Yukawas in F-theory GUTs

fernando marchesano

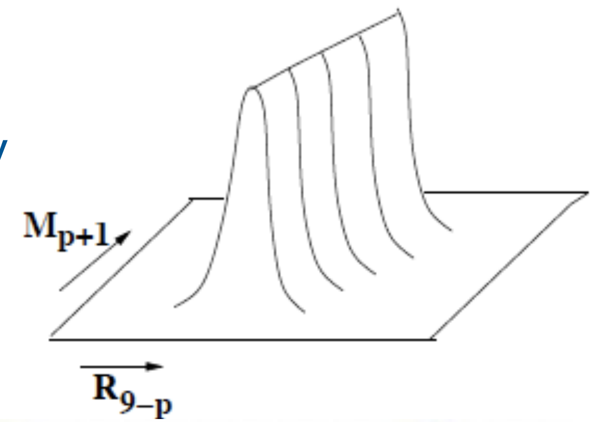
Instituto de Física Teórica
UAM-CSIC

GUTs from F-theory

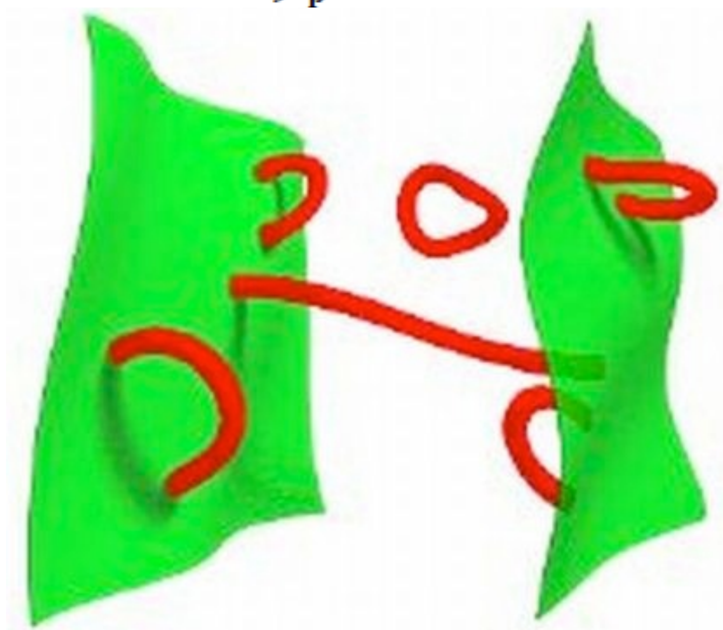
- F-theory GUT models have recently attracted a lot of attention, and shown to contain a number of phenomenological virtues
- They provide a new way to build realistic 4d models from string theory
- They contain the best of both heterotic and D-brane worlds.

Introducing D-branes

- Dp-branes are solitonic objects that appear in 10D superstring and supergravity theories
- In supergravity they are seen as lumps of energy extended along p+1 dimensions with certain
    - Charge
    - Tension



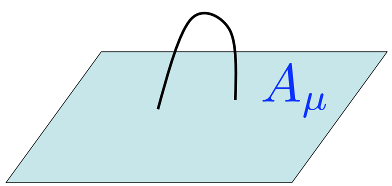
- In string theory, they can be described at a more fundamental level, namely as p+1 dimensional hypersurfaces where open string endpoints are confined



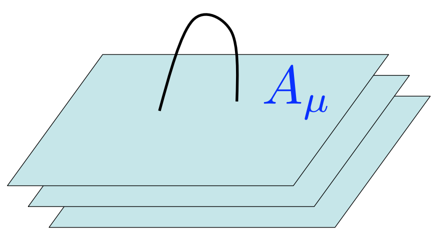
D = Dirichlet boundary cond.

Introducing D-branes

- This string theory description of D-branes allows to unveil one of their most fundamental properties



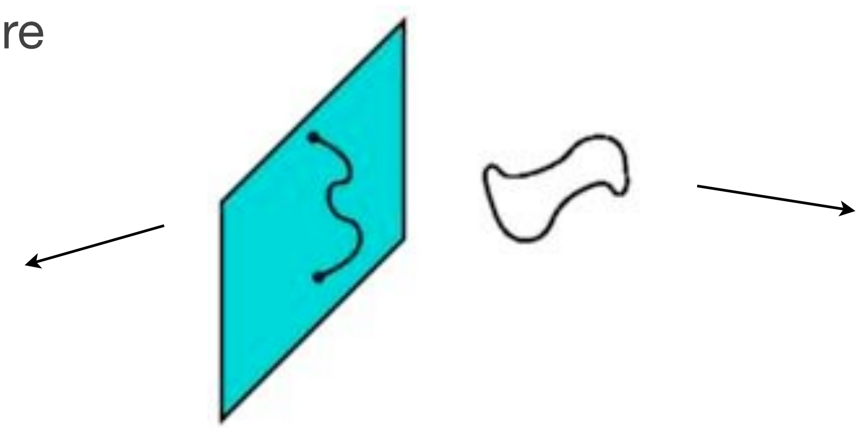
$\Rightarrow$ U(1) gauge theory



$\Rightarrow$ U(N) gauge theory

- General picture

confined
(p+1) dim.
gauge theory



unconfined
10 dim.
gravity

Why do we like D-branes?

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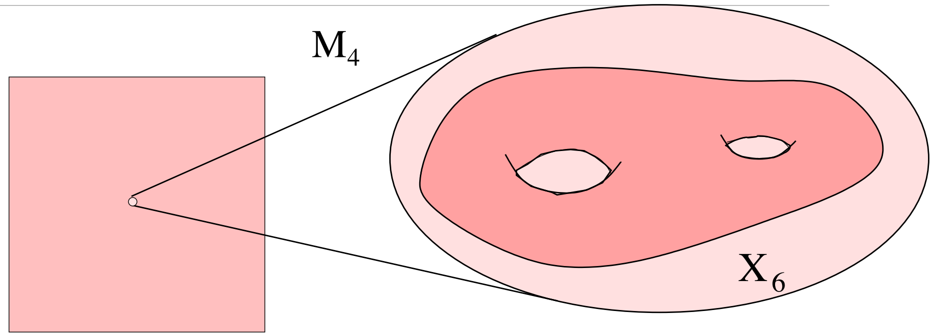

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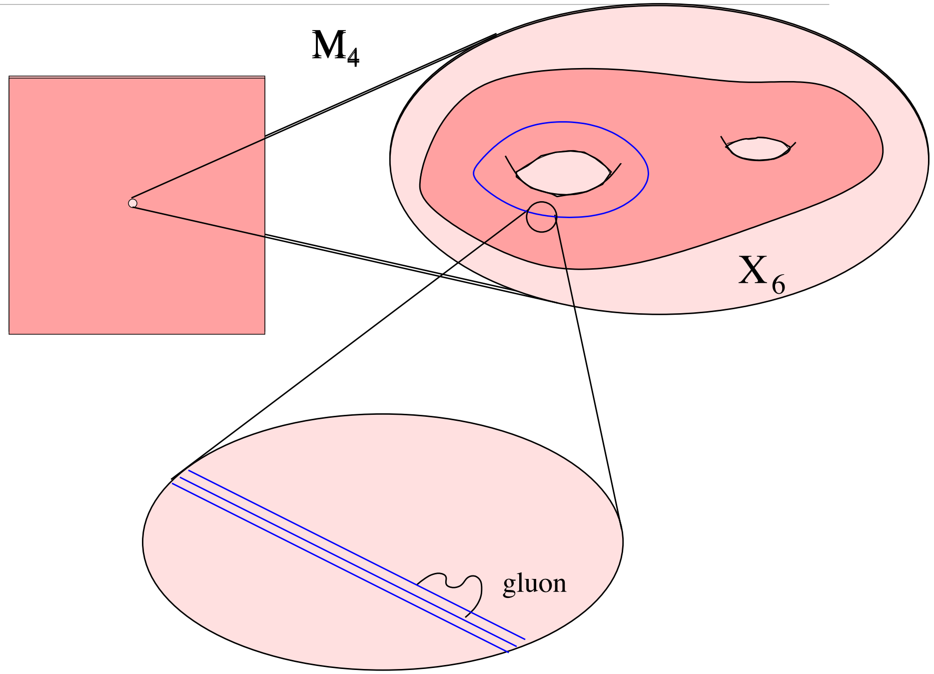
- Gauge interactions are localised on D*p*-branes which have
    - 3+1 observable dim.
    - p-3 internal dim.



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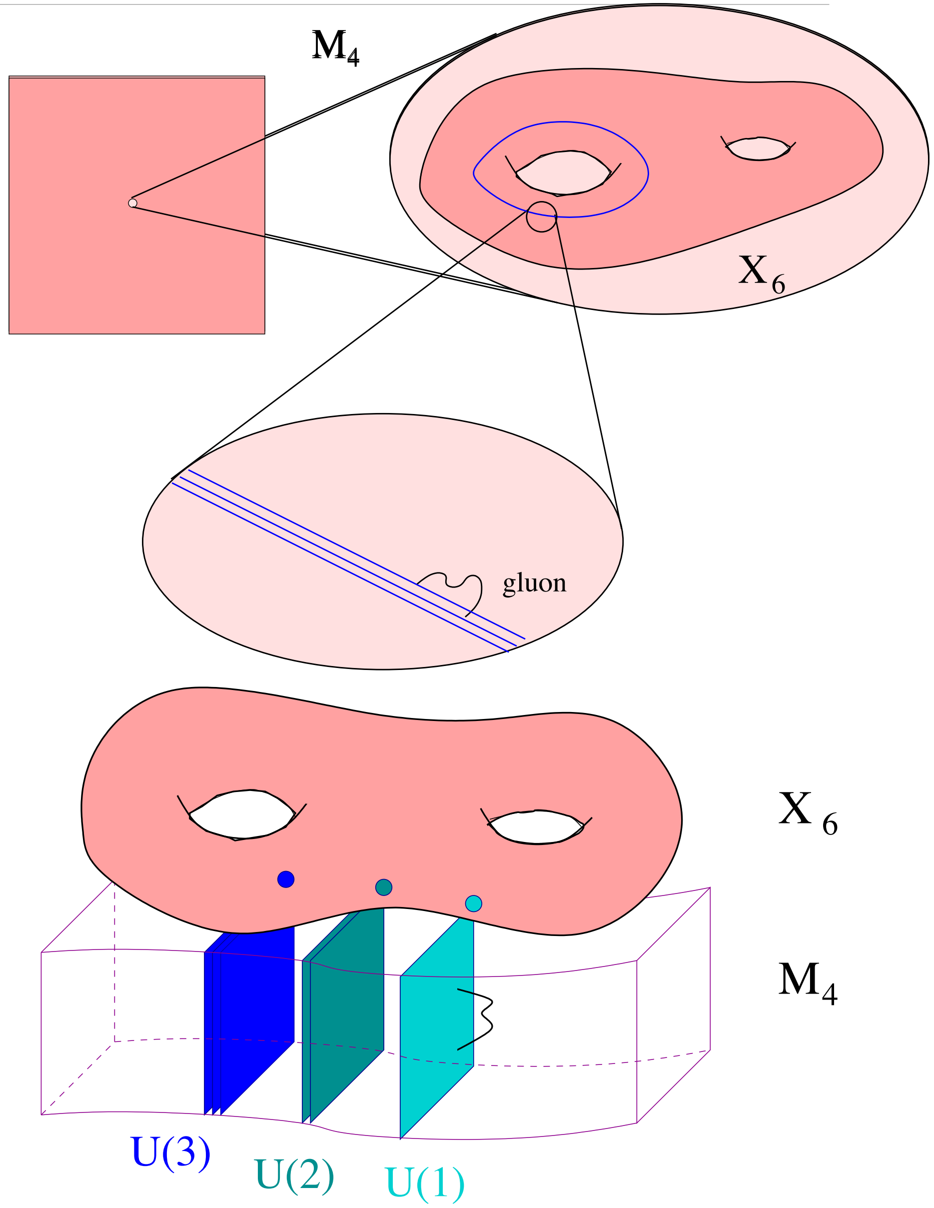
- Simplest example: 3 + 2 + 1 D3-branes

    ➡ 4D U(3) x U(2) x U(1) gauge theory



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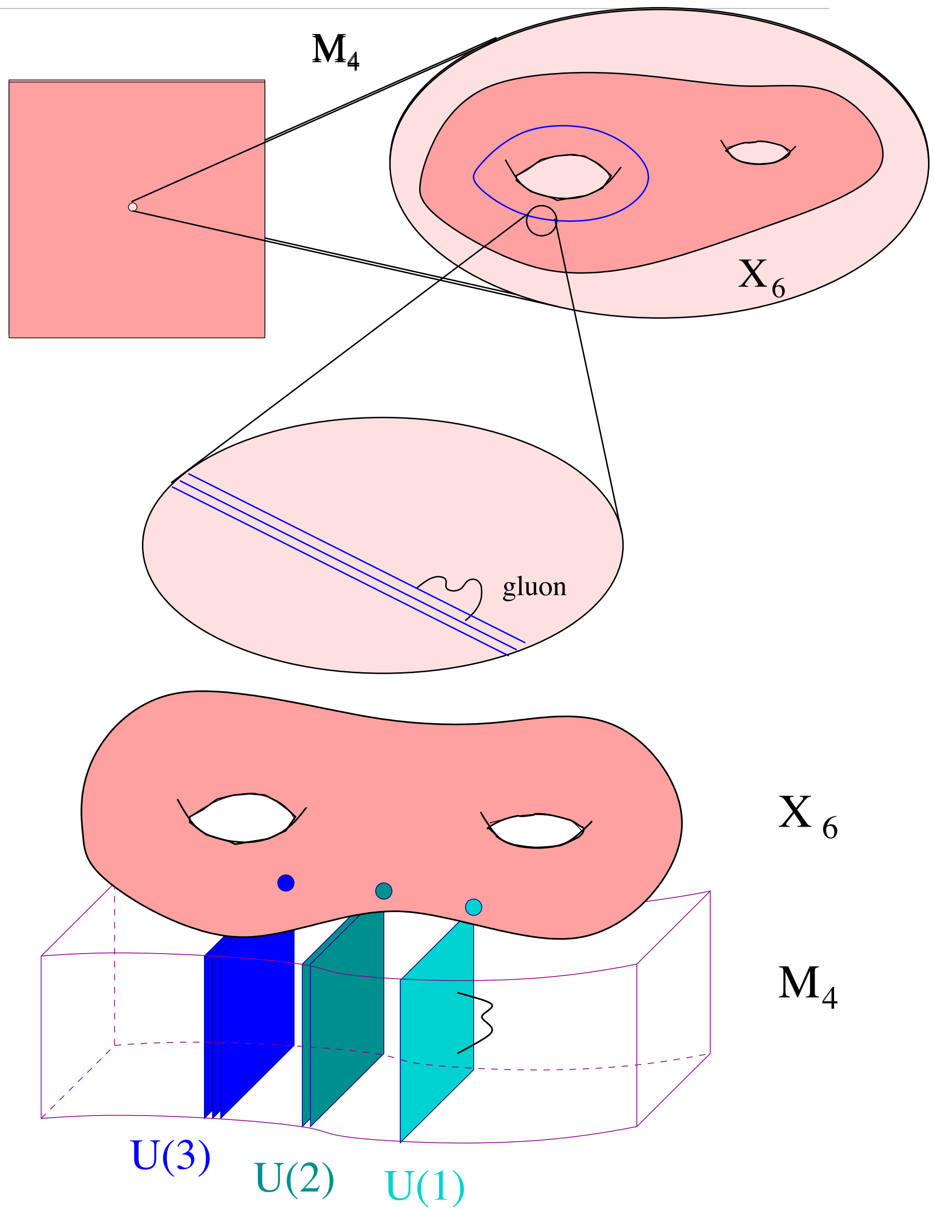
  - SM physics only depends on a certain region of X_6 (local model)
  - However, this example lacks of a key property of the SM: Chiral Fermions



The quest for the Standard Model

Question: Can we reproduce the Standard Model from D-branes?

✔ Four observable dimensions

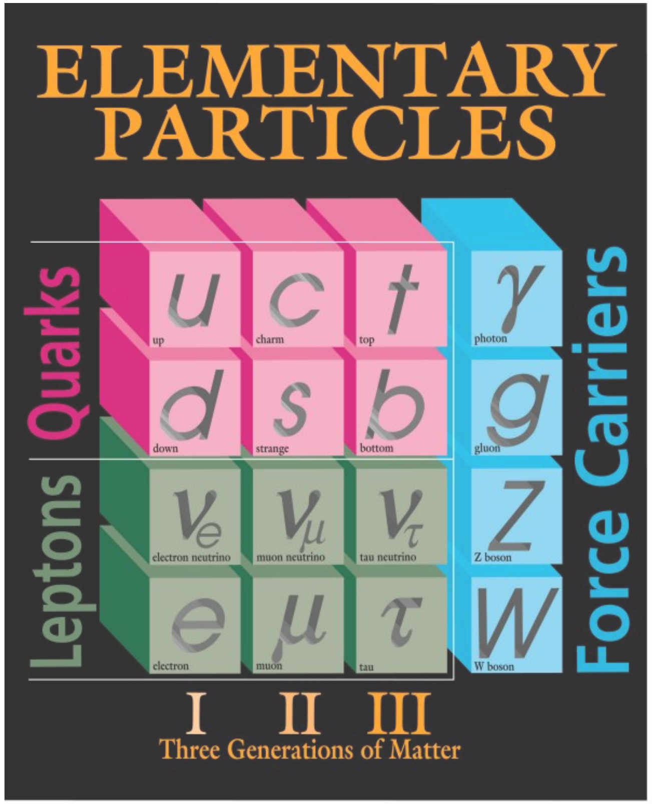
✔ Gauge group SU(3) x SU(2) x U(1)$_Y$

✘ Chiral fermions

3 Quarks & Leptons generations

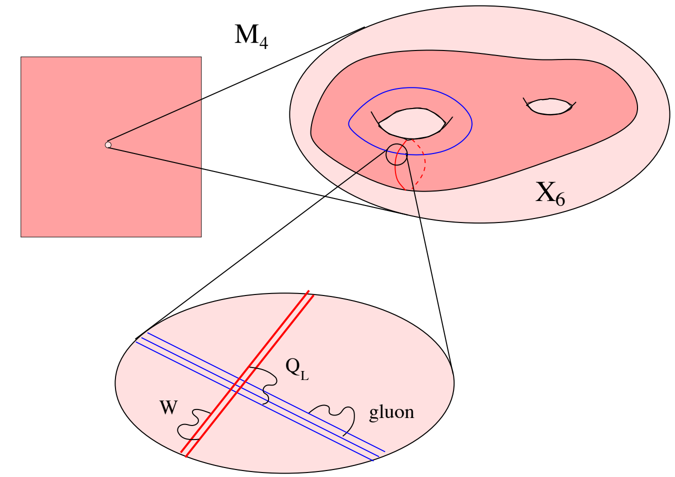
Gauge coupling constants

Yukawa couplings

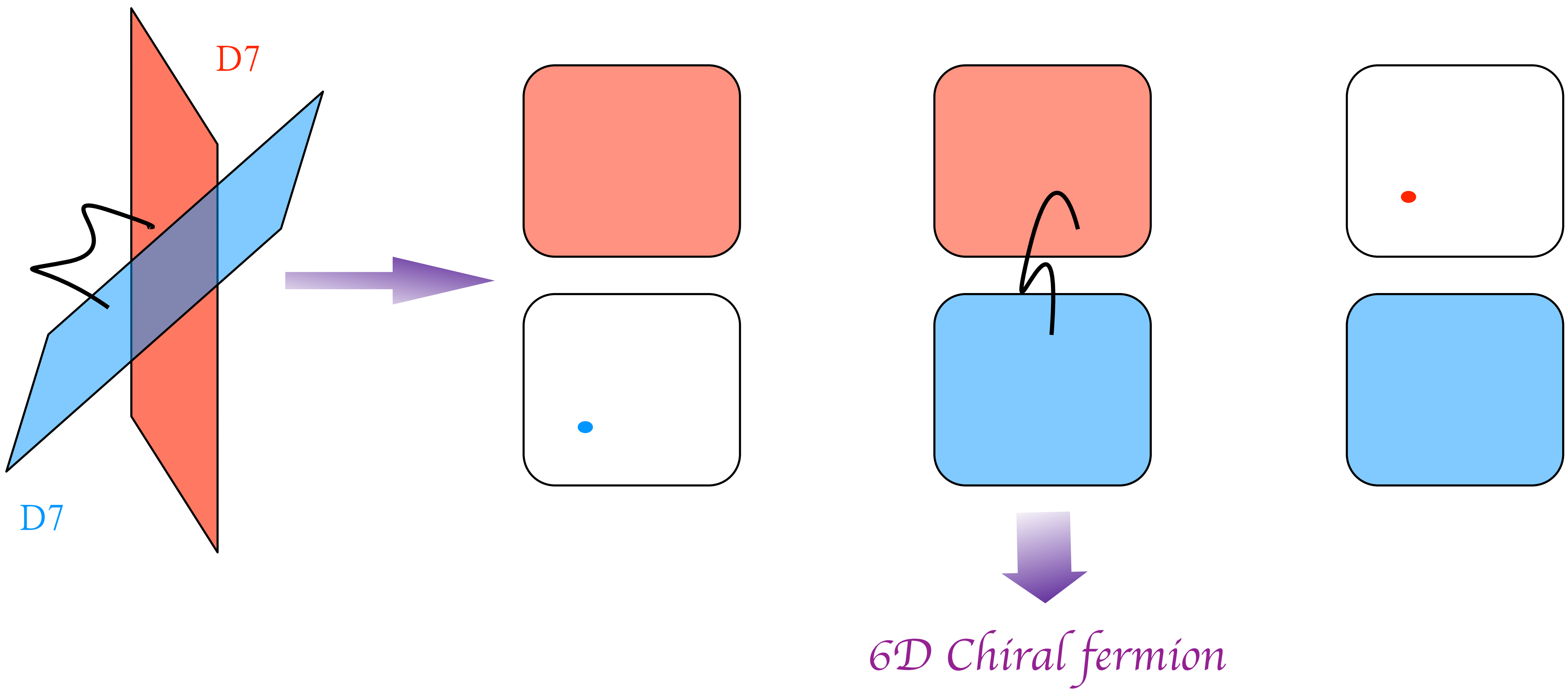


Fermilab 95-759

Getting chirality



- In type II string theory we get chirality via higher dimensional D-branes
- The same idea works for F-theory
  - 4d Chirality



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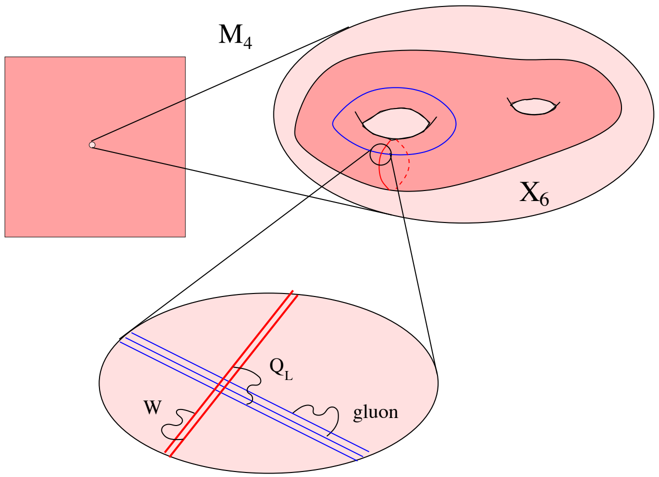

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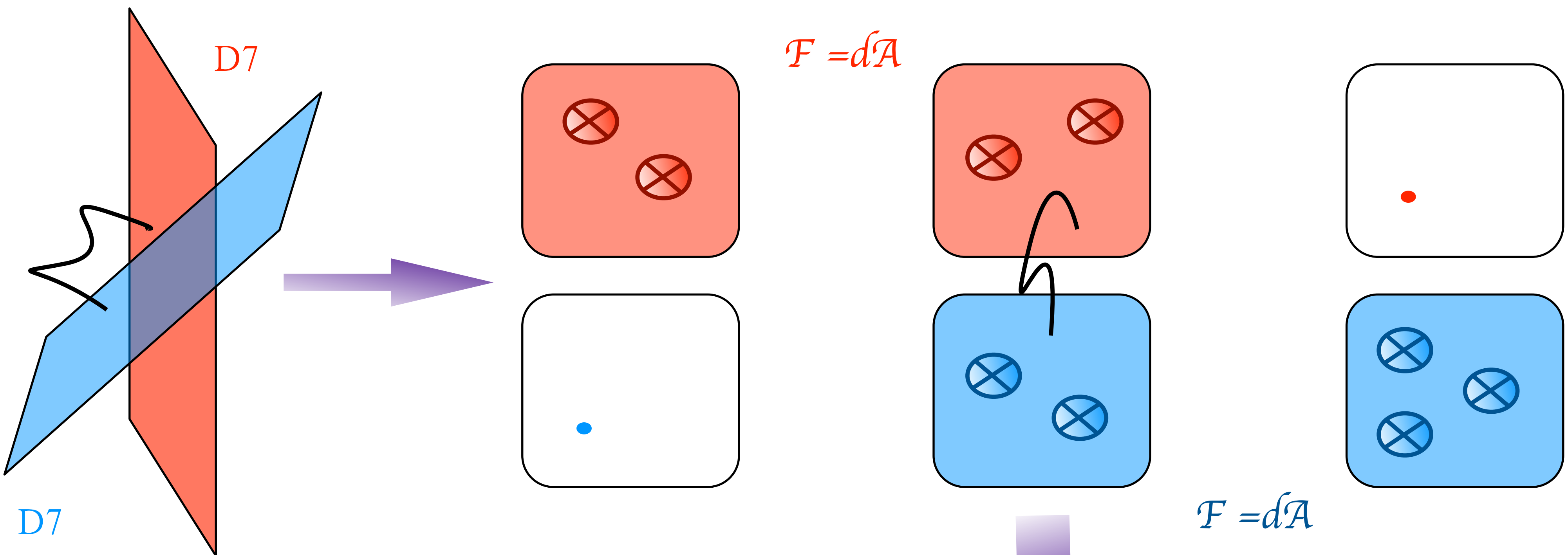

we modify the Dirac operator
to achieve chirality

$$D = \nabla + i\,(qA + qA)$$

Getting coupling unification

- In type II string theory we get similar set of gauge couplings when all the D-branes have the same internal size
- The same approach works in F-theory
    - Local Models → gauge coupling unification

$$\alpha_i^{-1} \sim \mathrm{Vol}(D_i) + \int_{D_i} F^2$$

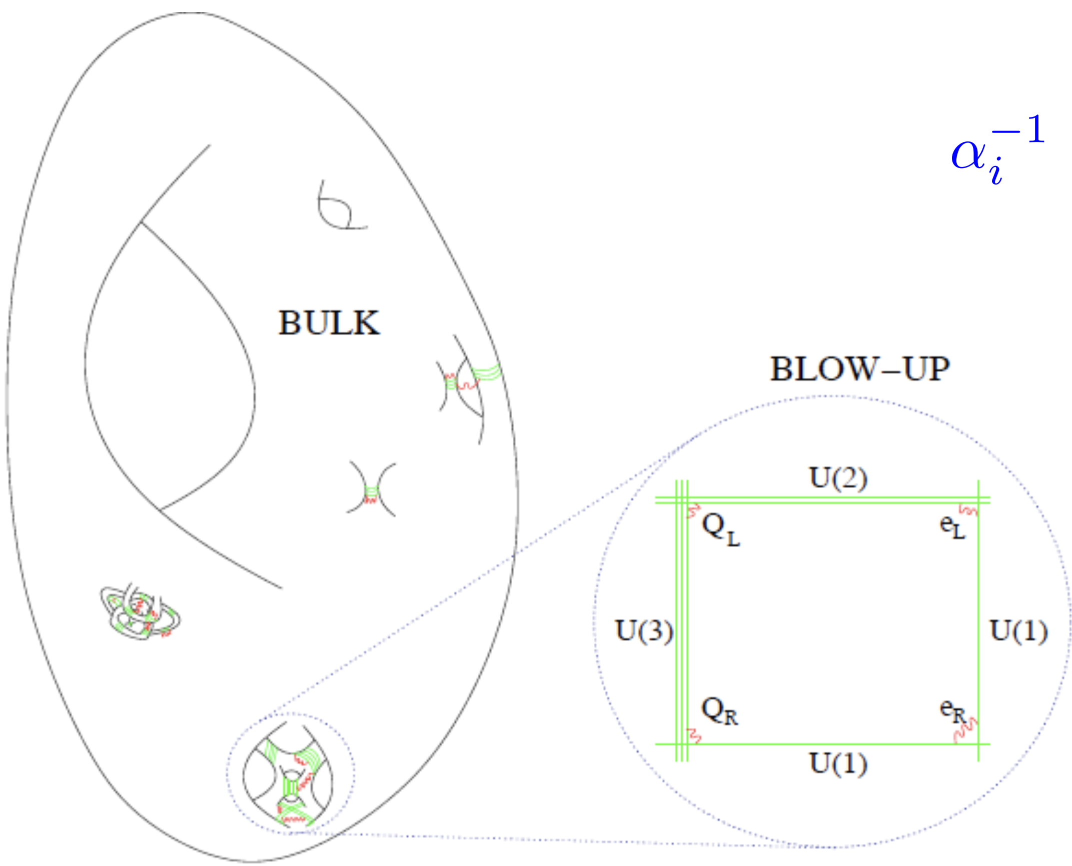

Figure taken from Conlon et al. '06

D-branes and GUTs

- Local D-brane models are very suggestive for constructing 4d GUTs
- In particular, one could consider a model made of 5+1 Dp-branes
  ⇒ $[SU(5) \times U(1)_a] \times U(1)_b$ gauge group, 10, 5* and 5_H representations
- However, the presence of $U(1)_a$ forbids the Yukawa coupling $10 \times 10 \times 5_H$
  ⇒ Top Yukawa needs to be generated by non-perturbative effects

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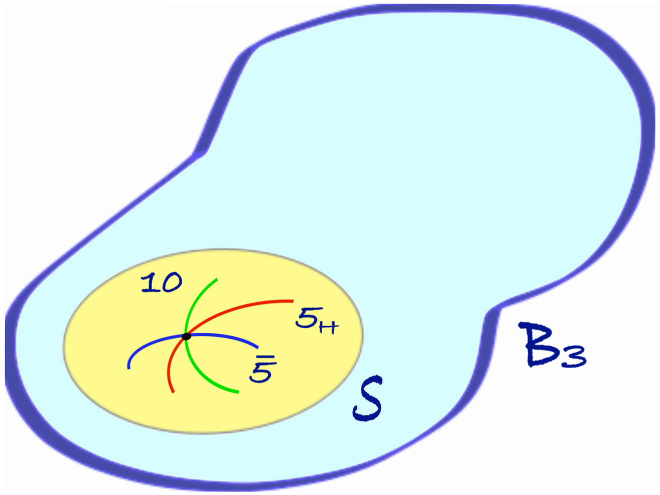
Solution: Consider F-theory GUTs

Beasley, Heckman, Vafa '08
Donagi & Wijnholt '08

F-theory:

- Relative of type IIB with D7-branes that allows for more general gauge groups like E_6, E_7, E_8

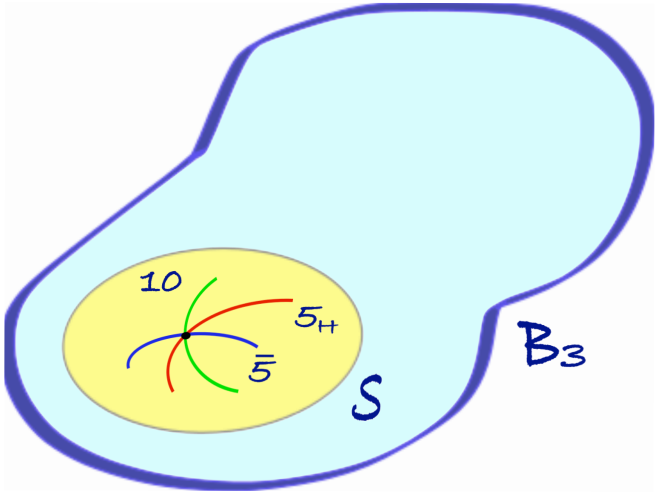
This translates into less selection rules for Yukawa couplings, as in heterotic strings

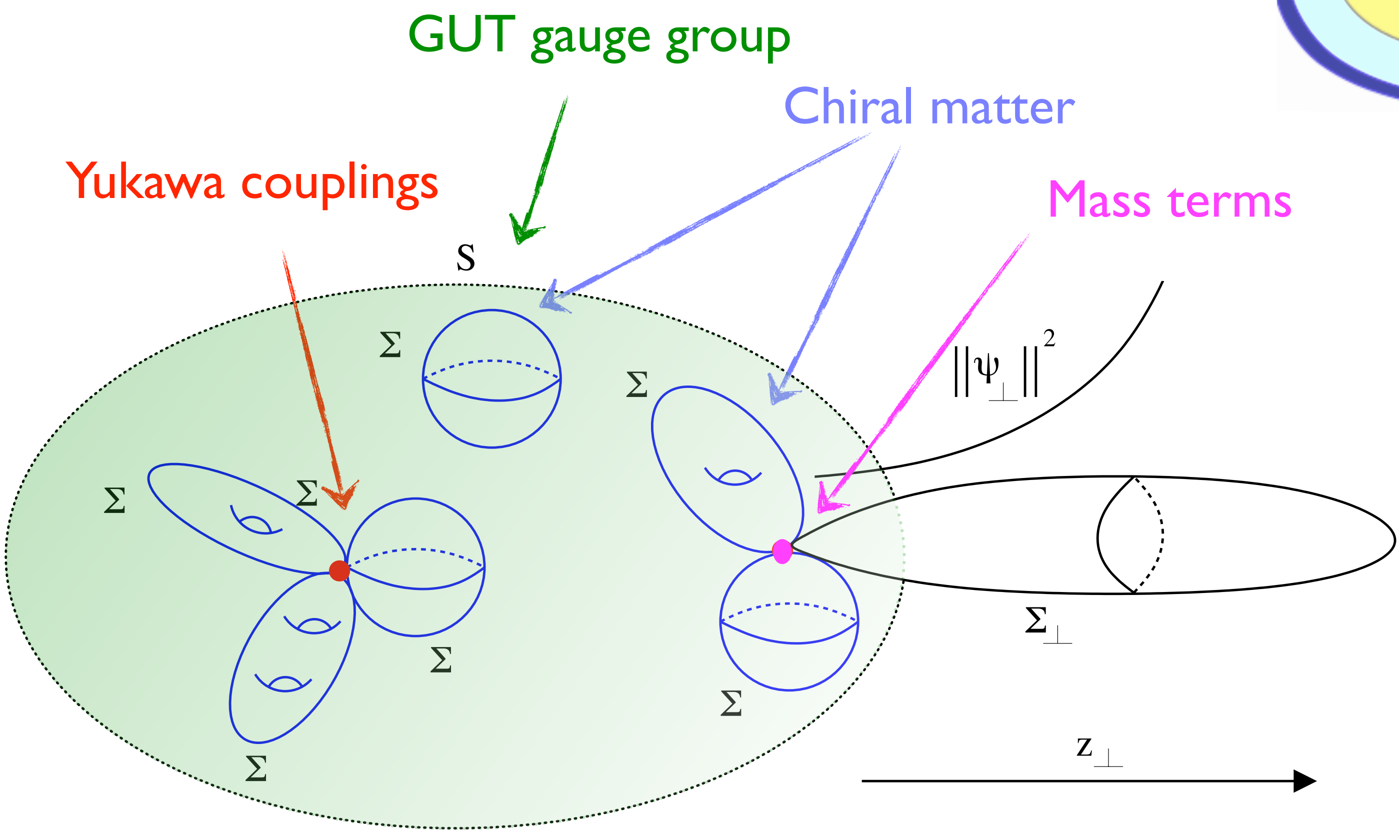


F-theory GUTs

Beasley, Heckman, Vafa '08
Donagi & Wijnholt '08

- Chirality: same as for D7-branes
  - GUT gauge group localised in 4-cycle S
  - Matter localised in 2-cycles Σ inside S
  - Yukawas arise at intersection points





F-theory GUTs

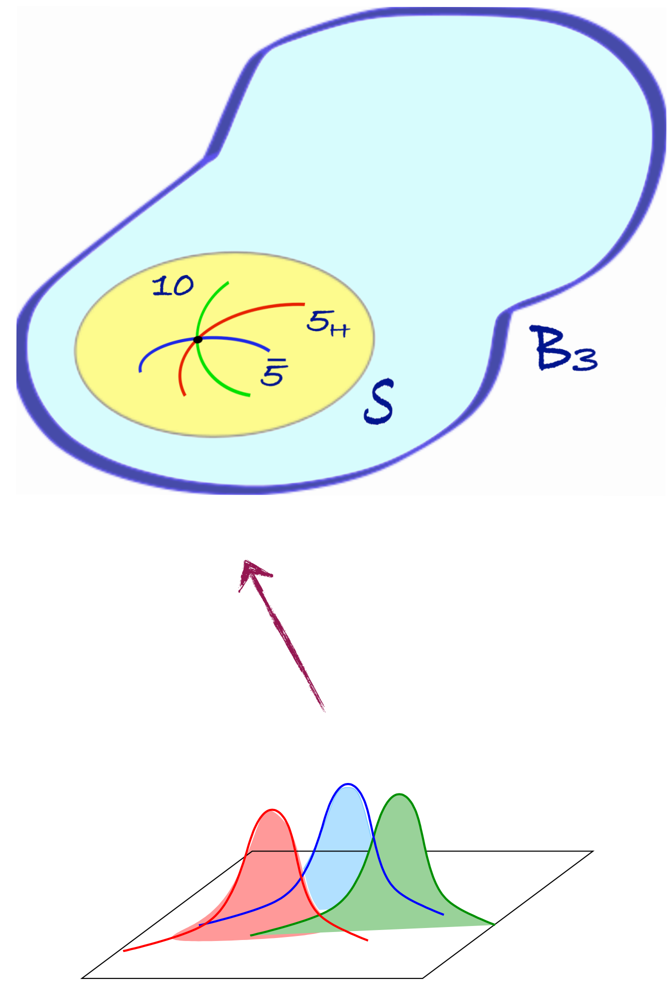
Beasley, Heckman, Vafa '08
Donagi & Wijnholt '08

- In order to achieve chirality and family replication, 7-brane fluxes on S are needed

- One of them can be taken along the hypercharge generator ⇒ breaking of the GUT gauge group

$$SU(5) \xrightarrow{F_Y} SU(3) \times SU(2)_L \times U(1)_Y$$

$$Q_Y = \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix} \subset SU(5)$$

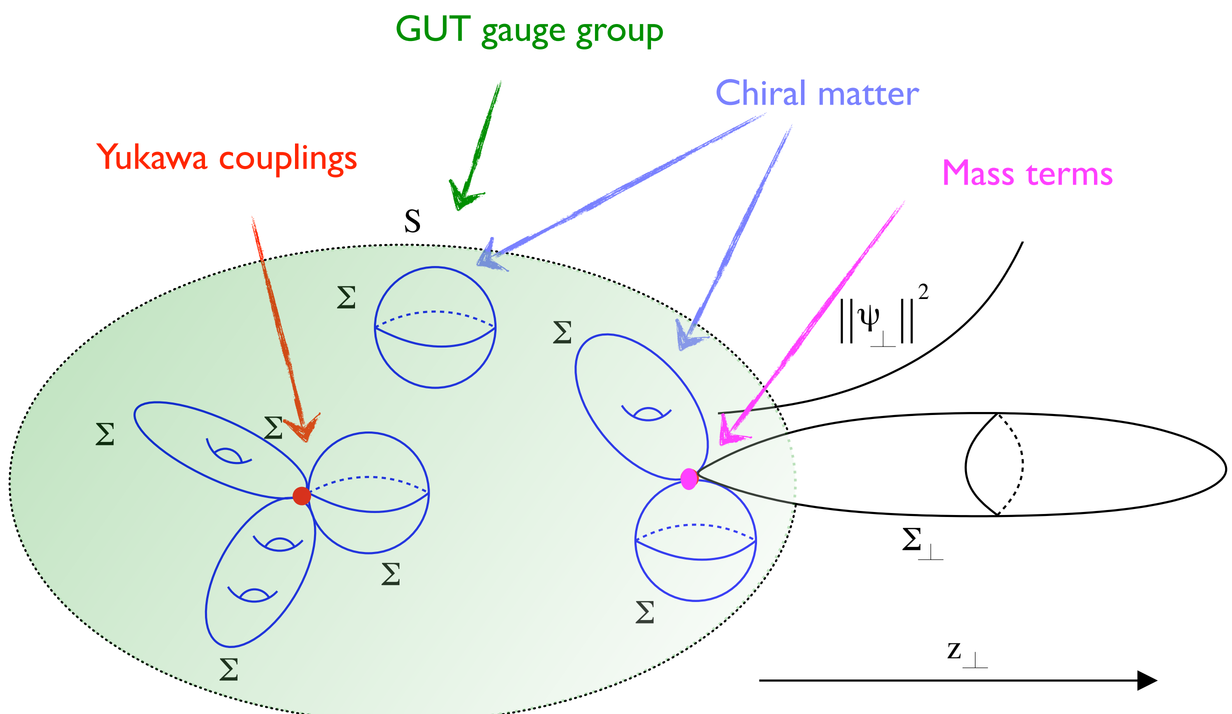


- This affects gauge couplings constants and Yukawa couplings...

$$\begin{array}{c} 10_M \times 10_M \times 5_H \\ 10_M \times \bar{5}_M \times \bar{5}_H \end{array} \xrightarrow{F_Y} \lambda_u^{ij} Q^i U^j H_u + \lambda_d^{ij} Q^i D^j H_d + \lambda_l^{ij} L^i E^j H_d$$

Key promises of F-theory GUTs @ 2008

- Gauge coupling unification
- Realistic Yukawa couplings
  - ✦ Tree level top Yukawa (b.t. type II)
  - ✦ Local computation (b.t. heterotic)
- Doublet-triplet splitting via hypercharge flux



Taken from Beasley, Heckman, Vafa '08

F-theory Yukawas

- Despite their differences, one can easily gain intuition in understanding F-theory Yukawas in terms of their type IIB and heterotic cousins

- Like for heterotic strings in CYs, one may compute Yukawas from dim. red. of a higher dimensional field theory

Beasley, Heckman, Vafa '08

Heterotic	F-theory
10d SYM	8d tw. YM
$W = \int_X \Omega \wedge \mathrm{Tr}\,(A \wedge F)$	$W = \int_S \mathrm{Tr}\,(F \wedge \Phi)$
$G_X = E_8 \mathrm{x} E_8$, $SO(32)$	$G_S = SO(2N), E_6, E_7, E_8 \ldots$

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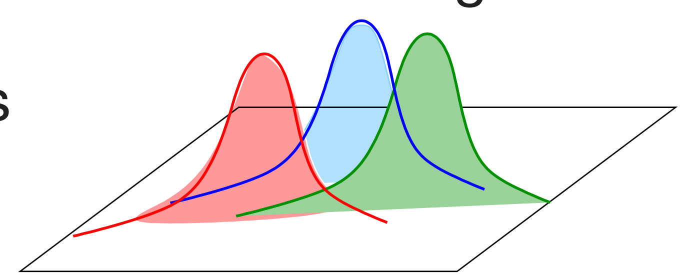
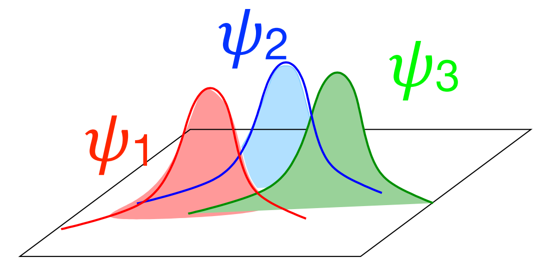
- Computation of zero mode wavefunctions in a certain background

- Yukawas = triple overlap of wavefunctions



Key features of Yukawa couplings

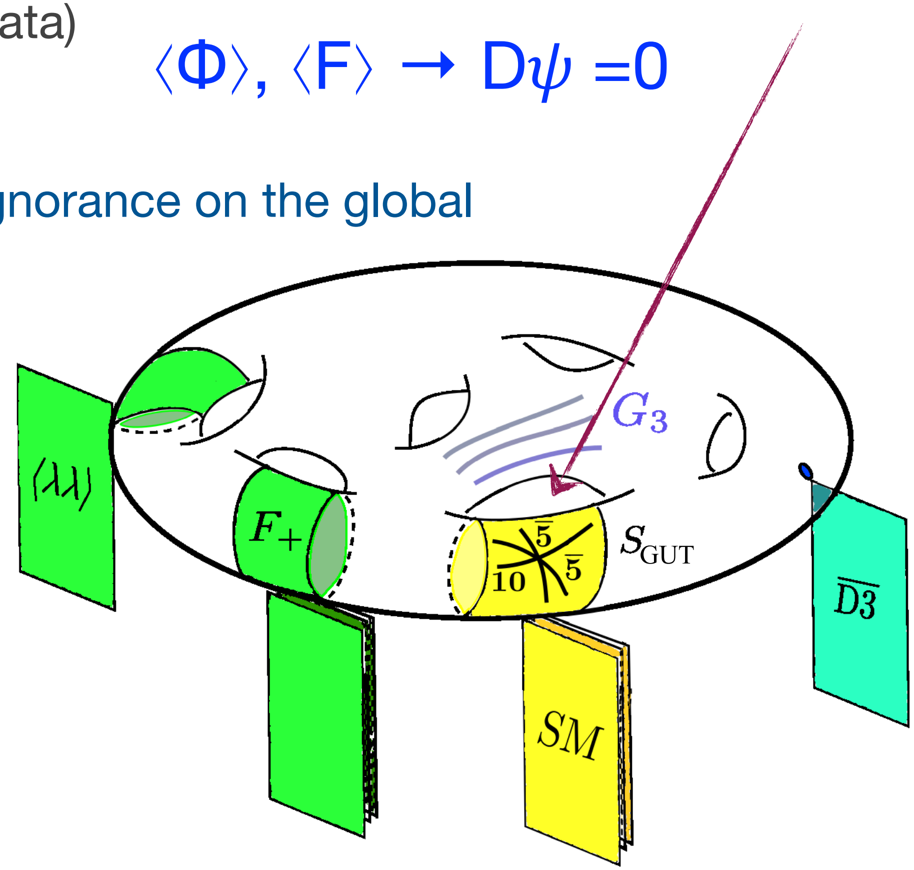
- Computed via dim. red. of a 8d gauge theory on S_{GUT}



- Depend on ultra-local data around some points in S_{GUT} (holomorphic Yukawas on fewer data)

$$\langle \Phi \rangle, \langle F \rangle \rightarrow D\psi = 0$$

- Such local data parametrise our ignorance on the global features of the compactification

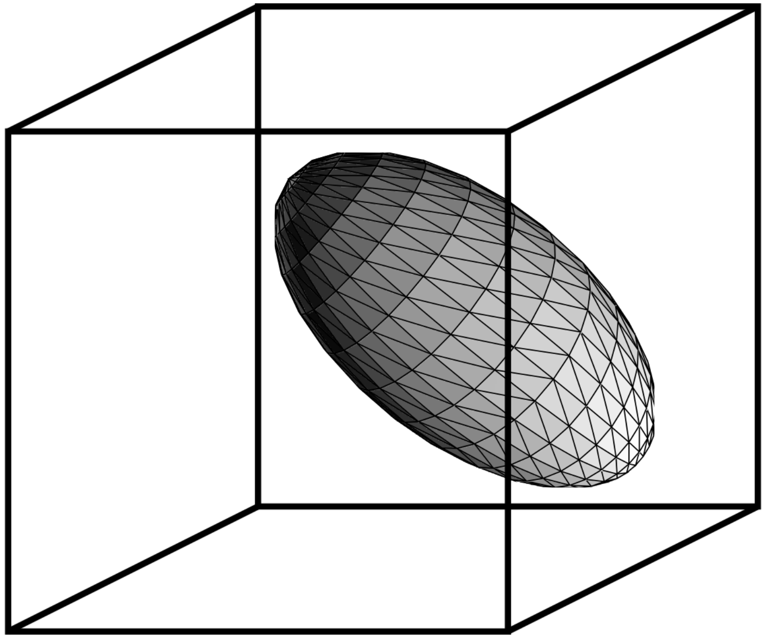


Taken from Camara, Ibañez, Valenzuela'14

Key questions on Yukawa couplings

Ok, nice but...

How easy is it to get realistic Yukawas in terms of local parameters?



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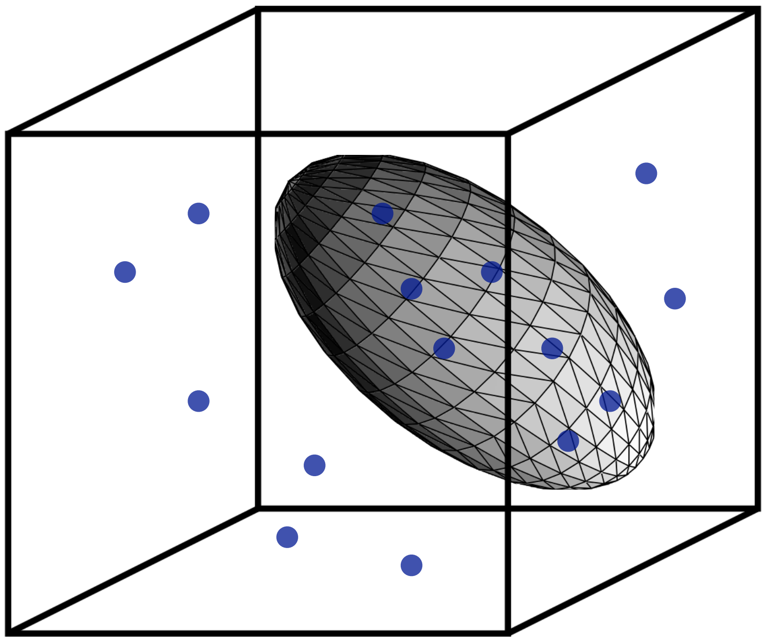

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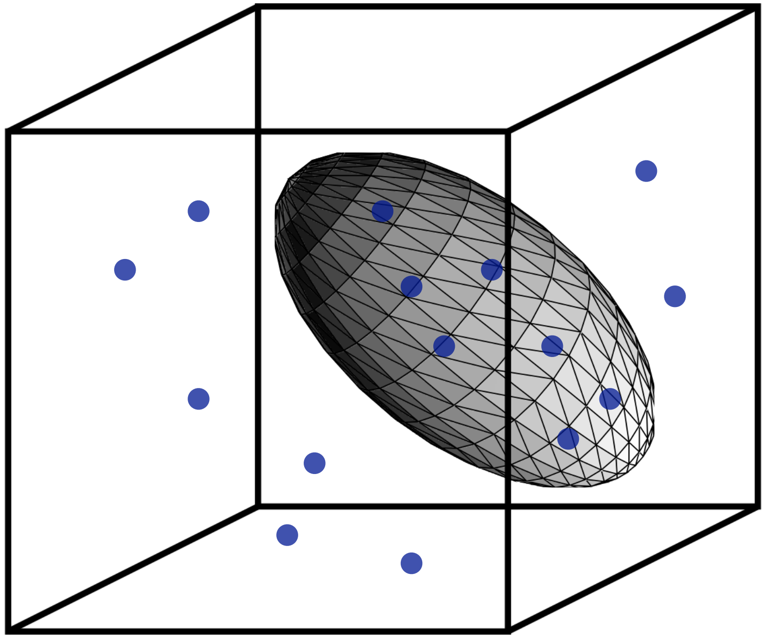


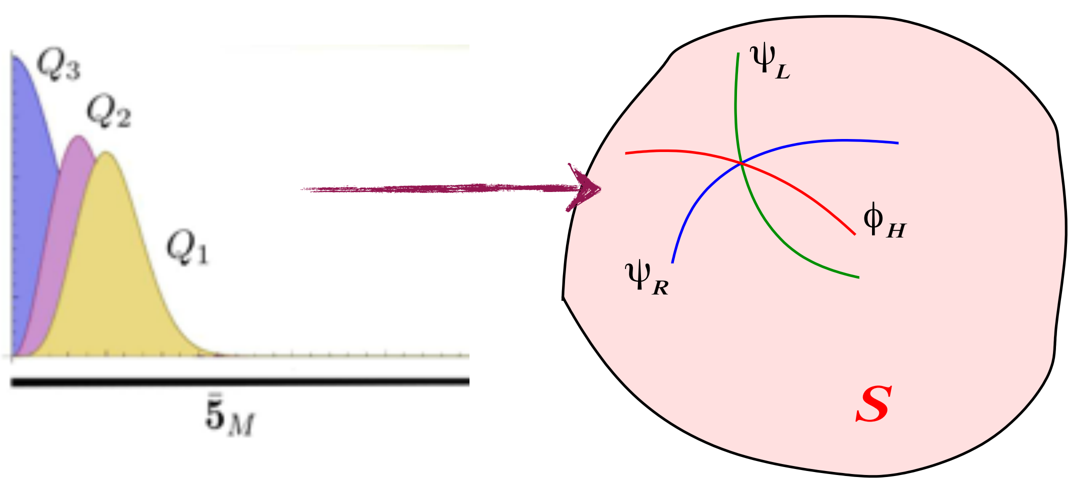
But generating a wide region of local data with realistic Yukawas is not as easy as it may seem…

First step: robust mechanism for family hierarchies

Rank one Yukawas

- F-theory comes with a mechanism to have one quark/lepton family much heavier than the other two
  - ✦ We may host several families of chiral fermions in a single matter curve by means of a worldvolume flux threading it
  - ✦ Holomorphic Yukawas independent of this flux. Their maximal rank only depends on a topological invariant: the curves intersections

Cecotti, Cheng, Heckman, Vafa '09



Single triple intersection

⇓

rank one Yukawas

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Cecotti, Cheng, Heckman, Vafa '09

If we break this rank one result by a small amount we may generate a hierarchical pattern of fermion masses

Heckman & Vafa '08

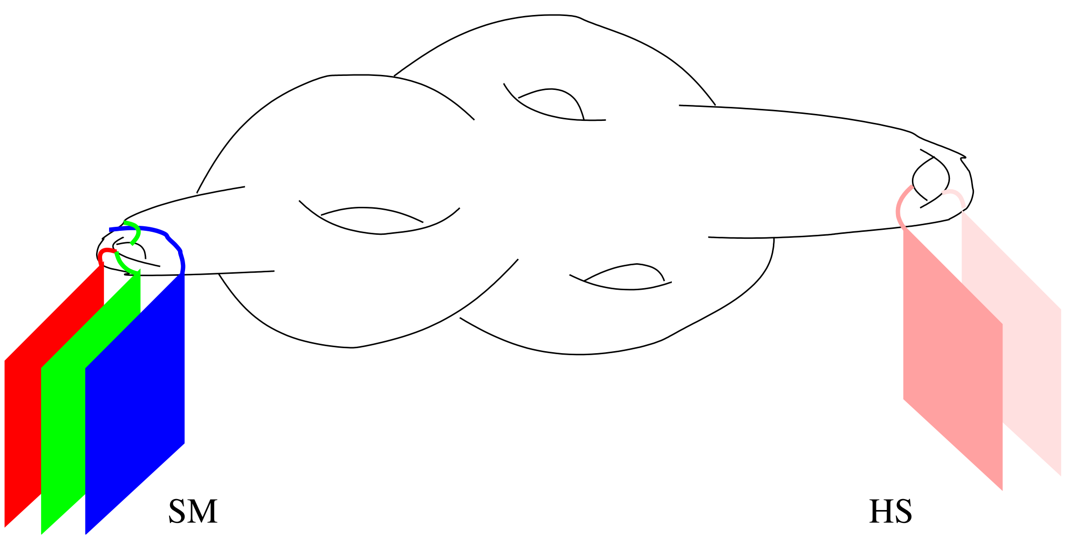
Single triple intersection

⇓

rank one Yukawas

Non-perturbative effects

- Even if the SM sector is localised in a particular region of the extra dimensions, some other gauge sectors can probe other regions/scales of X_6 :
  - Hidden gauge sectors made up of D-brane on other regions of X_6
  - D-branes which are point-like in 4d ⇒ D-brane instantons



$$Y_{ijk}\, e^{-2\pi T}$$

$$T = \rho + i\phi$$

Adding non-perturbative effects

- Non-perturbative effects like E3-brane instantons will increase the rank of the Yukawa matrix while maintaining the family mass hierarchy

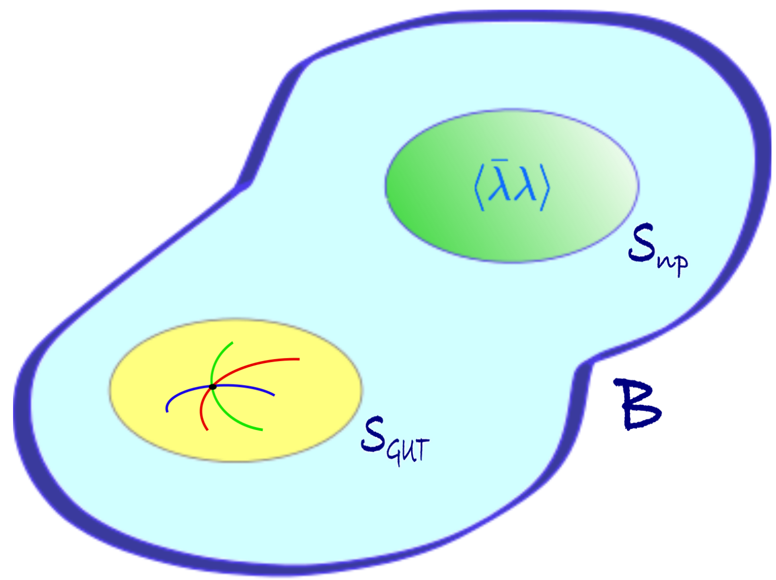
- In the case of plain D3-instantons we have

F.M. & Martucci '09

$$W_7 = W_7^{\rm tree} + W^{\rm np}$$

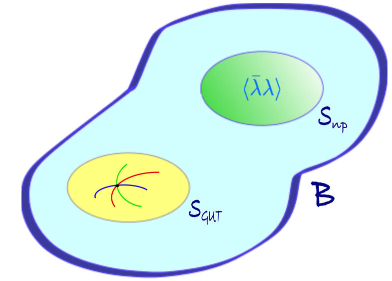
$$W_7 = \int_S {\rm Tr}\,(F \wedge \Phi) + \frac{\epsilon}{2} \int_S \theta_0 {\rm Tr}\,(F \wedge F)$$

$$\epsilon \sim e^{-T_{np}}$$



$$W^{\rm np} = m_*^4 \epsilon \left[\int_S \theta_0 {\rm Tr} F^2 + \int_S \theta_1 {\rm Tr}(\cancel{\Phi_{xy}} F^2) + \int_S \theta_2 {\rm STr}(\Phi_{xy}^2 F^2) + \ldots \right]$$

Adding non-perturbative effects



- The expression

$$W_7 = \int_S \mathrm{Tr}\,(F \wedge \Phi) + \frac{\epsilon}{2}\int_S \theta_0 \mathrm{Tr}\,(F \wedge F)$$

allows to carry the computation of np-corrected Yukawas at the local level

- Holomorphic Yukawas can also be computed via a residue formula. They depend on ϵ and θ_0 but not on worldvolume fluxes.

- Physical Yukawas are computed by solving for the MSSM fields internal wavefunctions and performing local dim. red. in the deformed theory.

  - ✦ SO(12) enhancement (down-type Yukawas) — *Font, Ibañez, F.M., Regalado '12*
  - ✦ E_6 enhancement (up-type Yukawas) — *Font, F.M., Regalado, Zoccarato '13*
  - ✦ E_7 and E_8 enhancement (both Yukawas +CKM) — *F.M., Regalado, Zoccarato '15*; *Carta, F.M., Zoccarato '15*

Local model data

holomorphic

- ✦ $\langle \Phi \rangle$ contains the 7-brane intersection angles: μ, m
- ✦ Non-perturbative effect encoded in ϵ, θ_0

physical

- ✦ $\langle F \rangle$ generates chirality and family replication at matter curves, enters via flux densities: N_i, M_j
- ✦ $\langle F_Y \rangle$ breaks $G_{GUT} \to G_{MSSM}$, enters via densities N_Y, $\tilde{N}_Y$

Example: SU(5)

$$\begin{matrix} 5_{H_u} \times 10 \times 10 \\ \bar{5}_{H_d} \times \bar{5} \times 10 \end{matrix} \xrightarrow{F_Y} \begin{matrix} \lambda_u^{ij} Q^i U^j H_u \\ \lambda_d^{ij} Q^i D^j H_d + \lambda_l^{ij} L^i E^j H_d \end{matrix}$$

The presence of $\langle F \rangle$ also localises wavefunctions along matter curves and allows an ultra-local computation of Yukawa couplings

Not all of these parameters will be independent in a global model

General results

- Assuming $\theta_0 = i(\theta_{00} + x\,\theta_x + y\,\theta_y)$ one obtains, at the holomorphic level

$$\frac{Y^{\rm hol}}{Y^{\rm hol}_{33}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \mathcal{O}(\epsilon) \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

and so a family hierarchy $(1, \epsilon, \epsilon^2)$, independent of worldvolume fluxes

General results

- On the other hand, the physical Yukawas

$$Y^{ij}_{\rm phys} = \gamma_i \gamma_j \gamma_H Y^{ij}_{\rm hol}$$

depend on fluxes via the normalisation factors γ_i

- Such γ_i's are the normalisation of the wavefunctions

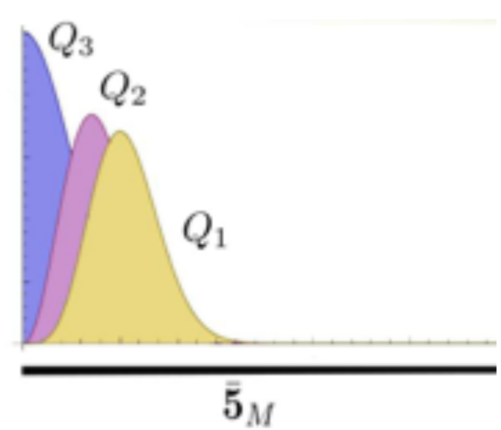
$$\gamma_i^{-2} \propto \int dy\, e^{-\pi |F||y|^2} |f^i(y)|^2$$

They depend on the family

They depend on the flux F $\quad \gamma_i \propto \left(\frac{\pi}{\sqrt{2}}|F|, \sqrt{\pi}|F|^{1/2}, 1 \right) \qquad F = N + N_Y$

- Higher hypercharge ⇒ thinner wavefunction

⇒ larger quotients ⇒ $\quad \frac{m_\mu}{m_\tau} \simeq 3 \frac{m_s}{m_b} \quad$ for $\quad \frac{N_Y}{N} \simeq 1.8$



The E_7 and E_8 stories

F.M., Regalado, Zoccarato '15
Carta, F.M., Zoccarato '15

- One may build a local model where all Yukawas arise from a single patch of S_{GUT}, described by either an E_7 or E_8 local symmetry broken by Φ to SU(5)
  - ✦ Several classes of models, classified by the structure of Φ
  - ✦ Few of them give the appropriate hierarchy $(1, \epsilon, \epsilon^2)$
  - ✦ Essentially only one model yields realistic Yukawas.
    Realised on E_7 and E_8, with same parametric dependence on local data
  - ✦ Yukawas have a complicated dependence on flux densities through the normalisation factors γ, but ratios of charged lepton mass ratios have simpler expressions

$$\frac{m_\mu/m_\tau}{m_s/m_b} = \sqrt{\frac{(x-1)\left(y-\frac{1}{2}\right)}{\left(x-\frac{1}{6}\right)\left(y-\frac{1}{3}\right)}}\,, \qquad x = -\frac{M_1}{\tilde{N}_Y}\,, \quad y = -\frac{M_2}{\tilde{N}_Y}$$

Fitting Yukawas for charged leptons

- In this model one is able to fit charged fermion masses for the 3rd and 2nd families at the GUT scale assuming an MSSM scheme

$\tan\beta$	10	38	50
m_d/m_s	$5.1 \pm 0.7 \times 10^{-2}$	$5.1 \pm 0.7 \times 10^{-2}$	$5.1 \pm 0.7 \times 10^{-2}$
m_s/m_b	$1.9 \pm 0.2 \times 10^{-2}$	$1.7 \pm 0.2 \times 10^{-2}$	$1.6 \pm 0.2 \times 10^{-2}$
m_e/m_μ	$4.8 \pm 0.2 \times 10^{-3}$	$4.8 \pm 0.2 \times 10^{-3}$	$4.8 \pm 0.2 \times 10^{-3}$
m_μ/m_τ	$5.9 \pm 0.2 \times 10^{-2}$	$5.4 \pm 0.2 \times 10^{-2}$	$5.0 \pm 0.2 \times 10^{-2}$
m_b/m_τ	0.73 ± 0.03	0.73 ± 0.03	0.73 ± 0.04
Y_τ	0.070 ± 0.003	0.32 ± 0.02	0.51 ± 0.04
Y_b	0.051 ± 0.002	0.23 ± 0.01	0.37 ± 0.02
Y_t	0.48 ± 0.02	0.49 ± 0.02	0.51 ± 0.04

Ross & Serna '07

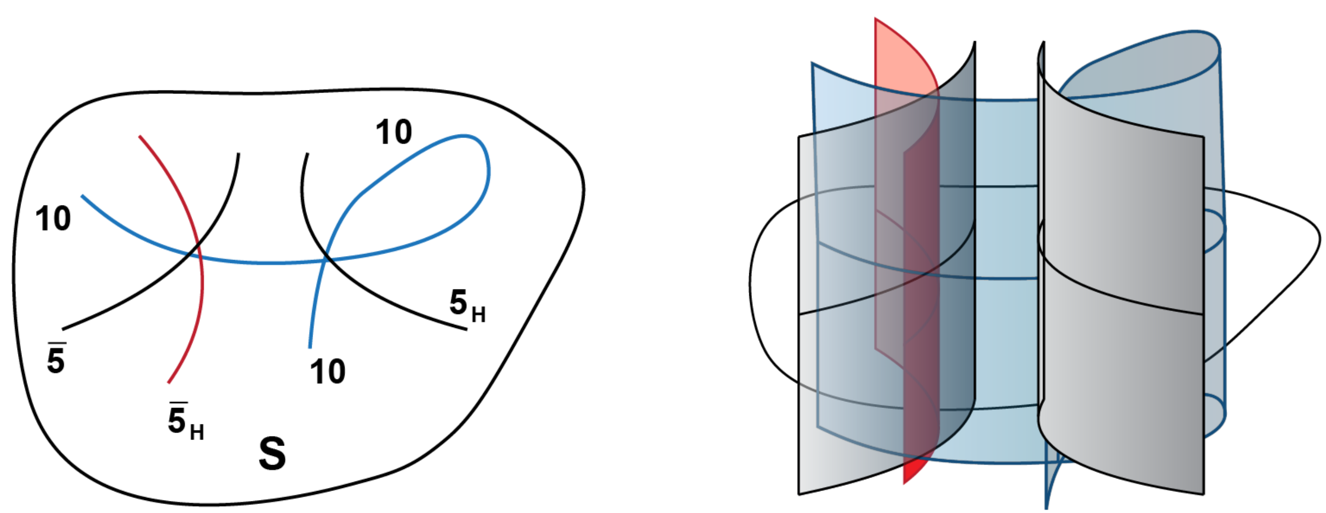
typical value:
$\tan\beta \sim$ 10-20

CKM matrix

- In this scheme one is also able to compute the CKM matrix
- If both Yukawa points p_{up} and p_{down} coincide there is no mixing, but when we separate them a source of family mixing is induced

Randall & Simmons-Duffins '09

comparing V_{tb} with the experimental value
sets the separation of points ~ $R_{GUT}/100$



Taken from Aparicio et al. '12

Conclusions

- To build a string model, we need to reproduce a "wish list" of SM features. The first items of the list are more universal, as well as more robust with respect to corrections. Further items are usually more model-dependent

- A key feature is chirality. One can classify models by how chiral fermions arise.

- A quantity difficult to reproduce are Yukawa couplings, but vacua based on F-theory local models can realise a hierarchical structure in a natural way.

- Key mechanism: stringy non-perturbative effects of strength ϵ. They increase the Yukawa rank from one to three → hierarchy of eigenvalues $\mathcal{O}(1), \mathcal{O}(\epsilon), \mathcal{O}(\epsilon^2)$

- Wavefunction normalization depends on local data like internal flux densities F. Allows to accommodate a large top Yukawa and realistic MSSM mass ratios via F_Y GUT breaking, more flexible than 4d GUTs

- Local result → hierarchy is universal upon global completion. Precise mass values depend on local parameters → determined by vacuum choice in the Landscape