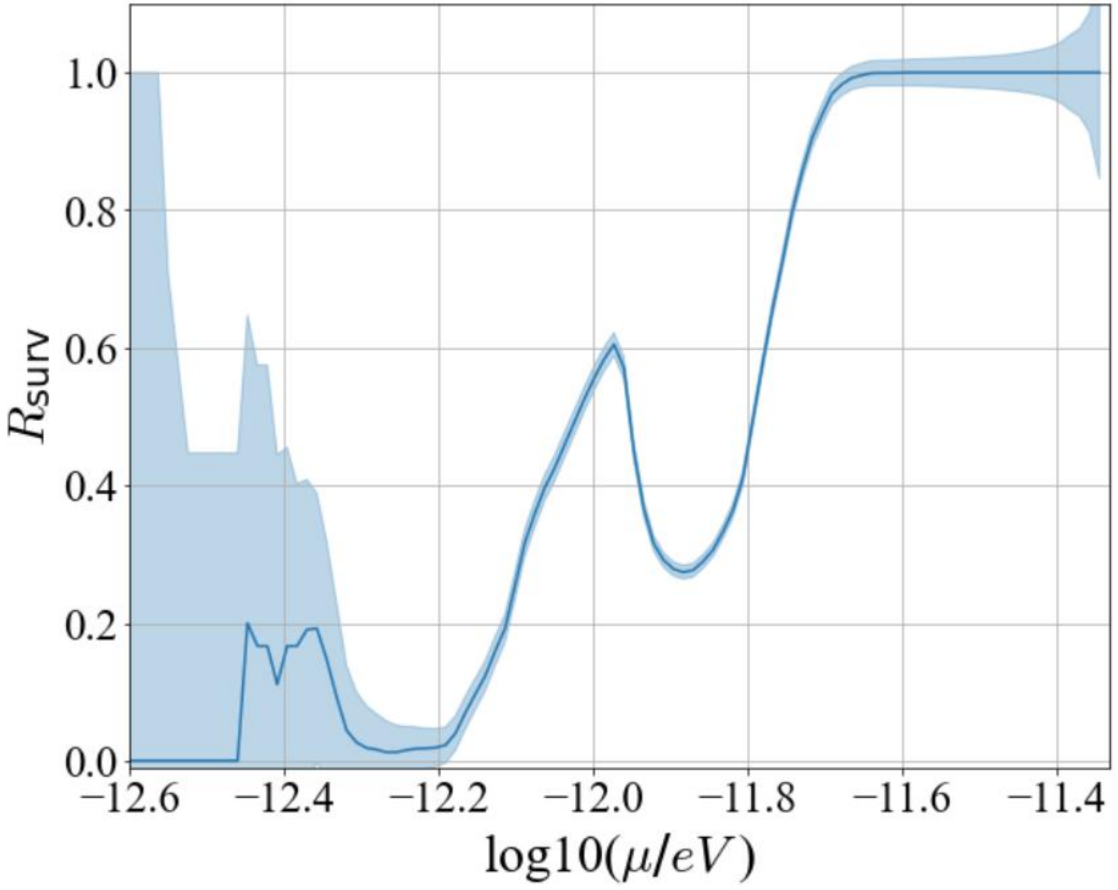


ST Statistic: Survival Rate of GA

[**Zhu**, Tong, Manzoni & Ma, 2024]

- Stellar Evolution for N-body (SEVN) for 8×10^6 BBH systems with **WR spin model**



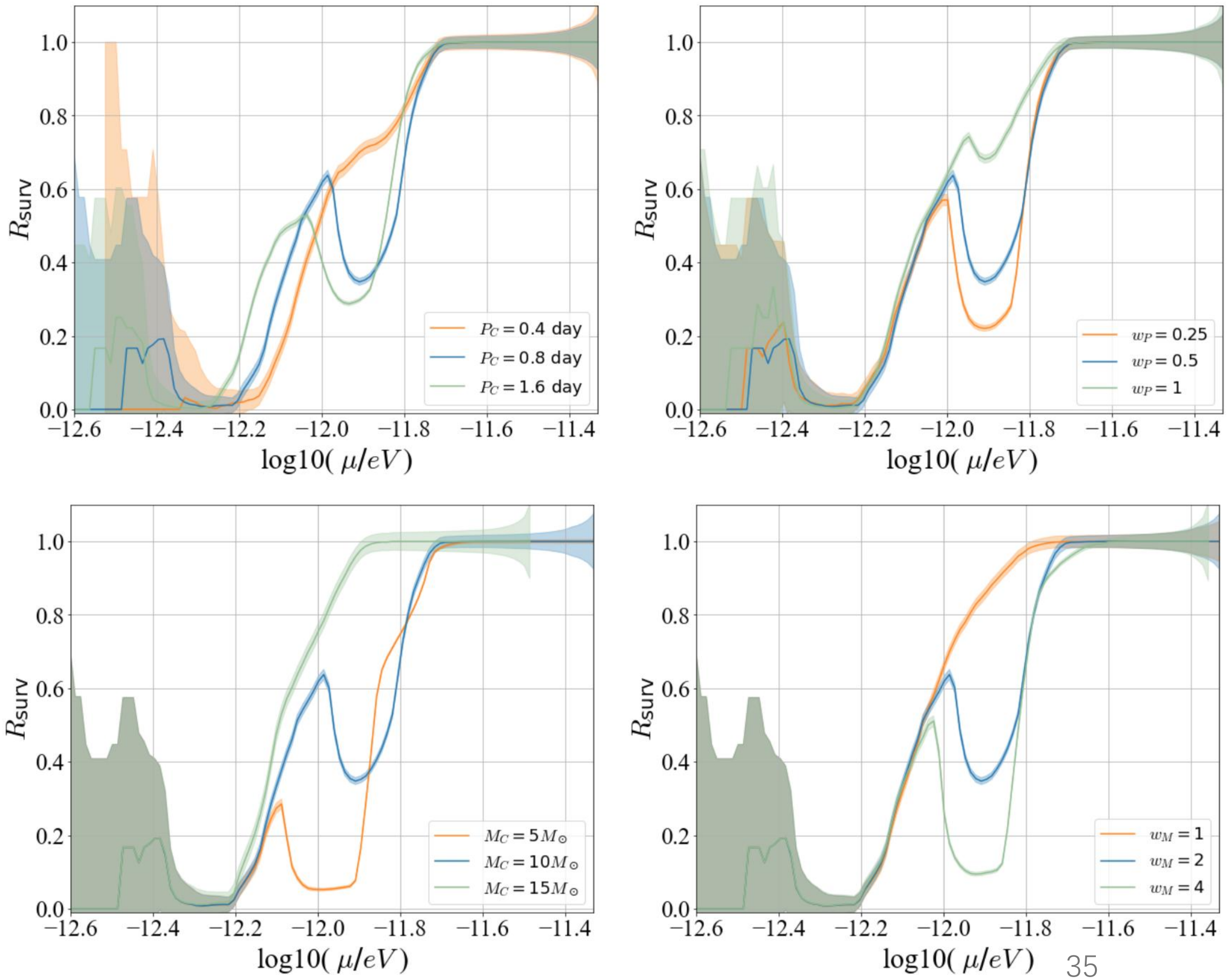


ST Statistic: Survival Rate of GA

[**Zhu**, Tong, Manzoni & Ma, 2024]

- Stellar Evolution for N-body (SEVN) for 8×10^6 BBH systems with **PT spin model**

- Common behavior :
  - The survival rate is approximately an increasing function of the boson mass.
  - For small boson masses (typically around $\mu < 0.5 \times 10^{-12}$ eV), the survival rate can drop below 10%.
  - BHs that survive come from the highly superradiant tail of the participant sample.



Summary and outlook

- ✓ BH superradiance instability
- ✓ GA enjoys a rich phenomenology
- ✓ Yet a binary companion can destabilize the cloud
- ✓ This leads to ST at a critical distance
- ✓ ST poses tight constraints on possible GCP transitions and have backreaction
- ✓ For small boson masses (typically around $\mu < 0.5 \times 10^{-12}$ eV), the survival rate can drop below 10%.

Thank you for listening!

Backup slides

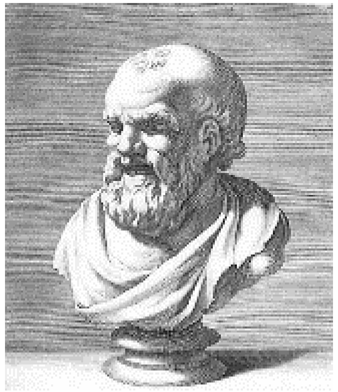
🤔 What is our world made of?

Starting From The Very Beginning

Everything is composed of indestructible ***atoms***

Atoms are composed of ***electrons*** and ***nucleus***

Nucleus are composed of ***Protons*** and ***Neutrons***



… **Standard Model**

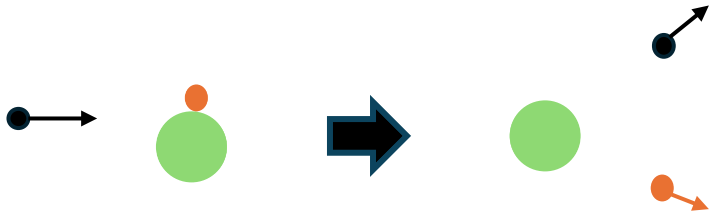
Democritus
(460 - 370 BC)

Thomson
(1856—1940)
& Rutherford
(1871—1937)

Chadwick
(1891-1974)

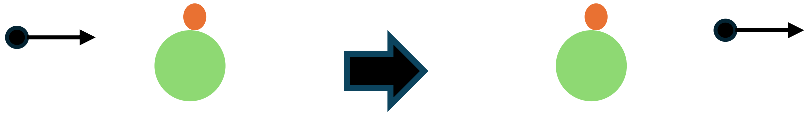
Why Not Particle Collider

Standard Model Particle



- Interaction described by three fundamental forces
- Test particle 'collide' with SM particle
- Can be observed by detector

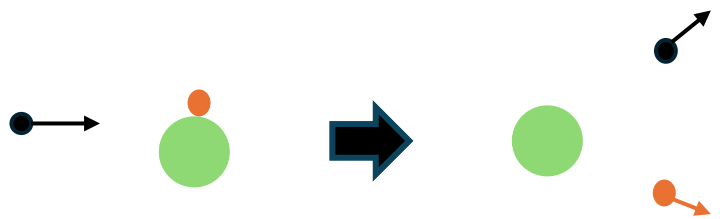
Dark Sector



- Interact mostly by gravity
- Can not 'collide' with test particle
- Can not be observed by the detector

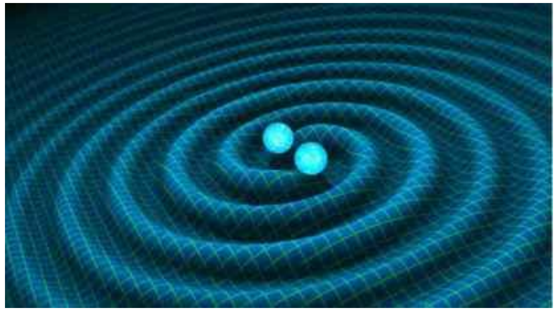
Why Not Particle Collider

Standard Model Particle



- Interaction described by three fundamental forces
- Test particle 'collide' with SM particle
- Can be observed by detector

Dark Sector



- Dark sector couple with rotational BH
- Gravitational wave signal carry the fingerprints of the properties of new particles
- **Gravitational Collider**

[Baumann et al, 2019]

Kerr Spacetime

Kerr spacetime in Boyer-Lindquist coordinates:

$$\mathrm{d}s^2 = -\left(\frac{\Delta - a^2\sin^2\theta}{\Sigma}\right)\mathrm{d}t^2 + \frac{\Sigma}{\Delta}\mathrm{d}r^2 + \Sigma\mathrm{d}\theta^2 + \left(\frac{(r^2+a^2)^2 - \Delta a^2\sin^2\theta}{\Sigma}\right)\sin^2\theta\mathrm{d}\phi^2 - \frac{2a\sin^2\theta(r^2+a^2-\Delta)}{\Sigma}\mathrm{d}t\mathrm{d}\phi\ ,$$

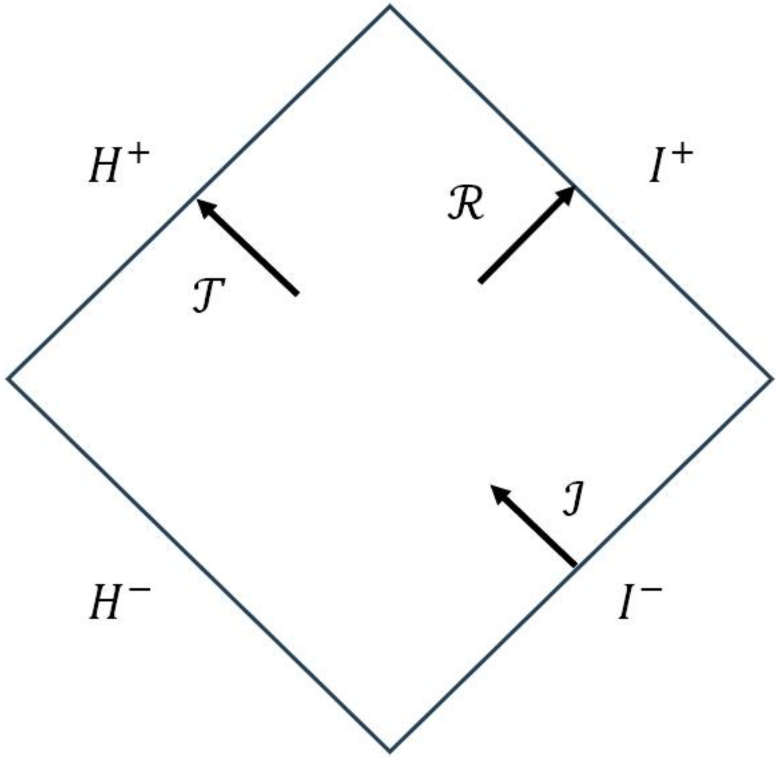
with

$$\Sigma = r^2 + a^2\cos^2\theta\ ,\ \ \Delta = r^2 - 2Mr + a^2\ .$$

Two horizons are

$$r_\pm = M \pm \sqrt{M^2 - a^2}\ .$$

Superradiant Scattering



KG Equation in Kerr with tortoise coordinate:

$$\mathrm{d}r_* = \frac{r^2 + a^2}{\Delta}\mathrm{d}r \ ,$$

Asymptotic behavior:

$$\chi = \begin{cases} e^{-i\omega r_*} + \mathcal{R}e^{i\omega r_*} \ , & r \to \infty \ ; \\ \mathcal{T}e^{-ik_H r_*} \ , & r \to r_+ \ , \end{cases}$$

with

$$k_H = \pm(\omega - m\Omega_H) \ .$$

Wronskian Conservation

For Differential Equation

$$y'' + p(x)y' + q(x)y = 0$$

Define Wronskian

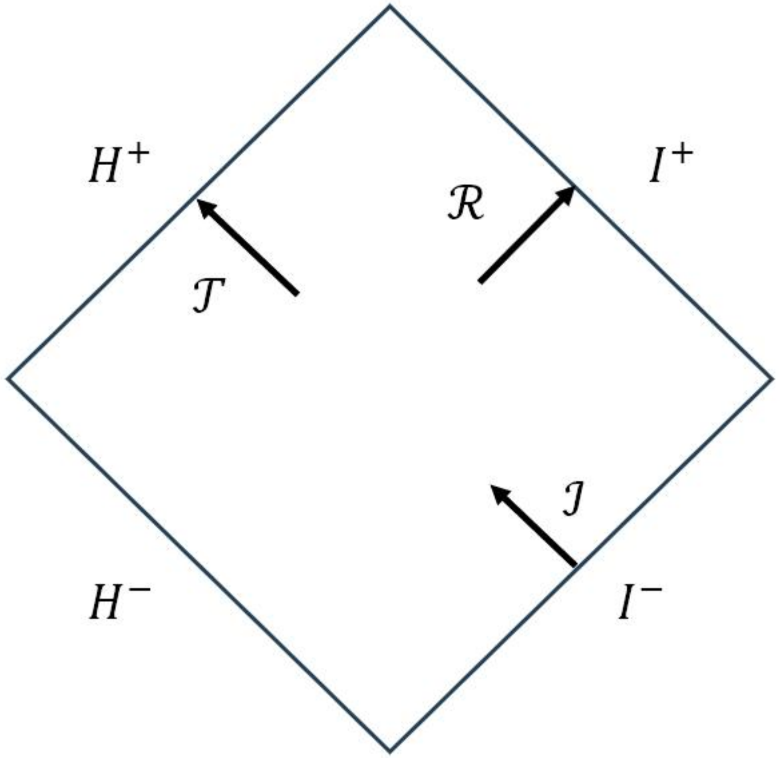
$$W = y_1 y_2' - y_2 {y_1}' \qquad \text{so} \qquad W' = -pW$$

For our EoM

$$\left[\frac{d^2}{dr_*^2} + V_{eff}\right]\chi(r_*) = 0$$

$p = 0 \longrightarrow W' = 0 \longrightarrow$ Wronskian Conserve

Superradiant Scattering



Wronskian conservation:

$$W = \chi \frac{\mathrm{d}\chi^*}{\mathrm{d}r} - \chi^* \frac{\mathrm{d}\chi}{\mathrm{d}r} \ ,$$

$$\Downarrow$$

$$|\mathcal{R}|^2 + \frac{\omega - m\Omega_H}{\omega} |\mathcal{T}|^2 = 1$$

When $\omega > m\Omega_H$, **superradiant!**

Selection Rules

- From angular integral

$$\mathcal{G}^{l' l_* l}_{-m' m_* m} = \int \mathrm{d}\Omega Y_{l' -m'}(\Omega) Y_{l_* m_*}(\Omega) Y_{lm}(\Omega)$$

$$\int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} Y_\ell^m \, Y_{\ell'}^{m'\,*} \, d\Omega = \delta_{\ell\ell'} \, \delta_{mm'} \,, \qquad \int |Y_\ell^m|^2 d\Omega = 1.$$

we obtain selection rules

$$\begin{cases} -m' + m_* + m = 0 \ , \\ l + l_* + l' = 2k, \ \text{for } k \in \mathbb{Z} \ , \\ |l - l'| \leqslant l_* \leqslant l + l' \ . \end{cases}$$

General Orbit Calculation Detail

ST torque comes from

$$\frac{\mathrm{d}S_{\mathrm{c}}(t)}{\mathrm{d}t} = \left(\frac{\mathrm{d}S_{\mathrm{c}}(t)}{\mathrm{d}t}\right)_{\mathrm{ST}} + \left(\frac{\mathrm{d}S_{\mathrm{c}}(t)}{\mathrm{d}t}\right)_{\mathrm{cGW}} .$$

$$\left(\frac{\mathrm{d}S_{\mathrm{c}}(t)}{\mathrm{d}t}\right)_{\mathrm{ST}} = 2\overline{\Delta\Gamma}_1^{(\mathrm{ACR})} S_{\mathrm{c}}(t) , \qquad \left(\frac{\mathrm{d}S_{\mathrm{c}}(t)}{\mathrm{d}t}\right)_{\mathrm{cGW}} = \gamma_1(S_{\mathrm{c}}) S_{\mathrm{c}}(t) , \qquad \tau_{\mathrm{c}} = -\left(\frac{\mathrm{d}S_{\mathrm{c}}(t)}{\mathrm{d}t}\right)_{\mathrm{ST}} .$$

Power from ST and GW reads

$$P_{\mathrm{c}} = \frac{1}{T}\int \mathrm{d}t\dot{\varphi}_* \tau_{\mathrm{c}} \cos\iota_* \approx -\frac{2\pi}{T}\left(\frac{\mathrm{d}S_{\mathrm{c}}(t)}{\mathrm{d}t}\right)_{\mathrm{ST}} \cos\iota_*$$

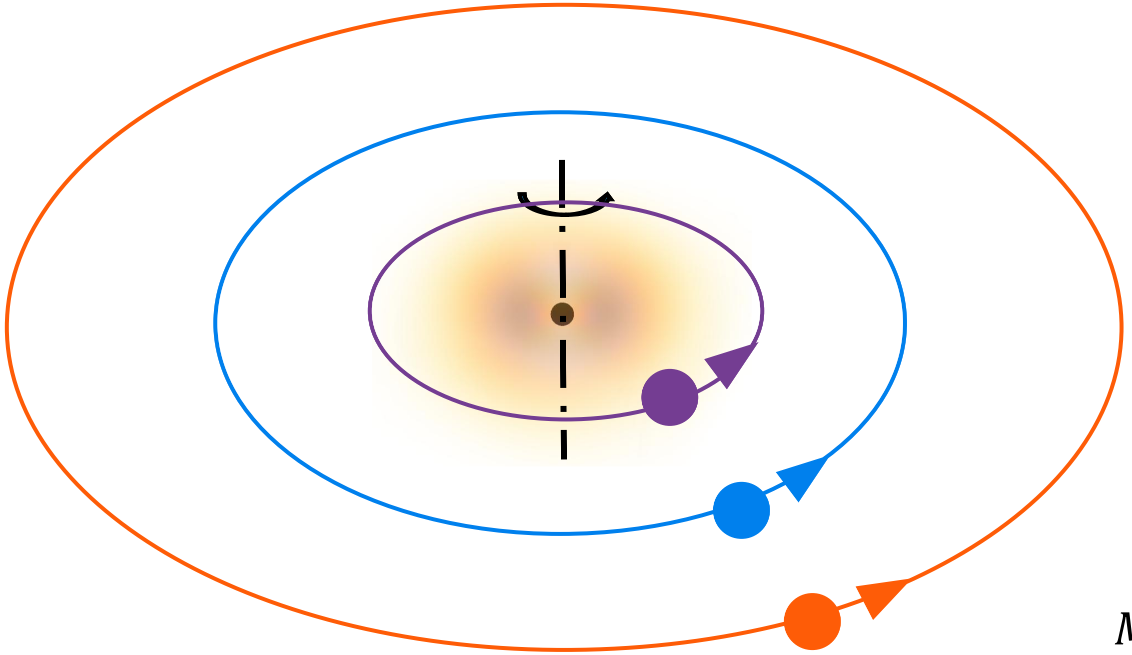
$$P_{\mathrm{bGW}} = -\frac{32}{5}\frac{M^5 q^2 (1+q)^{1/2}}{p^5}(1-e^2)\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right),$$

🤔 Gravitational wave observation is enough?

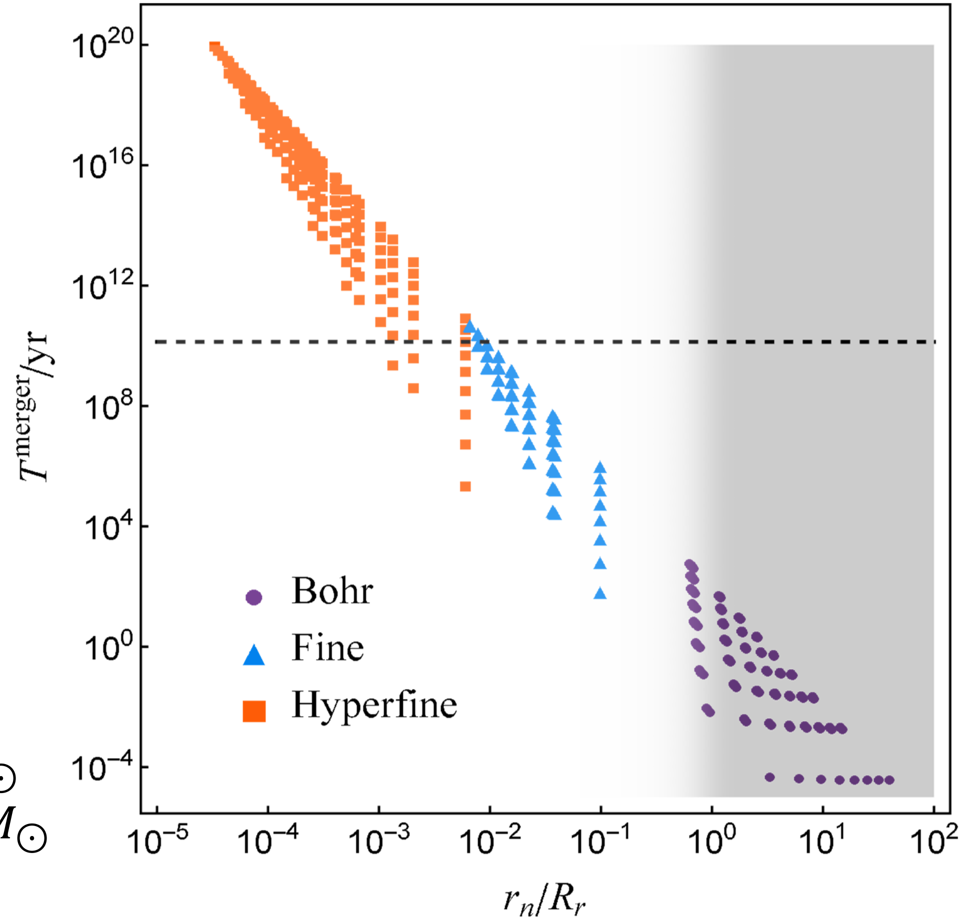
GA phenomenology in binaries

[Tong, Wang & **Zhu**, 2021]

- Atomic resonant transitions



$M = 5\ M_{\odot}$
$M_* = 1.4\ M_{\odot}$
$\alpha = 0.12$



GA phenomenology in binaries

[Tong, Wang & **Zhu**, 2021]

	Transition	P_r (hr)	Δt (yr)	Δt_c (yr)	r_n/R_r	$T^{(\text{growth})}$ (yr)	$T^{(\text{deplete})}$ (yr)	$T^{(\text{merge})}$ (yr)
Bohr	$\|322\rangle \to \|200\rangle$	6.4×10^{-4}	2.8×10^{-3}	2.2×10^{-3}	0.96	9600	10^{13}	7.9×10^{-3}
	$\|322\rangle \to \|100\rangle$	9.9×10^{-5}	1.9×10^{-5}	2.8×10^{-5}	3.3	9570	10^{13}	5.5×10^{-5}
	$\|311\rangle \to \|21-1\rangle$	7.0×10^{-4}	3.6×10^{-3}	1.4×10^{-2}	0.89	4.7×10^{-2}	10^{5}	1.0×10^{-2}
	$\|211\rangle \to \|31-1\rangle$	7.1×10^{-4}	3.6×10^{-3}	-1.4×10^{-2}	0.89	1.7×10^{-2}	10^{5}	1.0×10^{-2}
Fine	$\|322\rangle \to \|300\rangle$	1.9×10^{-2}	25	6.3	9.8×10^{-2}	9600	10^{13}	72
Hyperfine	$\|322\rangle \to \|320\rangle$	12	7.5×10^{8}	2.2×10^{7}	1.3×10^{-3}	9600	10^{13}	2.1×10^{9}
	$\|321\rangle \to \|32-1\rangle$	6.4	1.3×10^{8}	2.4×10^{7}	2.0×10^{-3}	6.4×10^{5}	10^{5}-10^{13}	3.8×10^{8}
	$\|311\rangle \to \|31-1\rangle$	1.3	1.8×10^{6}	5.6×10^{5}	6.0×10^{-3}	4.7×10^{-2}	10^{5}	5.1×10^{6}
	$\|211\rangle \to \|21-1\rangle$	0.38	7.0×10^{4}	3.3×10^{4}	6.0×10^{-3}	1.7×10^{-2}	10^{5}	2.0×10^{5}

Table. A comparison between Bohr transitions and fine/hyperfine transitions

New Approach: Pulsar Timing Method

- Mixing of cloud states backreacts on the binary orbit

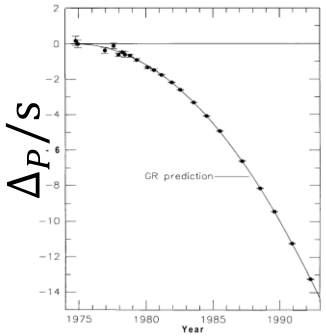
[Ding, Tong & Wang, 2020]
[Tong, Wang & **Zhu**, 2021]

$$\frac{(\dot{P})_{\mathrm{C}}}{(\dot{P})_{\mathrm{GR}}} \simeq -15|c_{322}|^2\left(\frac{\alpha}{0.1}\right)^{-9}\frac{q}{(1+q)^{7/3}}\left(\frac{M}{10M_\odot}\right)^{5/3}\left(\frac{P}{1\ \mathrm{hr}}\right)^{-5/3}$$

Extra period derivative due to the resonance

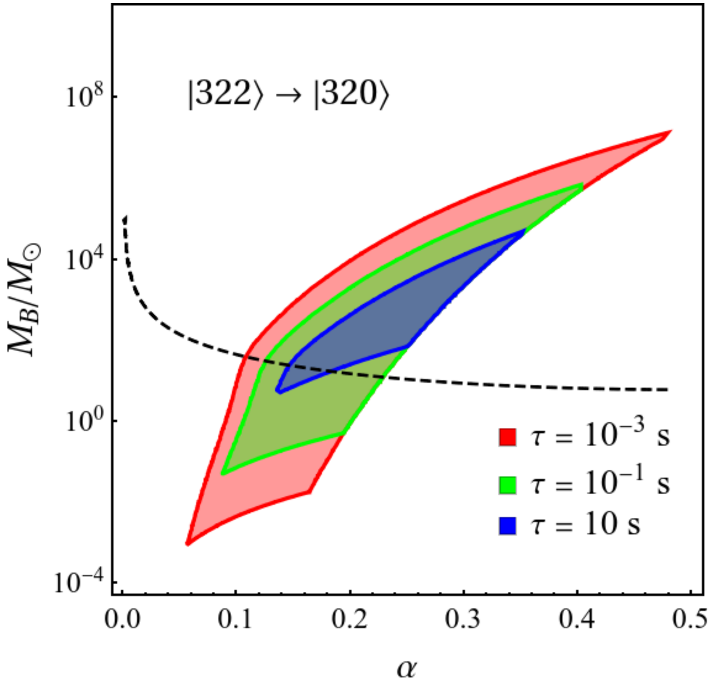
- Measuring the orbital derivative à la Hulse & Taylor

$$\Delta_P \equiv t - P(0)\int_0^t \frac{\mathrm{d}t'}{P(t')} \approx \frac{1}{2}\frac{\dot{P}}{P}t^2$$

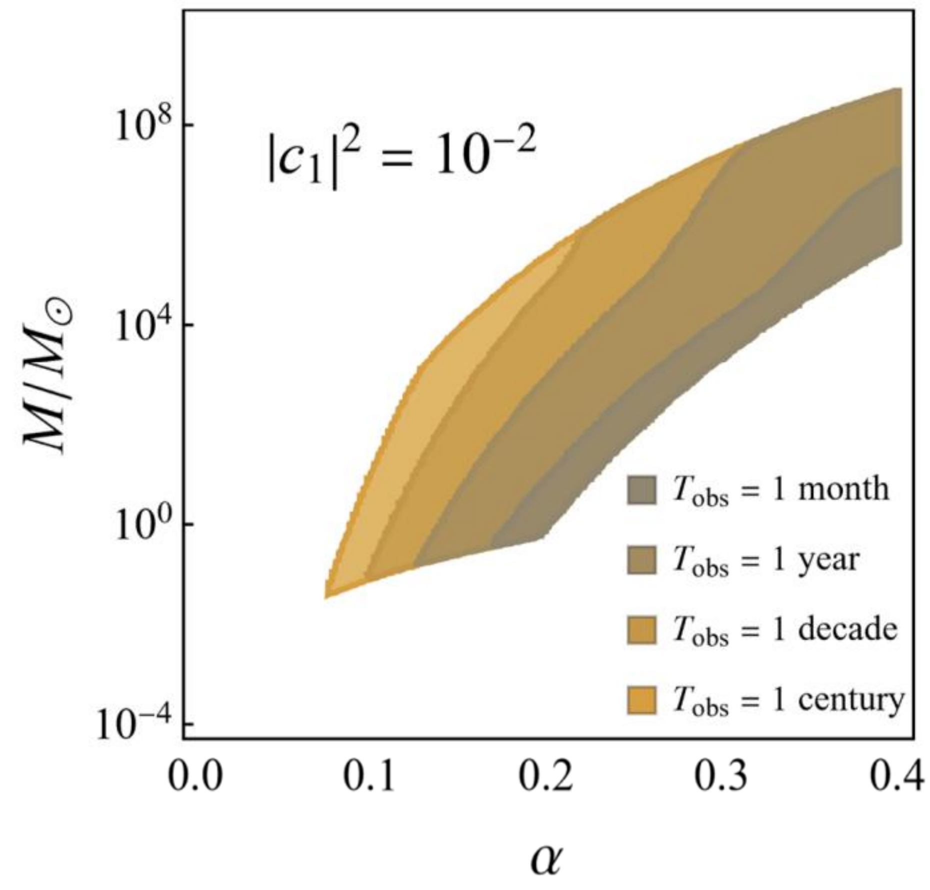


[Hulse & Taylor 1975]

(Pulsar-black hole binary)

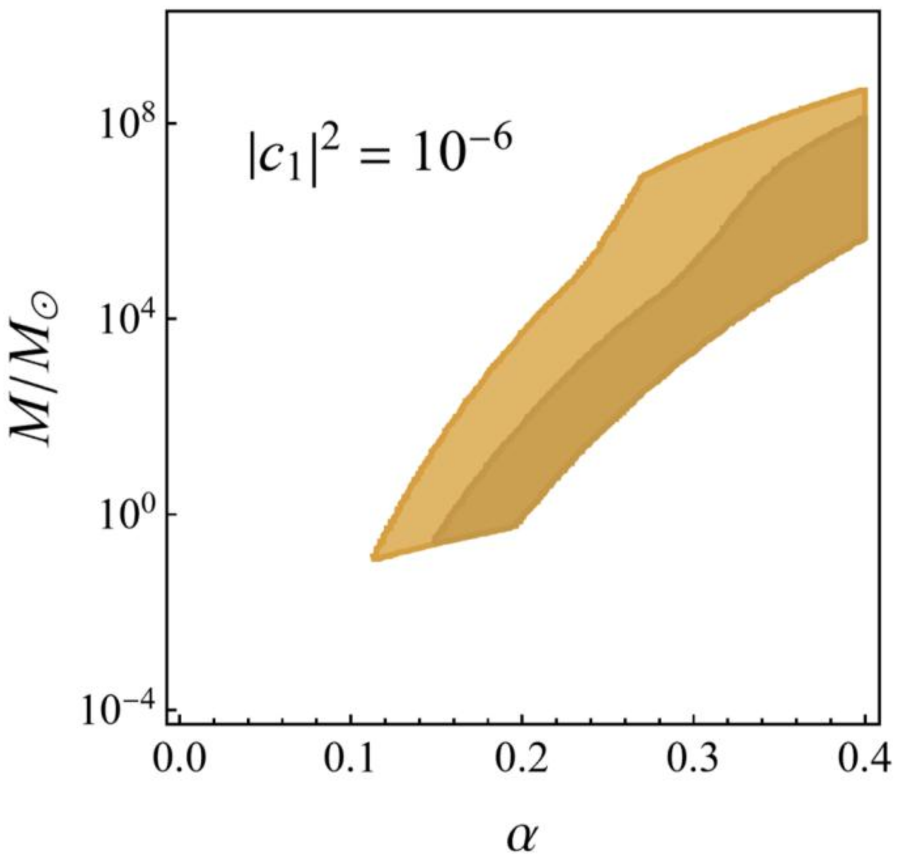


Backreaction of ST Trough Pulsar Timing



$|322\rangle$

$M_* = 1.4 \times 10^{-3}$



ST backreaction: Orbital flow of EMRIs ($q \ll 1$)

[Fan, Tong, Wang & **Zhu**, 2023]

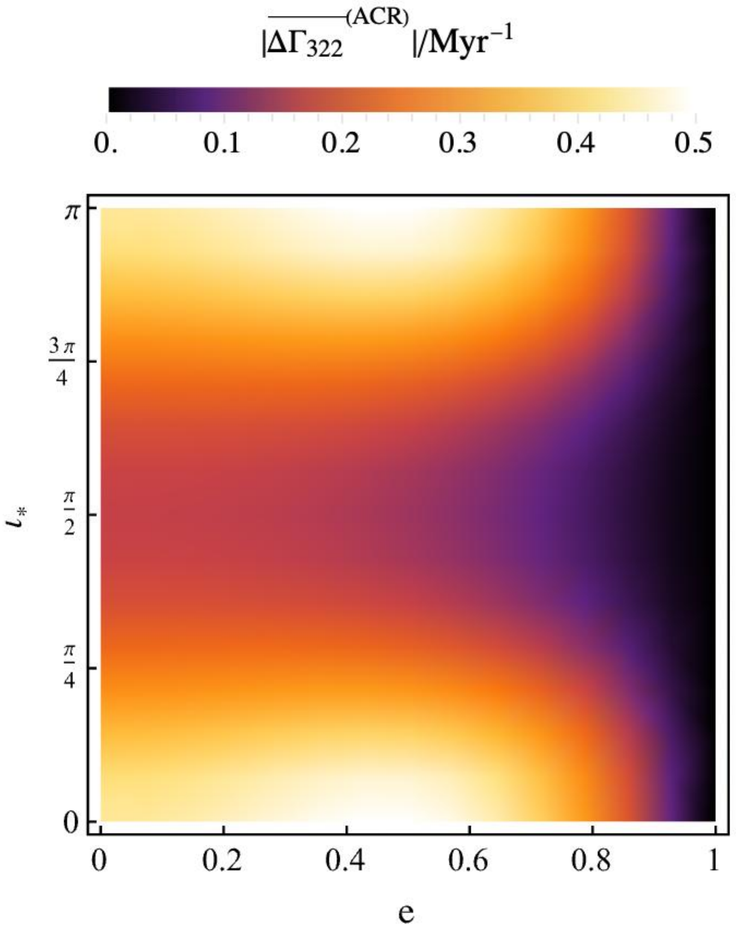
- General binary orbits: $\{p(t), e(t), \iota(t)\} \cup \{S_c(t)\}$

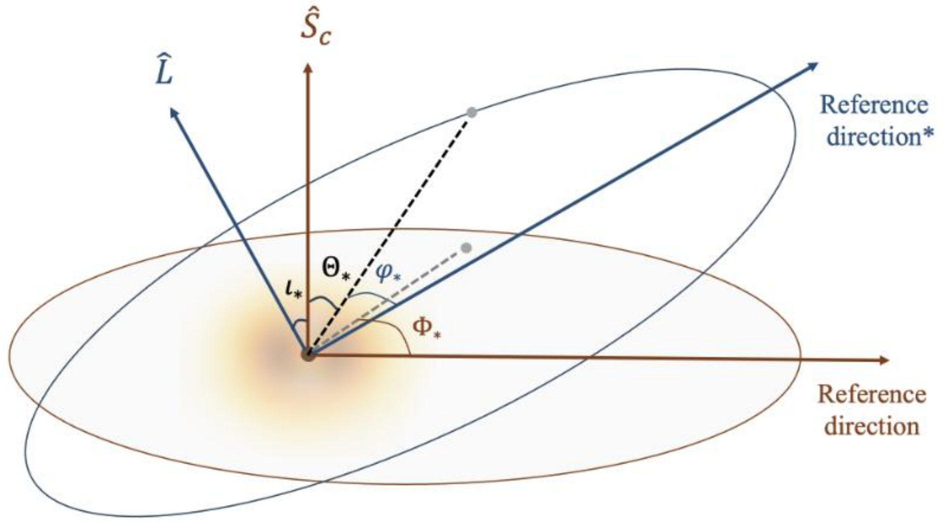
Cloud angular momentum

Inclination angle

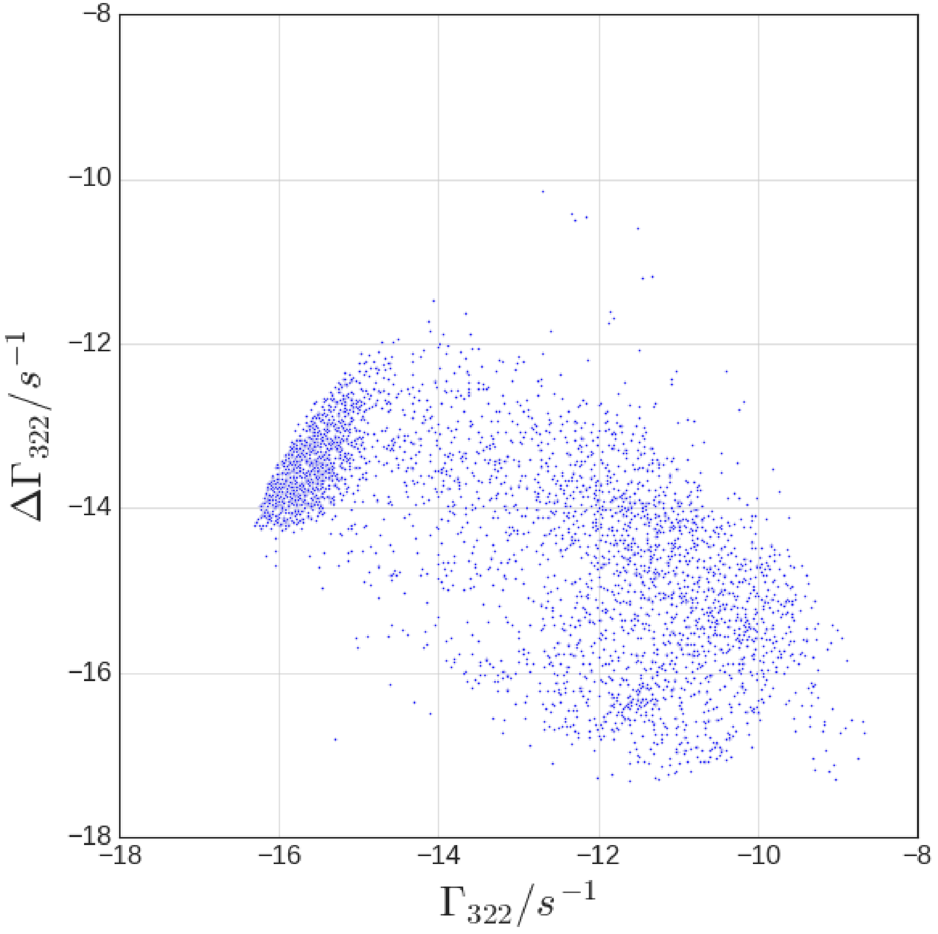
Eccentricity

Semi-latus rectum

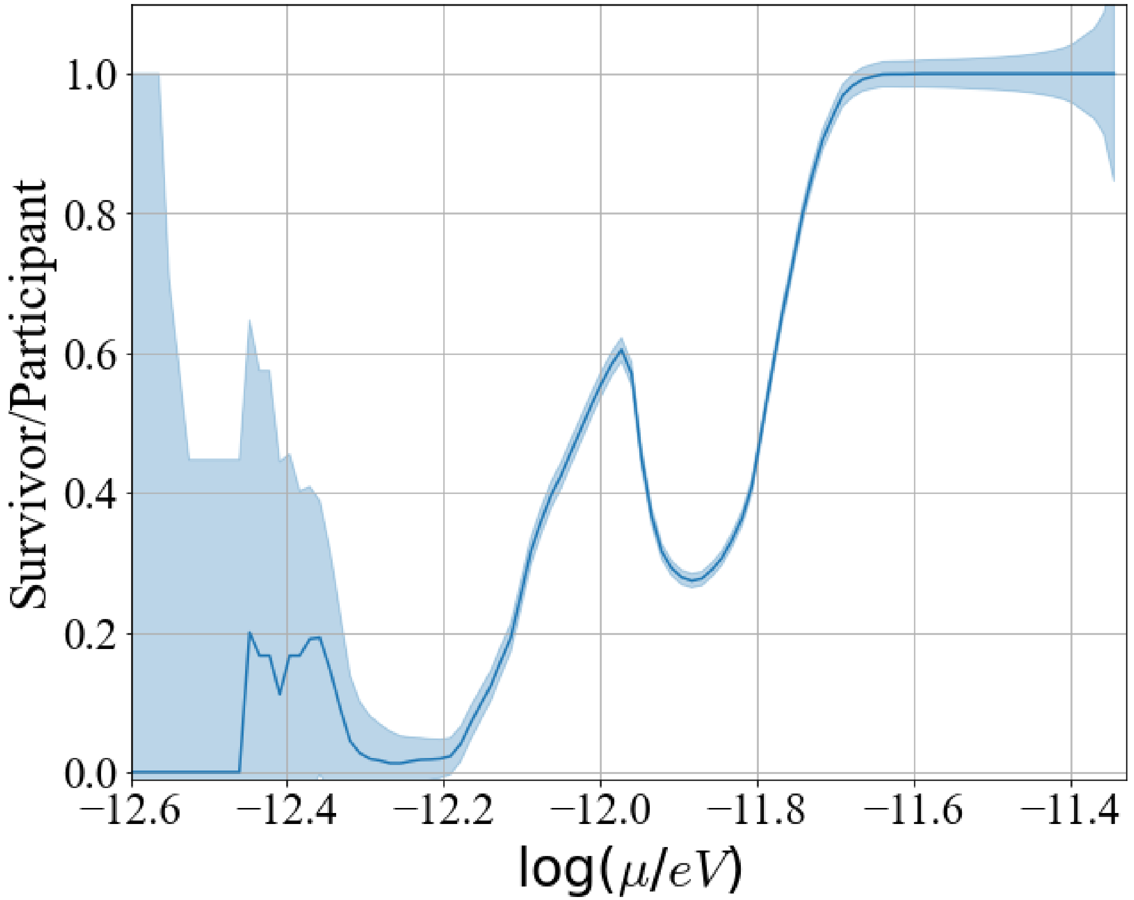




Statistical Test of ST



$\mu = 1.34 \times 10^{-12}$ eV

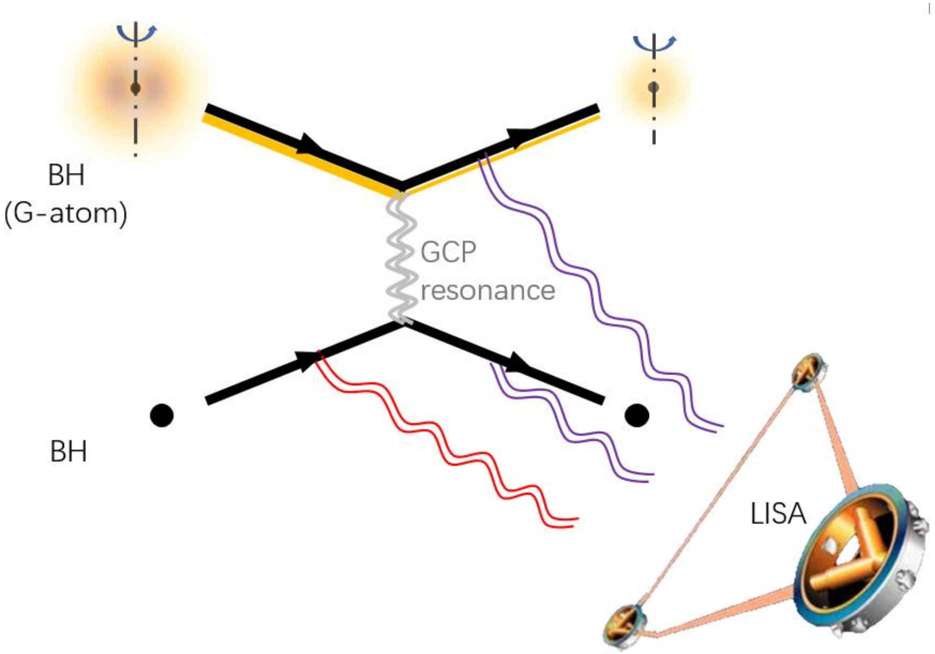


Pulsar Timing Accuracy

- Suppose we observe the pulsar for t_{obs} every day, and the pulse period τ.
- We can measure t_{obs}/P periods every day.
- The error for every single continuous measurement is $\tau/[\min(t_{obs}, t)/P]$.
- If we observe for $0 < t \leq T_{obs}$, where T_{obs} is the longest observation time. Then the uncertainty for Periastron time shift is

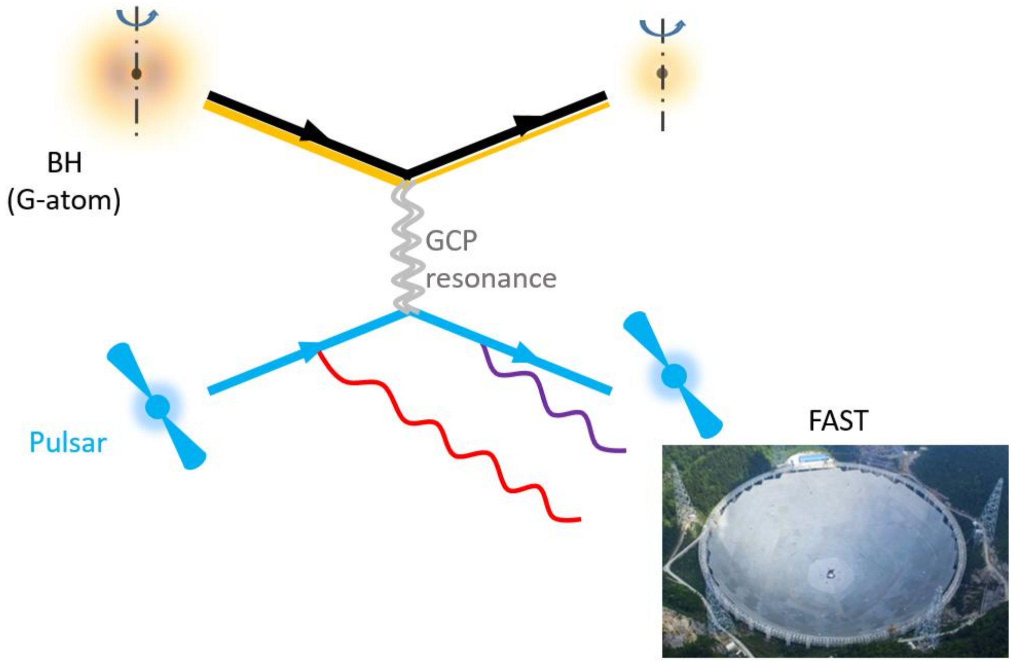
$$\sigma_{\Delta P} = \frac{1}{\sqrt{\left[\frac{t}{1day}\right]}} \frac{\tau}{\min(t_{obs}, t)\,/P}$$

GCP channels



GCP: The BH-BH-GW channel

[Baumann et al,2019,2020]



GCP: The BH-PSR-Radio channel

[Tong et al, 2021]

The GA spectrum

$$E_{nlm} = \mu\left(-\frac{\alpha^2}{2n^2} - \frac{\alpha^4}{8n^4} - \frac{(3n-2l-1)\alpha^4}{n^4(l+1/2)}\right) + \frac{2\tilde{a}m\alpha^5}{n^3 l(l+1/2)(l+1)} + \mathcal{O}(\alpha^6).$$

$$\Gamma_{n00} = -\frac{4}{n^3}\left(1+\sqrt{1-\tilde{a}^2}\right)\mu\alpha^5,$$
$$\Gamma_{nlm} = 2\tilde{r}_+ C_{nl} g_{lm}(\tilde{a},\alpha,\omega)(m\Omega_H - \omega_{nlm})\alpha^{4l+5}.$$

$$C_{nl} \equiv \frac{2^{4l+1}(n+l)!}{n^{2l+4}(n-l-1)!}\left[\frac{l!}{(2l)!(2l+1!)}\right]^2,$$

$$g_{lm}(\tilde{a},\alpha,\omega) \equiv \prod_{k=1}^{l}\left(k^2(1-\tilde{a}^2) + (\tilde{a}m - 2\tilde{r}_+ M\omega)^2\right).$$

Going to the co-rotating frame

$$H = \begin{pmatrix} \omega_1 + V_{11} & V_{12} \\ V_{21} & \omega_2 + V_{22} \end{pmatrix} \equiv \begin{pmatrix} \bar{E}_1 + i\Gamma_1 & \eta^* \\ \eta & \bar{E}_2 + i\Gamma_2 \end{pmatrix},$$

⇩

$$H_D = U(t)^\dagger (H(t) - i\partial_t) U(t),$$

$$\text{with} \quad U(t) \equiv e^{-i\varphi_*(t) L_z},$$

$$H_D = \begin{pmatrix} \bar{E}_1 + i\Gamma_1 - m_1\dot{\varphi}_* & |\eta| \\ |\eta| & \bar{E}_2 + i\Gamma_2 - m_2\dot{\varphi}_* \end{pmatrix}.$$

$$\frac{1}{[\bar{E}_1 - \bar{E}_i - (m_1 - m_i)\dot{\varphi}_*(R_*)]^2}$$

"Wick Rotation"

$$\to \frac{1}{(\bar{E}_1 - \bar{E}_i)^2 + [(m_1 - m_i)\dot{\varphi}_*(R_*)]^2}.$$