Decoherence of Primordial Perturbations in the View of a Local Observer

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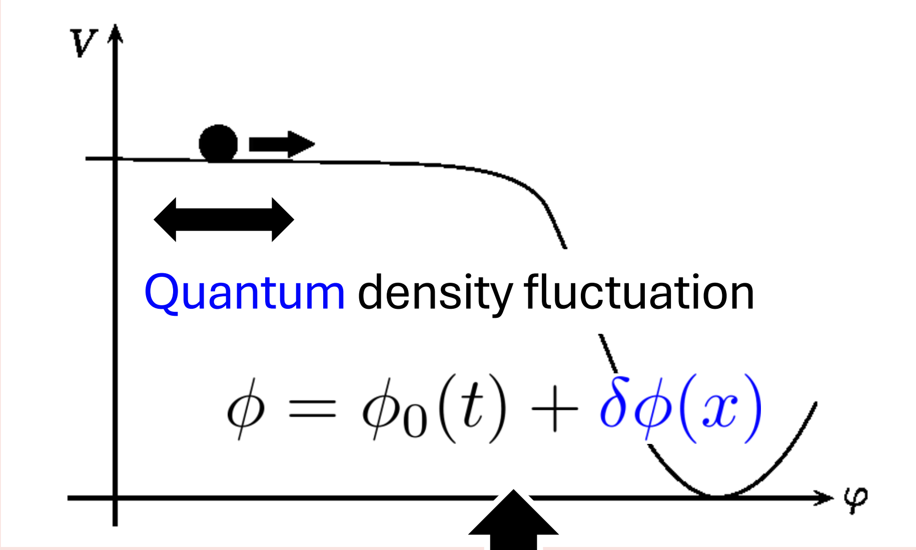
Seminar talk
August 21, 2025 at SI2025

Based on
2504.10472 with Junsei Tokuda (McGill University)

Inflation as a Source for Cosmological Perturbations



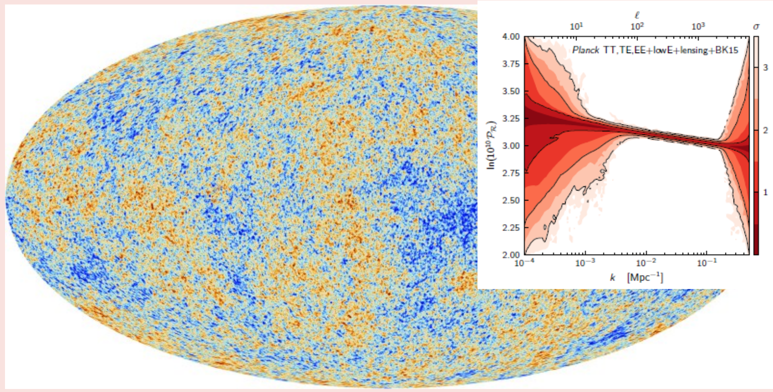
Quantum density fluctuation

$$\phi = \phi_0(t) + \delta\phi(x)$$

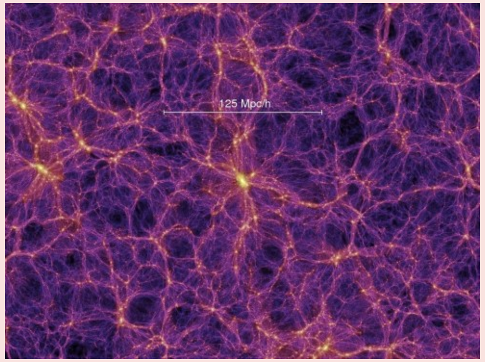
Quantum curvature perturbation

$$h_{ij} = (e^{\zeta(x)} a(t))^2 (\delta_{ij} + \gamma_{ij})$$

How (fast) classicalized?



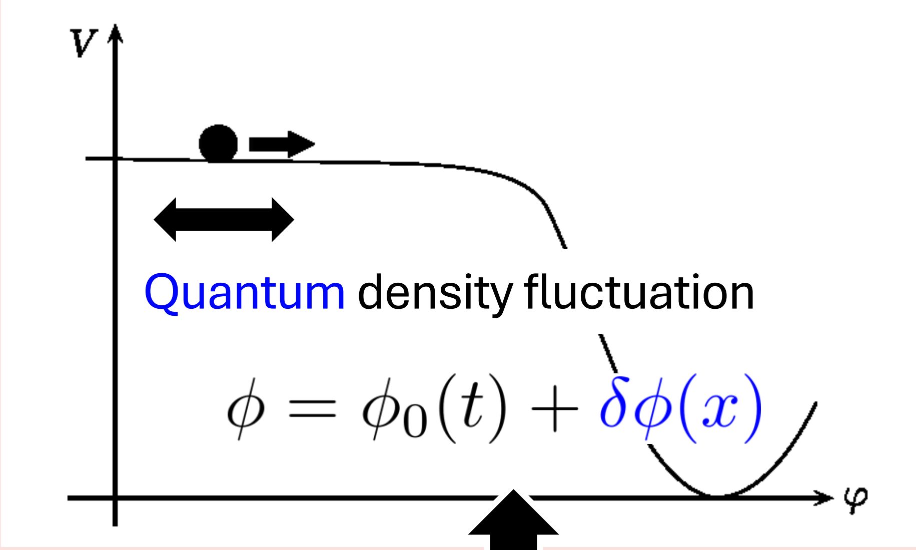
[Planck 1807.06211]



[Millennium Simulation 2005]

Classical anisotropy and inhomogeneity

Inflation as a Source for Cosmological Perturbations



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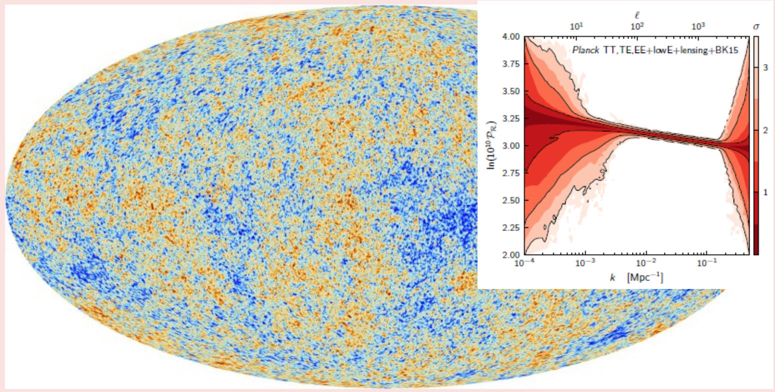
How (fast) classicalized?

Can we test quantum gravity by bottom-up?



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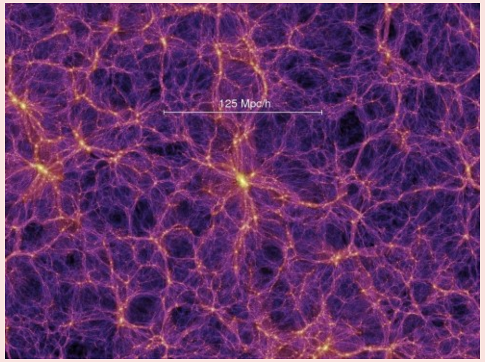

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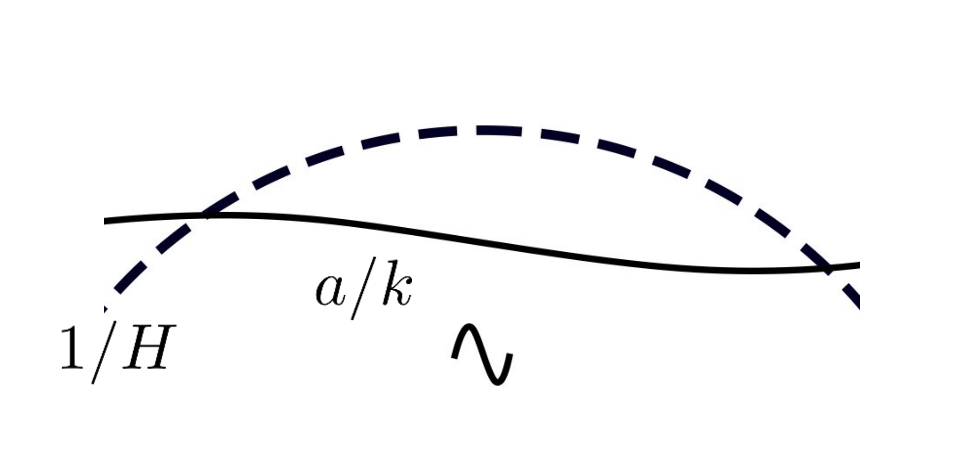
Classical anisotropy and inhomogeneity

“Quantumness” and “Classicalization”



- Intuitively...

Large scale $\longrightarrow$ Classical
$a/k \gg 1/H$

Formally?

- Coherence, Entanglement, Uncertainty, ...

$$\hat{\rho}[\zeta, \tilde{\zeta}] \text{ vs. } P(\zeta)$$

✓ Quantum vs. classical dist.
[Martin and Vennin 1801.09949, 1805.05609, Green and Porto 2001.09149, etc.]

✓ Stochastic formalism, PBH
[Weenink and Prokopec 1108.3994]

$$|\Psi\rangle_{AB} \neq |\phi\rangle_A \otimes |\varphi\rangle_B$$

✓ Bell test
[Martin and Vennin 1706.04516, 2203.03505 etc.
Sou et al. 2405.07141]

$$\mathcal{H}_{\mathrm{tot}} = \bigotimes_i \mathcal{H}_i$$

$i \longleftarrow k?\ x?\ \text{fields?}\ e^{ikx}\ \text{vs}\ Y_{lm}?$

Quantumness can be sensitive to the system.

$$\Delta\zeta\Delta\pi \gtrsim \hbar$$

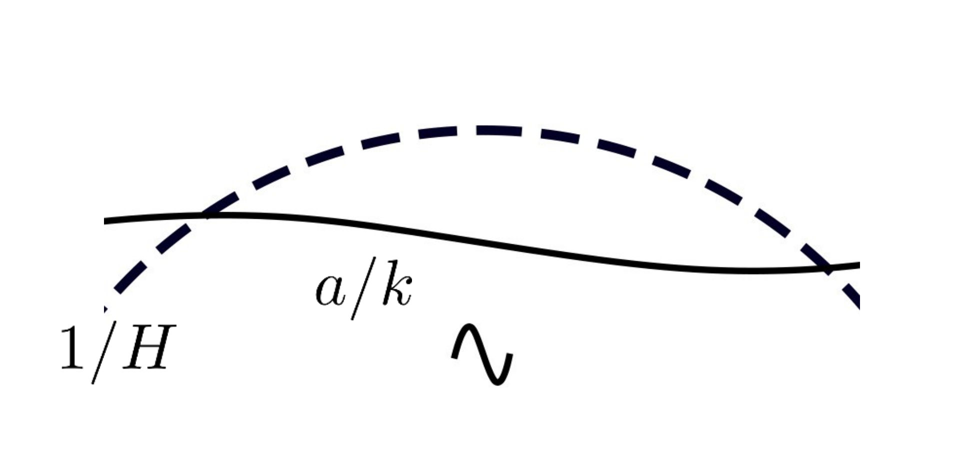
$$\Leftrightarrow [\zeta_{\mathbf{k}}, \pi_{\mathbf{k}'}] = i\hbar(2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')$$

✓ Gaussian $\longrightarrow$ minimal uncertainty

Two mode squeezed state
[Polarski and Starobinsky gr-qc/9504030]

$$|\Psi\rangle = \prod_{\mathbf{k}} \left(\sum_n \alpha_{n,\mathbf{k}} |n_{\mathbf{k}}\rangle \otimes |n_{-\mathbf{k}}\rangle \right)$$

"Quantumness" and "Classicalization"



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- Coherence, This talk

Entanglement,

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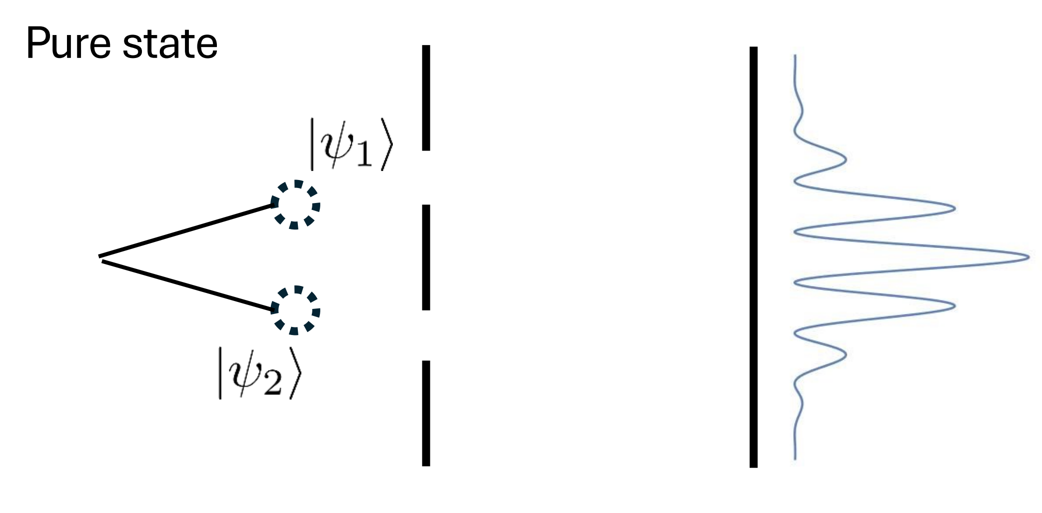
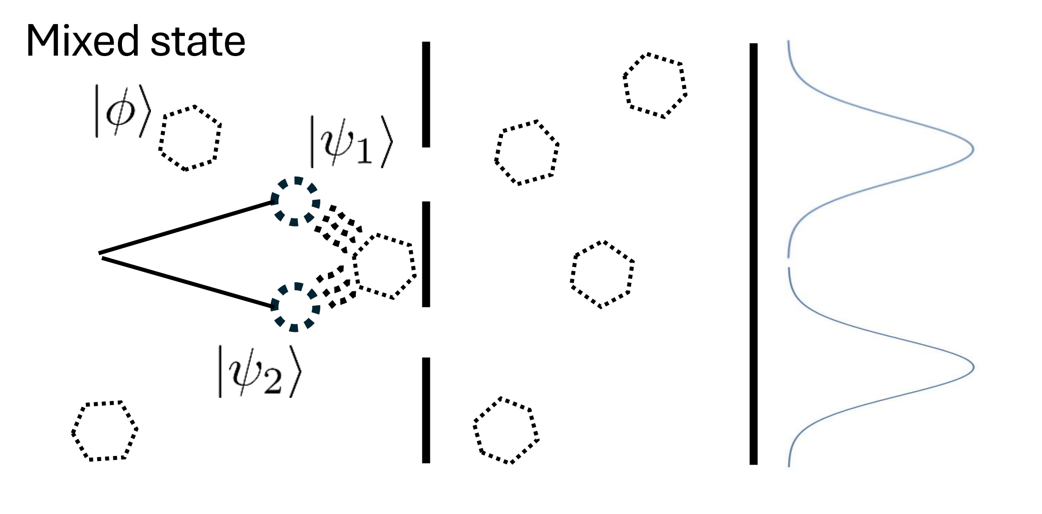
Quantum Interference and Decoherence



$$|\Psi\rangle = \alpha\,|\psi_1\rangle + \beta\,|\psi_2\rangle$$

$$\langle\Psi|\widehat{A}|\Psi\rangle = |\alpha|^2\,\langle\psi_1|\widehat{A}|\psi_1\rangle + |\beta|^2\,\langle\psi_2|\widehat{A}|\psi_2\rangle + \left(\alpha\beta^*\,\langle\psi_2|\widehat{A}|\psi_1\rangle + \text{c.c.}\right)$$



$$|\Psi\rangle = \alpha\,|\psi_1\rangle\,|\phi_1\rangle + \beta\,|\psi_2\rangle\,|\phi_2\rangle$$

$$\rho_\psi = \mathrm{Tr}_\phi[|\Psi\rangle\,\langle\Psi|] = \begin{pmatrix} |\alpha|^2 & \alpha\beta^*\langle\phi_2|\phi_1\rangle \\ \alpha^*\beta\langle\phi_1|\phi_2\rangle & |\beta|^2 \end{pmatrix}$$

$\langle\phi_2|\phi_1\rangle \sim 0$ if scattered to independent states.

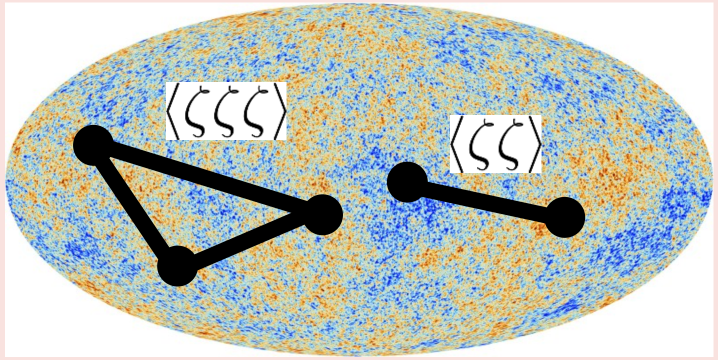
More scattering, more independent, less interference.

✓ Measure of coherence: $\rho_{ij}\Big|_{i\neq j}$

* Representation independent measure of coherence: Rényi entropy, purity, quantum discord, etc.
[Streltsov et al. 1612.07570, Henderson and Vedral quant-ph/0105028, etc. Comparison: Martin et al. 2211.10114]

Wavefunction Formalism

- ❑ **Observables: correlation functions**



$$\langle\Omega|\hat{\zeta}^n(t)|\Omega\rangle = \int \mathcal{D}\zeta(t)\langle\Omega|\zeta;t\rangle\,\langle\zeta;t|\Omega\rangle\zeta^n \equiv \int \mathcal{D}\zeta(t)|\Psi[\zeta(t)]|^2\zeta^n$$

$$\hat{\zeta}(t)\,|\zeta;t\rangle = \zeta(t)\,|\zeta;t\rangle$$

✓ E.g., $\mathcal{H} = \mathcal{H}_{\mathbf{k}_\mathrm{S}} \otimes \mathcal{H}_{\mathbf{k}_\mathrm{E}}$ with $k_\mathrm{S} \in \{k_\mathrm{CMB}\}$

- ❑ **Wavefunction at a certain time slice**

Gaussian

Gravitational non-linearity

$$\Psi[\zeta(t)] \equiv \langle\zeta;t|\Omega\rangle = \exp\left[-\frac{1}{2}\int\frac{d^3\mathbf{k}_1}{(2\pi)^3}\frac{d^3\mathbf{k}_2}{(2\pi)^3}\psi_2\zeta_{k_1}\zeta_{k_2} - \frac{1}{3!}\int\frac{d^3\mathbf{k}_1}{(2\pi)^3}\frac{d^3\mathbf{k}_2}{(2\pi)^3}\frac{d^3\mathbf{k}_3}{(2\pi)^3}\psi_3\zeta_{k_1}\zeta_{k_2}\zeta_{k_3} - \cdots\right]$$

$(\approx e^{iS_\mathrm{cl}[\zeta]})$

ψ_n : coefficient of the expansion

✓ Free propagation: $e^{-\int_\mathbf{k}\psi_2\zeta_\mathbf{k}\zeta_{-\mathbf{k}}}$ $\Longrightarrow$ no entanglement between $\mathbf{k}_\mathrm{S}$ and $\mathbf{k}_\mathrm{E}$
(no scattering)

$\Longrightarrow$ Non-linearities cause decoherence

Formulation of Decoherence

[Nelson 1601.03734, Sou et al. 2207.04435]

Another method: quantum master equation [Burgess et al. astro-ph/061646, Burgess et al. 2211.11046, etc.]

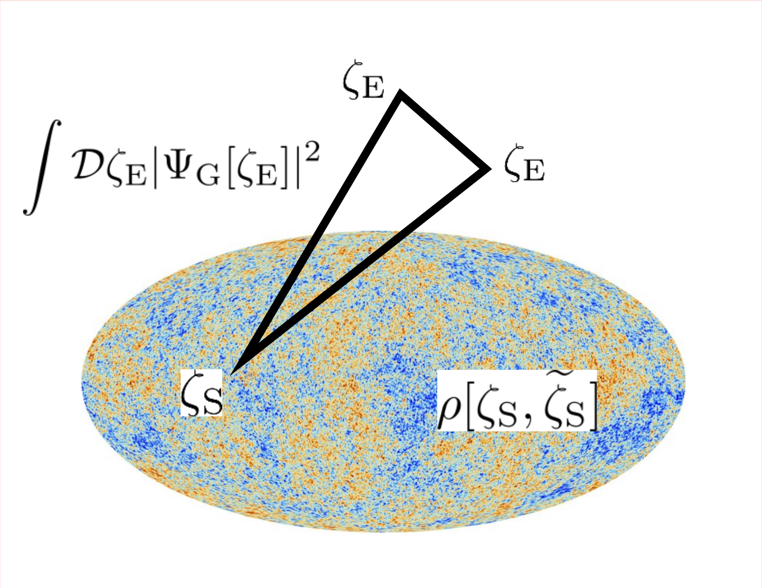
❑ Density matrix

$$\rho_{\rm S}[\zeta_{\rm S},\tilde\zeta_{\rm S}] = \int \mathcal{D}\zeta_{\rm E}(t)\Psi[\zeta_{\rm S},\zeta_{\rm E}]\Psi^*[\tilde\zeta_{\rm S},\zeta_{\rm E}]$$

$$\simeq \Psi_{\rm G}[\zeta_{\rm S}]\Psi^*_{\rm G}[\tilde\zeta_{\rm S}]\int \mathcal{D}\zeta_{\rm E}|\Psi_{\rm G}[\zeta_{\rm E}]|^2 e^{-\frac{1}{6}\int_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3}(\zeta_1\zeta_2\zeta_3\psi_3+\tilde\zeta_1\tilde\zeta_2\tilde\zeta_3\psi_3^*)}$$

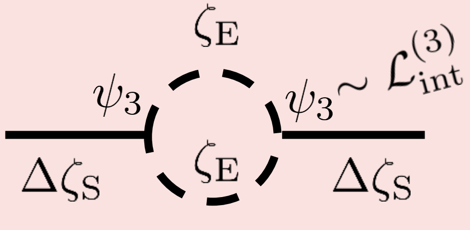
$$\equiv \rho_{\rm diag}\times \exp\left[-\int_{\mathbf{k}_{\rm S}}\Gamma\ \Delta\zeta^2_{\mathbf{k}_{\rm S}}\right]$$

$\equiv (\zeta-\tilde\zeta)^2$



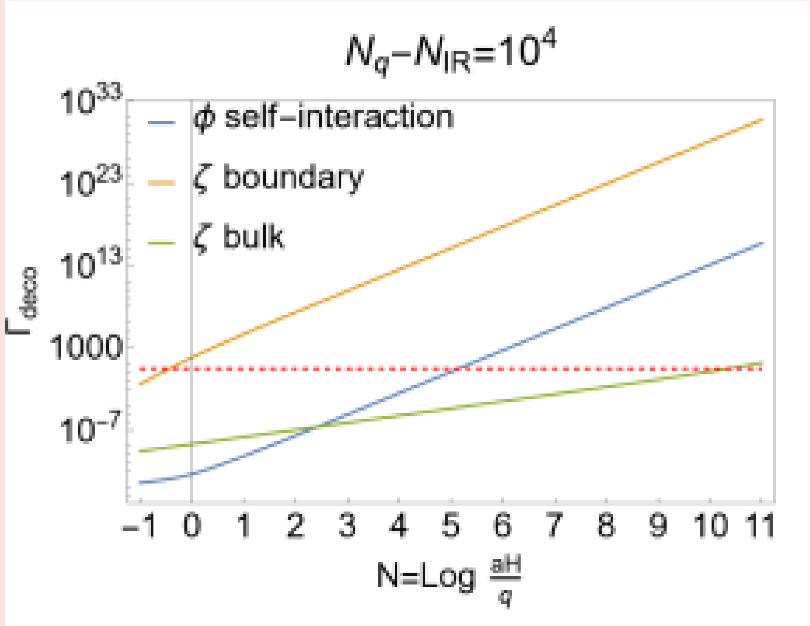
❑ Decoherence rate

(decay rate of off-diagonal component)

$\Gamma \approx$ 

$$\sim \frac{1}{\epsilon^2}\left(\frac{aH}{k_{\rm S}}\right)^6 + \epsilon^2\left(\frac{aH}{k_{\rm S}}\right)^3$$

Loop at the time slice — Boundary term — Bulk term



[Sou et al. 2207.04435]

WITH IR divergence and UV divergence $\Gamma \supset \log\frac{k_{\rm S}}{k_{\rm IR}}, \Lambda^{\#}_{\rm UV}$

Some cancellations? Regularization?

Consistency condition for loop calculations

[Sou et al. 2207.04435]

[Nelson 1601.03734]

$$\psi_3 \longleftarrow \quad S_3 = \int dt d^3x \Big\{ a^3\epsilon^2\zeta\dot{\zeta}^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2a\epsilon\dot{\zeta}\partial\zeta\partial\chi + 2f(\zeta)\frac{\delta\mathcal{L}}{\delta\zeta}\Big|_1 + \mathcal{L}_b \Big\}, \quad \partial^2\chi \equiv a^2\epsilon\dot{\zeta}$$

$$\begin{aligned}\mathcal{L}_b = \partial_t\Big[& -9a^3H\zeta^3 + \frac{a}{H}\zeta(\partial\zeta)^2 \\ & - \frac{1}{4aH^3}(\partial\zeta)^2\partial^2\zeta - \frac{a\epsilon}{H}\zeta(\partial\zeta)^2 \\ & - \frac{\epsilon a^3}{H}\zeta\dot{\zeta}^2 + \frac{1}{2aH^2}\zeta(\partial_i\partial_j\zeta\partial_i\partial_j\chi - \partial^2\zeta\partial^2\chi) \\ & - \frac{\eta a}{2}\zeta^2\partial^2\chi - \frac{1}{2aH}\zeta(\partial_i\partial_j\chi\partial_i\partial_j\chi - \partial^2\chi\partial^2\chi)\Big]\end{aligned}$$

Necessary for correlation function

- **Maldacena's consistency condition for wavefunction** [Pimentel 1309.1793]

$$\lim_{k_1\to 0}\psi_3(k_1,k_3) = \left(3 - k_3\frac{d}{dk_3}\right)\psi_2(k_3)$$

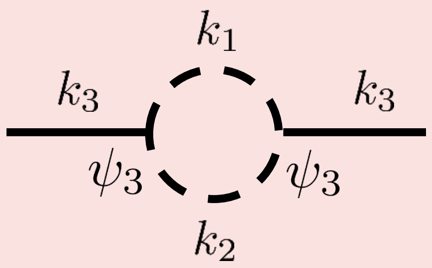
Cf.

$$\lim_{k_1\to 0}\langle\zeta_1\zeta_2\zeta_3\rangle = -\langle\zeta_1\zeta_1\rangle\left(3 + k_3\frac{d}{dk_3}\right)\langle\zeta_3\zeta_3\rangle$$

[Maldacena astro-ph/0210603]

$$\langle\zeta_1\zeta_2\rangle = \frac{1}{2\,\mathrm{Re}[\psi_2(k_1)]}, \quad \langle\zeta_1\zeta_2\zeta_3\rangle = -\frac{2\,\mathrm{Re}[\psi_3]}{\prod_{i=1}^3 2\,\mathrm{Re}[\psi_2(k_i)]}$$

- **Loop diagram at the time slice**



IR: $k_1 \ll k_2 \simeq k_3 \ll aH \longrightarrow \log k_1$ from $\int \langle\zeta_1\zeta_1\rangle\, k_1^2 dk_1$

UV: $k_1 \simeq k_2 \gg aH \gg k_3 \longrightarrow k_1^5$ from $\partial_t(a\zeta(\partial_i\zeta)^2/H)$

False decoherence during inflation

[Sano and Tokuda 2504.10472]

❑ IR and UV divergences $(\rho_{\text{off-diag}} \sim e^{-\Gamma})$

$$\Gamma \sim \left[\frac{1}{\epsilon^2}\left(\frac{aH}{k_\mathrm{S}}\right)^6 + \epsilon^2\left(\frac{aH}{k_\mathrm{S}}\right)^3\right]\underbrace{(1+\log(k_\mathrm{IR}/k_\mathrm{S}))}_{\text{IR cutoff}} + \underbrace{\frac{1}{\epsilon^2}\left(\frac{\Lambda}{aH}\right)^5}_{\text{UV cutoff}}$$

IR div.: Affected by very beginning of inflation?

UV div.: Divergent offset to decoherence exists. Never quantum?

✓ Proper observables should be insensitive to deep IR and deep UV contributions.
(e.g., adiabaticity: rapid modes decouple to slow modes. [Unruh 1110.2199 for QFT])

IR: local observer's coordinate

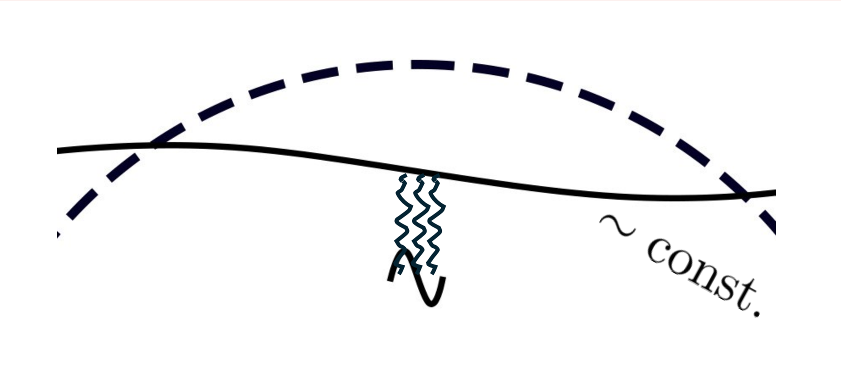
UV: time averaged observables as well as renormalization

Local Observer Effect in Correlation Function

[Tanaka and Urakawa 1103.1251, Pajer et al. 1305.0824]

$$\langle\zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle \supset \int_{k_1\ll k_3} \frac{k_1^2 dk_1\ k_3^2 dk_3}{k_1^3 k_3^3} \sim \log k_1\Big|_{k_1\to 0}$$

Short modes strongly correlates with constant long modes (?)



- **Conformal free-falling observer** $\mathbf{x}_{\mathrm{F}} \simeq (1+\zeta_{\mathrm{L}})\mathbf{x}\ ,\ \ ds^2 = a^2(-d\tau^2 + d\mathbf{x}_{\mathrm{F}}^2) + \cdots$
  (Conformal Fermi normal coordinate)

➡ $\zeta_{\mathrm{F},\mathbf{k}} \simeq \zeta_{\mathbf{k}} + \zeta_{\mathrm{L}}(3 + k\partial_k)\zeta_{\mathbf{k}}$

➡ $\lim_{k_1\to 0}\langle\zeta_1\zeta_2\zeta_3\rangle_{\mathrm{F}} = \lim_{k_1\to 0}\langle\zeta_1\zeta_2\zeta_3\rangle + \langle\zeta_1\zeta_1\rangle\left(3 + k_3\frac{d}{dk_3}\right)\langle\zeta_3\zeta_3\rangle = \underline{0}$ IR correlations are turned off

➡ $\langle\zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle_{\mathrm{F}} \xrightarrow[\mathrm{IR}]{} \int_{k_1\ll k_3} \frac{k_1}{k_3^3} dk_1\ dk_3$ Finite result

Local Observer Effect in Wavefunction Formalism

[Sano and Tokuda 2504.10472]

- ❑ **Wavefunction in free-falling coordinate**

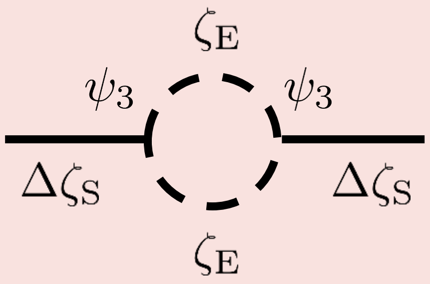
$$\Psi[\zeta] = \exp\left[-\frac{1}{2}\int_{\mathbf{k}_1,\mathbf{k}_2} \psi_2 \zeta_{k_1}\zeta_{k_2} - \frac{1}{3!}\int_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3} \psi_3 \zeta_{k_1}\zeta_{k_2}\zeta_{k_3} - \cdots\right]$$

$$\zeta_{\mathbf{k}} \simeq \zeta_{\mathrm{F},\mathbf{k}} - \zeta_{\mathrm{L}}(3 + k\partial_k)\zeta_{\mathrm{F},\mathbf{k}}$$

Changing the expansion basis

$$= \Psi_{\mathrm{F}}[\zeta_{\mathrm{F}}] = \exp\left[-\frac{1}{2}\int_{\mathbf{k}_1,\mathbf{k}_2} \psi_{\mathrm{F},2} \zeta_{\mathrm{F},k_1}\zeta_{\mathrm{F},k_2} - \frac{1}{3!}\int_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3} \psi_{\mathrm{F},3} \zeta_{\mathrm{F},k_1}\zeta_{\mathrm{F},k_2}\zeta_{\mathrm{F},k_3} - \cdots\right]$$

➡ $$\lim_{k_1\to 0}\psi_{\mathrm{F},3} = \lim_{k_1\to 0}\psi_3 - \left(3 - k_3\frac{d}{dk_3}\right)\psi_2 = \underline{0}$$

➡ $\Gamma_{\mathrm{IR}} \sim$ 

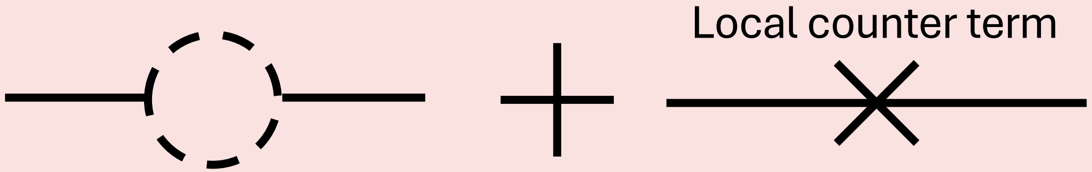
$\sim \cancel{\log(k_{\mathrm{IR}}/k_{\mathrm{S}})}$

$(k_{\mathrm{E}}/k_{\mathrm{S}})^2$ moderation for each ψ_3

$$\Gamma_{\mathrm{IR,F}} \sim (k_{\mathrm{IR}}/k_{\mathrm{S}})^4 \sim 0$$

UV divergence in Equal Time

- ❑ **Scattering**



$$S_{\rm ren} = S_0 + S_{\rm CT}$$

- ❑ **Equal time correlators**

[Balasubramanian et al. 1108.3568 and Bucciotti 2410.01903 for flat spacetime examples etc.]

$$\langle \mathcal{O}^{\mathbf{k}}_{1,\rm ren} \mathcal{O}^{-\mathbf{k}}_{2,\rm ren}(t) \rangle \sim \int d(\mathbf{x}_1 - \mathbf{x}_2) \frac{1}{|\mathbf{x}_1 - \mathbf{x}_2|^{2\Delta}}$$

diverges even after renormalization when $\Delta \geq \frac{3}{2}$.

"Equal time" is beyond IR effective theory?

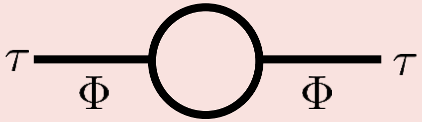
➡ Time averaged observables naturally shows decoupling of UV physics

(Observers do not have much time resolution to see the equal time correlators)

Time Averaged Observables

[Sano and Tokuda 2504.10472]

$\langle \Phi_1 \Phi_2(\tau) \rangle \supset$



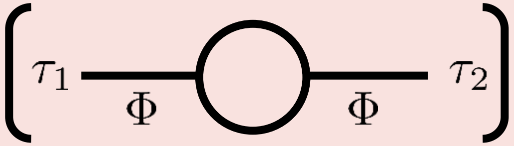
$\Phi = \zeta$ or π

$\tau:$ Conformal time

Time averaging

$$\langle \overline{\Phi}_1 \overline{\Phi}_2(\tau) \rangle = \int d\tau_1 d\tau_2 W_\tau(\tau_1) W_\tau(\tau_2) \langle \Phi_1(\tau_1) \Phi_2(\tau_2) \rangle$$

$$\supset \int d\tau_1 d\tau_2 W_\tau(\tau_1) W_\tau(\tau_2)$$



$$\int^{\Lambda} k^{\#} dk \longrightarrow \Lambda^{\#}$$

$$\frac{1}{|\tau_1 - \tau_2|^{\#}}$$

From time-ordered loop contributions.
This is (expected to be) renormalized.

$$\frac{e^{-ik(\tau_1 - \tau_2)}}{|\tau_1 - \tau_2|^{\#}}$$

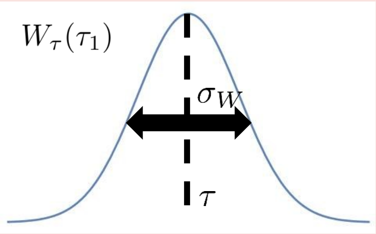
From Wightman function.
Not renormalized in standard procedure.

❑ Time averaging

$$W_\tau(\tau_1) = \frac{e^{-(\tau_1 - \tau)^2 / 2\sigma_W^2}}{\sqrt{2\pi\sigma_W^2}},$$

$$G(k; \tau_1, \tau_2) \propto e^{-ik_1(\tau_1 - \tau_2)}$$

Green function



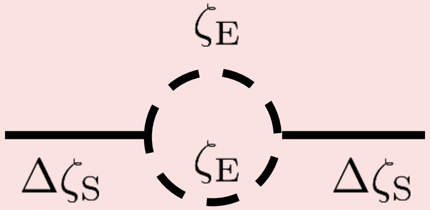
$$\Gamma_{\rm UV} \sim \int_{k > aH} dk\ k^{\#} e^{-k^2 \sigma_W^2}$$

Exponential decay in sub-horizon

Summary: Genuine decoherence during inflation

- **False contributions** $\quad (\rho_{\text{off-diag}} \sim e^{-\Gamma})$

$\Gamma_{\text{false}} \approx$



Loop at the time slice

$$\sim (1+\log(k_{\rm IR}/k_{\rm S}))\left[\frac{1}{\epsilon^2}\left(\frac{aH}{k_{\rm S}}\right)^6 + \epsilon^2\left(\frac{aH}{k_{\rm S}}\right)^3\right] + \frac{1}{\epsilon^2}\left(\frac{\Lambda}{aH}\right)^5$$

IR cutoff $\qquad$ UV cutoff

Long mode is absorbed in geodesic coordinate.

$$ds^2 = a^2(-d\tau^2 + e^{2\zeta}d\mathbf{x}^2) = a^2(-d\tau^2 + d\mathbf{x}_{\rm F}^2) + \cdots$$

$$\lim_{k_1\to 0}\psi_{{\rm F},3} = \lim_{k_1\to 0}\psi_3 - \left(3 - k_3\frac{d}{dk_3}\right)\psi_2 = 0$$

Classified to two components when averaging in time.

$$\frac{1}{|\tau_1-\tau_2|^{\#}} \qquad \frac{e^{-ik(\tau_1-\tau_2)}}{|\tau_1-\tau_2|^{\#}}$$

Renormalized $\qquad$ Averaged out

$$\Gamma_{\rm UV} \sim \int_{k>aH} dk\ k^{\#} e^{-k^2\sigma_W^2}$$

✓ Leading scaling in the previous work is genuine

$$\Gamma_{\rm genuine} \sim \underbrace{\frac{1}{\epsilon^2}\left(\frac{aH}{k_{\rm S}}\right)^6}_{\text{boundary term}} + \underbrace{\epsilon^2\left(\frac{aH}{k_{\rm S}}\right)^3}_{\text{bulk term}}$$

Outlook: Importance of late time evolutions

- **Boundary terms** [Sano and Tokuda ’25]

  ✓ During inflation

  $$\Gamma_{\rm inf} \sim \underbrace{\frac{1}{\epsilon^2}\left(\frac{aH}{k_{\rm S}}\right)^6}_{\text{Boundary term}} + \underbrace{\epsilon^2\left(\frac{aH}{k_{\rm S}}\right)^3}_{\text{Bulk term}}$$

  $\Longrightarrow$

  ✓ Late time universe (but before re-entry)

  $$\Gamma_{\rm rad.\ dom.} \sim \underbrace{\frac{1}{\epsilon^2}\left(\frac{a_{\rm f}H_{\rm f}}{k_{\rm S}}\right)^6\left(\frac{a}{a_{\rm f}}\right)^2}_{\text{Boundary term}} + \underbrace{\epsilon^2\left(\frac{a_{\rm f}H_{\rm f}}{k_{\rm S}}\right)^3\left(\frac{a}{a_{\rm f}}\right)^5}_{\text{Bulk term}}$$

- **Time averaging scale?**

- **High-frequency gravitational wave** [Takeda and Tanaka ’25]

  ✓ GW with frequency $f_{\rm GW} \gtrsim 100\ {\rm Hz}$ (?) may be quantum even today!

  * Estimation of thermal decoherence by a scalar field, keeping reheating in mind

- **Outlook**

  ✓ Systematic approaches to sub-horizon evolution for a more realistic model?

  ✓ Entanglement harvesting through detectors? Graviton-photon conversion?

  ✓ What is more than proving quantumness of gravity? QG from bottom up.

Back up slides

Jacobian and momentum correlators

- In general, correlation functions are expressed as

$$\langle\hat{\mathcal{O}}[\zeta,\pi]\rangle = \int \mathcal{D}\zeta_c \left(\mathcal{O}\left[\zeta_c, -i\frac{\delta}{\delta\zeta_\Delta}\right]\Psi\left[\zeta_c+\frac{\zeta_\Delta}{2}\right]\Psi^*\left[\zeta_c-\frac{\zeta_\Delta}{2}\right]\right)_{\zeta_\Delta=0}$$

$$\zeta_c = \frac{\zeta+\tilde{\zeta}}{2},\quad \zeta_\Delta = \zeta-\tilde{\zeta}$$

$$\xrightarrow{\text{Coord. Transf.}} \langle\hat{\mathcal{O}}[\zeta_\mathrm{F},\pi_\mathrm{F}]\rangle = \int \mathcal{D}\zeta_{c,\mathrm{F}} \underbrace{\left|\frac{\delta\zeta_c}{\delta\zeta_{c,\mathrm{F}}}\right|}_{\text{Jacobian}} \left(\mathcal{O}\left[\zeta_{c,\mathrm{F}}, -i\frac{\delta}{\delta\zeta_{\Delta,\mathrm{F}}}\right]\Psi_\mathrm{F}\left[\zeta_{c,\mathrm{F}}+\frac{\zeta_{\Delta,\mathrm{F}}}{2}\right]\Psi^*_\mathrm{F}\left[\zeta_{c,\mathrm{F}}-\frac{\zeta_{\Delta,\mathrm{F}}}{2}\right]\right)_{\zeta_{\Delta,\mathrm{F}}=0}$$

- **Momentum correlators in the geodesic coordinate**

$$\left.\begin{aligned}
\lim_{k_1\to 0}\langle\pi_{1,\mathrm{F}}\zeta_{2,\mathrm{F}}\zeta_{3,\mathrm{F}}\rangle &= -\frac{(3-k_3\partial_{k_3})\,\mathrm{Im}\,\psi_2(k_3)}{4(\mathrm{Re}\,\psi_2(k_3))^2}\\
\lim_{k_1\to 0}\langle\pi_{1,\mathrm{F}}\pi_{2,\mathrm{F}}\zeta_{3,\mathrm{F}}\rangle &= \frac{\mathrm{Re}[\psi_2(k_3)(3-k_3\partial_{k_3})\psi_2(k_3)]}{4(\mathrm{Re}\,\psi_2(k_3))^2}\\
\lim_{k_1\to 0}\langle\pi_{1,\mathrm{F}}\pi_{2,\mathrm{F}}\pi_{3,\mathrm{F}}\rangle &= -\frac{\mathrm{Im}[\psi_2^2(k_3)(3-k_3\partial_{k_3})\psi_2(k_3)]}{4(\mathrm{Re}\,\psi_2(k_3))^2}
\end{aligned}\right\}$$

Non-vanishing contributions in squeezed limit when the conjugate momentum is soft.

$\Rightarrow$ corresponding to Jacobian?

Tomographic approach to quantum state

[Sano and Tokuda 2504.10472]

- Wavefunction $\Psi[\zeta(t)] = \langle \zeta(t) | \psi \rangle$: defined in equal time. How to consider time averaging?

- **Quantum state tomography**

$$\langle \zeta_1 \zeta_2 \rangle = \frac{1}{2\,\mathrm{Re}[\psi_2(k_1)]}, \quad \langle \zeta_1 \zeta_2 \zeta_3 \rangle = -\frac{2\,\mathrm{Re}[\psi_3]}{\prod_{i=1}^{3} 2\,\mathrm{Re}[\psi_2(k_i)]}$$

$$\langle \pi_1 \zeta_2 \rangle = -\frac{\mathrm{Im}[\psi_2(k_1)]}{2\,\mathrm{Re}[\psi_2(k_1)]}, \quad \langle \pi_1 \zeta_2 \zeta_3 \rangle = \frac{2\,\mathrm{Im}[\psi_2(k_1)\psi_3^*]}{\prod_{i=1}^{3} 2\,\mathrm{Re}[\psi_2(k_i)]}$$

$$\Psi[\zeta] = \exp\left[-\frac{1}{2}\int_{k_1,k_2} \psi_2 \zeta_{k_1}\zeta_{k_2} - \frac{1}{3!}\int_{k_1,k_2,k_3} \psi_3 \zeta_{k_1}\zeta_{k_2}\zeta_{k_3} - \cdots\right]$$

Quantum state is reconstructed from observables

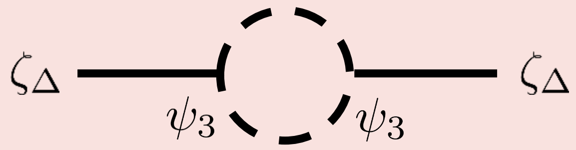
➡ Quantum state is identified as a probability distribution of canonical variables.

✓ E.g., tree-level of averaged quantum fields

$$\langle \overline{\zeta}_1 \overline{\zeta}_2 \rangle \equiv \frac{1}{2\mathrm{Re}[\overline{\psi}_2(k_1)]}, \quad \langle \overline{\pi}_1 \overline{\zeta}_2 \rangle \equiv \frac{\mathrm{Im}[\overline{\psi}_2(k_1)]}{2\,\mathrm{Re}[\overline{\psi}_2(k_1)]}, \quad \cdots \quad \Longleftrightarrow \quad \Psi[\overline{\zeta}] \equiv \exp\left[-\frac{1}{2}\int_{k_1,k_2} \overline{\psi}_2 \overline{\zeta}_{k_1}\overline{\zeta}_{k_2} - \cdots\right]$$

Mathematical identity

with $[\overline{\zeta}_{\mathbf{k}}, \overline{\pi}_{\mathbf{k}'}] \approx i\hbar(2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')$

➡ 

is included in one-loop corrections of correlation functions

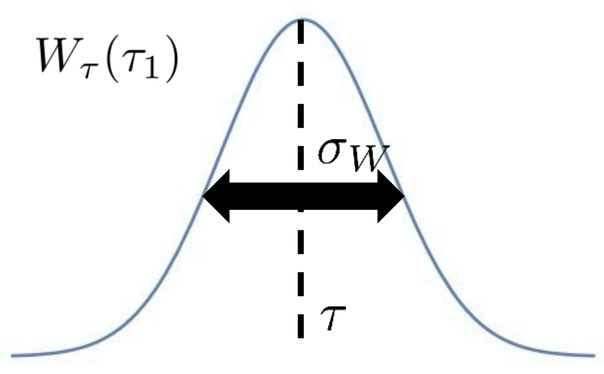
Averaging Scale

[Sano and Tokuda 2504.10472]

$$\Gamma_{\rm UV} \sim \int_{k>aH} dk\ k^{\#} e^{-k^2\sigma_W^2}$$

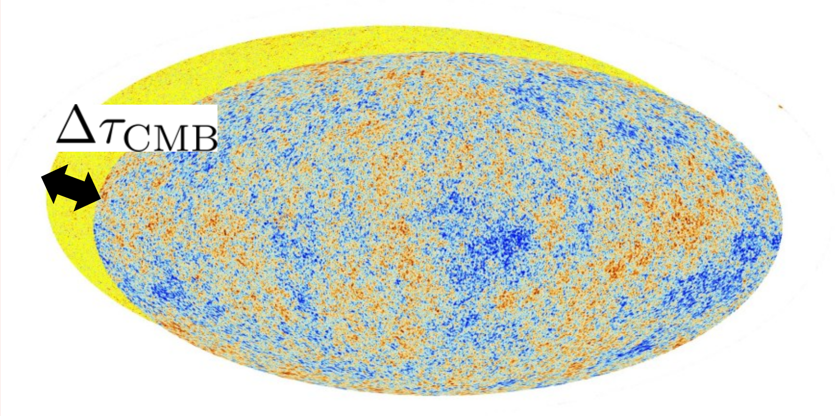
$(a\Lambda_{\rm UV})^{-1} \lesssim \sigma_W \ll k_{\rm S}^{-1}$

to ensure the time smearing only affect UV contributions



- What is σ_W ?
  - ✓ Theoretical resolution $\sigma_W \sim \dfrac{1}{a\Lambda_{\rm UV}}$
  - ✓ Phenomenological scale? E.g., $\Delta\tau_{\rm CMB}$
  - ✓ Observational device's resolution?

When is ζ "measured"?



Observational effect on UV, rather than theoretical resolution, may affect signals