Searching for ways to probe

pseudo-Nambu-Goldstone-Boson Dark Matter

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Today's goal

1. Explain why we are interested in pNGB DM
2. Provide brief background & motivation
3. Present our current status.

Introduction

The why of Dark Matter

Evidence:

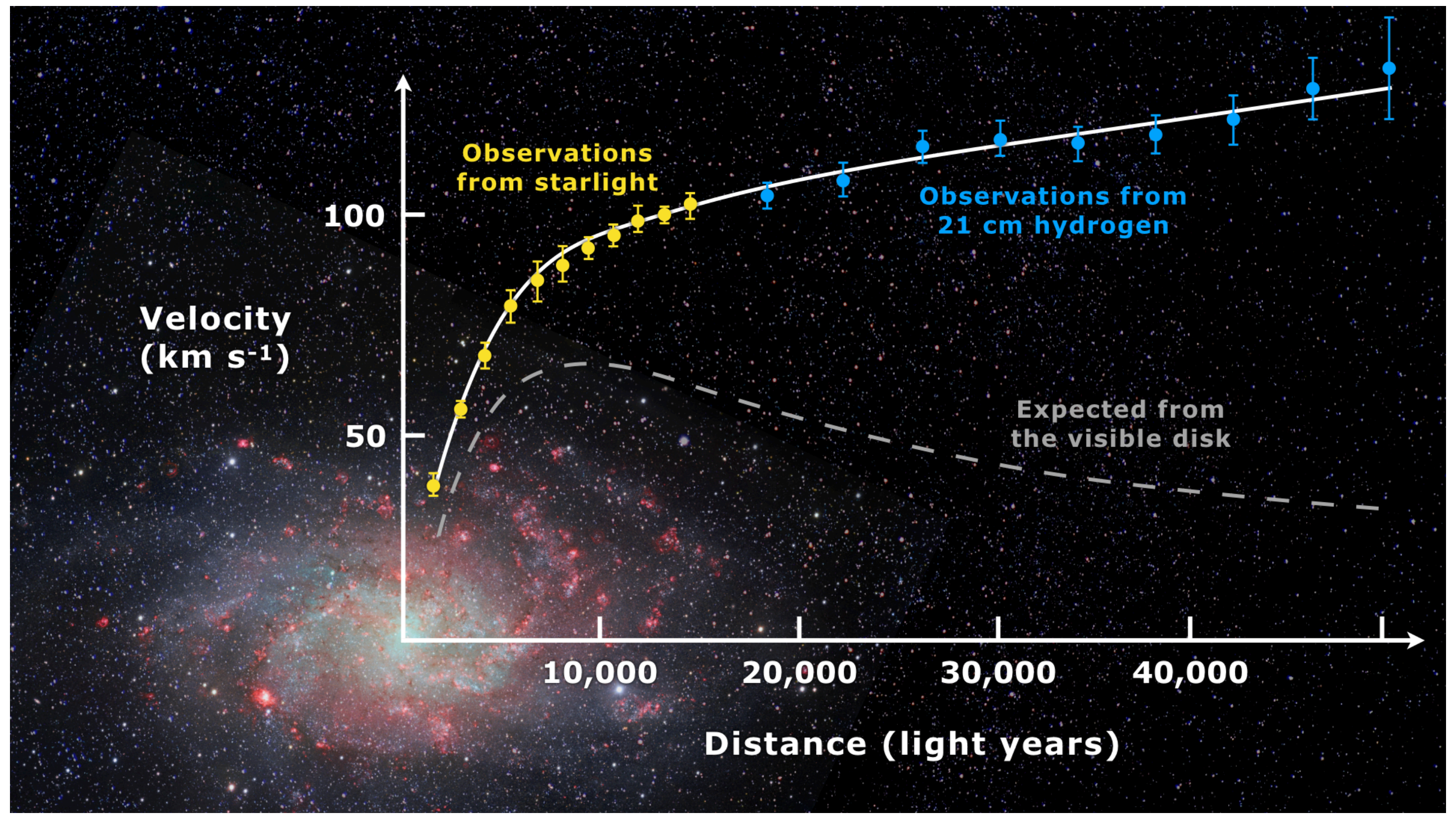
- Cosmic microwave background
- Galactic rotation curve
- ……

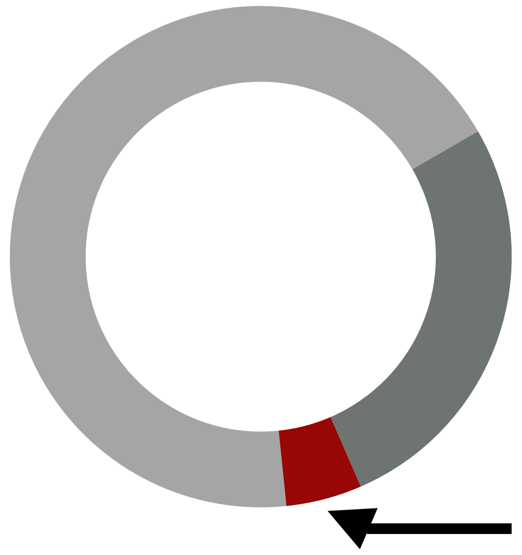
Nature:

- Electrically neutral
- Stable
- Non-relativistic (cold).

Candidates:

- Primordial Black Hole
- Weakly Interacting Massive Particle (WIMP)
- Strongly Interacting Massive Particle (SIMP)
- ……





Galactic rotation curve deviation.

WIMPs role in DM search

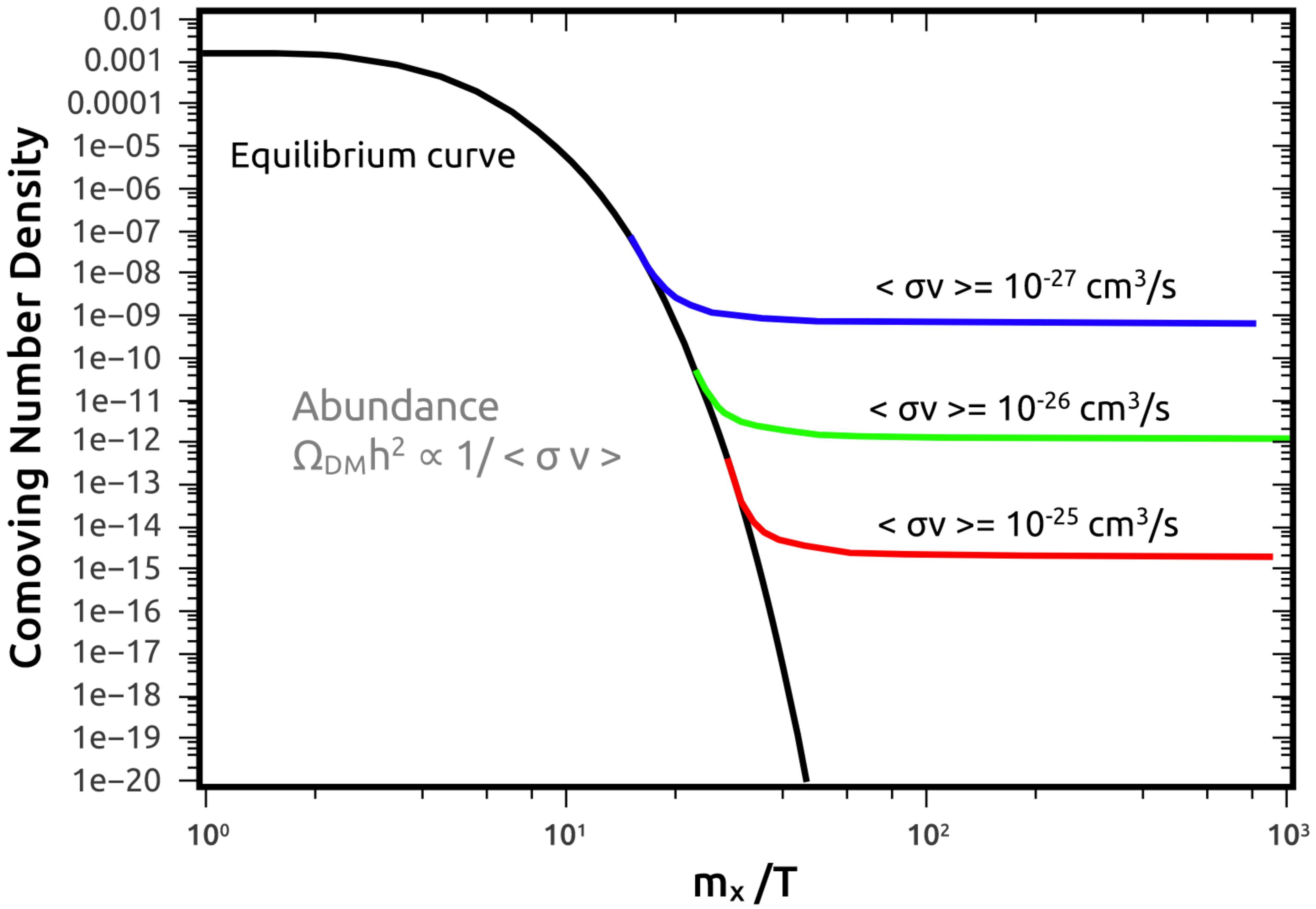
- DM is produced thermally in chemical equilibrium.

$$\frac{dn_a}{dt} + 3H(T)n_a = -\langle \sigma_{\text{ann.}} v \rangle \left(n_a^2 - n_{a,eq}^2 \right)$$

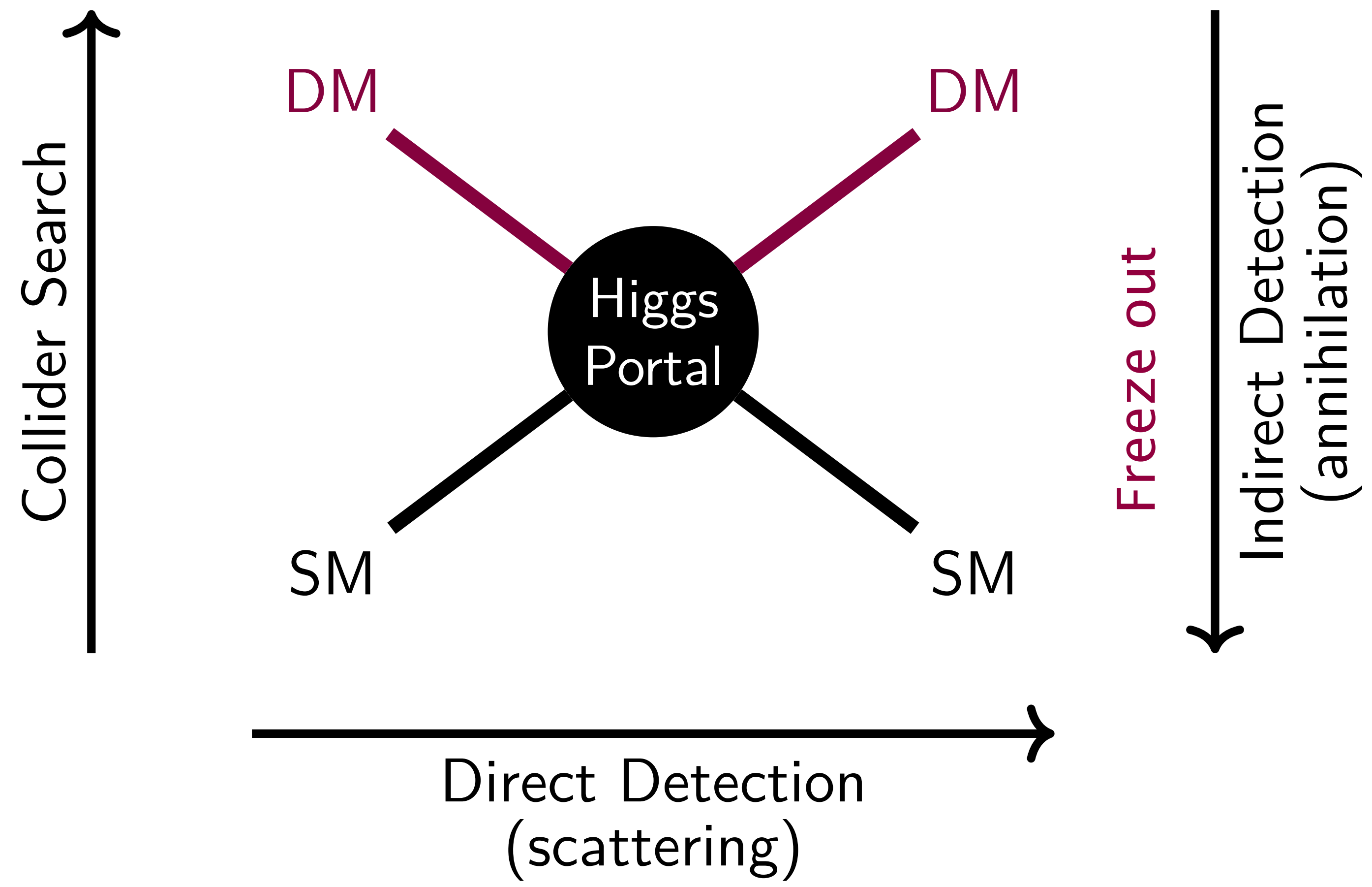
$$\langle \sigma_{\text{ann.}} v \rangle \propto \frac{g^4}{m_{\text{DM}}^2}$$

- Freeze out abundance attained i.e.,

$$\Omega h^2 \simeq \frac{0.1\,\text{pb}}{\langle \sigma_{\text{ann.}} v \rangle} \simeq 0.12 \pm 0.001$$



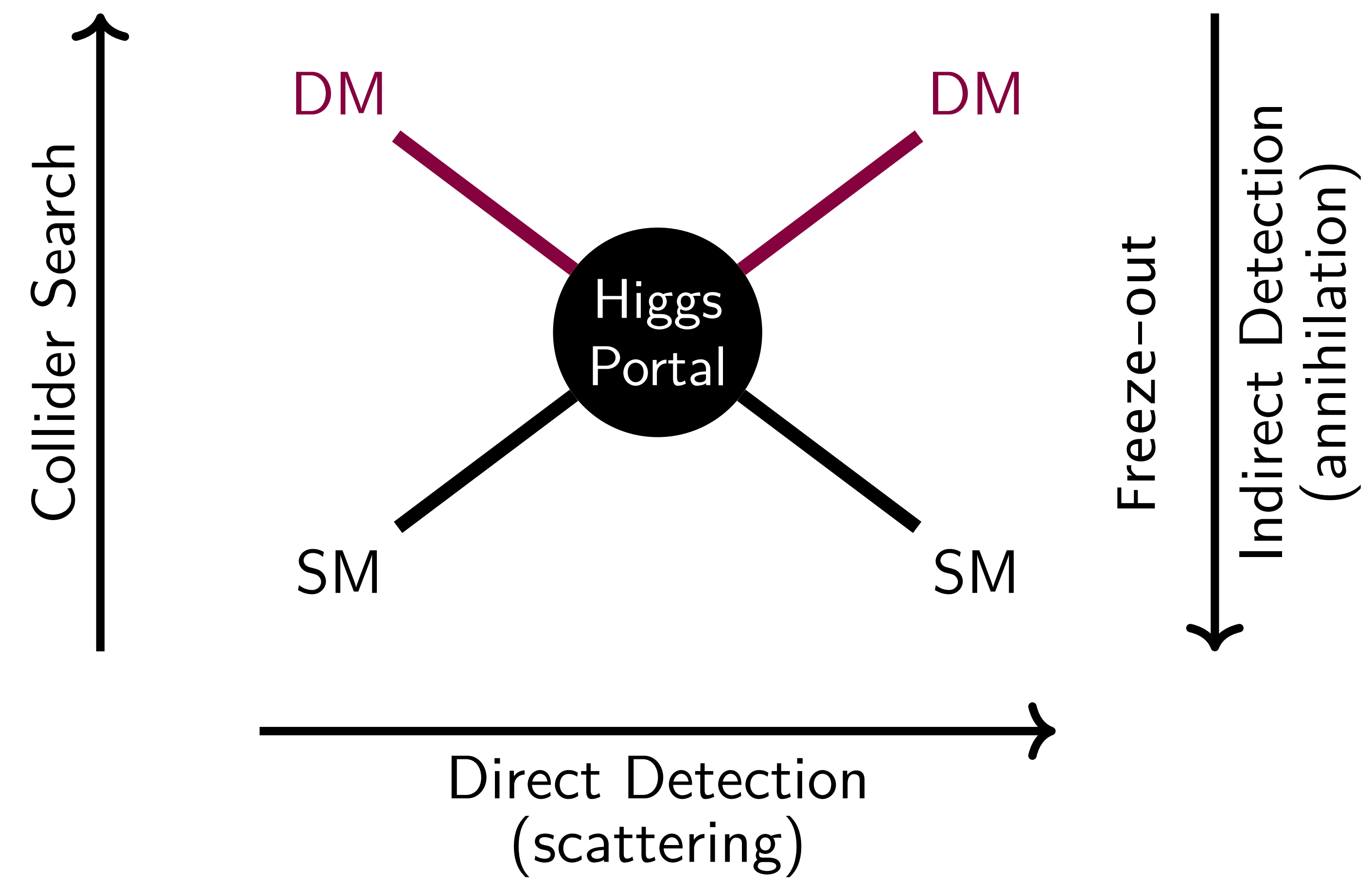
Number density evolution. [Arcadi et al. 2018]

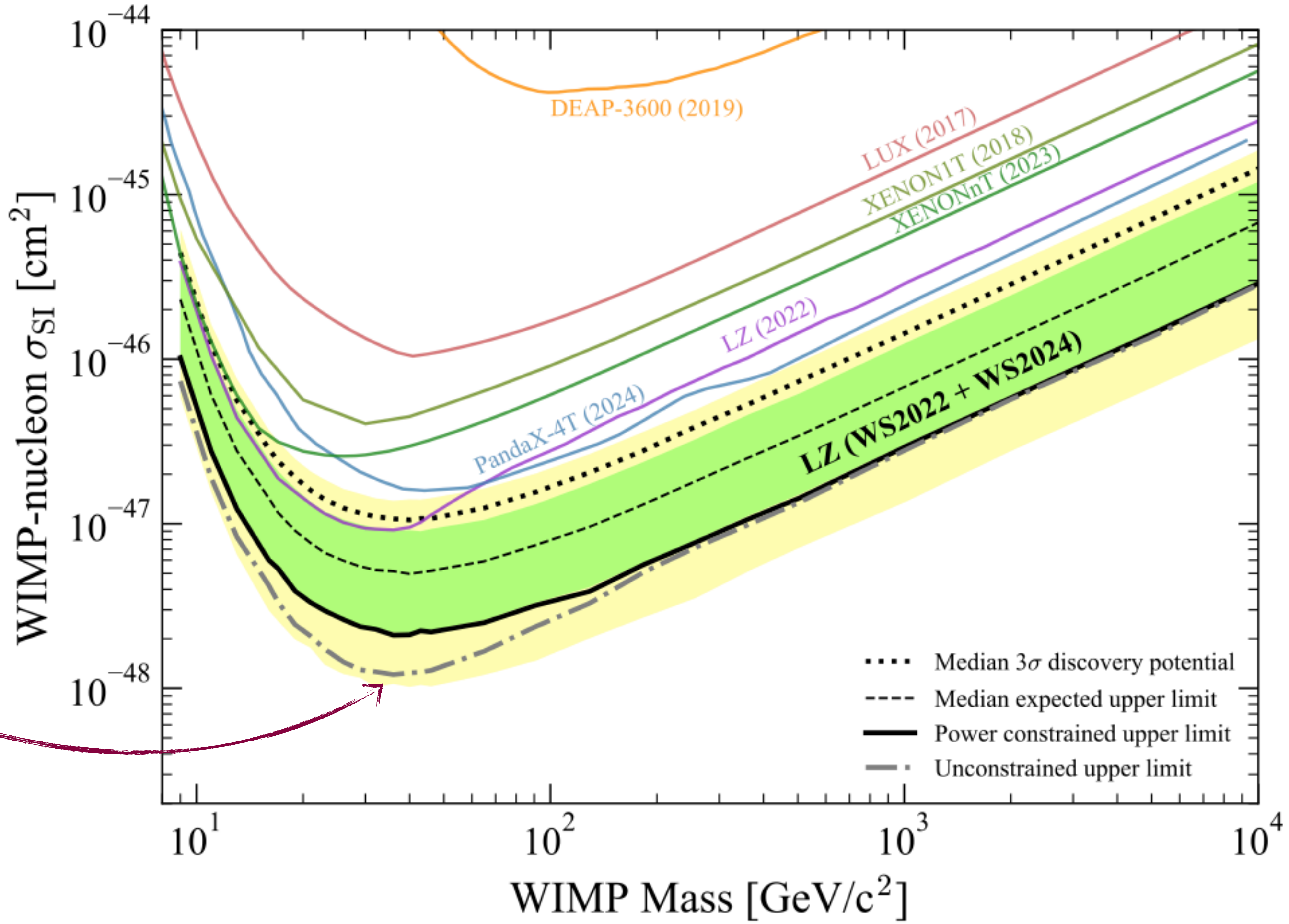


"Problem" with WIMPs!

Cross section bounds are getting stringent

$\sigma_{\rm SI}^{\rm DD} \approx 10^{-48}\ {\rm cm}^2$ $\quad m_{\rm DM} \sim 40\ {\rm GeV}$

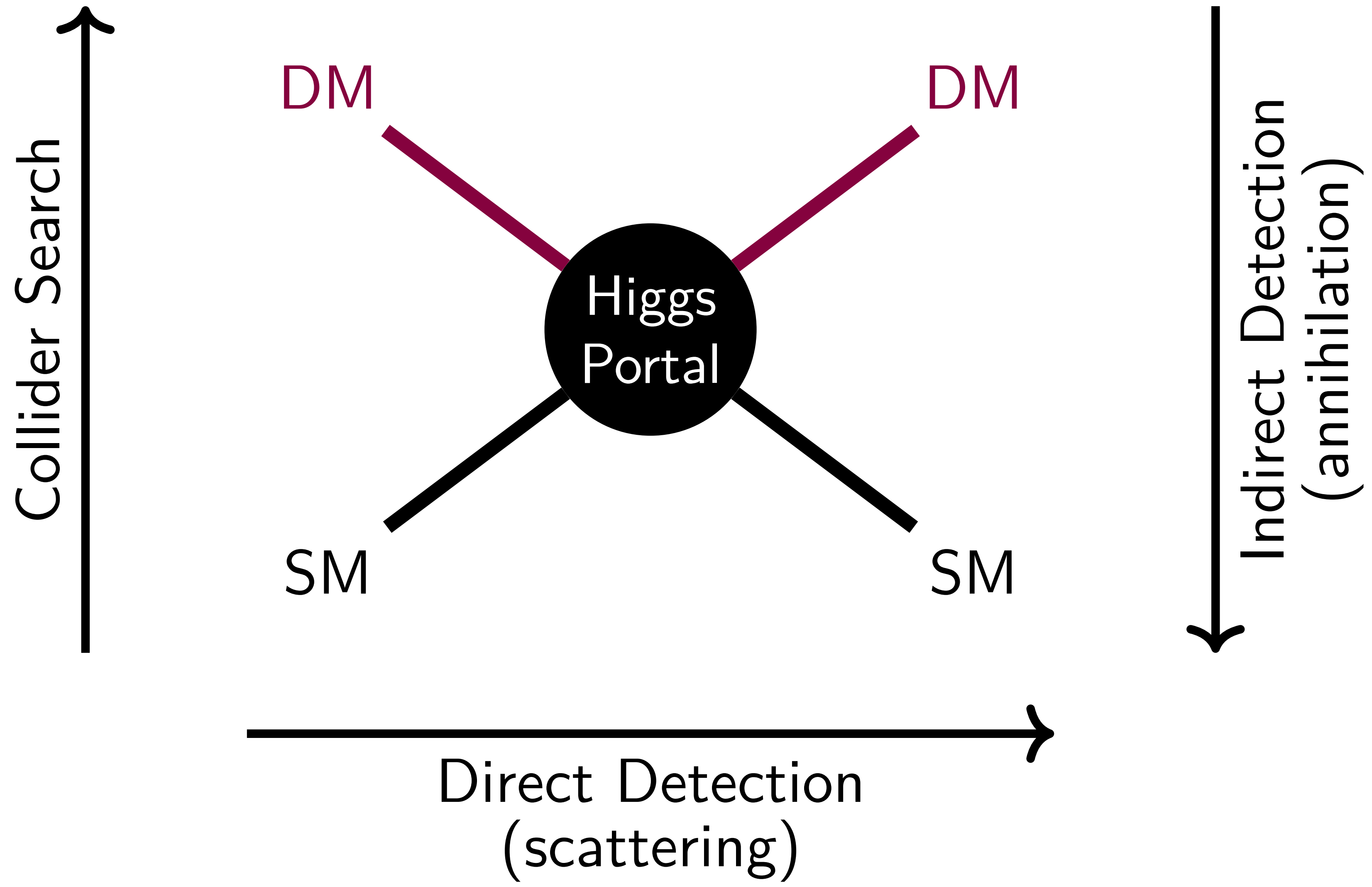




DD constraint [LZ 2024].

Why pNGBs?

- Arises from spontaneous <u>and soft</u> breaking of global symmetries.
- Produces <u>derivative interactions</u> i.e, velocity suppressed.
- <u>Naturally evades</u> the stringent <u>Direct Detection bound.</u>



$$\mathrm{E}_{\mathrm{ann.}} \approx 2m_{\mathrm{DM}}c^2$$

$$\mathrm{E}_{\mathrm{recoil}} \approx \frac{2m_N m_{\mathrm{DM}}^2}{(m_N + m_{\mathrm{DM}})^2} v_{\mathrm{DM}}^2$$

Background

Original Idea

C. Gross, O. Lebedev, T. Toma, PRL (2017) [arXiv:1708.02253]

- One SM Singlet complex scalar S.
- Invariant under global $\mathrm{U}(1)$ sym. i.e, $S \to e^{i\alpha} S$.

$$\mathcal{V}(S,\Phi) = -\frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_S}{2}|S|^4 \underbrace{- \mu_\Phi^2|\Phi|^2 + \frac{\lambda_\Phi}{2}|\Phi|^4}_{\text{SM Higgs}} + \underbrace{\lambda_{\Phi S}|\Phi|^2|S|^2}_{\text{Higgs portal}}$$

$$\underbrace{- \frac{\mu_S'^2}{4} S^2 + \text{h.c.}}_{\text{soft U(1) breaking } (\to \mathbb{Z}_2)}$$

Original Idea

C. Gross, O. Lebedev, T. Toma, PRL (2017) [arXiv:1708.02253]

- After introducing VEVs:

CP-odd
(DM candidate)

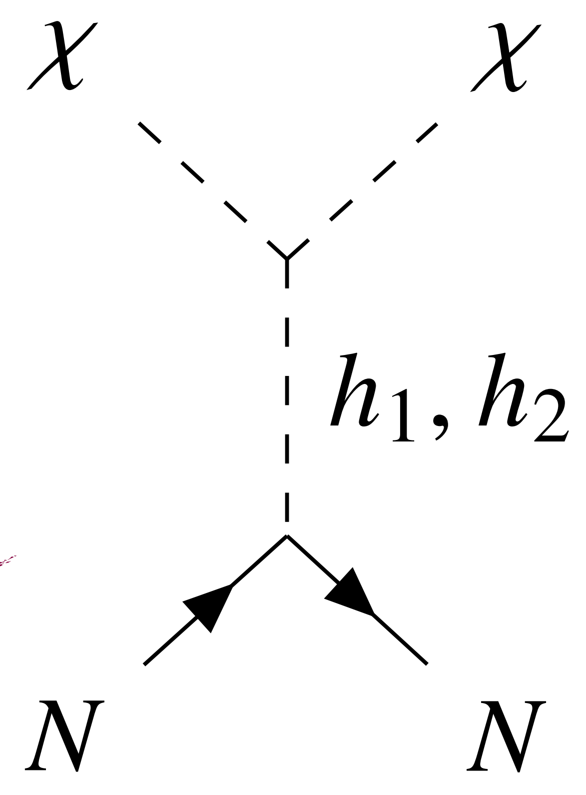
$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h \end{pmatrix}, \qquad S = \frac{1}{\sqrt{2}}(v_s + s + i\chi),$$

we have scalar mixing:

CP-even

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} h \\ s \end{pmatrix}.$$

- Higgs bosons (h_1, h_2) acts as mediator.



$$\mathcal{M} \propto \left(\frac{m_1^2}{t - m_1^2} - \frac{m_2^2}{t - m_2^2}\right) \simeq \frac{t\,(m_1^2 - m_2^2)}{m_1^2 m_2^2} \to 0$$

Original Idea, Not enough!

C. Gross, O. Lebedev, T. Toma, PRL (2017) [arXiv:1708.02253]

- Why?

  → No signal!

  → Domain Wall Problem! (SSB of discrete $\mathbb{Z}_2$ symmetry)

- How was the DW problem fixed?

  → Soft breaking of global sym. + SSB of gauge sym. → New species of pNGB DM

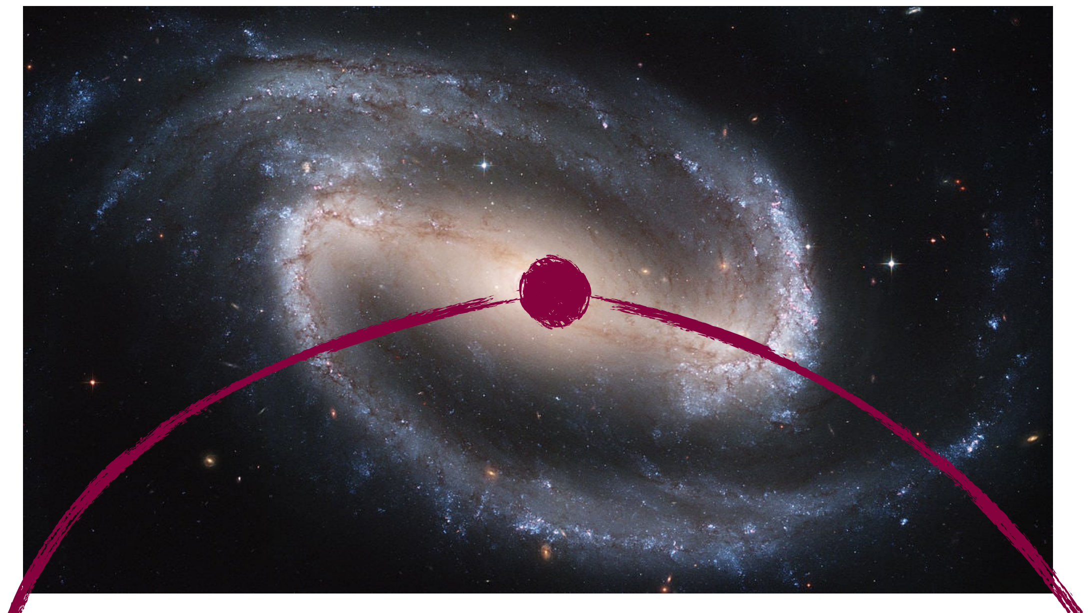
  [Abe et. al. 2020, 2023, 2024, …]

- Still no viable signal from pNGB DM!

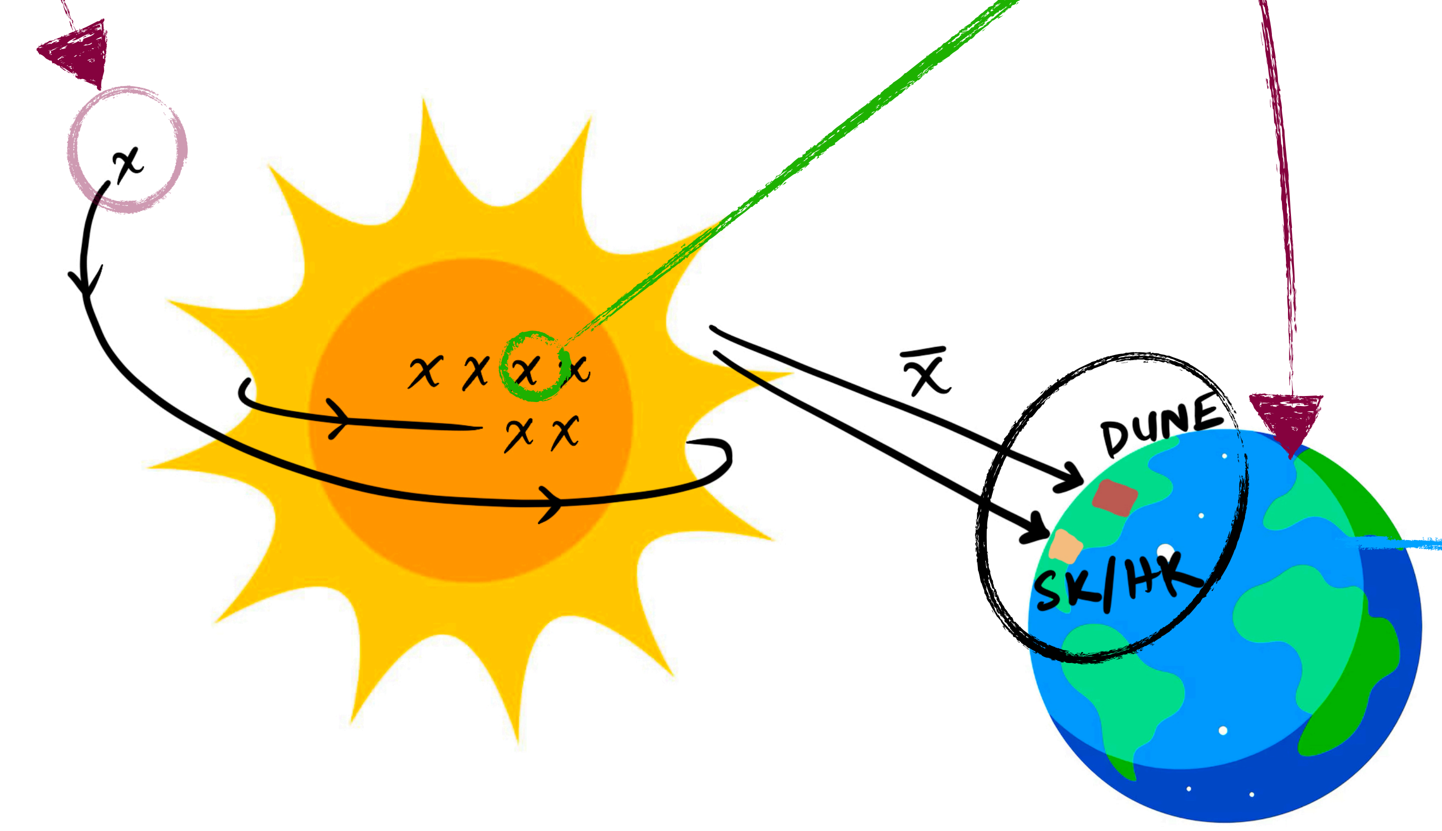
Motivation

Boosted Dark Matter (BDM)

(DM on Steroid?)



Galactic center producing DM

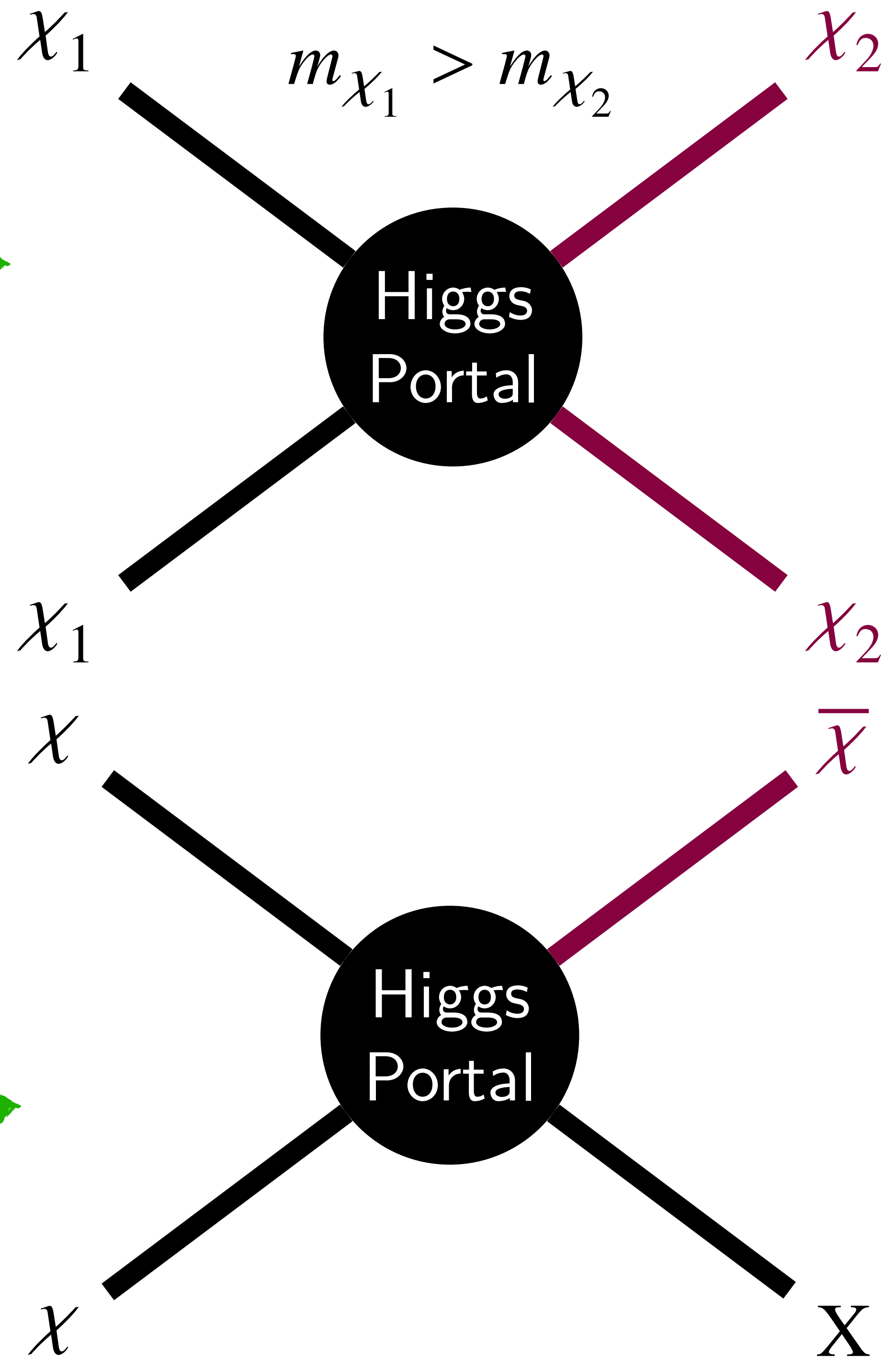


BDM produced in celestial bodies.

Production Mechanisms

Annihilation of heavy particles

Semi-annihilations

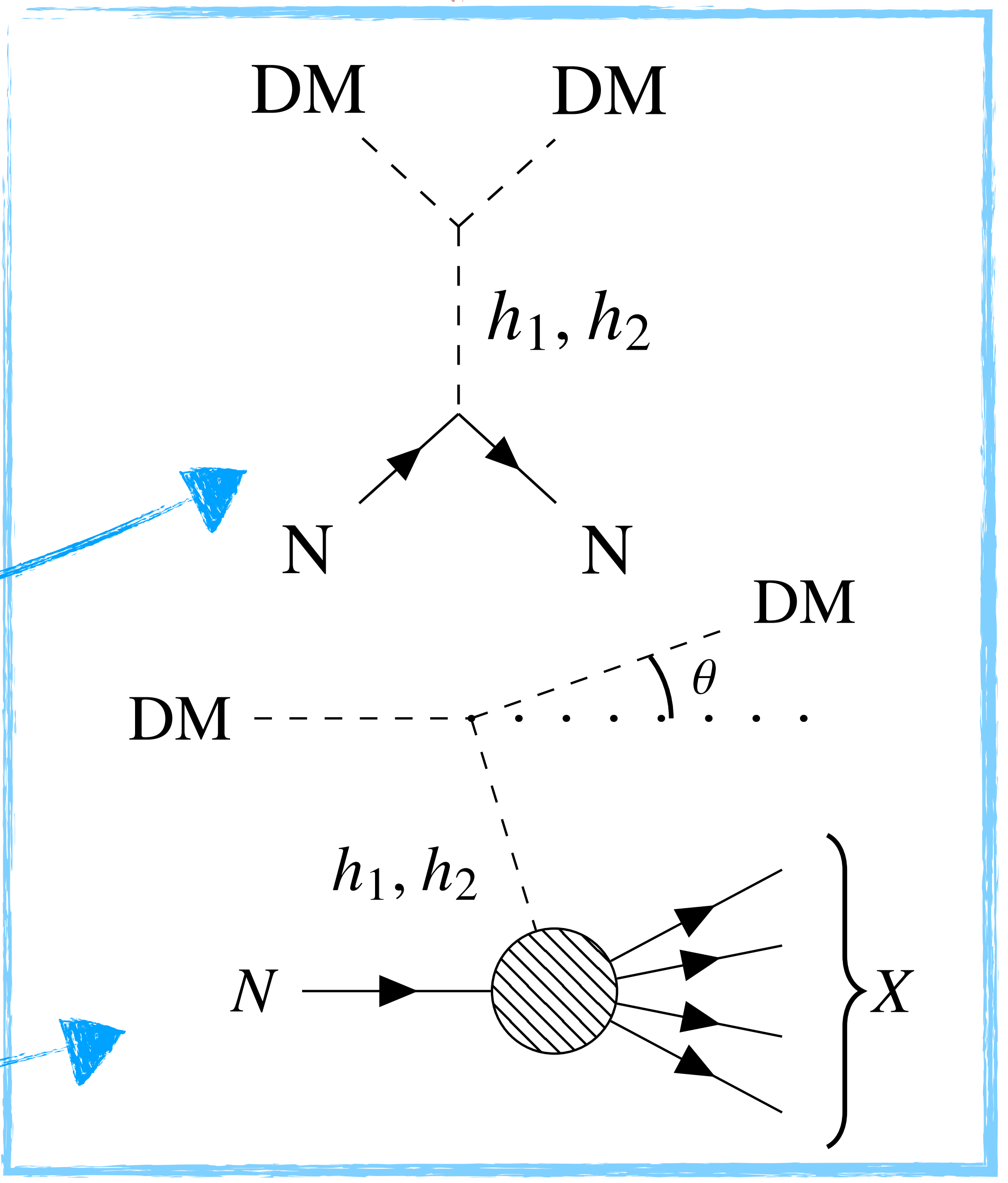


Directional Search

Elastic scattering

(Deep) inelastic scattering

$$E_{\mathrm{BDM}} \equiv \gamma m_{\mathrm{BDM}}$$



Signal

Detector choice based on boost

$$\gamma \geq \begin{cases} 1.51 & \rightarrow \text{SK/HK} \\ 1.55 & \rightarrow \text{IceCube/DeepCore} \\ 1.25 & \rightarrow \text{DUNE} \end{cases}$$

Our approach!

First attempt

Semi-annihilation driven pNGB BDM

Pseudo-Nambu-Goldstone-boson Dark Matter from Three Complex Scalars

RS, T. Toma, K. Tsumura, JHEP (2025) [arXiv:2504.19886]

The Model

RS, T. Toma, K. Tsumura, JHEP (2025) [arXiv:2504.19886]

- Three SM Singlet complex scalars $S_j\,(j = 1{,}2{,}3)$.
- Gauge $\mathrm{U}(1)_V$ sym. Invariant: $S_j \to e^{i\theta_V(x)} S_j$.
- Softly-broken global $\mathrm{U}(1)_A$ sym.: $S_j \to e^{i\theta_A^j} S_j$.

$$\mathcal{V}(S_1, S_2, S_3, \Phi) = \mu_S^2\left(|S_1|^2 + |S_2|^2 + |S_3|^2\right) - \underbrace{\frac{m_{12}^2}{3}\left(S_1^* S_2 + S_2^* S_3 + S_3^* S_1 + \text{h.c.}\right)}_{\text{soft breaking}}$$

$$+ \frac{\lambda_S}{2}\left(|S_1|^4 + |S_2|^4 + |S_3|^4\right) + \lambda_S'\left(|S_1|^2|S_2|^2 + |S_2|^2|S_3|^2 + |S_3|^2|S_1|^2\right)$$

$$\underbrace{- \mu_\Phi^2|\Phi|^2 + \frac{\lambda_\Phi}{2}|\Phi|^4}_{\text{SM Higgs}} + \underbrace{\lambda_{\Phi S}|\Phi|^2\left(|S_1|^2 + |S_2|^2 + |S_3|^2\right)}_{\text{Higgs portal}}.$$

The Model

RS, T. Toma, K. Tsumura, JHEP (2025) [arXiv:2504.19886]

$$\mathcal{V}_{\text{soft}} = -\frac{m_{12}^2}{3}\left(S_1^* S_2 + S_2^* S_3 + S_3^* S_1 + \text{h.c.}\right)$$

$$\Sigma_1 = \frac{1}{\sqrt{2}}\left(v_s + s_1' + iz\right),$$

$$m_a^2 = m_{12}^2 \equiv m_{\text{DM}}^2$$

Cx DM

$$\overbrace{\mathrm{U}(1)_A^{\text{global}} \times \mathrm{U}(1)_V^{\text{gauge}}}^{S_1,\ S_2,\ S_3} \xrightarrow[\text{2 (p)NGBs}]{\text{soft break}} \text{Higgs basis } (\Sigma_1,\ \Sigma_2,\ \Sigma_3) \quad \underbrace{\mathbb{Z}_3}_{} \times \mathrm{U}(1)_V^{\text{gauge}} \xrightarrow[\text{1 (would-be)NGB}]{\text{SSB}} \mathbb{Z}_3 \text{ (DM stability)}$$

$\supset S(3)$

mass eigenbasis $(h,\ s_1',\ z,\ \Sigma_\omega,\ a_\omega)$

$$\langle S_1 \rangle = \langle S_2 \rangle = \langle S_3 \rangle = \frac{v_s}{\sqrt{6}}$$

Eaten by dark gauge boson Z'

Third Higgs Boson

Discrete S(3) permutation exchange sym.

$$S_1 \leftrightarrow S_2 \quad S_2 \leftrightarrow S_3 \quad S_3 \leftrightarrow S_1.$$

$$m_\Sigma^2 = m_{12}^2 + \frac{1}{3}v_s^2\left(\lambda_S - \lambda_S'\right)$$

Results RS, T. Toma, K. Tsumura, JHEP (2025) [arXiv:2504.19886]

Unique Channels

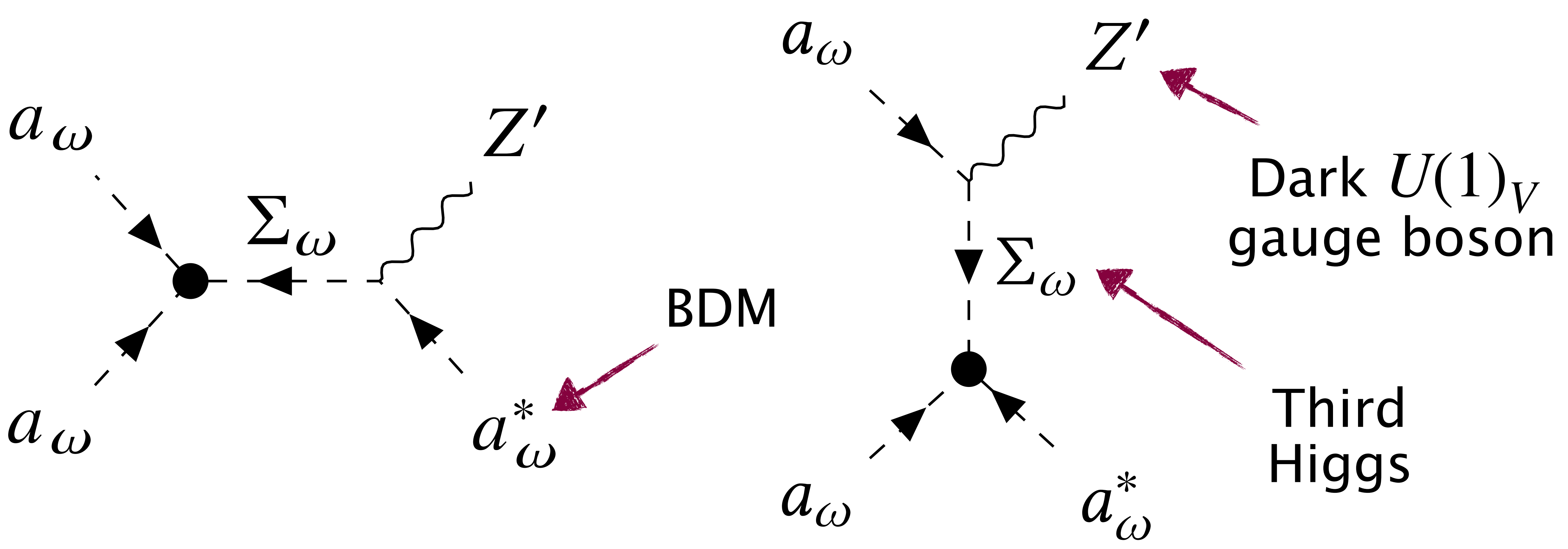
$$D_\mu S_j = (\partial_\mu - i g_V V_\mu) S_j$$

$$D_\mu \Phi = \left(\partial_\mu - i\frac{g}{2} W^a_\mu \sigma^a - i\frac{g_Y}{2} Y_\mu\right)\Phi$$

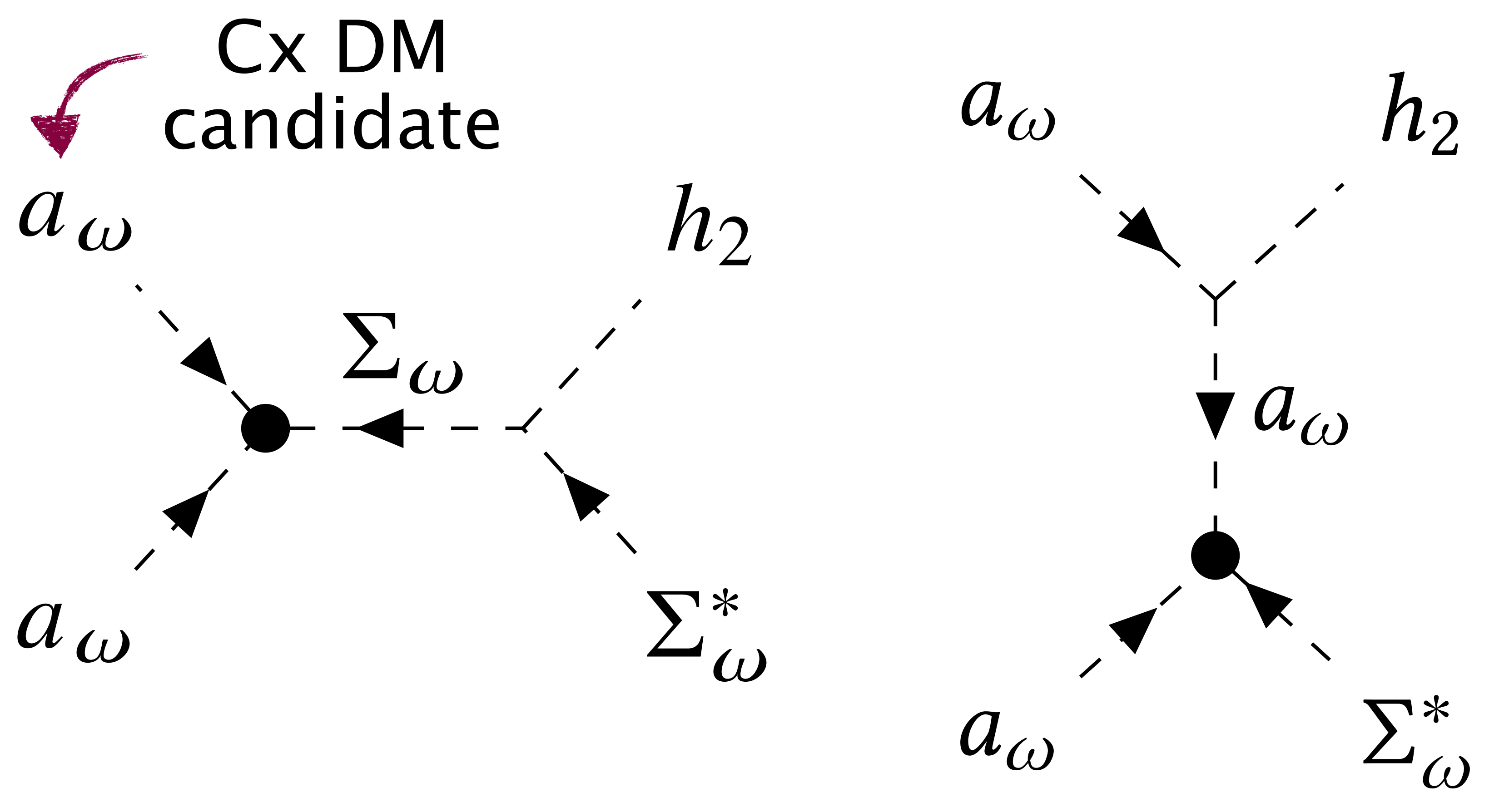
$$\propto \frac{m_\Sigma^2 - m_a^2}{v_s}$$

$$\mathcal{L} \supset \kappa_1 \left(\Sigma_\omega \overleftrightarrow{\partial}_\mu a^*_\omega + a_\omega \overleftrightarrow{\partial}_\mu \Sigma^*_\omega\right) Z'^\mu - \kappa_2 \underbrace{\left(\Sigma^3_\omega + \Sigma^{*3}_\omega - \Sigma_\omega a^2_\omega - \Sigma^*_\omega a^{*2}_\omega\right)}_{\text{New cubic interactions}} - \kappa_3 |\Sigma_\omega|^2 h_2$$

$$\propto g_V \qquad\qquad \propto \frac{m_2^2 + 2(m_\Sigma^2 - m_a^2)}{v_s}\cos\theta$$



Semi-annihilation channels

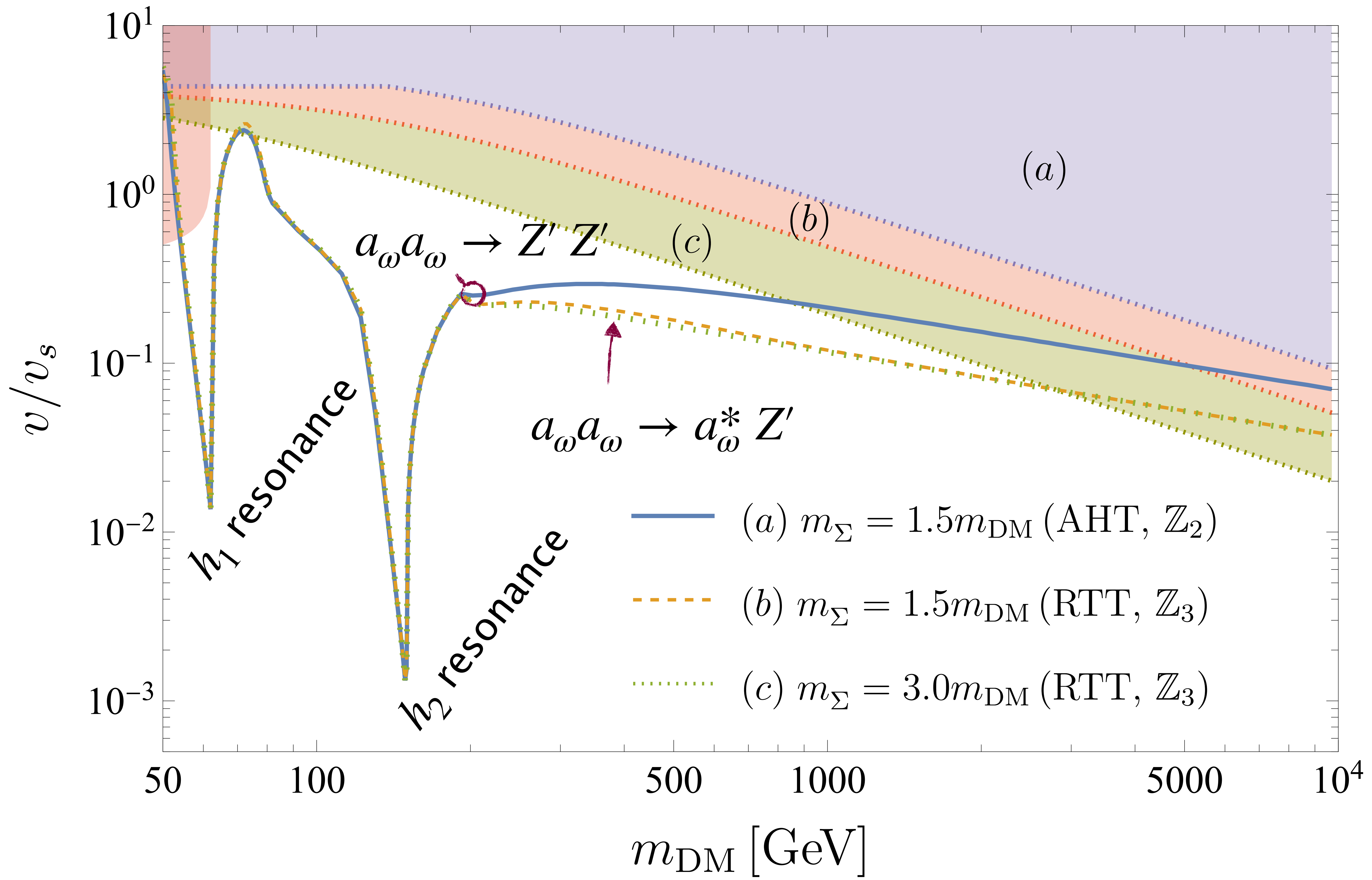


Semi-annihilation-like channels

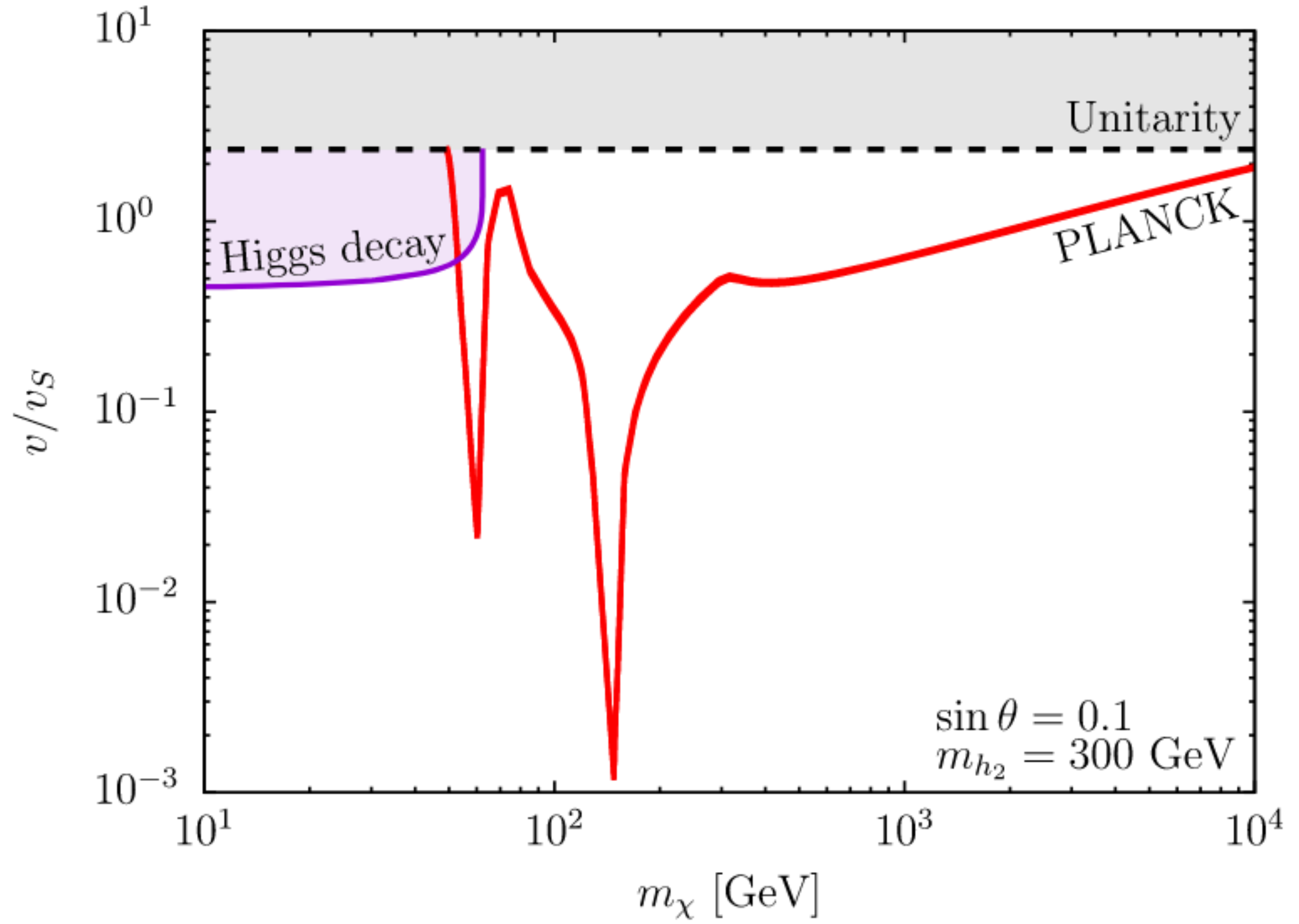
Results
RS, T. Toma, K. Tsumura, JHEP (2025) [arXiv:2504.19886]

Signature of the model

$\sin\theta = 0.1,\ \sin\epsilon = 10^{-4},\ m_{h_2} = 300\ \mathrm{GeV},\ m_{Z'} = 200\ \mathrm{GeV}$



Original pNGB idea [Gross et al., 2017]



Viable parameter space satisfying $\Omega_{\mathrm{DM}} h^2 \approx 0.12$ and comparison with the AHT model.

T. Abe, Y. Hamada, K. Tsumura, JHEP (2024).

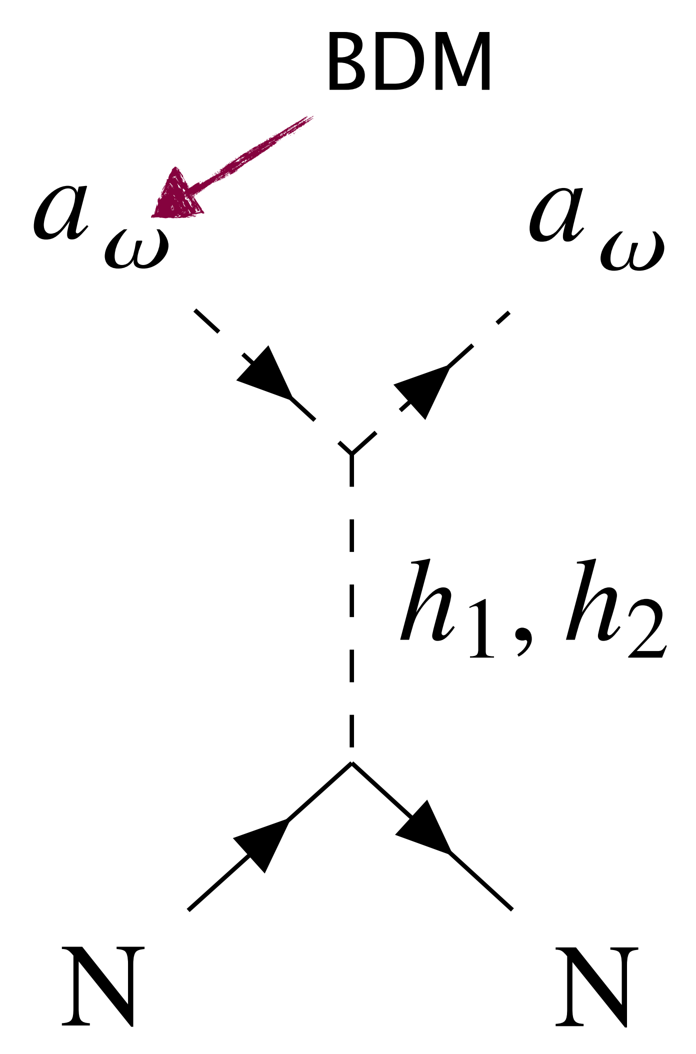
Results RS, T. Toma, K. Tsumura, JHEP (2025) [arXiv:2504.19886]

Not enough!

- Cross-section well below current sensitivity of DUNE.

$$v_{\rm DM} = \sqrt{1 - \frac{m_{\rm DM}^2}{E_{\rm DM}^2}}$$

$$\sigma_{\rm el} \approx \frac{f_N^2 \sin^2 2\theta\, m_N^4}{24\pi v^2 v_s^2 s^3} \frac{\left(m_1^2 - m_2^2\right)^2}{m_1^4 m_2^4} \left(s - m_{\rm DM}^2 - m_N^2\right)^4 v_{\rm DM}^4$$

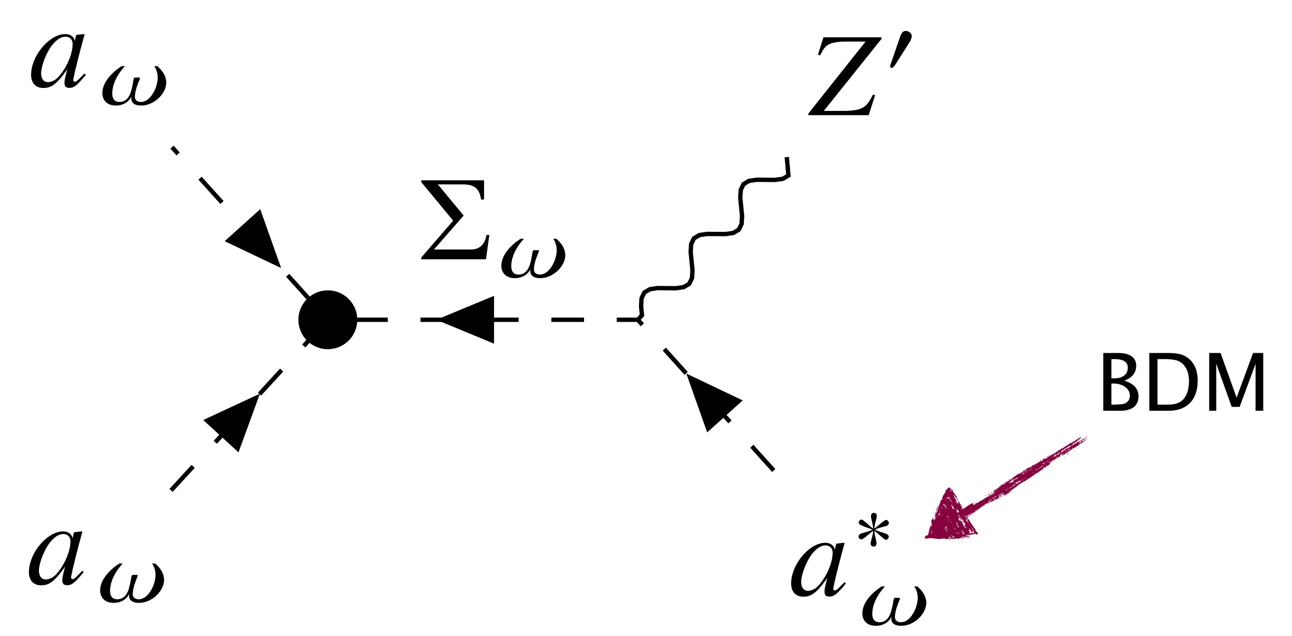


- Using this typical values

$$\sin\theta = 0.1, \quad v_s = v, \quad m_2 = 300\ \text{GeV}, \quad m_{\rm DM} = 100\ \text{GeV}, \quad m_{Z'} \ll m_{\rm DM}$$

$$\boxed{\sigma_{\rm el} \sim 10^{-54}\ \text{cm}^2 < 10^{-45}\ \text{cm}^2}$$

- Reason? $\gamma \equiv \frac{E_{\rm DM}}{m_{\rm DM}} = \frac{5m_{\rm DM}^2 - m_{Z'}^2}{4m_{\rm DM}^2} \leq \frac{5}{4} = 1.25$



Conclusion

Not yet!

But for this talk

- Natural DD suppression → pNGB DM good!
- But, detection is very stubborn!
- First attempt

  → Unique semi-annihilation driven pNGB BDM model.
  → Not enough boost!
- Second attempt (W.I.P.)

  → Multi-component pNGB BDM model
  (Should produce good signal — Stay tuned!)

Backups

Lagrangian

$$D_\mu S_j = (\partial_\mu - i g_V V_\mu) S_j$$

$$D_\mu \Phi = \left(\partial_\mu - i\frac{g}{2} W_\mu^a \sigma^a - i\frac{g_Y}{2} Y_\mu\right)\Phi$$

The Lagrangian of our model

$$\mathcal{L} = \mathcal{L}_{SM(\text{w/o Higgs potential})}$$
$$+ |D_\mu S_1|^2 + |D_\mu S_2|^2 + |D_\mu S_3|^2 + |D_\mu \Phi|^2$$
$$- \frac{1}{4} V^{\mu\nu} V_{\mu\nu} - \underbrace{\frac{\sin\epsilon}{2} V^{\mu\nu} Y_{\mu\nu}}_{\text{gauge kinetic mixing}}$$
$$- \mathcal{V}(S_1, S_2, S_3, \Phi)$$

Field strength tensors

$U(1)_Y \rightarrow Y_\mu$

$U(1)_V \rightarrow V_\mu$

Minimization of potential

Parameterize the most general VEVs for the singlets

$$\langle S_1 \rangle = \frac{v_{s_1}}{\sqrt{2}}, \quad \langle S_2 \rangle = \frac{v_{s_2}}{\sqrt{2}}, \quad \langle S_3 \rangle = \frac{v_{s_3}}{\sqrt{2}},$$

To have SSB one of the VEV must be non-zero

$$\frac{\partial \langle \mathcal{V} \rangle}{\partial v} = \left\{ -\mu_\Phi^2 + \frac{\lambda_\Phi}{2} v^2 + \frac{\lambda_{\Phi S}}{2} \left(v_{s_1}^2 + v_{s_2}^2 + v_{s_3}^2 \right) \right\} v = 0,$$

$$\frac{\partial \langle \mathcal{V} \rangle}{\partial v_{s_1}} = \left\{ \mu_S^2 + \frac{\lambda_S}{2} v_{s_1}^2 + \frac{\lambda'_S}{2} \left(v_{s_2}^2 + v_{s_3}^2 \right) + \frac{\lambda_{\Phi S}}{2} v^2 - \frac{m_{12}^2}{3} \left(\frac{v_{s_2} + v_{s_3}}{v_{s_1}} \right) \right\} v_{s_1} = 0,$$

$$\frac{\partial \langle \mathcal{V} \rangle}{\partial v_{s_2}} = \left\{ \mu_S^2 + \frac{\lambda_S}{2} v_{s_2}^2 + \frac{\lambda'_S}{2} \left(v_{s_1}^2 + v_{s_3}^2 \right) + \frac{\lambda_{\Phi S}}{2} v^2 - \frac{m_{12}^2}{3} \left(\frac{v_{s_3} + v_{s_1}}{v_{s_2}} \right) \right\} v_{s_2} = 0,$$

$$\frac{\partial \langle \mathcal{V} \rangle}{\partial v_{s_3}} = \left\{ \mu_S^2 + \frac{\lambda_S}{2} v_{s_3}^2 + \frac{\lambda'_S}{2} \left(v_{s_1}^2 + v_{s_2}^2 \right) + \frac{\lambda_{\Phi S}}{2} v^2 - \frac{m_{12}^2}{3} \left(\frac{v_{s_1} + v_{s_2}}{v_{s_3}} \right) \right\} v_{s_3} = 0.$$

Two non-trivial solutions

$$v_{s_1} \neq 0, \; v_{s_2} = 0, \; v_{s_3} = -v_{s_1}$$

$$v_{s_1} = v_{s_2} = v_{s_3} \neq 0$$

$$v_{s_1} = v_{s_2} = v_{s_3} = \frac{v_s}{\sqrt{3}},$$

Higgs basis?

After introducing the VEVs

$$\langle\Phi\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \qquad \langle S_1\rangle_0 = \langle S_2\rangle_0 = \langle S_3\rangle_0 = \frac{v_s}{\sqrt{6}}$$

we can move to this basis

$$\begin{pmatrix}\Sigma_1\\ \Sigma_2\\ \Sigma_3\end{pmatrix} \to R\cdot\left\{\frac{v_s}{\sqrt{6}}\begin{pmatrix}1\\1\\1\end{pmatrix} + \begin{pmatrix}S_1\\ S_2\\ S_3\end{pmatrix}\right\} = \frac{v_s}{\sqrt{2}}\begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}\Sigma_1\\ \Sigma_2\\ \Sigma_3\end{pmatrix}$$

where, $R = \frac{1}{\sqrt{3}}\begin{pmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{pmatrix}$ with $\omega = e^{i2\pi/3}$

$$\Sigma_1 = \frac{1}{\sqrt{2}}\big(v_s + s_1'(x) + iz(x)\big), \quad \Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+h(x)\end{pmatrix}$$

Understand the charges

The potential in the Higgs basis

$$\begin{aligned}\mathcal{V}(\Sigma_1,\Sigma_2,\Sigma_3,\Phi) = m_{12}^2\left(|\Sigma_2|^2+|\Sigma_3|^2\right)+\frac{\lambda_\Phi}{2}\left(|\Phi|^2-\frac{v^2}{2}\right)^2 \\ +\frac{\lambda_S}{6}\left\{\left(|\Sigma_1|^2+|\Sigma_2|^2+|\Sigma_3|^2-\frac{3v_s^2}{2}\right)^2+2\left|\Sigma_1^*\Sigma_2+\Sigma_2^*\Sigma_3+\Sigma_3^*\Sigma_1\right|^2\right\} \\ +\frac{\lambda_S'}{3}\left\{\left(|\Sigma_1|^2+|\Sigma_2|^2+|\Sigma_3|^2-\frac{3v_s^2}{2}\right)^2-\left|\Sigma_1^*\Sigma_2+\Sigma_2^*\Sigma_3+\Sigma_3^*\Sigma_1\right|^2\right\} \\ +\lambda_{\Phi S}\left\{\left(|\Sigma_1|^2+|\Sigma_2|^2+|\Sigma_3|^2-\frac{3v_s^2}{2}\right)\left(|\Phi|^2-\frac{v^2}{2}\right)\right\}\end{aligned}$$

It is clear that

$$\begin{aligned}\mathbb{Z}_3: &\quad \Phi\to\Phi, \quad \Sigma_1\to\Sigma_1, \quad \Sigma_2\to\omega\Sigma_2, \quad \Sigma_3\to\omega^2\Sigma_3 \\ (CP)_S: &\quad \Phi\to\Phi, \quad \Sigma_1\to\Sigma_1^*, \quad \Sigma_2\to\Sigma_2^*, \quad \Sigma_3\to\Sigma_3^*\end{aligned}$$

Mass spectrum

In the mass eigenstate the $(CP, \mathbb{Z}_3)$ charges for $\begin{pmatrix} h & s_1' & z & \Sigma_\omega & a_\omega \end{pmatrix}$

$$(h_1, s_1')_{2\times 2} : (+, 1)_{2\times 2}, \quad z : (-, 1), \quad \Sigma_\omega : (+, \omega), \quad a_\omega : (-, \omega)$$

Separated even sector

$$M^2_{\text{even}} = \begin{pmatrix} v^2 \lambda_\Phi & v_s v \lambda_{\Phi S} \\ v_s v \lambda_{\Phi S} & v_s^2(\lambda_S + 2\lambda_S')/3 \end{pmatrix}$$

A third Higgs boson $\quad m_\Sigma^2 = m_{12}^2 + v_s^2(\lambda_S - \lambda_S')/3$

1 Cx pNGB DM $\quad m_{\text{DM}}^2 = m_a^2 = m_{12}^2$

Parameters

Dependent parameters

$$\lambda_\Phi = \frac{m_1^2 c_\theta^2 + m_2^2 s_\theta^2}{v^2}$$

$$\lambda_{\Phi S} = \frac{(m_1^2 - m_2^2) s_\theta c_\theta}{v_s v}$$

$$\lambda_S = \frac{m_1^2 s_\theta^2 + m_2^2 c_\theta^2}{v_s^2} + \frac{2(m_\Sigma^2 - m_a^2)}{v_s^2}$$

$$\lambda'_S = \frac{m_1^2 s_\theta^2 + m_2^2 c_\theta^2}{v_s^2} - \frac{m_\Sigma^2 - m_a^2}{v_s^2}$$

Free parameters

$$m_1 (= 125 \text{ GeV}), \quad m_2, \quad \sin\theta, \quad m_{\rm DM}, \quad m_\Sigma, \quad v, \quad v_s, \quad m_{Z'}, \quad \sin\epsilon.$$

Signal at detectors

The signal can basically be calculated as

$$N_{\text{signal}} \sim N_{\text{target}} \times T \times \sigma_{\text{DM}N} \times \Phi_{\text{DM}}$$

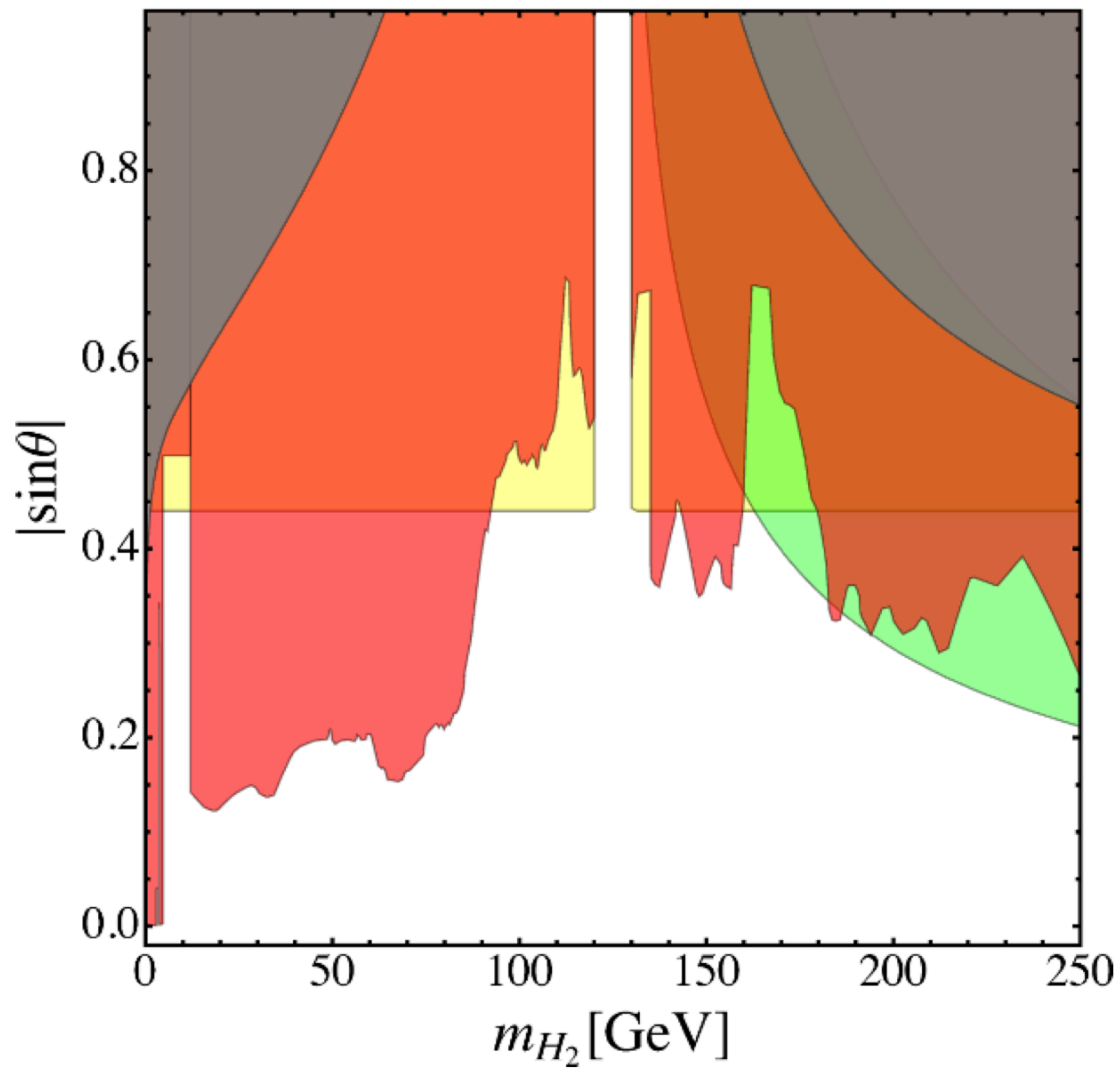
where,

- N_{target} : Number of target nuclei
- T : Total exposure time
- $\sigma_{\text{DM}N}$: Cross-section of DM—Nucleon scattering
- Φ_{DM} : Flux of incoming DM

Similar for atmospheric ν background.

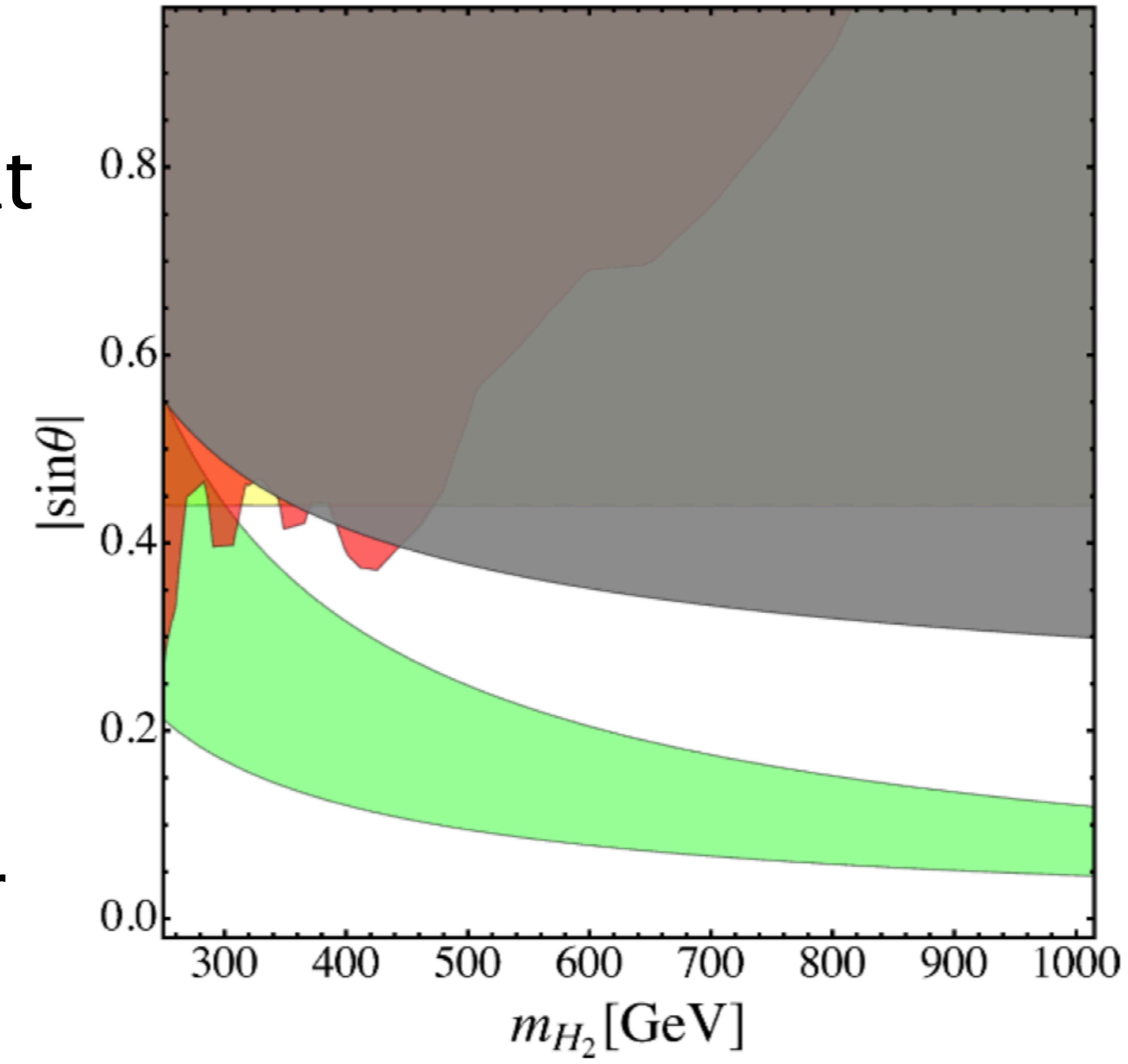
Bound on scalar mixing $(\sin\theta)$

Falkowski, Gross, Lebedev, JHEP 1505 (2015) [arxiv:1502.01361]



Parameter space for $m_{h_2} \leq 2m_{h_1}$

- Red: h_2 direct search at LHC
- Yellow: h_1 coupling measurements
- Green: Favored region from stability of scalar potential
- Gray: EW precision tests



Parameter space for $m_{h_2} > 2m_{h_1}$

Constraints RS, T. Toma, K. Tsumura, JHEP (2025) [arXiv:2504.19886]

Higgs invisible decay

Contribution from

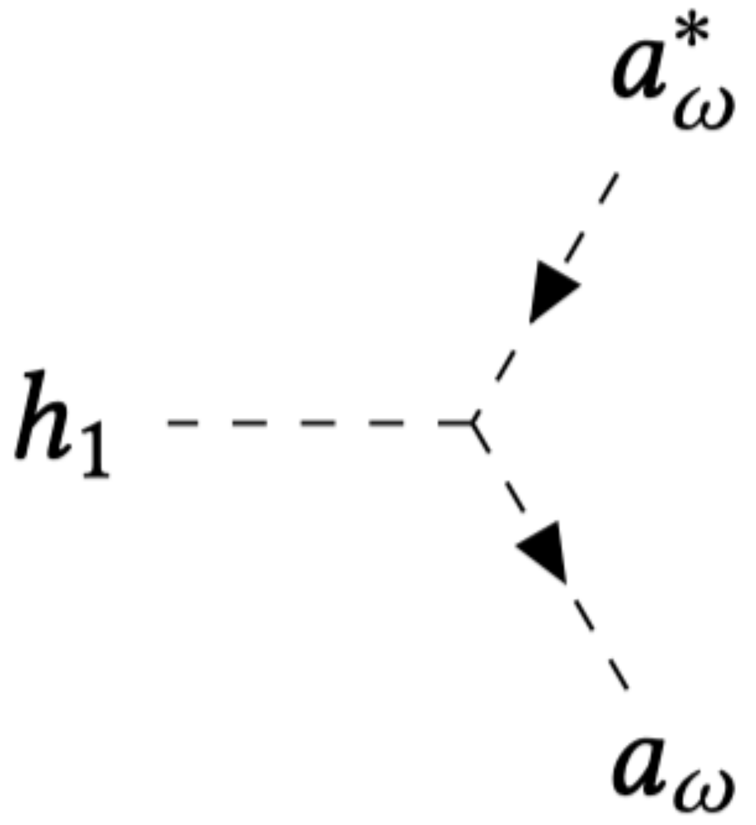
$$\Gamma(h_1 \to a_\omega a_\omega) = \frac{1}{32\pi}\frac{\kappa_{aah_1}^2}{m_1}\sqrt{1-\frac{4m_{\rm DM}^2}{m_1^2}}\,\Theta(m_1 - 2m_{\rm DM}).$$

where,

$$\kappa_{aah_1} = v\lambda_{\Phi S}\cos\theta + \frac{1}{3}v_s\left(\lambda_S + 2\lambda_S'\right)\sin\theta.$$

and the bounds

$$\mathrm{BR_{inv}} < \begin{cases} 0.107 & \text{(ATLAS)} \\ 0.15 & \text{(CMS)} \end{cases}.$$



$$= i\kappa_{aah_1}$$

Constraints RS, T. Toma, K. Tsumura, JHEP (2025) [arXiv:2504.19886]

Perturbative Unitarity

Bounds on all the parameters are as follows:

$$|\lambda_\Phi| < 8\pi,$$

$$|\lambda'_S - 2\lambda_S| < 8\pi,$$

$$|\lambda'_S| < 8\pi,$$

$$\left|3\lambda_\Phi + 2\lambda'_S + 2\lambda_S \pm \sqrt{24\lambda_{\Phi S}^2 + (-3\lambda_\Phi + 2\lambda'_S + 2\lambda_S)^2}\right| < 16\pi.$$

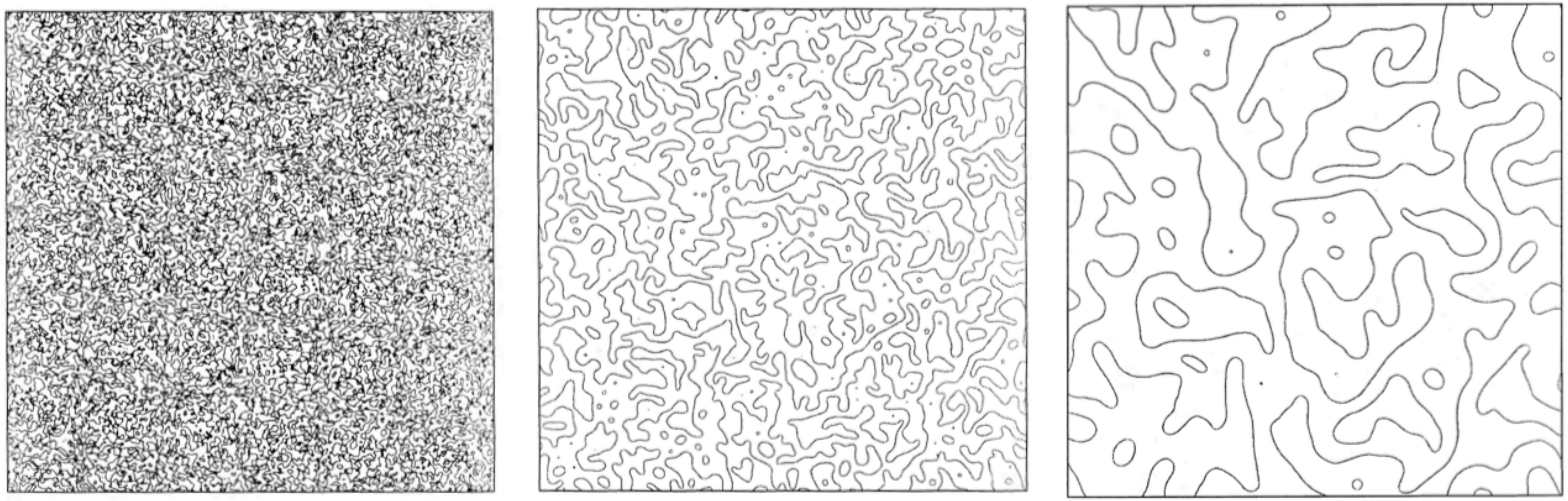
$$|\lambda_S| < 8\pi,$$

$$|\lambda_{\Phi S}| < 8\pi.$$

$$g_V < \sqrt{4\pi}.$$

Domain wall problem

Domain Walls from SSB of a discrete symmetry, $\mathbb{Z}_2$ in the original idea, would distort the CMB



Time evolution →

Press, Ryden, Spergel ApJ (1989)