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Detectability of the phase transition gravitational waves in the DFSZ axion model

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Strong CP problem

Why is the CP violation parameter $\bar{\theta}$ so small?

$$\mathcal{L}_{\mathrm{QCD}} \supset \bar{\theta}\,\frac{g_s^2}{32\pi^2} G^{a\mu\nu}\tilde{G}_{a\mu\nu}$$

EDM of neutron:

$$d_n^{(th)} \simeq 2.4(1.0)\times 10^{-16}\,\bar{\theta}\,\mathrm{e\,cm}$$

M. Pospelov and A. Ritz, Phys. Rev. Lett. 83, 2526-2529 (1999)

Experimental upper limit on the neutron EDM:

$$|d_n^{(exp)}| < 1.8\times 10^{-26}\,\mathrm{e\,cm}$$

C. A. Baker, D. D. Doyle, P. Geltenbort, K. Green, M. G. D. van der Grinten, P. G. Harris,
P. Iaydjiev, S. N. Ivanov, D. J. R. May and J. M. Pendlebury, et al. Phys. Rev. Lett. 97,
131801 (2006)

$$\bar{\theta} < 10^{-10}$$

U(1) PQ symmetry

Roberto D Peccei and Helen R Quinn. CP conservation in the presence of pseudoparticles. Physical Review Letters, 38(25):1440, 1977.

PQWW model

Peccei & Quinn

PQ symmetry

Goldstone's theorem

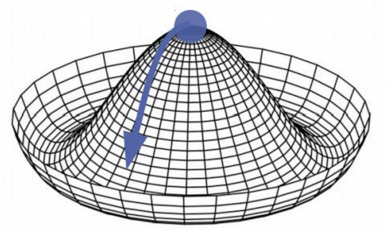
Spontaneous breakdown

Weinberg & Wilczek

Axion

(pseudo-NG boson)

Steven Weinberg. A new light boson? Physical Review Letters, 40(4):223,1978.
Frank Wilczek. Problem of strong p and t invariance in the presence of instantons. Physical Review Letters, 40(5):279, 1978

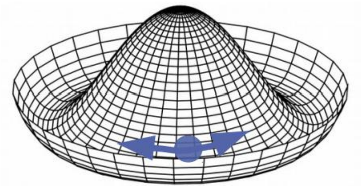


[Phenomenology of Axion Dark Matter](#)

[Andreas Pargner](#) ([KIT, Karlsruhe, IKP](#))

$$\bar{\theta} \longrightarrow a(x)$$

$$\mathcal{L} = \left(\frac{a}{f_a} - \bar{\Theta} \right) \frac{\alpha_s}{8\pi} G^{\mu\nu a} \tilde{G}_{a\mu\nu}$$



Solved the strong CP problem

Axion model

Strong CP problem → **PQWW** *ruled out by* $K^+ \to \pi^+ + \phi$

instanton

t Hooft, G. Phys. Rev.Lett. 37, 8 (1976)

Invisible axion model

DFSZ

KSVZ

M. Dine, W. Fischler, and M. Srednicki, Physics letters B 104, 199 (1981)

$f_a \gg 246$ GeV

J. E. Kim, Phys. Rev. Lett. 43, 103 (1979)
Jihn E Kim. Light pseudoscalars, particle physics and cosmology. Physics Reports, 150(1-2):1–177, 1987

U(1) PQ symmetry breaking

Question 1: How to experimentally verify the process of U(1)PQ symmetry breaking?

Limitations of traditional collider experiments

High energy scale

Axion detection is difficult

$f_a \gg$ **246 GeV**

Other traditional detection methods:

Astrophysical observations: indirect evidence, difficult to verify definitively

Dark matter experiments: Axion dark matter detection sensitivity is limited

New Window: Gravitational Wave

In 2015, LIGO-Virgo made the first direct detection of gravitational waves

The Unique Advantages of Gravitational Wave Detection

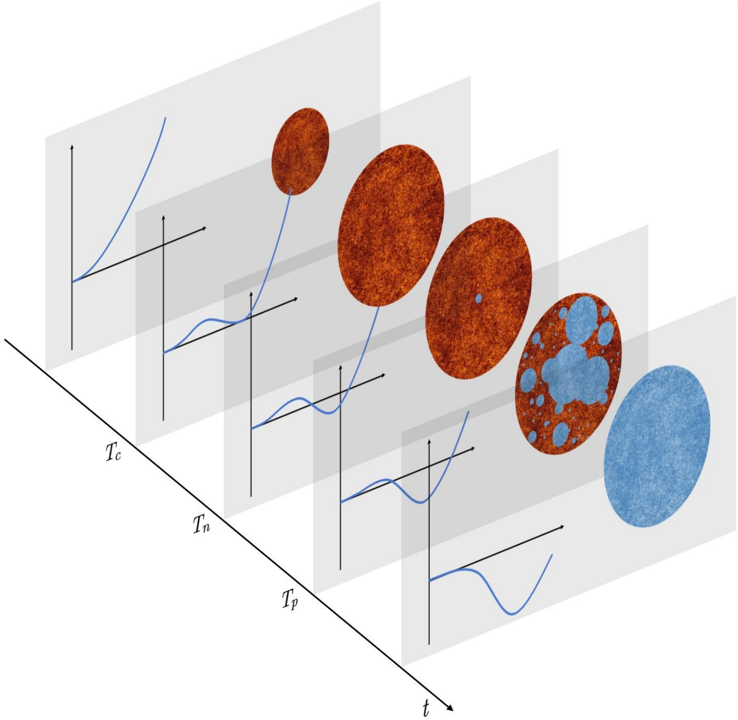
It allows for the detection of phase transition processes occurring at high energy scales

PQ symmetry breaking could be associated with a first-order phase transition

Gravitational Wave

Question 2: What are the mechanisms for gravitational wave generation during the PQ phase transition?

Phase transition gravitational waves

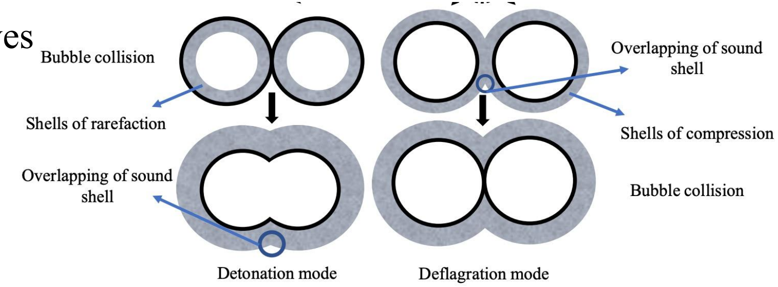


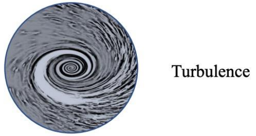
Finite Temperature Effective Field Theory

$$V_{\mathrm{eff}}\left(v_\sigma, T\right) \equiv V_{\mathrm{tree}}\left(v_\sigma\right) + V_{\mathrm{CW}}\left(v_\sigma\right) + V_{\mathrm{T}}\left(v_\sigma, T\right)$$

Sources of Phase Transition Gravitational

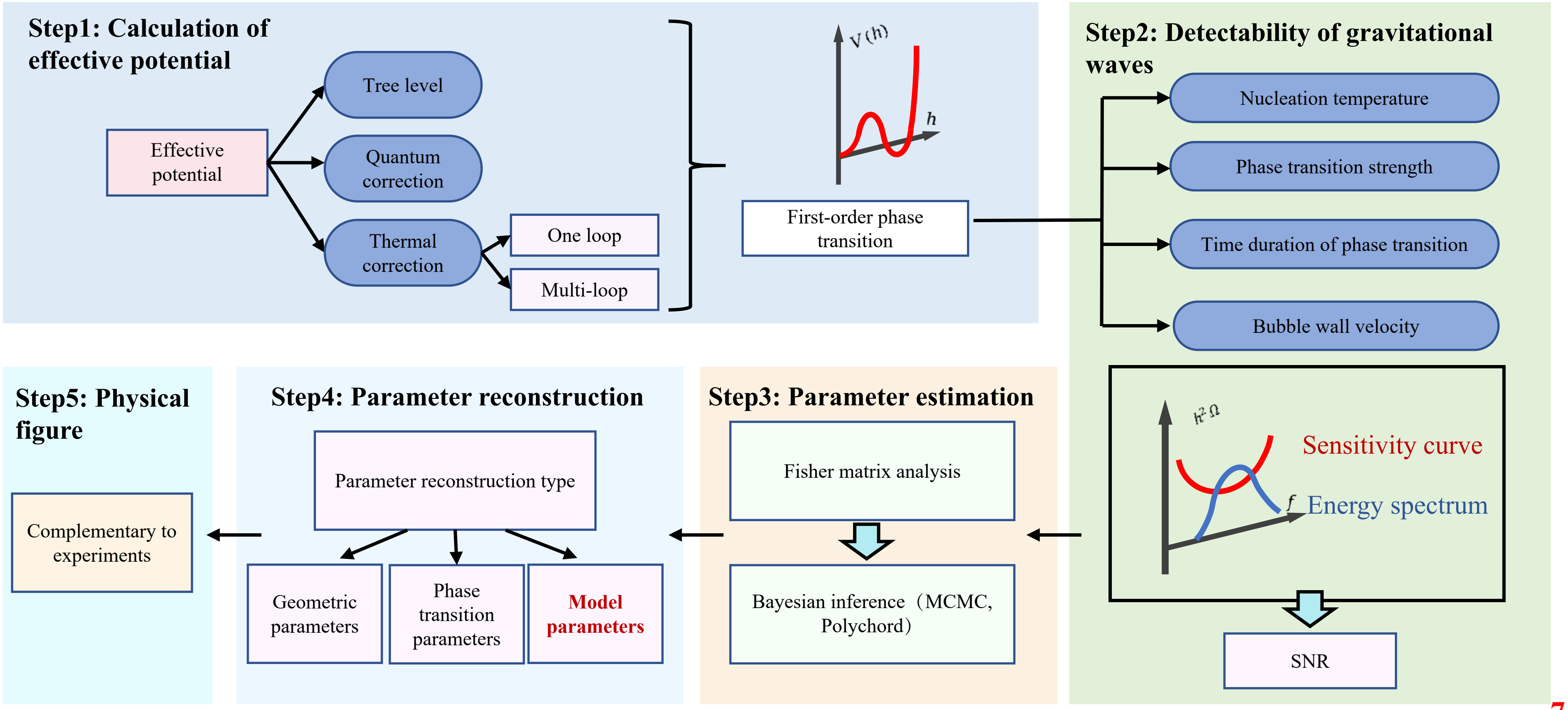
1. Bubble Collisions
2. Sound Waves
3. Turbulence





X. Wang, F. P. Huang and X. Zhang, JCAP 05, 045 (2020)

Phase transition gravitational wave calculation and parameter reconstruction flow



Minimal Dine-Fischler-Srednicki-Zhitnitsky model

$$\begin{aligned} V_{\text{tree}} = & -\mu_1^2 |H_u|^2 - \mu_2^2 |H_d|^2 + \lambda_1 |H_u|^4 + \lambda_2 |H_d|^4 + \lambda_4 \left| H_u^\dagger H_d \right|^2 \\ & - \mu_3^2 |\sigma|^2 + \lambda_3 |\sigma|^4 + \lambda_{12} |H_u|^2 |H_d|^2 + \lambda_{13} |\sigma|^2 |H_u|^2 \\ & + \lambda_{23} |\sigma|^2 |H_d|^2 + \left(\lambda_5 \sigma^2 \tilde{H}_u^\dagger H_d + \text{h.c.} \right) \end{aligned}$$

Extended research on axion models mainly focuses on improving the KSVZ and DFSZ models. For example, adding additional particles or interactions can further reduce the coupling between axions and ordinary matter.

M Dine, W Fischler, and M Srednicki. A simple solution to the strong cp problem with a harmless axion

Minimal Dine-Fischler-Srednicki-Zhitnitsky model

$$\sigma = \frac{1}{\sqrt{2}}\left(v_\sigma + \sigma^0 + i\eta_\sigma^0\right)$$

$$H_u = \begin{pmatrix} h_u^R + ih_u^I \\ \frac{1}{\sqrt{2}}\left(h_u^0 + i\eta_u^0\right) \end{pmatrix}$$

$$H_d = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(h_d^0 + i\eta_d^0\right) \\ h_d^R + ih_d^I \end{pmatrix}$$

$$V_{\text{tree}}\left(\sigma^0\right) = -\frac{1}{2}\mu_3^2\left(\sigma^0\right)^2 + \frac{1}{4}\lambda_3\left(\sigma^0\right)^4$$

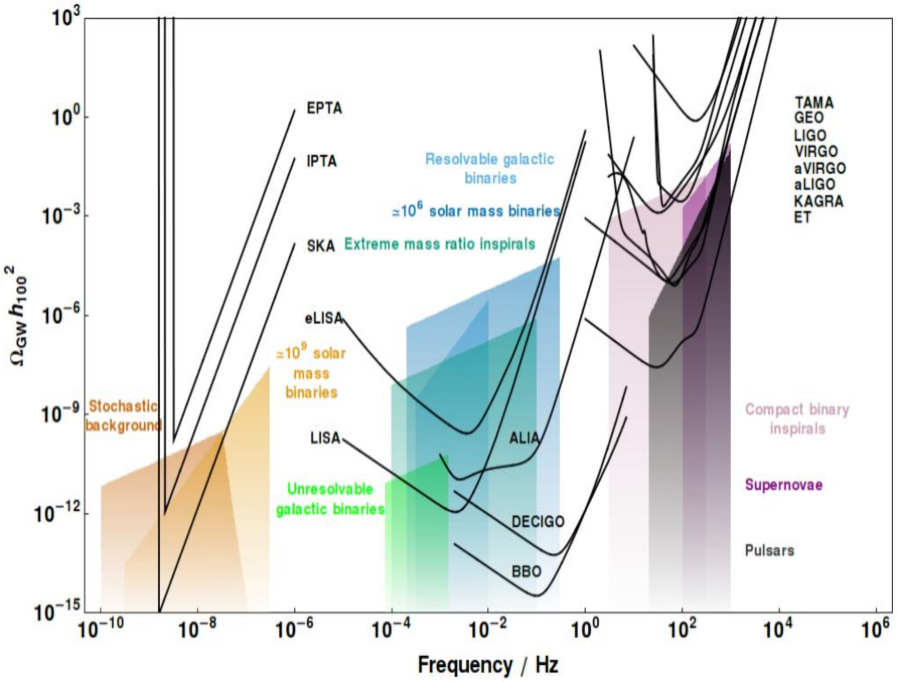
$$V_{\text{CW}}\left(\sigma^0\right) = \frac{1}{64\pi^2}\sum_S n_S m_S^4\left(\sigma^0\right)\left[\log\frac{m_S^2\left(\sigma^0\right)}{\mu^2} - \frac{3}{2}\right]$$

$$V_T\left(\sigma^0, T\right) = \sum_S \frac{n_S T^4}{2\pi^2} J_B\left[\frac{m_S^2\left(\sigma^0\right)}{T^2}\right]$$

Parwani scheme: $m_S^2 \rightarrow \bar{m}_S^2 = m_S^2 + \Pi_S^{ij}$

$$V_{\text{eff}}\left(\sigma^0, T\right) \equiv V_{\text{tree}}\left(\sigma^0\right) + V_{\text{CW}}\left(\sigma^0\right) + V_T\left(\sigma^0, T\right)$$

Gravitational wave detectors



Ground-based gravitational wave detectors:

Second-generation detectors: LIGO, Virgo, and KAGRA

Third-generation detectors: ET and CE

Space-based gravitational wave detectors:

LISA, China's Tianqin and Taiji, and Japan's DECIGO

Pulsar timing arrays:

NANOGrav in North America, EPTA in Europe, PPTA in Australia, and CPTA in China

Gravitational wave detectors

Sensitivity curves: characteristic strain, power spectrum density, and energy density

$$\Omega_{\mathrm{gw}}(f) = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f) = \frac{2\pi^2 S_n(f)}{3H_0^2} f^3$$

$$\begin{tikzcd}
 & S_n(f) \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] & \\
h_c(f) \arrow[rr, leftrightarrow] & & \Omega_{\mathrm{gw}}(f)
\end{tikzcd}$$

E. Thrane and J. D. Romano, Physical Review D 88, 124032 (2013)

SNR

$$SNR = \sqrt{T_i \int_{f_{\min}}^{f_{\max}} df \left(\frac{h_{100}^2 \Omega_{\mathrm{gw}}}{h_{100}^2 \Omega_{\mathrm{noise}}} \right)^2}$$

Detectability

$$SNR \geq threshold\ \ value$$

Gravitational wave detectors

Detector output signal

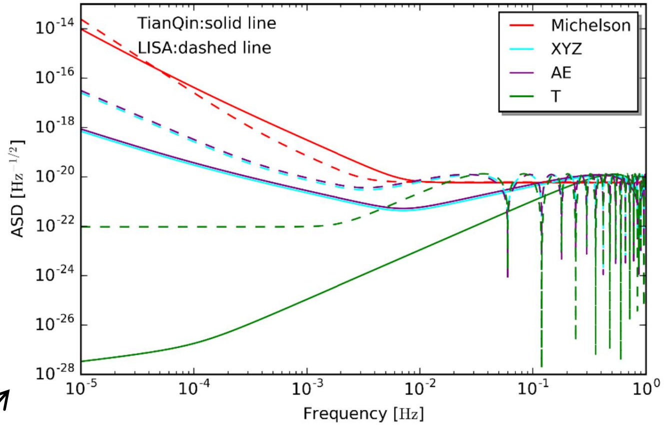
$$s_{gw}(t) = h_{gw}(t) + n_{gw}(t)$$

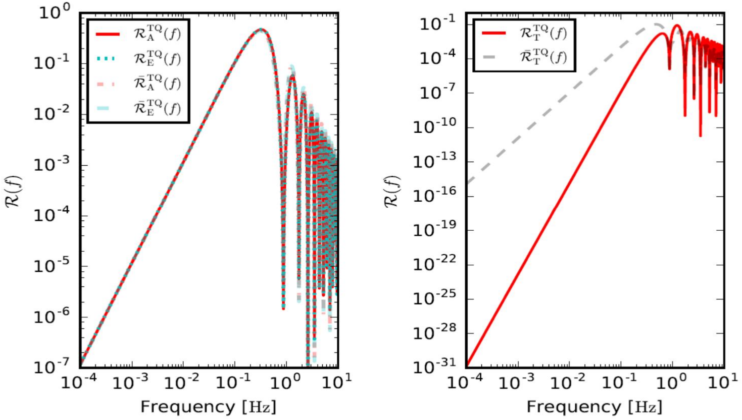
Statistical properties of noise

$$\left\langle n_{gw}^2(t) \right\rangle = \int_0^\infty df\, S_n(f)$$

Sensitivity curves

$$S_{n_I}(f) = h_n^2(f) = \frac{P_{n_I}(f)}{\mathcal{R}_I(f)}$$





E. Thrane and J. D. Romano, Physical Review D 88, 124032 (2013)

Gravitational wave detectors

Cross-correlation techniques:

Using two or more geographically separate and independently operated detectors for simultaneous observations.

Overlap reduction function

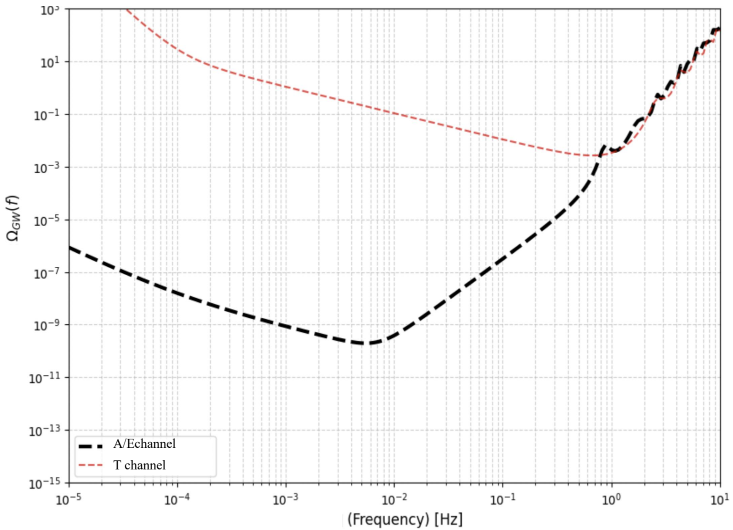
$$\Gamma(f) = \frac{1}{8\pi} \sum_{A=+,x} \int_{S^2} d\hat{\Omega} e^{i2\pi f \hat{\Omega}\cdot\Delta x} F_1^A(\hat{\Omega}) F_2^A(\hat{\Omega})$$

Optimized (SNR)

$$SNR_{\text{optimized}} = T_t^{1/2} \left[\int_{-\infty}^{\infty} df \, \frac{S_h^2(f)\Gamma^2(f)}{S_{2n}^2(f)} \right]^{1/2}$$

Sensitivity curves

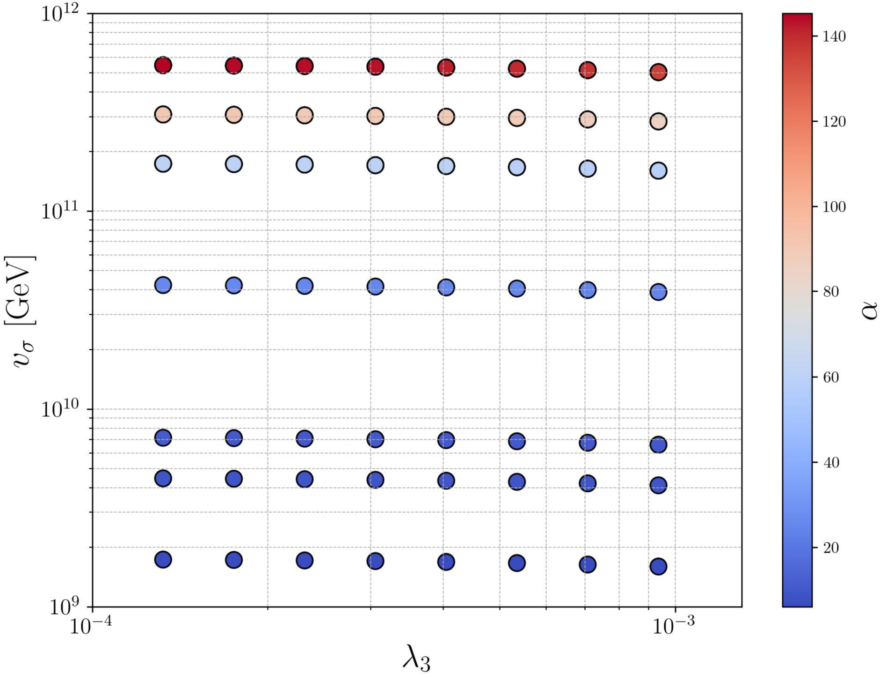
$$S_{n_{IJ}}(f) = \frac{\sqrt{P_{n_I}(f) P_{n_J}(f)}}{\Gamma_{IJ}(f)}$$



By using matched filtering techniques, the SNR of the detector network can be maximized

Detectability of the phase transition gravitational waves in the DFSZ axion model

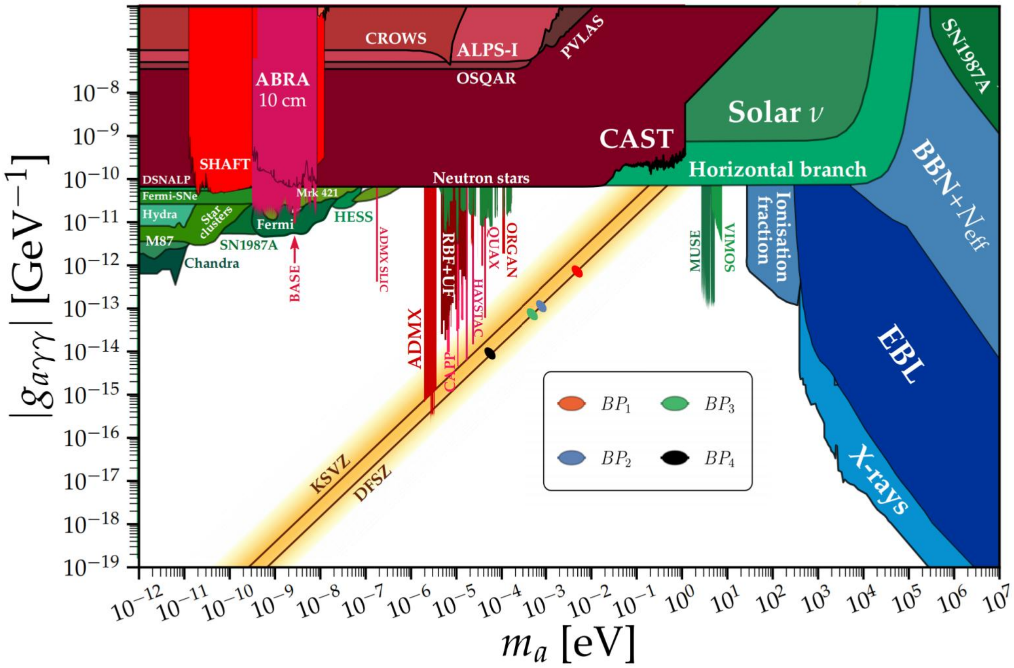
Parameter space exploration results of the SFOPT in the DFSZ axion model



Aidi Yang, FPH*, arXiv:2404.18703, JCAP (2025)

Each point plotted in the figure represents a set of parameters for which a SFOPT occurs. The color of each point in the figure indicates the value of α, with the color bar ranging from blue to red.

Detectability of the phase transition gravitational waves in the DFSZ axion model



Aidi Yang, FPH*, arXiv:2404.18703, JCAP (2025)

All four points fall within the expected yellow region which represents the range of axion masses and coupling constants that have not yet been excluded by ALP-photon coupling experiments.

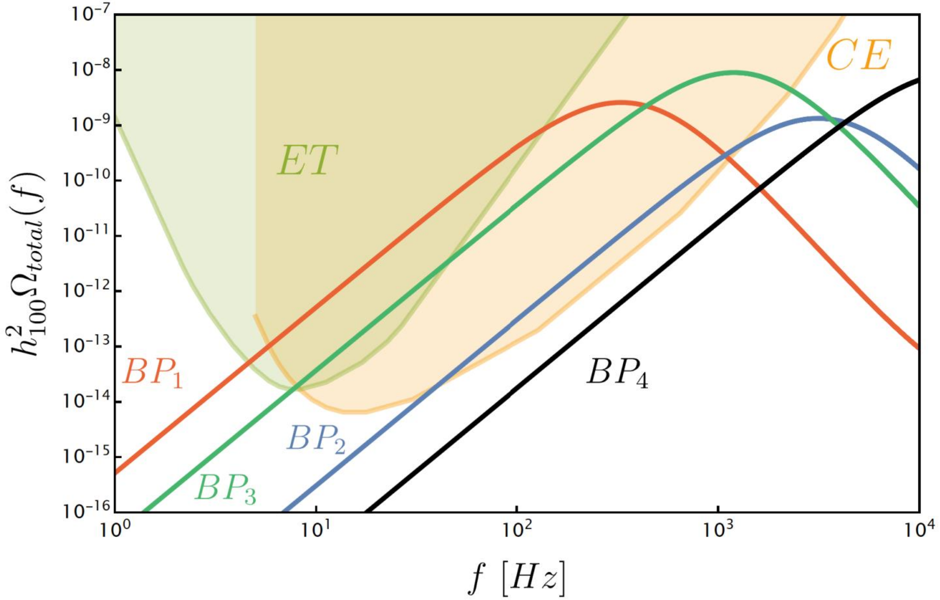
Detectability of the phase transition gravitational waves in the DFSZ axion model

0.001-0.00001 eV DFSZ axion

	$f_{\rm PQ}$ [GeV]	m_a [eV]	$g_{a\gamma\gamma}$ [GeV^{-1}]
BP$_1$	1.24×10^{9}	4.59×10^{-3}	7.01×10^{-13}
BP$_2$	7.01×10^{9}	8.12×10^{-4}	1.21×10^{-13}
BP$_3$	1.62×10^{10}	3.51×10^{-4}	5.37×10^{-14}
BP$_4$	1.57×10^{11}	3.63×10^{-5}	5.54×10^{-15}

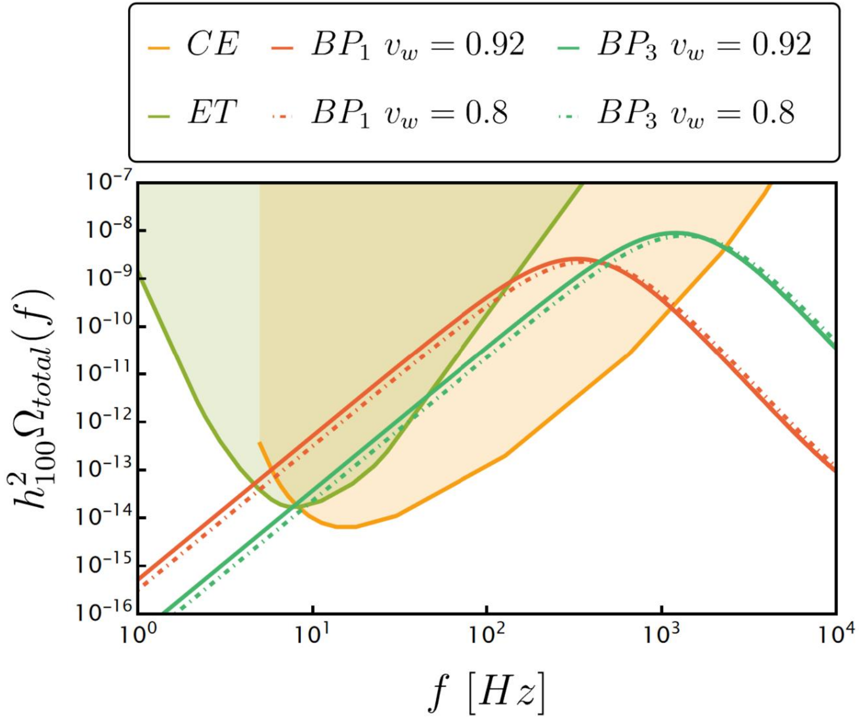
Aidi Yang, FPH*,
arXiv:2404.18703, JCAP

$$\mathrm{SNR_{gw}} = \sqrt{T_t \int_{f_{\min}}^{f_{\max}} df \left(\frac{h_{100}^2 \Omega_{\rm gw}}{h_{100}^2 \Omega_{\rm sens}} \right)^2}$$



SNR : $BP_1(21794.10)$ $BP_2(46.19)$
$BP_3(3964.97)$ $BP_4(0.15)$

Detectability of the phase transition gravitational waves in the DFSZ axion model



The uncertainties of each parameter?

FM analysis is a powerful statistical tool widely used to estimate the precision with which model parameters can be determined from a given set of observations.

Aidi Yang, FPH*,
arXiv:2404.18703,JCAP (2025)

Fisher analysis

Probability distribution function $f(x;\Theta)$

Log-likelihood function $l(\Theta;x) = \ln f(x;\Theta)$

The Fisher matrix element is defined as the expected value of the partial derivative of the log-likelihood function with respect to the parameters Θi and Θj:

$$F_{ij} = E\left[\frac{\partial l(\Theta;x)}{\partial \Theta_i}\frac{\partial l(\Theta;x)}{\partial \Theta_j}\right]$$

Covariance matrix:

$$C_{ij} = F_{ij}^{-1}$$

Diagonal elements represent the variance of the parameters.

Off-diagonal elements represent the covariance between the parameters.

Ronald A Fisher. On the mathematical foundations of theoretical statistics

Detectability of the phase transition gravitational waves in the DFSZ axion model

Likelihood function under Gaussian approximation

$$l_G = -\frac{1}{2}\sum_{b=1}^{N_b} \frac{n_b\left(\overline{E_b} - A_b\right)^2}{A_b^2} - \sum_{b=1}^{N_b} \ln\left(2A_b\right) + \text{const.}$$

Substituting into:

$$F_{ij} = \sum_{b=1}^{N_b} \frac{n_b}{A_b^2} \frac{\partial A_b}{\partial \Theta_i} \frac{\partial A_b}{\partial \Theta_j}$$

Use the total power spectrum instead :

$$F_{ij} = T_{obs} \sum_{b=1}^{N_b} \frac{\Delta f_b}{\Omega_t^2} \frac{\partial \Omega_t}{\partial \Theta_i} \frac{\partial \Omega_t}{\partial \Theta_j}$$

Total power spectrum:

Contains the gravitational wave signal and all associated noise sources, such as detector noise and foreground noise from compact binary star mergers.

$$\Omega_t = \Omega_n + \Omega_{cbc} + \Omega_{gw}$$

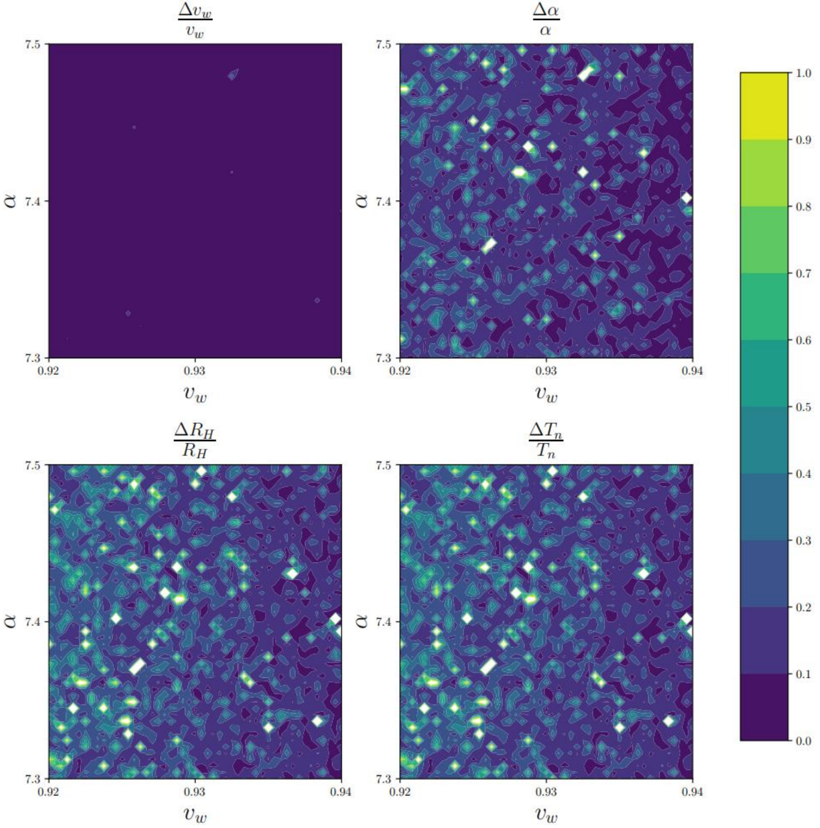
Ronald A Fisher. On the mathematical foundations of theoretical statistics

Detectability of the phase transition gravitational waves in the DFSZ axion model

Relative uncertainty with phase transition dynamics parameters bubble wall speed v_w, transition strength α, Hubble-scaled mean R_H bubble spacing T_n and nucleation temperature

Aidi Yang, FPH*, arXiv:2404.18703,JCAP (2025)

If the detector observes a signal, v_w will be the first and most accurately measured parameter。



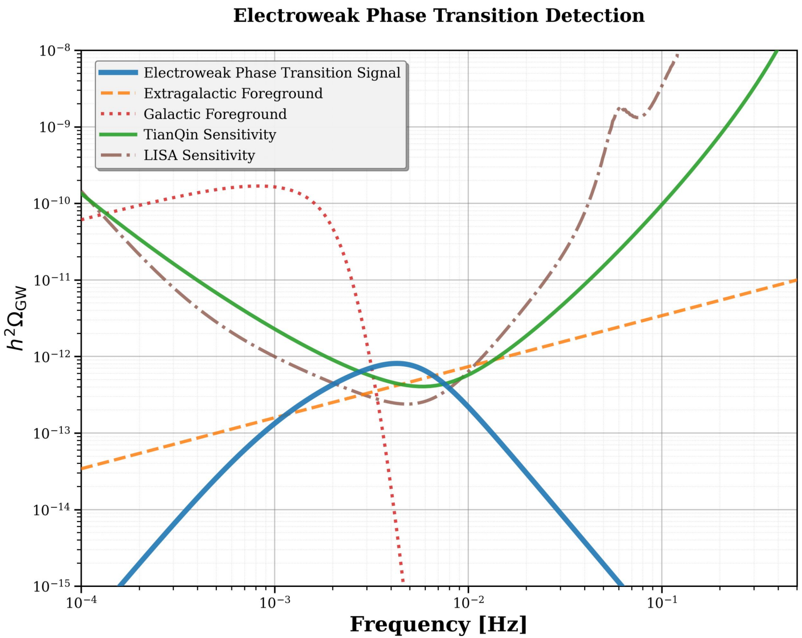
Parameter reconstruction

Reconstruction Type

- Geometric parameters: Describe the spectral shape (amplitude, frequency breakpoints, spectral index)

- Thermodynamic parameters: Describe the phase transition physics (phase transition temperature, phase transition duration, etc.)

- Basic model parameters: Particle mass, coupling constant, etc



Parameter reconstruction links gravitational wave observations to fundamental physical models, providing constraints complementary to particle physics experiments.

Parameter reconstruction

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- Geometric parameters: Describe the spectral shape (amplitude, frequency breakpoints, spectral index)

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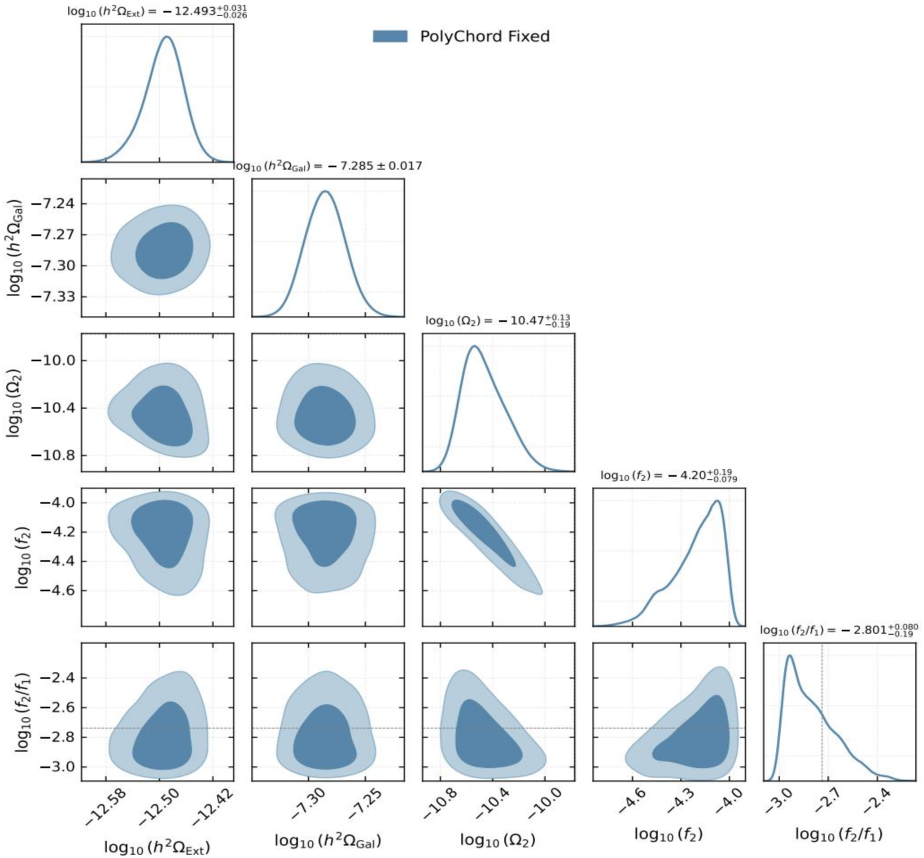


Summary and Outlook

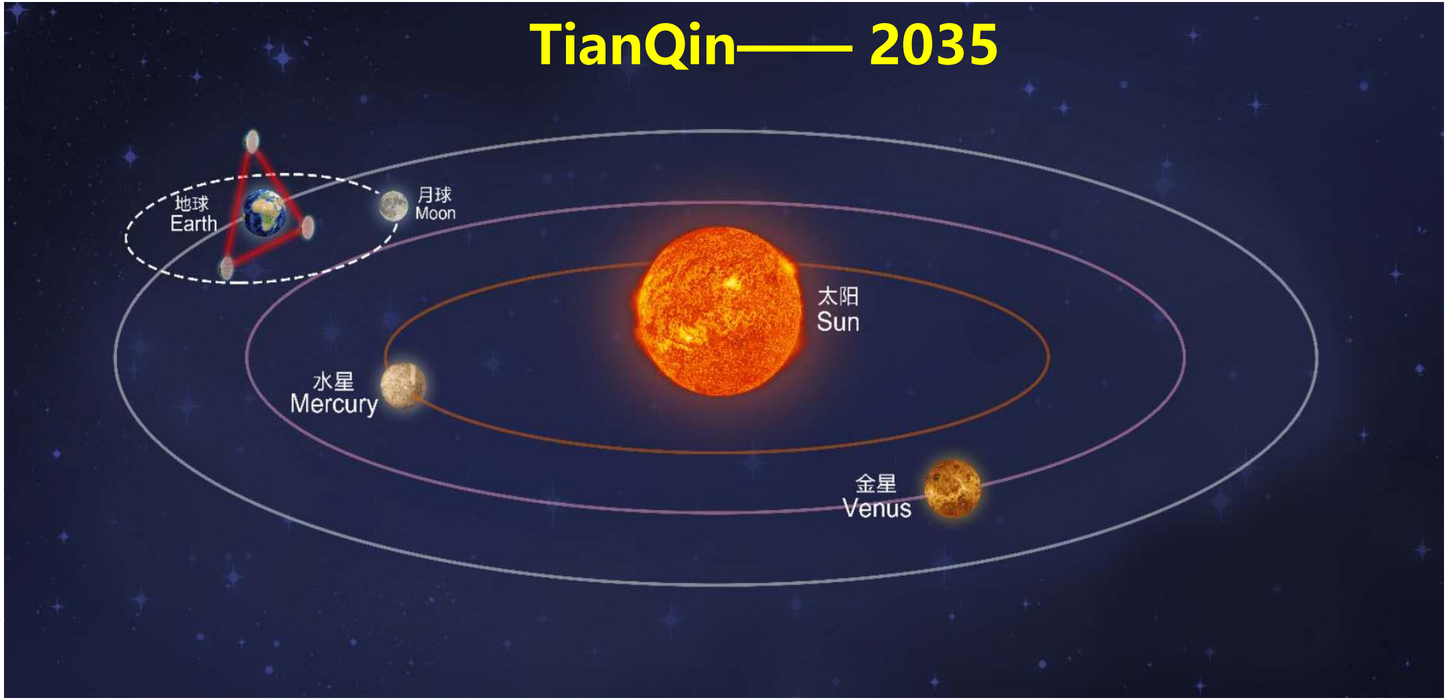
Summary:

1. This work establishes the detectability of gravitational waves from phase transitions that produce axions. We demonstrate how gravitational waves can be used to detect PQ symmetry violation. This demonstrates that gravitational waves are currently the newest means of detecting high-energy scale violation.

2. Through Fisher matrix analysis, we found that the relative uncertainty of the bubble wall velocity is smaller than that of other parameters, which means that if the detector observes a signal, the bubble wall velocity will be the first and most accurately measured parameter.

3. Outlook: Use MCMC to reconstruct the parameters of the axion model and the properties of axions.

Thanks!



Thanks!