Impact of the electroweak Weinberg operator on the electric dipole moment of electron

JHEP02(2025)082 [arXiv2408.02375]

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SI 2025

Electric Dipole Moment(EDM)

Non-relativistic

$$H_{\rm int} = -d\,\vec{S}\cdot\vec{E}$$

The interaction with
Spin($\boldsymbol{S}$) & Electric field($\boldsymbol{E}$)

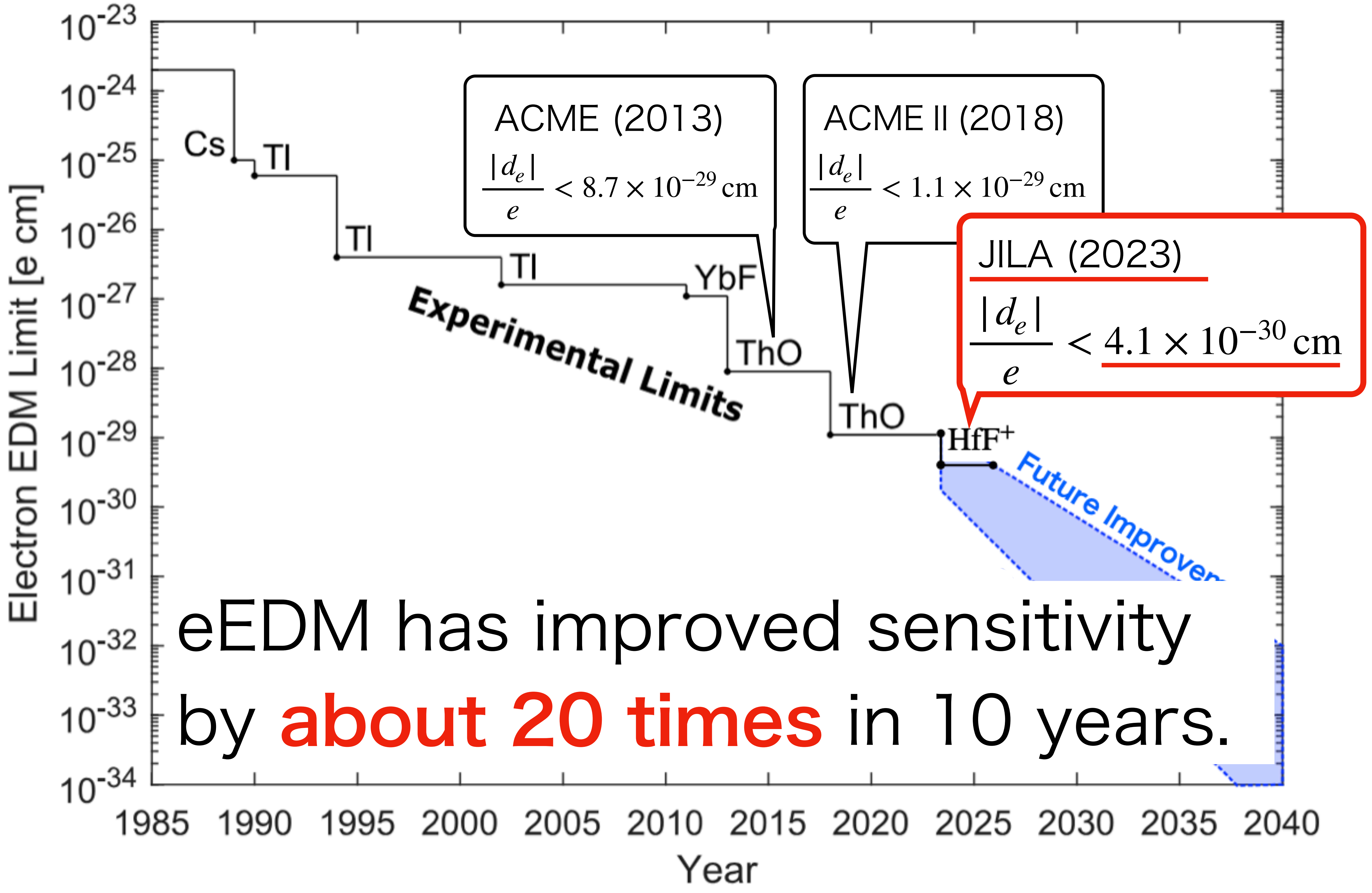
Relativistic

$$\mathcal{L} = -d\frac{i}{2}\bar{\psi}\sigma_{\mu\nu}F^{\mu\nu}\gamma_5\psi$$

Parity(P) & Time reversal($T = CP$) symmetries are violated!!

under the CPT theorem

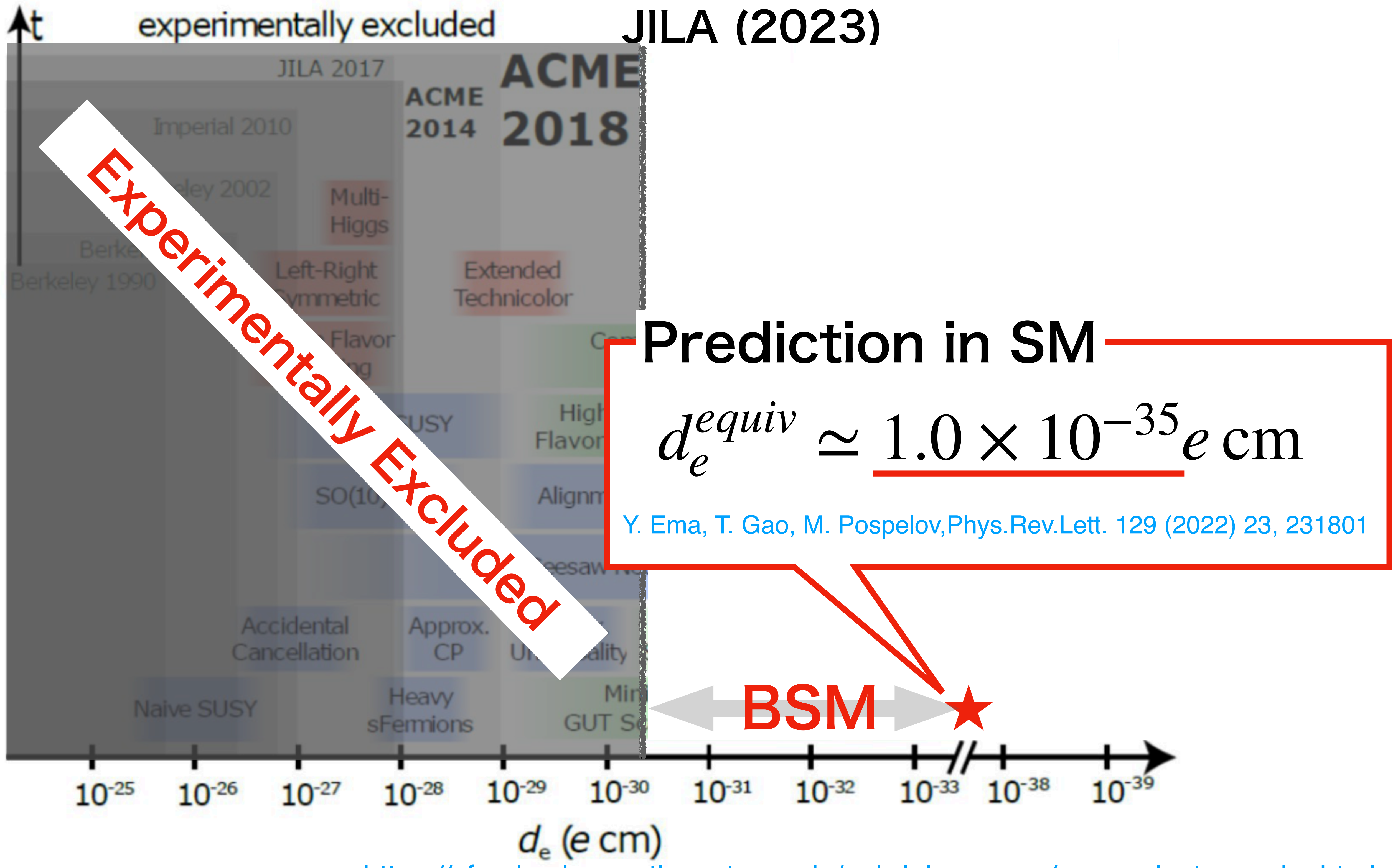
Electron EDM (eEDM)



eEDM has improved sensitivity by **about 20 times** in 10 years.

R. Alarcon et al. "Electric dipole moments and the search for new physics" in Snowmass,(2021),(2022)

T. S. Roussy et al. Science 381 (2023)



Prediction in SM

$$d_e^{equiv} \simeq 1.0 \times 10^{-35} e\,\mathrm{cm}$$

Y. Ema, T. Gao, M. Pospelov, Phys.Rev.Lett. 129 (2022) 23, 231801

https://cfp.physics.northwestern.edu/gabrielse-group/acme-electron-edm.html

M. Pospelov, A. Ritz, Phys.Rev.D 89 (2014) 5, 056006

eEDM is attractive observable to probe BSM!

Motivation in our work

- **order estimate on the eEDM (n-loop)**

$$\frac{d_e}{e} \approx \left(\frac{\lambda^2}{16\pi^2}\right)^n \frac{m_e}{\Lambda^2} \sin\phi_{\rm CP} \quad \left(\begin{array}{ll} n: \text{loop order} & \Lambda: \text{BSM scale} \\ \lambda: \text{coupling constants} & \sin\phi_{\rm CP}: \text{CP phase} \end{array}\right)$$

- **constraints to BSM scale**

Current

$$\frac{|d_e|}{e} < 4.1 \times 10^{-30}\,\mathrm{cm}$$

$$\left(\begin{array}{l} \lambda \simeq g \ \text{(SU(2)}_L \text{ gauge coupling)} \\ \sin\phi_{\rm CP} = O(1) \end{array}\right)$$

$\Lambda \gtrsim 80$ TeV (n = 1)

$\Lambda \gtrsim 4$ TeV (n = 2)

$\Lambda \gtrsim 200$ GeV (n = 3)

Future Experiments

$\frac{|d_e|}{e} < O(10^{-31})\,\mathrm{cm}$ ▶ $\Lambda \gtrsim 1$ TeV (n = 3)

$\frac{|d_e|}{e} < O(10^{-32})\,\mathrm{cm}$ ▶ $\Lambda \gtrsim 3$ TeV (n = 3)

sensitive to TeV scale at 3-loop level

In the future eEDM experiments,

TeV-scale BSM that induces the eEDM at 3-loop can be probed!

Set Up

BSM(TeV)

- $SU(2)_L$ multiplets : ψ_A, ψ_B (fermion), S (scalar)

CP violating Yukawa interaction $\mathcal{L} \supset -\bar{\psi}_B g_{\bar{B}AS} \psi_A S - \bar{\psi}_A g_{\bar{A}B\bar{S}} \psi_B S^*$

$$g_{\bar{B}AS} = X_{\bar{B}AS}(s + \boxed{\gamma_5 a})$$
$$g_{\bar{A}B\bar{S}} = X_{\bar{A}B\bar{S}}(s^* - \boxed{\gamma_5 a^*})$$

~~CP~~

(s, a : complex number)

($X_{\bar{B}AS}, X_{\bar{A}B\bar{S}}$: coefficients that depend on representation)

※assumption: $SU(2)_L$ multiplets couple to electron only through W

SM

- **The motivation of the $SU(2)_L$ multiplets**
  - **Minimal Dark Matter model** M. Cirelli and A. Strumia, New J. Phys. 11(2009) 105005
    - The neutral component is DM candidate.
    - mass $\sim O(\mathrm{TeV})$ (Relic abundance)
    - quintuplet fermion, septet scalar are favor (DM stability)

Set Up

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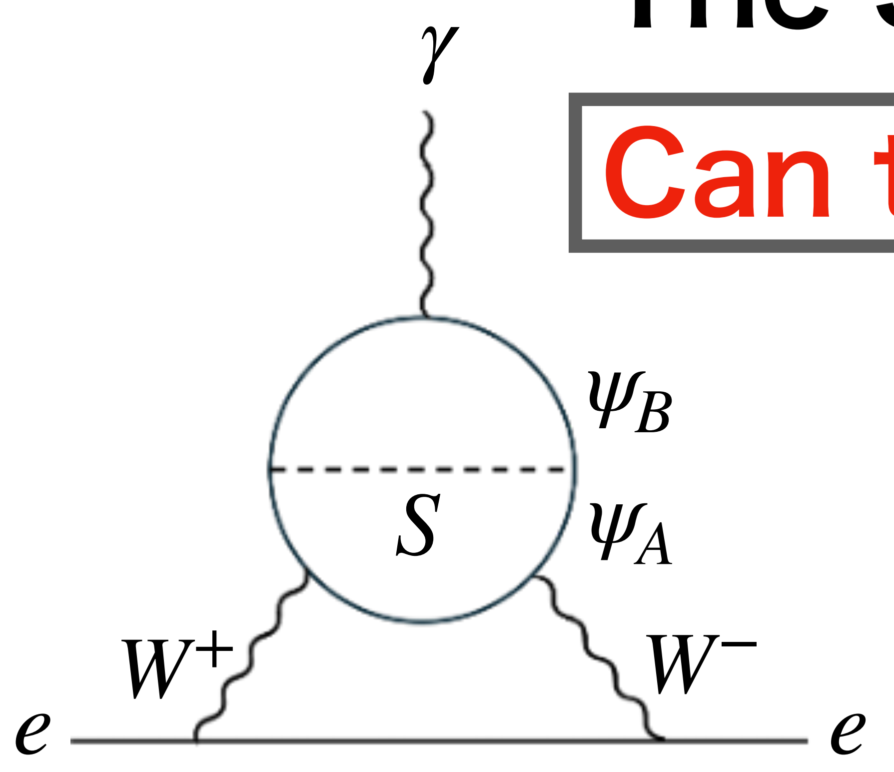
Our Work : Evaluate the eEDM in this model

SM

The 3-loop contribution is Leading Order(LO)!

Can this model de probed in the future experiment?

eEDM



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~~CP~~

※assumption: $SU(2)_L$ multiplets couple to electron only through W

SM

Strategy : EFT approach

2-loop

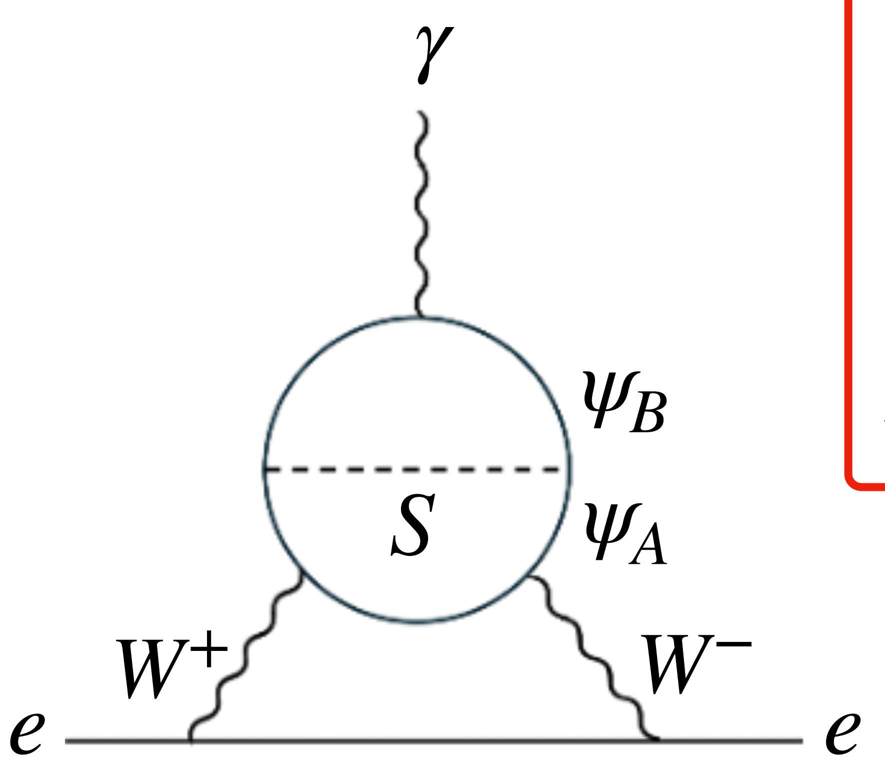
Electroweak-Weinberg Operator

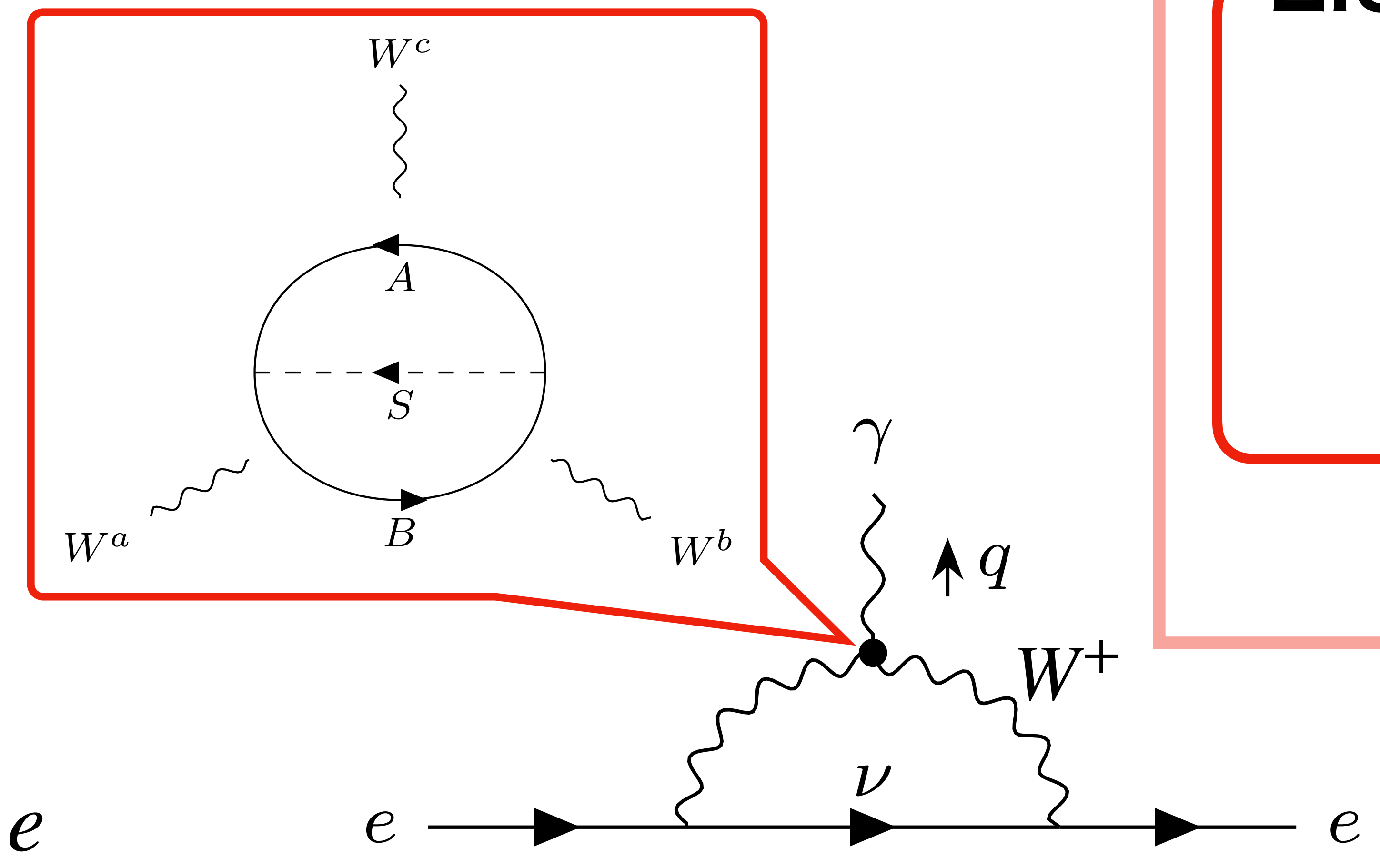
$$\mathcal{L}_W = -\frac{g^3}{3} C_W f^{abc} W^a_{\mu\nu} W^{b\nu}{}_{\rho} \tilde{W}^{c\rho\mu}$$

$$\tilde{W}^a_{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} W^a_{\rho\sigma}$$

1-loop

eEDM



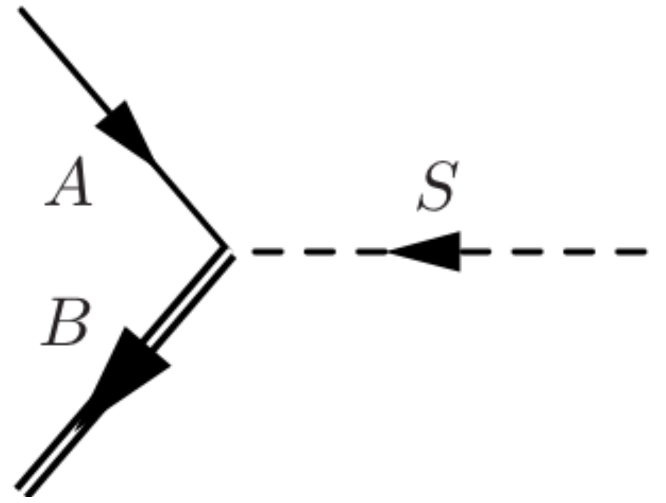


eEDM

eEDM induced by EW-Weinberg Operator

Calculate by EFT approach

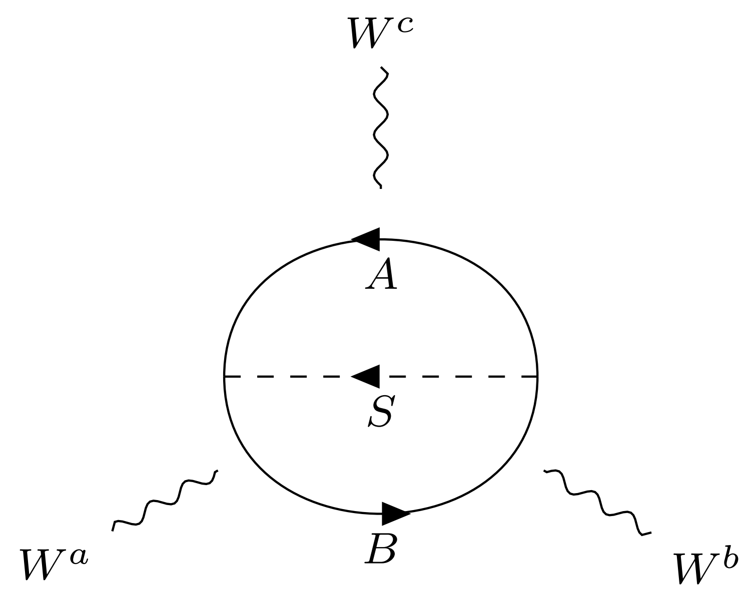
BSM(TeV)



Yukawa interaction

$$\mathcal{L} \supset -\bar{\psi}_B g_{\bar{B}AS} \psi_A S - \bar{\psi}_A g_{\bar{A}B\bar{S}} \psi_B S^*$$

SM



EW-Weinberg Operator

$$\mathcal{L}_W = -\frac{g^3}{3} C_W f^{abc} W^a_{\mu\nu} W^{b\nu}{}_{\rho} \tilde{W}^{c\rho\mu}$$

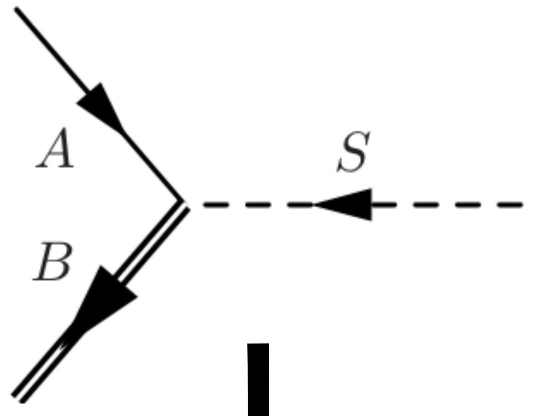
T. Abe, J. Hisano, and R. Nagai, JHEP 03 (2018) 175

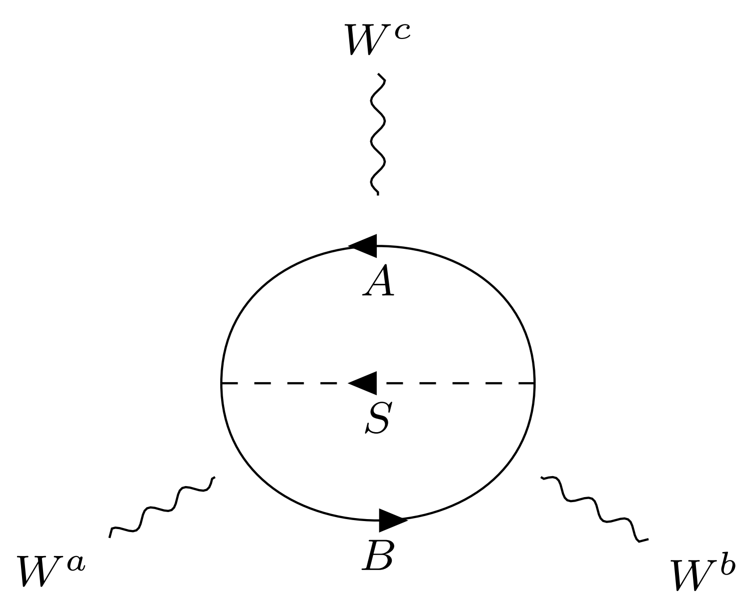
eEDM

$$\mathcal{L} = -\frac{i}{2} d_e \bar{e} (\sigma \cdot F) \gamma_5 e$$

F. Boudjema, Hagiwara et al, Phys. Rev. D43(1991) 2223–2232

Yukawa interaction → EW-Weinberg Operator





✔ Yukawa interaction→ QCD Weinberg Operator

T. Abe, J. Hisano, and R. Nagai, JHEP 03 (2018) 175

- We derive the EW-Weinberg Operator **with 2 changes.**

JHEP 03 (2018) 175		Our Work
① SU(3)	→	SU(N) → N=2
② Numerical	→	Analytic

S.P.Martin,Phys.Rev.D.65 (2002) 116003

eEDM induced by EW-Weinberg Operator

Calculate by EFT approach

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Yukawa interaction

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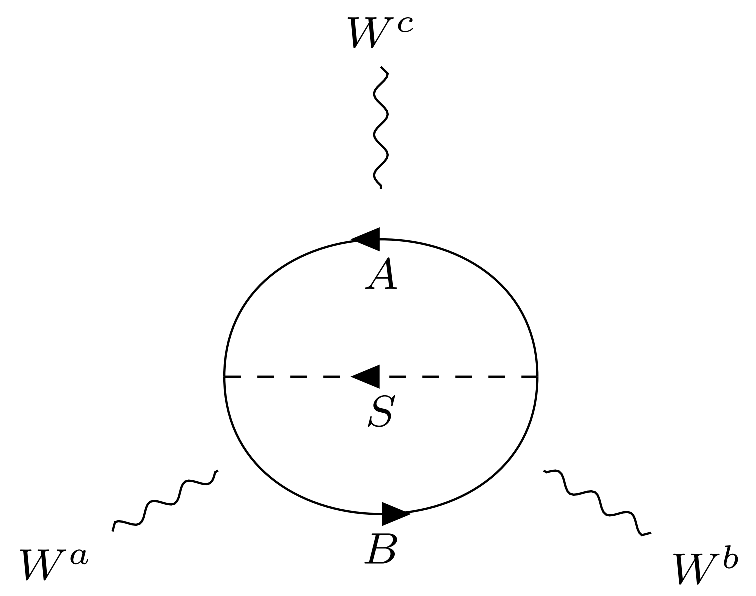

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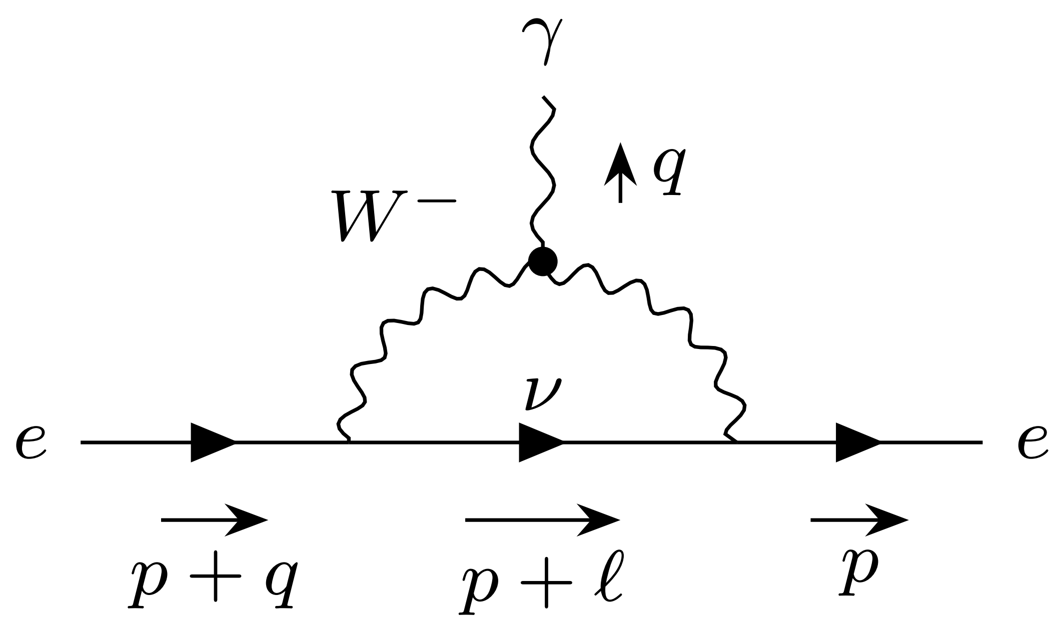

eEDM

$$\mathcal{L} = -\frac{i}{2} d_e \bar{e}(\sigma \cdot F)\gamma_5 e$$

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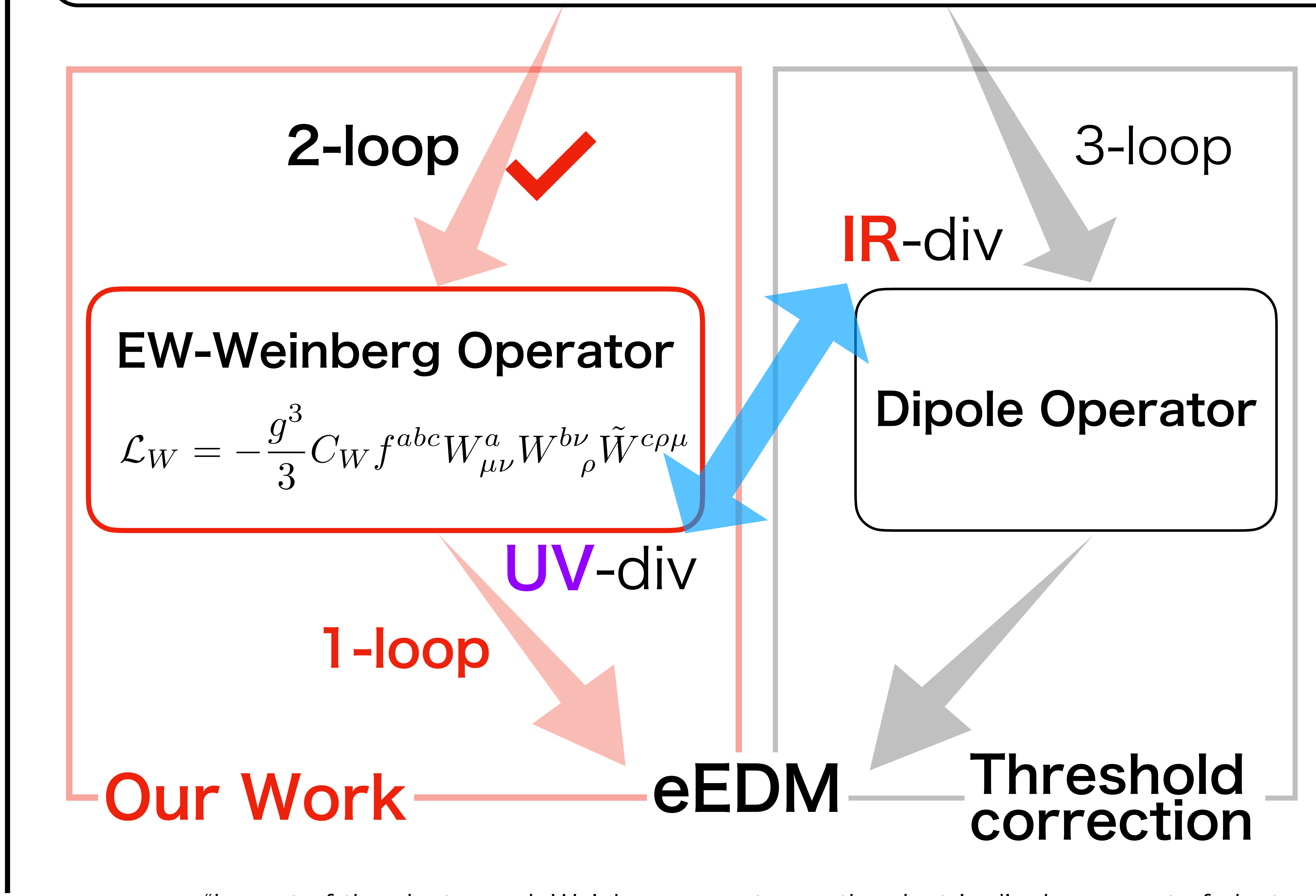
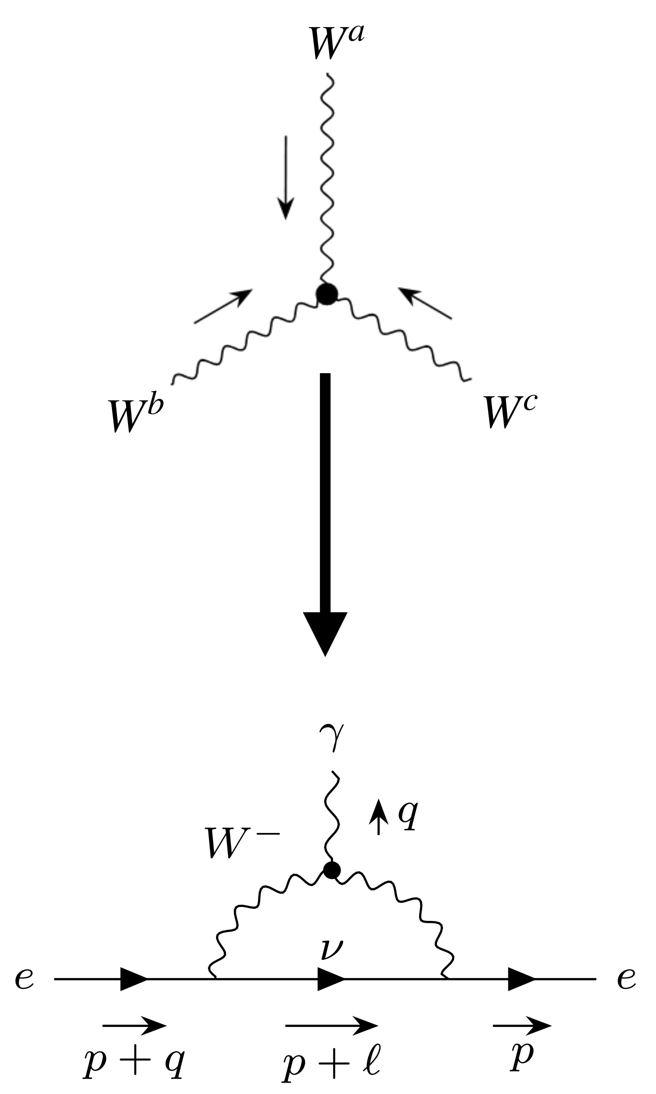


UV & IR-div must be removed by the consistent regularization.

dimensional regularization(DR)

EW-Weinberg Operator → eEDM



- **Evaluate 1-loop diagram in DR**

▶ **Lagrangian 4-dim → d-dim** $\mathcal{L}^{(d)} = \mathcal{L}^{(4)} + \mathcal{L}^{(d-4)}$

Evanescent operator is added

▶ Treatment for γ_5 & Levi-Chivita symbol

Bollini-**M**éndez-**H**oleman-**V**eltman(**BMHV**) Scheme

G. 't Hooft and M. J. G. Veltman, Nucl. Phys. B 44 (1972) 189–213.

- **γ_5 & Levi-Chivita symbol are defined as 4-dimensional objects**

$$\gamma_5 = -\frac{i}{4!}\bar{\epsilon}^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma \qquad \hat{\epsilon}^{\mu\nu\rho\sigma} = 0$$

- **give up anticommutation property**

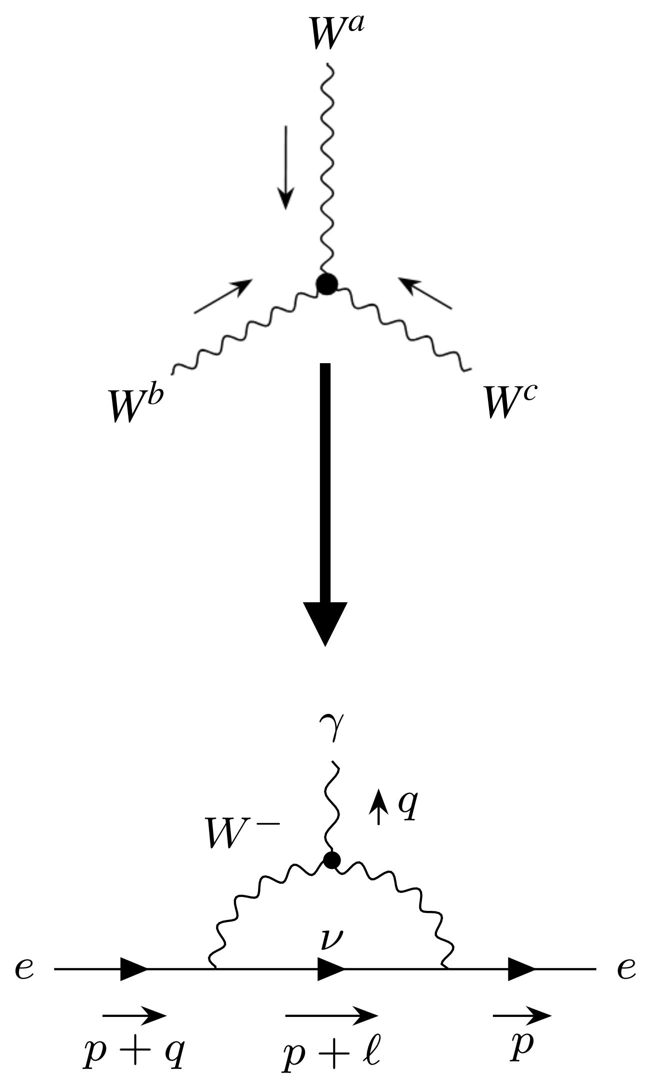
$$\{\bar{\gamma}^\mu, \gamma_5\} = 0, \ [\hat{\gamma}^\mu, \gamma_5] = 0$$

※ bars and hats conventionally represent 4 and (d – 4)-dim.

EW-Weinberg Operator in d-dim

$$\mathcal{L}_W^d = -\frac{g^3}{3}C_W^{(4)} f^{abc}\bar{W}^a_{\mu\nu}\bar{W}^{b\nu}{}_\rho\bar{\tilde{W}}^{c\rho\mu} - \frac{g^3}{3}C_W^{(d-4)} f^{abc} W^a_{\bar{\mu}\hat{\nu}} W^{b\hat{\nu}}{}_{\bar{\rho}}\bar{\tilde{W}}^{c\rho\mu}$$

EW-Weinberg Operator → eEDM



- **Evaluate 1-loop diagram in DR**

✔ F. Boudjema, Hagiwara et al, Phys. Rev. D43(1991) 2223–2232

The formula for Levi-Civita symbol

$$\bar\epsilon_{\mu\nu\rho\sigma}\bar\epsilon^{\alpha\beta\gamma\sigma} = -\bar g^\alpha_\mu \bar g^\beta_\nu \bar g^\gamma_\rho + \bar g^\beta_\mu \bar g^\alpha_\nu \bar g^\gamma_\rho + \bar g^\alpha_\mu \bar g^\gamma_\nu \bar g^\beta_\rho + \bar g^\gamma_\mu \bar g^\beta_\nu \bar g^\alpha_\rho - \bar g^\gamma_\mu \bar g^\alpha_\nu \bar g^\beta_\rho - \bar g^\beta_\mu \bar g^\gamma_\nu \bar g^\alpha_\rho$$

EW-Weinberg Operator in d-dim

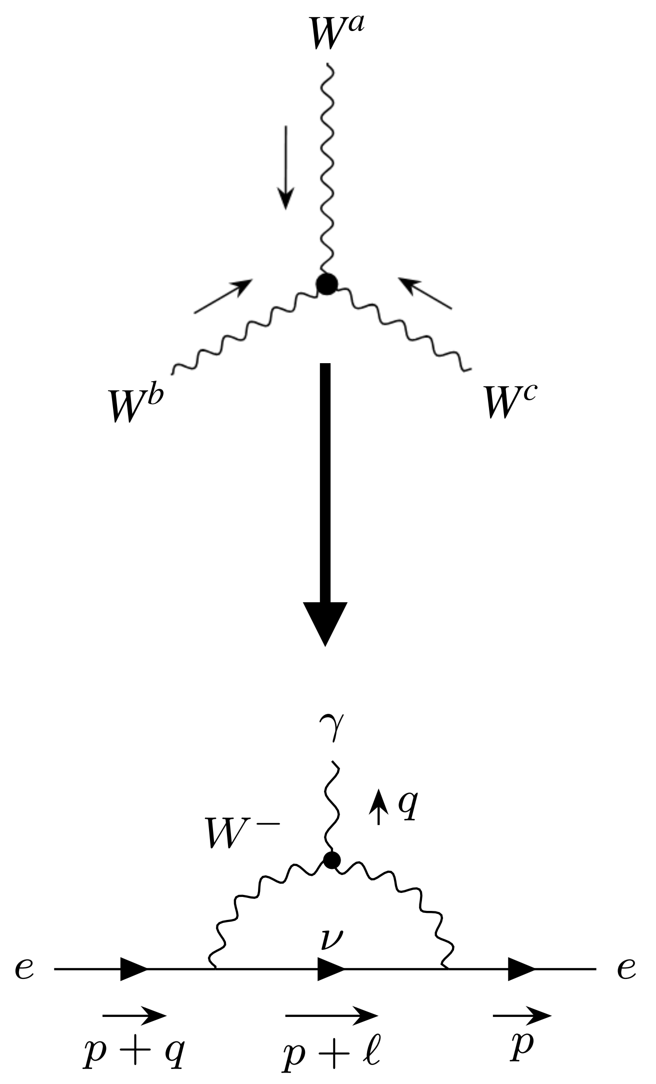
$$\mathcal{L}_W^d = -\frac{g^3}{3} C_W^{(4)} f^{abc} \bar W^a_{\mu\nu} \bar W^{b\nu}{}_{\rho} \tilde{\bar W}^{c\rho\mu} - \frac{g^3}{3} C_W^{(d-4)} f^{abc} W^a_{\bar\mu\hat\nu} W^{b\hat\nu}{}_{\bar\rho} \tilde{\bar W}^{c\rho\mu}$$

$$= -\frac{2ieg^2}{3} C_W^{(4)} \left[\bar W^-_{\mu\nu} \bar W^{+\nu}{}_{\lambda} \tilde{\bar F}^{\lambda\mu} + \bar F_{\mu\nu} \left(\bar W^{-\nu}{}_{\lambda} \widetilde{\bar W}^{+\lambda\mu} - \bar W^{+\nu}{}_{\lambda} \widetilde{\bar W}^{-\lambda\mu} \right) \right] = 3 \bar W^-_{\mu\nu} \bar W^{+\nu}{}_{\lambda} \tilde{\bar F}^{\lambda\mu}$$

$$- \frac{2ieg^2}{3} C_W^{(d-4)} \left[W^-_{\bar\mu\hat\nu} W^{+\hat\nu}{}_{\bar\lambda} \tilde{\bar F}^{\lambda\mu} + F_{\bar\mu\hat\nu} \left(W^{-\hat\nu}{}_{\bar\lambda} \widetilde{\bar W}^{+\lambda\mu} - W^{+\hat\nu}{}_{\bar\lambda} \widetilde{\bar W}^{-\lambda\mu} \right) \right] = 3 W^-_{\bar\mu\hat\nu} W^{+\hat\nu}{}_{\bar\lambda} \tilde{\bar F}^{\lambda\mu}$$

In the previous study, the 4-dim formula was naively applied to the evanescent operator.

EW-Weinberg Operator → eEDM



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The formula for Levi-Civita symbol

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EW-Weinberg Operator in d-dim

$$\begin{aligned}
\mathcal{L}_W^d &= -\frac{g^3}{3} C_W^{(4)} f^{abc} \bar W^a_{\mu\nu} \bar W^{b\nu}{}_{\rho} \tilde{\bar W}^{c\rho\mu} - \frac{g^3}{3} C_W^{(d-4)} f^{abc} W^a_{\bar\mu\hat\nu} W^{b\hat\nu}{}_{\bar\rho} \tilde{\bar W}^{c\rho\mu} \\
&= -\frac{2ieg^2}{3} C_W^{(4)} \left[\bar W^-_{\mu\nu} \bar W^{+\nu}{}_{\lambda} \tilde{\bar F}^{\lambda\mu} + \bar F_{\mu\nu} \left(\bar W^{-\nu}{}_{\lambda} \widetilde{\bar W}^{+\lambda\mu} - \bar W^{+\nu}{}_{\lambda} \widetilde{\bar W}^{-\lambda\mu} \right) \right] = 3 \bar W^-_{\mu\nu} \bar W^{+\nu}{}_{\lambda} \tilde{\bar F}^{\lambda\mu} \\
&\quad - \frac{2ieg^2}{3} C_W^{(d-4)} \left[W^-_{\bar\mu\hat\nu} W^{+\hat\nu}{}_{\bar\lambda} \tilde{\bar F}^{\lambda\mu} + F_{\bar\mu\hat\nu} \left(W^{-\hat\nu}{}_{\bar\lambda} \widetilde{\bar W}^{+\lambda\mu} - W^{+\hat\nu}{}_{\bar\lambda} \widetilde{\bar W}^{-\lambda\mu} \right) \right] = \cancel{3 W^-_{\bar\mu\hat\nu} W^{+\hat\nu}{}_{\bar\lambda} \tilde{\bar F}^{\lambda\mu}}
\end{aligned}$$

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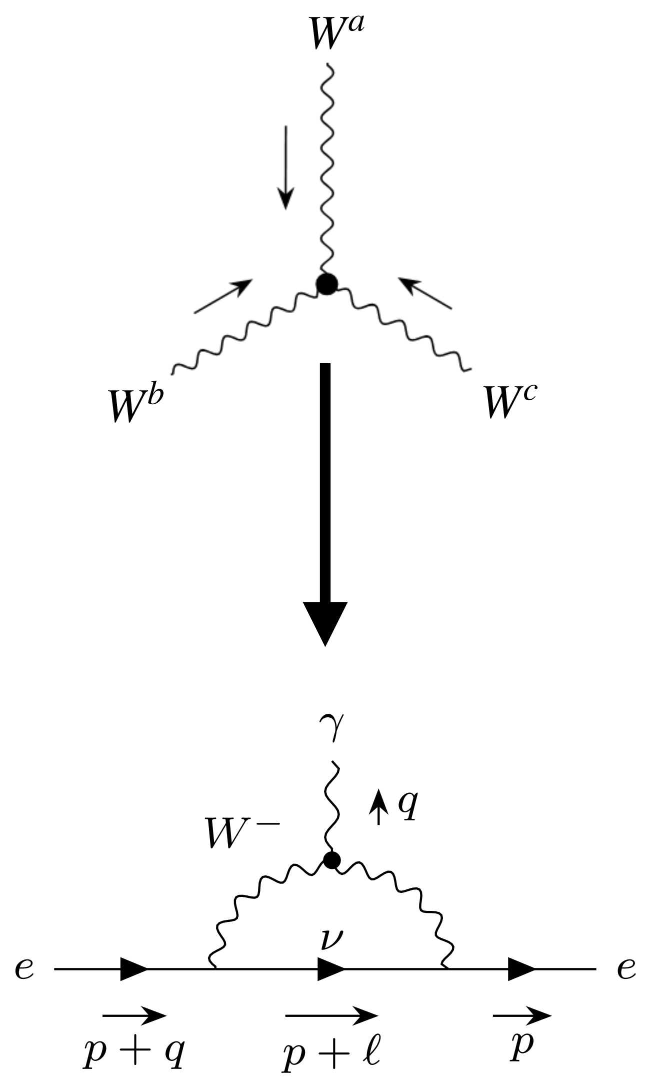
EW-Weinberg Operator → eEDM



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EW-Weinberg Operator in d-dim

$$\mathcal{L}_W^d = -\frac{2ieg^2}{3} C_W^{(4)} \left[\bar{W}^-_{\mu\nu} \bar{W}^{+\nu}{}_{\lambda} \widetilde{\bar{F}}^{\lambda\mu} + \bar{F}_{\mu\nu}\left(\bar{W}^{-\nu}{}_{\lambda} \widetilde{\bar{W}}^{+\lambda\mu} - \bar{W}^{+\nu}{}_{\lambda} \widetilde{\bar{W}}^{-\lambda\mu}\right)\right] = 3\bar{W}^-_{\mu\nu}\bar{W}^{+\nu}{}_{\lambda} \widetilde{\bar{F}}^{\lambda\mu}$$

$$-\frac{2ieg^2}{3} C_W^{(d-4)} \left[W^-_{\bar{\mu}\hat{\nu}} W^{+\hat{\nu}}{}_{\bar{\lambda}} \widetilde{\bar{F}}^{\lambda\mu} + F_{\bar{\mu}\hat{\nu}}\left(W^{-\hat{\nu}}{}_{\bar{\lambda}} \widetilde{\bar{W}}^{+\lambda\mu} - W^{+\hat{\nu}}{}_{\bar{\lambda}} \widetilde{\bar{W}}^{-\lambda\mu}\right)\right]$$

Result1

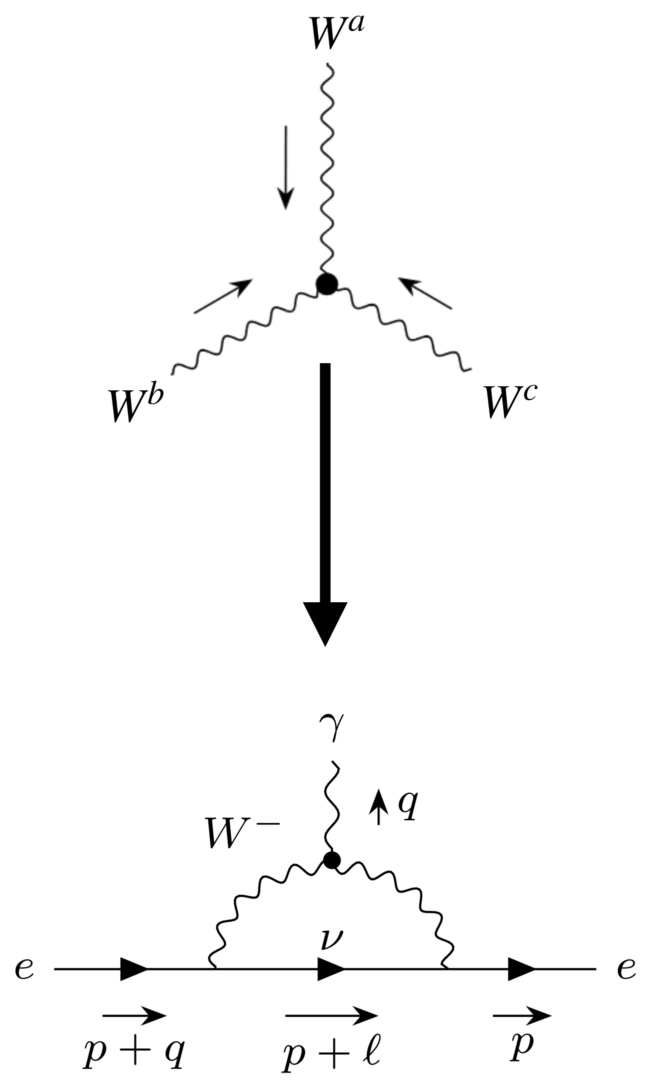
We separately evaluate the 4- & (d-4)-dim contributions.

$$\frac{d_e}{e} = \frac{\alpha_2^2}{6} m_e C_W^{(4)} + \frac{\alpha_2^2}{9} m_e C_W^{(d-4)}$$

In $C_W^{(4)} = C_W^{(d-4)}$ case, this result is consistent to the Naive Dimensional Regularization.

W. Dekens and P. Stoffer, JHEP 10 (2019) 197

EW-Weinberg Operator → eEDM



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$$-\frac{2ieg^2}{3}C_W^{(d-4)}\left[W^-_{\hat{\mu}\hat{\nu}}W^{+\hat{\nu}}{}_{\bar{\lambda}} \widetilde{\bar{F}}^{\lambda\mu} + F_{\bar{\mu}\hat{\nu}}\left(W^{-\hat{\nu}}{}_{\bar{\lambda}} \widetilde{\bar{W}}^{+\lambda\mu} - W^{+\hat{\nu}}{}_{\bar{\lambda}} \widetilde{\bar{W}}^{-\lambda\mu}\right)\right]$$

Result1

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$$\frac{d_e}{e} = \frac{\alpha_2^2}{6}m_e C_W^{(4)} + \frac{\alpha_2^2}{9}m_e C_W^{(d-4)}$$

We do not understand the Wilson coefficient for the evanescent operator $C_W^{(d-4)}$.

In our study, we analyze the 4-dim result.

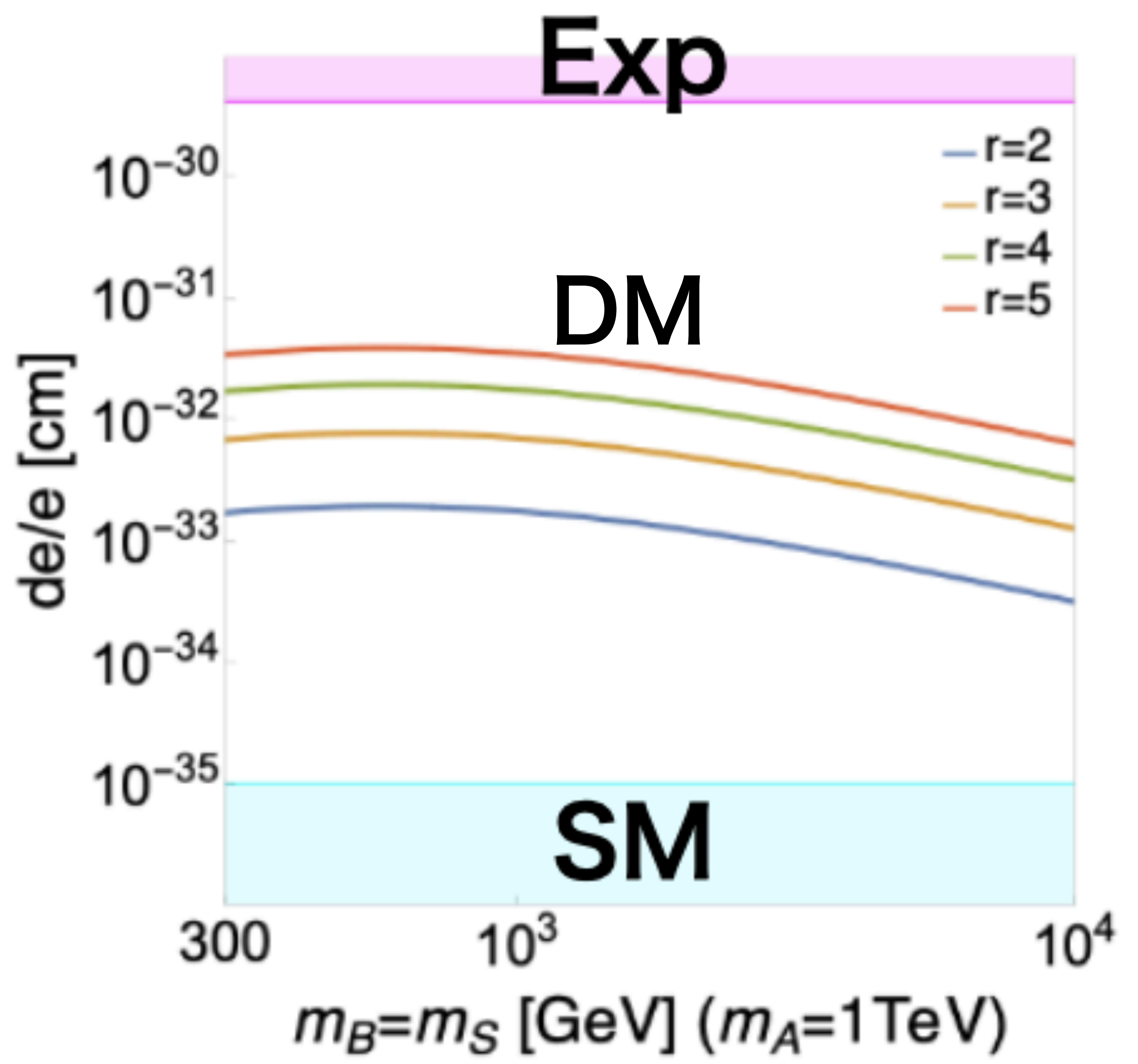
Can our scenario be tested in the future eEDM?

$$(A, B, S) = (r,1,r) \quad \mathrm{Im}(sa^*) = 0.25$$

- **EW-Weinberg contribution is the window between the experimental constraints and SM.**

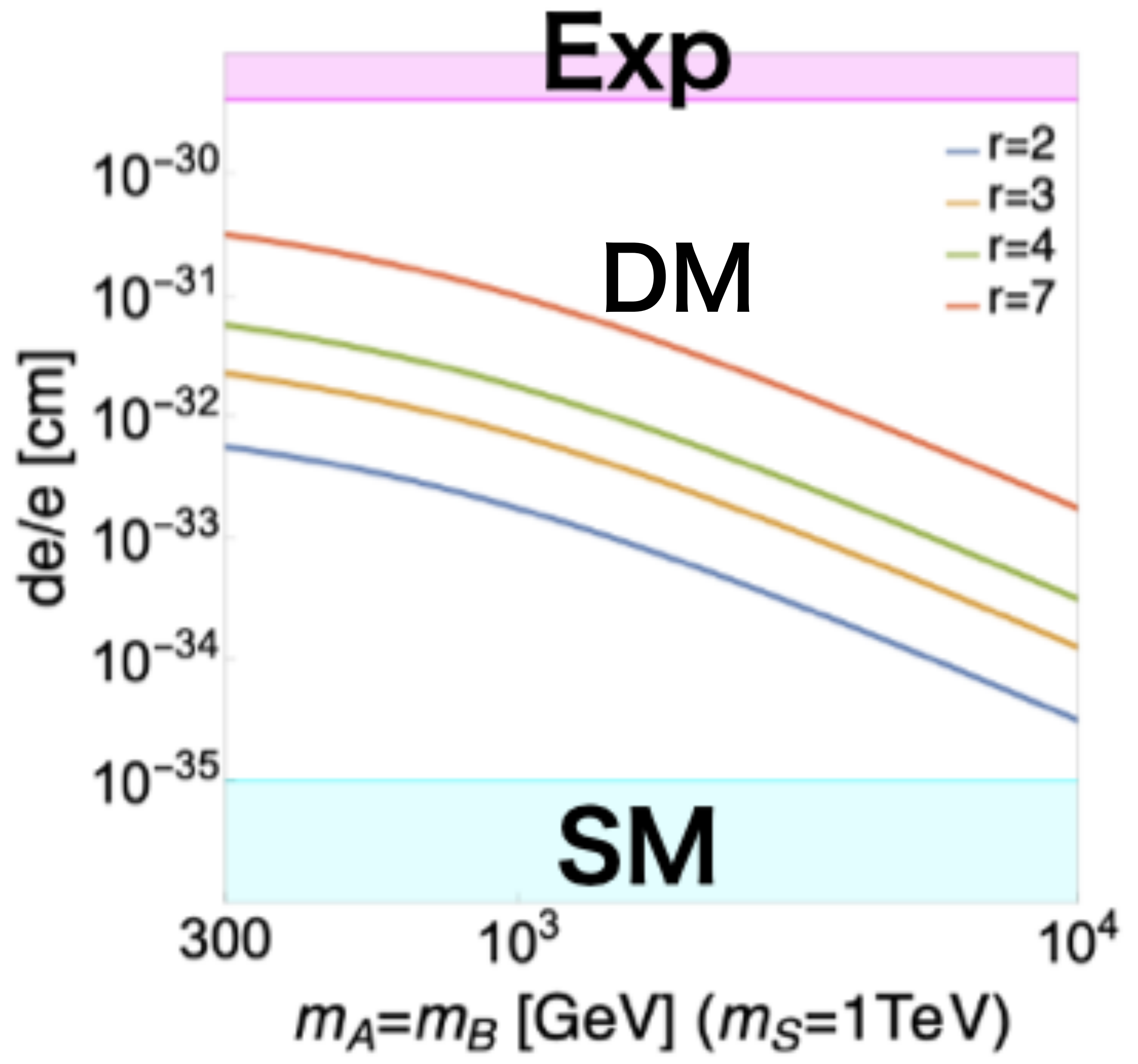
Result2

This scenario can be tested in the future eEDM!



- In Minimal Dark Matter model, higher representation is favored.

Minimal DM might be tested in 203X.



Conclusion

TeV scale physics which induces eEDM by 3-loop level is attractive!

- **Our Work**

We consider the eEDM induced by Yukawa interaction of $SU(2)_L$ multiplets.

Result1 We separately evaluate the 4- & (d-4)-dim contributions.

$$\frac{d_e}{e} = \frac{\alpha_2^2}{6} m_e C_W^{(4)} + \frac{\alpha_2^2}{9} m_e C_W^{(d-4)}$$

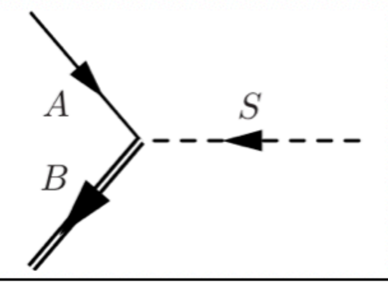
Result2 This scenario can be tested in the future eEDM!!

Calculate by EFT approach

BSM(TeV)

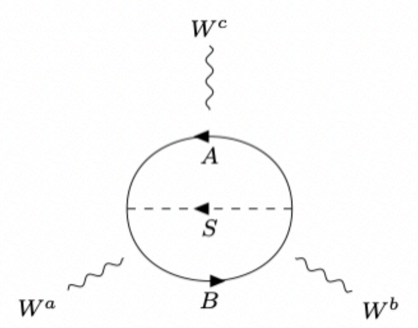
SM

Yukawa interaction



$$\mathcal{L} \supset -\bar{\psi}_B g_{\bar{B}AS} \psi_A S - \bar{\psi}_A g_{\bar{A}B\bar{S}} \psi_B S^*$$

EW-Weinberg Operator



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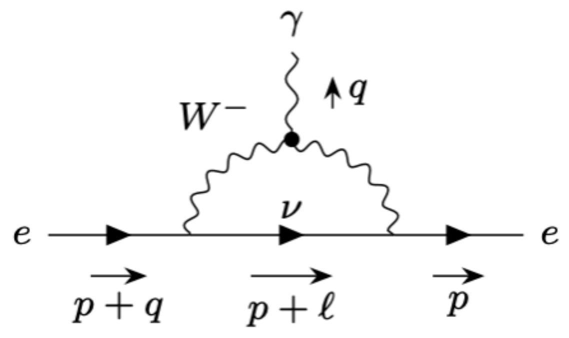
T. Abe, J. Hisano, and R. Nagai, JHEP 03 (2018) 175

eEDM



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Back Up

Electric Dipole Moment(EDM)

- EDM has many observables (nucleon, pala-magnetic atom, etc)

- The most recently results

Neutron EDM (2018)

$$|d_n| < 1.8 \times 10^{-26} e\mathrm{cm}$$ C. Abel, *et al.*, Phys. Rev. Lett. 124 (2020) 081803

Electron EDM (2023)

$$|d_e| < 4.1 \times 10^{-30} e\mathrm{cm}$$ T. S. Roussy et al. Science 381 (2023)

Mercury EDM (2023)

$$|d_{\mathrm{Hg}}| < 7.4 \times 10^{-30} e\mathrm{cm}$$ B. Graner et al., Phys. Rev. Lett. **116** (2016) 161601

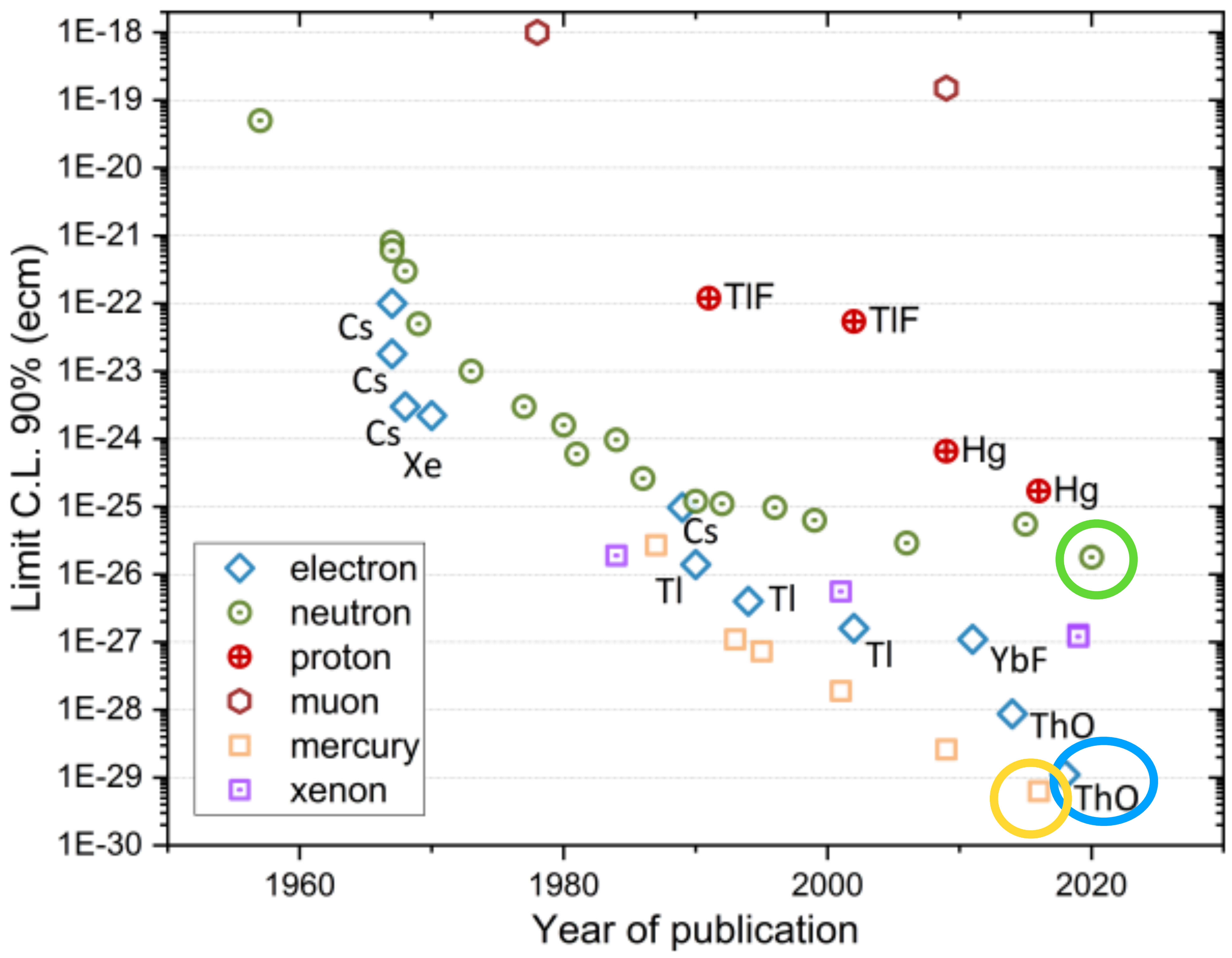


Figure 2. Plot of the history of upper EDM limits (CL 90%) as function of the year of publication.

Future Experiment in eEDM

- ACME III

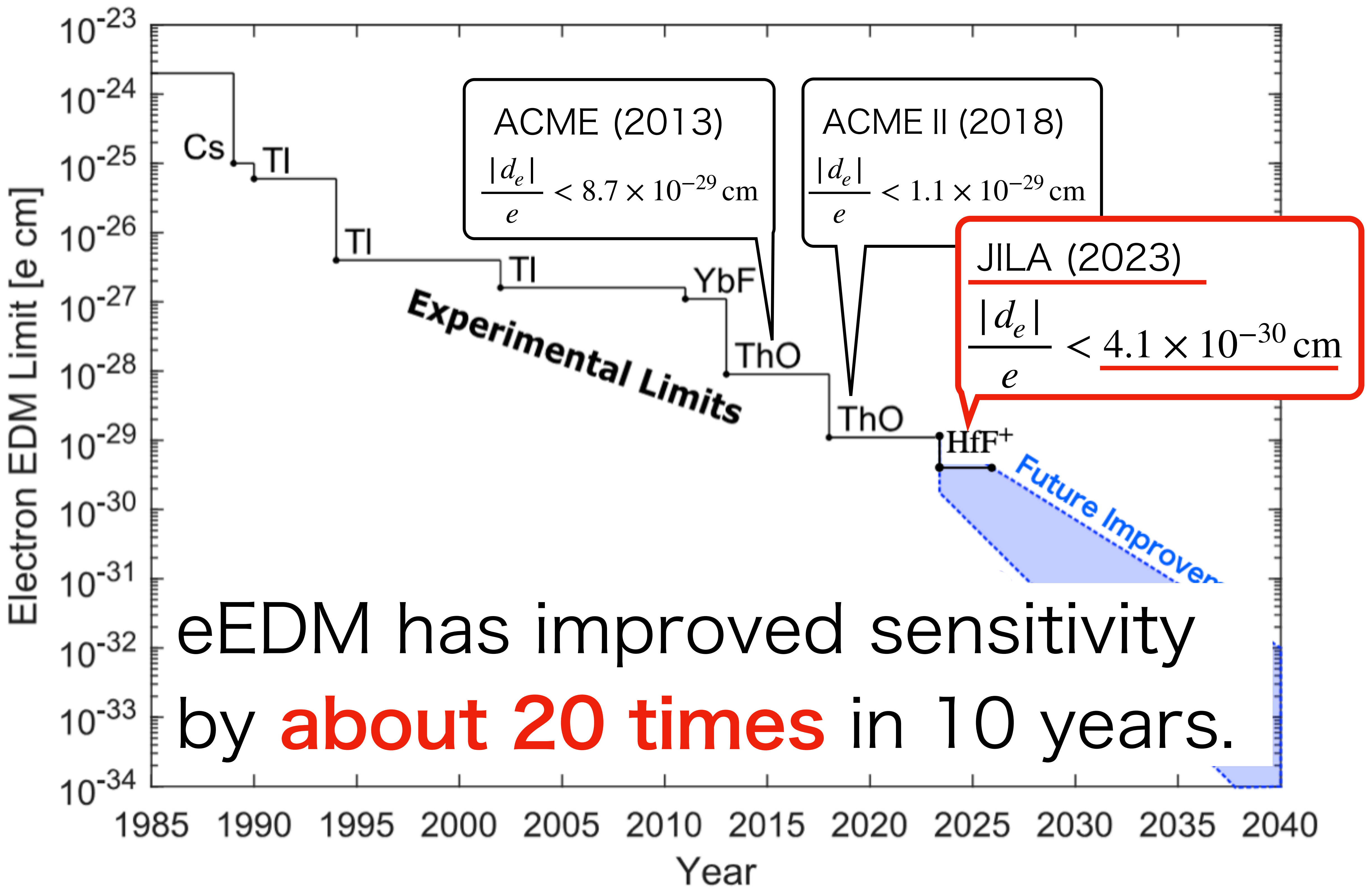
  The aim is to improve sensitivity by a factor of 30 over previous(ACME II) results.

  A.Hiramoto et al., Nucl. Instrum. Meth. A 1045 (2023) 167513

- JILA

  $HfF^+ \rightarrow ThF^+$

  Aim for sensitivity improvements of more than one order of magnitude.



R. Alarcon et al. "Electric dipole moments and the search for new physics" in Snowmass,(2021),(2022)

T. S. Roussy et al. Science 381 (2023)

Equivalent EDM

- In theoretically, We have to discuss eEDM including **nuclei effect**
- **Equivalent EDM** is defined as the eEDM including nuclei effect
  - Effective Lagrangian

$$\mathscr{L} = -i\frac{d_e}{2}\bar{e}\sigma_{\mu\nu}\gamma_5 e F^{\mu\nu} + \frac{G_F}{\sqrt{2}} C_S \bar{e} i \gamma_5 e \bar{N} N \qquad N = \binom{p}{n}$$

M. Pospelov and A. Ritz, Annals Phys. 318 (2005) 119 [hep-ph/0504231]

$$d_e^{\mathrm{equiv}} = d_e + C_S \times X(\text{molcule depend})\ \mathrm{e\ cm}$$

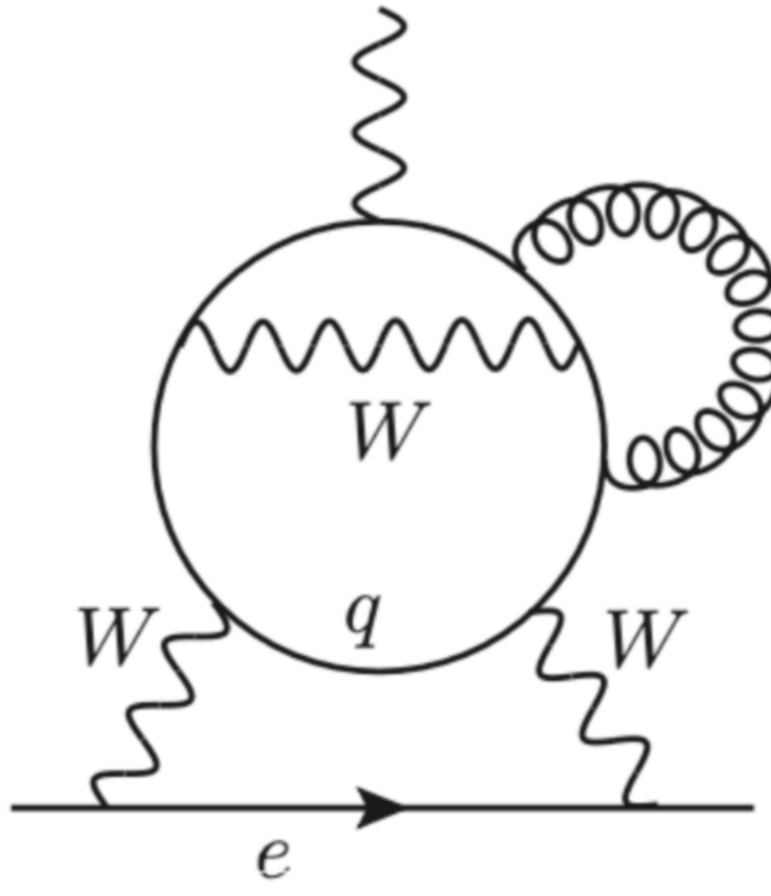
e.g.) $X = 0.9 \times 10^{-20}$ ($\mathrm{HfF^+}$)

$X = 1.5 \times 10^{-20}$ (ThO)

Equivalent EDM induced by SM

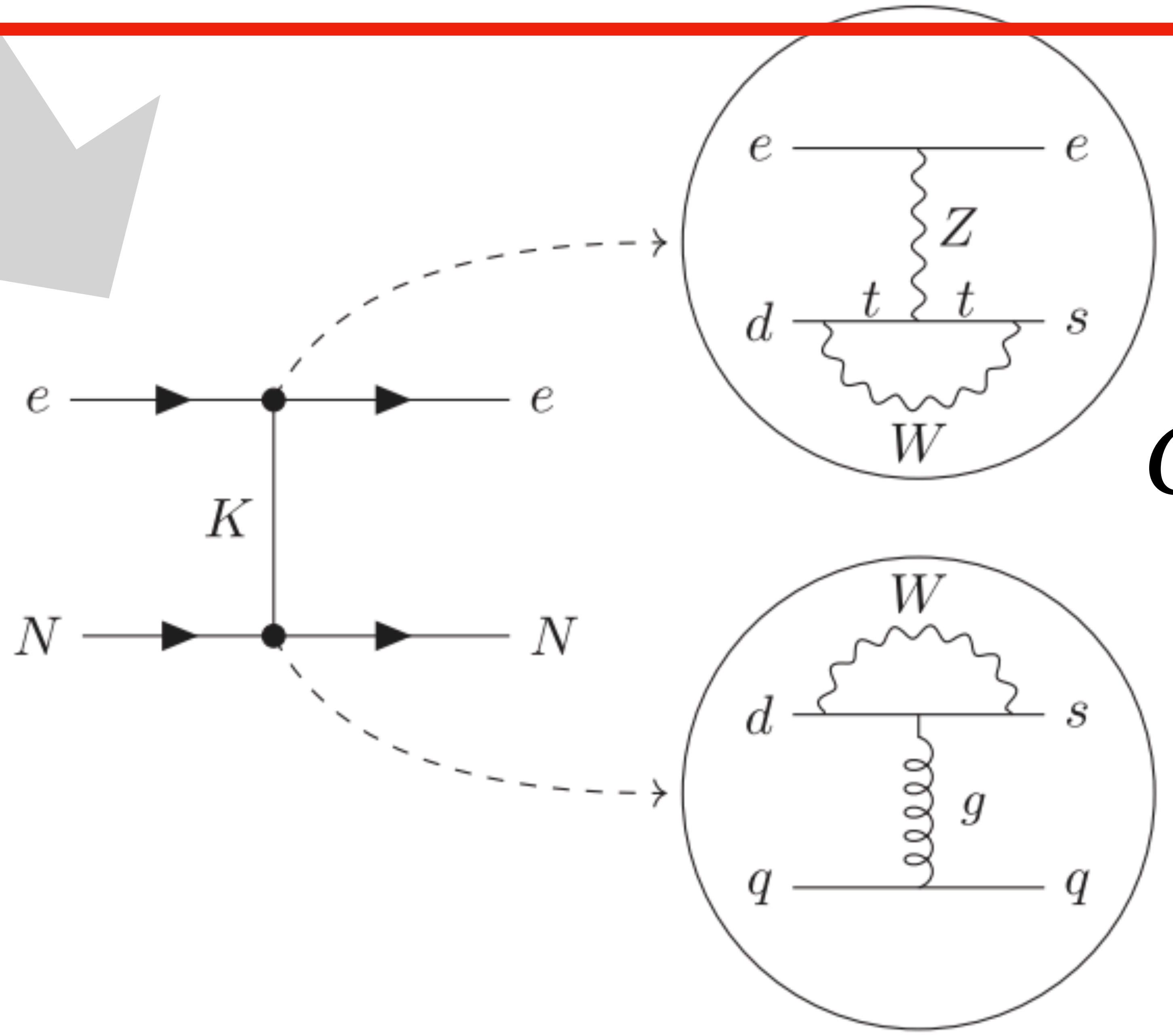
- Equivalent EDM predicted by Standard Model
  - In ThO case

$$d_e^{\text{equiv}} = d_e + \boxed{C_S \times 1.5 \times 10^{-20} e\text{ cm} \simeq 1.0 \times 10^{-35} e\text{cm}}$$



$$d_e \sim \mathcal{O}(10^{-44}) e\text{cm}$$

M. Pospelov, A. Ritz, Phys.Rev.D 89 (2014) 5, 056006



$$C_S \simeq 6.9 \times 10^{-16}$$

Y. Ema, T. Gao, M. Pospelov,Phys.Rev.Lett. 129 (2022) 23, 231801

$$d_e^{equiv} \simeq 1.0 \times 10^{-35} e\text{cm}$$

In the SM, C_s contribution is dominant!

Minimal Dark Matter Model

M. Cirelli, A. Strumia, and M. Tamburini, Nucl. Phys. B 787 (2007) 152–175.

- **Representation**

Quantum numbers $SU(2)_L$	$U(1)_Y$	Spin	DM can decay into	DD bound?	Stable?
2	1/2	S	EL	×	×
2	1/2	F	EH	×	×
3	0	S	HH^*	√	×
3	0	F	LH	√	×
3	1	S	HH, LL	×	×
3	1	F	LH	×	×
4	1/2	S	HHH^*	×	×
4	1/2	F	(LHH^*)	×	×
4	3/2	S	HHH	×	×
4	3/2	F	(LHH)	×	×
5	0	S	(HHH^*H^*)	√	×
5	0	F	–	√	√
5	1	S	$(HH^*H^*H^*)$	×	×
5	1	F	–	×	√
5	2	S	$(H^*H^*H^*H^*)$	×	×
5	2	F	–	×	√
6	1/2, 3/2, 5/2	S	–	×	√
7	0	S	–	√	√
8	1/2, 3/2 ...	S	–	×	√

- **Thermal relic abundance of the DM**

$$\Omega_{\rm DM}h^2 \leq 0.11 \Rightarrow m_{\rm DM} \leq 10\text{TeV} \quad m_{\rm DM} \leq 25\text{TeV}$$

- **Collider search**

Mass splitting

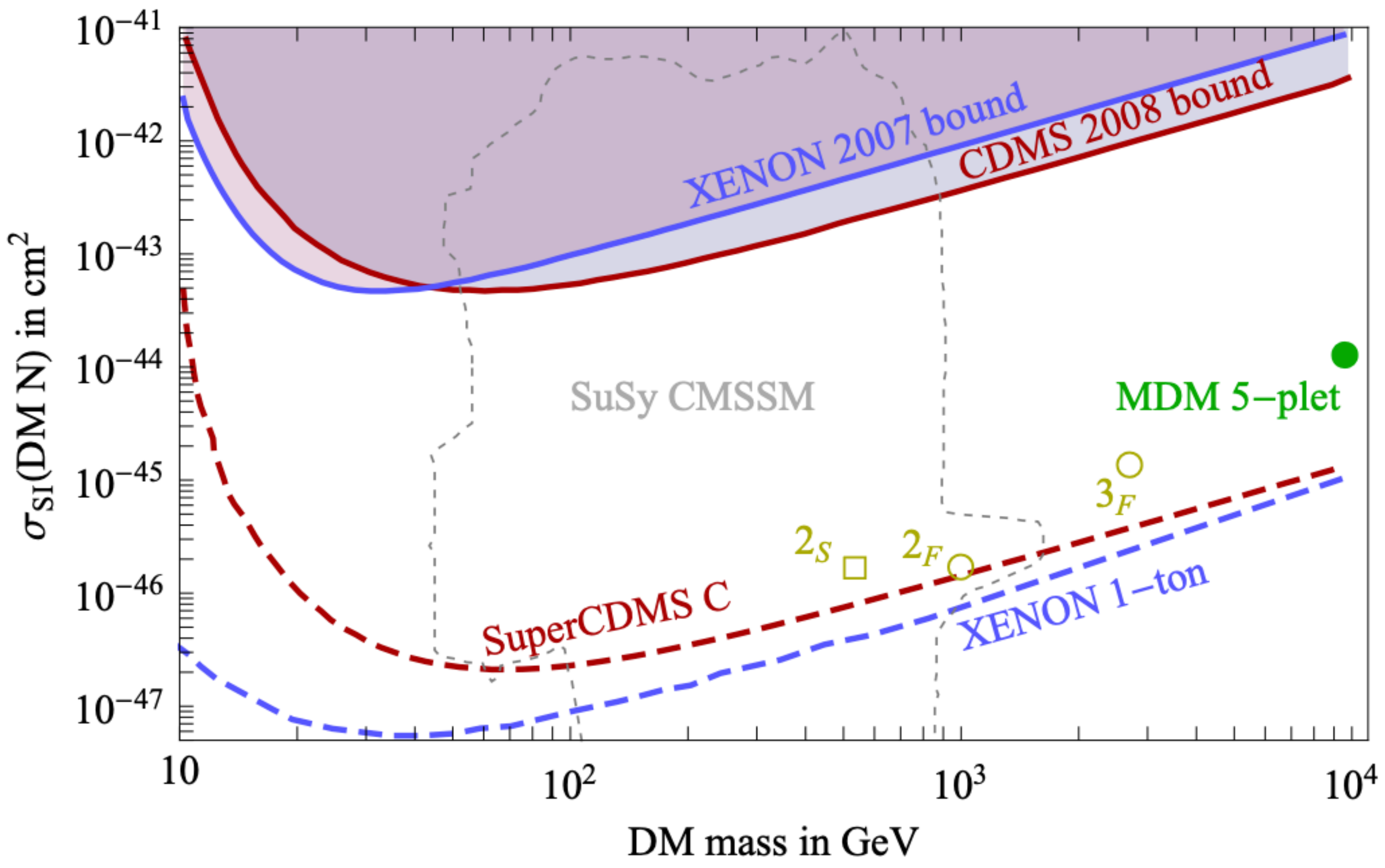
$$M_{\chi^\pm} - M_\chi = \Delta M = O(100)\ \text{MeV}$$

The constraints to $SU(2)_L$ triplets by LHC

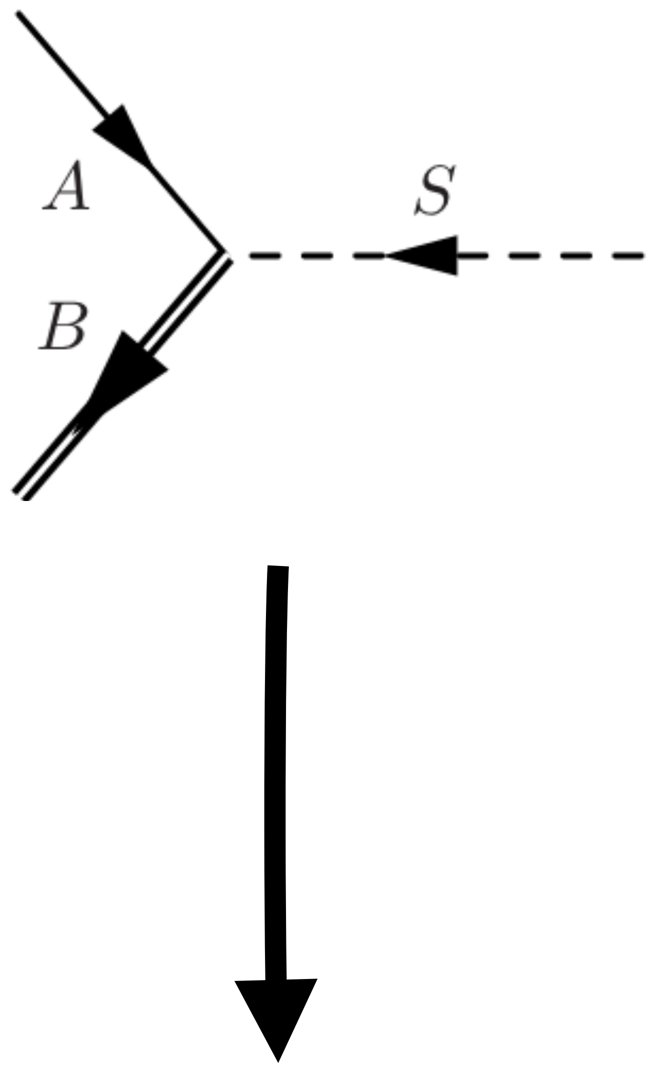
$$m_{r=3} \geq 660\text{GeV}$$

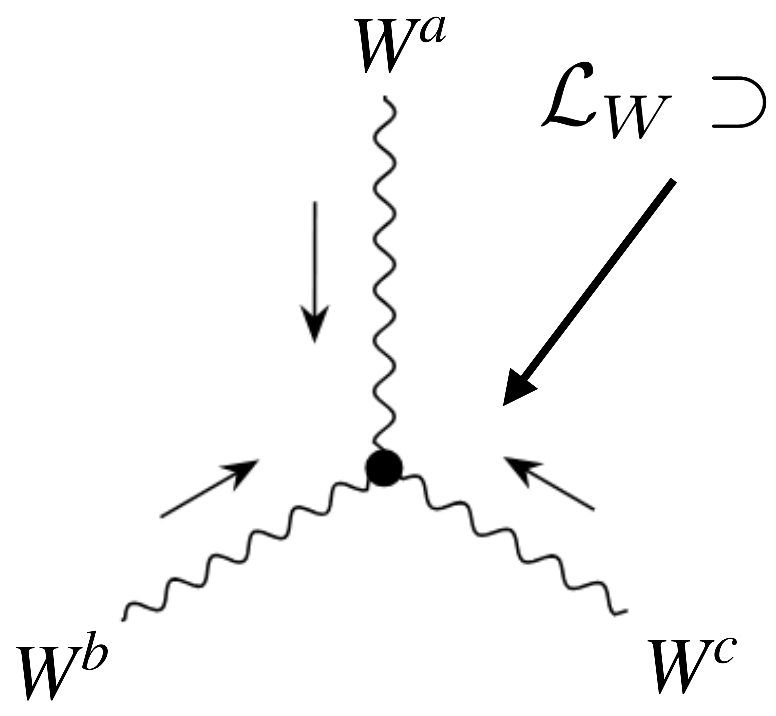
ATLAS,Aad et al, 2201.02472,CERN-EP-2021-209,Eur.Phys.J.C,(2022)

- **Direct Detection**



How to derive EW-Weinberg Operator

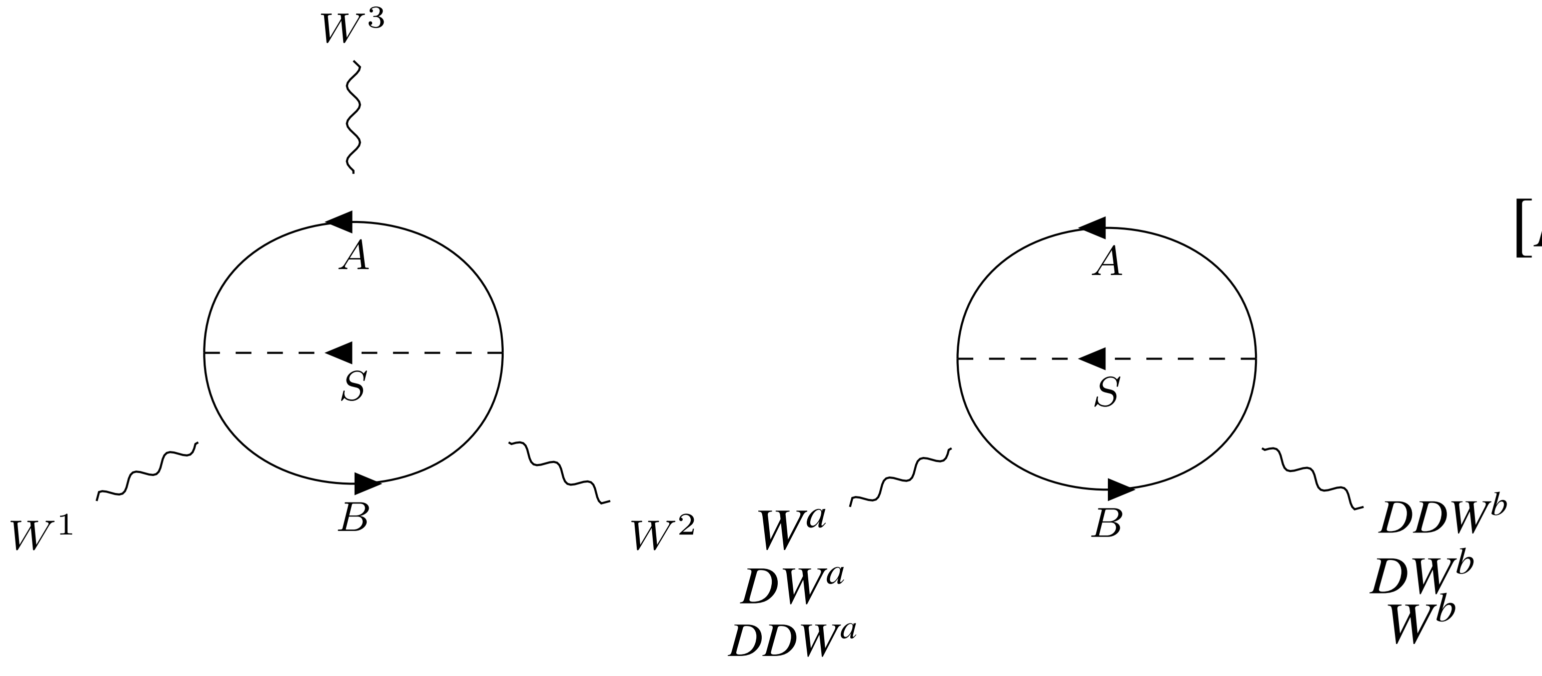




$$\mathcal{L}_W \supset -\frac{g^3}{3} C_W f^{abc} W^a_{\mu\nu} W^{b\nu}{}_{\rho} \widetilde{W}^{c\rho\mu}$$

- evaluate by Fock-Schwinger gauge
  - gauge fixing : $(x - x_0)^\mu A^a_\mu(x) = 0$
  - For gauge-invariant quantities, translational symmetry is restored

$$\tilde{A}^a_\mu(k) = \frac{1}{2} W^a_{\rho\mu} \frac{1}{i} \frac{\partial}{\partial k_\rho} \delta^4(k) + \frac{1}{3} (D_\alpha W^a_{\rho\mu}) \frac{1}{i} \frac{\partial}{\partial k_\rho} \frac{1}{i} \frac{\partial}{\partial k_\alpha} \delta^4(k)$$
$$+ \frac{1}{8} (D_\beta D_\alpha W^a_{\rho\mu}) \frac{1}{i} \frac{\partial}{\partial k_\rho} \frac{1}{i} \frac{\partial}{\partial k_\alpha} \frac{1}{i} \frac{\partial}{\partial k_\beta} \cdots$$



$$[D_\alpha, D_\beta]\, W^a_{\beta\nu} = ig[W^a_{\alpha\beta}, W^a_{\beta\nu}]$$

$$DWD\tilde{W} = 27gWW\tilde{W}$$
$$WDD\tilde{W} = -18gWW\tilde{W}$$

T. Abe, J. Hisano, and R. Nagai, JHEP 03 (2018) 175

(A,B,S) Group factors

(A,B,S)	$\psi_A \psi_B S$	$X_{\bar{B}AS}$	$XT_AT_AX^\dagger$	$XT_AX^\dagger T_B$	$XX^\dagger T_BT_B$
$(2,2,1)$	$(\psi_A)_a(\psi_B)_bS$	δ^{ab}	$1/2$	$1/2$	$1/2$
$(2,1,2)$	$(\psi_A)_a\psi_BS_i$	δ^{ai}	$1/2$	0	0
$(3,3,1)$	$(\psi_A)_\alpha(\psi_B)_\beta S$	$\delta^{\alpha\beta}$	2	2	2
$(3,1,3)$	$(\psi_A)_\alpha\psi_BS_\gamma$	$\delta^{\alpha\gamma}$	2	0	0
$(3,2,2)$	$(\psi_A)_\alpha(\psi_B)_bS_i$	$(T^\alpha)^{bi}$	1	$1/2$	$3/8$
$(2,2,3)$	$(\psi_A)_a(\psi_B)_bS_\gamma$	$(T^\gamma)^{ba}$	$3/8$	$-1/8$	$3/8$
$(3,3,3)$	$(\psi_A)_\alpha(\psi_B)_\beta S_\gamma$	$\epsilon^{\beta\alpha\gamma}$	4	2	4
$(r,r,1)$	$(\psi_A)_{a_r}(\psi_B)_{b_r}S$	$\delta^{a_rb_r}$	$r(r^2-1)/12$	$r(r^2-1)/12$	$r(r^2-1)/12$
$(r,1,r)$	$(\psi_A)_{a_r}\psi_BS_{i_r}$	$\delta^{a_ri_r}$	$r(r^2-1)/12$	0	0

Table 2: Group factors in the $\mathrm{SU}(2)_L$ representations. The last two rows show the simple cases of the r-dimensional representations.

How to derive EW-Weinberg Operator

- The Wilson coefficient for EW-Weinberg operator

$$C_W = \frac{6}{(4\pi)^4}\,\mathrm{Im}(sa^*) m_A m_B$$
$$\times\left\{\boxed{\left(XT_AT_AX^\dagger\right)} g_1(m_A^2, m_B^2, m_S^2) + \boxed{\left(XX^\dagger T_BT_B\right)} g_1(m_B^2, m_A^2, m_S^2)\right.$$
$$\left. + \boxed{\left(XT_AX^\dagger T_B\right)}\left[g_2(m_A^2, m_B^2, m_S^2) + g_2(m_B^2, m_A^2, m_S^2)\right]\right\}.$$

$\boxed{}$: group factor

loop functions

$$g_1(x_1, x_2, x_3) = \left(2\bar{I}_{(4;1)} + 4x_1\bar{I}_{(5;1)}\right)(x_1; x_2; x_3)\,,$$
$$g_2(x_1, x_2, x_3) = \left(\bar{I}_{(3;2)} + x_1\bar{I}_{(4;2)}\right)(x_1; x_2; x_3)\,,$$

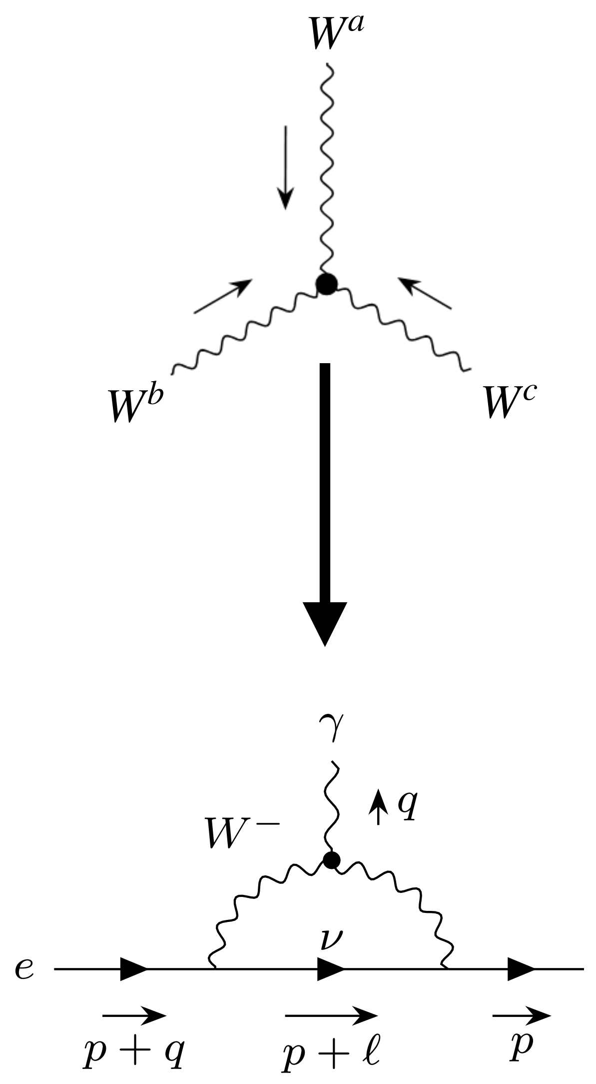
$$\bar{I}_{(n;m)}(x_1; x_2; x_3) = \frac{1}{(n-1)!(m-1)!}\frac{d^{n-1}}{dx_1^{n-1}}\frac{d^{m-1}}{dx_2^{m-1}}\bar{I}(x_1; x_2; x_3)\,,$$

Master Integral

$$I(x_1; x_2; x_3) = \int d^dk \int d^dq \frac{1}{(k^2 - x_1)(q^2 - x_2)[(k-q)^2 - x_3]}$$
$$= \bar{I}(x_1; x_2; x_3) + I_{div}(x_1; x_2; x_3)$$

S.P.Martin,Phys.Rev.D.65 (2002) 116003

EW-Weinberg Operator → eEDM



- Evaluate 1-loop diagram in DR

Result1

We separately evaluate the 4- & (d-4)-dim contributions.

	4-dim.	$(d-4)$-dim.
$-ieg^2\frac{2}{3}W^-W^+\widetilde{F}$	$\bar{O}_1$: $(1/18)C_W$	$\hat{O}_1$: $(1/9)C_{W^{(d-4)}}$
$-ieg^2\frac{2}{3}F(W^-\widetilde{W}^+ - W^+\widetilde{W}^-)$	$\bar{O}_2$: $(1/18+1/18)C_W$	$\boxed{\hat{O}_2 : \quad 0}$
total	$(1/6)C_W$	$(1/9)C_{W^{(d-4)}}$

$$\hat{O}_2 \neq 2\hat{O}_1$$

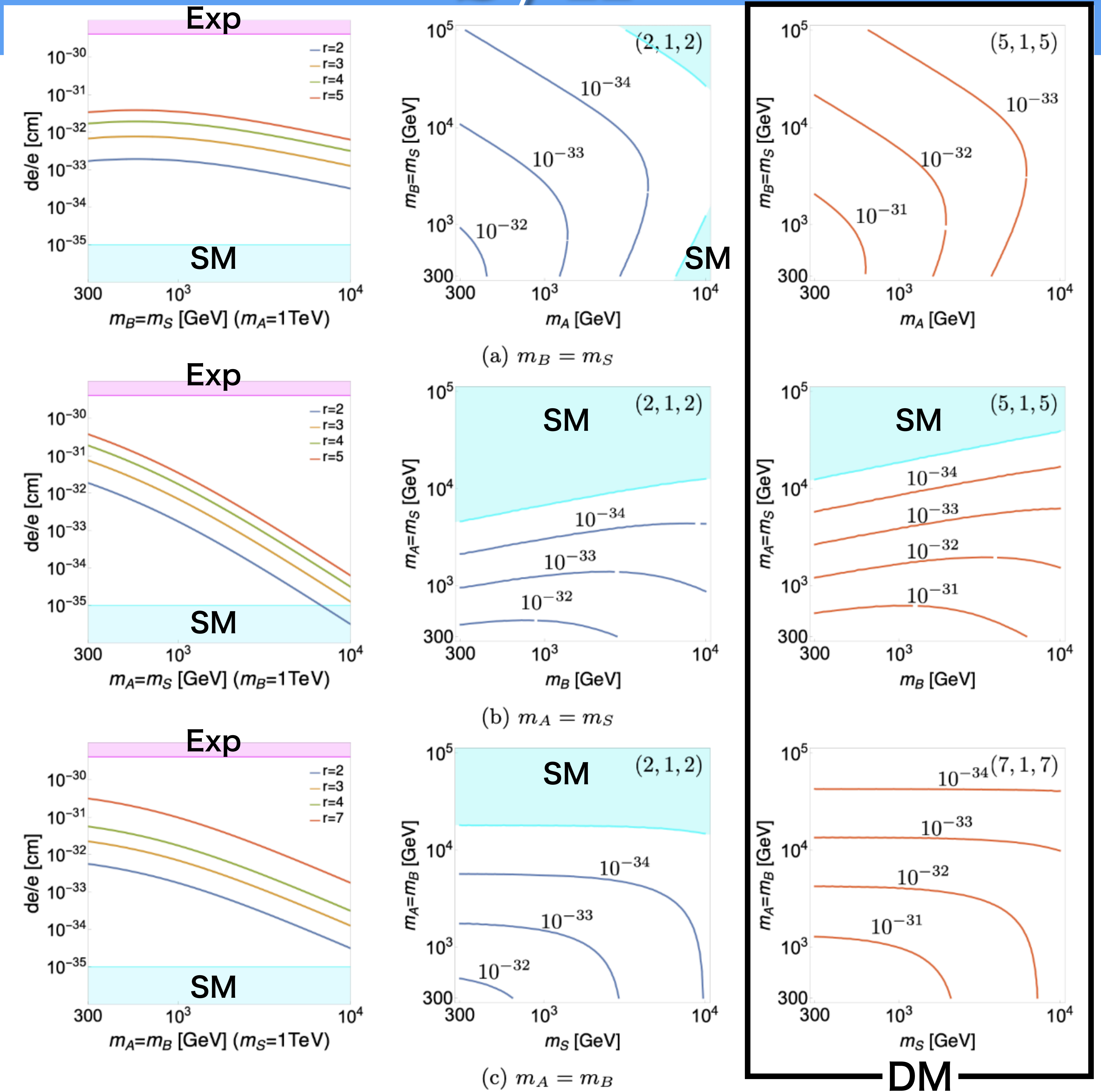
$$F_{\bar{\mu}\hat{\nu}}\left(W^{-\hat{\nu}}{}_{\bar{\lambda}}\widetilde{W}^{+\lambda\mu} - W^{+\hat{\nu}}{}_{\bar{\lambda}}\widetilde{W}^{-\lambda\mu}\right)$$

⇒ $\hat{O}_2$ do not contribute because $F_{\bar{\mu}\hat{\nu}}$ has (d-4)-dimensional momentum or polarization.

$S \neq H$

$$(A, B, S) = (r, 1, r)$$

$$\mathrm{Im}(sa^*) = 0.25$$



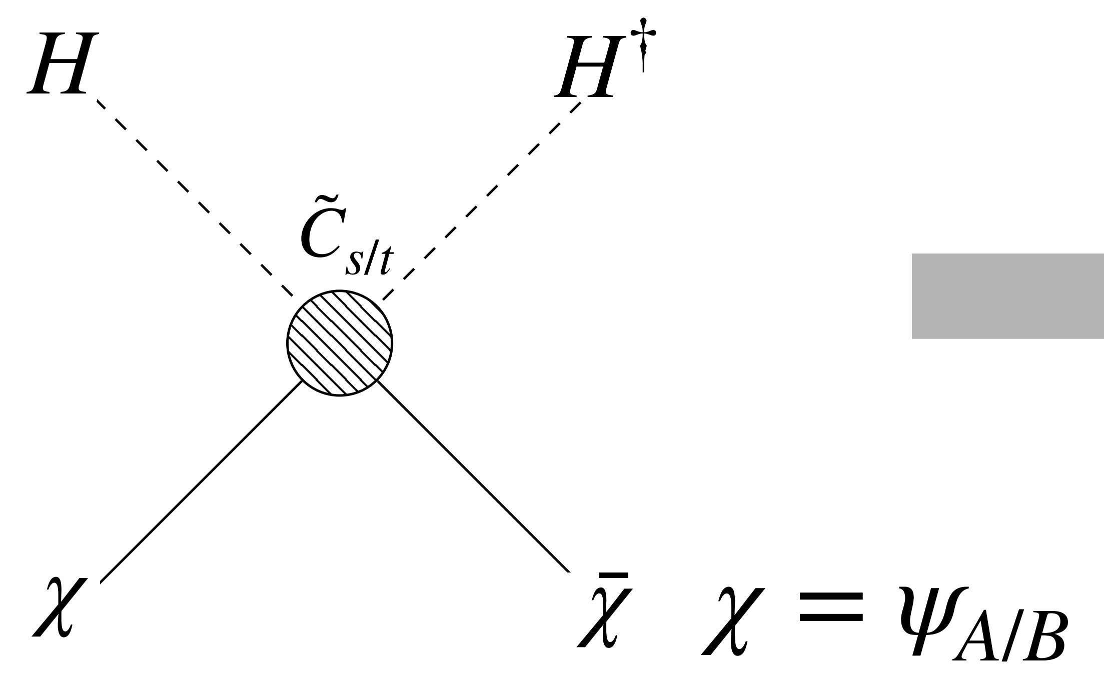
EW-Weinberg Operator

$(A, B, S) = (r, 1, r)$

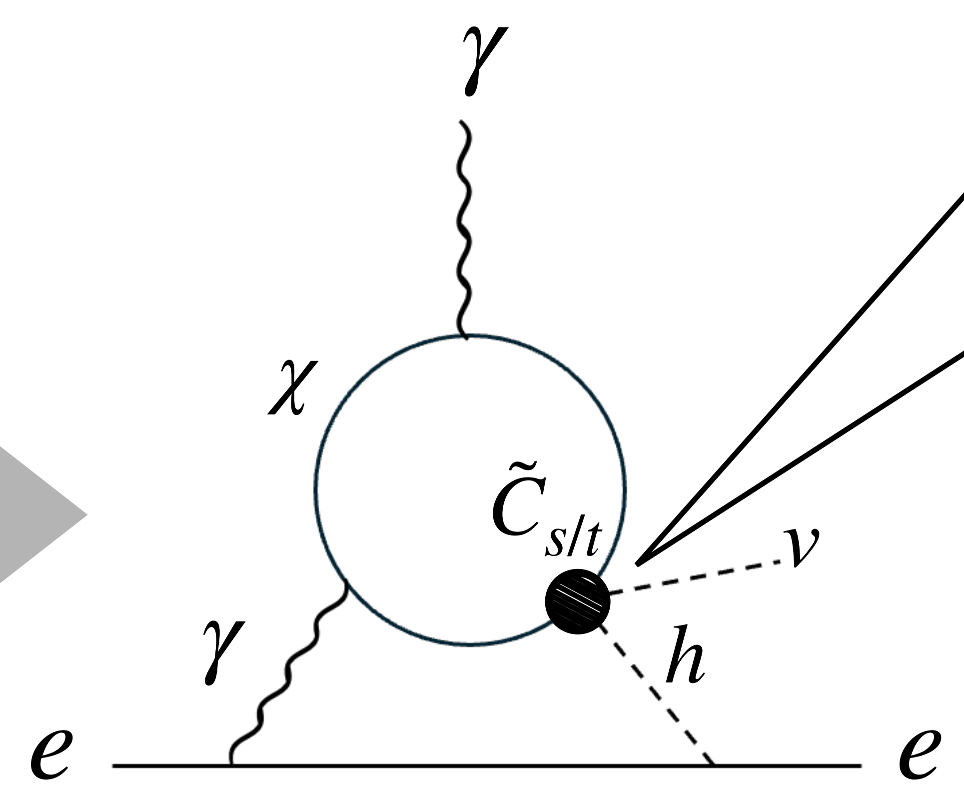
$$
\begin{aligned}
C_W^{(r,1,r)} &= \frac{1}{2(4\pi)^2}\,\mathrm{Im}(sa^*)m_A m_B r(r^2-1)g_1(m_A, m_B, m_S) \\
&\simeq \begin{cases}
\frac{1}{36(4\pi)^2}\,\mathrm{Im}(sa^*)r(r^2-1)\frac{1}{m_A m_B} & (m_A < m_B = m_S) \\
\frac{1}{2(4\pi)^2}\,\mathrm{Im}(sa^*)r(r^2-1)\frac{m_B}{m_A^3} & (m_B < m_A = m_S) \\
\frac{1}{12(4\pi)^2}\,\mathrm{Im}(sa^*)r(r^2-1)\frac{1}{m_A m_B} & (m_S < m_A = m_B)
\end{cases}
\end{aligned}
$$

In the case of S = SM Higgs

CP violating operator



2-loop level



Barr-Zee diagram(LO)

N. Nagata and S. Shirai, JHEP 01 (2015)

3-loop level

RG effect for CP violating operator at 1-loop level

$$\bar{\gamma}_{ss}^{(r,Y_r)} = -\left[6\alpha_2\left(C_2(r)+\frac{3}{4}\right)+6\alpha_Y\left(Y_r^2+\frac{1}{4}\right)-3\lambda'-6\alpha_t\right]$$

$$\bar{\gamma}_{tt}^{(r,Y_r)} = -\left[6\alpha_2\left(C_2(r)-\frac{1}{4}\right)+6\alpha_Y\left(Y_r^2+\frac{1}{4}\right)-\lambda'-6\alpha_t\right]$$

$\alpha_2/\alpha_Y/\alpha_t/\lambda'$: SM coupling $C_2(r)$: casimir operator

Barr-Zee diagram(NLO)

W. Kuramoto, T. Kuwahara, and R. Nagai, Phys. Rev. D99 (2019) 095024

vs

EW-Weinberg

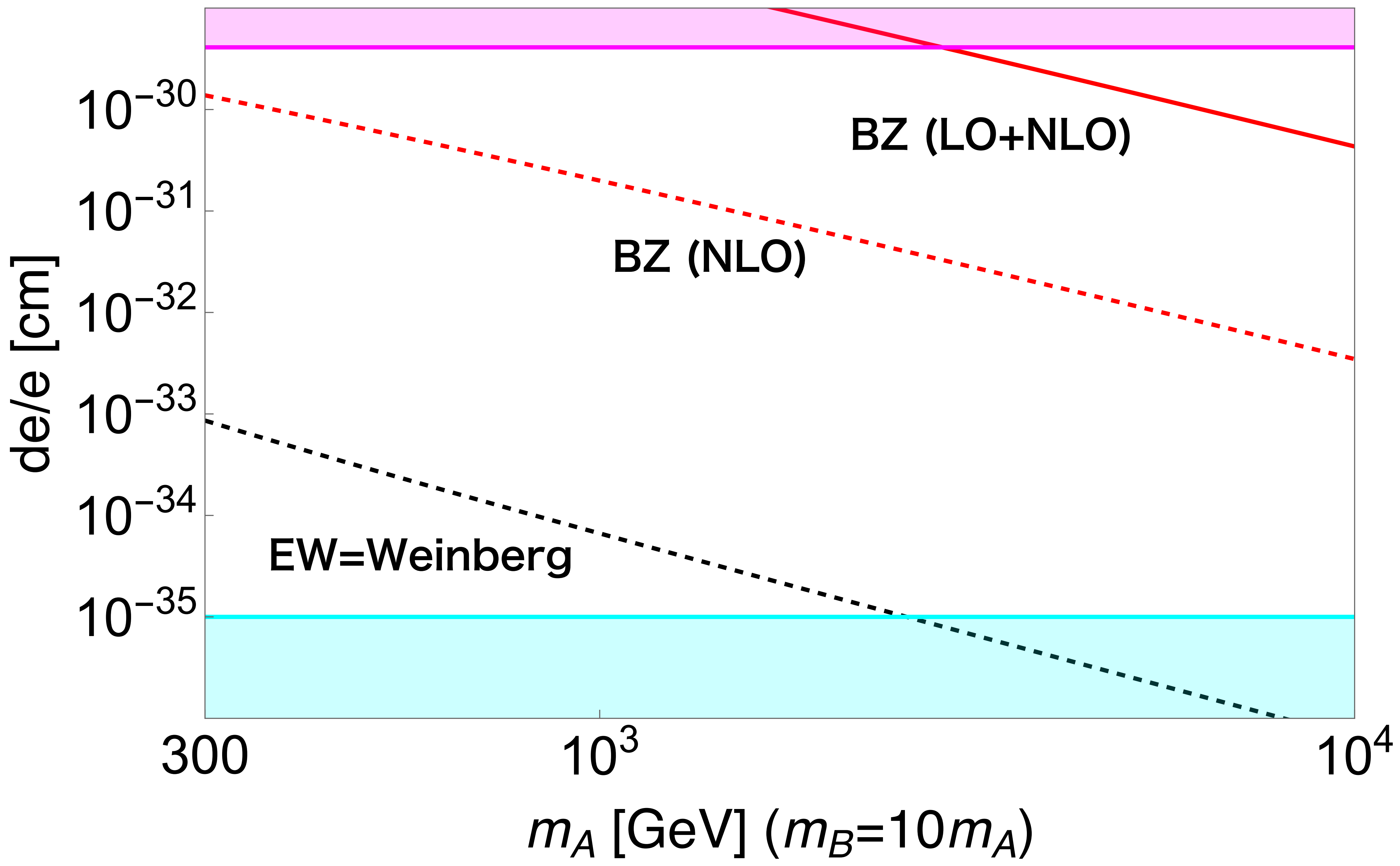
(Our Work)

In the case of S = SM Higgs

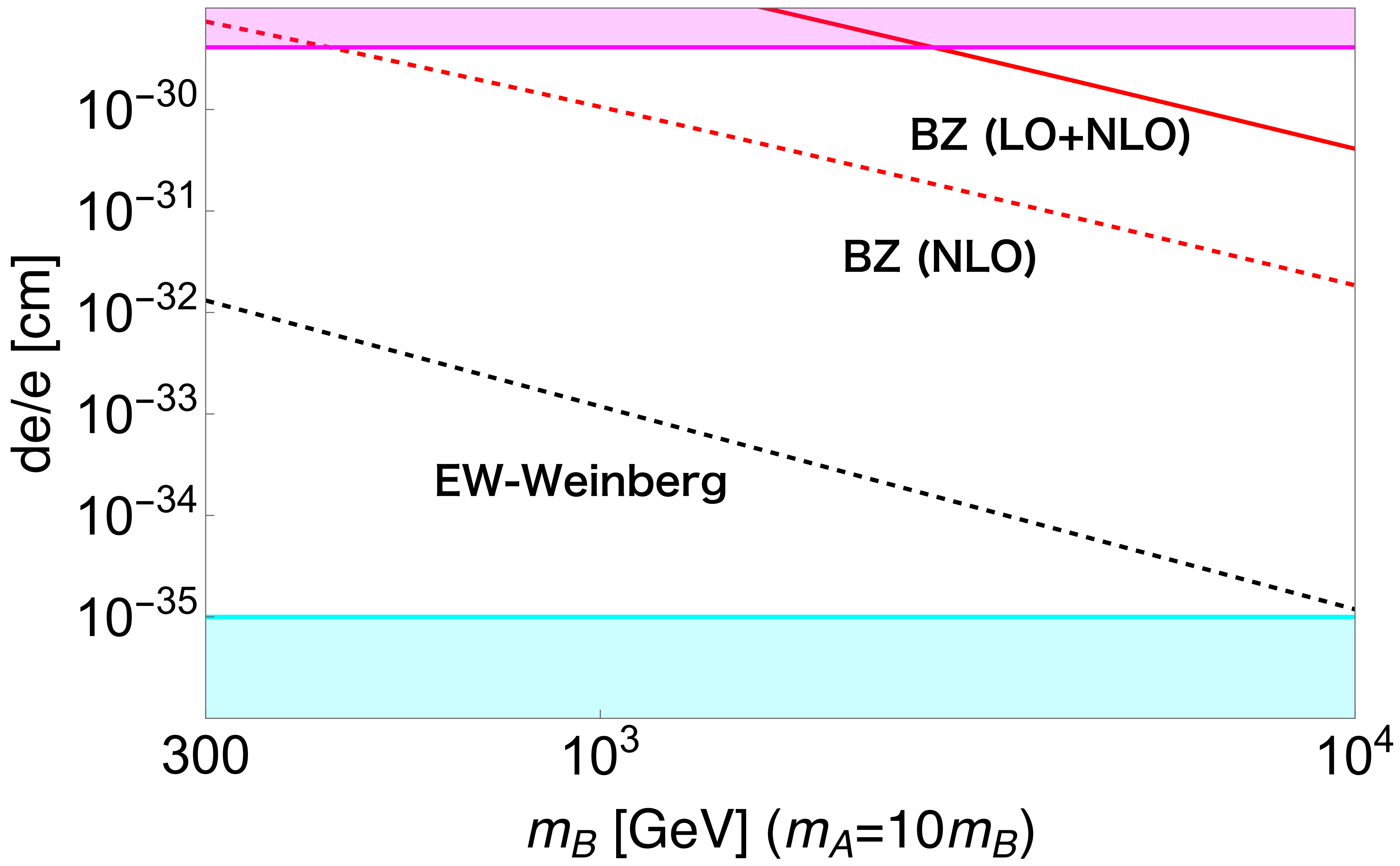
Barr-Zee diagram(NLO) vs EW-Weinberg

$(A, B, S) = (3,2,2^H)$

$(m_H \ll m_A < m_B)$



$(m_H \ll m_B < m_A)$



EW-Weinberg $\ll$ Barr-Zee diagram(NLO)

In the Yukawa interaction with Higgs, EW-Weinberg contribution can be neglected!

Barr-Zee diagram(NLO) vs Electroweak Weinberg

$$\frac{d_e^{(W)}}{\delta d_e^{(BZ)}} \simeq \begin{cases} \frac{1}{64}\frac{\alpha_2^2}{\alpha_e}\left(\bar{\gamma}_{ss}^{(3,0)} \ln\frac{m_B}{m_A}\right)^{-1} \frac{m_A^2}{m_B^2} \frac{11 + 8\ln\frac{m_A^2}{m_B^2}}{2 + \ln\frac{m_A^2}{m_H^2}} & (m_H \ll m_A < m_B) \\ \frac{5}{16}\frac{\alpha_2^2}{\alpha_e}\left[\left(3\bar{\gamma}_{ss}^{(2,1/2)} + \bar{\gamma}_{tt}^{(2,1/2)}\right) \ln\frac{m_A}{m_B}\right]^{-1} \frac{1}{2 + \ln\frac{m_B^2}{m_H^2}} & (m_H \ll m_B < m_A) \end{cases}$$

$$(\gamma_{ss}^{(3,0)})^{-1} = 284.4 \qquad \left(3\gamma_{ss}^{(2,1/2)} + \gamma_{tt}^{(2,1/2)}\right)^{-1} = 13.3$$