



Black holes, environments, and bosonic fields

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Outline

- **General motivations**
- **Physical setups**
- **Binaries in environments**
- **Final remarks**

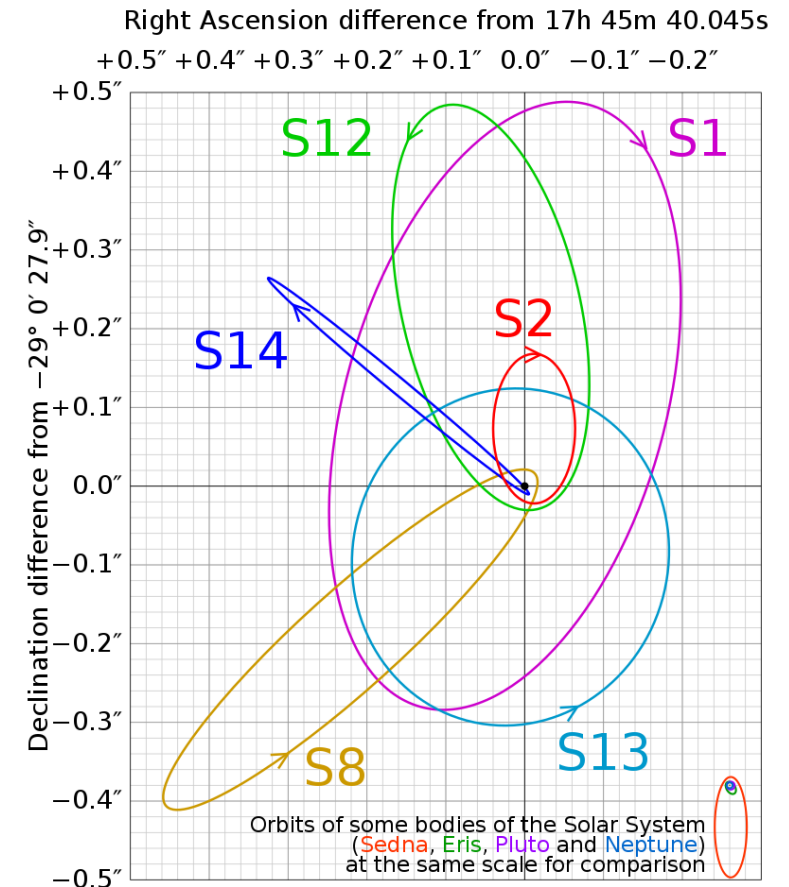
General motivations

Accretion disks, plasmas and dark matter

Barausse et. al PRD **89**, 104059 (2014), Wayne Hu, Rennan Barkana, and Andrei Gruzinov Phys. Rev. Lett. 85, 1158 (2000)

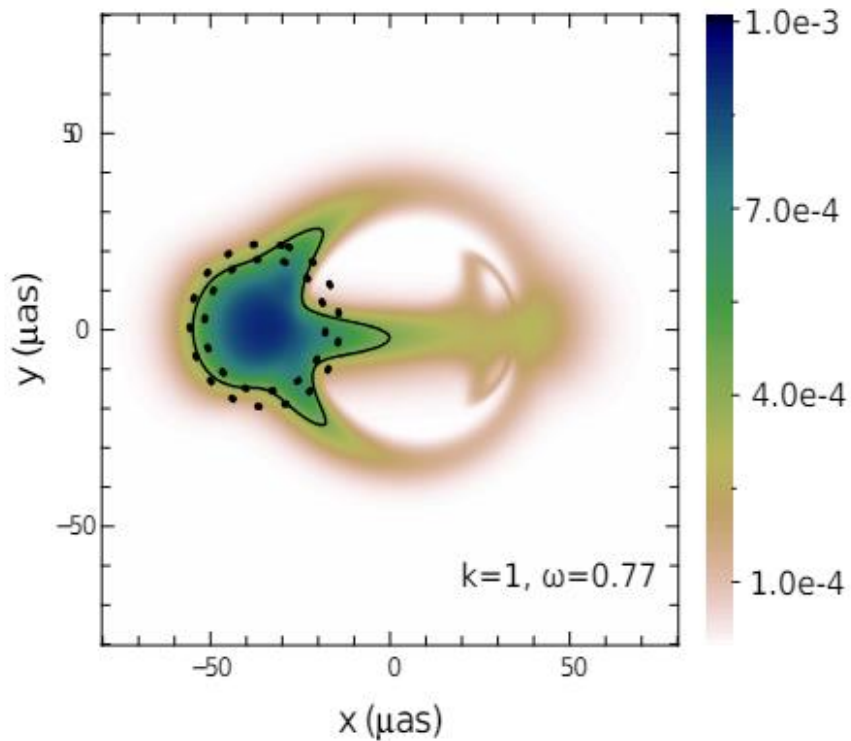


*Environmental forces
play a role*

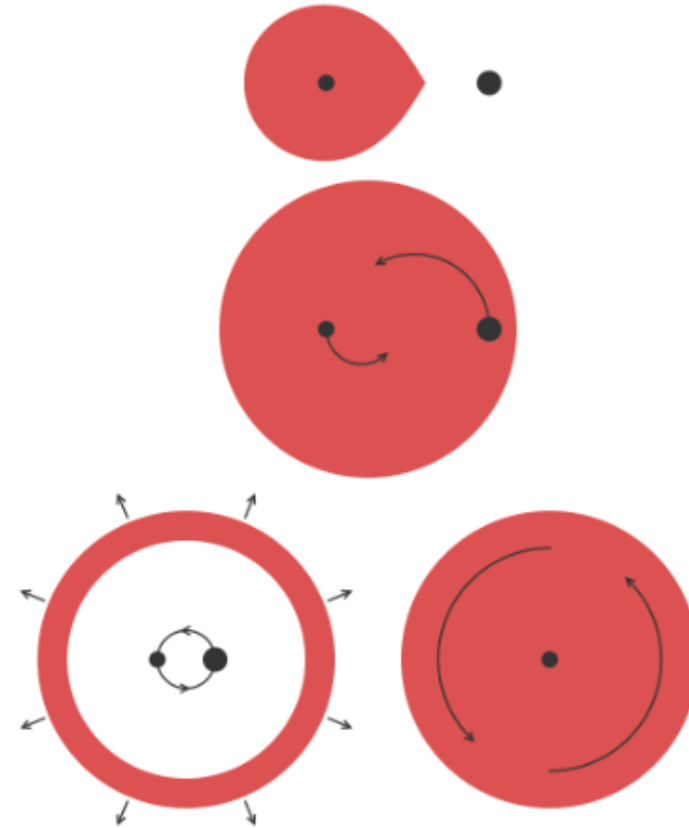


GRAVITY Collaboration

Boson stars and common envelopes



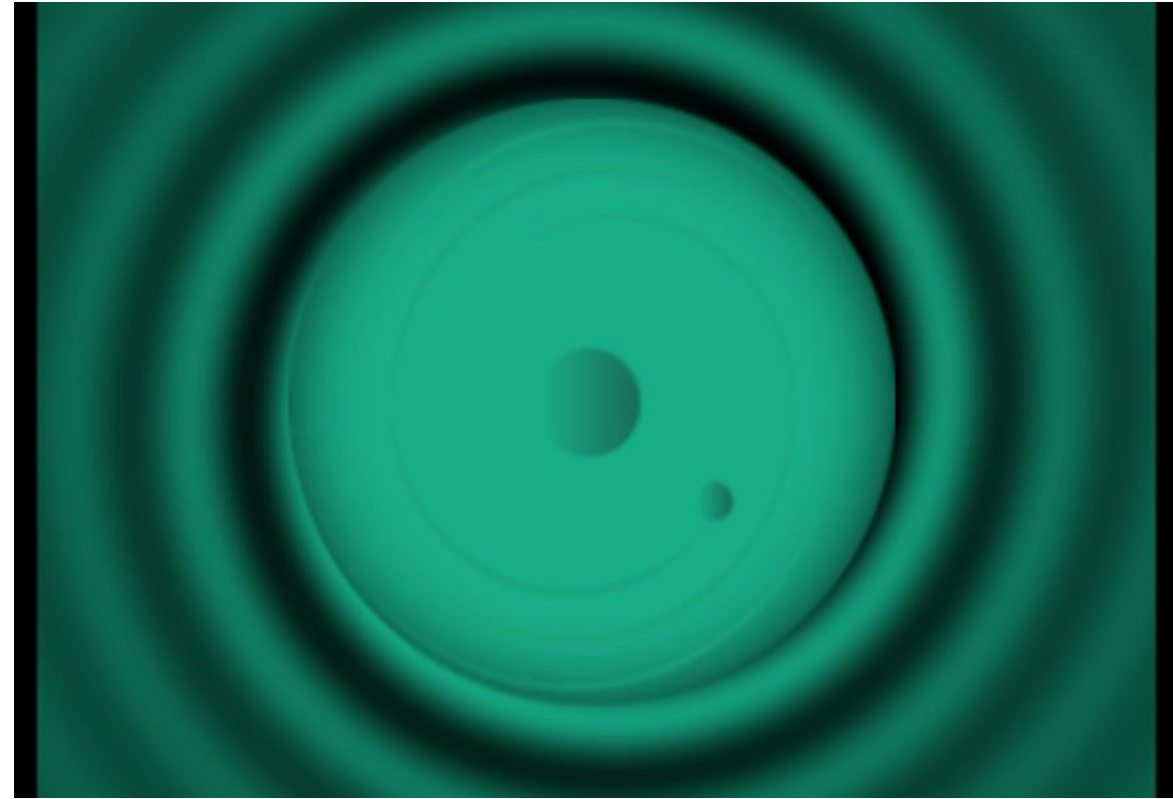
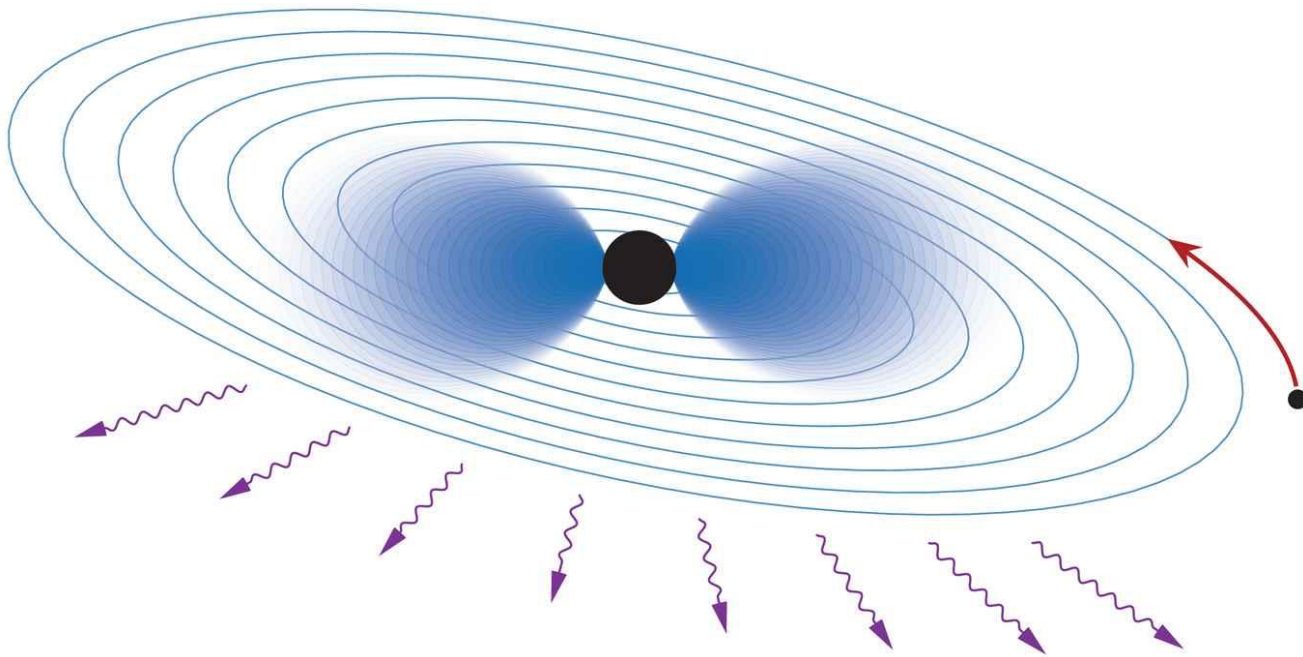
Vicent et al. Class.Quant.Grav. 33 (2016) 10, 105015



Ivanova et al., AARev., 21, 59 (2013)

Gravitational atoms and parasitic BHs

See the lectures of Monday



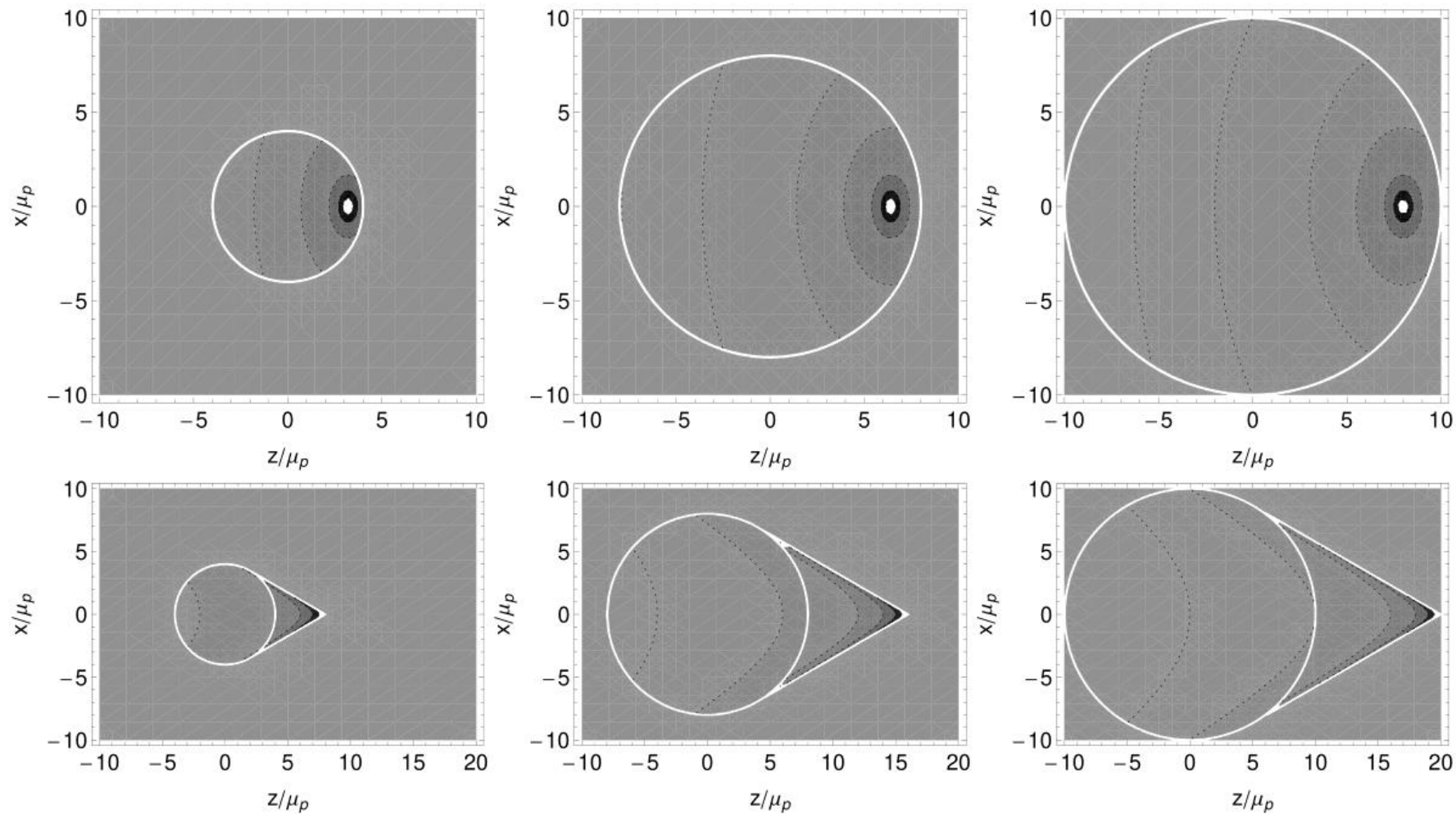
Apart from understanding axions, it is important to test BH paradigm. See Cardoso&Pani 2019

Physical Setups

Physical setups

Dynamical friction

(Chandrasekhar, APJ (1943)), (Ostriker, astro-ph/9810324 (1998))

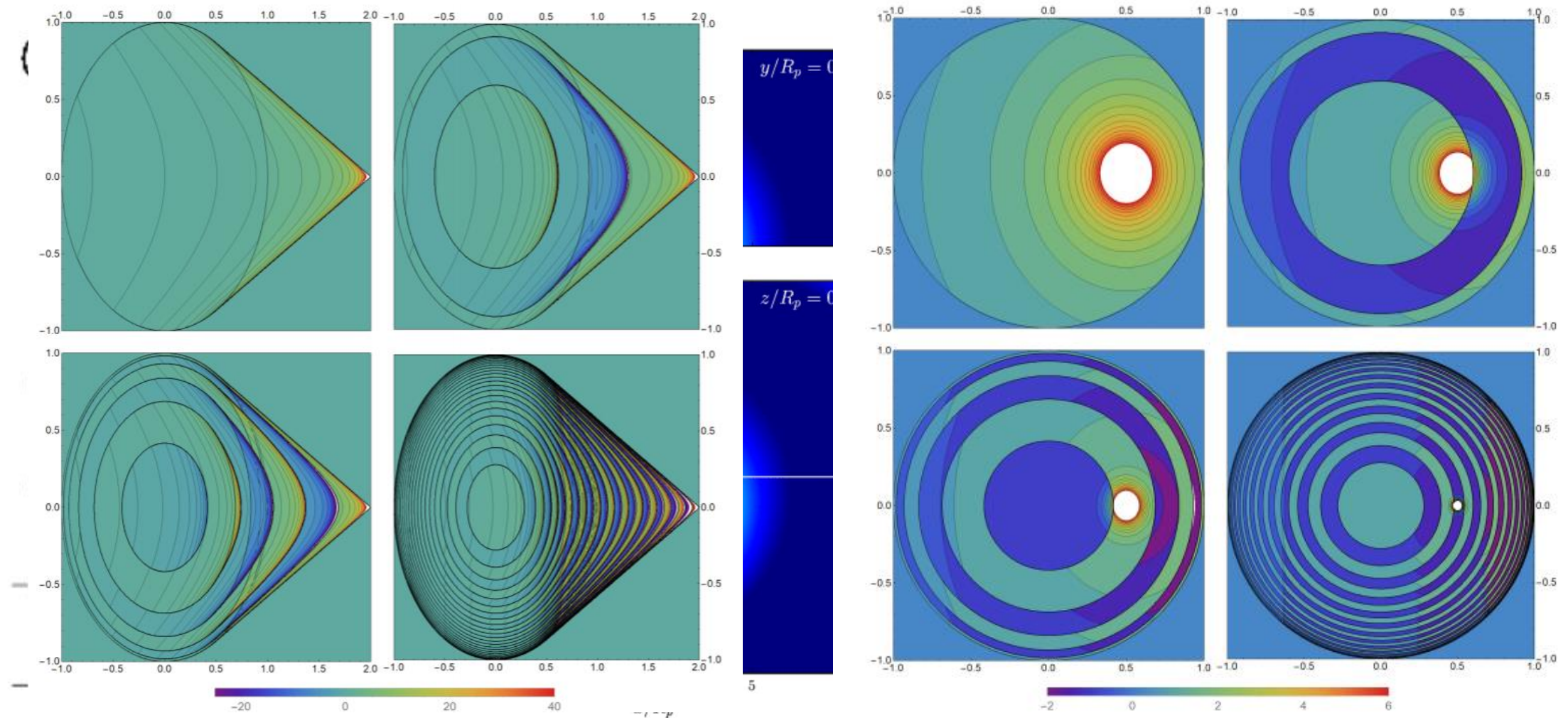


Physical setups

Dynamical friction

Kim&Kim, 0705.0084 (2007); Kim et al. 0804.2010 (2018); Vicente et. al 1905.06353 (2019)

“These are my principles. If you don’t like them, I have others”.



Physical setups

Dynamical friction and scattering amplitudes

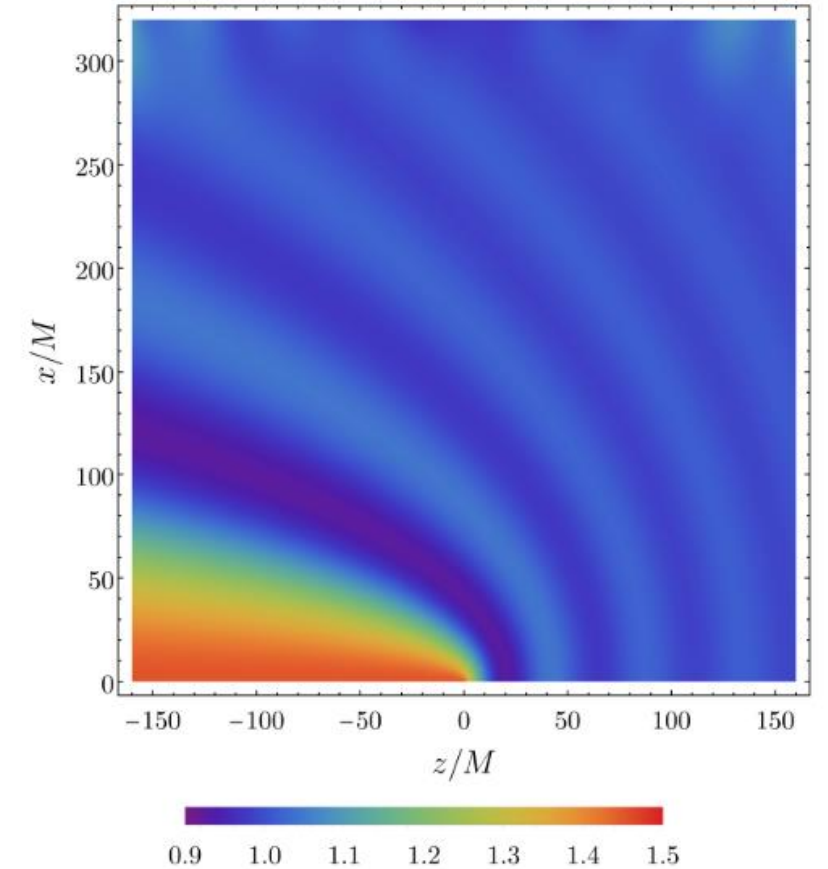
Traykova et al. *Phys. Rev. D* 104, 103014 (2021), Vicente and Cardoso. *Phys.Rev.D* 105 8, 083008 (2022), Traykova et al., *Phys.Rev.D* 108 12, L121502 (2023)

$$\dot{E}_{\text{BH}} = \frac{\pi \hbar \omega n}{\mu k_{\infty}} \sum_{\ell, m} (2\ell + 1) \frac{(\ell - m)!}{(\ell + m)!} (\text{Ps}_{\ell}^m)^2 \left(1 - \left|\frac{R}{I}\right|^2\right).$$

$$P_S^i(t') = \int_{S_{t'}} dV_3 T^{\alpha i} N_{\alpha}.$$



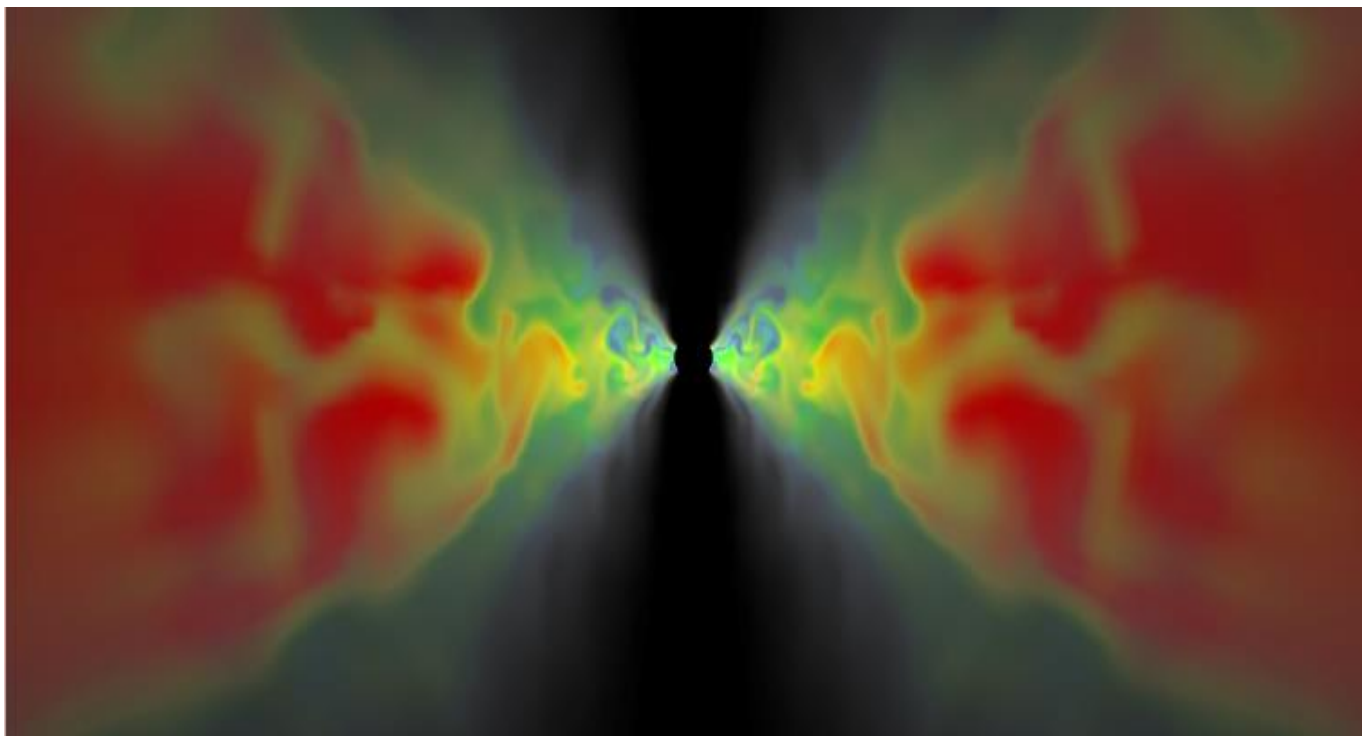
$$\mathbf{F}' = -\frac{4\pi M^2 \rho \mathbf{v}}{v^3} \log \left(\sqrt{1 + \frac{b_{\text{max}}^2}{(M/v^2)^2}} \right)$$



Note that the Chandrasekhar case is recovered

Physical setups

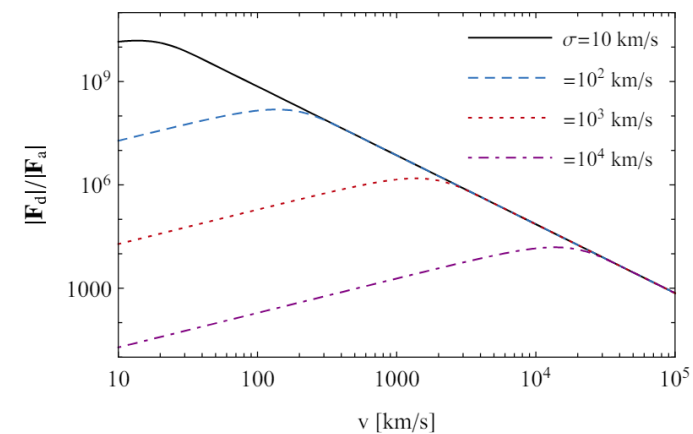
Accretion



$$m_i \ddot{\mathbf{r}}_i + \dot{m}_i \dot{\mathbf{r}}_i = \pm \frac{G m_1 m_2}{r^3} \mathbf{r} + \mathbf{F}_i$$

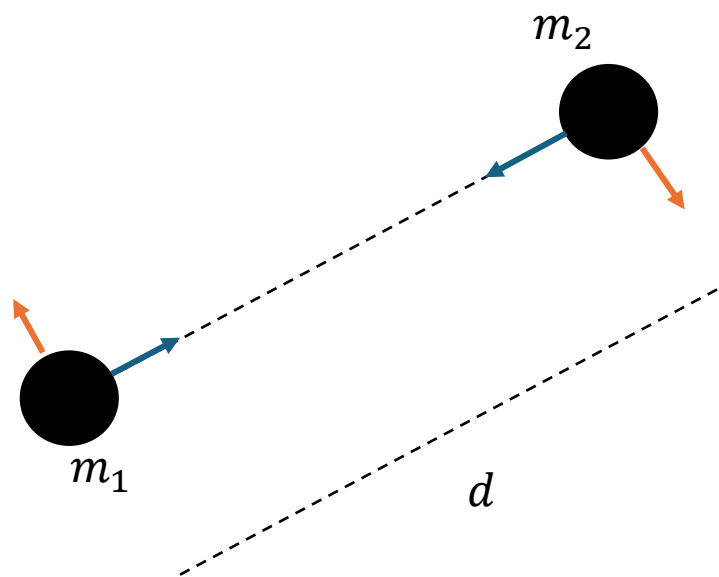
$$\dot{m}_i = 4\pi G^2 \rho \frac{m_i^2}{(v_i^2 + c_s^2)^{3/2}}$$

E.g. Bondi-Hoyle-Littleton



Physical setups

Newtonian equations



$$\ddot{\mathbf{r}} = f_1 \dot{\mathbf{r}} + f_2 \dot{\mathbf{R}} + f_3 \mathbf{r}$$

$$\ddot{\mathbf{R}} = f_4 \dot{\mathbf{r}} + f_5 \dot{\mathbf{R}} + f_6 \mathbf{r}$$

$$\mathbf{F}_{d,i} = -G^2 m_i^2 \rho I_d(v_i) \dot{\mathbf{r}}_i$$

$$f_1 = -\frac{G^2 M q \rho (I_{a1} + I_{a2} + I_{d1} + I_{d2})}{(q+1)^2},$$

$$f_2 = \frac{G^2 M \rho [I_{a1} + I_{d1} - q(I_{a2} + I_{d2})]}{q+1},$$

$$f_3 = GM \left\{ \frac{G^3 M q \rho^2 (I_{a1} - q I_{a2}) [I_{a1} + I_{d1} - q(I_{a2} + I_{d2})]}{(q+1)^4} - \frac{1}{r^3} \right\},$$

$$f_4 = \frac{G^2 M q \rho [q(I_{a2} - I_{d2}) - I_{a1} + I_{d1}]}{(q+1)^3},$$

$$f_5 = -\frac{G^2 M \rho [q^2(I_{a2} + I_{d2}) + I_{a1} + I_{d1}]}{(q+1)^2},$$

$$f_6 = -\frac{G^4 M^2 q \rho^2 (I_{a1} - q I_{a2}) [q^2(I_{a2} + I_{d2}) + 2q(I_{a1} + I_{a2}) + I_{a1} + I_{d1}]}{(q+1)^5}.$$

Physical setups

quadrupole approximation

For **gravitational waves**, we have

$$\langle \dot{E} \rangle = -\frac{32}{5} \frac{G^4 m_1^2 m_2^2 M}{a^5 (1 - e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right)$$

$$\langle \dot{L} \rangle = -\frac{32}{5} \frac{G^{7/2} m_1^2 m_2^2 M^{1/2}}{a^{7/2} (1 - e^2)^2} \left(1 + \frac{7}{8} e^2 \right).$$

$$\frac{da}{dt} = -\frac{64}{5} \frac{G^3 \mu M^2}{c^5 a^3 (1 - e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right), \quad \downarrow$$

$$\frac{de}{dt} = -\frac{304}{15} \frac{G^3 \mu M^2}{c^5 a^4 (1 - e^2)^{5/2}} \left(1 + \frac{121}{304} e^2 \right). \quad \downarrow$$



Emission of gravitational waves
circularizes orbits.

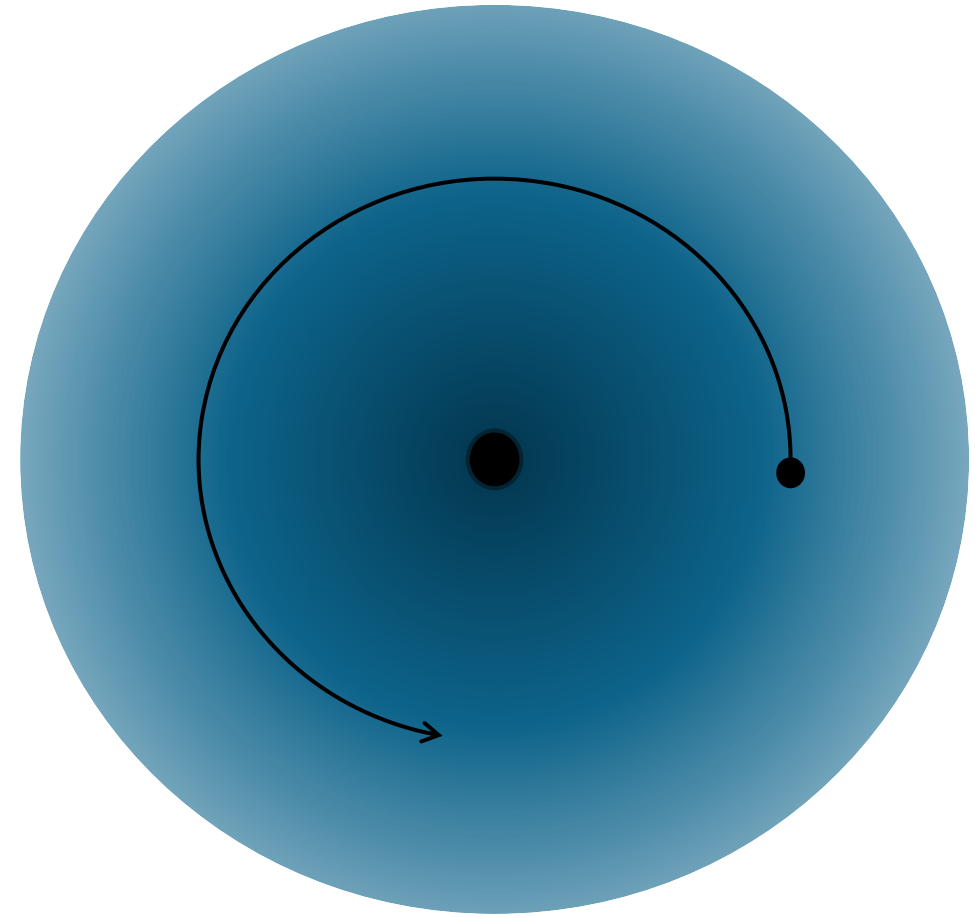
Physical setups

Radiation: scalar fields, black holes and particles

Macedo et al. 2013, Duque et al. 2023

$$G_{\mu\nu} = 8\pi T_{\mu\nu}, \quad \square_g \Phi = \frac{\partial V}{\partial \Phi^*}$$

$$g_{\mu\nu} \approx \hat{g}_{\mu\nu} + q \delta g_{\mu\nu}, \quad \Phi \approx \hat{\Phi} + q \delta \Phi$$



See Francisco's talk

Physical setups

Radiation: scalar fields, black holes and particles

Macedo et al. 2013, Duque et al. 2023

$$\begin{aligned}
 & -\frac{\partial^2 K}{\partial^2 t} + \frac{\partial^2 K}{\partial^2 r_*} + 2\frac{\sqrt{AB}}{r} \frac{\partial K}{\partial r_*} - \left[\frac{A}{r^2} (\ell(\ell+1) + 2B - 2 + 2rB') - \frac{2A'B}{r} + 8\pi\omega^2\phi_0^2 + 8\pi AB\phi_0'^2 \right] K \\
 & = \left[\frac{2B}{r} + 2B' - \frac{2A'B}{A} + 8\pi\frac{r}{A}\omega^2\phi_0^2 + 8\pi Br\phi_0'^2 \right] S \\
 & + 16\pi\frac{AB}{r^2}\phi_0'(\phi_+ + \phi_-) - 8\pi A\delta V - 8\pi\mathcal{A}_{\ell m}^0 - \frac{8\pi AB r}{\sqrt{n(n+1)/2}} \frac{\partial \mathcal{F}_{\ell m}}{\partial r} \\
 & + \frac{4\pi}{\sqrt{n(n+1)/2}} \left[2rA'B - A(\ell(\ell+1) + 2 + 2B + 2rB') - 8\pi r^2\omega^2\phi_0^2 - 8\pi r^2 AB\phi_0'^2 + 8\pi r^2 A\hat{V} \right] \mathcal{F}_{\ell m}
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{\partial^2 S}{\partial^2 t} + \frac{\partial^2 S}{\partial^2 r_*} - \left[\frac{A}{r^2} (\ell(\ell+1) + 2B - 2 + \frac{r}{2}B') + \frac{A'B}{2r} - 8\pi\omega^2\phi_0^2 + 8\pi AB\phi_0'^2 + 8\pi A\hat{V} \right] S \\
 & = \frac{A}{r} \left[\frac{2A'B}{r} - A'B' - 2A''B \right] K + \frac{8\pi}{r^2} \left[\left(A^2 \left(B' - \frac{2B}{r} \right) + 2AA'B \right) \phi_0' + 2A^2 B\phi_0'' \right] (\phi_+ + \phi_-) \\
 & + 8\pi\sqrt{2} \frac{A^2}{r} \mathcal{G}_{\ell m} - \frac{8\pi A r}{\sqrt{n(n+1)/2}} \frac{\partial^2 \mathcal{F}_{\ell m}}{\partial t^2} + \frac{4\pi AB}{\sqrt{n(n+1)/2}} (2A + rA') \frac{\partial \mathcal{F}_{\ell m}}{\partial r} \\
 & + \frac{4\pi}{\sqrt{n(n+1)/2}} \frac{A}{r} \left[2rA'B + r^2 B \frac{A'^2}{A} - A(\ell(\ell+1) - 4 + 4B) + 16\pi r^2\omega^2\phi_0^2 - 16\pi r^2 AB\phi_0'^2 - 16\pi r^2 A\hat{V} \right] \mathcal{F}_{\ell n}
 \end{aligned}$$

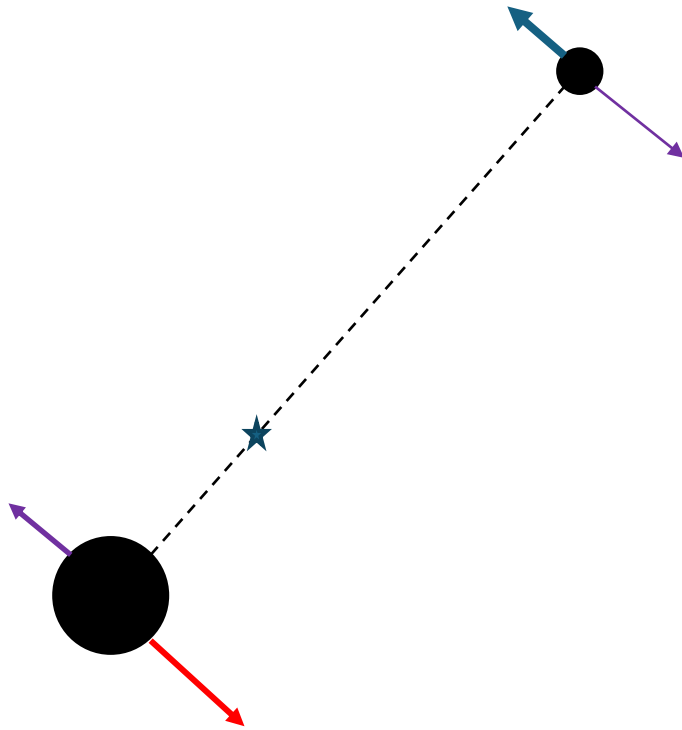
$$\begin{aligned}
 & -\frac{\partial^2 \phi_+}{\partial^2 t} + \frac{\partial^2 \phi_+}{\partial^2 r_*} + 2i\omega \frac{\partial \phi_+}{\partial t} + \left[\omega^2 - \frac{A}{r^2} \ell(\ell+1) - \frac{AB' + A'B}{2r} - A\hat{U} \right] \phi_+ - 8\pi AB\phi_0'^2 (\phi_+ + \phi_-) - rA\phi_0 \delta U \\
 & = -2ir\omega\phi_0 \frac{\partial K}{\partial t} - i\frac{r^2}{A}\omega\phi_0 \frac{\partial S}{\partial t} + i\frac{r^2}{A}\sqrt{\frac{B}{A}}\omega\phi_0 \frac{\partial \tilde{H}_1}{\partial r_*} - \left[\frac{r^2}{A}\omega^2\phi_0 + rB\phi_0' \left(r\frac{A'}{2A} - r\frac{B'}{2B} - 2 \right) - r^2 B\phi_0'' \right] \left(S + \frac{A}{r}K \right) \\
 & + i\frac{r^2\omega}{2A} \left[\left(6\frac{B}{r} + B' - 3\frac{A'B}{A} \right) \phi_0 + 4B\phi_0' \right] H_1 \\
 & + i\frac{4\pi r^3\omega\phi_0}{\sqrt{n(n+1)/2}} \frac{\partial \mathcal{F}_{\ell m}}{\partial t} - \frac{4\pi r^3 AB\phi_0'}{\sqrt{n(n+1)/2}} \frac{\partial \mathcal{F}_{\ell m}}{\partial r} - \frac{4\pi r^2 AB}{\sqrt{n(n+1)/2}} \left[\left(4 + r\frac{B'}{B} \right) \phi_0' + 2r\phi_0'' \right] \mathcal{F}_{\ell m}, \tag{30}
 \end{aligned}$$

Binaries in environments

Let us first look into $q \neq 1$.

Accelerating the CM

Cardoso&CFBM, 2008.01091 (2020)



$$\ddot{\mathbf{R}} = -\frac{G^2 M \rho}{(q+1)^2} (I_1 + q^2 I_2) \dot{\mathbf{R}} + \frac{G^2 M \rho q}{(q+1)^3} (I_1 - q I_2) \dot{\mathbf{r}},$$

$$\ddot{\mathbf{r}} = -\frac{G^2 M \rho q}{(q+1)^2} (I_1 + I_2) \dot{\mathbf{r}} + \frac{G^2 M \rho}{q+1} (I_1 - q I_2) \dot{\mathbf{R}} - \frac{GM}{r^3} \mathbf{r}.$$

Given the initial conditions, we have the key expressions:

CM “acceleration”: $\alpha \equiv \frac{G^2 M^2 \rho}{(q+1) \sigma^2},$

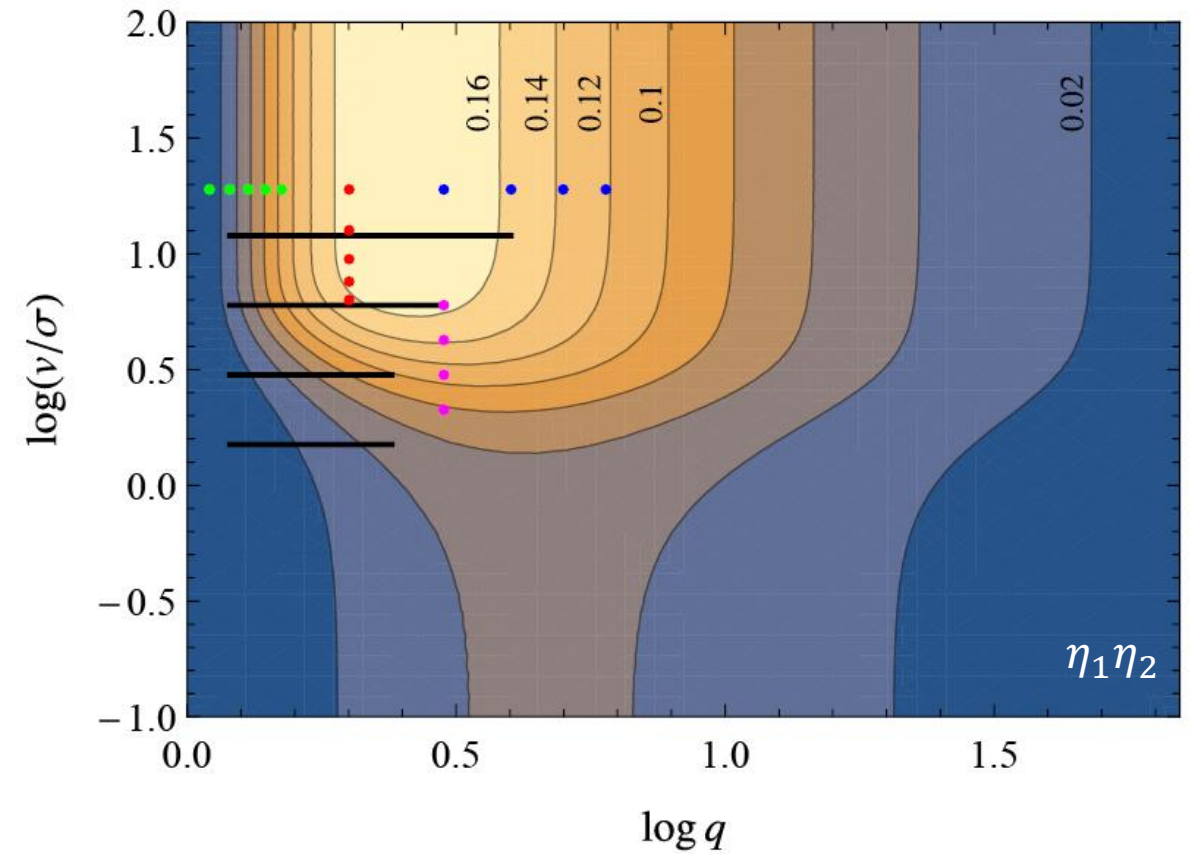
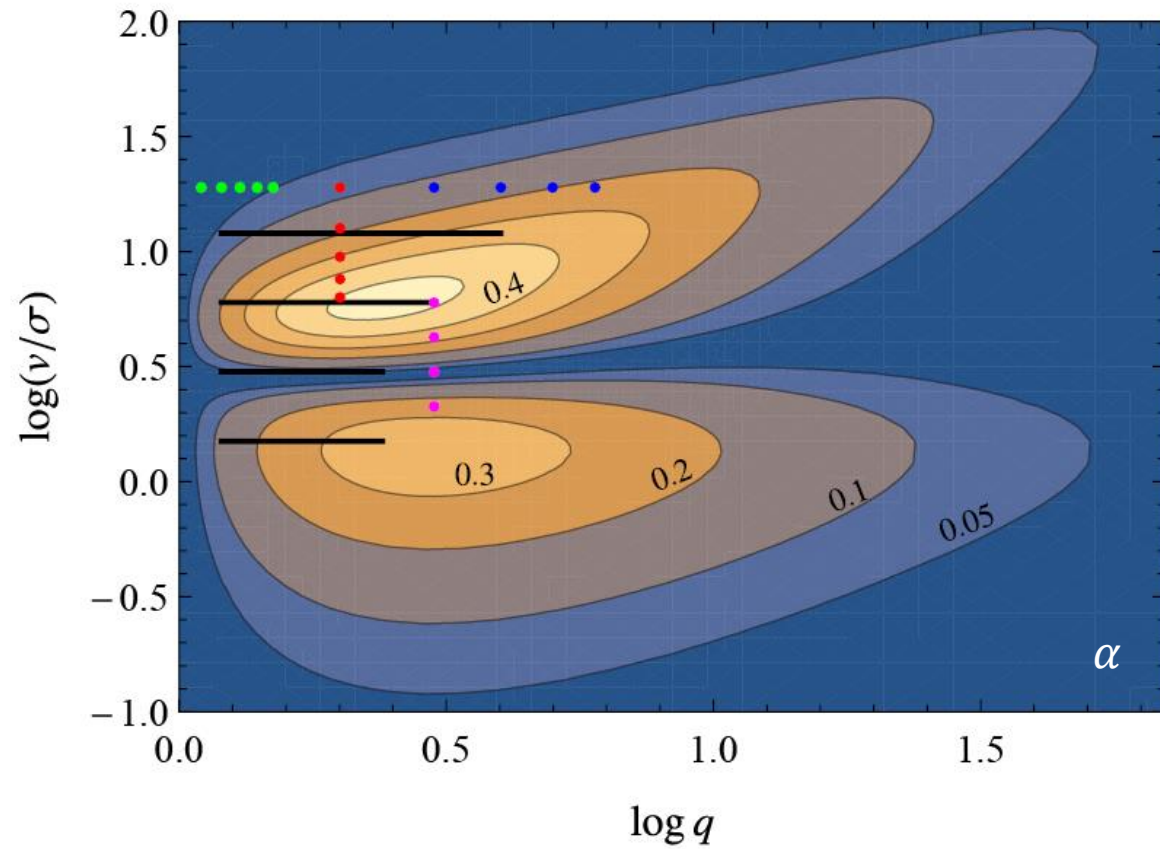
CM/drag: $\eta_1 = \frac{1}{q+1} \frac{(I_1 - q I_2)}{(I_1 + I_2)}.$

CM boost/drag: $\eta_2 = \frac{q}{q+1} \frac{(I_1 - q I_2)}{(I_1 + q^2 I_2)}.$

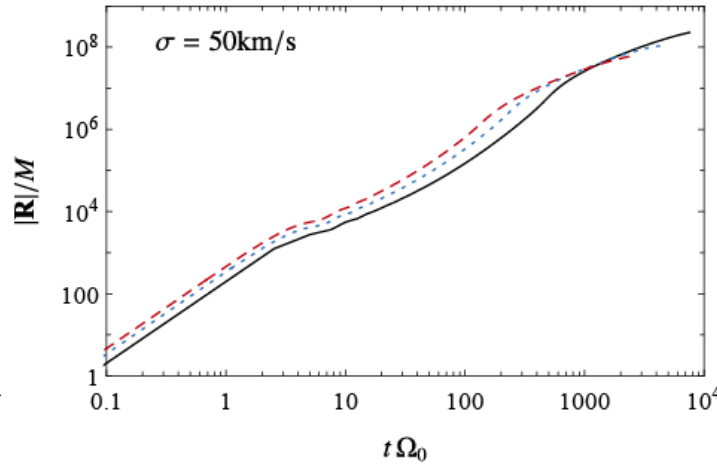
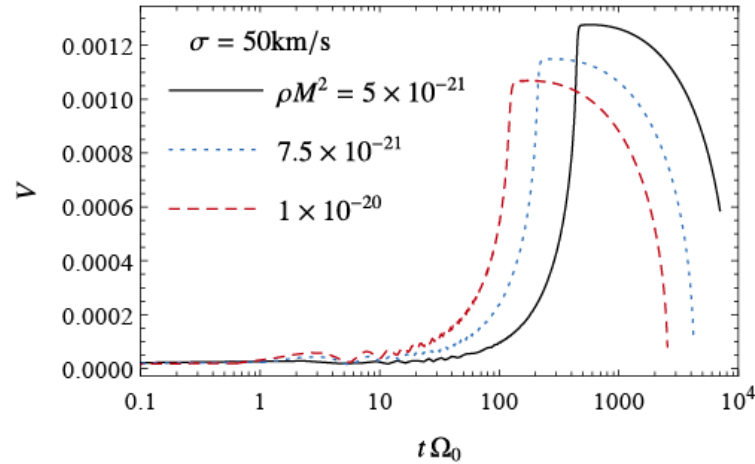
Let us first look into $q \neq 1$.

Accelerating the CM

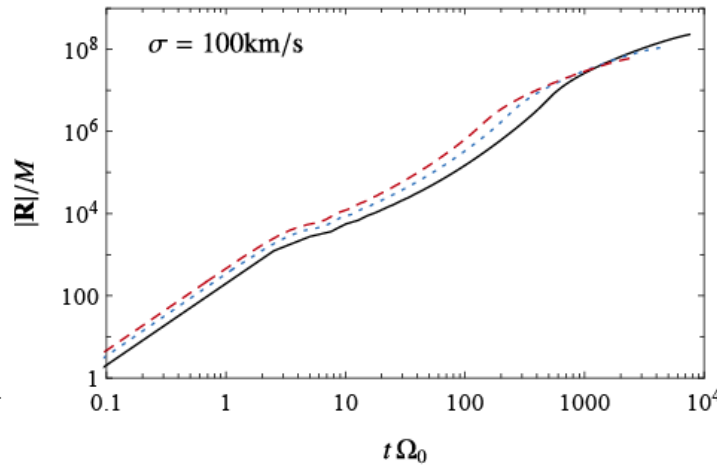
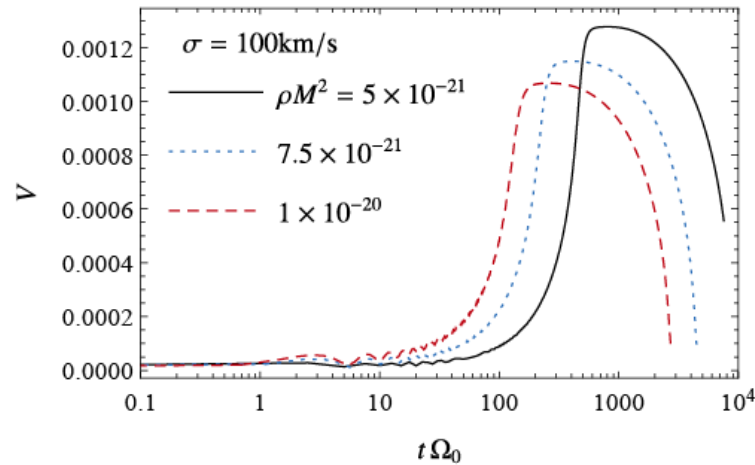
Cardoso&CFBM, 2008.01091 (2020)



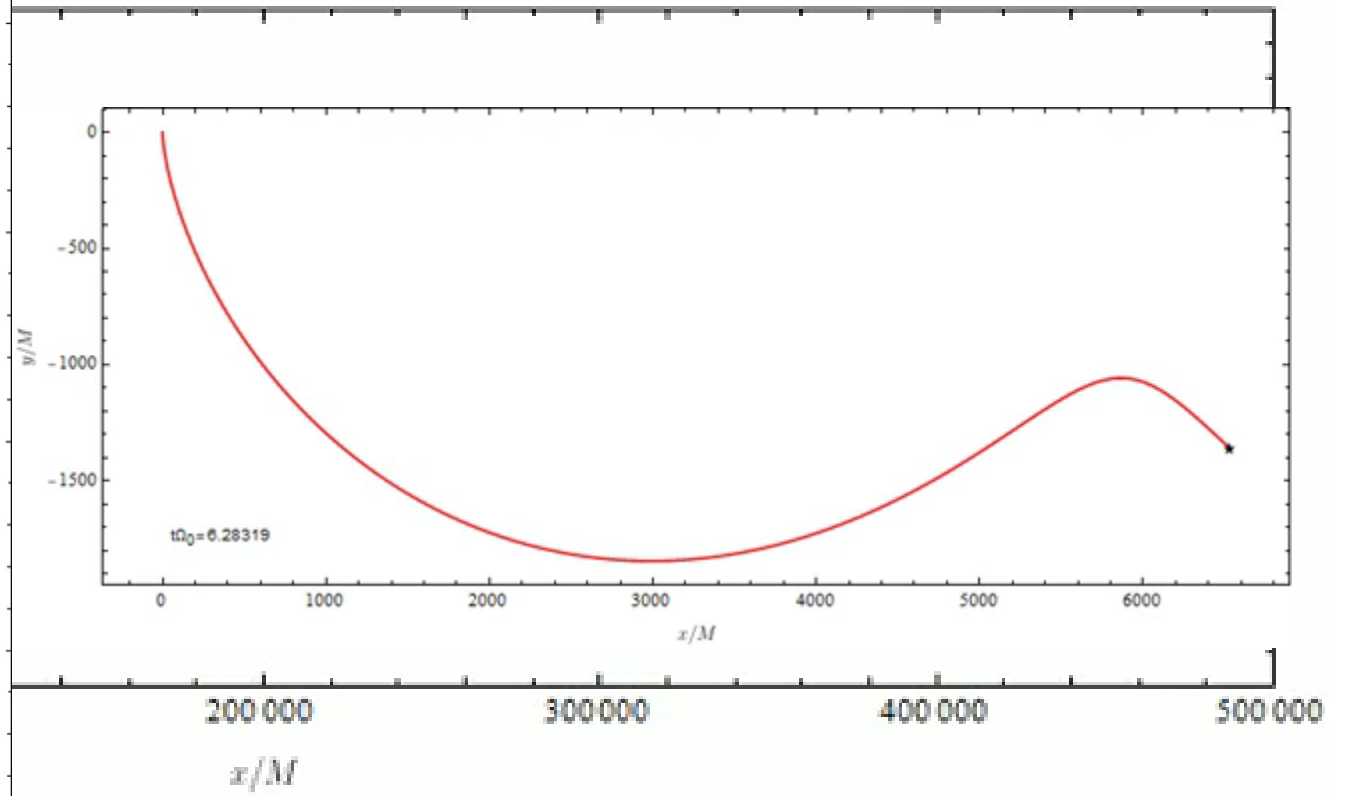
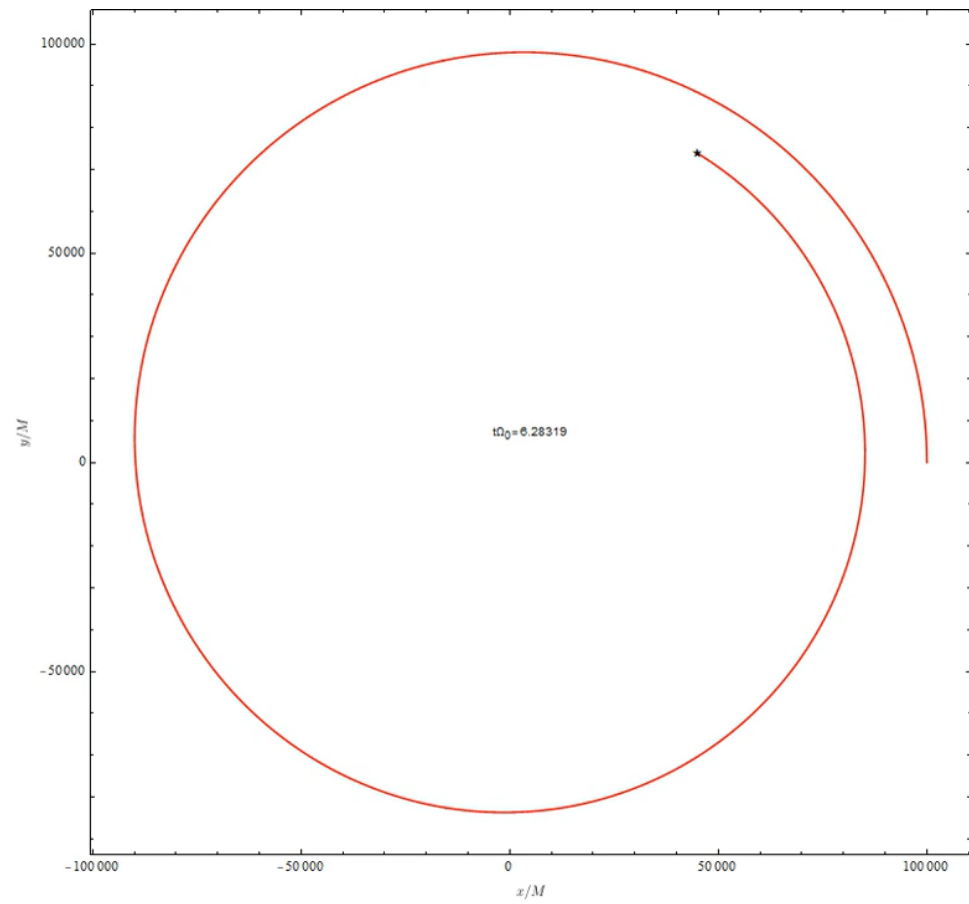
Boost in the center of mass

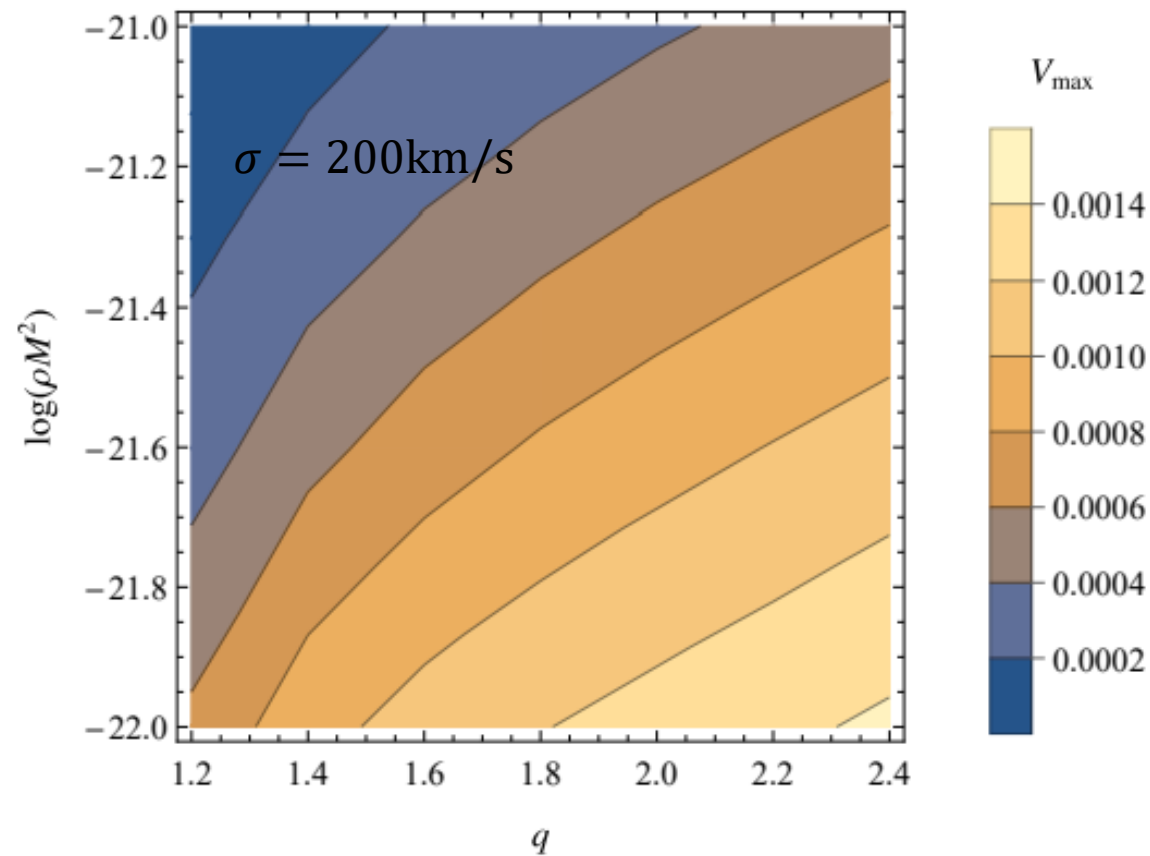
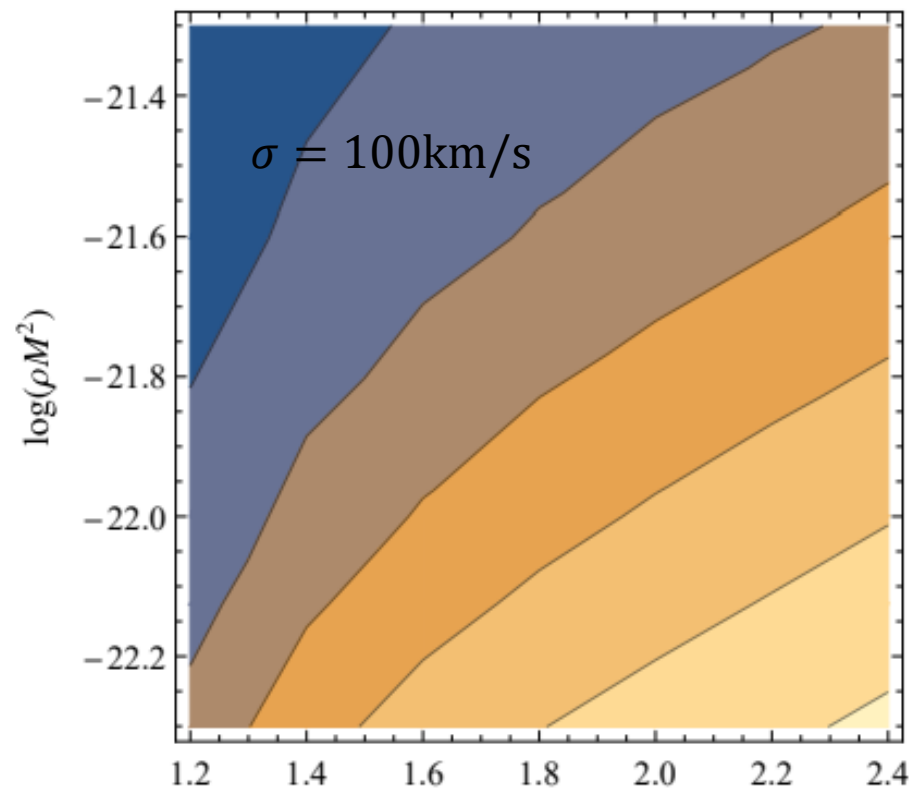


$$V \sim 350 \text{ km/s}$$

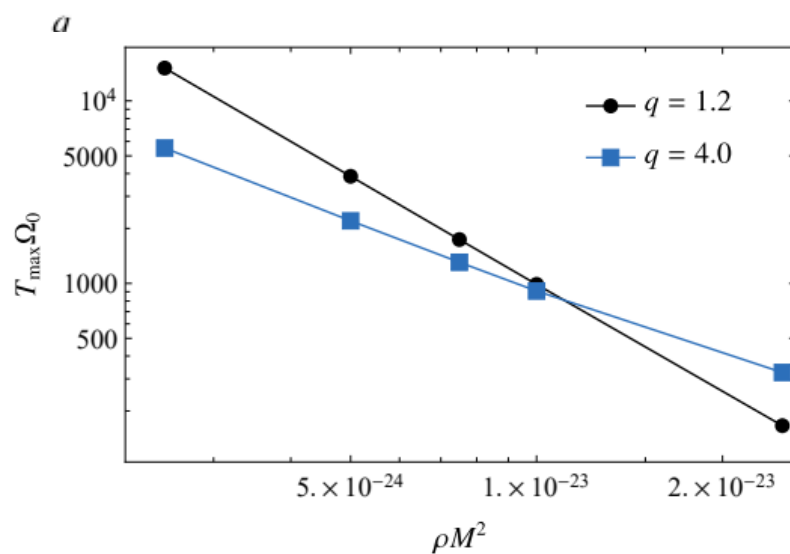


Binaries could escape galaxies





Time to the max speed



$$T_{\text{max}} = 6 \left(\frac{M_{\odot}}{M} \right)^3 \left(\frac{10^{-6} \rho_{\text{water}}}{\rho} \right)^2 \text{ years}$$

Eccentricity evolution and dynamical friction

Cardoso, CFBM, Vicente 2010.1515 (2021) Roedig&Sesana 1111.3742 (2012)

Considering dominant terms

$$\left. \begin{aligned} \langle \dot{a} \rangle &= -k\rho \sqrt{\frac{G a^5}{M}} \left(1 + \frac{3e^2}{4} + \mathcal{O}(e^4) \right), \\ \langle \dot{e} \rangle &= \frac{3}{2} k\rho \sqrt{\frac{G a^3}{M}} e \left(1 + \frac{3e^2}{8} + \mathcal{O}(e^4) \right). \end{aligned} \right\} \quad \frac{da}{de} = -\frac{2a}{3e} \left(1 + \frac{3}{8}e^2 + \mathcal{O}(e^3) \right)$$

Therefore, the environment favors eccentric motion. Considering both GW and the environment

$$\frac{da}{de} = \frac{6a \left(5c^5 k\rho \sqrt{GM a^{11}} + 32G^3 M^4 \right)}{e \left(304G^3 M^4 - 45c^5 k\rho \sqrt{GM a^{11}} \right)}$$



***Critical
distance***

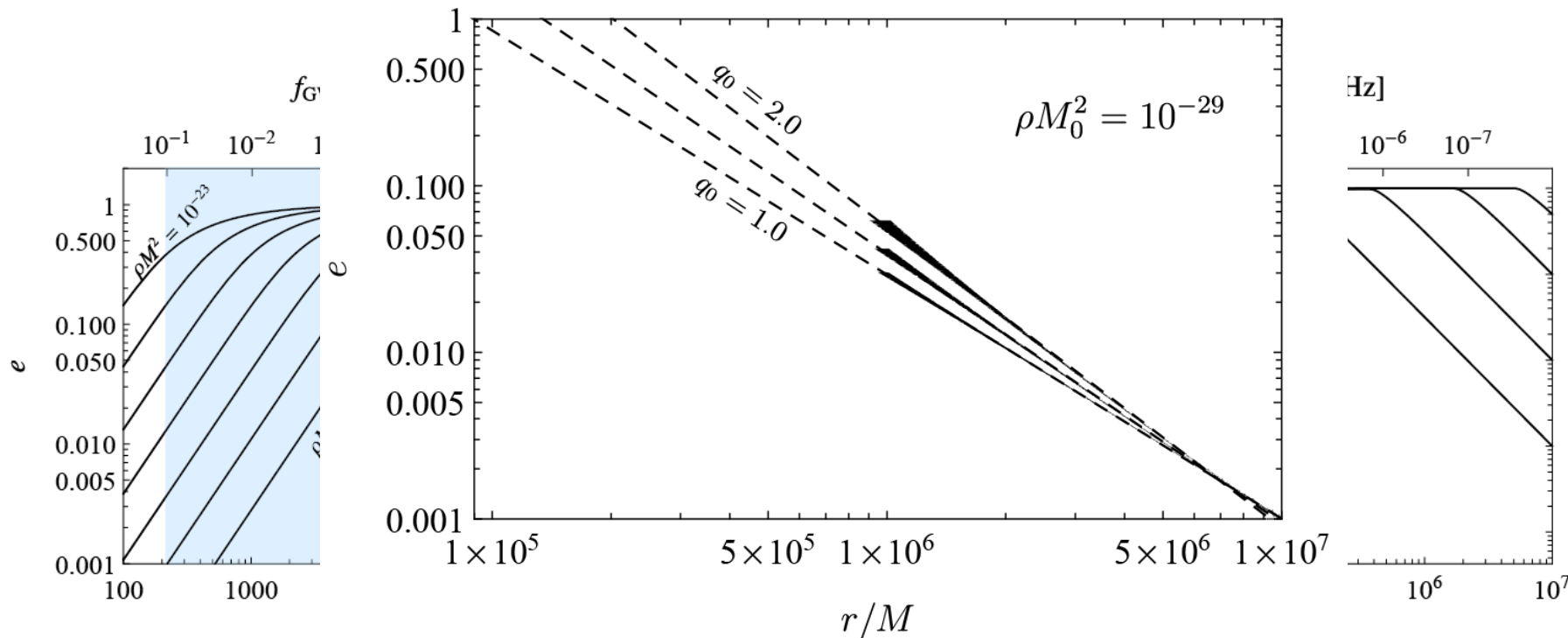
Eccentricity evolution and dynamical friction

Cardoso, CFBM, Vicente 2010.1515 (2021) Roedig&Sesana 1111.3742 (2012)

$$\frac{a_c}{\left(\frac{100GM_\odot}{c^2}\right)} = 3 \times 10^4 k^{-2/11} \left(\frac{M}{100M_\odot}\right)^{7/11} \left(\frac{\rho_{10}}{\rho}\right)^{2/11}$$

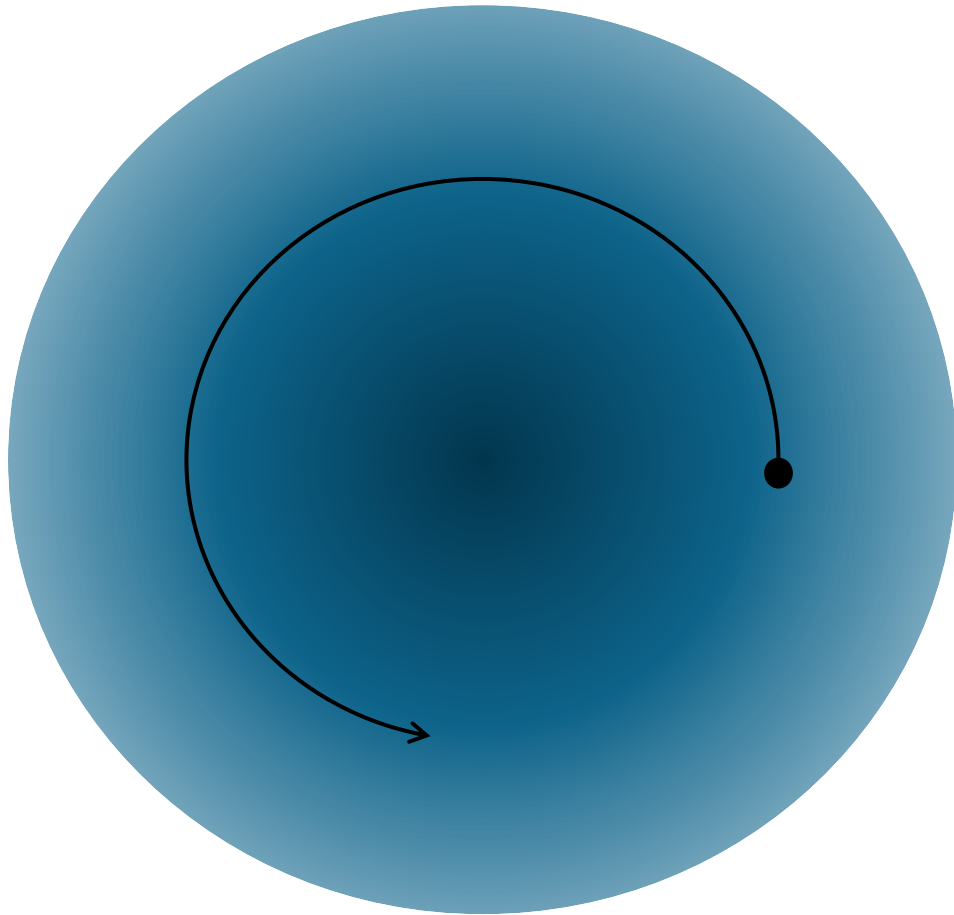
$$\rho_{10} = 10^{-10} \text{g cm}^{-3}$$

What about the evolution?



$$\frac{G^3}{c^6} \rho M^2 = 1.6 \times 10^{-24} \frac{\rho}{\rho_{10}} \left(\frac{M}{100M_\odot}\right)^2$$

A specific model with dynamical friction: boson stars



$$G_{\mu\nu} = 8\pi T_{\mu\nu}, \quad \square_g \Phi = \frac{\partial V}{\partial \Phi^*}$$

$$g_{\mu\nu} \approx \hat{g}_{\mu\nu} + q \delta g_{\mu\nu}, \quad \Phi \approx \hat{\Phi} + q \delta \Phi$$

Important to test BH paradigm. See Cardoso&Pani 2019

A specific model with dynamical friction: boson stars

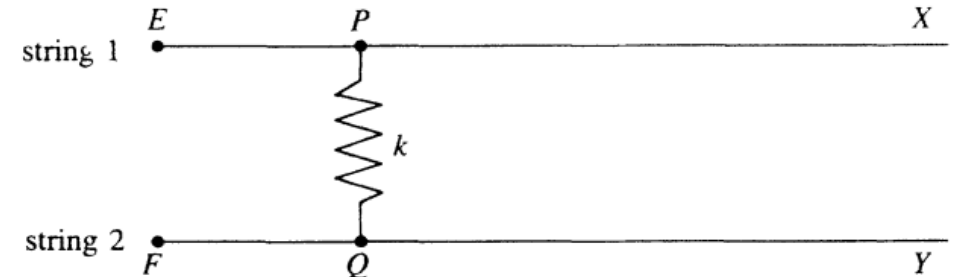
$$\begin{aligned}
 & -\frac{\partial^2 K}{\partial t^2} + \frac{\partial^2 K}{\partial^2 r_*} + 2\frac{\sqrt{AB}}{r} \frac{\partial K}{\partial r_*} - \left[\frac{A}{r^2} (\ell(\ell+1) + 2B - 2 + 2rB') - \frac{2A'B}{r} + 8\pi\omega^2\phi_0^2 + 8\pi AB\phi_0'^2 \right] K \\
 & = \left[\frac{2B}{r} + 2B' - \frac{2A'B}{A} + 8\pi\frac{r}{A}\omega^2\phi_0^2 + 8\pi Br\phi_0'^2 \right] S \\
 & + 16\pi\frac{AB}{r^2}\phi_0'(\phi_+ + \phi_-) - 8\pi A\delta V - 8\pi\mathcal{A}_{\ell m}^0 - \frac{8\pi AB r}{\sqrt{n(n+1)/2}} \frac{\partial \mathcal{F}_{\ell m}}{\partial r} \\
 & + \frac{4\pi}{\sqrt{n(n+1)/2}} \left[2rA'B - A(\ell(\ell+1) + 2 + 2B + 2rB') - 8\pi r^2\omega^2\phi_0^2 - 8\pi r^2 AB\phi_0'^2 + 8\pi r^2 A\hat{V} \right] \mathcal{F}_{\ell m}
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{\partial^2 S}{\partial t^2} + \frac{\partial^2 S}{\partial^2 r_*} - \left[\frac{A}{r^2} (\ell(\ell+1) + 2B - 2 + \frac{r}{2}B') + \frac{A'B}{2r} - 8\pi\omega^2\phi_0^2 + 8\pi AB\phi_0'^2 + 8\pi A\hat{V} \right] S \\
 & = \frac{A}{r} \left[\frac{2A'B}{r} - A'B' - 2A''B \right] K + \frac{8\pi}{r^2} \left[\left(A^2 \left(B' - \frac{2B}{r} \right) + 2AA'B \right) \phi_0' + 2A^2 B\phi_0'' \right] (\phi_+ + \phi_-) \\
 & + 8\pi\sqrt{2} \frac{A^2}{r} \mathcal{G}_{\ell m} - \frac{8\pi Ar}{\sqrt{n(n+1)/2}} \frac{\partial^2 \mathcal{F}_{\ell m}}{\partial t^2} + \frac{4\pi AB}{\sqrt{n(n+1)/2}} (2A + rA') \frac{\partial \mathcal{F}_{\ell m}}{\partial r} \\
 & + \frac{4\pi}{\sqrt{n(n+1)/2}} \frac{A}{r} \left[2rA'B + r^2 B \frac{A'^2}{A} - A(\ell(\ell+1) - 4 + 4B) + 16\pi r^2\omega^2\phi_0^2 - 16\pi r^2 AB\phi_0'^2 - 16\pi r^2 A\hat{V} \right] \mathcal{F}_{\ell m}
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{\partial^2 \phi_+}{\partial t^2} + \frac{\partial^2 \phi_+}{\partial^2 r_*} + 2i\omega \frac{\partial \phi_+}{\partial t} + \left[\omega^2 - \frac{A}{r^2} \ell(\ell+1) - \frac{AB' + A'B}{2r} - A\hat{U} \right] \phi_+ - 8\pi AB\phi_0'^2 (\phi_+ + \phi_-) - rA\phi_0 \delta U \\
 & = -2ir\omega\phi_0 \frac{\partial K}{\partial t} - i\frac{r^2}{A}\omega\phi_0 \frac{\partial S}{\partial t} + i\frac{r^2}{A}\sqrt{\frac{B}{A}}\omega\phi_0 \frac{\partial \tilde{H}_1}{\partial r_*} - \left[\frac{r^2}{A}\omega^2\phi_0 + rB\phi_0' \left(r\frac{A'}{2A} - r\frac{B'}{2B} - 2 \right) - r^2 B\phi_0'' \right] \left(S + \frac{A}{r}K \right) \\
 & + i\frac{r^2\omega}{2A} \left[\left(6\frac{B}{r} + B' - 3\frac{A'B}{A} \right) \phi_0 + 4B\phi_0' \right] H_1 \\
 & + i\frac{4\pi r^3\omega\phi_0}{\sqrt{n(n+1)/2}} \frac{\partial \mathcal{F}_{\ell m}}{\partial t} - \frac{4\pi r^3 AB\phi_0'}{\sqrt{n(n+1)/2}} \frac{\partial \mathcal{F}_{\ell m}}{\partial r} - \frac{4\pi r^2 AB}{\sqrt{n(n+1)/2}} \left[\left(4 + r\frac{B'}{B} \right) \phi_0' + 2r\phi_0'' \right] \mathcal{F}_{\ell m}, \tag{30}
 \end{aligned}$$

Simple model: Semi-infinity strings and a massless spring

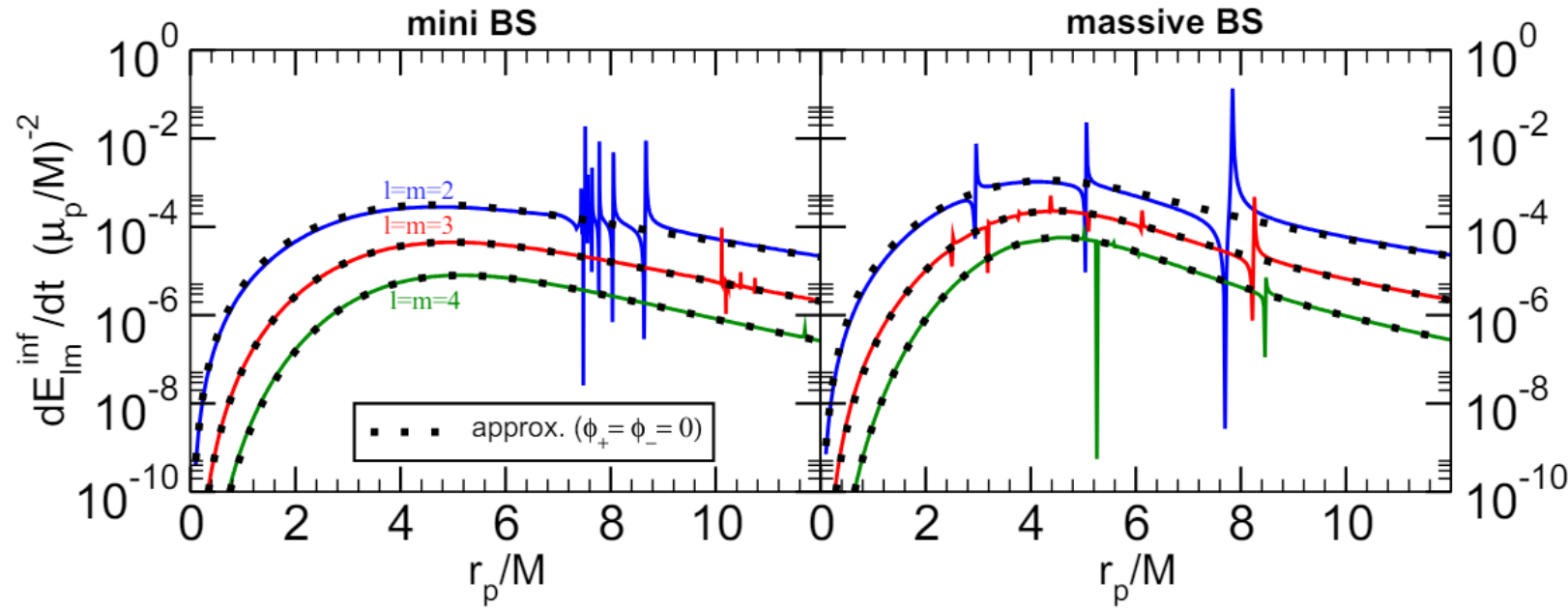
Kokkotas&Schutz 1986, Yoshida 1994



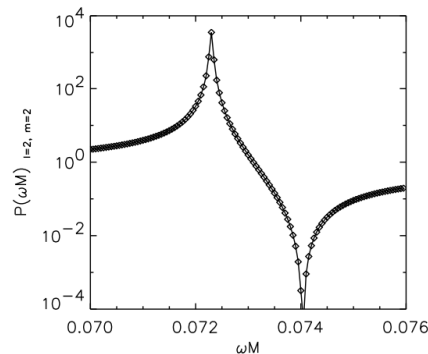
Mode excitation: Resonances

Macedo et al. Phys.Rev.D 88 6, 064046 (2013) & Astrophys.J. 774 (2013).

See also
Y. Kojima 1987
J. Pons et al. 2002



$$\omega_r \approx \omega_{DW} \pm \omega_{BS}$$

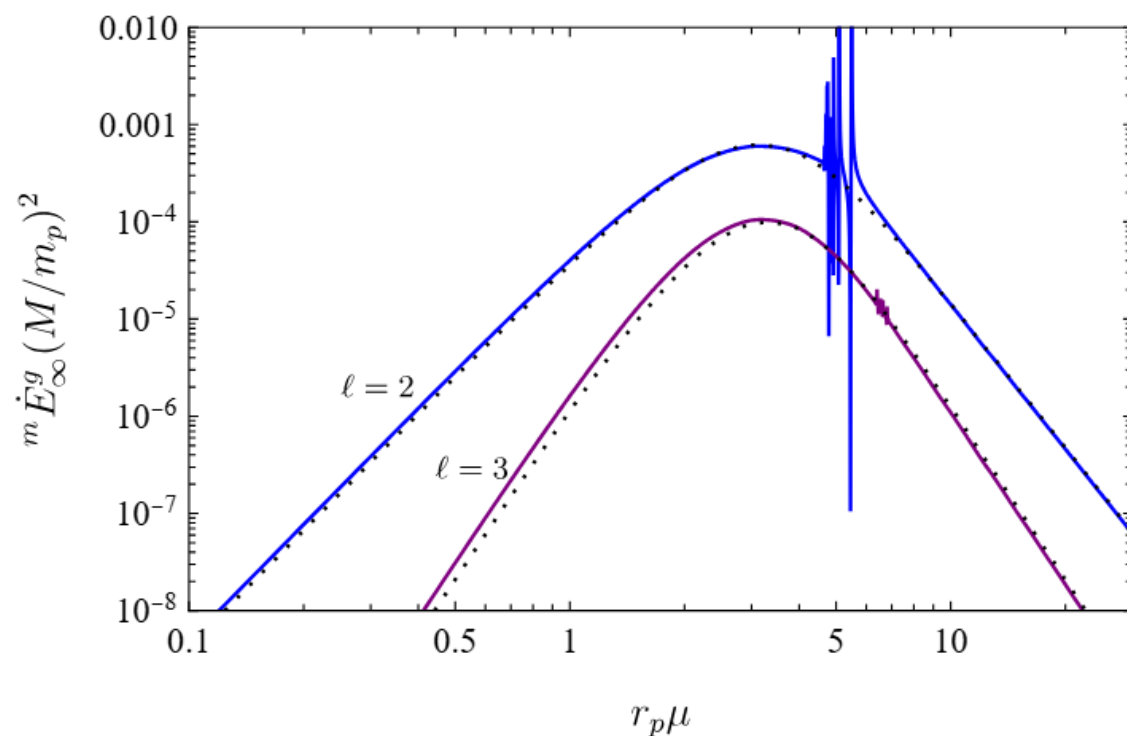


In order to understand physical picture of this resonance, we consider the following **forced harmonic oscillator with damping**, mimicking the system for the star and gravitational waves:

$$\ddot{\xi}(t) + 2\Gamma\dot{\xi}(t) + \omega_0^2\xi(t) = \alpha\omega^2 e^{-i\omega t}, \quad (51)$$

Mode excitation: Resonances

Fitting



$$\dot{E}_\infty^g \approx |g_{tt}(r_p)|^\alpha \dot{E}_{\text{NGW}}$$

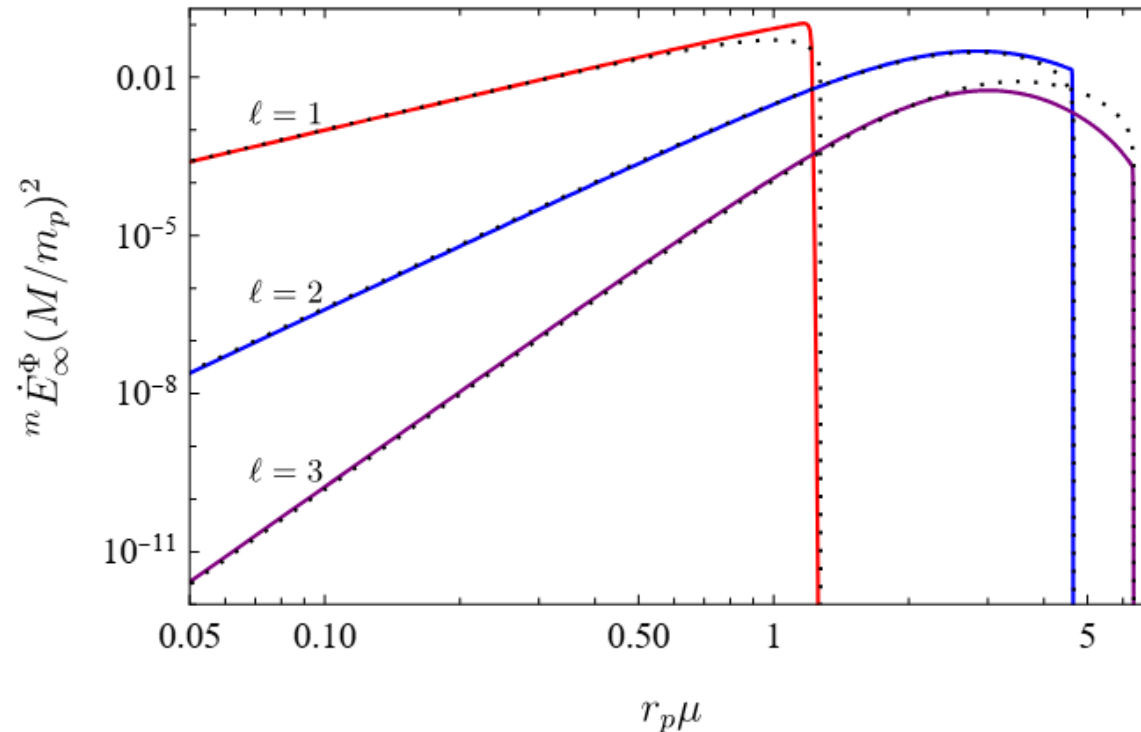
$$\alpha \approx 1$$

Therefore, we can have a good semi-analytical fitting for the GW part.

Bosonic fluxes

Fitting

[Annulli et al. 2020, Duque et al. 2024]

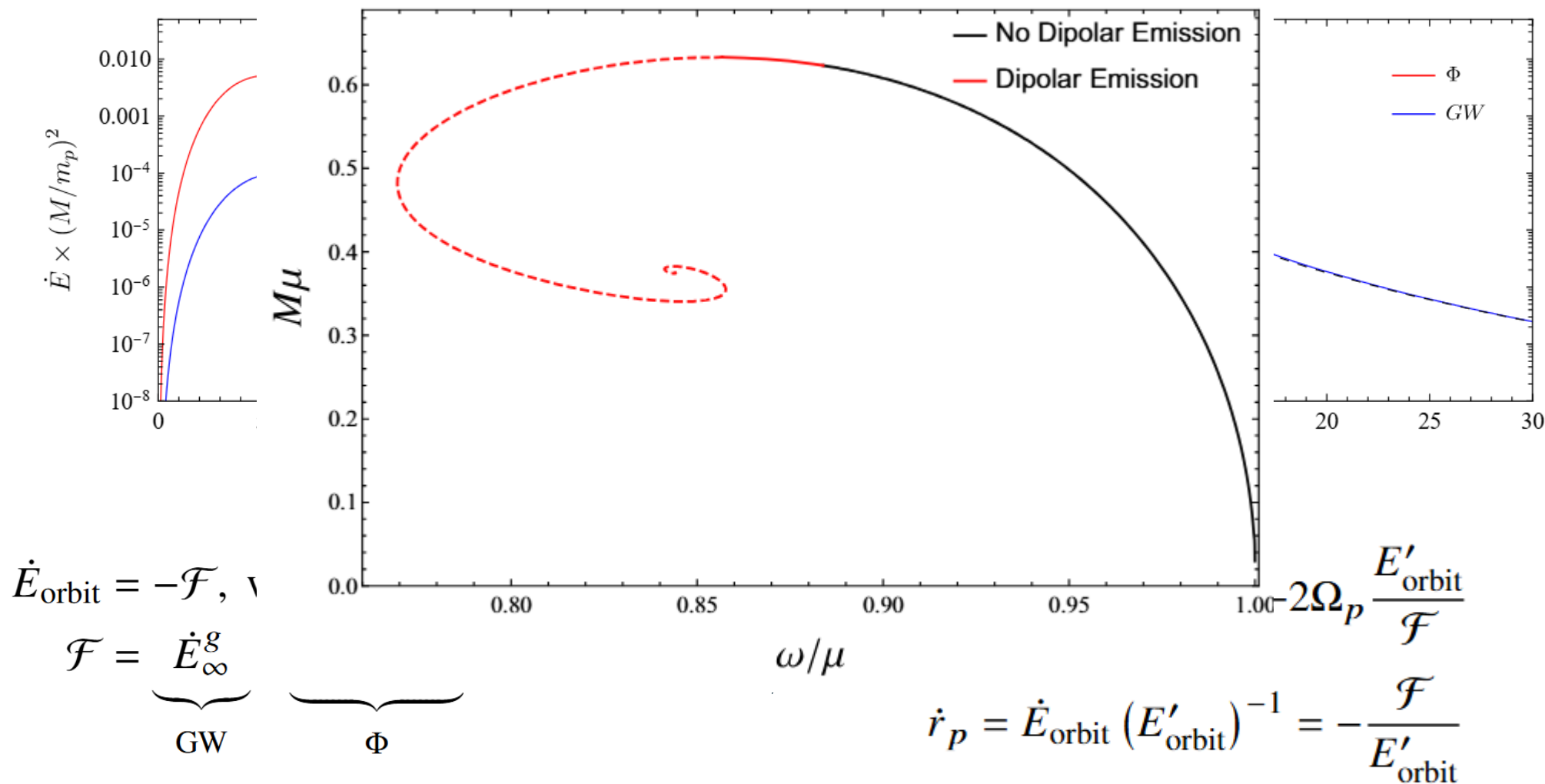


$$m \dot{E}_\infty^\Phi = \frac{m \dot{E}_{\text{N}\Phi}}{|g_{tt}(r_p)|^\beta}$$

Given a BS, we can find semi-analytical fits for both GW and bosonic fluxes.

Binary evolution and waveforms

(in prep.)

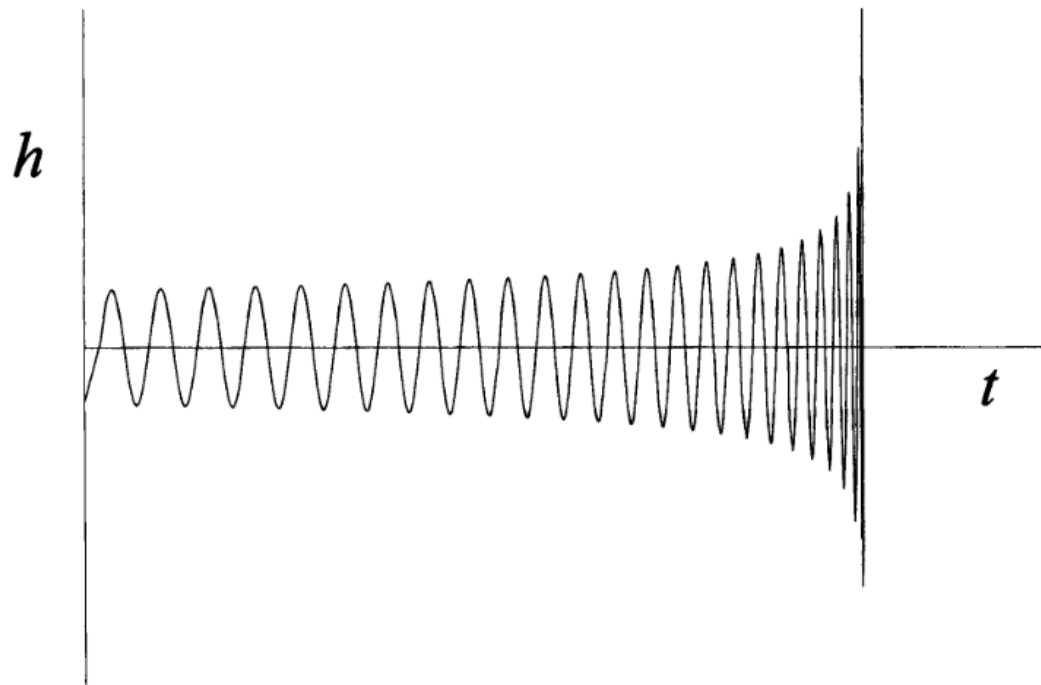


Binary evolution and waveforms

(in prep.)

If we look into the quadupolar approximation

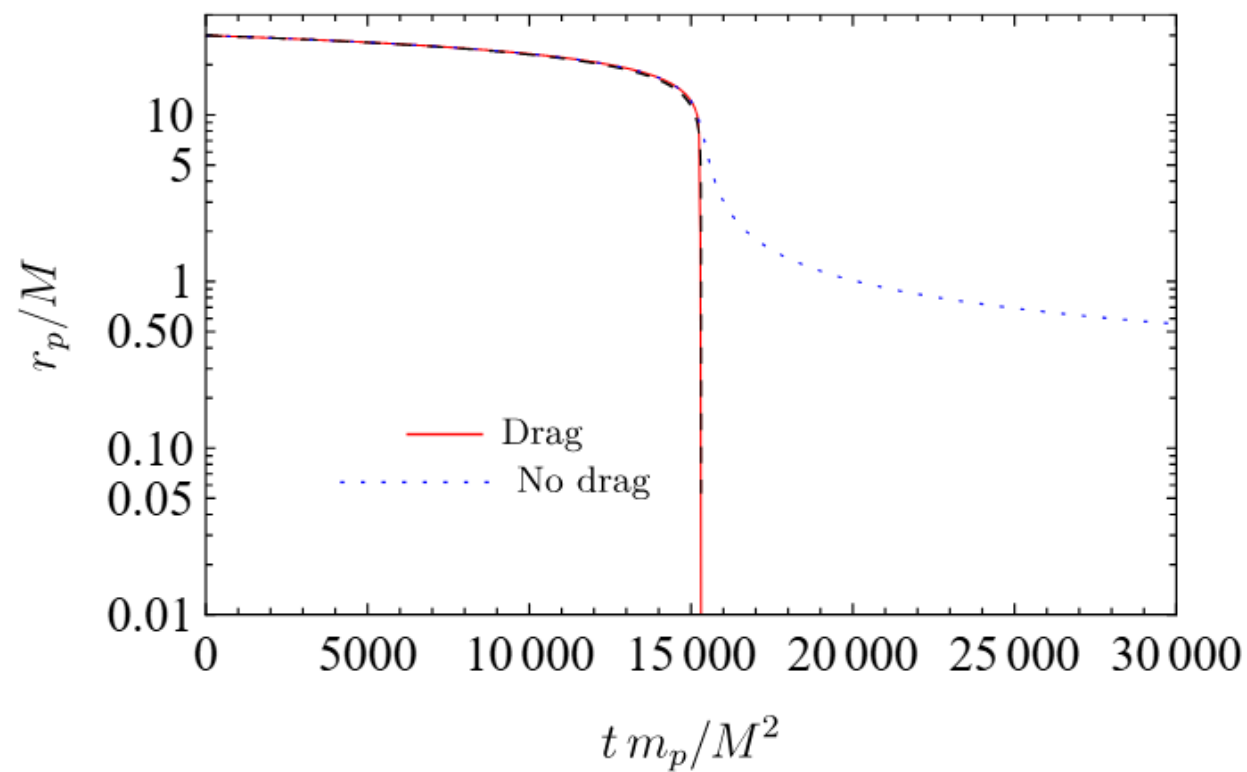
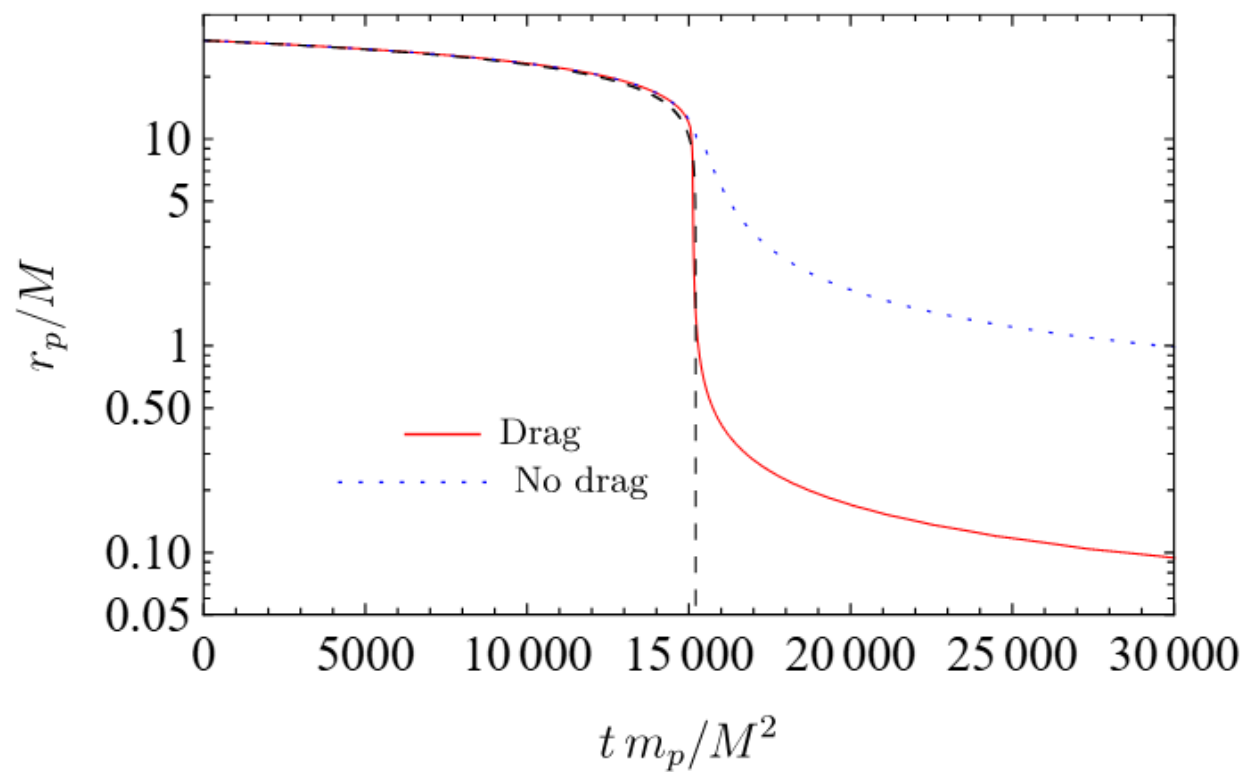
$$r_{\text{quad}} = r_0 \left(1 - \frac{t}{t_c} \right)^{1/4}$$



A similar result holds considering the flux computed with the Schwarzschild BH.

Binary evolution and waveforms

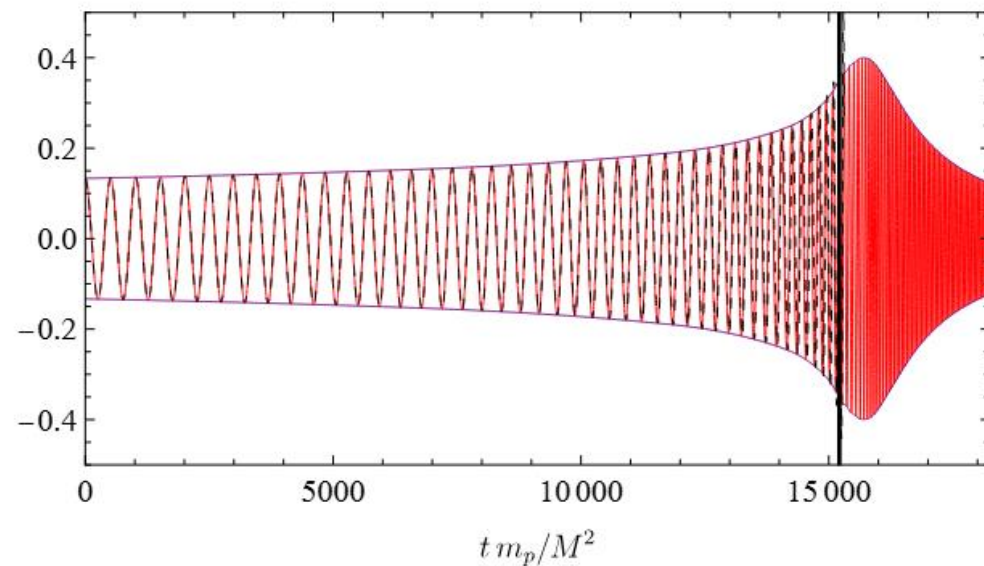
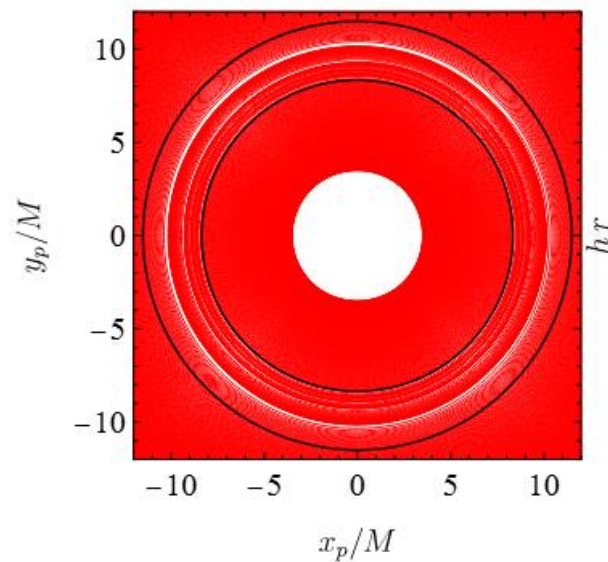
(in prep.)



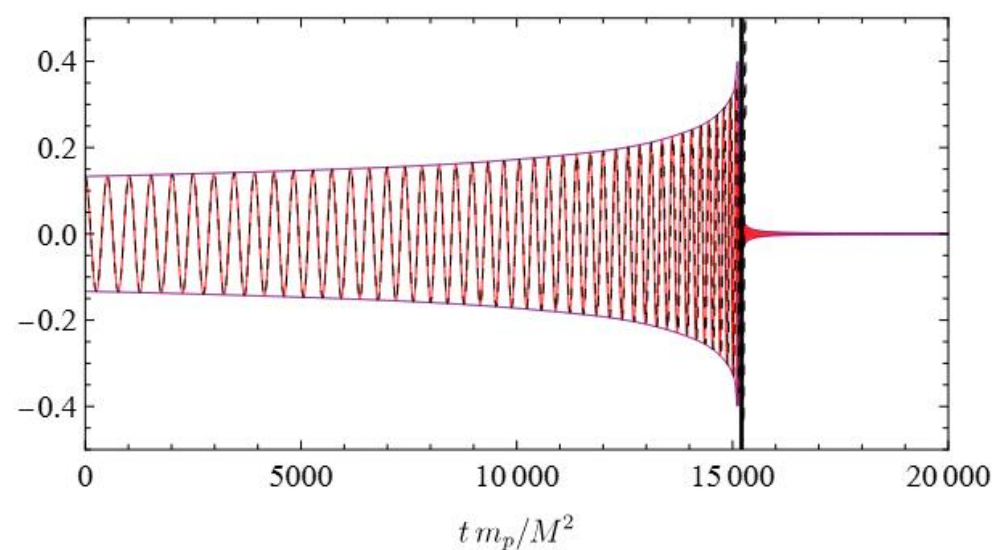
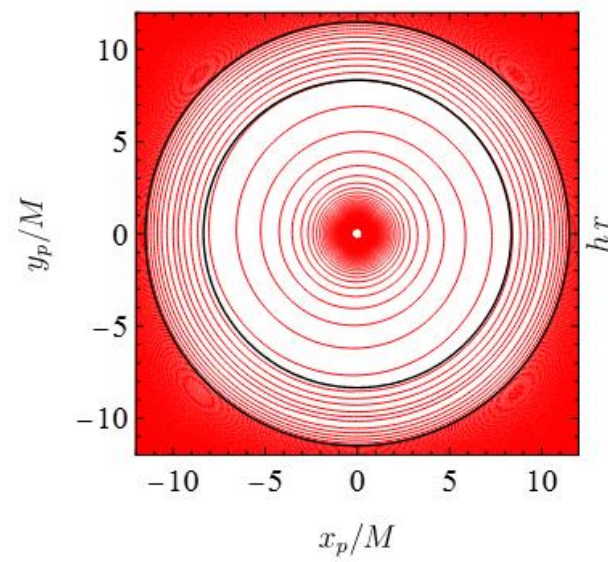
Binary evolution and waveforms

(in prep.)

No DF:



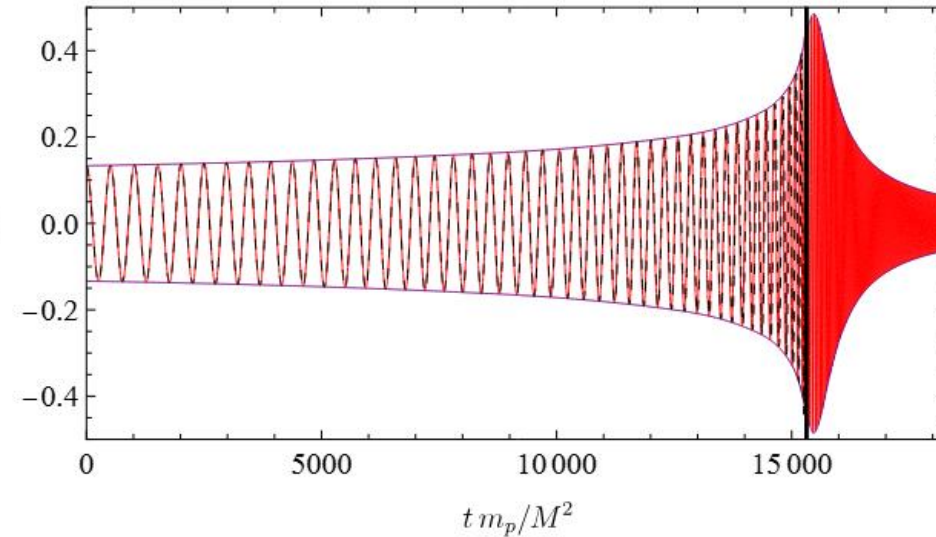
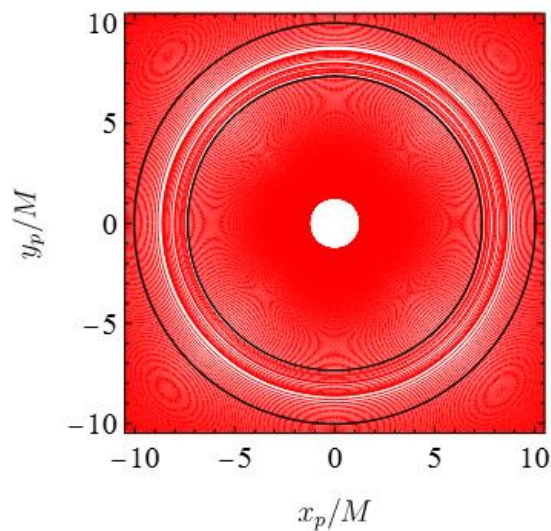
With DF:



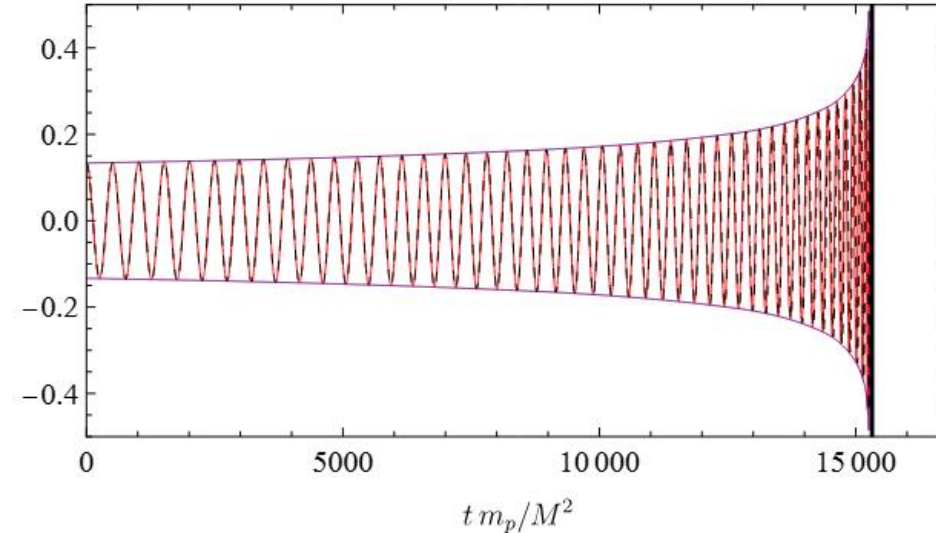
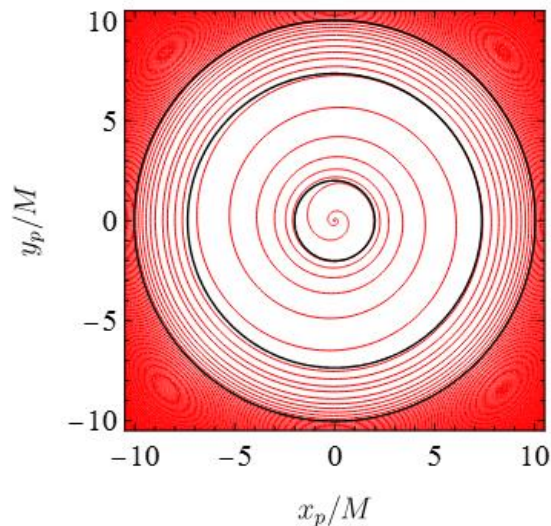
Binary evolution and waveforms

(in prep.)

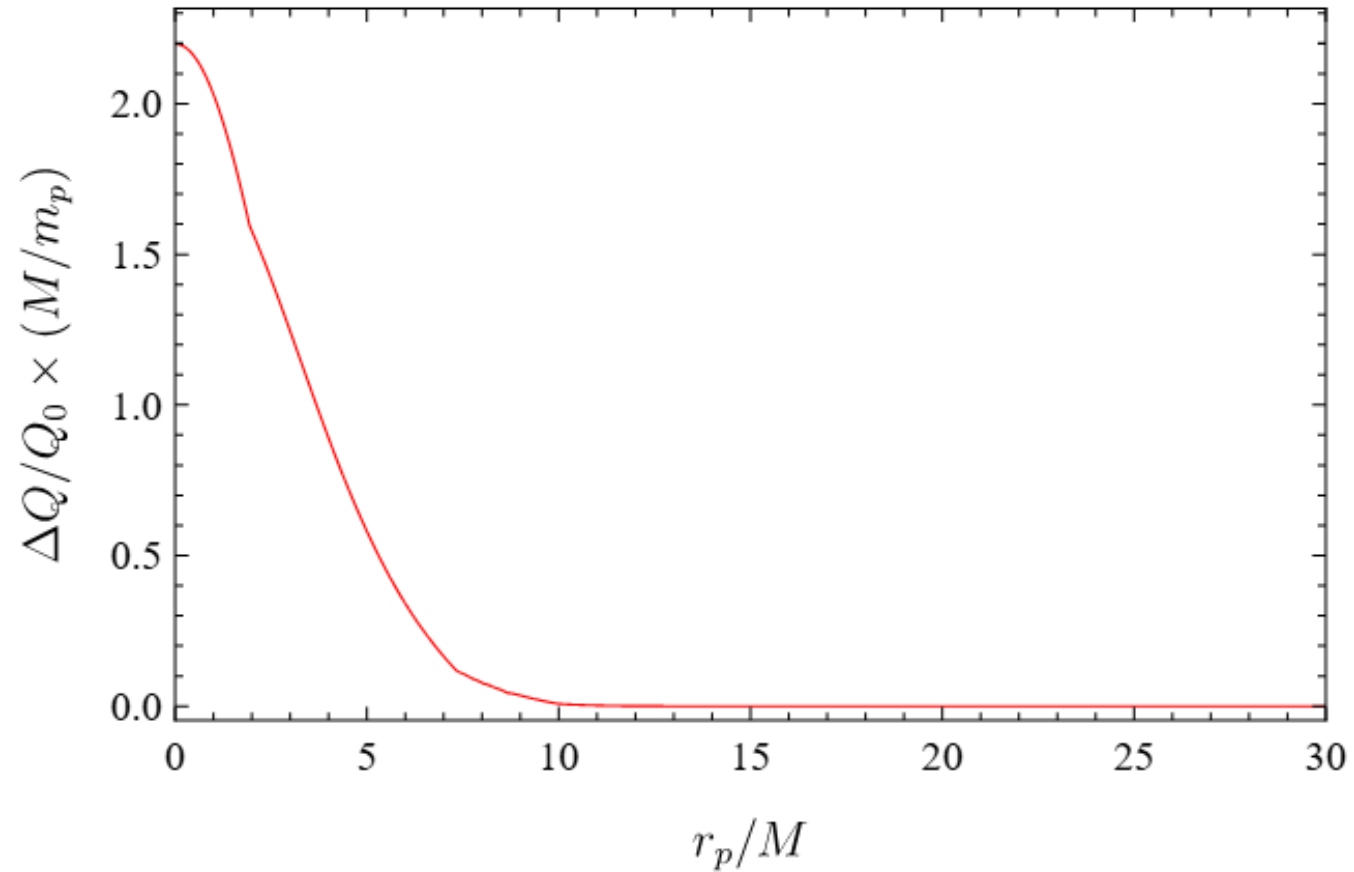
No DF:



With DF:

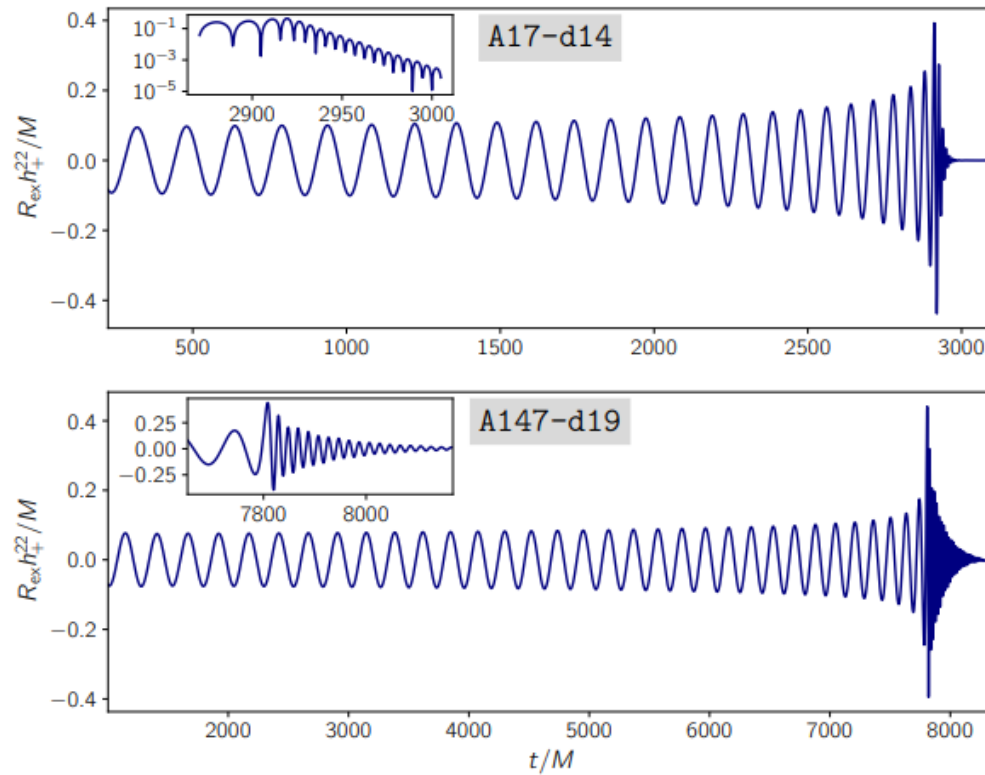


Does the BS survive?

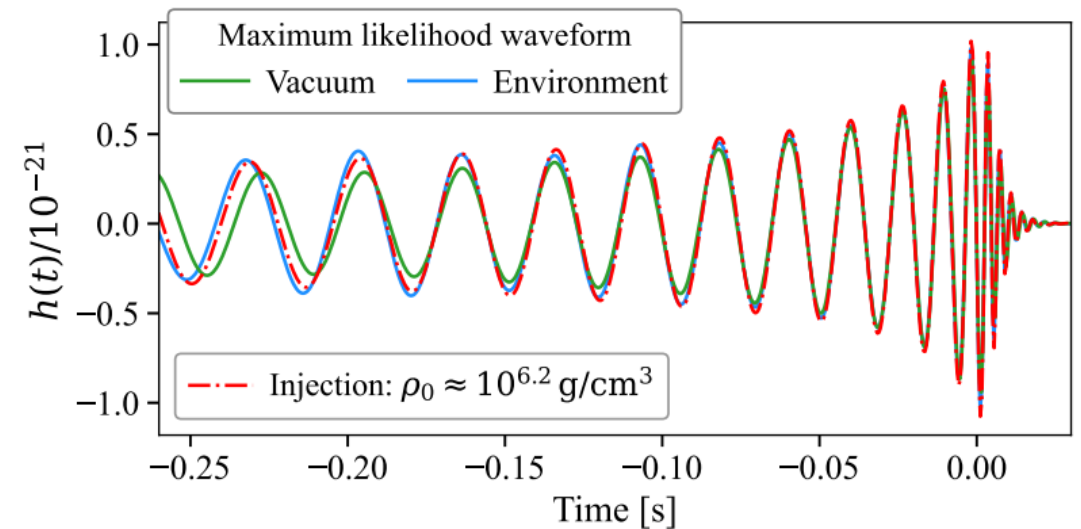


$$\frac{\Delta Q}{Q_0} \sim (m_p/M)$$

Can dynamical friction give support for non-UCO BH mimickers?



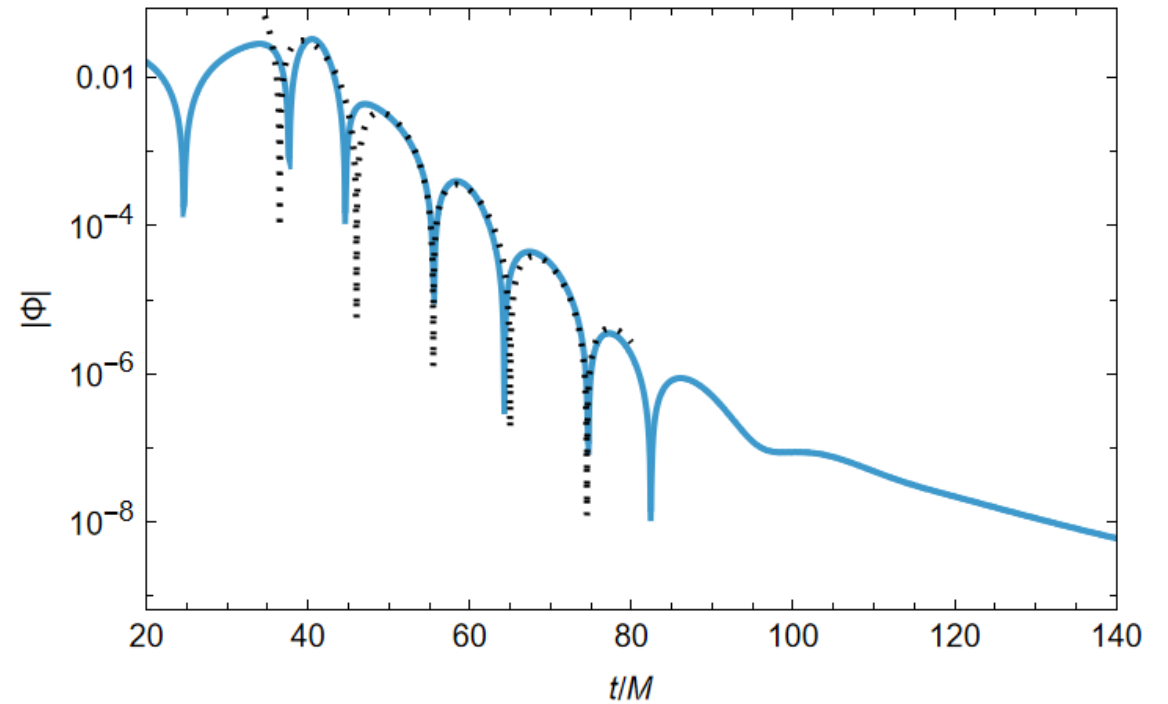
Evstafyeva et al. 2024



Roy&Vicente 2025

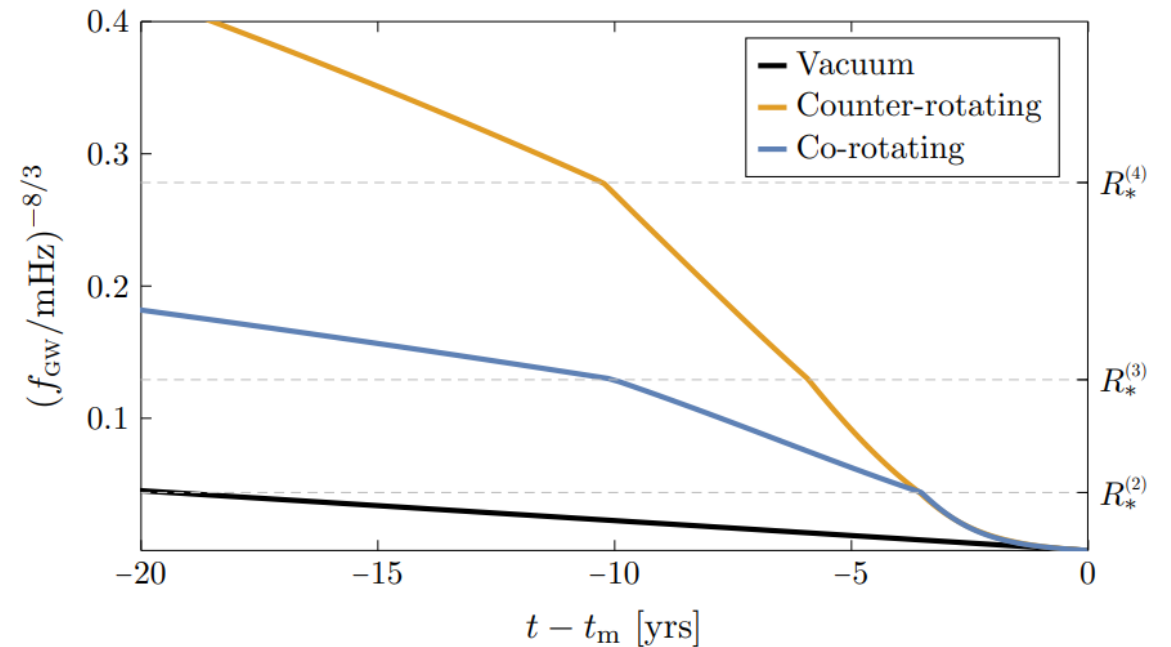
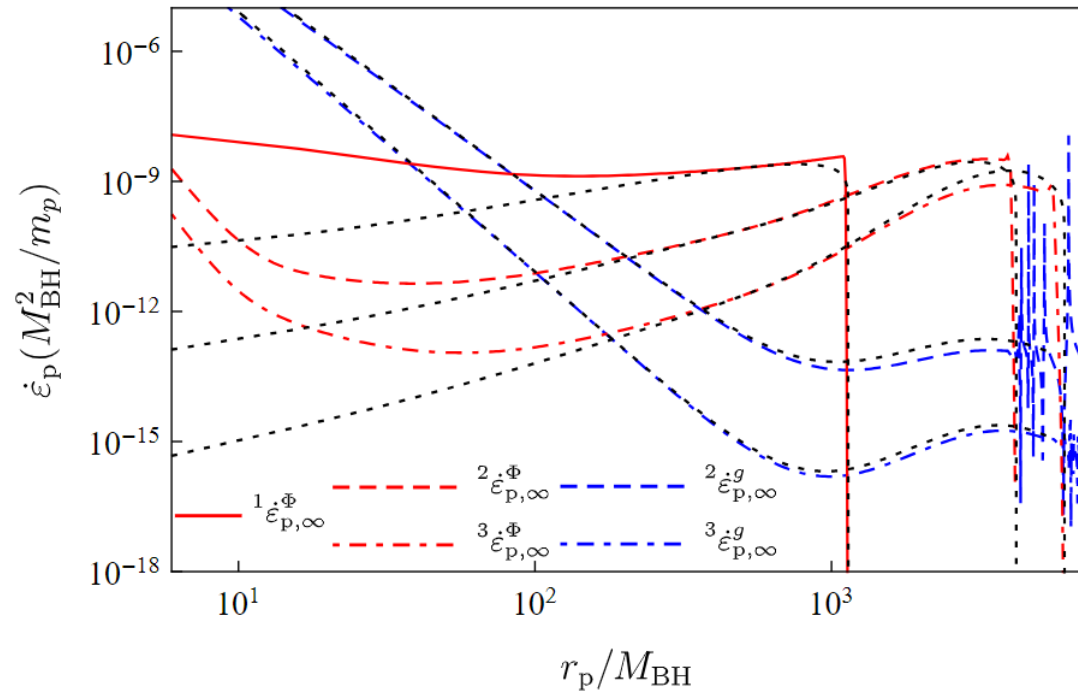
What about the ringdown in BS? I don't know.

- Additional modes? How do they appear?
- Can it mimick *a* BH?
- What is the role of the self interaction?
- In BS+BS, what is the role of the phase?



EMRI: BH surrounded by (scalar) dark matter

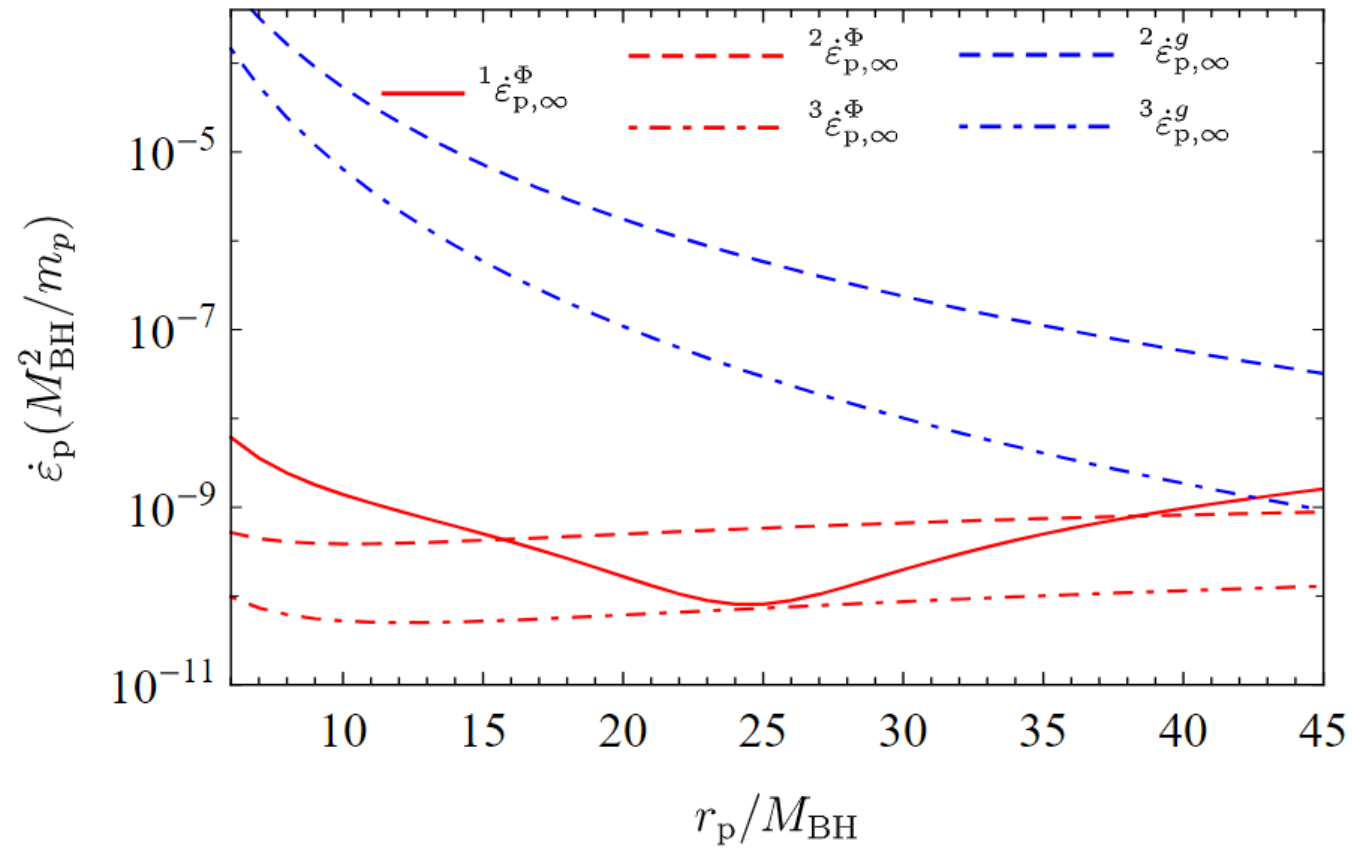
Duque et al. arXiv:2312.06767 (2023)



Baumann et al. 2022

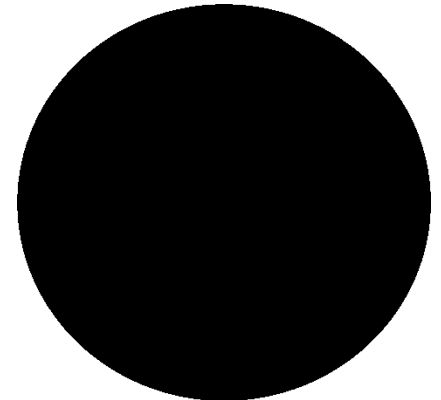
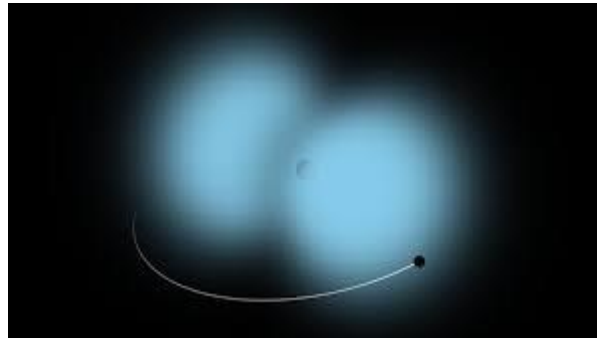
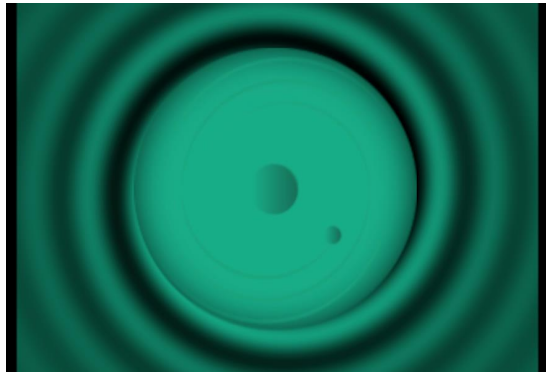
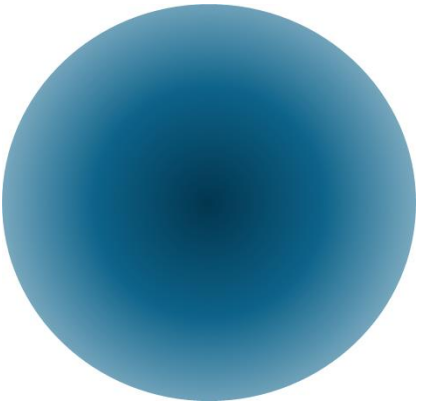
EMRI: BH surrounded by a cloud

Duque et al. arXiv:2312.06767 (2023). Brito, Shah Phys.Rev.D 108 8, 084019 (2023)



Take away

- Environmental effects are important;
- Binaries can be kicked from their host galaxies;
- Circularization is challenged;
- Non-UCO mimickers;
- Dephasing on LISA band.



Thank you!

Obrigado!

감사합니다!