A Supersymmetric two-field relaxion model

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J. L. Evans, T. Gherghetta, N. Nagata, Z. Thomas, JHEP **1609**, 150 (2016).
J. L. Evans, T. Gherghetta, N. Nagata, M. Peloso, in preparation.

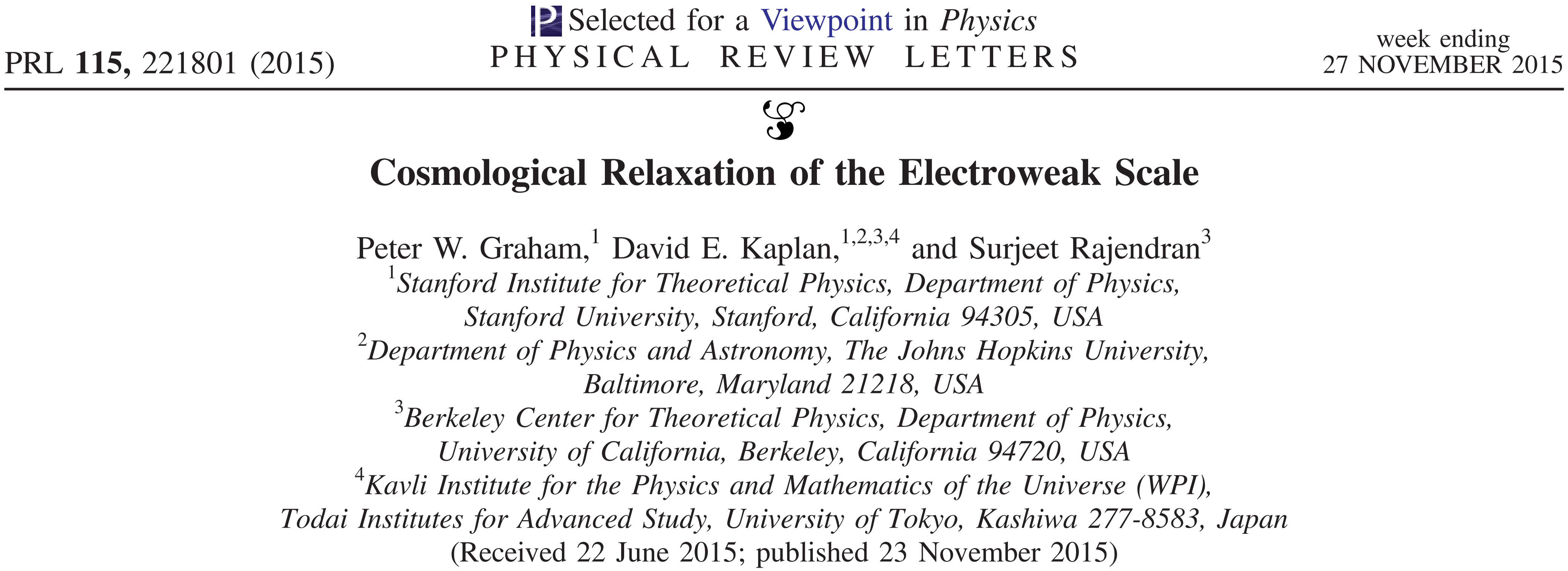
Outline

- Introduction
- Supersymmetry and relaxion
- Relaxion and Inflation
- Conclusion

Introduction

Relaxion mechanism

Relaxion and Electroweak fine-tuning



Selected for a Viewpoint in *Physics*
PRL **115,** 221801 (2015) PHYSICAL REVIEW LETTERS week ending 27 NOVEMBER 2015

Cosmological Relaxation of the Electroweak Scale

Peter W. Graham,[1] David E. Kaplan,[1,2,3,4] and Surjeet Rajendran[3]
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A new approach to the electroweak fine-tuning problem!

Relaxion model

See also L. F. Abbott (1985),
G. Dvali and A. Vilenkin (2013)
G. Dvali (2014)

P. W. Graham, D. E. Kaplan, S. Rajendran, Phys. Rev. Lett. **115**, 221801 (2015).

Relaxion ϕ

- Axion-like field (but takes non-compact [−∞, +∞] field values).
- Scans the Higgs mass parameter via its coupling with the Higgs field.

Scalar potential

$$V = \underline{-g\Lambda^3\phi} + \left(\Lambda^2 - \underline{g\Lambda\phi}\right)|H|^2 + \frac{\lambda}{2}|H|^4 + \underline{f_\pi^2 m_\pi^2}\cos\left(\frac{\phi}{f}\right)$$

Slightly breaks the shift symmetry.

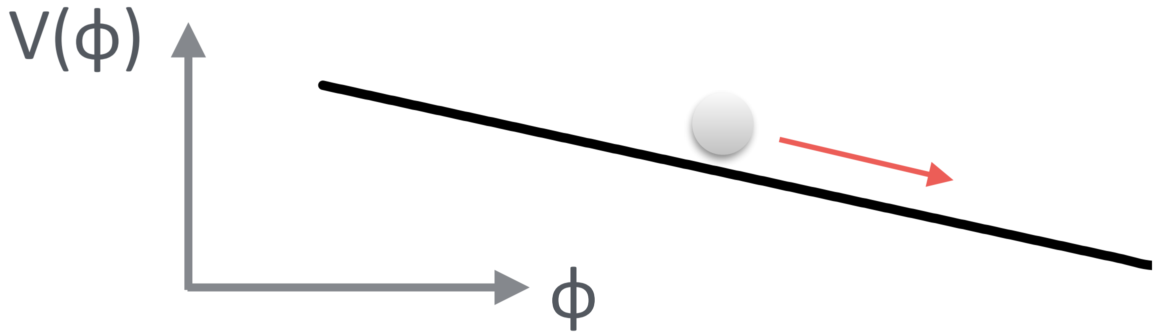
Proportional to Higgs VEV

→ ϕ rolls down on the potential.

→ Generate a barrier on ϕ after EWSB.

(Λ: cut-off, f: relaxion decay constant)

Relaxion model



Rolls down during inflation.

Scalar potential

$$V = \underline{-g\Lambda^3\phi} + \left(\Lambda^2 - \underline{g\Lambda\phi}\right)|H|^2 + \frac{\lambda}{2}|H|^4 + \underline{f_\pi^2 m_\pi^2} \cos\left(\frac{\phi}{f}\right)$$

Slightly breaks the shift symmetry.

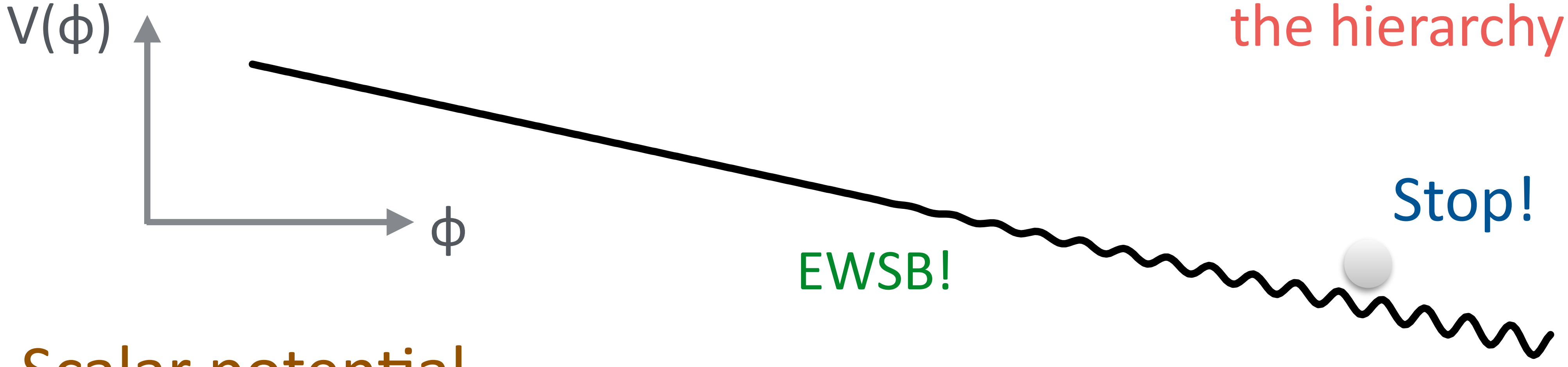
Proportional to Higgs VEV

→ ϕ rolls down on the potential.

→ Generate a barrier on ϕ after EWSB.

Relaxion model

Dynamical solution to the hierarchy problem.



Scalar potential

$$V = -\underline{g\Lambda^3\phi} + \left(\Lambda^2 - \underline{g\Lambda\phi}\right)|H|^2 + \frac{\lambda}{2}|H|^4 + \underline{f_\pi^2 m_\pi^2}\cos\left(\frac{\phi}{f}\right)$$

Slightly breaks the shift symmetry.

→ ϕ rolls down on the potential.

Proportional to Higgs VEV

→ Generate a barrier on ϕ after EWSB.

$$g \simeq \frac{f_\pi^2 m_\pi^2}{\Lambda^3 f}$$

← Extremely small but “technically natural”.

Tuning

→ ϕ stops right after the EWSB.

Motivation

- Definitely new idea.
- May offer a profound big picture.
- A lot of problems:
  - UV completion
  - Strong CP problem/coincidence problem
  - Inflation models

Self-organized criticality

etc.

Can we construct a promising and attractive model comparable to existing various ideas such as SUSY, composite Higgs, …?

SUSY and Relaxion

Supersymmetric two-field relaxion model

J. L. Evans, T. Gherghetta, N. Nagata, Z. Thomas, JHEP **1609**, 150 (2016).

Supersymmetry and relaxion

Original relaxion model considers an effective theory with cut-off Λ.

- UV completion ?
- Meaning of cut-off Λ ?

Here we assume SUSY as UV completion.

- In this case, the cut-off corresponds to SUSY scale.
- Relaxion mechanism explains the little hierarchy problem in SUSY models.

SUSY two-field relaxion model

We introduce two singlet chiral superfields S, T:

$$S = \frac{s + i\phi}{\sqrt{2}} + \sqrt{2}\,\widetilde{\phi}\,\theta + F_S\theta\theta \ ,$$

$$T = \frac{\tau + i\sigma}{\sqrt{2}} + \sqrt{2}\,\widetilde{\sigma}\,\theta + F_T\theta\theta \ ,$$

ϕ: relaxion

σ: the 2nd field (amplitudon)

Shift symmetry

$$\mathcal{S}_S : \ \begin{aligned} S &\to S + i\alpha f_\phi \ , \\ T &\to T \ , \\ Q_i &\to e^{iq_i\alpha} Q_i \ , \\ H_u H_d &\to e^{iq_H\alpha} H_u H_d \ , \end{aligned}$$

$$\mathcal{S}_T : \ \begin{aligned} S &\to S \ , \\ T &\to T + i\beta f_\sigma \ , \\ Q_i &\to Q_i \ , \\ H_u H_d &\to H_u H_d \ , \end{aligned}$$

J. L. Evans, T. Gherghetta, N. Nagata, Z. Thomas, JHEP **1609**, 150 (2016).

SUSY two-field relaxion model

Superpotential

$$W_{S,T} = \frac{1}{2} m_S S^2 + \frac{1}{2} m_T T^2 \ ,$$ (Shift-symmetry breaking terms)

$$W_\mu = \mu_0 e^{-\frac{q_H S}{f_\phi}} H_u H_d$$

$$W_{\rm gauge} = \left(\frac{1}{2g_a^2} - i \frac{\Theta_a}{16\pi^2} - \frac{c_a S}{16\pi^2 f_\phi} \right) {\rm Tr}(\mathcal{W}_a \mathcal{W}_a)$$ (a: SM, SU(N))

$$W_N = m_N N \bar{N} + i g_S S N \bar{N} + i g_T T N \bar{N} + \frac{\lambda}{M_L} H_u H_d N \bar{N} \ ,$$

- Kahler potential preserves the shift symmetry. $K(S+S^*, T+T^*)$
- σ does not couple to the Higgs fields directly.
- N and N̄ has a new SU(N) interaction.

Scalar potential

Suppose that ϕ and σ have large field values at the outset.

- ϕ and σ slowly rolls down due to the shift-symmetry breaking terms.
- On the other hand, s and τ are stuck by Kahler potential.

Scalar potential

$$V = \boldsymbol{m}^{\dagger}\mathcal{K}^{-1}\boldsymbol{m}\ . \qquad \mathcal{K} = \frac{1}{2}\begin{pmatrix} \frac{\partial^2\kappa}{\partial s^2} & \frac{\partial^2\kappa}{\partial s\partial\tau} \\ \frac{\partial^2\kappa}{\partial s\partial\tau} & \frac{\partial^2\kappa}{\partial\tau^2} \end{pmatrix}, \quad \boldsymbol{m} = \frac{1}{\sqrt{2}}\begin{pmatrix} m_S(s+i\phi) \\ m_T(\tau+i\sigma) \end{pmatrix}\ .$$

Minimum conditions with respect to s and τ

$$\frac{\partial}{\partial s}\mathcal{K}^{-1}(s,\tau) \simeq \frac{\partial}{\partial\tau}\mathcal{K}^{-1}(s,\tau) \simeq 0\ ,$$

As long as ϕ and σ have large field values, the minima of s and τ do not depend on ϕ and σ.

➡ s and τ can be regarded as constant.

Soft masses

During the field excursion, the potential energy is non-zero.

➡ $F_S \neq 0\ , \quad F_T \neq 0\ .$ ➡ These fields break SUSY.

Scalar masses

(e.g.) $$\int d^4\theta \frac{1}{M_*^2}(S+S^*)^2 Q_i Q_i^* \quad \Rightarrow \quad \tilde{m} \sim \frac{F_S}{M_*} \sim \frac{m_S \phi}{M_*}$$

Gaugino masses

$$\int d^2\theta \frac{c_a S}{16\pi^2 f_\phi} \mathrm{Tr}(\mathcal{W}_a \mathcal{W}_a) \quad \Rightarrow \quad M_a \sim \frac{c_a F_S}{16\pi^2 f_\phi} \sim \frac{c_a m_S \phi}{16\pi^2 f_\phi}$$

“Axion mediation”

M. Baryakhtar, E. Hardy, J. March-Russell, JHEP **1307**, 096 (2013).

A-terms

$$\int d^2\theta \frac{S+S^*}{M_*} Q_i Q_j Q_k \quad \Rightarrow \quad A \sim \frac{F_S}{M_*} \sim \frac{m_S \phi}{M_*}$$

Relaxion field scans these values.

B. Batell, G. F. Giudice, M. McCullough, JHEP **1512**, 162 (2015).

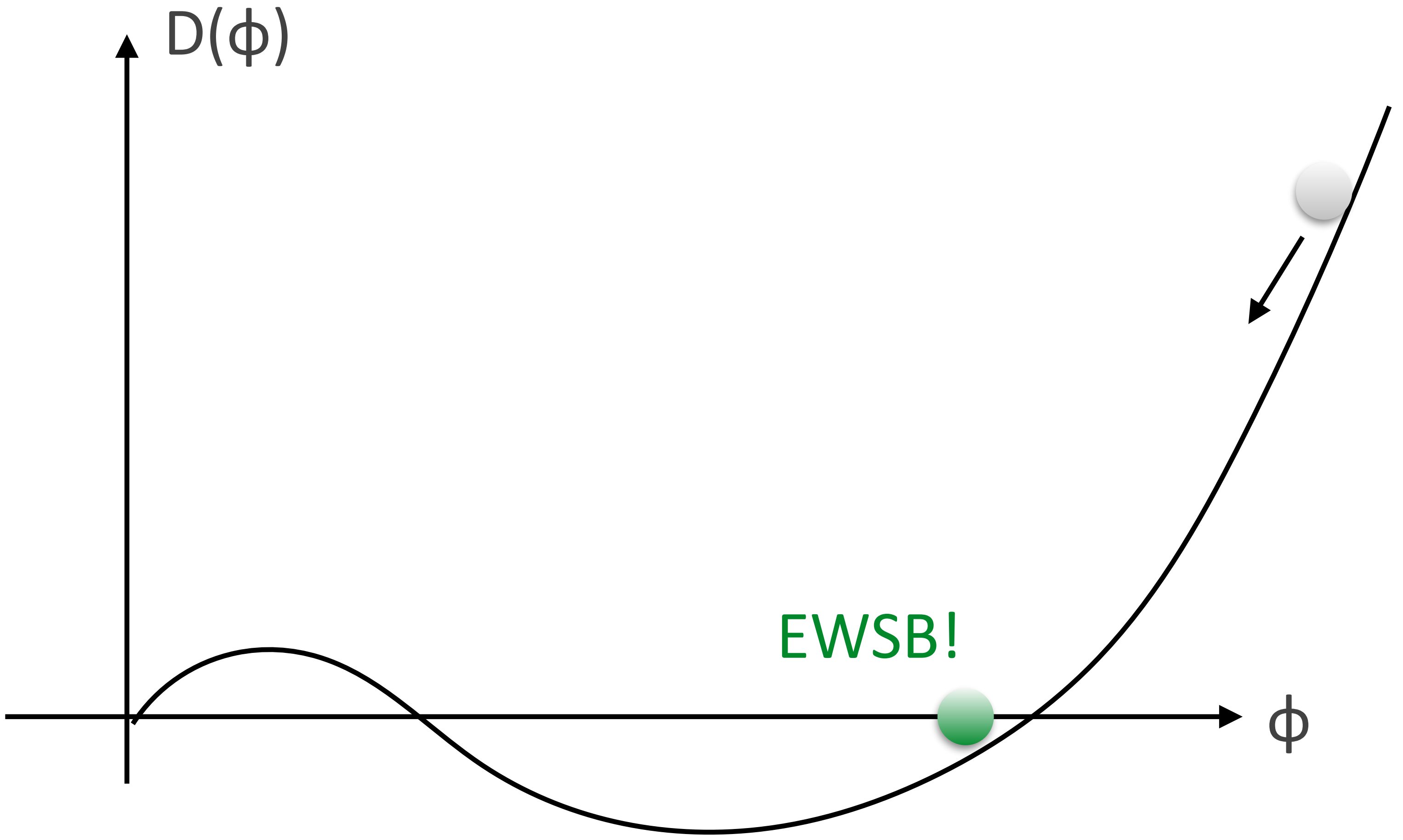
EWSB

EWSB condition

$$\mathcal{D}(\phi) \equiv (m_{H_u}^2 + |\mu|^2)(m_{H_d}^2 + |\mu|^2) - |B\mu|^2 < 0$$

We assume D(ϕ) > 0 at the outset.

Relaxion scans D(ϕ) while rolling down, and at a certain point the above condition is satisfied and EWSB occurs.



Critical value [D(ϕ*) = 0]

$$\mu_0 \sim \frac{m_S \phi_*}{f_\phi} \equiv m_{\text{SUSY}}$$

Cosmological evolution

Relevant potential

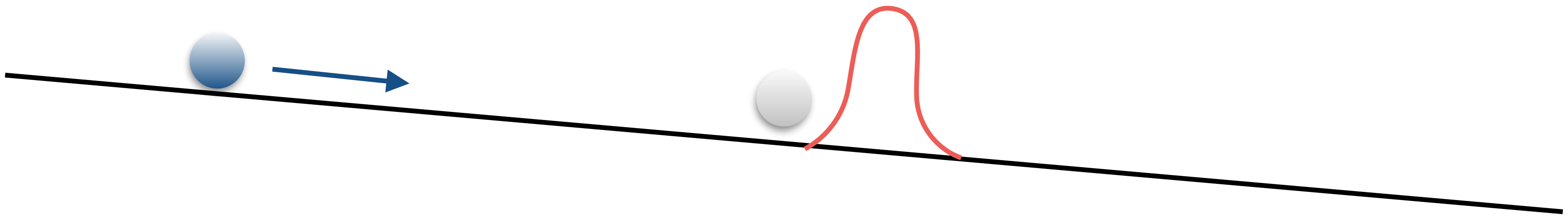
$$V_{\phi,\sigma}(\phi,\sigma,H_uH_d) = \frac{1}{2}|m_S|^2\phi^2 + \frac{1}{2}|m_T|^2\sigma^2 + \mathcal{A}\left(\phi,\sigma,H_uH_d\right)\underbrace{\Lambda_N^3}_{\langle\overline{\psi_N}\psi_N\rangle}\cos\left(\frac{\phi}{f_\phi}\right) \ ,$$

with

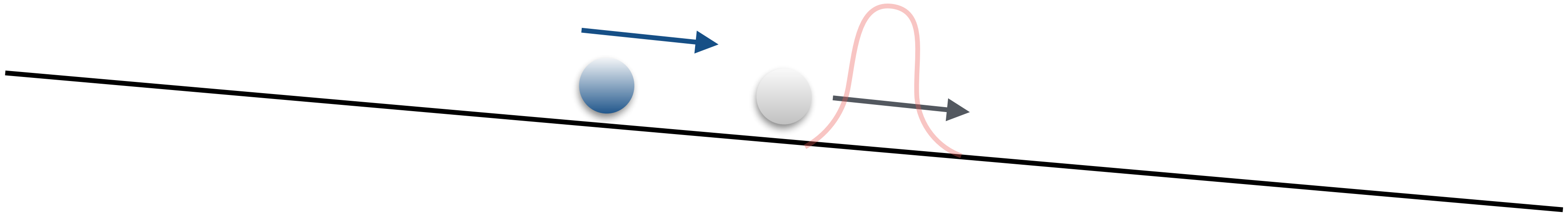
$$\mathcal{A}\left(\phi,\sigma,H_uH_d\right) = \overline{m}_N - \frac{g_S}{\sqrt{2}}\phi - \frac{g_T}{\sqrt{2}}\sigma + \frac{\lambda}{M_L}H_uH_d \ .$$

$(\overline{m}_N, g_S > 0,\ \ g_T < 0,\ \ \lambda < 0)$

I) At the beginning, only σ rolls while ϕ stops.



II) At some point, the barrier vanishes [A = 0] and both fields roll.



ϕ follows σ

$|m_T| < |m_S|$

Cosmological evolution

III) At some point, D(ϕ) < 0 and the electroweak symmetry is broken.

IV) The periodic potential stops ϕ. σ keeps rolling to the minimum.

Amplitude of the barrier gets larger and larger.

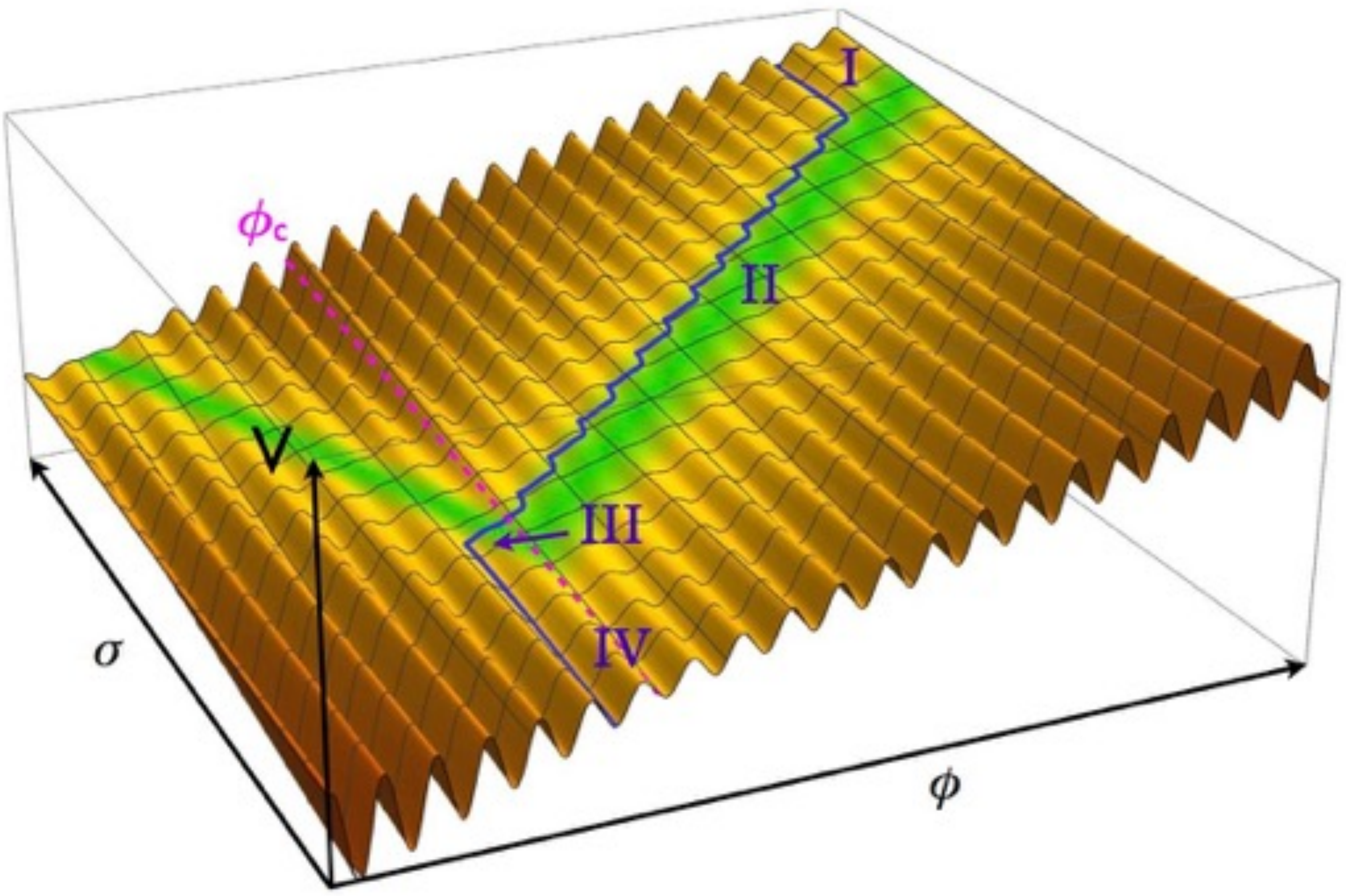
Two-field relaxion mechanism

I) ϕ stops. σ rolls.

II) Both ϕ and σ roll [A = 0].

III) EWSB (D(ϕ)<0).

IV) ϕ stops while σ keeps rolling.



J. Espinosa, C. Grojean, G. Panico, A. Pomarol, O. Pujolas, G. Servant (2015).

Constraints

Inflation is driven by another inflaton field.

Slow-roll condition

$$|m_S| \ll H_I$$

(H_I : Hubble parameter during inflation)

The energy of ϕ and σ is smaller than the inflaton energy

$$\frac{1}{2}|m_S|^2\phi^2 \ , \ \frac{1}{2}|m_T|^2\sigma^2 \ll 3H_I^2 M_P^2$$

(M_P : Planck mass)

Effects on SUSY breaking from the inflation sector are small

$$H_I \lesssim v$$

Low scale inflation

Classical rolling condition

$$\left|\frac{d\sigma}{dt}H_I^{-1}\right| \sim \frac{|m_T|^2\sigma}{3H_I^2} \gg H_I$$

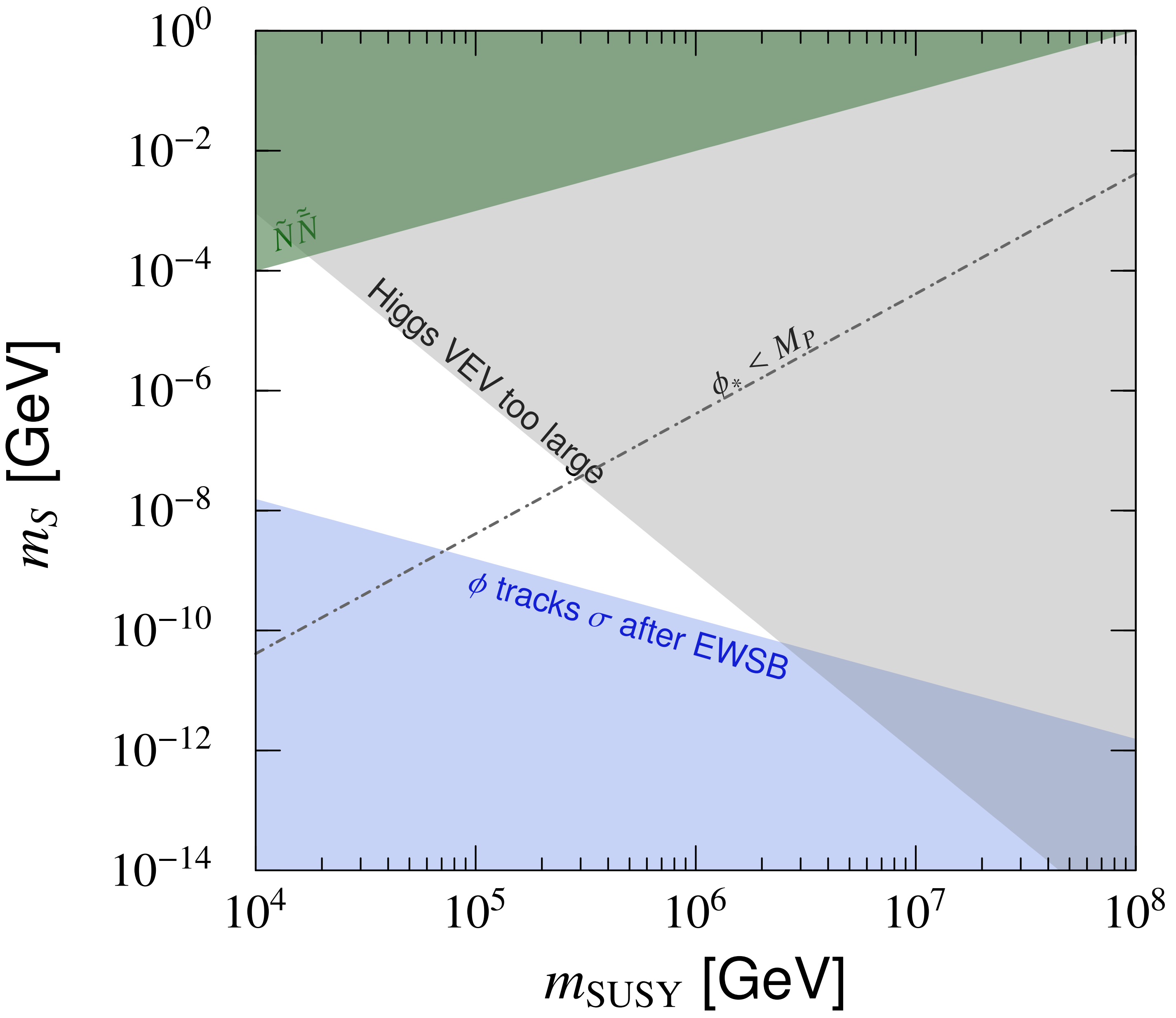
Typical quantum fluctuation during inflation

Displacement of σ over Hubble time

Number of *e*-folds

$$N_e \simeq \frac{H_I\Delta\phi}{\left|\frac{d\phi}{dt}\right|} \simeq \frac{3H_I^2\Delta\phi}{\left|\frac{\partial V}{\partial\phi}\right|} \gtrsim \frac{H_I^2}{|m_S|^2} = 10^{14} \times \left(\frac{H_I}{1\ \mathrm{GeV}}\right)^2 \left(\frac{10^{-7}\ \mathrm{GeV}}{|m_S|}\right)^2 .$$

Results



$$m_{\rm SUSY} = \Lambda_N = M_L = f_\phi = f_\sigma$$

$$m_T = 0.1 m_S$$

$$\frac{g_S f_\phi}{m_S} = \frac{g_T f_\sigma}{m_T} = 10^{-8}$$

$$\phi_* \sim \frac{m_{\rm SUSY} f_\phi}{m_S}$$

Relaxion mechanism can explain Little Hierarchy in 100 TeV SUSY.

J. L. Evans, T. Gherghetta, N. Nagata, Z. Thomas, JHEP **1609**, 150 (2016).

Relaxion and Inflation

Low-scale D-term inflation and relaxion

J. L. Evans, T. Gherghetta, N. Nagata, M. Peloso, in preparation.

Inflation models

Can we find an inflation model compatible with relaxion mechanism?

Hubble parameter

$$H_I < \min\left\{ v,\ 4.6\ \mathrm{GeV} \times \left(\frac{m_T/m_S}{0.1}\right)^{\frac{1}{3}} \left(\frac{f_\phi}{m_{\mathrm{SUSY}}}\right)^{\frac{1}{3}} \left(\frac{|m_S|}{10^{-7}\ \mathrm{GeV}}\right)^{\frac{1}{3}} \left(\frac{m_{\mathrm{SUSY}}}{10^5\ \mathrm{GeV}}\right)^{\frac{2}{3}} \right\} .$$

e-folding number

$$N_e \gtrsim 10^{14} \times \left(\frac{H_I}{1\ \mathrm{GeV}}\right)^2 \left(\frac{10^{-7}\ \mathrm{GeV}}{|m_S|}\right)^2 .$$

Can we find such a low-scale inflation model?

D-term inflation

E. D. Stewart (1995); P. Binetruy and G. R. Dvali (1996); E. Halyo (1996).

Introduce a new U(1) gauge interaction.

Superpotential

$$W = \kappa T \Phi_+ \Phi_-$$

$\Phi_\pm$ has U(1) charge ±1, and the rest of the fields do not have U(1) charge.

Tree-level potential

$$V_{\rm tree} = \kappa^2 \left[\frac{\tau^2 + \sigma^2}{2} \left(|\phi_-|^2 + |\phi_+|^2 \right) + |\phi_+ \phi_-|^2 \right] + \frac{g^2}{2} \left[|\phi_+|^2 - |\phi_-|^2 - \xi \right]^2 \, .$$

Fayet-Iliopoulos (FI) term

When σ has a large field value, $\phi_+ = \phi_- = 0$ becomes a local minimum.

Masses of $\phi_\pm$

$$m_\pm^2 = \frac{\kappa^2 \sigma^2}{2} \mp g^2 \xi \, .$$

Critical value

$$\sigma_c \equiv \frac{g}{\kappa} \sqrt{2\xi} \, .$$

For $\sigma > \sigma_c$

$$V_{\rm tree} = \frac{g^2 \xi^2}{2}$$

D-term inflation

E. D. Stewart (1995); P. Binetruy and G. R. Dvali (1996); E. Halyo (1996).

One-loop potential

$$V \simeq \frac{g^2\xi^2}{2}\left[1 + \underbrace{\frac{g^2}{8\pi^2}\ln\left(\frac{\kappa^2\sigma^2}{2Q^2}\right)}\right]$$

Coleman-Weinberg potential

Slow-roll parameters

$$\epsilon = \frac{M_P^2}{2V^2}\left(\frac{\partial V}{\partial\sigma}\right)^2 = \frac{g^4}{32\pi^4}\left(\frac{M_P}{\sigma}\right)^2 , \qquad \eta = \frac{M_P^2}{V}\frac{\partial^2 V}{\partial\sigma^2} = -\frac{g^2}{4\pi^2}\left(\frac{M_P}{\sigma}\right)^2 .$$

ε << η

CMB

$$\sigma_{\rm CMB} \simeq \frac{gM_P}{\pi}\sqrt{\frac{N_{\rm CMB}}{2}}$$

$$\epsilon_{\rm CMB} = \frac{g^2}{16\pi^2}\frac{1}{N_{\rm CMB}} , \qquad \eta_{\rm CMB} = -\frac{1}{2N_{\rm CMB}} .$$

$$n_s - 1 = 2\eta_{\rm CMB} - 6\epsilon_{\rm CMB} \simeq 2\eta_{\rm CMB} = -\frac{1}{N_{\rm CMB}}$$

Low-scale D-term inflation

$N_{\rm CMB}$ is given as a function of Hubble parameter and the energy scale of reheating.

e-folding number

$$e^{N_e(k)} \equiv \frac{a_{\rm end}}{a_k}$$

$a_{\rm end}$: Scale factor at the end of inflation

$a_k \equiv k/H_I$

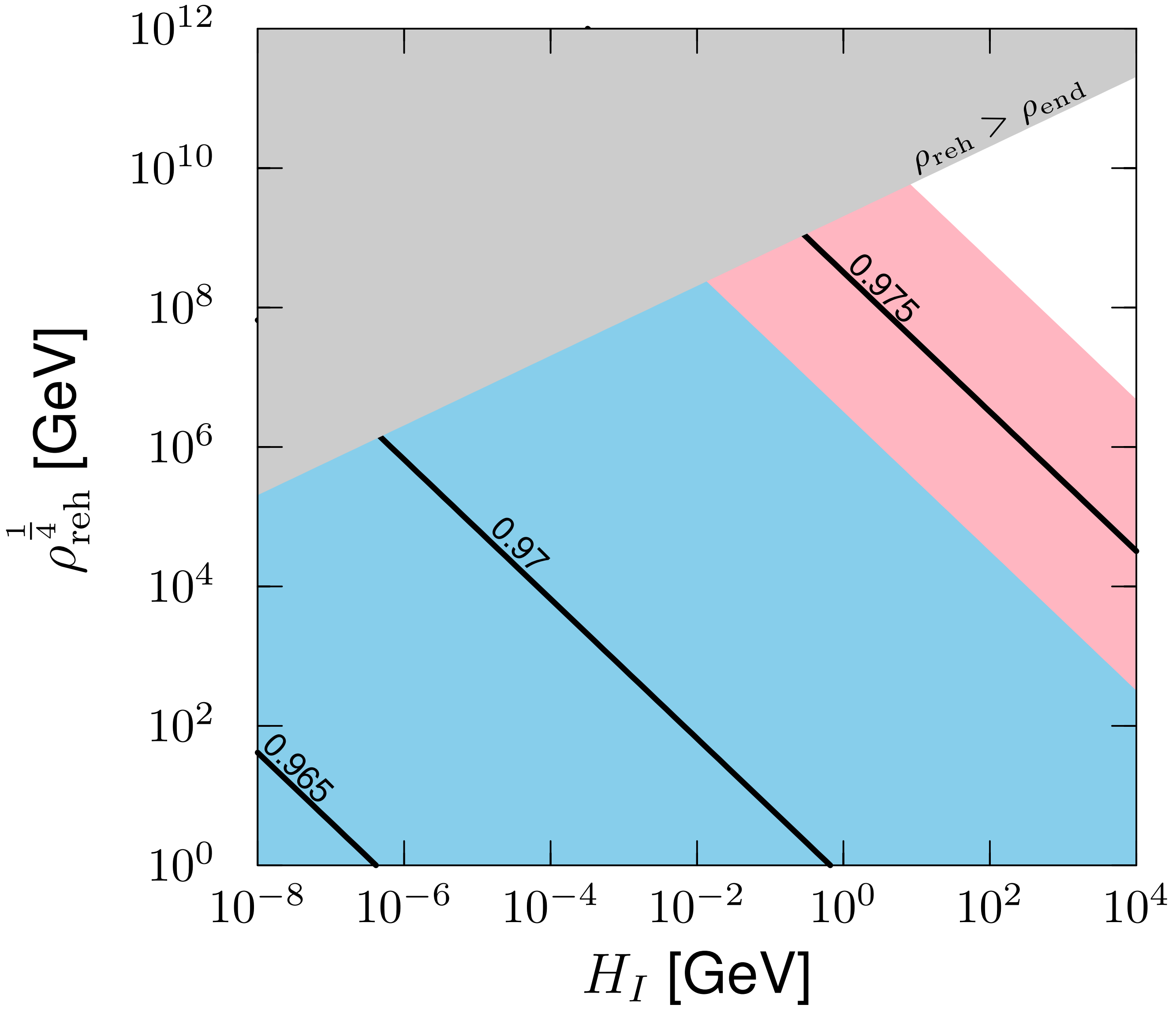
$N_{\rm CMB}$

$$N_{\rm CMB} \equiv N_e(\underbrace{k = 0.05\ {\rm Mpc}^{-1}}_{\text{Default pivot scale of Planck}}) \simeq 35.8 + \frac{1}{3}\ln\left(\frac{H_I}{1\ {\rm GeV}}\right) + \frac{1}{3}\ln\left(\frac{\rho_{\rm reh}^{\frac{1}{4}}}{1\ {\rm TeV}}\right)$$

$$\boxed{n_s - 1 = -\frac{1}{N_{\rm CMB}}}$$

Look into the parameter region consistent with the observation.

Low-scale D-term inflation



Planck TT + lowPP (1σ)

+ BICEP2 & Keck Array (1σ)

Low inflation scale is consistent with the observation.

Low-scale D-term inflation

FI-term ξ and the gauge coupling g are determined via the following quantities.

Power spectrum

$$A_s \simeq \frac{V}{24\pi^2 M_P^4 \epsilon_{\rm CMB}} \simeq \frac{\xi^2}{3(1-n_s)M_P^4}$$

$$\sqrt{\xi} \simeq 9 \times 10^{15} \times \left(\frac{1-n_s}{0.03}\right)^{\frac{1}{4}} \left(\frac{A_s}{2.1 \times 10^{-9}}\right)^{\frac{1}{4}} \ {\rm GeV}\ .$$

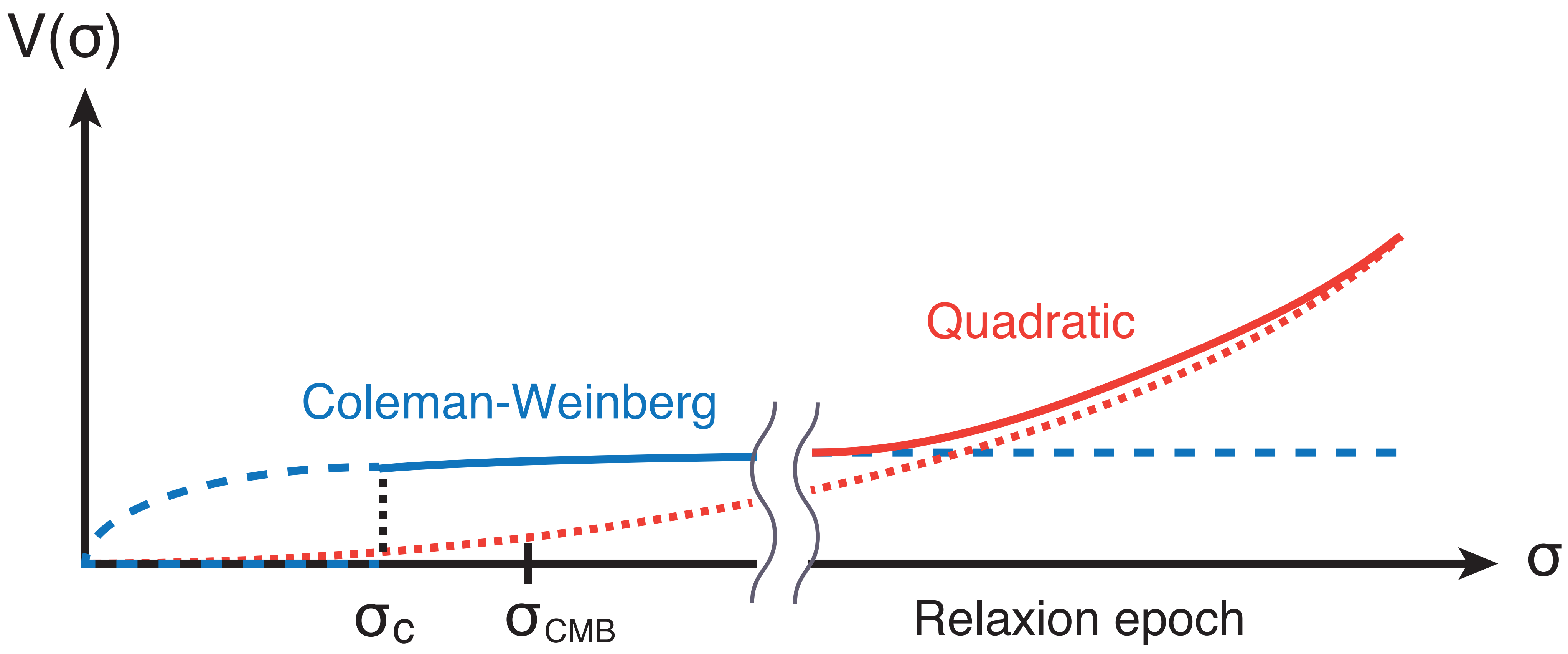
Energy scale of inflation

$$3M_P^2 H_I^2 \simeq \frac{g^2\xi^2}{2}$$

$$g \simeq \sqrt{6}\frac{M_P H_I}{\xi} \simeq 7.3 \times 10^{-14} \times \left(\frac{H_I}{1\ {\rm GeV}}\right)\left(\frac{1-n_s}{0.03}\right)^{-\frac{1}{2}} \left(\frac{A_s}{2.1 \times 10^{-9}}\right)^{-\frac{1}{2}}\ .$$

D-term inflation with amplitudon

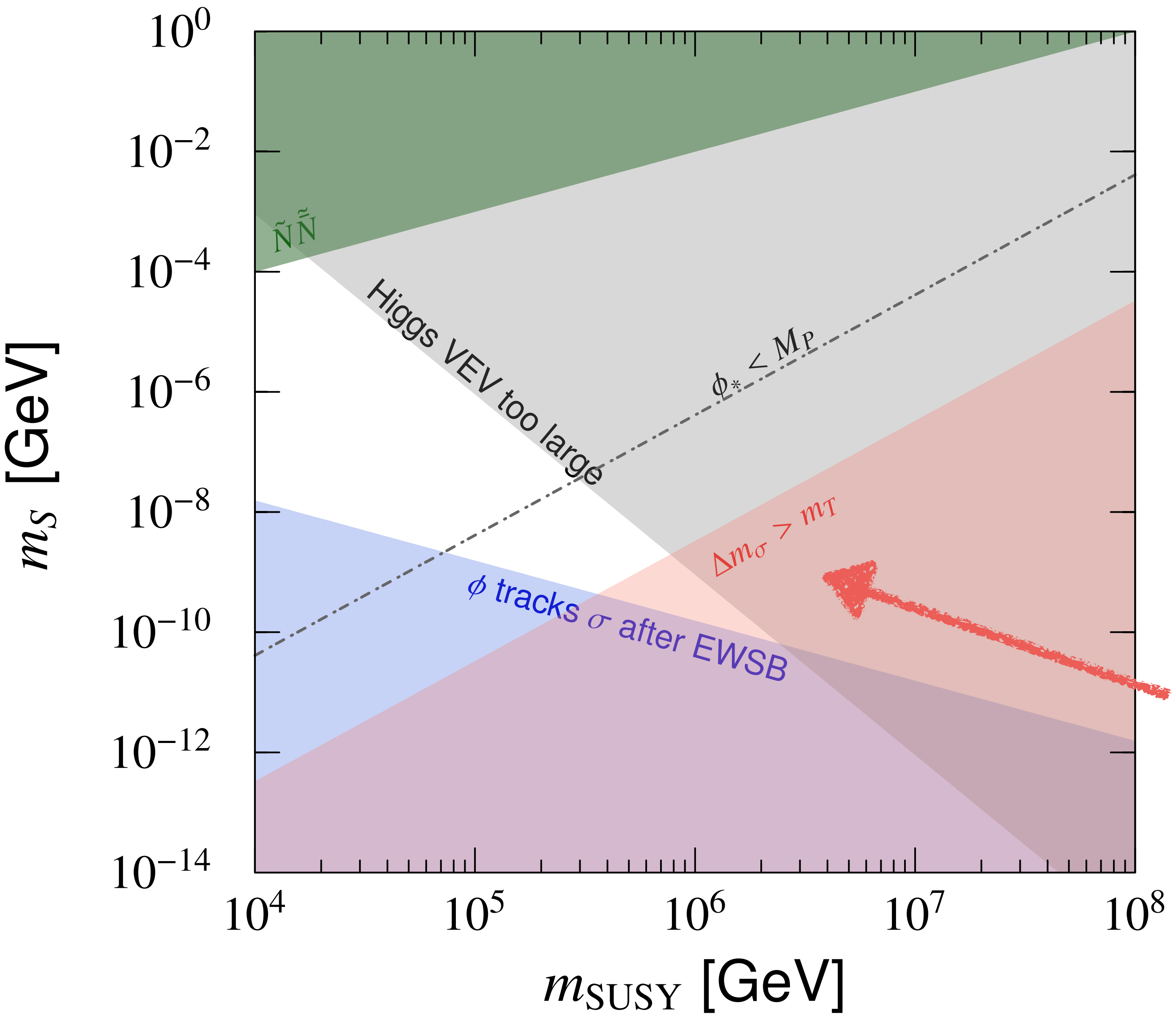
Use amplitudon σ as inflaton.



- For $N_e > 10^{14}$, σ acts as amplitudon.
- At later time, σ plays a role of inflaton ($N_e \sim 35$: CMB).

During inflation, $\phi_+ = \phi_- = 0$, and thus the extra fields do not affect the relaxion mechanism.

Results



$$m_{\rm SUSY} = \Lambda_N = M_L = f_\phi = f_\sigma$$

$$m_T = 0.1 m_S$$

$$\frac{g_S f_\phi}{m_S} = \frac{g_T f_\sigma}{m_T} = 10^{-8}$$

$$|\kappa| = 10^{-2}$$

Shift-symmetry breaking effects from loop corrections.

$$\Delta m_\sigma \simeq \frac{|\kappa|}{4\pi} \frac{m_{\rm SUSY} f_\phi}{M_P}$$

We obtain a new limit, but a wide range of parameter region is still allowed.

J. L. Evans, T. Gherghetta, N. Nagata, M. Peloso, in preparation.

Conclusion

Conclusion

- Relaxion mechanism

  New idea but still immature. There are a lot of things to study.

- Two-field SUSY relaxion model

  Solves the little hierarchy problem in the PeV-scale SUSY.

- D-term inflation

  Low-scale D-term inflation is consistent with the CMB observation.

  Amplitudon σ can be inflaton.

Digression

Near criticality

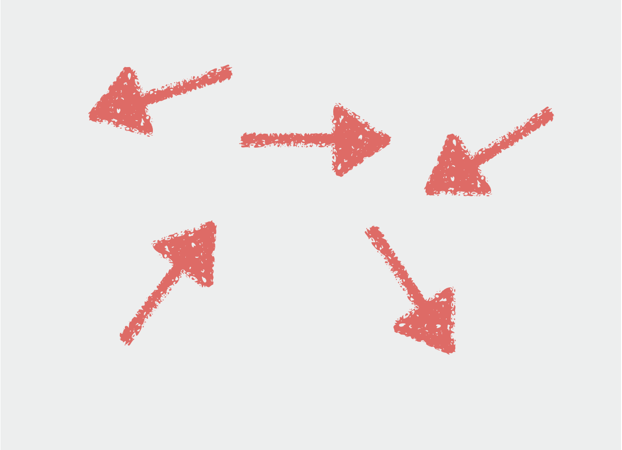
Another viewpoint on hierarchy problem:

$m^2 = 0$ is critical point for the Higgs mechanism.

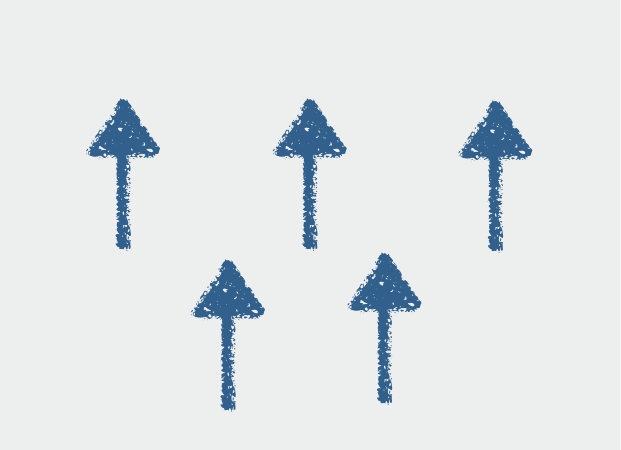
Why our vacuum sits near the critical point?

Ex.) Ferromagnet

To set the system near the critical point of phase transition, we need to tune the temperature T around the transition temperature T_C .



$T > T_C$



$T < T_C$

Does Nature execute such fine-tuning?

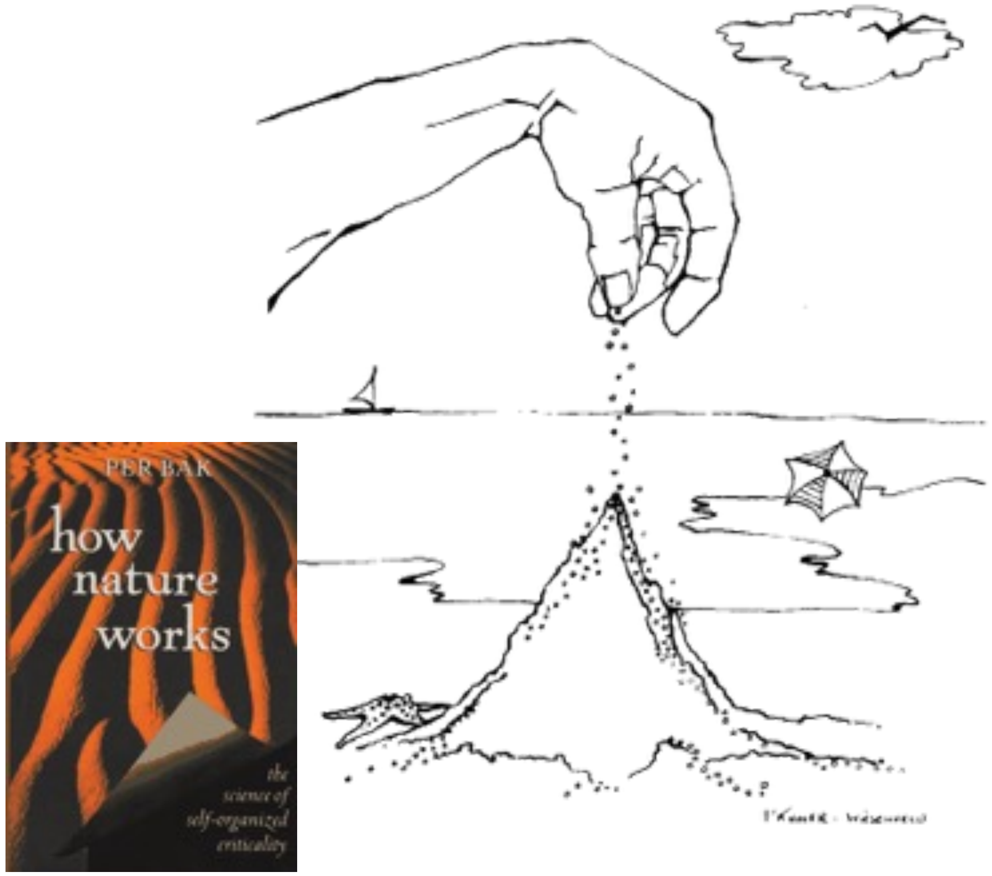
Self-organized criticality

P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987).

Self-organized criticality

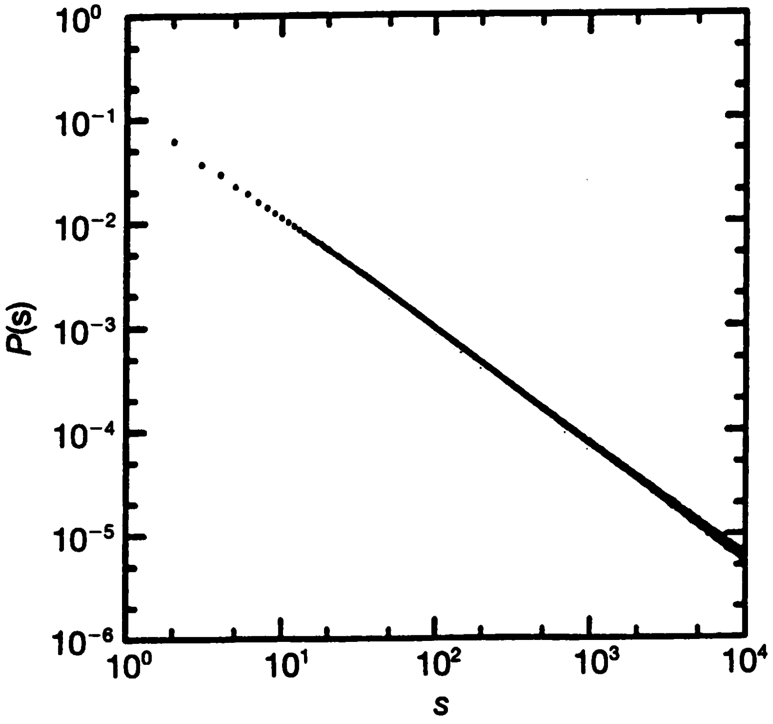
A property of dynamical systems that evolves toward the critical point without tuning of external parameters to a particular value.

Ex.) Sandpile



P. Bak, “How nature works”.

Pile up → Criticality → Avalanche



Distribution of size of avalanches follow a power low.

The system automatically evolves to criticality.

P. Bak and M. Paczuski, Proc. Natl. Acad. Sci. USA, **92**, 6689 (1995).

Self-organized criticality

A similar dynamical mechanism sets the Higgs mass parameter such that our vacuum is driven to near criticality?

Features of self-organized criticality		Relaxion mechanism
i) Dissipation	→	Hubble friction • To stop relaxion easily • Insensitive to initial conditions
ii) Many metastable states	→	Periodic potential • To evade fine-tuning
iii) Slow and long-time process	→	Large *e*-folding number • For relaxion to scan whole range of Higgs mass parameters.

We may find a lot of models (other than relaxion) that realize the self-organized criticality.

Backup

Problems in the original model

- Strong CP Problem

Original model uses QCD axion as relaxion.

➡ θ_{QCD} becomes too large.

- A simple extension

Introduce new vector-like fermions which have new strong interactions.

➡ This new strong interaction generates a periodic potential which stops relaxion rolling.

In order for the Higgs VEV to give a sizable effect on the relaxion potential to stop relaxion, this new strong dynamics should be around the TeV scale.

➡ Coincidence problem

Origin of relaxion and shift symmetry

Relaxion has non-compact field values, which is different from usual Nambu-Goldstone bosons (such as axion) with compact field space.

➡ It seems difficult to realize relaxion in field theories.

R. S. Gupta, Z. Komargodski, G. Perez, L. Ubaldi, JHEP **1602**, 166 (2016).

Nice ideas have already been proposed for inflation models.

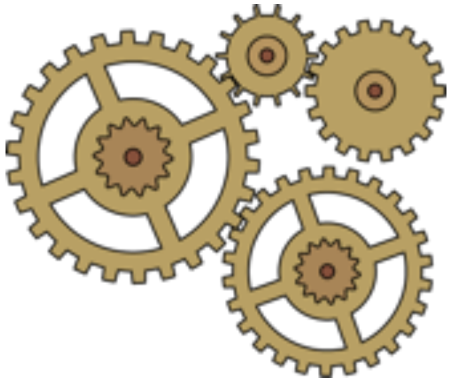
- String origin (monodromy)

E. Silverstein and A. Westphal, Phys. Rev. D**78**, 106003 (2008); L. McAllister, E. Silverstein, and A. Westphal, Phys. Rev. D**82**, 046003 (2010); N. Kaloper, L. Sorbo, Phys. Rev. Lett. **102**, 121301 (2009).

However, see L. McAllister, P. Schwaller, G. Servant, J. Stout, A. Westphal [1610.05320].

- Many axion fields (clockwork)

J. E. Kim, H. P. Nilles, and M. Peloso (2005); K. Harigabya and M. Ibe (2014);
K. Choi, H. Kim and S. Yun (2014); T. Higaki and F. Takahashi (2014);
K. Choi and S. H. Im (2015); D. E. Kaplan and R. Rattazzi (2015);
G. F. Giudice, M. McCullough (2016).



Clockwork mechanism

J. E. Kim, H. P. Nilles, and M. Peloso (2005); K. Harigabya and M. Ibe (2014);
K. Choi, H. Kim and S. Yun (2014); T. Higaki and F. Takahashi (2014);
K. Choi and S. H. Im (2015); D. E. Kaplan and R. Rattazzi (2015);
G. F. Giudice, M. McCullough (2016).

Suppose that N+1 global U(1) symmetries are spontaneously broken at a scale f:

- N+1 Nambu-Goldstone bosons: π_j (j = 0, 1, …, N)

$$U_j(x) = e^{i\pi_j(x)/f} \qquad j = 0, .., N$$

$Q_i\,[U_j] = \delta_{ij}$, Q_j : $U(1)_j$ charge

Then, the $U(1)^{N+1}$ symmetry is explicitly (softly) broken by mass parameters m_j:

(j = 0, 1, …, N-1)

Regard them as spurion fields:

$$Q_i[m_j^2] = \delta_{ij} - q\,\delta_{i\,j+1}$$

$m_j^2 << f^2$

- Only one U(1) symmetry is left unbroken.

$$\mathcal{Q} = \sum_{j=0}^{N} \frac{Q_j}{q^j}$$

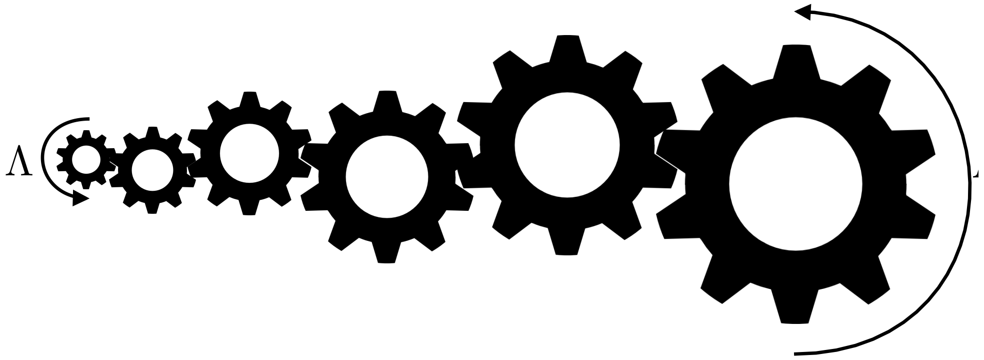
$Q_i\,[m_j^2] = 0$

Lagrangian

$$\mathcal{L} = -\frac{f^2}{2}\sum_{j=0}^{N}\partial_\mu U_j^\dagger\,\partial^\mu U_j + \frac{m^2 f^2}{2}\sum_{j=0}^{N-1}\left(U_j^\dagger\,U_{j+1}^q + \mathrm{h.c.}\right) .$$

$m_j^2 = m^2$

Clockwork mechanism



Low-energy effective Lagrangian

$$\mathcal{L} = -\frac{1}{2}\sum_{j=0}^{N} \partial_\mu \pi_j \partial^\mu \pi_j - V(\pi) \qquad V(\pi) = \frac{m^2}{2}\sum_{j=0}^{N-1} (\pi_j - q\,\pi_{j+1})^2 + \mathcal{O}(\pi^4) = \frac{1}{2}\sum_{i,j=0}^{N} \pi_i\, M^2_{\pi\,ij}\, \pi_j + \mathcal{O}(\pi^4)\,.$$

with

$$M_\pi^2 = m^2 \begin{pmatrix} 1 & -q & 0 & \cdots & & 0 \\ -q & 1+q^2 & -q & \cdots & & 0 \\ 0 & -q & 1+q^2 & \cdots & & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ & & & & 1+q^2 & -q \\ 0 & 0 & 0 & \cdots & -q & q^2 \end{pmatrix}.$$

One massless NG boson

$$a_0 = \sum_j \frac{\mathcal{N}_0}{q^j}\pi_j$$

Now couple the gauge fields only to the Nth pion.

$$\mathcal{L} = \frac{\pi_N}{16\pi^2 f} G_{\mu\nu}\widetilde{G}^{\mu\nu}\,. \quad \Rightarrow \quad \mathcal{L} = \frac{a_0}{16\pi^2 f_0} G_{\mu\nu}\widetilde{G}^{\mu\nu} + \ldots$$

⇒ $$\boxed{f_0 = \frac{f q^N}{\mathcal{N}_0}}$$

Effectively huge decay constant.

Inflation model

Relaxion mechanism needs to proceed during inflation, and the inflation is assumed to occur independently.

Requirement for the inflation sector

- Relaxion excursion should be classical and slow-roll.
- Inflaton should dominate the energy density.
- The inflation should persist long enough so that the relaxion. can scan the whole region for the Higgs mass parameter.

➡ Extremely low inflation scale and huge *e*-folding number

$H_{inf} < O(1)$ MeV, $N_e > 10^{42}$, ….

Can we find a plausible inflation model…?

Lagrangian

Kahler potential

$$K = \kappa(S+S^*, T+T^*) + Z_i(S+S^*, T+T^*)\Phi_i^* e^{2V}\Phi_i$$
$$+ \left[U(S+S^*, T+T^*)e^{-\frac{q_H S}{f_\phi}} H_u H_d + \text{h.c.}\right] \ ,$$

where $\Phi_i = Q_i, H_u, H_d, N, \bar{N}$

Super potential

$$W_{\text{gauge}} = \left(\frac{1}{2g_a^2} - i\frac{\Theta_a}{16\pi^2} - \frac{c_a S}{16\pi^2 f_\phi}\right)\text{Tr}\mathcal{W}_a\mathcal{W}_a \ ,$$

$$W_{\text{Yukawa}} = y_u Q\overline{U}H_u + y_d Q\overline{D}H_d + y_e L\overline{E}H_d \ ,$$

$$W_\mu = \mu_0 e^{-\frac{q_H S}{f_\phi}} H_u H_d \ ,$$

$$W_{S,T} = \frac{1}{2}m_S S^2 + \frac{1}{2}m_T T^2 \ ,$$

$$W_N = m_N N\bar{N} + ig_S SN\bar{N} + ig_T TN\bar{N} + \frac{\lambda}{M_L}H_u H_d N\bar{N} \ .$$

Absence of the σ-Higgs coupling

In the two-field relaxion mechanism, σ should not have a direct coupling to the Higgs fields. (Otherwise, the late time excursion of σ changes the Higgs mass.)

➡ In our model, there is no such a coupling at renormalizable level.

(The Kahler potential depends on T + T*.)

The σ-Higgs couplings are generated by SUSY-breaking effects.

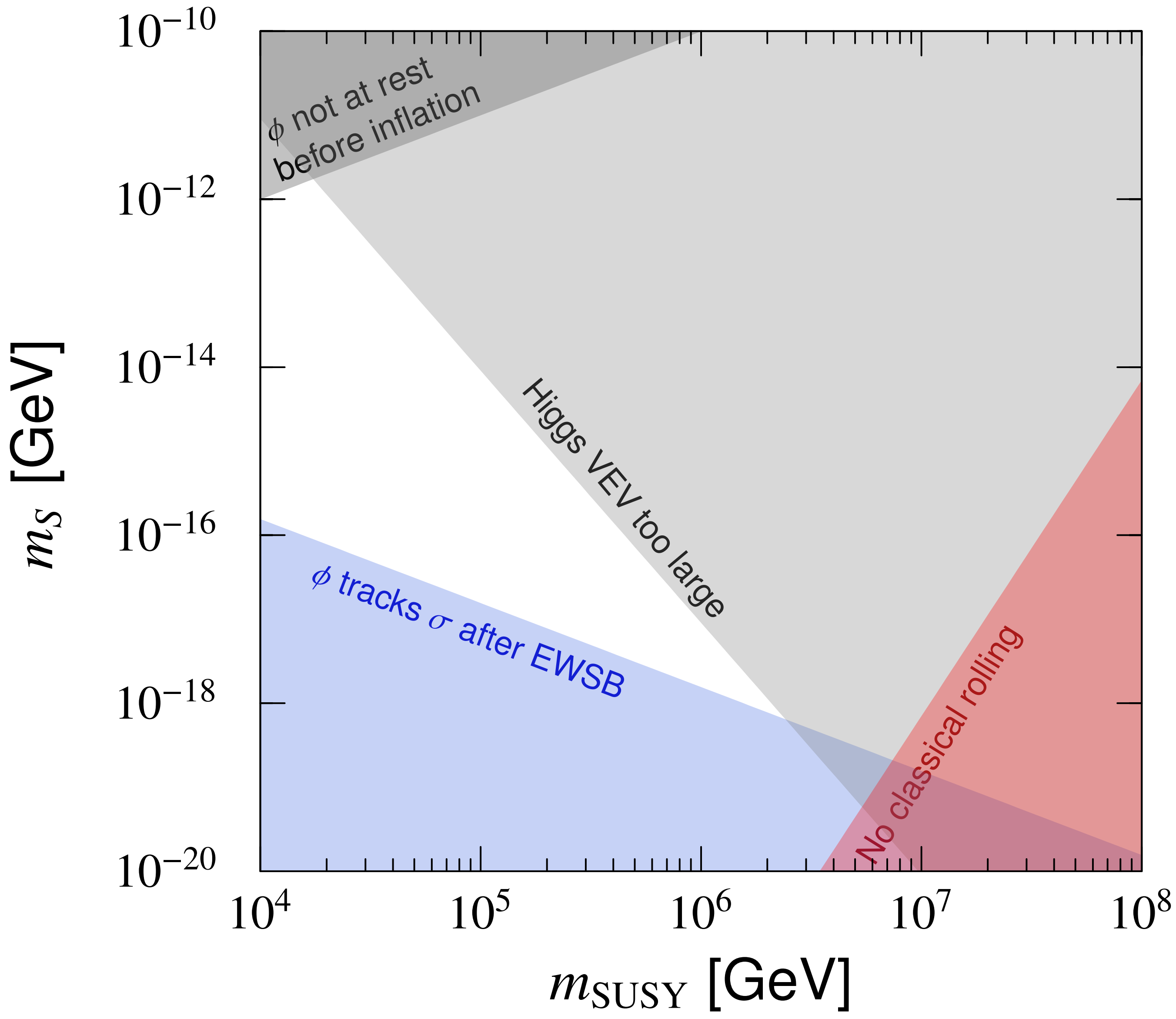
- $m_T << m_S$

  $F_T << F_S$. In this case, F_S is the dominant source of the SUSY-breaking.

- $M_* >> f$

  Again, F_S is the dominant source of the SUSY-breaking.

Results



$$m_{\rm SUSY} = \Lambda_N = M_L = 10^{-4} f_{\phi,\sigma}$$

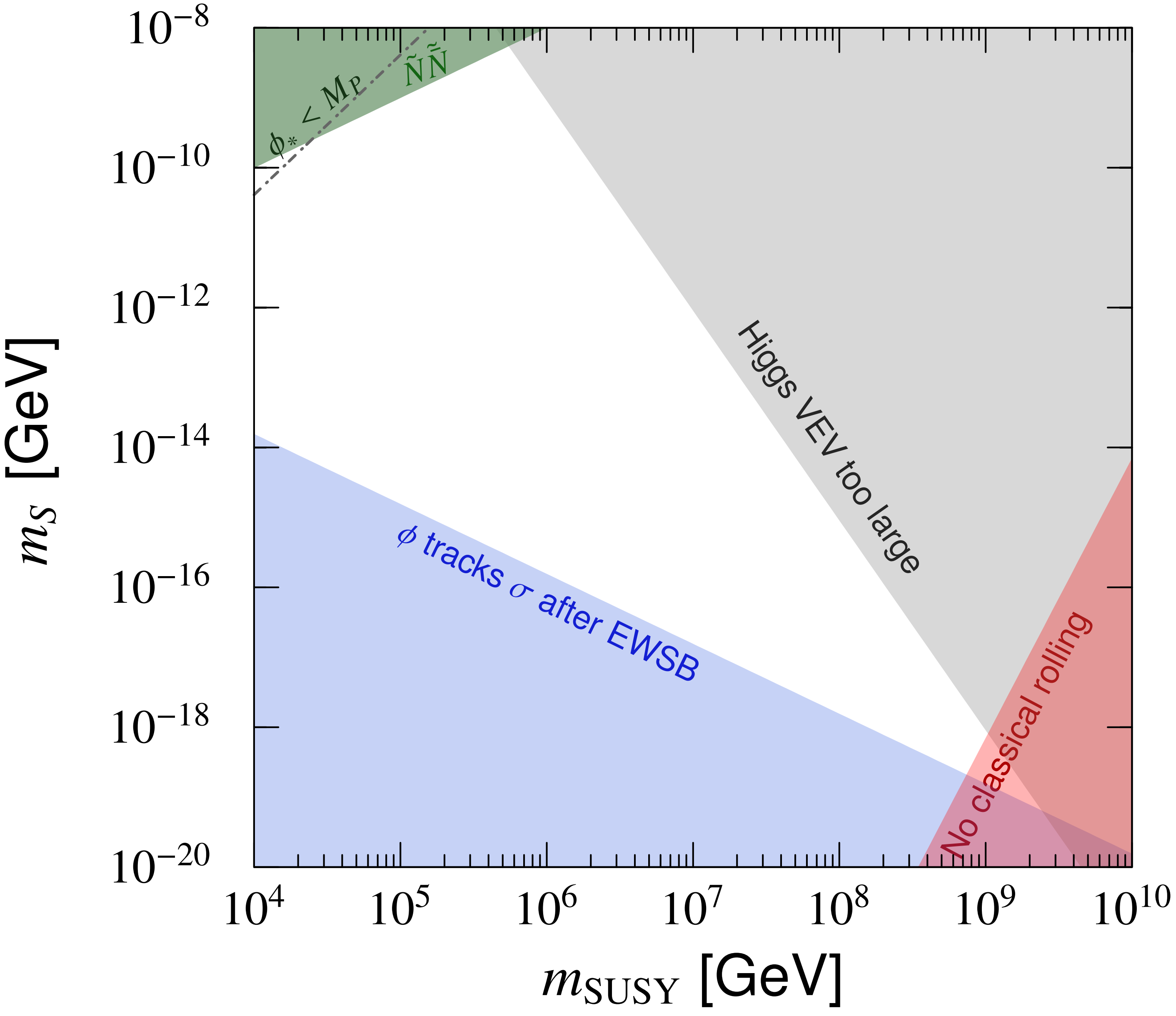
$$m_T = 0.1 m_S$$

$$\frac{g_S f_\phi}{m_S} = \frac{g_T f_\sigma}{m_T} = 10^{-8}$$

f_ϕ can be as large as 10^{10} GeV.

J. L. Evans, T. Gherghetta, N. Nagata, Z. Thomas, JHEP **1609**, 150 (2016).

Results



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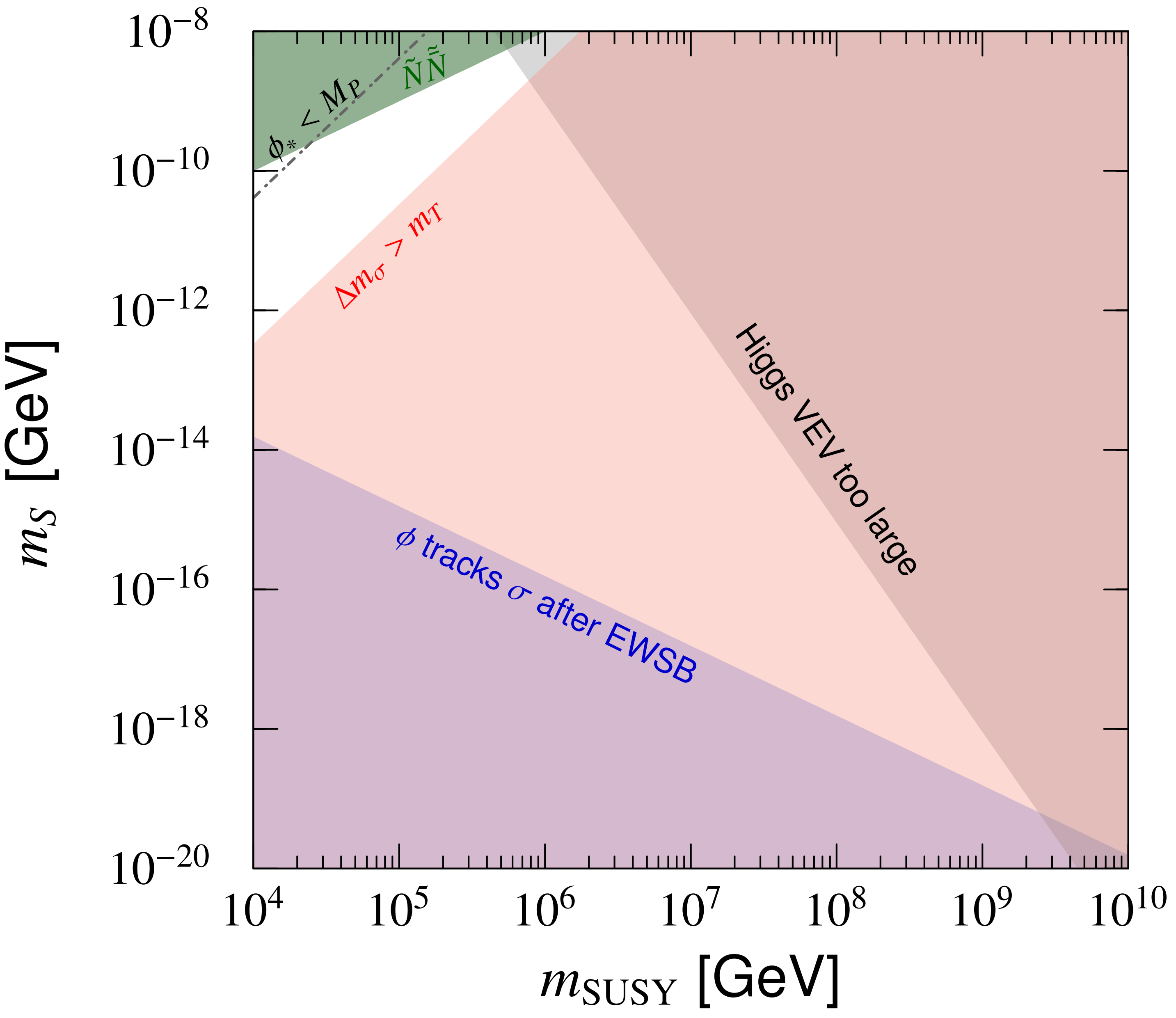

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SUSY spectrum

Scalar masses

(e.g.) $$\int d^4\theta \frac{1}{M_*^2}(S+S^*)^2 Q_i Q_i^* \quad \Rightarrow \quad \tilde{m} \sim \frac{F_S}{M_*} \sim \frac{m_S \phi}{M_*}$$

Gaugino masses

$$\int d^2\theta \frac{c_a S}{16\pi^2 f_\phi} \mathrm{Tr}(\mathcal{W}_a \mathcal{W}_a) \quad \Rightarrow \quad M_a \sim \frac{c_a F_S}{16\pi^2 f_\phi} \sim \frac{c_a m_S \phi}{16\pi^2 f_\phi}$$

A-terms

$$\int d^2\theta \frac{S+S^*}{M_*} Q_i Q_j Q_k \quad \Rightarrow \quad A \sim \frac{F_S}{M_*} \sim \frac{m_S \phi}{M_*}$$

- $M_* \sim f_\phi$

→ Gaugino masses are suppressed by a loop factor compared with scalar masses.

Mini split [But with large A-terms]

- $M_* \gg f_\phi$

→ Scalar masses are induced from gaugino masses through RGEs (gaugino mediation/no-scale scenario).

Mass spectrum (Relaxion sector)

- $\phi \quad \cdots$ Periodic potential gives a mass to ϕ.

$$m_\phi \simeq \sqrt{\frac{\Lambda_N^3 \mathcal{A}(\phi_*)}{f_\phi}} \simeq 10 \text{ GeV} \times \left(\frac{g_S f_\phi / m_S}{10^{-8}}\right)^{\frac{1}{2}} \left(\frac{\Lambda_N}{f_\phi}\right)^{\frac{3}{2}} \left(\frac{m_{\text{SUSY}}}{f_\phi}\right)^{\frac{1}{2}} \left(\frac{f_\phi}{10^5 \text{ GeV}}\right)$$

Since the amplitude of the periodic potential becomes large in the end, relaxion is heavy.

Remark: If relaxion is light, it is severely constrained by axion bounds.

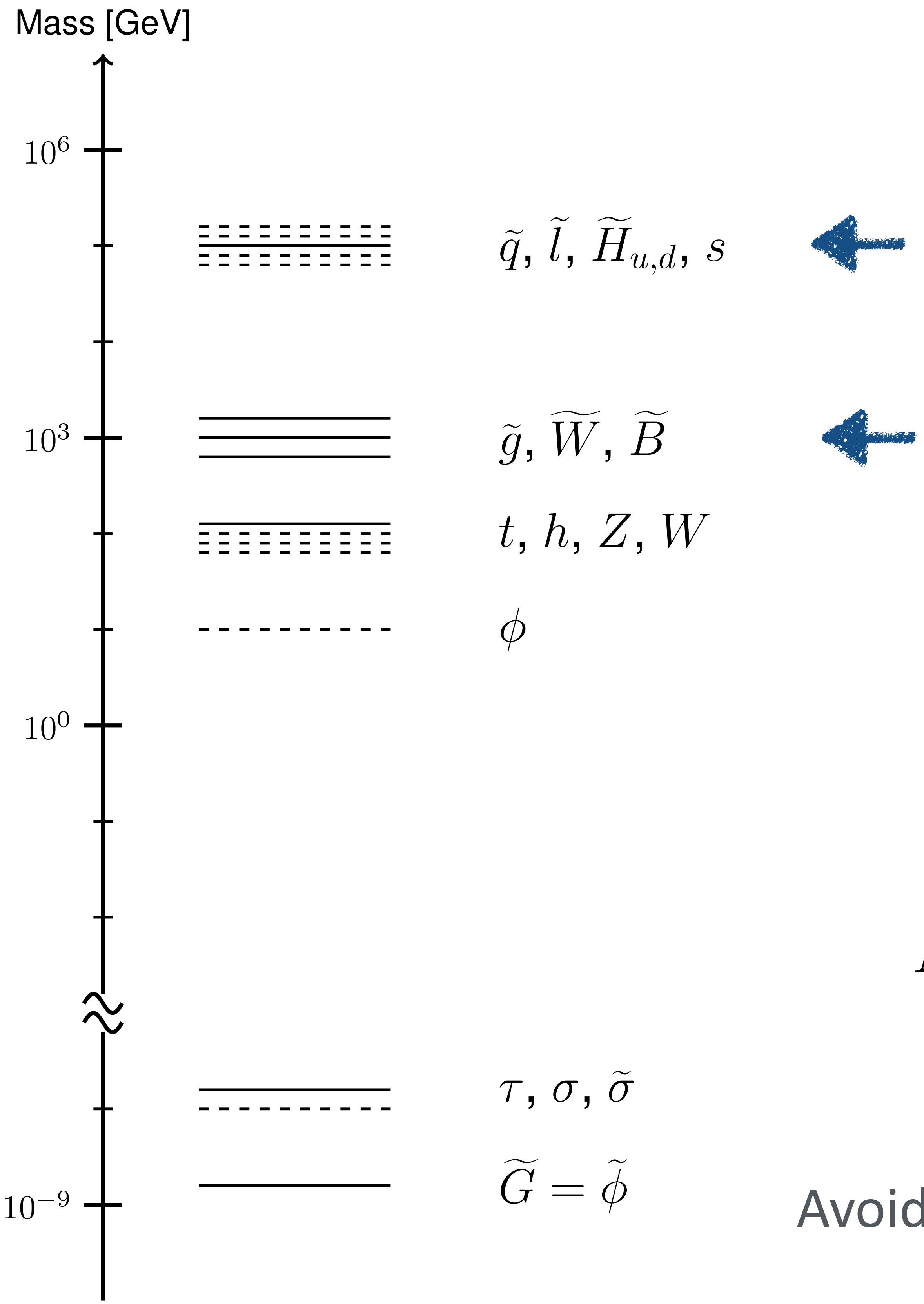
T. Kobayashi, O. Seto, T. Shimomura, and Y. Urakawa [1605.06908].

- $\widetilde{\phi} \quad \cdots$ Relaxino is eaten by gravitino (goldstino).

$$m_{3/2} = \frac{F}{\sqrt{3} M_P} \simeq 2 \text{ eV} \times \left(\frac{m_{\text{SUSY}}}{10^5 \text{ GeV}}\right) \left(\frac{f_\phi}{10^5 \text{ GeV}}\right) .$$

- $s \quad \cdots$ SUSY scale.
- $\tau, \ \widetilde{\sigma} \quad \cdots$ Can be as light as gravitino (depending on Kahler potential).
- $\sigma \quad \cdots$ m_T

Particle spectrum



Target of EDM and flavor physics.

Target of collider experiments.

$$m_{\rm SUSY} = 10^5 \ {\rm GeV}$$

$$M_* \sim f_{\phi,\sigma} = 10^5 \ {\rm GeV}$$

Avoids gravitino problem.

Mass spectrum after D-term inflation

After inflation, the fields settle in the SUSY-preserving minimum.

➡ Superfield treatment is still valid.

$$\langle \Phi_+ \rangle = \sqrt{\xi}$$

T, ϕ-

Form a massive vector-like chiral multiplet with a mass of $\kappa\sqrt{\xi}$

ϕ+

Eaten by the U(1) gauge vector superfield:

$$m_{Z'} = g\sqrt{2\xi} = 9.3 \times 10^2 \times \left(\frac{H_I}{1\ \mathrm{GeV}}\right)\left(\frac{1-n_s}{0.03}\right)^{-\frac{1}{4}}\left(\frac{A_s}{2.1\times 10^{-9}}\right)^{-\frac{1}{4}}\ \mathrm{GeV}$$

Cosmic string problem

Tree-level potential

$$V_{\rm tree} = \kappa^2 \left[\frac{\tau^2 + \sigma^2}{2} \left(|\phi_-|^2 + |\phi_+|^2 \right) + |\phi_+ \phi_-|^2 \right] + \frac{g^2}{2} \left[|\phi_+|^2 - |\phi_-|^2 - \xi \right]^2 .$$

$\tau = \sigma = \phi_- = 0$, $\phi_+ = \sqrt{\xi}$ is the absolute minimum. ➡ U(1) breaking

Since the U(1) symmetry is broken after inflation, cosmic strings can be problematic.

String mass per unit length

$$\mu = 2\pi \langle \phi_+ \rangle^2 = 2\pi\xi$$

➡ $$G\mu \simeq 3.4 \times 10^{-6} \times \left(\frac{1 - n_s}{0.03} \right)^{\frac{1}{2}} \left(\frac{A_s}{2.1 \times 10^{-9}} \right)^{\frac{1}{2}} .$$

Planck bound

$$G\mu < 3.3 \times 10^{-7}$$

[Planck 2015]

Minimal D-term inflation model is disfavored.

Cosmic string problem

Various solutions to the cosmic string problem have been proposed.

- Curvaton

  M. Endo, M. Kawasaki, and T. Moroi (2003).
- Non-minimal Kahler potential

  O. Seto and J. Yokoyama (2006); J. Rocher and M. Sakellariadou (2006).
- Sub-critical D-term inflation

  W. Buchmuller and K. Ishiwata (2014).
- Dynamical D-term generation

  V. Domcke, K. Schmitz and T. T. Yanagida (2014).
- Semi-local string

  J. Urrestilla, A. Achucarro, and A. C. Davis (2004).

It is not easy to find a solution which is compatible with low-scale inflation and relaxion mechanism.

Semi-local strings and textures are also constrained by CMB.

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← We use this to break U(1) during inflation.

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