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Pseudo NG bosons from finite modular symmetry

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based on **arXiv: 2402.02071 [JHEP]** 2405.03996 [JHEP], 2409.19261 [JHEP]

in collaboration with

T.Higaki (Keio U.), T.H.Jung (IBS), T.Kobayashi (Hokkaido U.)

Pseudo Nambu-Goldstone [NG] boson

➤ “pseudo” NG boson

Goldstone theorem :

massless NG boson appears when **continuous symmetry is broken**

but if a symmetry is not exact...

- **pseudo** NG boson is not exactly massless, but **light**
- mass is induced by symmetry breaking effects

➤ Examples

	pion π	axion a	majoron J
symmetry	chiral $U(1)$	global $U(1)_{PQ}$	global $U(1)_{B-L}$
breaking	quark mass	QCD + ?	gravity ?

in SM

maybe dark mater [DM], this talk

Outline

1. Introduction
- 2. Quick review of finite modular symmetry Γ_N**
3. Finite modular axion and radiative stabilization
4. Summary

Modular transformation

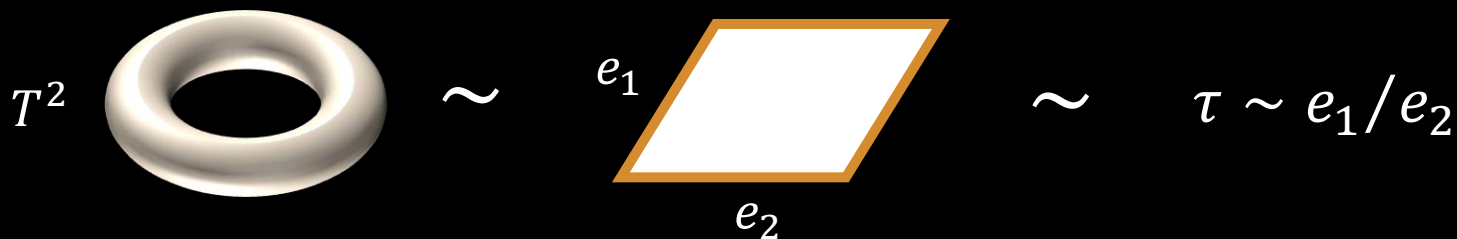
➤ **modulus τ** : complex scalar with $\text{Im } \tau > 0$ $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma$

$$\tau \xrightarrow{\gamma} \frac{a\tau + b}{c\tau + d} \qquad \tau \xrightarrow{S} -1/\tau \qquad \tau \xrightarrow{T} \tau + 1$$

complex scalar field τ is transformed under $\Gamma := \text{SL}(2, \mathbb{Z})$, so its VEV breaks Γ

T.Kobayashi, S.Nagamoto et.al. '17 '18 '20
J.Lauer, J.Mas, H.P.Nilles '89, '91, e.t.c.

➤ UV origin of modulus string theory predicts 6D extra dimension



- theory on torus has modular symmetry Γ
- it is broken by matters to **finite** modular sym. $\Gamma_N = \Gamma/\Gamma(N)$

Modular form of Γ_N

is a **holomorphic function of τ** transforms as

$$Y_r^{(k)} = Y_r^{(k)}(\tau) \rightarrow (c\tau + d)^k \rho(r) Y_r^{(k)}(\tau) \quad \text{modulus } \tau \xrightarrow{\Gamma} \frac{a\tau + b}{c\tau + d}$$

$\rho(r)$: **representation matrix of r** under e.g. $A_4 \simeq \Gamma_3$

k : modular weight, positive integer valued

➤ What if Yukawa couplings are modular forms ?

Feruglio, "Are neutrino mass modular form ?" 17'

- Yukawa's are **holomorphic functions**, easily implemented in SUSY
- may couple to multiple flavors, so **more predictive**
- **only one modulus**, less than non-modular flavor symmetries w/ flavons

Residual $\mathbb{Z}_N^T$ symmetry Novichkov, Penedo, Petkov, 21'

$$\Gamma_N \text{ algebra: } (ST)^3 = S^2 = 1, \quad \mathbf{T}^N = \mathbf{1}$$

➤ At $\tau \sim i\infty$

τ is insensitive to $\tau \xrightarrow{T} \tau + 1 \rightarrow \mathbb{Z}_N^T \text{ symmetry is unbroken}$

➤ Modular forms at $\text{Im}\tau \gg 1$

$$Y_3^{(2)}(\tau) \sim \begin{pmatrix} 1 \\ -6q^{1/3} \\ -18q^{2/3} \end{pmatrix} \begin{matrix} \mathbb{Z}_3^T\text{-charge} \\ 0 \\ 1 \\ 2 \end{matrix}$$

$$q = \exp(2\pi i\tau)$$

powers of $q^{1/3} = e^{2\pi i\tau/3} \ll 1$ is controlled by $\mathbb{Z}_3^T$ charge

➔ Froggatt-Nielsen mechanism $\left(\frac{\langle\phi\rangle}{\Lambda}\right)^n \Leftrightarrow q^{n/3}$

Γ_N for quark and lepton flavor hierarchies

Summary of Γ_N

➤ Finite modular symmetry $\Gamma_N \sim$



- is the quotient group $\bar{\Gamma}/\bar{\Gamma}(N)$
- modular form transforms as

$$Y = Y(\tau) \rightarrow (c\tau + d)^k \rho(r) Y(\tau)$$

- is found in string models

T.Kobayashi, S.Nagamoto et.al. '17 '18 '20

J.Lauer, J.Mas, H.P.Nilles '89, '91,

S.Ferrara, D.Lust, S.Theisen, '89

A.Baur, H.P.Nilles, A.Trautner, PKS.Vaudrevange S.Ramos-Sanches, '19, '20

➤ Applications for particle physics

- quark/lepton **Yukawa couplings** (masses) **are modular forms**
- residual symmetry can explain **flavor hierarchies**
- value of modulus τ plays crucial role in those models

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KSVZ axion model

'79 J.E.Kim, '80 M.A.Shifman, A.I.Vainshtein, V.I.Zakharov

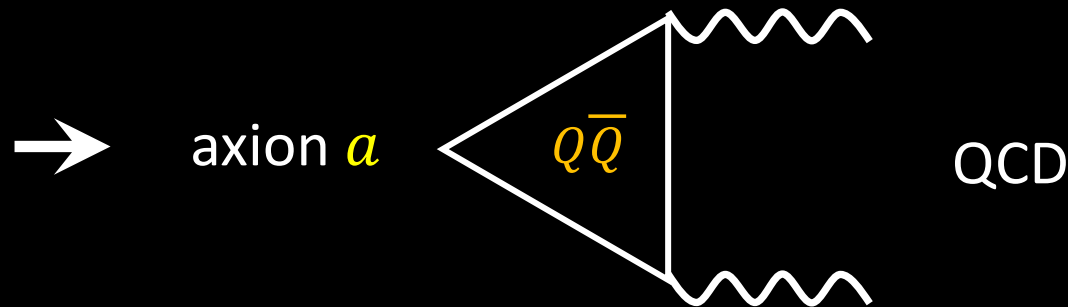
$$\mathcal{L} = y P \bar{Q} Q$$

PQ-charge: $+1 \ -1$

- complex scalar $P \propto e^{ia/f_a}$
- vector-like pair of non-SM quarks $(Q, \bar{Q})$

➤ PQ symmetry and QCD anomaly

PQ transformation: $a \rightarrow a + f_a \alpha, Q \bar{Q} \rightarrow e^{-i\alpha} Q \bar{Q}, \alpha \in \mathcal{R}$



➔ $\left(\theta + \frac{a}{f_a} \right) F_{\mu\nu} \tilde{F}^{\mu\nu}$ with decay constant $f_a \sim \langle P \rangle \gg \text{EW scale}$

Finite modular axion

2402.02071 T.Higaki, JK, T.Kobayashi

c.f. KSVZ axion

$$\mathcal{L} = \Lambda_Q Y_r^{(k)}(\tau) \bar{Q} Q \quad \longleftrightarrow \quad \mathcal{L} = y P \bar{Q} Q$$

scalar $P \ni$ axion is replaced by a **modular form** $Y_r^{(k)}(\tau)$

➤ Accidental $U(1)_{PQ}$ assume $r = 1_t$ where t is a charge of $Z_N^T \subset \Gamma_N$

$$Y_{1_t}^{(k)}(\tau) \bar{Q} Q \sim \exp\left(\frac{2\pi i t \tau}{N}\right) (1 + \mathcal{O}(q)) Q \bar{Q} \quad \text{at } \text{Im}\tau \gg 1$$

has **discrete** sym. $Z_N^T : \tau \rightarrow \tau + \mathbf{1}, Q \bar{Q} \rightarrow \exp\left(-\frac{2\pi i t}{N}\right) Q \bar{Q}$

➔ **accidental continuous** $U(1)_{PQ} : \tau \rightarrow \tau + \alpha, Q \bar{Q} \rightarrow \exp\left(-\frac{2\pi i t}{N} \alpha\right) Q \bar{Q}$
 $\alpha \in \mathcal{R}$

A model for FM axion

2402.02071 T.Higaki, JK, T.Kobayashi

For concreteness, $\Gamma_3 \simeq A_4$ which has residual Z_3^T

➤ A_4 anomaly-free model with supersymmetry (SUSY)

$$W = \Lambda_{Q_1} \mathbf{Y}_{1_1}^{(k)}(\boldsymbol{\tau}) \bar{Q}_1 Q_1 + \Lambda_{Q_2} \mathbf{Y}_{1_2}^{(k)}(\boldsymbol{\tau}) \bar{Q}_2 Q_2$$

discrete anomaly $\mathcal{A}_{A_4-SU(3)^2} = 1 + 2 \equiv \mathbf{0 \bmod 3}$

➤ QCD θ -angle

$$\bar{\theta} = \underset{\text{bare}}{\theta_0} + \underset{\text{modular forms}}{\text{Arg} \left(\mathbf{Y}_{1_1}^{(k)} \mathbf{Y}_{1_2}^{(k)} \right)} \sim \theta_0 + \boldsymbol{\phi} + \mathcal{O}(|q|)$$

effective QCD angle depends on $\boldsymbol{\phi} = \mathbf{2\pi Re\tau}$, KSVZ-like axion

PQ quality

$$Y_{1_t}^{(k)}(\tau) \bar{Q}_t Q_t \sim e^{-i\frac{t}{3}\phi} \bar{Q}_t Q_t \quad Z_3^T \text{ (=PQ)-charge} = t = 1, 2$$

non-linear realization of $U(1)_{PQ}$, accidentally by Z_3^T

➤ PQ-violation

$q = \exp(2\pi i\tau)$ is **invariant under Z_3^T** : $\tau \rightarrow \tau + 1$,

but not invariant under $U(1)_{PQ}$: $\tau \rightarrow \tau + \alpha$, $\alpha \in \mathbb{R}$

➔ PQ violation is accompanied with $|q| = e^{-2\pi \text{Im}\tau}$

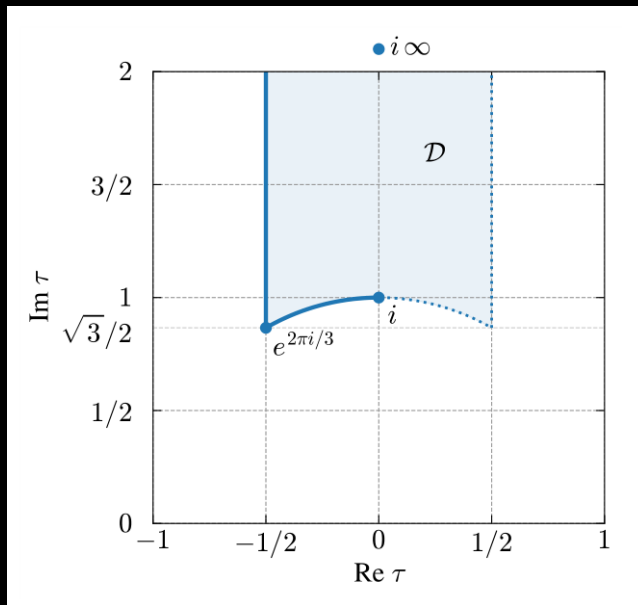
$$V = \underbrace{V_0}_{\text{PQ-invariant}} + \underbrace{V_1|q|}_{\text{PQ-violation}} + V_2|q|^2 + \dots \quad \Gamma_N\text{-invariant}$$

PQV is small if **Im τ is large** and $|q| = e^{-2\pi \text{Im}\tau} \ll 1$

Moduli stabilization

Where is the value of modulus τ ?

fundamental domain



- τ is stabilized at the **min. of potential**
- somewhere in the fundamental dom.

2006.03058, P.Novichkov, J.Penedo, S.Petcov

Radiative stabilization

2402.02071 T.Higaki, JK, T.Kobayashi

We do not need to extend the model,

$$W = \Lambda_{Q_t} Y_{1_t}^{(k)}(\tau) \bar{Q}_t Q_t \text{ generates modulus potential}$$

➤ Coleman-Weinberg potential

$$V \sim \underbrace{\left(m_0^2 + m_Q^2(\tau)\right)^2 \left(\log \frac{m_0^2 + m_Q^2(\tau)}{\mu^2} - \frac{3}{2}\right)}_{\text{from scalar quarks}} - \underbrace{\left(m_Q^2(\tau)\right)^2 \left(\log \frac{m_Q^2(\tau)}{\mu^2} - \frac{3}{2}\right)}_{\text{from quarks}}$$

$$\text{where VLQ mass } m_Q^2(\tau) = \Lambda_Q^2 (2\text{Im}\tau)^k \left| Y_{1_t}^{(k)}(\tau) \right|^2$$

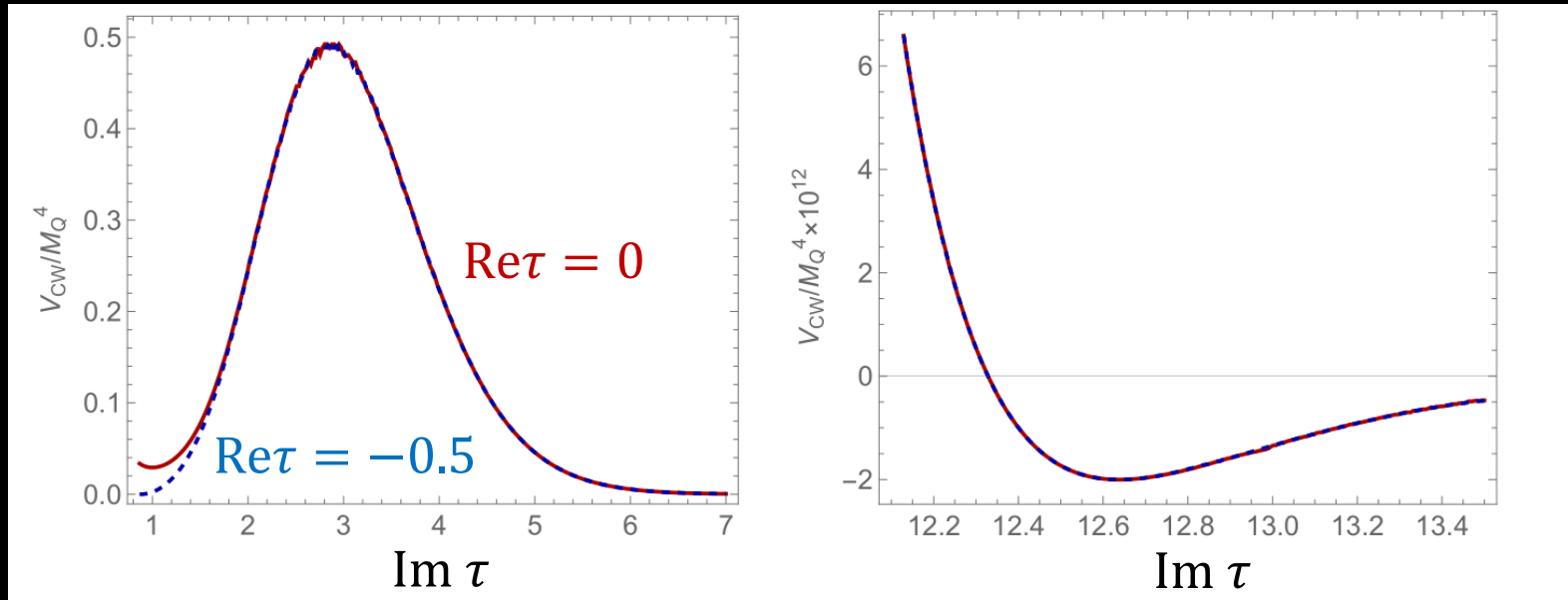
m_0^2 : soft SUSY breaking for squark, **assumed to be τ -independent**

μ : renormalization scale in $\overline{MS}$

Potential shape

2402.02071 T.Higaki, JK, T.Kobayashi

$$W = \Lambda_Q Y_{1_1}^{(12)} \overline{Q} Q \text{ under } \Gamma_3 \simeq A_4$$



$$m_0^2/M_Q^2 = 10^{-8}, \mu/\Lambda_Q = 0.01$$

- potential has **global minimum at $\text{Im } \tau \sim 13 \gg 1$**
- this minimum exists only for **non-trivial** singlet
- potential is **almost flat for $\text{Re } \tau \sim \text{axion}$** around minimum

PQ quality in the model

from CW, PQ violation

$$V(\phi) \sim \Lambda_{QCD}^4 \cos(\theta_0 + \phi) + \frac{m_0^2 \Lambda_Q^2}{8\pi^2} c_0 c_1 (2\text{Im}\tau_0)^k |q|^{7/3} \cos\phi$$

$$\Lambda_{QCD} \sim 100 \text{ MeV}$$

$$\text{axion} \sim \phi = 2\pi \text{Re}\tau$$

solve strong CP

spoil strong CP

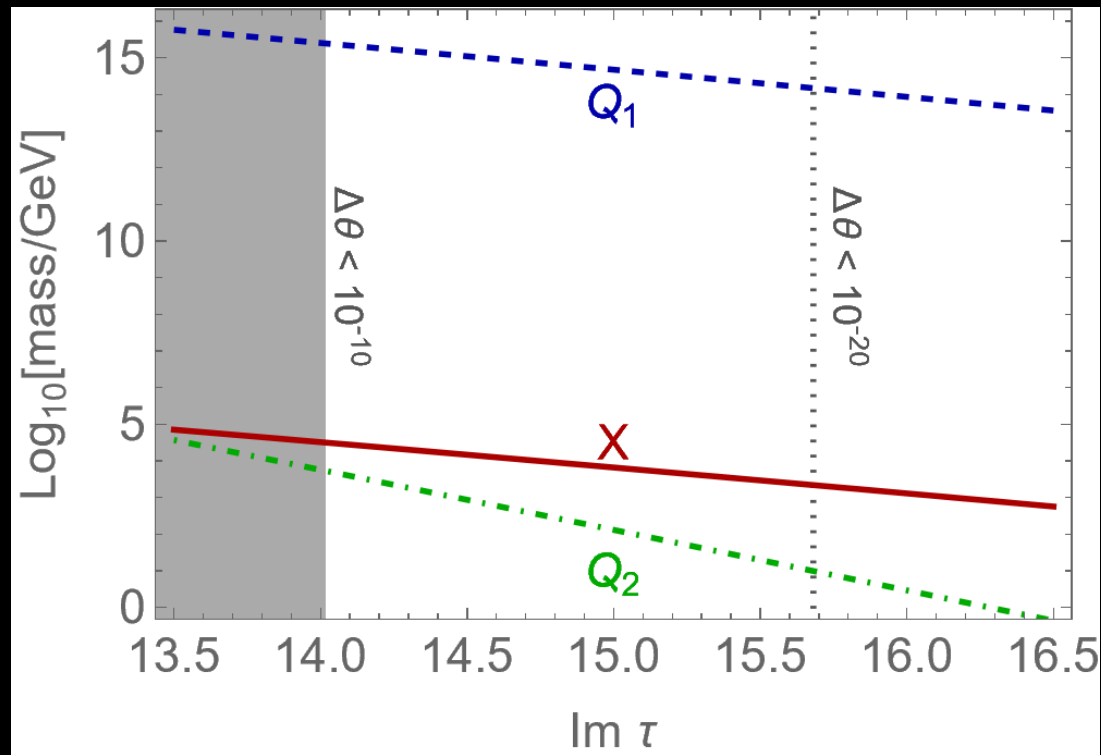


➤ shift of $\langle \bar{\theta} \rangle$ from zero:

$$\Delta\theta \sim 10^{-10} \times \sin\theta_0 \left(\frac{m_0}{10^7 \text{ GeV}} \right)^2 \left(\frac{\Lambda_Q}{10^{18} \text{ GeV}} \right)^2 \left(\frac{\text{Im}\tau_0}{14} \right)^{12} \left(\frac{|q|^{1/3}}{10^{-12}} \right)^7$$

- **consistent with neutron EDM limit** for $|q|^{1/3} < \mathcal{O}(10^{-12})$
- can **solve the strong CP** problem for $\text{Im}\tau_0 \gtrsim 14$

Modulus and vector-like quark masses



$$14 \lesssim \text{Im } \tau \lesssim 15$$

Axion quality $\Delta\theta < 10^{-10}$

Q_2 heavier than TeV

saxion $\sim \text{Im } \tau$ heavier than 10 TeV

Summary of finite modular axion

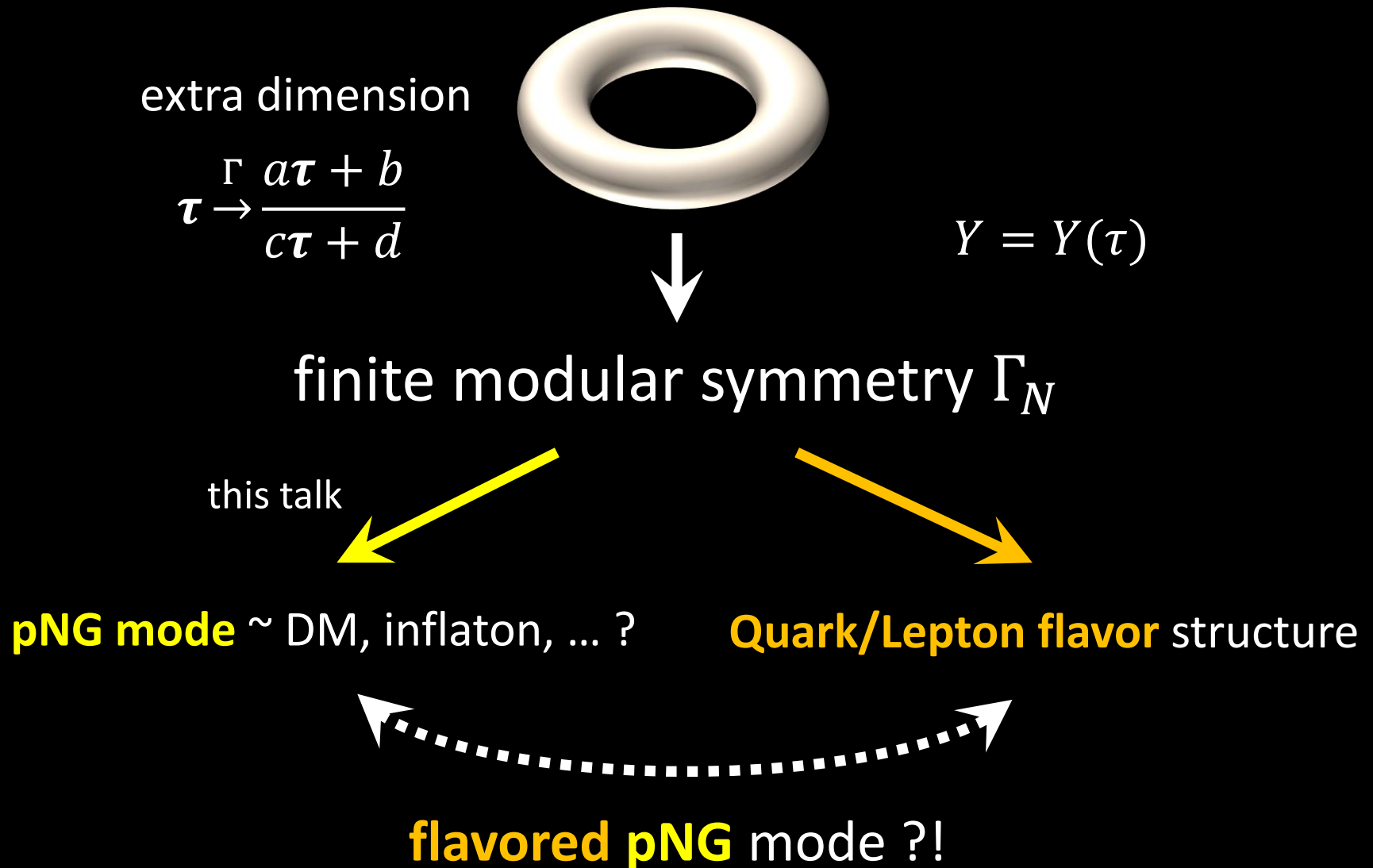
➤ Summary

- **accidental $U(1)_{PQ}$** is realized from residual Z_N^T in Γ_N
- modulus can be **stabilized by CW** in KSVZ-type model
- PQ quality is ensured by $\text{Im } \tau \gg 1$
- vector-like quark and modulus are at $O(\text{TeV})$ scale

➤ Discussions

- other applications to **accidental $U(1)$**
- **cosmological implications**, especially DM and modulus ?

Summary



backups

Finite modular symmetry

➤ **modular group** $\Gamma \Leftrightarrow$ special linear group $SL(2, \mathbb{Z})$

$$\Gamma := SL(2, \mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbb{Z}, ad - bc = 1 \right\}$$

we often consider $\bar{\Gamma} := \Gamma / \{\pm 1\} = PSL(2, \mathbb{Z})$

➤ **Finite** modular group $\Gamma_N := \bar{\Gamma} / \bar{\Gamma}(N)$

$$\text{where } \bar{\Gamma}(N) := \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \bar{\Gamma} \mid \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}$$

principal congruence group with level N

$$(ST)^3 = S^2 = 1, T^N = 1 \quad \text{with} \quad S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

➔ isomorphic to **non-Abelian discrete symmetry**, e.g. $\Gamma_3 \simeq A_4$
used for lepton/quark flavor

A modular form of $\Gamma_{N=3}$

➤ For $\mathbf{r = 3}$, $\mathbf{k = 2}$
rep. weight

$$c_{ij} = \frac{i}{2\pi} \begin{pmatrix} 1 & 1 & 1 \\ -2 & -2w^2 & -2w \\ -2 & -2w & -2w^2 \end{pmatrix} \quad w = \exp\left(\frac{2\pi i}{3}\right)$$

$$Y_3^{(2)}(\tau) = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} \quad Y_i(\tau) = \sum_{j=0}^2 c_{ij} \frac{\eta'((\tau + j)/3)}{\eta((\tau + j)/3)} - 27 \delta_{1i} \frac{\eta'(3\tau)}{\eta(3\tau)}$$

with Dedekind Eta function $\eta(\tau) := q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1 - q^n)$

➤ q -expansion $\mathbf{q := e^{2\pi i \tau}}$

$$\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} 1 + 12q + 36q^2 + \dots \\ -6q^{1/3}(1 + 7q + \dots) \\ -18q^{2/3}(1 + 2q + \dots) \end{pmatrix} \quad * |q| \ll 1 \text{ for } \text{Im}\tau \gg 1$$

Minimum of the potential

2402.02071 T.Higaki, JK, T.Kobayashi

- For $\text{Im}\tau \gg 1$ t: Z_N^T -charge of modular form Y

$$Y_{1_t}^{(k)}(\tau) \sim q^{t/3}(c_0 + c_1 q + c_2 q^2 + \cdots) \quad q = \exp(2\pi i\tau)$$

- Derivative along $x := 2\text{Im}\tau$

$$\frac{\partial V}{\partial x} = \frac{c_0^2 m_0^2 \Lambda_Q^2}{16\pi^2} x^{k-1} e^{-2\pi t x/3} \left(\underbrace{k - \frac{2\pi t x}{3}}_{\text{maximum}} \right) \left(\underbrace{\log \frac{c_0^2 \Lambda_Q^2}{e\mu^2} + k \log x - \frac{2\pi t x}{3}}_{\text{minimum}} \right)$$

$$x_{\min} = \frac{3k}{2\pi t} \mathcal{W} \left(-\frac{2\pi t}{3k} \left(\frac{\mu}{c_0 \Lambda_Q} \right)^{2/k} \right) > 1$$

minimum exists for t > 0

↔ non-trivial singlet under Γ_N

* Lambert fct. $\mathcal{W}(z)e^{\mathcal{W}(z)} = z$

Finite modular majoron '24 JK and T.H.Jung

Finite modular symmetry can be used for the accidental $U(1)_{B-L}$

➤ Type-I seesaw model

$$\mathcal{L} = \Lambda_N Y_r^{(k)}(\tau) N N \quad \text{c.f. FM axion} \quad \longleftrightarrow \quad \mathcal{L} = \Lambda_Q Y_r^{(k)}(\tau) \bar{Q} Q$$

$J \sim \text{Re } \tau$ is pNGB of B-L, so it is **finite modular majoron**

➤ Majoron cosmology

- majoron contributes both **dark matter** and **dark radiation**
- modulus $\sim \text{Im } \tau$ also play significant role
- **additional matter domination** looks preferable

Other examples of pNG mode

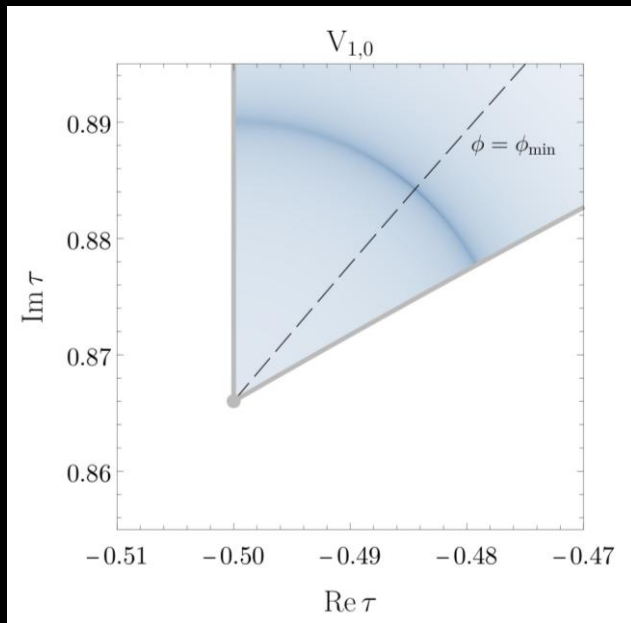
➤ Klein J function

* J is modular-inv function

'22 P.P.Novichkov, J.T. Penedo, S.T.Petcov

'91 M. Cvetič, A. Font, L.E.Ibanez et.al.

$$W \sim (J(\tau) - 1)^{m/2} J(\tau)^{n/3} P(J(\tau))$$

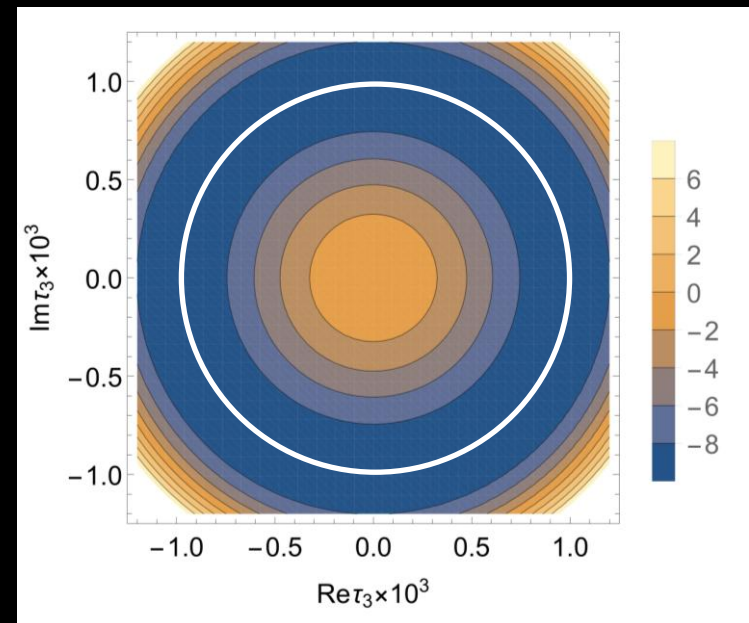


$$Z_3^{ST}: \tau \rightarrow -1/(\tau + 1)$$

➤ Siegel group $Sp(4, Z)$

'24 JK, H. Otsuka et.al.

$$W \sim Y(\tau_1, \tau_3)X + Y'(\tau_1, \tau_3)\bar{Q}Q$$



$$Z_2: \tau_3 \rightarrow -\tau_3$$

pseudo flat direction appears around a **fixed point**

Axion

$$\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

- strong CP problem

$$\mathcal{L}_{QCD} \ni \theta F_{\mu\nu} \tilde{F}^{\mu\nu}$$

is not forbidden by sym., so $\theta \sim \mathcal{O}(1)$

but neutron EDM limit is $\theta < 10^{-10}$

Why is θ so small ?

- Axion solution ...if global $U(1)_{PQ}$ has mixed anomaly with QCD

$$\left(\theta + \frac{a}{f_a}\right) F_{\mu\nu} \tilde{F}^{\mu\nu}, \text{ where } a \text{ is axion as pNG of } U(1)_{PQ}$$

➔ $V(a) = \Lambda_{QCD}^4 \left[1 - \cos\left(\theta + \frac{a}{f_a}\right)\right]$ by QCD effect

➔ axion is stabilized at $\left\langle \theta + \frac{a}{f_a} \right\rangle = 0$, thus no EDM

$U(1)_{PQ}$ symmetry ??

$U(1)_{PQ}$ has mixed QCD anomaly, so it is broken by QCD

➤ PQ (axion) quality

- if QCD breaks $U(1)_{PQ}$, why not others ?
- in general, global symmetry will be broken by gravity

$$V(a) = \Lambda_{QCD}^4 \left[1 - \cos \left(\theta + \frac{a}{f_a} \right) \right] + [\text{PQ violating terms}]$$

PQV effects should be so small that $\langle \theta + a/f_a \rangle < 10^{-10}$



PQ quality problem

This talk

finite modular symmetry Γ_N realizes pNG

➤ Finite modular axion and strong CP problem

- residual Z_N^T symmetry of Γ_N realizes **accidental** $U(1)_{PQ}$
- in KSVZ-like scenario, modulus is **stabilized by 1-loop potential**
- PQ quality is ensured by the Z_N^T symmetry

➤ Other examples

- majoron in type-I seesaw
- pNG modes in other stabilization

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4. Other examples and summary

Modular group

➤ **modular group** $\Gamma \iff$ special linear group $SL(2, \mathbb{Z})$

$$\Gamma := SL(2, \mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbb{Z}, ad - bc = 1 \right\}$$

generators

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad R = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S^2 = R, \quad (ST)^3 = R^2 = 1, \quad TR = RT$$

➤ action to **modulus** τ : complex scalar with $\text{Im } \tau > 0$

$$\tau \xrightarrow{\Gamma} \frac{a\tau + b}{c\tau + d} \quad \tau \xrightarrow{S} -1/\tau \quad \tau \xrightarrow{T} \tau + 1 \quad \tau \xrightarrow{R} \tau$$

We often consider $\bar{\Gamma} := \Gamma / Z_2^R = PSL(2, \mathbb{Z})$

Finite modular group Γ_N

➤ Congruence group $\Gamma(N)$ level $N \in \mathbb{N}$

$$\bar{\Gamma}(N) := \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \bar{\Gamma} := \text{PSL}(2, \mathbb{Z}) \mid \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}$$

$$\text{ex) } T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \rightarrow T^N = \begin{pmatrix} 1 & N \\ 0 & 1 \end{pmatrix} \in \Gamma(N)$$

➤ **Finite modular group** $\Gamma_N := \bar{\Gamma}/\bar{\Gamma}(N)$

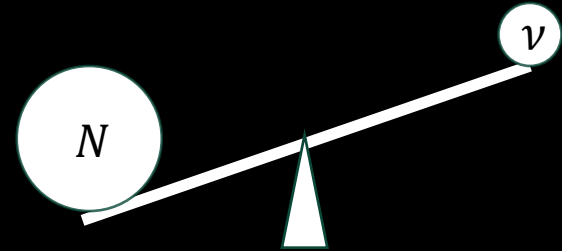
$$(ST)^3 = S^2 = 1, \quad \mathbf{T^N = 1}$$

➔ **isomorphic to non-Abelian discrete symmetries** for $N \leq 5$

$$\Gamma_2 \simeq S_3, \quad \mathbf{\Gamma_3 \simeq A_4}, \quad \Gamma_4 \simeq S_4, \quad \Gamma_5 \simeq A_5$$

Type-I seesaw

$$\mathcal{L} = \frac{1}{2} m_N N N + H_u L^c Y_d N$$



$$\rightarrow m_\nu \sim \frac{v_H^2 y_D^2}{m_N} \sim 0.1 \text{ eV} \times \left(\frac{10^{14} \text{ GeV}}{m_N} \right) \left(\frac{y_D v_H}{100 \text{ GeV}} \right)^2$$

➤ $U(1)_{B-L}$ symmetry

- is the anomaly-free symmetry in the SM + RH neutrinos
- **forbids Majorana mass**, since N has lepton number 1

➔ Majorana mass m_N should be related to B-L breaking

$$m_N \sim v_{BL}$$

Majoron

'81 Y.Chikashige, R.N.Mohapatra, R.D.Peccei; '81 G.B.Gelmini, M.Roncadelli

If B-L symmetry is **global** (not gauged),

→ there exists a pseudo NG boson, named **majoron J**

➤ Majoron DM

If majoron only couples to RH-neutrinos via $e^{iJ/f_J} m_N N N$

- decay width $\Gamma_J \sim m_\nu^2 m_J / f_J^2$ is small enough to be DM
- but it is a **decaying DM** particle

→ we may see neutrino flux from the DM decay

'17 C.Garcia-Cely, J.Heeck, '23 K.Akita, M.Niibo

B-L quality ?

majoron is a **pseudo**-NG boson of $U(1)_{B-L}$ symmetry

→ majoron gets its mass from explicit $U(1)_{B-L}$ breaking

$$\text{Ex)} \quad \mathcal{L}_{BLV} \sim \frac{\phi^n}{M_p^{n-4}} + h.c. \sim \frac{f_J^n}{M_p^{n-4}} \cos\left(n \frac{J}{f_J}\right)$$

➤ Question: How $U(1)_{B-L}$ is broken ?

need a mass term, while do not need interaction e.g. Jee

→ $U(1)_{B-L}$ should be **broken in a proper way** to be DM

B-L quality may matter, as for the PQ quality of axion

Finite modular majoron

'24 JK and T.H.Jung

Finite modular symmetry can be used for the accidental $U(1)_{B-L}$

➤ Model

c.f. FM axion

$$\mathcal{L} = \Lambda_N Y_r^{(k)}(\tau) NN \quad \longleftrightarrow \quad \mathcal{L} = \Lambda_Q Y_r^{(k)}(\tau) \bar{Q} Q$$

residual symmetry $Z_N^T : \tau \rightarrow \tau + 1, NN \rightarrow \exp\left(-\frac{2\pi i}{N}\right) NN$

➔ **accidental continuous** $U(1)_{B-L} : \tau \rightarrow \tau + \alpha, NN \rightarrow \exp\left(-\frac{2\pi i}{N} \alpha\right) NN$
 $\alpha \in \mathcal{R}$

$J \sim \text{Re } \tau$ is pNGB of B-L, so it is **finite modular majoron**

Radiative stabilization

the same **CW stabilization** works for the majoron

➤ Potential

$$V \sim \left(m_0^2 + M_N^2(\tau)\right)^2 \left(\log \frac{m_0^2 + M_N^2(\tau)}{\mu^2} - \frac{3}{2}\right) - \left(M_N^2(\tau)\right)^2 \left(\log \frac{M_N^2(\tau)}{\mu^2} - \frac{3}{2}\right)$$

$$\text{where } M_N^2(\tau) = \Lambda_N^2 (2\text{Im}\tau)^k \left|Y_{1_t}^{(k)}(\tau)\right|^2$$

➤ Modulus $X \sim \text{Im } \tau$

	soft mass	Majorana mass
mass	$m_X \sim \frac{m_0 m_N}{4\pi M_p} \sim 10 \text{ TeV} \times \left(\frac{m_0}{10^{10} \text{ GeV}}\right) \left(\frac{m_N}{10^{14} \text{ GeV}}\right)$	
decay	$\Gamma_X \sim \Gamma(X \rightarrow JJ) \sim (20 \text{ s})^{-1} \times \left(\frac{m_\phi}{20 \text{ TeV}}\right)^3$ via Kähler potential	

both majoron and modulus are important for cosmology

$\sim \text{Re } \tau$

$\sim \text{Im } \tau$

Majoron as dark matter

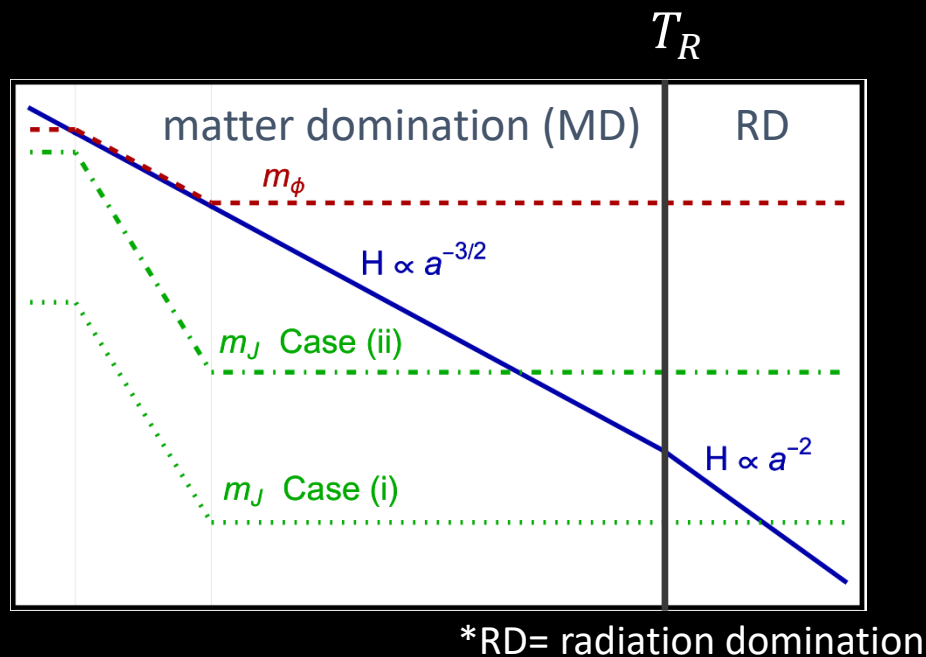
- Majoron potential from B-L breaking

$$V_{CW} \supset \frac{m_0^2 m_N^2}{16\pi^2} e^{-4\pi I m \tau} \left[1 + \cos\left(\frac{J}{f_J}\right) \right]$$

where **decay constant** $f_J \sim \frac{M_p}{4\pi I m \tau} \sim \mathcal{O}(10^{16})$ GeV

- Two scenarios for DM

1. oscillation starts **during RD**
majoron mass is 10^{-17} eV,
ultralight DM
2. oscillation starts **during MD**
majoron mass is keV-GeV,
diluted by reheating after MD



Majoron as dark radiation [DR]

➤ Modulus decay

modulus **decays** $X \rightarrow JJ$ **after BBN** [Big Bang Nucleosynthesis]

➔ produced majorons are **relativistic**, so contribute to DR

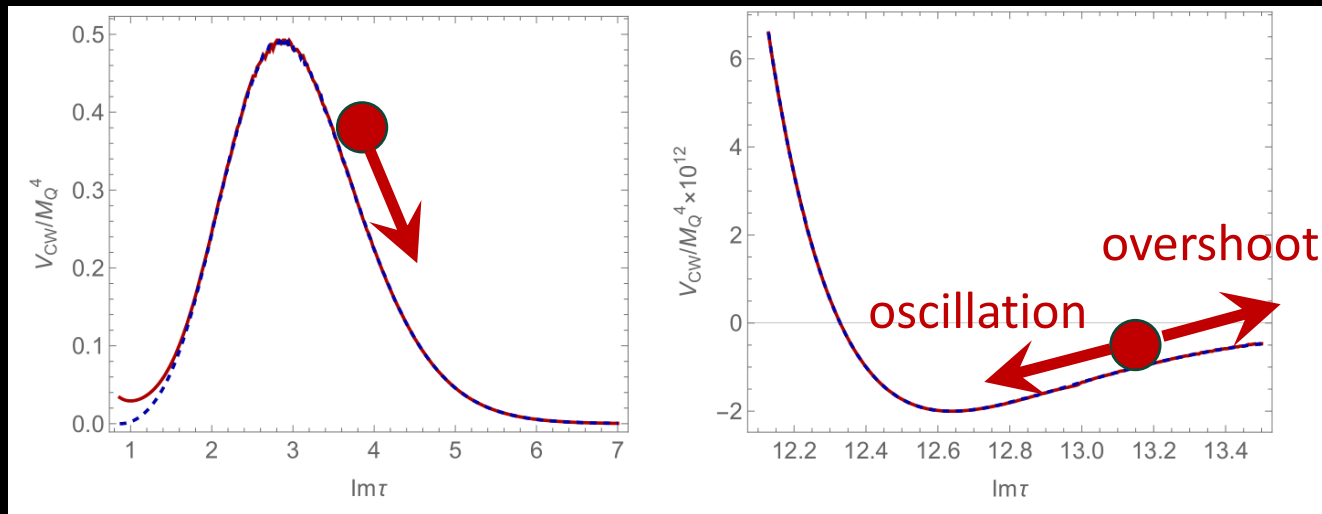
➤ Effective number of neutrinos, $N_{\text{eff}} \sim 3 + \Delta N_{\text{eff}}$

$$\Delta N_{\text{eff}} \sim 0.6 \times \left(\frac{\rho_X / \rho_{\text{rad}}}{10^{-3}} \right) \left(\frac{T_R}{10 \text{ MeV}} \right) \left(\frac{10 \text{ TeV}}{m_X} \right)^3$$

- assuming **MD by another particle** χ , reheating at T_R
- **modulus energy per radiation energy** should be small for $\Delta N_{\text{eff}} \sim \mathcal{O}(0.1)$
- limit from CMB is < 0.3 , but **Hubble tension** may prefer $\Delta N_{\text{eff}} \sim 0.4$

Moduli dynamics and energy density

if no additional energy, modulus will **overshoot**



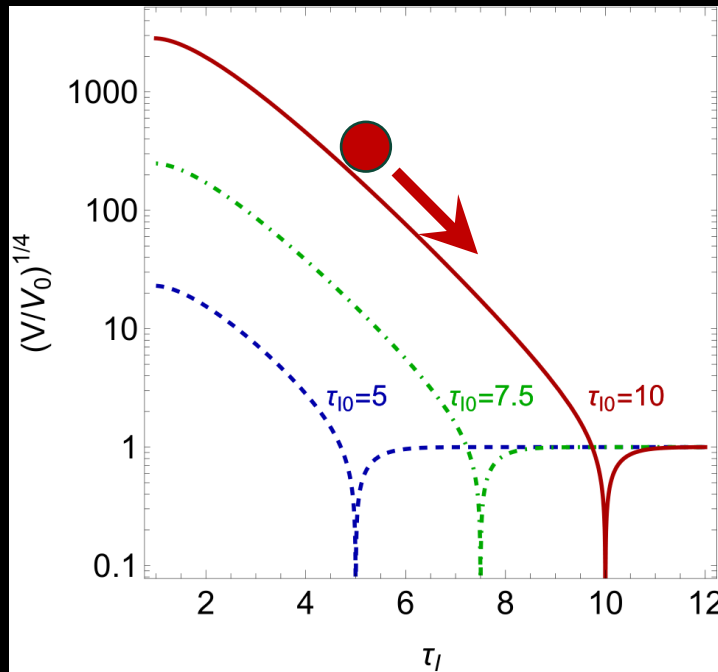
However, with the MD field χ , **Hubble friction** stops too fast rolling

$$\text{EOM: } \ddot{X} + 3H\dot{X} + V_X = 0$$

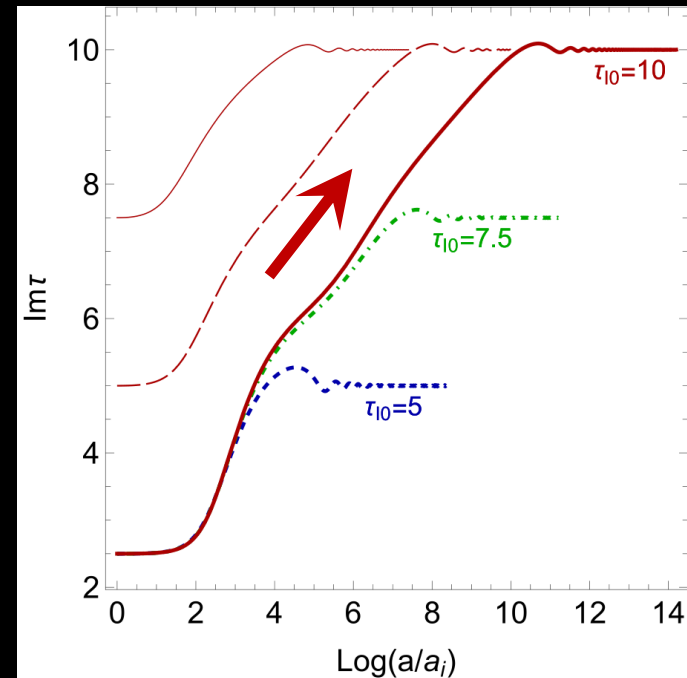
- then, modulus starts to oscillate around the minimum
- effective amplitude is small, so that $\rho_X/\rho_{rad} \sim \mathcal{O}(10^{-4})$

Moduli dynamics, numerically

potential shape in log-scale



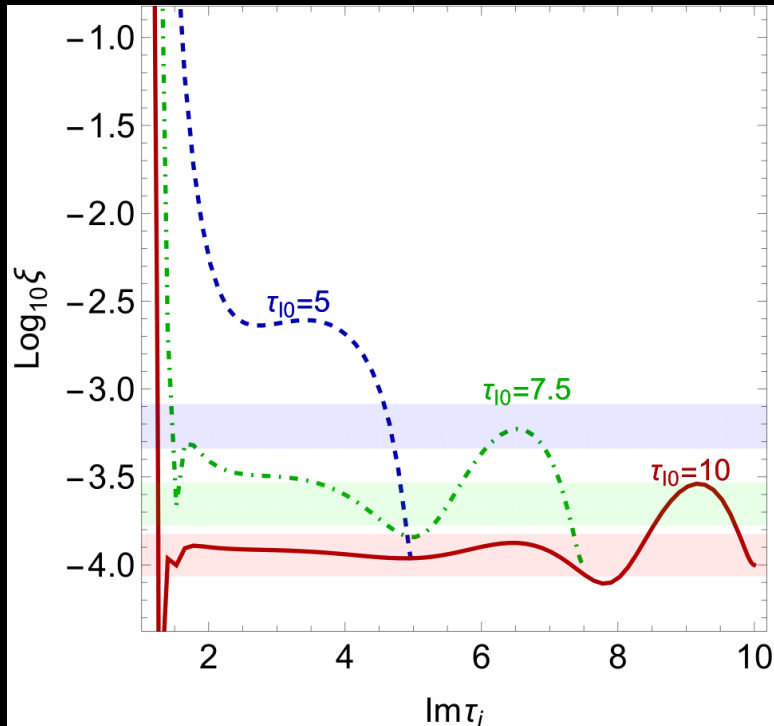
evolution of modulus



- **no overshoot**, if it starts from the **exponential slope**
- behavior of **oscillation** looks the **same for various cases**

Modulus energy density

Values of $\xi := \rho_X / \rho_\chi = \rho_X / \rho_{\text{rad}}(T_R)$



➤ Analytical formula

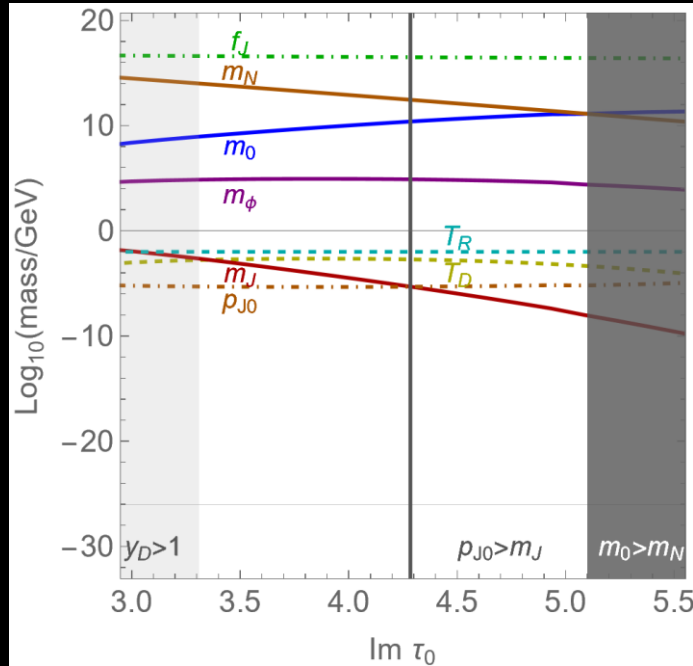
$$\xi \sim \frac{3}{2e^2} \left(k - \frac{4\pi t}{N} \text{Im}\tau_{\min} \right)^{-2}$$

$$\sim 10^{-4} \text{ for } \text{Im}\tau_{\min} \sim 0$$

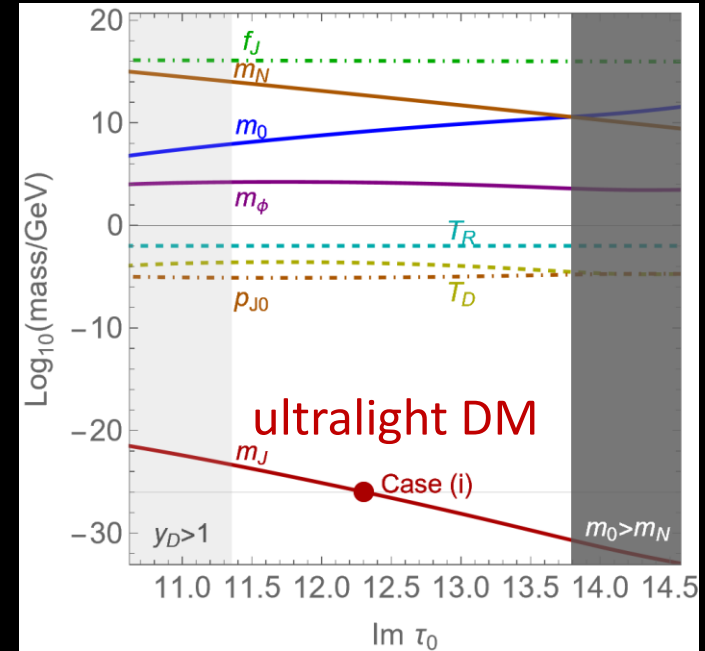
- **energy ratio is 10^{-4}** independent of initial position
- true for long slopes, and it can be larger for shorter slopes

Masses when $\Delta N_{\text{eff}} = 0.3$

$k = 4$



$k = 24$



- RH neutrino is $10^{10 \sim 14}$ GeV, soft mass is $10^{8 \sim 10}$ GeV
- modulus mass is 10 TeV for $\Delta N_{\text{eff}} \sim 0.3$
- majoron mass can be in a **wide range** $m_J \in [10^{-30}, 1.]$ GeV

Summary of finite modular majoron

➤ Summary

- **accidental $U(1)_{B-L}$** is realized from residual Z_N^T in Γ_N
- modulus can be **stabilized by CW** in type-I seesaw
- modulus does **not overshoot** because of Hubble friction
- majoron contributes to both **DM and DR**

➤ Discussions

- can we probe majoron with **keV-GeV mass** and $f_J \sim 10^{16}$ GeV ?
- application to **flavor models** ?
- relation to other cosmology, e.g. baryogenesis/inflation ?

Majoron limits

'23 K.Akita, N.Michiru

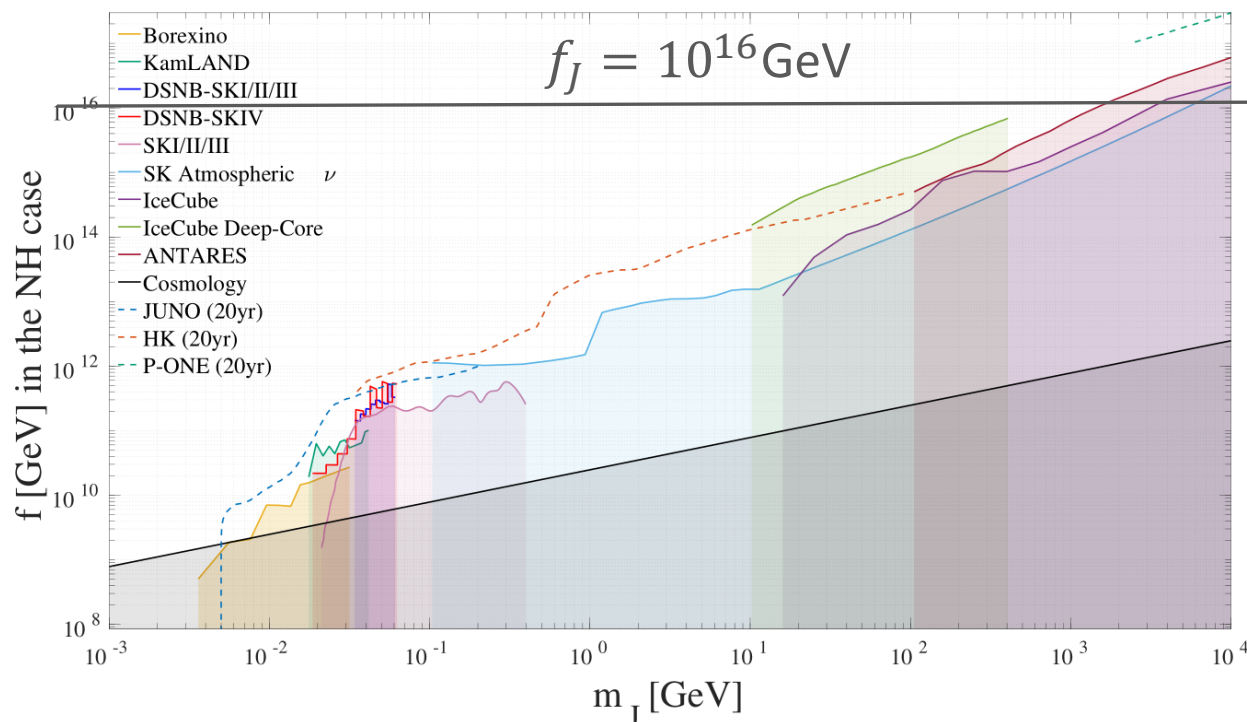


Figure 3: Lower bounds on the energy scale of the spontaneous lepton number symmetry breaking f in NH case, as a function of the majoron DM mass. The black region corresponds to the cosmological constraint on the DM lifetime comes from CMB+BAO analysis as ≤ 250 Gyr [58–62]. The other colored regions with solid curves describe the current constraints from Borexino [53] (yellow), KamLAND [54] (green), Super-Kamiokande [31–33, 55, 56] (red, blue, pink, light-blue), IceCube [35, 36] (light-green, purple), and ANTARES [38, 57], and the dashed curves describe the expected sensitivities of future neutrino detectors, JUNO [40] (blue), HK [39] (orange) and P-ONE [38, 44] (green).

Known mechanisms for stabilization

- $SL(2, \mathbb{Z})$ invariant '91 M.Cvetič, A.Font, L.E.Ibanez, D.Lust, F.Quevedo
2006.03058, P.Novichkov, J.Penedo, S.Petcov

$$W \sim (j(\tau) - 1728)^{\frac{m}{2}} j(\tau)^{\frac{n}{3}} \mathcal{P}(j(\tau))$$

$j(\tau)$: Klein function

non-perturbative effects

- Γ_N invariant potential 1909.05139, 1910.11553
T.Kobayashi, Y.Shimizu, K.Takagi, M.Tanimoto, T.Tatsuishi, H.Uchida

$$W \sim X_1^{(k_X)} \left(Y_1^{(k_Y)} \right)^p \quad X \text{ has } \mathbf{non-zero weight}, \text{ additional field}$$

- 3-form flux potential 2011.09154, 2206.04313 K.Ishiguro H.Okada, T.Kobayashi, H.Otsuka

$$W \sim \sum_{n=0}^3 c_n \tau^n \quad \text{Polynomial of } \tau, \mathbf{coefficients transform under } SL(2, \mathbb{Z})$$

Canonical normalization

➤ Modular invariant kinetic term

$$\begin{array}{lll} \text{kinetic term} & \frac{\bar{Q}Q}{(-i\tau + i\bar{\tau})^{k_q}} \rightarrow \bar{Q}Q & \text{canonical basis} \\ \text{Yukawa coup.} & Y^{(k_Y)}(\tau) \rightarrow (2\text{Im}\tau)^{k_Y/2} Y^{(k_Y)} & \end{array}$$

➤ When $\epsilon(\tau) \ll 1$

$$\epsilon(\tau) \sim 0.05 \rightarrow t := 2\text{Im } \tau \sim 5 \text{ gives additional structure}$$

another FN-like mechanism controlled by **modular weights**

Axion decay constant

$$K = -h \log(-i\tau + i\tau^*) \longrightarrow \mathcal{L}_{kin} = \frac{h M_p^2}{(2\text{Im}\tau)^2} \partial_\mu \tau^* \partial^\mu \tau$$

$$M_p \simeq 2 \times 10^{18} \text{ GeV} : \text{reduced Planck mass}$$

➤ After canonical normalization,

$$f_\phi = \frac{\sqrt{h} M_p}{4\pi \text{Im}\tau} \sim 2 \times 10^{16} \text{ GeV} \times \left(\frac{h}{3}\right)^{\frac{1}{2}} \left(\frac{14}{\text{Im}\tau}\right)$$

need entropy production maybe by saxion X and/or fine-tuned initial condition

$$m_X = \frac{c_0 m_0 M_Q}{2\sqrt{2h\pi} M_p^2} (2\text{Im}\tau)^k e^{-\frac{2\pi \text{Im}\tau}{3}} \left| k - \frac{4\pi \text{Im}\tau}{3} \right|$$

Modular forms of A4

$$Y_{1_1}^{(12)}(\tau) = (Y_1^2 + 2 Y_2 Y_3)^2 (Y_3^2 + 2 Y_1 Y_2)$$

$$Y_{1_2}^{(12)}(\tau) = (Y_1^2 + 2 Y_2 Y_3)(Y_3^2 + 2 Y_1 Y_2)^2$$

where

$$\begin{aligned} Y_1(\tau) &= \frac{i}{2\pi} \left[\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} - 27 \frac{\eta'(3\tau)}{\eta(3\tau)} \right], \\ Y_2(\tau) &= \frac{-i}{\pi} \left[\frac{\eta'(\tau/3)}{\eta(\tau/3)} + w^2 \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + w \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} \right], \\ Y_3(\tau) &= \frac{-i}{\pi} \left[\frac{\eta'(\tau/3)}{\eta(\tau/3)} + w \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + w^2 \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} \right], \end{aligned}$$

➤ q-expansion

$$Y_{1_1}^{(12)} = -12q^{1/3} (1 + 472q + \mathcal{O}(q^2)), \quad Y_{1_2}^{(12)} = 144q^{2/3} (1 + 224q + \mathcal{O}(q^2)).$$