(Heterotic) String Theory and the Standard Model

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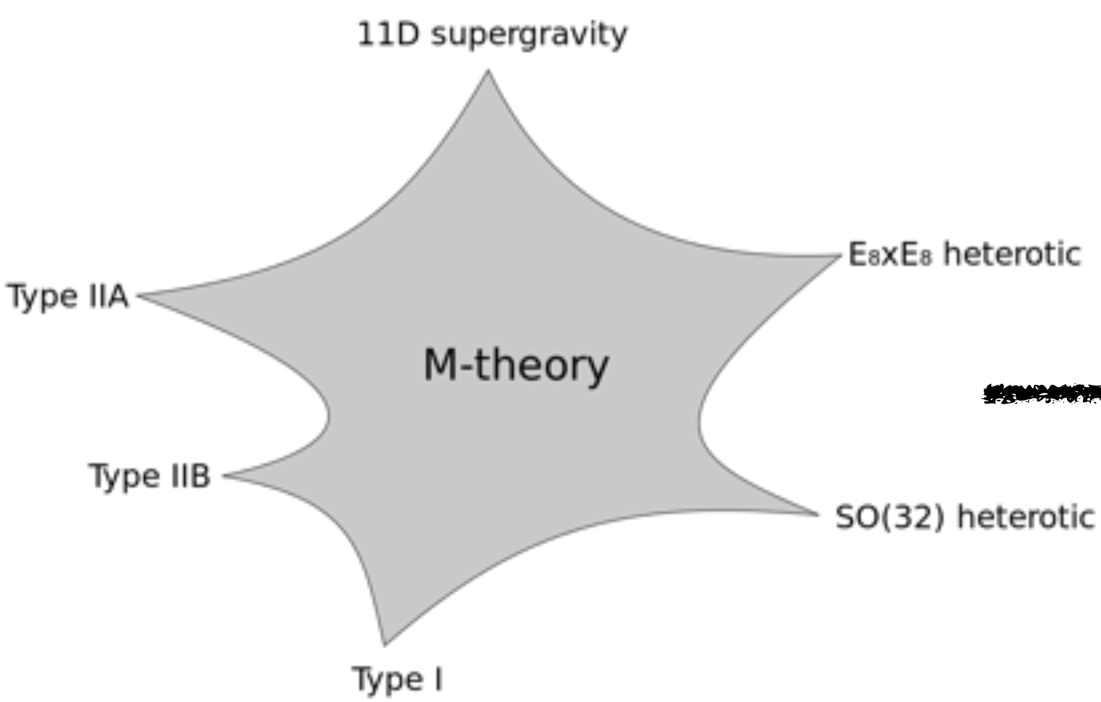
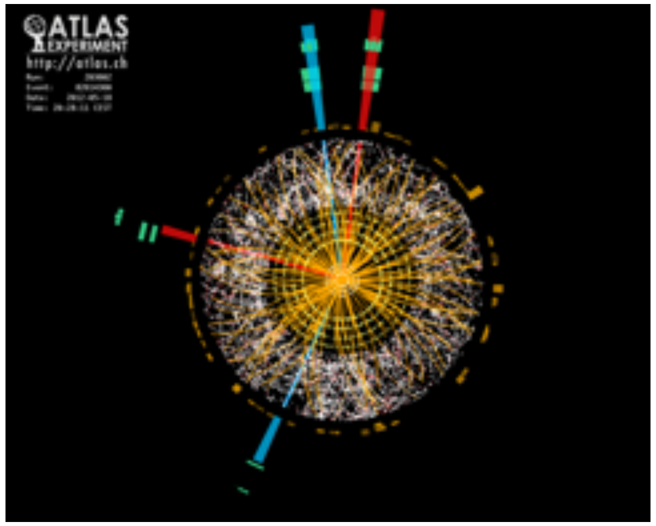


? ?



based on work in collaboration with

Lara Anderson, Stefan Blasneag, Evgeny Buchbinder, Philip Candelas, Andrei Constantin, James Gray, Alexander Haupt, Yang-Hui He, Seung-Joo Lee, Challenger Mishra, Burt Ovrut, Eran Palti, Chuang Sun

which has appeared in

1607.03461, 1607.01830, 1606.04032, 1512.05322, 1509.02729, 1412.8696, 1411.0034, 1409.2412, 1404.2767, 1311.1941, 1307.4787, 1304.2704, 1202.1757, 1106.4804,...

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Heterotic Calabi-Yau model building

10-dim. effective (N=1) supergravity:

gravity multiplet	SYM multiplet
(g_{IJ}, Ψ_I)	$(A_I, \chi) \in E_8 \times E_8$

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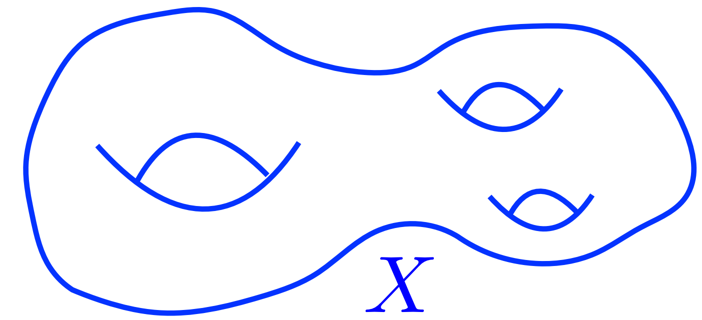

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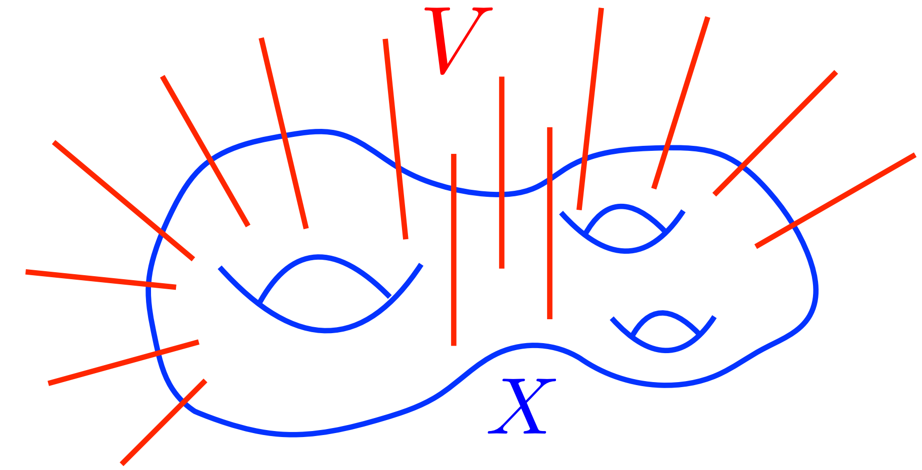

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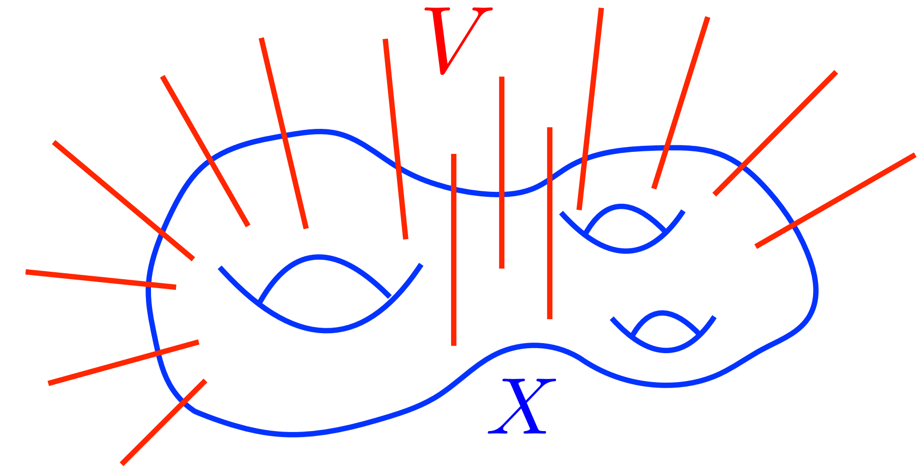

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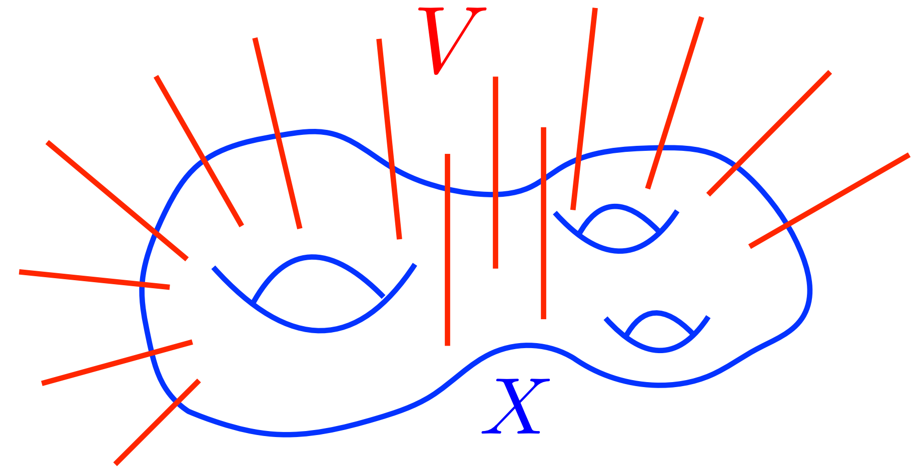

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Resulting 4d N=1 theory:

$(g_{\mu\nu}, \psi_\mu, \text{moduli})$

$(A_\mu, \chi, \text{matter fields})$

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-> heterotic vacuum is determined by a Calabi-Yau 3-fold X and a holomorphic, poly-stable vector bundle V on X.

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$(\mathbf{1},\mathbf{24})$: 4d gauge fields

$(\mathbf{5},\overline{\mathbf{10}})$: $(\tilde{Q}, \tilde{u}, \tilde{e})$

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4d gauge fields ↑ $(\mathbf{1}, \mathbf{24})$

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$(\mathbf{10}, \mathbf{5})$: $(\tilde{d}, \tilde{L})$

$(\overline{\mathbf{10}}, \overline{\mathbf{5}})$: (d, L)

$(\mathbf{24}, \mathbf{1})$: bundle moduli

c) Multiplicity of 4d matter multiplets:

First pass:

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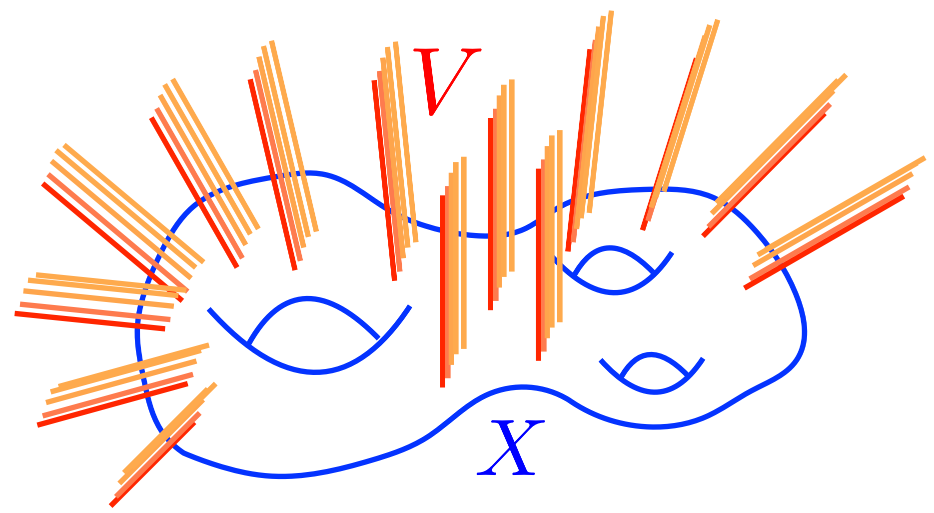
In practice: $X \to X/\Gamma$, $V \to X$ descends to $\hat{V} \to X/\Gamma$

For many years is was difficult to construct models, due to technical problems with vector bundles on CY manifolds.

How can (quasi-realistic) models be constructed?

Line bundle standard models

Focus on bundles with Abelian structure group.



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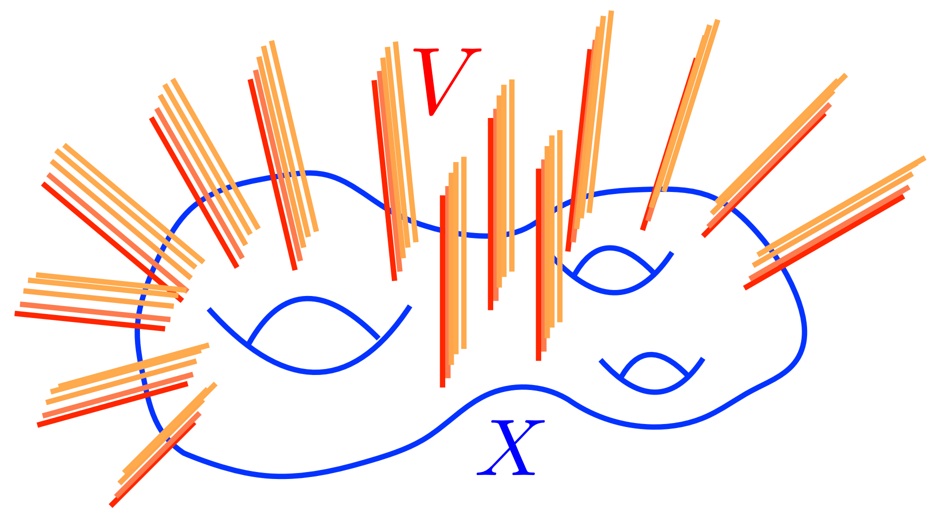

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is a sum of five line bundles, specified by integer vectors $\mathbf{k}_a$.

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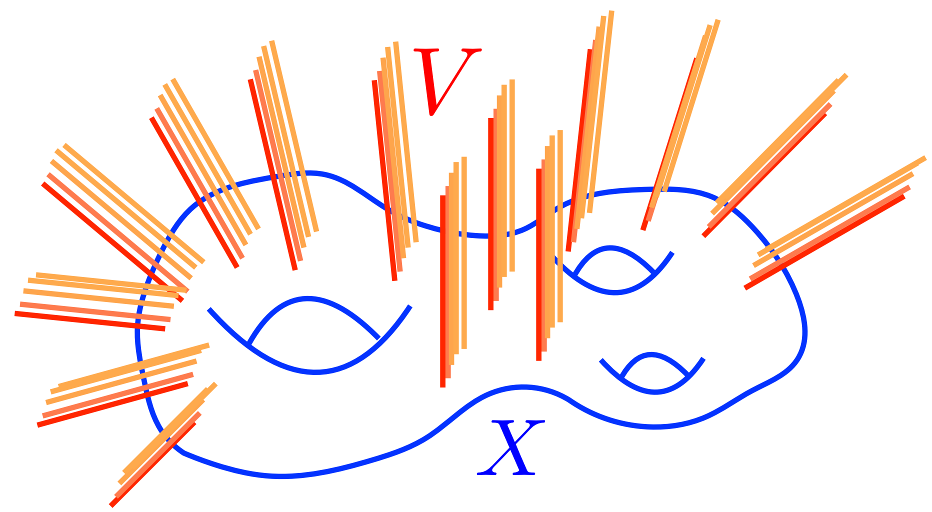

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Line bundles are classified by their first Chern class:

$$c_1(L_a) = k_a^i J_i \quad \Leftrightarrow \quad L_a = \mathcal{O}_X(\mathbf{k}_a)$$

. . . results in GUT models with gauge group

$$SU(5) \times S(U(1)^5)$$

typically anomalous

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$$SU(5) \times S(U(1)^5)$$ ← typically anomalous

. . . and matter multiplets

$$\mathbf{10}_a,\ \overline{\mathbf{10}}_a,\ \mathbf{5}_{a,b},\ \bar{\mathbf{5}}_{a,b},\ \mathbf{1}_{a,b} = S_{a,b}$$

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multiplet	$S(U(1)^5)$ charge	associated line bundle L	contained in
$\mathbf{10}_{\mathbf{e}_a}$	$\mathbf{e}_a$	L_a	V
$\bar{\mathbf{10}}_{-\mathbf{e}_a}$	$-\mathbf{e}_a$	L_a^*	V^*
$\bar{\mathbf{5}}_{\mathbf{e}_a+\mathbf{e}_b}$	$\mathbf{e}_a + \mathbf{e}_b$	$L_a \otimes L_b$	$\wedge^2 V$
$\mathbf{5}_{-\mathbf{e}_a-\mathbf{e}_b}$	$-\mathbf{e}_a - \mathbf{e}_b$	$L_a^* \otimes L_b^*$	$\wedge^2 V^*$
$\mathbf{1}_{\mathbf{e}_a-\mathbf{e}_b}$ $\mathbf{1}_{-\mathbf{e}_a+\mathbf{e}_b}$	$\mathbf{e}_a - \mathbf{e}_b$ $-\mathbf{e}_a + \mathbf{e}_b$	$L_a \otimes L_b^*$ $L_a^* \otimes L_b$	$V \otimes V^*$

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families and mirror families

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$\overline{\mathbf{10}}_{-\mathbf{e}_a}$	$-\mathbf{e}_a$	L_a^*	V^*	$= 0$
$\bar{\mathbf{5}}_{\mathbf{e}_a+\mathbf{e}_b}$	$\mathbf{e}_a + \mathbf{e}_b$	$L_a \otimes L_b$	$\wedge^2 V$	$\Rightarrow 3\lvert\Gamma\rvert$
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$\mathbf{1}_{\mathbf{e}_a-\mathbf{e}_b}$	$\mathbf{e}_a - \mathbf{e}_b$	$L_a \otimes L_b^*$	$V \otimes V^*$
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bundle moduli S^α

$\longleftarrow = 3|\Gamma|$

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$\Rightarrow 3|\Gamma|$

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	multiplet	$S(U(1)^5)$ charge	associated line bundle L	contained in	
families and mirror families	$\mathbf{10}_{\mathbf{e}_a}$	$\mathbf{e}_a$	L_a	V	$\longleftarrow = 3\lvert\Gamma\rvert$
	$\overline{\mathbf{10}}_{-\mathbf{e}_a}$	$-\mathbf{e}_a$	L_a^*	V^*	$\longleftarrow = 0$
	$\bar{\mathbf{5}}_{\mathbf{e}_a+\mathbf{e}_b}$	$\mathbf{e}_a + \mathbf{e}_b$	$L_a \otimes L_b$	$\wedge^2 V$	$\Rightarrow 3\lvert\Gamma\rvert$
	$\mathbf{5}_{-\mathbf{e}_a-\mathbf{e}_b}$	$-\mathbf{e}_a - \mathbf{e}_b$	$L_a^* \otimes L_b^*$	$\wedge^2 V^*$	
bundle moduli S^α	$\mathbf{1}_{\mathbf{e}_a-\mathbf{e}_b}$	$\mathbf{e}_a - \mathbf{e}_b$	$L_a \otimes L_b^*$	$V \otimes V^*$	
	$\mathbf{1}_{-\mathbf{e}_a+\mathbf{e}_b}$	$-\mathbf{e}_a + \mathbf{e}_b$	$L_a^* \otimes L_b$		

Can lead to standard models after taking Γ- quotient and including Wilson line.

The additional U(1) symmetries are relevant in a number of ways:

- They constrain the low-energy theory, in particular:
- Together with bundle moduli singlets, they generate Yukawa-textures (Frogatt-Nielsen)
- They can forbid dim. 4/5 operators inducing proton decay
- They can help to solve the μ - problem
- Line bundles can help stabilise moduli

Search for SU(5) line bundle models

Arena: complete intersection CYs (with $h^{1,1} \leq 6$)

line bundles: described by $h^{1,1} \times 5$ integer matrices

Scan over $\sim 10^{40}$ models, imposing consistency and correct number of chiral families leads to:

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$h^{1,1}(X)$	1	2	3	4	5	6	total
#models	0	0	6	552	21731	41036	63325

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Roughly, a factor 10 more models per CY for each additional Kahler parameter!

After demanding absence of $\bar{10}$ and presence of $5-\bar{5}$ pair:

34989 SU(5) GUT models

Available at:

http://www-thphys.physics.ox.ac.uk/projects/CalabiYau/linebundlemodels/index.html

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So far we have further analyzed of those:

~ 200 GUT models leading to ~ 2000 standard models*

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So far we have further analyzed of those:

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*standard model: SM gauge group times (anomalous) U(1)s, exact MSSM matter spectrum, one or more pairs of Higgs doublets, no exotics charged under standard model group.

The standard model example

CY data:

- **Cicy 7862, Symmetry 3**

$$X = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix}$$

$\eta(X) = -128$ $h^{1,1}(X) = 4$ $h^{2,1}(X) = 68$ $c_2(TX) = \{24, 24, 24, 24\}$

$\kappa = 12\, t_1\, t_2\, t_3 + 12\, t_1\, t_2\, t_4 + 12\, t_1\, t_3\, t_4 + 12\, t_2\, t_3\, t_4$

symmetry: 3 order: 4

Abelian: True block diagonal: True factors: {2, 2}

Action on coordinates: $\left\{ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \right\}$

Action on polynomials: $\{(\ 1\ ),\ (\ 1\ )\}$

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Action on polynomials: $\{(\,1\,), (\,1\,)\}$

$\mathbb{Z}_2 \times \mathbb{Z}_2$ generators

bundle data:

■ Basic properties

standard model? True massless U(1): 1 number of $5\,\overline{5}$ pairs: 3 $c_2(V) = \{24, 8, 20, 12\}$

$$V:\ (k^i{}_a) = \begin{pmatrix} -1 & -1 & 0 & 1 & 1 \\ 0 & -3 & 1 & 1 & 1 \\ 0 & 2 & -1 & -1 & 0 \\ 1 & 2 & 0 & -1 & -2 \end{pmatrix}$$

Cohomology of V:

L_2	=	$\{-1, -3, 2, 2\}$	$h[L_2]$	=	$\{0, 8, 0, 0\}$	$h[L_2,R]$	= $\{\{0, 0, 0, 0\}, \{2, 2, 2, 2\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$
L_5	=	$\{1, 1, 0, -2\}$	$h[L_5]$	=	$\{0, 4, 0, 0\}$	$h[L_5,R]$	= $\{\{0, 0, 0, 0\}, \{1, 1, 1, 1\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$
$L_2\times L_4$	=	$\{0, -2, 1, 1\}$	$h[L_2\times L_4]$	=	$\{0, 4, 0, 0\}$	$h[L_2\times L_4,R]$	= $\{\{0, 0, 0, 0\}, \{1, 1, 1, 1\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$
$L_2\times L_5$	=	$\{0, -2, 2, 0\}$	$h[L_2\times L_5]$	=	$\{0, 3, 3, 0\}$	$h[L_2\times L_5,R]$	= $\{\{0, 0, 0, 0\}, \{0, 1, 1, 1\}, \{0, 1, 1, 1\}, \{0, 0, 0, 0\}\}$
$L_4\times L_5$	=	$\{2, 2, -1, -3\}$	$h[L_4\times L_5]$	=	$\{0, 8, 0, 0\}$	$h[L_4\times L_5,R]$	= $\{\{0, 0, 0, 0\}, \{2, 2, 2, 2\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$
$L_1\times L_2^*$	=	$\{0, 3, -2, -1\}$	$h[L_1\times L_2^*]$	=	$\{0, 0, 12, 0\}$	$h[L_1\times L_2^*,R]$	= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{3, 3, 3, 3\}, \{0, 0, 0, 0\}\}$
$L_1\times L_5^*$	=	$\{-2, -1, 0, 3\}$	$h[L_1\times L_5^*]$	=	$\{0, 0, 12, 0\}$	$h[L_1\times L_5^*,R]$	= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{3, 3, 3, 3\}, \{0, 0, 0, 0\}\}$
$L_2\times L_3^*$	=	$\{-1, -4, 3, 2\}$	$h[L_2\times L_3^*]$	=	$\{0, 20, 0, 0\}$	$h[L_2\times L_3^*,R]$	= $\{\{0, 0, 0, 0\}, \{5, 5, 5, 5\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$
$L_2\times L_4^*$	=	$\{-2, -4, 3, 3\}$	$h[L_2\times L_4^*]$	=	$\{0, 12, 0, 0\}$	$h[L_2\times L_4^*,R]$	= $\{\{0, 0, 0, 0\}, \{3, 3, 3, 3\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$
$L_3\times L_5^*$	=	$\{-1, 0, -1, 2\}$	$h[L_3\times L_5^*]$	=	$\{0, 0, 4, 0\}$	$h[L_3\times L_5^*,R]$	= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{1, 1, 1, 1\}, \{0, 0, 0, 0\}\}$

Wilson line: $\{\{0, 0\}, \{0, 1\}\}$ Equivariant structure: $\{\{0, 0\}, \{0, 0\}, \{0, 0\}, \{0, 0\}, \{0, 0\}\}$ Higgs pairs: 1

Downstairs spectrum: $\left\{2\,10_2,\ 10_5,\ \overline{5}_{2,4},\ 2\,\overline{5}_{4,5},\ H_{2,5},\ \overline{H}_{2,5},\ 3\,S_{2,1},\ 3\,S_{5,1},\ 5\,S_{2,3},\ 3\,S_{2,4},\ S_{5,3}\right\}$ Phys. Higgs: $\left\{H_{2,5},\ \overline{H}_{2,5}\right\}$

Transfer format: $\{\{6, 1, 1, 4, 6, 5, 9, 9, 8, 10, 1, 7, 17\}, \{6, 6, -1, -1, -1, -1\}\}$

$\mathrm{rk}(Y^{(u)}) = \{2, 2\}$ $\mathrm{rk}(Y^{(d)})) = \{0, 0\}$ dim. 4 operators absent: {True, True} dim. 5 operators absent: {True, True}

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integer matrix defining line bundle sum

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$L_2\times L_4$	=	{0, −2, 1, 1}	$h[L_2\times L_4]$	=	{0, 4, 0, 0}	$h[L_2\times L_4,R]$	= {{0, 0, 0, 0}, {1, 1, 1, 1}, {0, 0, 0, 0}, {0, 0, 0, 0}}
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$L_1\times L_2^*$	=	{0, 3, −2, −1}	$h[L_1\times L_2^*]$	=	{0, 0, 12, 0}	$h[L_1\times L_2^*,R]$	= {{0, 0, 0, 0}, {0, 0, 0, 0}, {3, 3, 3, 3}, {0, 0, 0, 0}}
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Wilson line: {{0, 0}, {0, 1}} Equivariant structure: {{0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}} Higgs pairs: 1

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$L_2\times L_3^*$	=	{−1, −4, 3, 2}	$h[L_2\times L_3^*]$	=	{0, 20, 0, 0}	$h[L_2\times L_3^*,R]$	= {{0, 0, 0, 0}, {5, 5, 5, 5}, {0, 0, 0, 0}, {0, 0, 0, 0}}
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spectrum: $\mathbf{10}_2,\ \mathbf{10}_2,\ \mathbf{10}_5,\ \overline{\mathbf{5}}_{2,4},\ \overline{\mathbf{5}}_{4,5},\ \overline{\mathbf{5}}_{4,5},\ H_{2,5},\ \bar{H}_{2,5}$

$$3\,\mathbf{1}_{2,1},\ 3\,\mathbf{1}_{5,1},\ 5\mathbf{1}_{2,3},\ 3\,\mathbf{1}_{2,4},\ \mathbf{1}_{5,3}$$

allowed operators:

■ Operators

basic superpotential terms:

$\overline{H}10^p10^q$: $Y^{(u)} = \begin{pmatrix} (0) & (0) & (1) \\ (0) & (0) & (1) \\ (1) & (1) & (0) \end{pmatrix}$

$H\overline{5}^p10^q$: $Y^{(d)} = \begin{pmatrix} (0) & (0) & (0) \\ (0) & (0) & (0) \\ (0) & (0) & (0) \end{pmatrix}$

$H\overline{H}$: $\mu = \{1\}$

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R−parity violating terms in superpotential:

$\overline{H}L^p$: $\rho = \begin{pmatrix} 0 \\ S_{2,4} \\ S_{2,4} \end{pmatrix}$

$10^p\overline{5}^q\overline{5}^r$: $\lambda = \{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\}$

Dimension 5 operators in superpotential:

$\overline{5}^p10^q10^r10^s$: $\lambda' = \{\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\},$
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D−terms:

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$$H\overline{5}^p10^q:\ Y^{(d)} = \begin{pmatrix} (0) & (0) & (0) \\ (0) & (0) & (0) \\ (0) & (0) & (0) \end{pmatrix}$$

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- **Operators**

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Dimension 5 operators in superpotential:

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Dimension 5 operators in superpotential:

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What about more detailed properties, i.e. Yukawa couplings?

Yukawa couplings

Holomorphic Yukawa couplings:

$$\lambda(\nu_1, \nu_2, \nu_3) = \int_X \Omega \wedge \nu_1 \wedge \nu_2 \wedge \nu_3 \qquad \nu_i \in H^1(X, E_i)$$

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Can be computed without knowing Ricci-flat metric

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Physical Yukawa couplings depend on normalization:

$$(\nu_i, \mu_i) := \int_X \nu_i \wedge \bar{\star}_{E_i} \mu_i = \frac{1}{2} \int_X J \wedge J \wedge \nu_i \wedge (H \bar{\mu}_i)$$

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Requires harmonic representative and Ricci-flat metric

Yukawa unification?

Consider, for example, $SU(5)$ GUT with $\Gamma = \mathbb{Z}_2$ and down-Yukawa:

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upstairs: 6 families

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Wilson line described by Γ-representations χ_2, χ_3 satisfying $\chi_2^2 \otimes \chi_3^3 = 1$. For $\Gamma = \mathbb{Z}_2$ we have $\chi_2 = (1)$ and $\chi_3 = (0)$.

$$\chi_H = \chi_2^* = (1)$$

$$\chi_d = \chi_3^* = (0) \qquad \chi_Q = \chi_2 \otimes \chi_3 = (1)$$

$$\chi_L = \chi_2^* = (1) \qquad \chi_e = \chi_2 \otimes \chi_2 = (0)$$

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$$(\lambda_{IJ}) = \begin{pmatrix} \lambda_{(0,1)} & \lambda_{(0,0)} \\ \lambda_{(1,1)} & \lambda_{(1,0)} \end{pmatrix}$$

$$\chi_H = \chi_2^* = (1)$$

$$\chi_d = \chi_3^* = (0) \qquad \chi_Q = \chi_2 \otimes \chi_3 = (1)$$

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This holds for any symmetry Γ and all types of Yukawa couplings.

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Yukawa unification may arise from additional symmetries which constraints the upstairs Yukawa couplings λ_{IJ} .

Example: up-Yukawa couplings on tetra-quadric

Relevant line bundles for up-Yukawa coupling:

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\begin{array}{lll}
K_1 = L_2^* \otimes L_5^* & 3\ \mathbf{5}^H_{2,5} & \hat{\nu}_1 = \kappa_3^{-2} Q_{(0,2,-2,0)} d\bar{z}_3 \\
K_2 = L_5 & 4\ \mathbf{10}_2 & \hat{\nu}_2 = \kappa_4^{-2} R_{(1,1,0,-2)} d\bar{z}_4 \\
K_3 = L_2 & 8\ \mathbf{10}_5 & \hat{\omega} = \kappa_1^{-3} \kappa_2^{-5} S_{(-3,-5,0,0)} d\bar{z}_1 \wedge d\bar{z}_2
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where

$$
\begin{aligned}
Q &= q_0 + q_1 z_2 + q_2 z_2^2 \\
R &= r_0 + r_1 z_1 + r_2 z_2 + r_3 z_1 z_2 \\
S &= s_0 + s_1 \bar{z}_2 + s_2 \bar{z}_2^2 + s_3 \bar{z}_2^3 + s_4 \bar{z}_1 + s_5 \bar{z}_1 \bar{z}_2 + s_6 \bar{z}_1 \bar{z}_2^2 + s_7 \bar{z}_1 \bar{z}_2^3
\end{aligned}
$$

holomorphic Yukawa couplings:

$$\lambda(Q,R,S) = \frac{1}{\pi}\int_{\mathbb{C}^4} \frac{QRS}{\kappa_1^3\kappa_2^5\kappa_3^2\kappa_4^2} d^4z\, d^4\bar{z}$$

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After taking quotient by $\Gamma = \mathbb{Z}_2 \times \mathbb{Z}_2$ and adding Wilson line:

$$\lambda^{(u)} = \frac{\pi^3}{3}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

A vanishing theorem

For $X \subset \mathcal{A} = \mathbb{P}^{n_1} \times \cdots \times \mathbb{P}^{n_m}$ each one-form ν_i is described by a p_i-form on the ambient space $\mathcal{A}$, where $i = 1, 2, 3$.

If $p_1 + p_2 + p_3 < \dim(\mathcal{A})$ then $\lambda(\nu_1, \nu_2, \nu_3) = 0$.

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If $p_1 + p_2 + p_3 < \dim(\mathcal{A})$ then $\lambda(\nu_1, \nu_2, \nu_3) = 0$.

This is a topological vanishing with no apparent explanation by a symmetry in the 4d theory.

What about physical Yukawa couplings?

$$\nu \in H^1(X, \mathcal{O}_X(k_1, k_2, k_3, 0)) \qquad (k_1 \leq -2,\ k_2, k_3 \geq 0)$$

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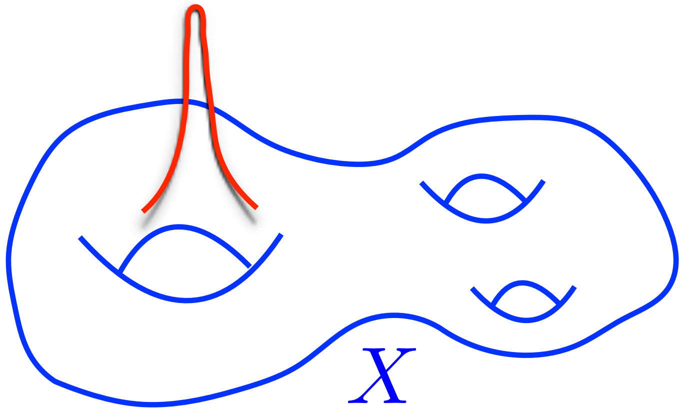

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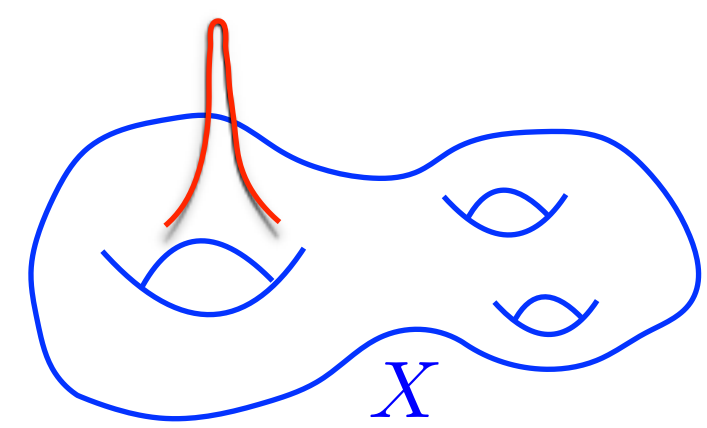
For large flux integrand localised -> normalization can be computed approximately without knowing Ricci-flat metric.

$$K_{IJ}^{(matter)} = \frac{1}{V} \langle \nu_I, \nu_J \rangle \simeq \frac{N_I}{t_1} \delta_{IJ}$$



Conclusion

- We can now systematically search for and construct heterotic models with the correct low-energy spectrum.

- We find 10 times more such models for $h^{1,1}(X) \to h^{1,1}(X) + 1$

- It is unclear if this continues until $h^{1,1}(X) \sim 500$.
  Large $h^{1,1}(X)$ are a computational challenge due to the growing number of bundles.

- Anomalous U(1) symmetries constrain the low-energy theory and can help to solve physical problems, e.g. proton stability.

- The underlying GUT symmetry of heterotic model does not lead to Yukawa unification.

- Yukawa couplings have unexpected topological vanishing properties.

- We can work out holomorphic Yukawa couplings, and there is progress in calculating the matter field Kahler potential

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Thanks