Phenomenology of Heterotic Compactifications with Torsion

Stephen Angus
IBS-CTPU, Daejeon
in collaboration with
Cyril Matti (City Univ., London) and Eirik Eik Svanes (LPTHE, Paris)

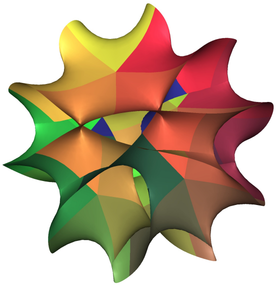
IBS Focus Workshop on Particle Physics and Cosmology
07 December 2016

Outline

1. **Background: heterotic strings and torsion**
   - Moduli stabilization overview
   - SU(3) structure and half-flat manifolds
   - Domain wall vacuum
2. *Spin*(7) **structures**
   - *Spin*(7) and generalized half-flat manifolds
   - 10*d* flow equations and matching to 4*d* 1/4-BPS equations
3. **Inflation**
   - Background: Anomalous $U(1)$ symmetries
   - Inflation: simple case
   - Extension: two-axion model

Calabi–Yau compactification

- Superstring theory is self-consistent only in 10 spacetime dimensions.
- Assume the extra 6 spatial dimensions are compactified.
- Lots of supersymmetry in $d = 10$ $\rightarrow$ want to break most of it.
- Amount of broken SUSY $\Rightarrow$ holonomy group of compactification manifold.



- Maximum holonomy is SO(6) $\cong$ SU(4) $\Rightarrow$ no SUSY preserved.
- Calabi–Yau manifold: SU(3) holonomy $\Rightarrow$ 1/4 SUSY preserved
  - e.g. heterotic Calabi-Yau: 4 of 16 supercharges unbroken.

Problems with heterotic moduli stabilization

- In heterotic string theory, only have NS-NS flux H_3.
- Can stabilize complex structure moduli... then what?
- Dilaton can be stabilized by gaugino condensation.
- Limited options for remaining moduli (worldsheet instantons...)
- In fact, problem is even worse:

Strominger, 1986

If a heterotic compactification on a manifold Y has a maximally symmetric (e.g. Poincaré) vacuum and non-vanishing H_3, Y is non-Calabi–Yau.

- Hence for a Calabi–Yau compactification, $H_3 = 0$!

What is an SU(3) structure manifold?

- Mirror dual of H_3: manifold with SU(3) structure
  hep-th/0008142 (Vafa), hep-th/0211102 (Gurrieri *et al*).
- SU(3) structure: there is a globally-defined spinor ζ that leaves 1/4 of the SUSY unbroken.
- Calabi–Yau case: ζ is covariantly constant with respect to the Levi-Civita connection ∇.
- Non-CY case: $\nabla\zeta \sim T^0\zeta$ (note: Γ matrices/indices suppressed).
- T^0 is the intrinsic torsion of the manifold.
- SU(3) decomposition: torsion splits into 5 torsion classes,

$$T^0 \in \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3 \oplus \mathcal{W}_4 \oplus \mathcal{W}_5 \ .$$

Half-flat manifolds

Two (not mutually exclusive) ways to satisfy Strominger's theorem:

Option 1:

Study compactifications on SU(3) structure manifolds with torsion.

- Has been studied in eg. hep-th/0408121 (Gurrieri, Lukas, Micu), hep-th/0507173 (de Carlos, Gurrieri, Lukas, Micu).
- Torsion quantization understood for half-flat manifolds.
- Expanding the SU(3) invariant forms on appropriate bases, the only non-closed basis forms in the half-flat case satisfy

$$d\omega_i = e_i \beta^0 \ , \quad d\alpha_0 = e_i \tilde{\omega}^i \ .$$

- For half-flat manifolds, torsion falls into the SU(3) classes

$$T^0 \in \mathcal{W}_1^+ \oplus \mathcal{W}_2^+ \oplus \mathcal{W}_3 \ ,$$

where $+$ denotes the real part of $\mathcal{W}$.

Domain wall vacuum

Option 2:

Break maximal symmetry of $d = 4$ spacetime.

- Compactification with H-flux on a half-flat or Calabi-Yau manifold.
- There exist 1/2-BPS domain wall solutions 1305.0594 (Klaput, Lukas, Svanes).
- 1/2-BPS: 2 of the 4 SUSY generators in $d = 4, \mathcal{N} = 1$ unbroken.
- $d = (2 + 1)$ Poincaré symmetry preserved; DW breaks symmetry in transverse y direction.
- Moduli satisfy flow equations in the y coordinate.
- 10d perspective: SU(3) fibred over $y \rightarrow G_2$ structure arXiv:1005.5302 (Lukas, Matti).

Spin(7): two transverse coordinates

- The domain wall solution is a special case of the metric

$$ds_4^2 = e^{-2B(x_a)} \left(\eta_{\alpha\beta} d\tilde{x}^\alpha d\tilde{x}^\beta + g_{ab} dx^a dx^b \right) .$$

- We can consider more general codimension-2 topological defects.
- 10d perspective: looks like an 8-dimensional *Spin*(7) structure.
- For the corresponding 6*d* compact SU(3) structure manifold, consider a generalized half-flat manifold, which satisfies

$$d\omega_i = p_{Ai}\beta^A - q_i^A \alpha_A , \quad d\alpha_A = p_{Ai}\tilde{\omega}^i , \quad d\beta^A = q_i^A \tilde{\omega}^i , \quad d\tilde{\omega}^i = 0 ,$$

where ω_i and (α_A, β^B) are basis 2- and 3-forms, respectively.
- Relevant SU(3) torsion classes are now

$$T^0 \in \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3 .$$

Flow equations: ten-dimensional perspective

- Ten-dimensional perspective:
  6 compact dimensions $+$ 2 non-compact directions x and y
  $\rightarrow$ 8d $Spin(7)$ structure.
- Killing spinor can be written in terms of invariant Cayley 4-form Ψ.
- Decompose Ψ under the 6d SU(3) structure as

$$\Psi = \mathrm{Re}(\mathrm{d}z \wedge \Omega) + \frac{1}{2} J \wedge J + \mathrm{dvol}_2 \wedge J \ ,$$

  where $\mathrm{d}z = \mathrm{d}x + i\mathrm{d}y$, $\mathrm{dvol}_2 = \mathrm{d}x \wedge \mathrm{d}y$.
- J is a 6d Kähler (1,1)-form, and Ω is a holomorphic (3,0)-form (for a Calabi–Yau they are harmonic, $\mathrm{d}J = \mathrm{d}\Omega = 0$).
- 10d supersymmetry transformations give the 8d flow equations,

$$*_8 \hat{H} = -e^{2\hat{\phi}} \mathrm{d}_8 (e^{-2\hat{\phi}} \Psi) \ , \quad 12 \mathrm{d}_8 \hat{\phi} = \Psi \lrcorner \mathrm{d}_8 \Psi = - *_8 \left(\Psi \wedge *_8 \mathrm{d}_8 \Psi \right) \ ,$$

  where $\hat{\phi}$ is the 10d dilaton and $\hat{H}$ is the 10d NS-NS flux.

Compactification

- Expand J and Ω in terms of the basis forms:

$$J = v^i \omega_i \ , \quad \Omega = \mathcal{Z}^A \alpha_A - \mathcal{G}_A \beta^A \ .$$

- The NS-NS 2-form potential and 3-form flux can be expanded as

$$\hat{B} = B + b^i \omega_i \ , \quad \hat{H} = H + \mathrm{d}b^i \wedge \omega_i + b^i \mathrm{d}\omega_i + H_{\text{flux}} \ ,$$

where we have introduced $H_{\text{flux}} = \mu^A \alpha_A - \epsilon_A \beta^A$.

- Moduli superfields, including 4*d* dilaton ϕ:

$$S = a + ie^{-2\phi} \ , \quad T^i = b^i + iv^i \ , \quad Z^a \equiv \mathcal{Z}^a / \mathcal{Z}^0 = c^a + iw^a \ .$$

- Consider *H*-flux only on the internal space $\Rightarrow a \sim b^i \sim$ constant.
- Under SU(3)-structure decomposition, flow equations reduce to:

$$\mathrm{d}J = 2\mathrm{Im}\left(\partial_{\bar{z}}\Omega - 2\partial_{\bar{z}}\hat{\phi}\Omega\right) - *\hat{H} \ ;$$

$$\mathrm{d}\Omega = \partial_z(J \wedge J) - 2\partial_z\hat{\phi} J \wedge J \ ; \qquad \Omega \wedge \hat{H} = 4i * \partial_z\hat{\phi} \ .$$

Four-dimensional perspective

- Let us also consider an ansatz for the 4*d* theory.
- Unbroken supercharge in singlet of *Spin*(7) $\Rightarrow$ 1/4-BPS in 4*d*.
- 1/4-BPS ansatz: $\bar{\zeta} = \sigma^2\zeta = i\sigma^3\zeta$ gives Killing spinor equations

$$
\begin{aligned}
(\partial_x + i\partial_y)\, A^I &= -ie^{-B}e^{K/2}K^{IJ^*}D_{J^*}W^* \ , \\
(\partial_x + i\partial_y)\, B &= -ie^{-B}e^{K/2}W^* \ , \\
0 &= \mathrm{Im}(K_I\partial_a A^I) \ , \\
2\partial_a\zeta &= -\partial_a B\zeta \ ,
\end{aligned}
$$

where $a \subset \{x, y\} \equiv \{2, 3\}$, and $A^I = (S, T^i, Z^a)$.
- GVW superpotential (for generalized half-flat manifolds):

$$
W = \sqrt{8}\int \Omega \wedge (\hat{H} + i\mathrm{d}J) = \sqrt{8}\,(\tilde{\mu}^A\mathcal{G}_A - \tilde{\epsilon}_A\mathcal{Z}^A) \ ,
$$

with modified flux parameters $\tilde{\epsilon}_A \equiv \epsilon_A - T^i p_{Ai}$, $\tilde{\mu}^A \equiv \mu^A - T^i q_i^A$.

Matching the 10*d* and 4*d* equations

- Consistency: need to match the 10*d* flow equations to the 4*d* 1/4-BPS Killing spinor equations 1512.02812 (SA, Matti, Svanes).

Summary of key points:

- 4*d* dilaton equation $\Rightarrow$ warp factor $B = \phi$ (up to a constant).
- $\mathrm{d}\Omega$ equation in 10*d* $\rightarrow$ KSE in 4*d* for the Kahler moduli T^i.
- $\mathrm{d}J$ equation in 10*d* $\rightarrow$ KSE for complex structure moduli Z^a.
- $\mathrm{d}J \wedge \bar{\Omega}$ with $\Omega \wedge \hat{H}$ $\rightarrow$ dilaton equation, $2\partial_{\bar{z}}\phi = -ie^{-\phi}e^{K/2}W^*$, BUT only if we also impose the additional 10*d* constraint

$$\int \partial_z \Omega \wedge \bar{\Omega} = \int \Omega \wedge \partial_z \bar{\Omega}\,.$$

- Actually, $\partial_z \Omega = K_z \Omega + \chi_z^{(2,1)}$; we are free to choose K_z real.
- Reducing this constraint to 4*d* $\rightarrow$ axion constraint, $K_a \partial_z c^a = 0$.

Summary I

- String compactifications generate moduli, which must be stabilized. This can be done using fluxes.
- For the heterotic string, only H_3 present. One can compactify on SU(3) structure manifolds which are not Calabi–Yau, and/or sacrifice maximal symmetry in $d = 4$. Domain wall solutions have been studied.
- We considered the more general codimension-2 case: from a Spin(7) ansatz compactified on generalized half-flat manifolds, the flow equations correspond to 1/4-BPS solutions in 4d.

Outlook:

- Still need to find explicit solutions... possible connection to Spin(7) compactifications of F-theory? 1307.5858 (Bonetti, Grimm, Pugh)
  $\Rightarrow$ Work in progress!

Moduli stabilization with aligned D-terms: a review

- Consider a half-flat manifold with axio-dilaton $S = s + i\sigma$ and two Kähler moduli, $T = t + i\tau$ and $U = u + i\nu$.
- Assume complex structure moduli absent, or stabilized already.
- Allow geometric flux on the 2-cycle t only, giving a perturbative superpotential
$$W_{\rm P} = w + e_1 T \ ,$$
where $T = t + i\tau$. (NOTE: phenomenology conventions!)
- Here w is generated by either NS flux or α' corrections.
- Assume an anomalous $U(1)$ symmetry under which only S and U transform, giving D-terms of the form $D = b/s + c/u$ (b, c real).
- Resulting scalar potential stabilizes all moduli, minimum is non-supersymmetric AdS. 1504.06978 (Lukas, Lalak, Svanes)

Inflation: simple case

- A successful model of inflation must:
  - allow a de Sitter phase, $V > 0$;
  - satisfy the slow-roll conditions, $\epsilon, |\eta| \ll 1$.
- Need to identify a flat direction for the inflaton field.
- Promising candidates: two axions (σ and ν) remain unstabilized.
- One linear combination is gauge-dependent; it will be absorbed by the $U(1)$ gauge boson via the Stueckelberg mechanism.
- Lift the remaining gauge-invariant axionic direction with a non-perturbative term,

$$W_{\mathrm{NP}} = Ae^{-\alpha(S-\beta U)} ,$$

which can arise from gaugino condensation in the hidden sector.

- We expect such models to have a Kähler potential of the form

$$K = -\ln s - \ln \kappa\ , \quad \kappa = \kappa_{ijk} t^i t^j t^k$$

  where d_{ijk} are intersection numbers on the mirror manifold.
- In the particular model above, we choose $\kappa = t^2 u$ in order to stabilize all moduli perturbatively.
- The final vacuum is non-supersymmetric AdS, so need to uplift: assume this can be done such that $V_{\text{final}} = 0$.
- Resulting potential has a natural inflation form

$$V = V_0 \left(1 - \cos\left(\frac{\hat{\theta}}{f}\right)\right)\ ,$$

  where $\hat{\theta}$ is canonically normalized and the axion decay constant

$$f = \frac{1}{\alpha\sqrt{2(1+\beta^2)}}\ .$$

Two-axion model

- Natural inflation with a single axion field requires a trans-Planckian field excursion to match observations.
- Furthermore we did not explicitly achieve de Sitter uplifting.

How to address these issues?

- First consider adding an additional Kähler modulus, $X = x + i\xi$.
- Stabilize ξ with a worldsheet instanton superpotential,

$$W_{\mathrm{NP2}} = Be^{-n_1 U - n_2 X} \ ,$$

  which is gauge-invariant by construction.
- Second, choose a Kähler potential of the form $\kappa = txu$.
- With this choice, the geometric flux induces vanishing of the tree-level potential (at the cost of stabilizing moduli perturbatively).

- The leading contributions to the scalar potential are

$$V = \frac{1}{s}\left[\frac{b}{s} + \frac{c}{u} + \frac{d}{x}\right]^2 + \frac{1}{stux}\Big\{(w - e_1 t)^2 + e_1^2\tau^2$$
$$-2|A|e^{-\hat{s}}\Big[(w - e_1 t + 2\hat{s}(w + e_1 t))\cos\hat{\sigma} - (1 + 2\hat{s})e_1\tau\sin\hat{\sigma}\Big]$$
$$-2|B|e^{-\hat{x}}\Big[(w - e_1 t + 2\hat{x}(w + e_1 t))\cos\hat{\xi} - (1 + 2\hat{x})e_1\tau\sin\hat{\xi}\Big]\Big\}\ ,$$

where $\hat{S} = \alpha(S - \beta U)$ and $\hat{X} = n_1 U + n_2 X$ are gauge-invariant.
- To leading order, the components of T are stabilized at

$$t \simeq \frac{w}{e_1}\ , \quad \tau \simeq 0\ .$$

- Subleading corrections give even further subleading terms in the scalar potential, which we can neglect for now.

- For the D-term potential to be minimized, we find that

$$\frac{1}{s}\left[\frac{b}{s}+\frac{c}{u}+\frac{d}{x}\right]^2=\frac{8e_1}{sxu}\left[\frac{|A|\hat{s}(\hat{s}+2)e^{-\hat{s}}}{3}+\frac{|B|\hat{x}(\hat{x}+2)e^{-\hat{x}}}{3}\right].$$

- Hence the remaining inflation potential takes the form

$$V=\frac{8e_1}{sxu}\left[|A|\hat{s}e^{-\hat{s}}\left(\frac{\hat{s}+2}{3}-\cos\hat{\sigma}\right)+|B|\hat{x}e^{-\hat{x}}\left(\frac{\hat{x}+2}{3}-\cos\hat{\xi}\right)\right].$$

- Supergravity approximation: $\hat{s},\hat{x}\gtrsim 1\Rightarrow$ de Sitter!
- For $n_1\gg n_2$ and $\beta\gg 1$, the axions are almost aligned along the ν direction $\rightarrow$ variation of aligned natural inflation possible. hep-ph/0409138 (Kim, Nilles, Peloso)
- However, recall that this violates the Weak Gravity Conjecture applied to axions/instantons.
- Also, to completely stabilize s and x, need g_s and α' corrections.

Summary II

- Using aligned D-terms all moduli can be stabilized, leaving only massless axions. Resulting vacuum is non-supersymmetric AdS.
- Including non-perturbative contributions from gaugino condensation and worldsheet instantons, natural inflation can be realized eg. 1409.8436 (Abe, Kobayashi, Otsuka).
- With two light axions, aligned natural inflation is possible; de Sitter uplifting can be achieved by geometric flux and D-terms.
- However, some moduli become destabilized — saved by quantum corrections?