Quantum Entanglement of Ions for Light Dark Matter Detection

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arXiv:2311.11632

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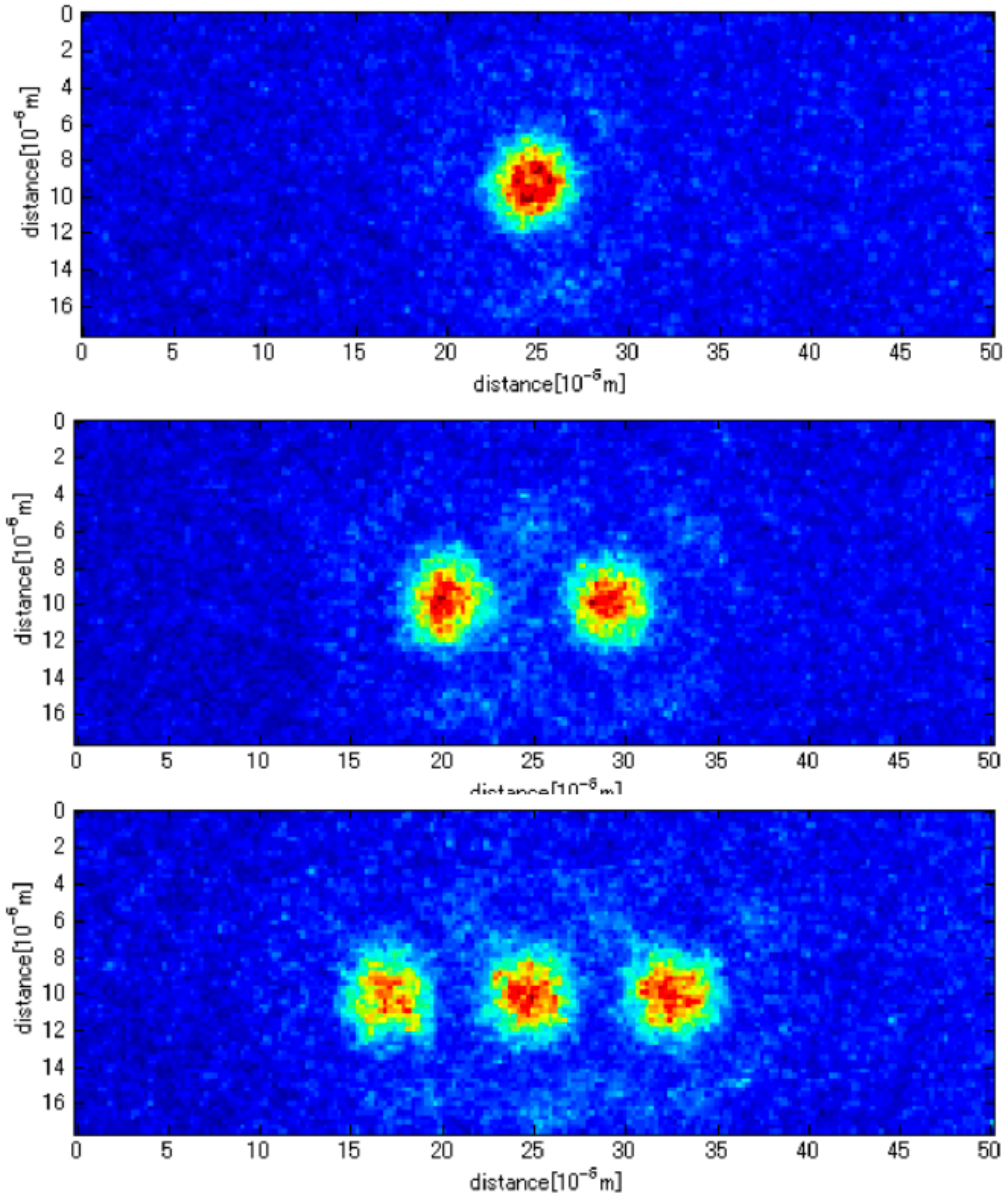
Contents

Why Atomic Ions?

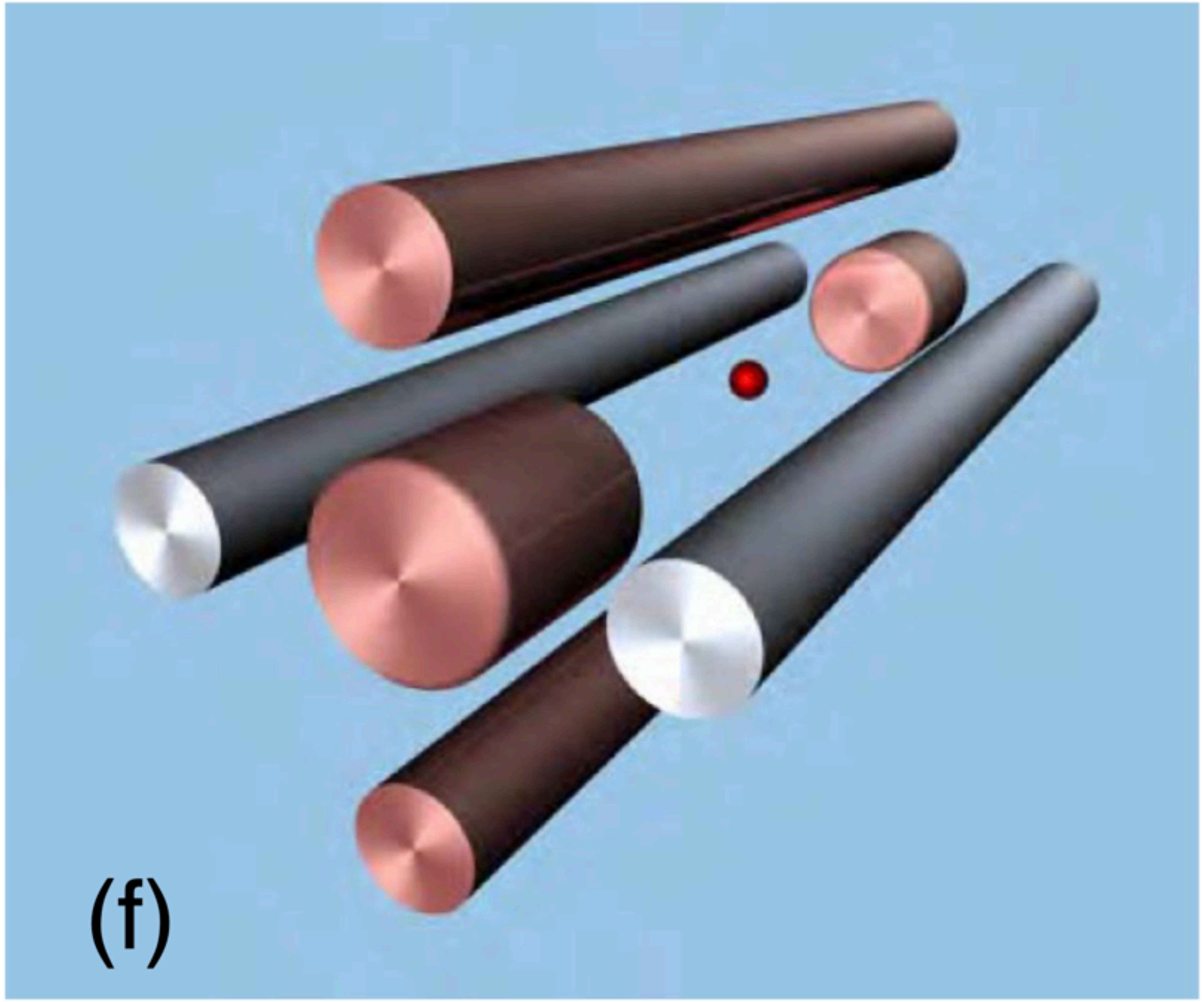
All Ions work same way!

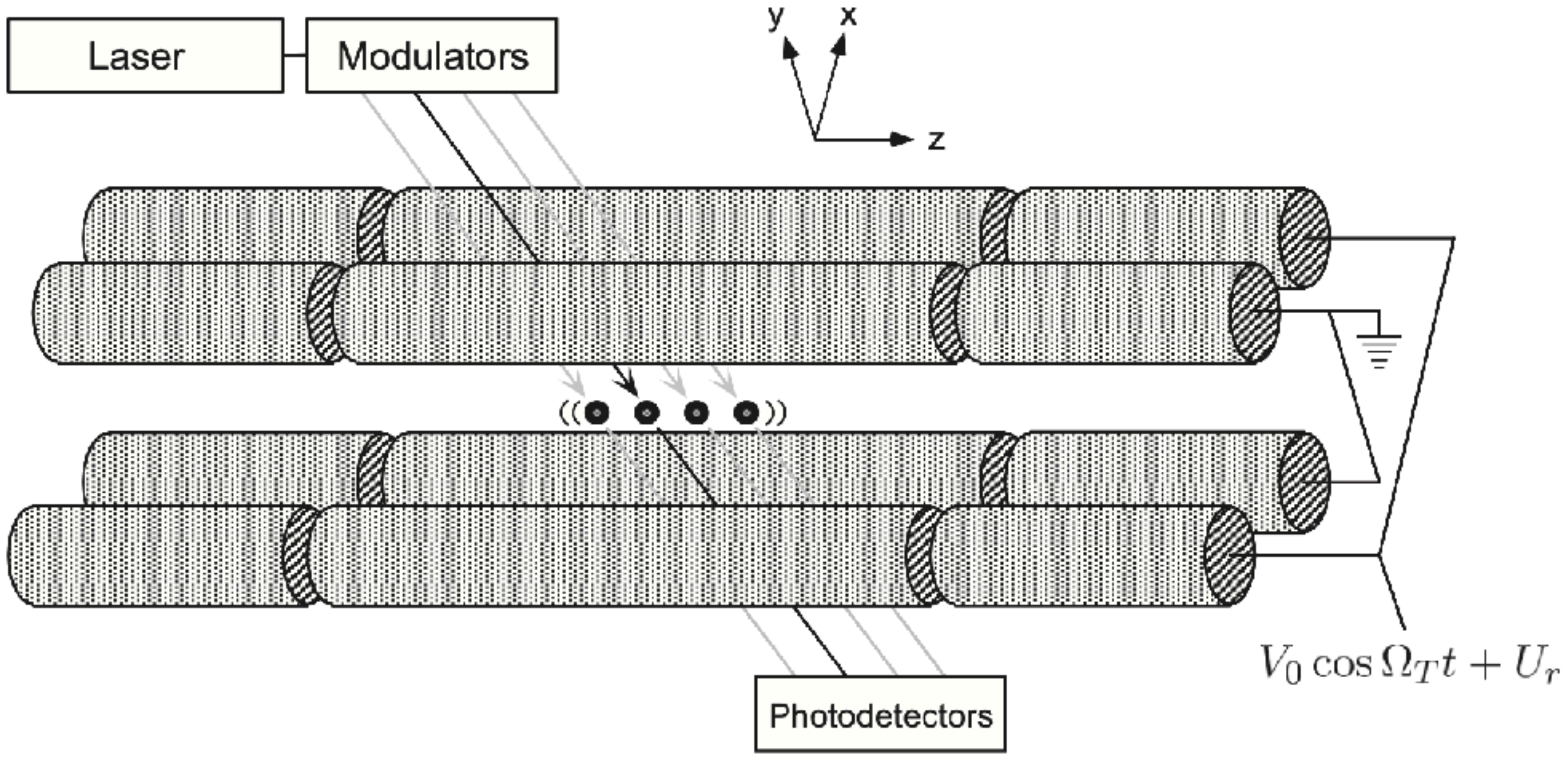


SQUID qubits have quality diff.

Nielsen, Chuang, 「Quantum Computation and Quantum Information」
Brownnutt et. al., Rev. Mod. Phys. (2015)

Linear Ion Trap





Linear Ion Trap

- **Scalar potential** $\varphi = (U + V\cos\omega t)\dfrac{x^2 - y^2}{r_0^2}$

- **Ansatz** $x(t) = X(t)(1 + q/2\cos(2\xi)) + \mathcal{O}(q^2)$ **and time average reduce EOM as**

$$\ddot{X}(t) + \omega_x^2 X(t) = 0$$

i.e., motion of harmonic oscillator

- **The Hamiltonian is written by using pseudo potential as**

$$H = \sum_{i=1}^{N} \frac{M}{2}\left(\omega_x^2 x_i^2 + \omega_y^2 y_i^2 + \omega_z^2 z_i^2 + \frac{|\vec{p}_i|^2}{M^2}\right) + \sum_{i=1}^{N}\sum_{j>i} \frac{e^2}{4\pi\epsilon_0 |\vec{r}_i - \vec{r}_j|}$$

https://www.youtube.com/watch?v=KwQdM8OK450

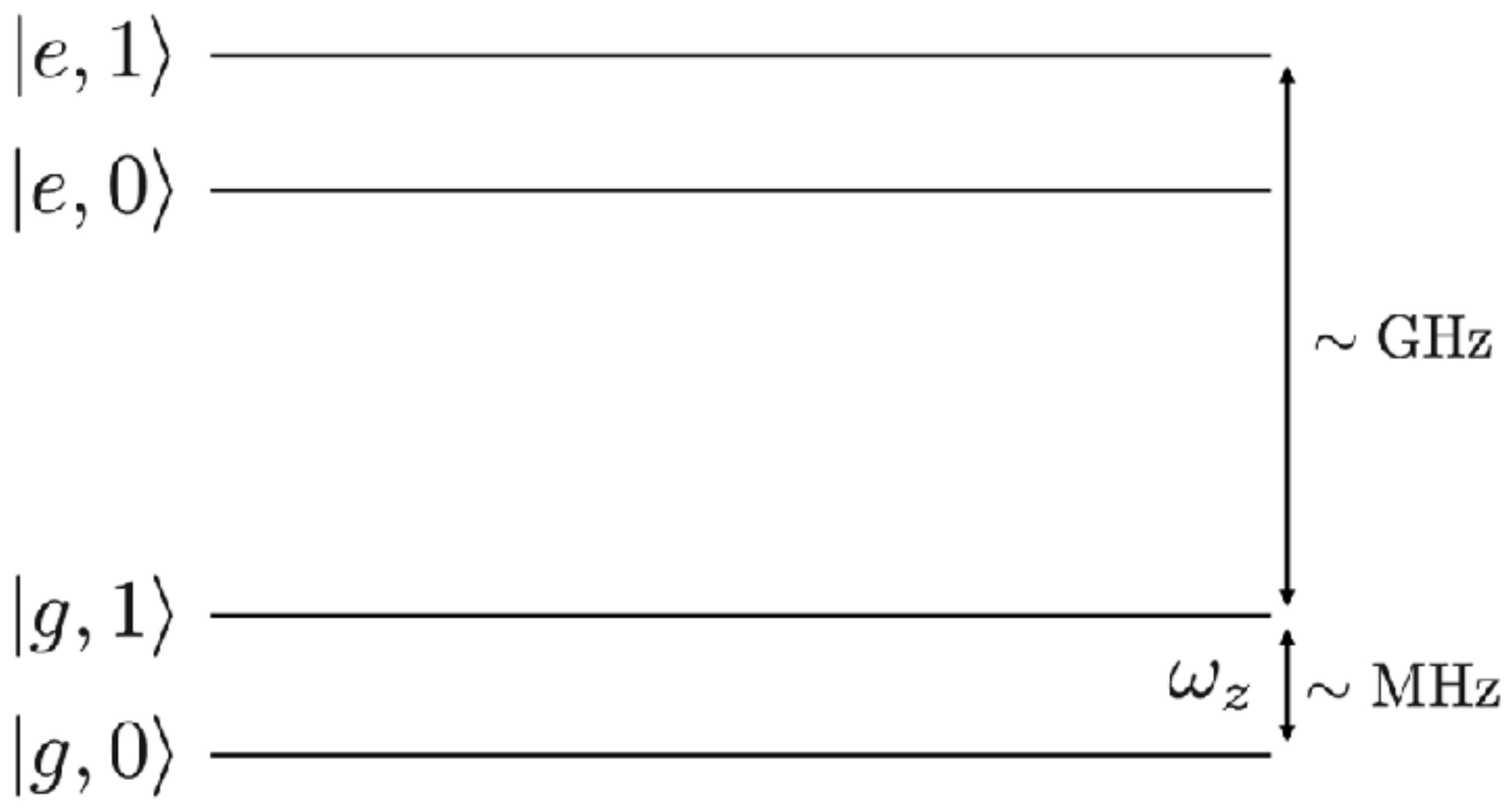
States of Ion

- **Trapped ions have states of**

Qubit

of z direction is important

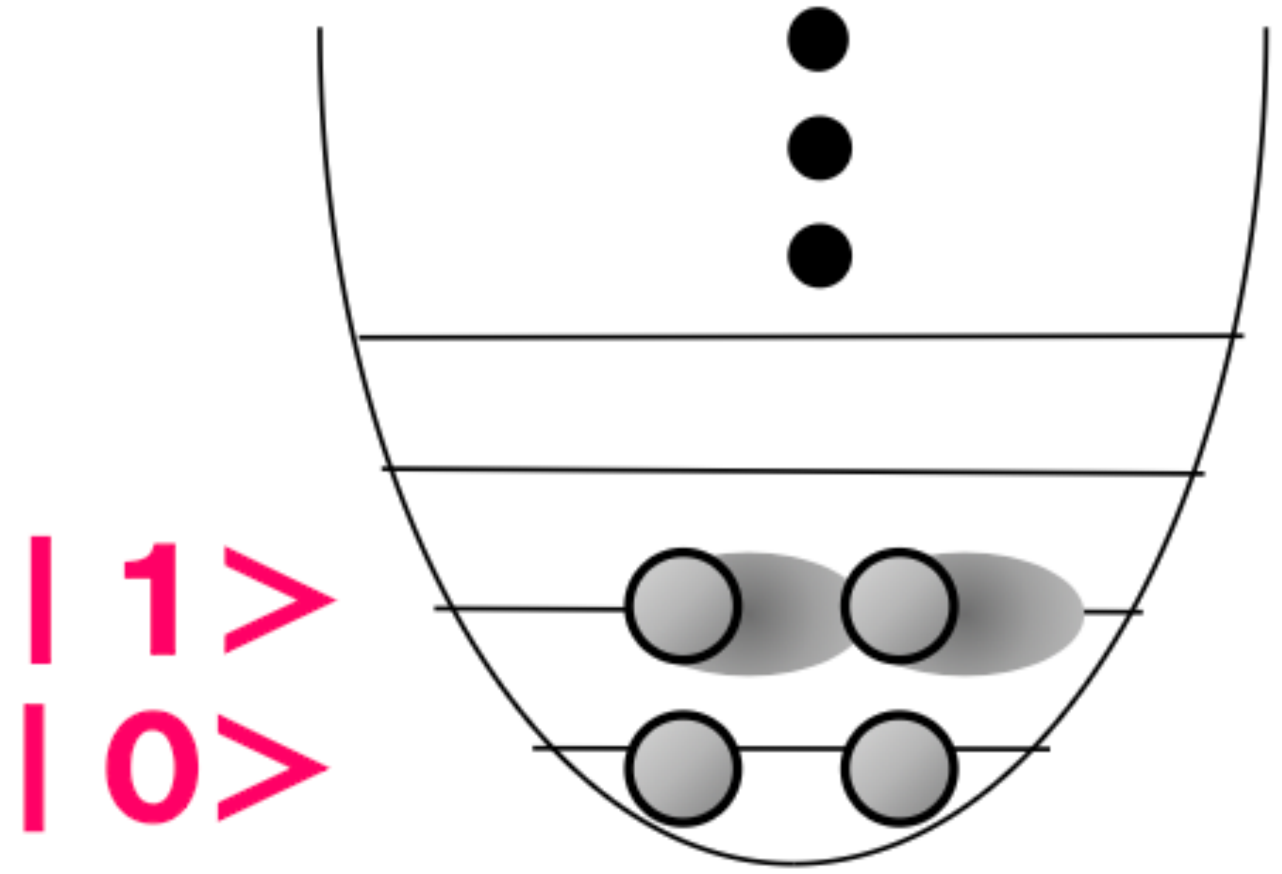
$$|\psi\rangle = \underbrace{|\mathrm{spin}\rangle}_{|g\rangle,\,|e\rangle} \otimes \underbrace{|\mathrm{osc.}\rangle}_{|0\rangle,\,|1\rangle,\,\cdots}$$



- **The oscillating states are collective modes**

e.g., two-qubit state $|g,g\rangle \otimes |1\rangle = |gg1\rangle$

- **Set by ground state by cooling the ion**



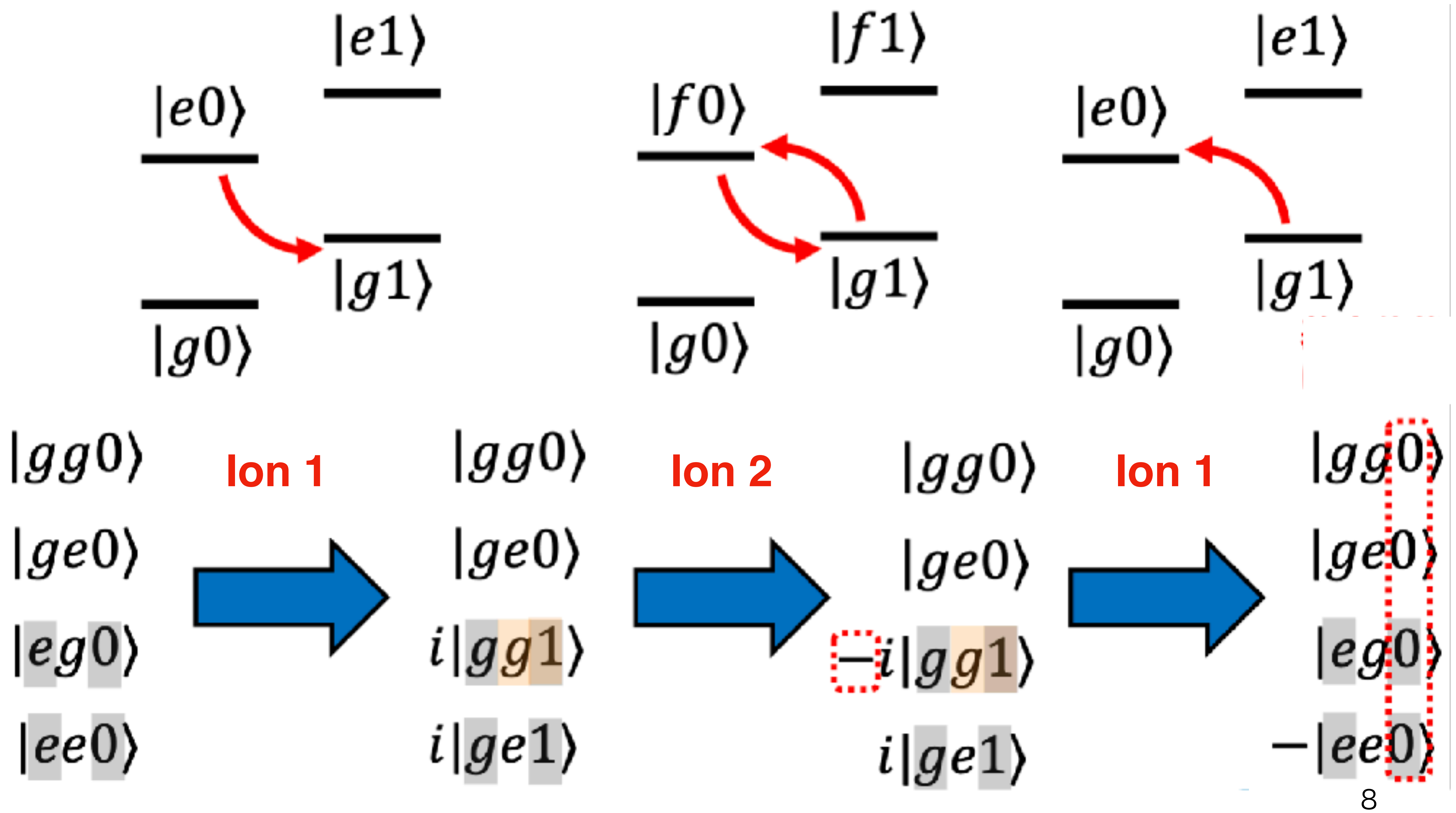
https://kds.kek.jp/event/48400/contributions/261318/attachments/177783/236136/20240329_金茶会_公開用.pdf

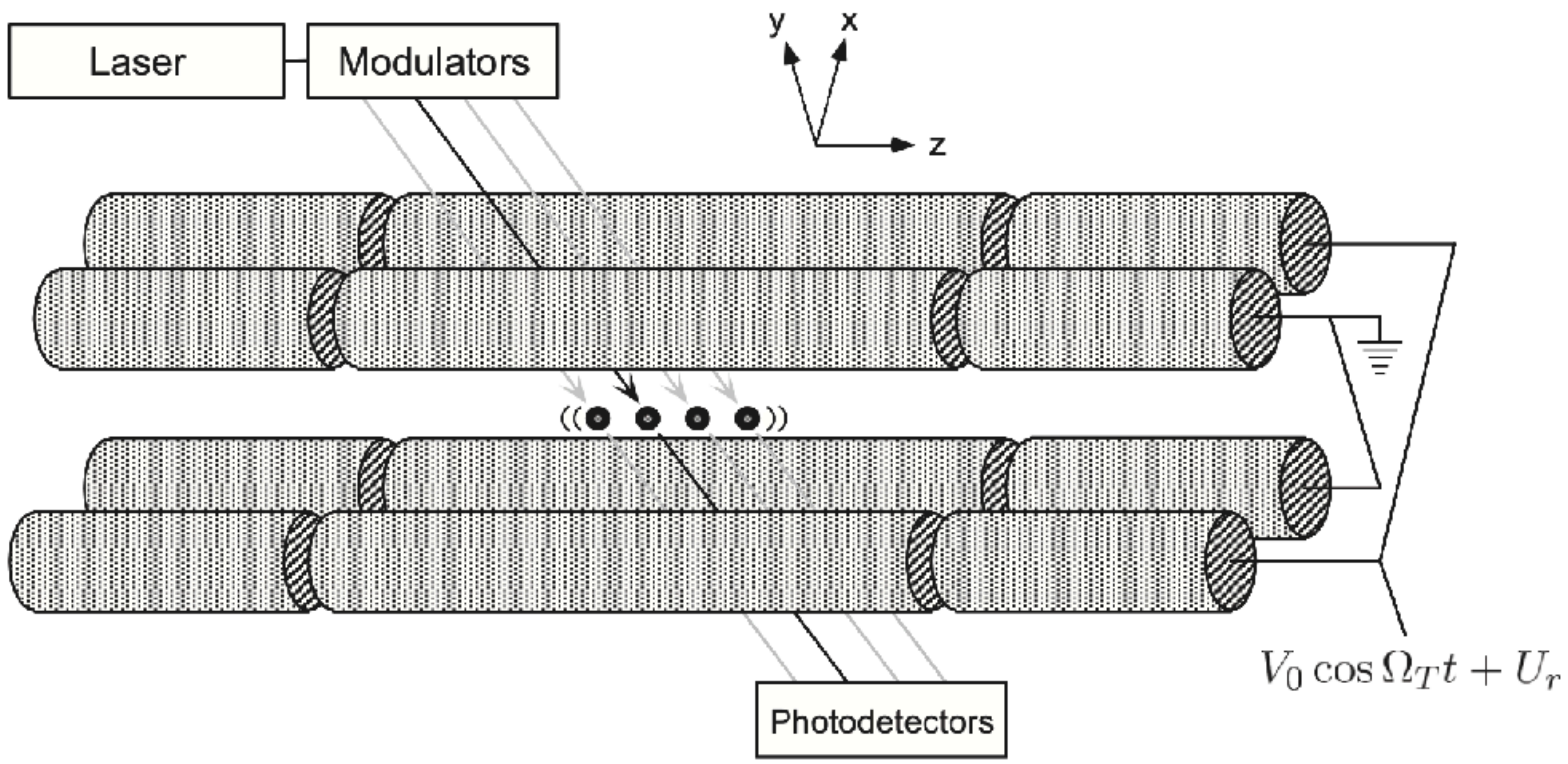
https://ocw.u-tokyo.ac.jp/lecture_files/11447/3/notes/ja/QEd_03_2_Takahashi.pdf

Nielsen, Chuang, 「Quantum Computation and Quantum Information」

Entanglement Operation

- **Qubit states are entangled by a coherent laser tuned with the energy difference**





Quantum Entanglement of Ions for Light Dark Matter Detection

Description of DM

- **Coherent ALP and dark photon induce the electric field as**

$$E_{a,z} = \epsilon_a \sqrt{2\rho_{\mathrm{DM}}} \sin\left(m_a t - \phi_a\right)$$

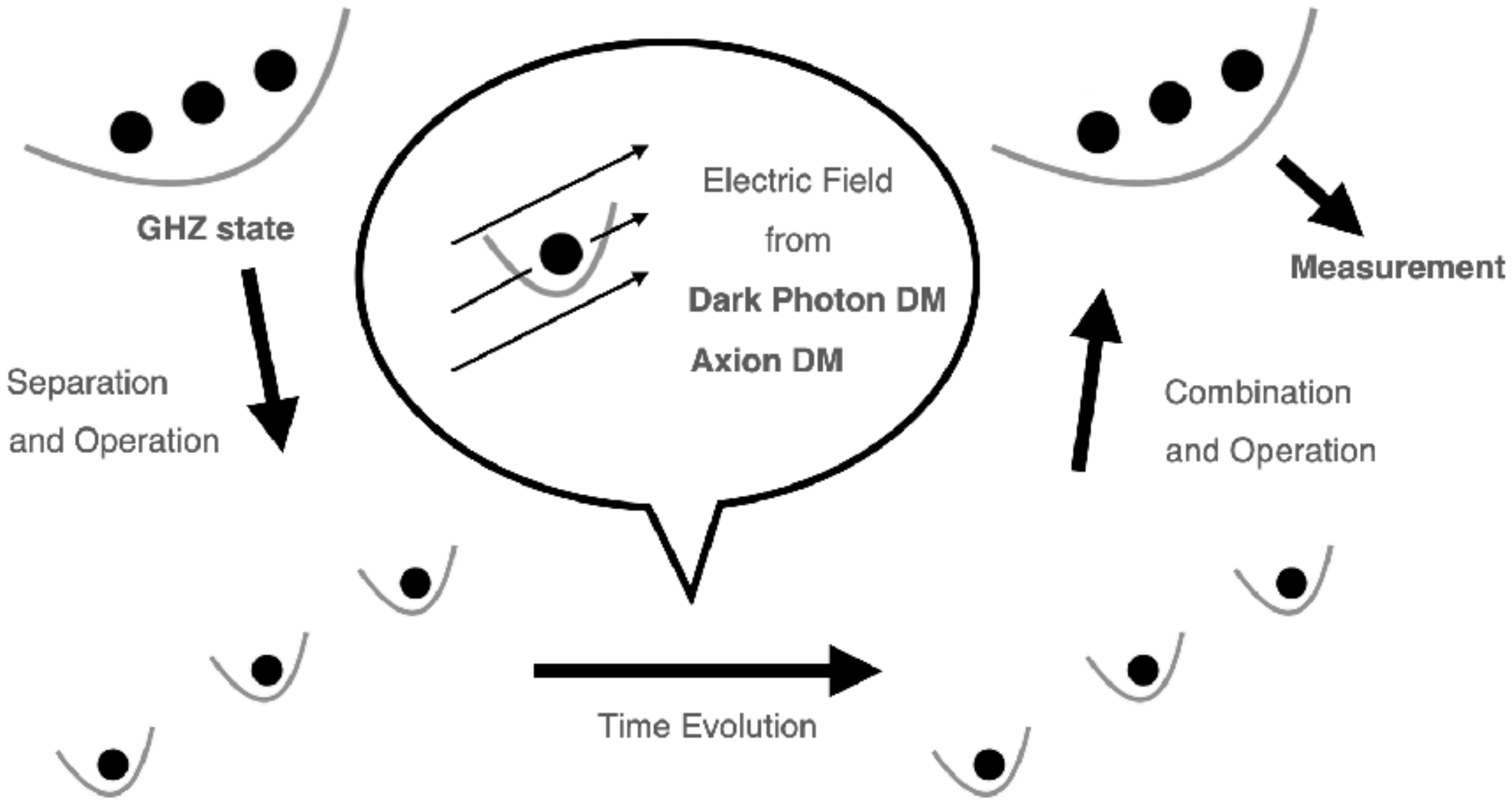
$$E_{\mathrm{DP},z} = \epsilon_{\mathrm{DP}} \sqrt{2\rho_{\mathrm{DM}}} \sin\left(m_{\mathrm{DP}} t - \phi_{\mathrm{DP}}\right)$$

- **Hamiltonian of both DM case is simply written as**

$$\hat{H}_X = e E_{X,z} \hat{z}$$

Detection by Multi-Ion

- The eigenstate of DM interaction can be realized by entanglement



- Then, as rough picture,

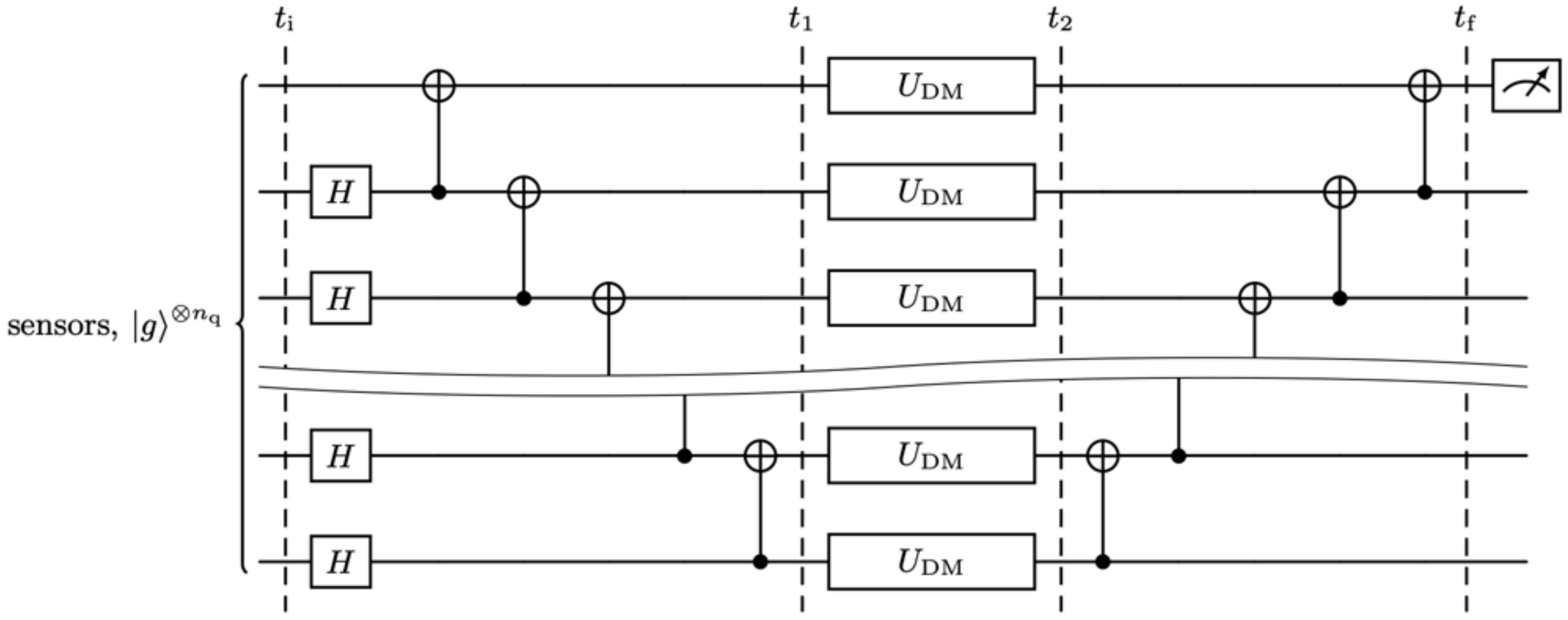
$$(\underline{N} \times \mathrm{DM\ amplitude})^2 > \underline{N} \times (\mathrm{DM\ amplitude})^2$$

can be realized for the probability

Entanglement Operation

~μeV

- **The operation of DM detection for spin state (GHz region)**



- **For the oscillation state, MHz DM mass region can be searched (~neV)**

Signal to Noise Ratio

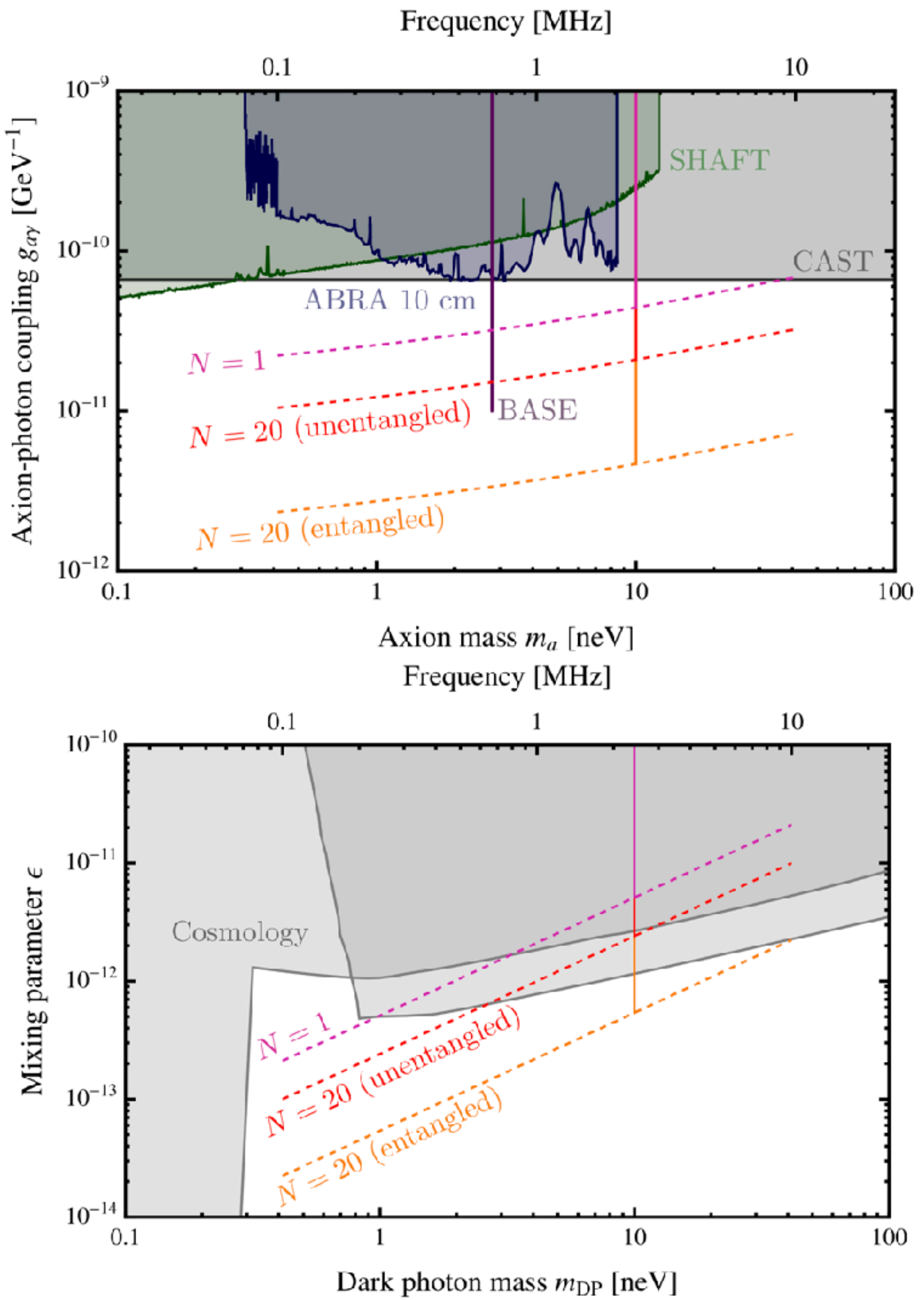
- **Assuming background $B \propto N$, the SNR becomes**

$$\boxed{\frac{S}{\sqrt{B}} \propto N^{3/2}}$$

$$\propto N^{1/2}$$

w/o entanglement

- **The mass is scanned by changing the trap frequency**
- **$N = 1$ case shows already good sensitivity**



Summary

- Amplification of DM signal by using
  the quantum entanglement (sensing) is discussed

- Quantum information is now quickly developing region
  and there will be other applications to particle physics region

- We can use public quantum computer and
  test this quantum operation

Appendix

Qubit

- (Classical) bit: Two-level system of classical states

  e.g., On/Off of electric currents

- Qubit: Two-level system of quantum states

$$|g\rangle = (1,0)^T, \ \ |e\rangle = (0,1)^T$$

- Qudit: d-level system of quantum states

Bloch Sphere

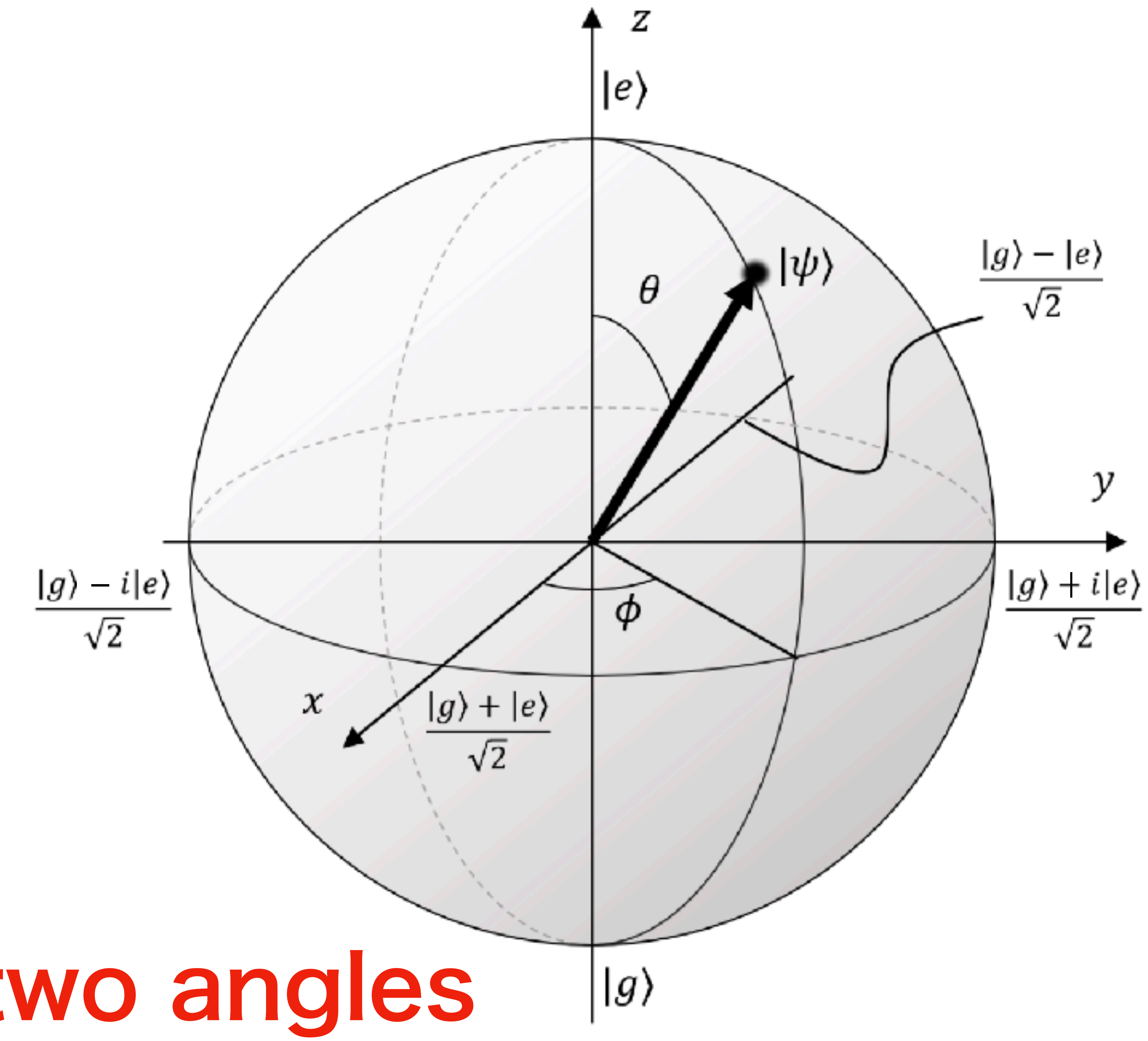
- Decomposition of density matrix of single qubit

$$\rho = |\psi\rangle\langle\psi| = \frac{I + \vec{r}\cdot\vec{\sigma}}{2}$$

- e.g.,

$$\frac{I + \vec{\sigma}_z}{2} = |g\rangle\langle g|$$

$$\frac{I + \vec{\sigma}_y}{2} = \left(\frac{|g\rangle + i|e\rangle}{\sqrt{2}}\right)\left(\frac{\langle g| - i\langle e|}{\sqrt{2}}\right)$$



- Single qubit states are expressed by two angles

https://www.sqei.c.u-tokyo.ac.jp/qed/QEd_textbook.pdf

Single Qubit Operations

- **Unitary** matrices to conserve probability

Hadamard	$\boxed{H}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$
Pauli-X	$\boxed{X}$	$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$
Pauli-Y	$\boxed{Y}$	$\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$
Pauli-Z	$\boxed{Z}$	$\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$
Phase	$\boxed{S}$	$\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$
$\pi/8$	$\boxed{T}$	$\begin{bmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{bmatrix}$

e.g., $$H|g\rangle = \frac{|g\rangle + |e\rangle}{\sqrt{2}}$$

$$H^2 = I$$

Nielsen, Chuang, 「Quantum Computation and Quantum Information」

Single Qubit Operations

- Pauli matrices give rotation operators as

$$R_x(\theta) \equiv e^{-i\theta X/2} = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}X = \begin{bmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

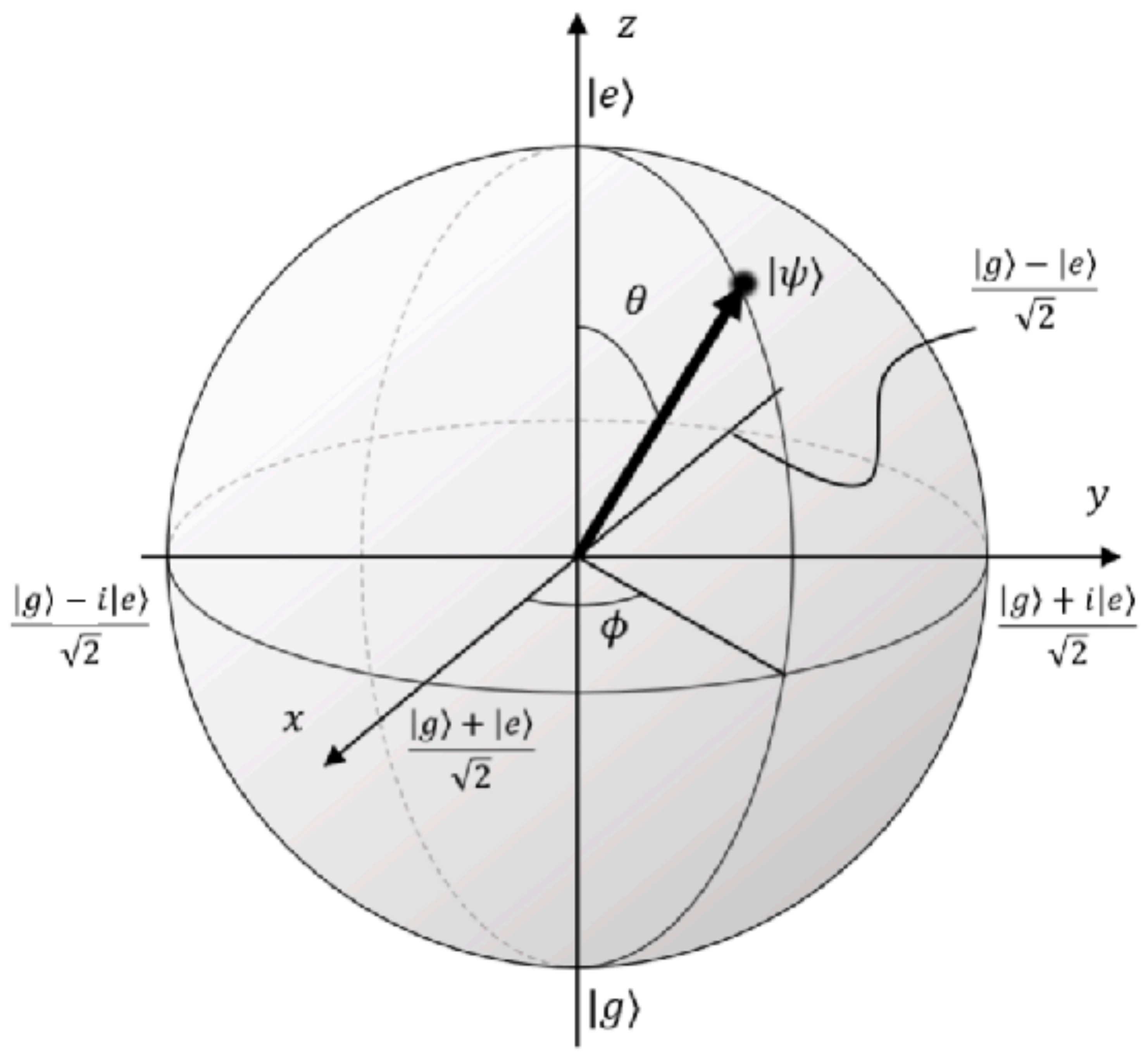
$\theta = 2\pi$ returns minus sign

$$R_y(\theta) \equiv e^{-i\theta Y/2} = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}Y = \begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

$$R_z(\theta) \equiv e^{-i\theta Z/2} = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}Z = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}.$$

- They act as rotation around their axis,

e.g., $R_z(\pi/2)\left(\frac{|g\rangle + |e\rangle}{\sqrt{2}}\right) = e^{-i\pi/4}\frac{|g\rangle + e^{i\pi/2}|e\rangle}{\sqrt{2}}$



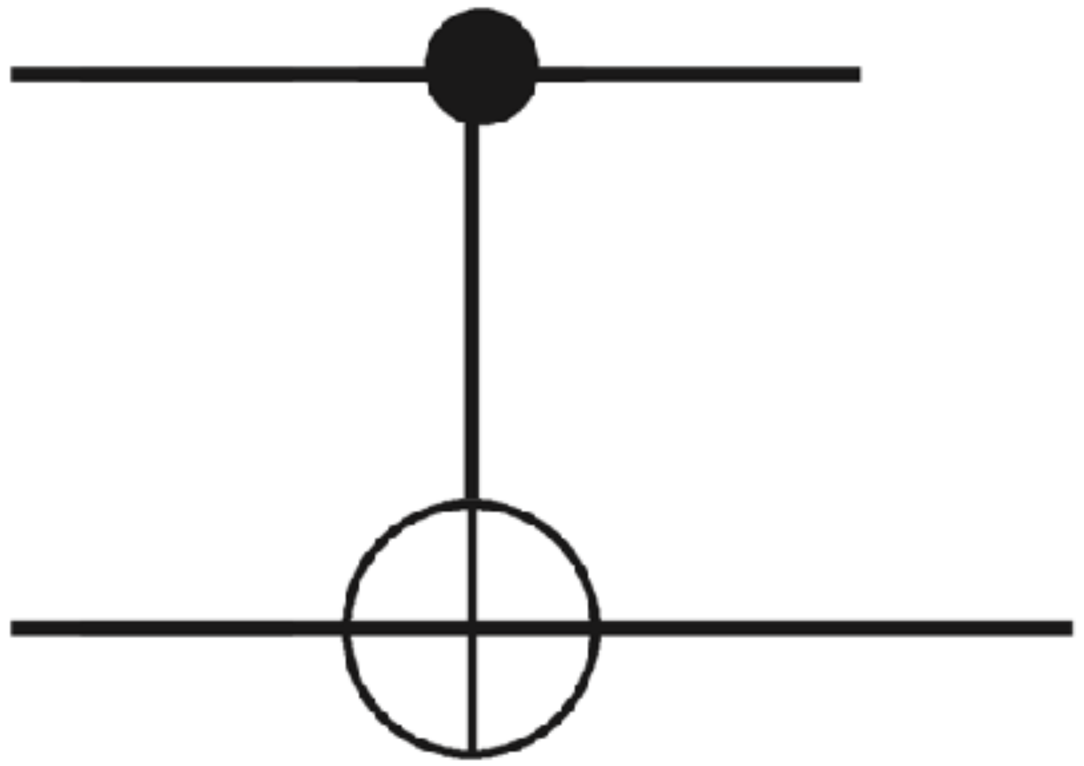
https://www.sqei.c.u-tokyo.ac.jp/qed/QEd_textbook.pdf

Two Qubit Operation: CNOT

- If $|c\rangle = |1\rangle$, then operate NOT gate for $|t\rangle$

Control qubit $|c\rangle$

Target qubit $|t\rangle$

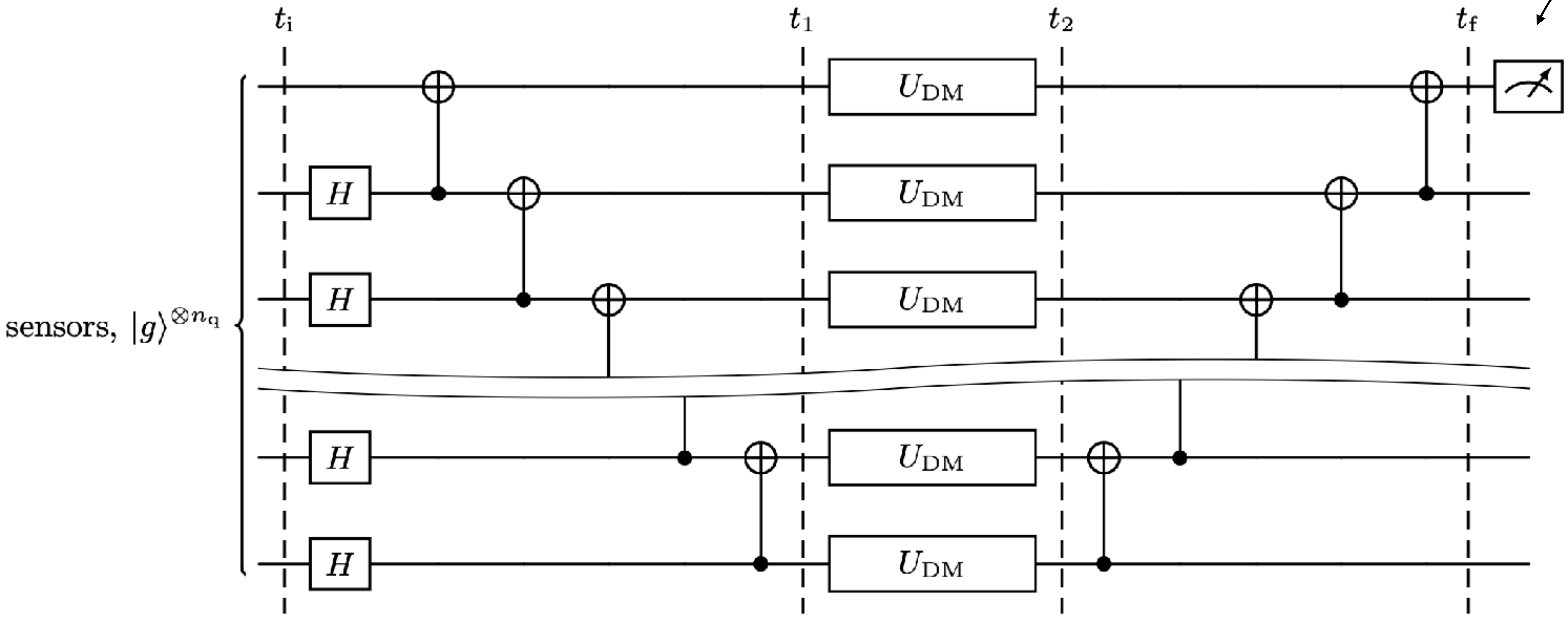


$$\begin{array}{cc} & |ct\rangle \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} & \begin{array}{c} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} \end{array}$$

Order of binary number counting

Quantum Circuit

- Code of unitary operations



Measurement

Chen et. al., 2311.10413

Universal Operations

- An arbitrary unitary operator is expressed exactly using single qubit and CNOT gates

- Single qubit operation is approximated to arbitrary accuracy using $H, S,$ and T gates (discrete gates)

Two rotation operators $R_{\hat{n}}(\alpha) = THTH$, $R_{\hat{m}}(\alpha) = HR_{\hat{n}}(\alpha)H$ can cover any region

$$\alpha = 2\pi \times \text{irrational number}$$

Nielsen, Chuang, 「Quantum Computation and Quantum Information」

Entanglement States

- Composite states: Expressed by tensor product (separable)

$$|\psi_1\rangle \otimes |\psi_2\rangle, \text{ or } |\psi_1\psi_2\rangle$$

e.g., $\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) \otimes \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)$

- Entangled states: Cannot expressed by tensor product

e.g., $\frac{|00\rangle + |11\rangle}{\sqrt{2}}$

Entanglement States

- Pure state: A quantum system whose state is known exactly

$$\mathrm{tr}(\rho^2) = 1$$

- Mixed state: Otherwise, or mixture of different pure states

$$\mathrm{tr}(\rho^2) < 1$$

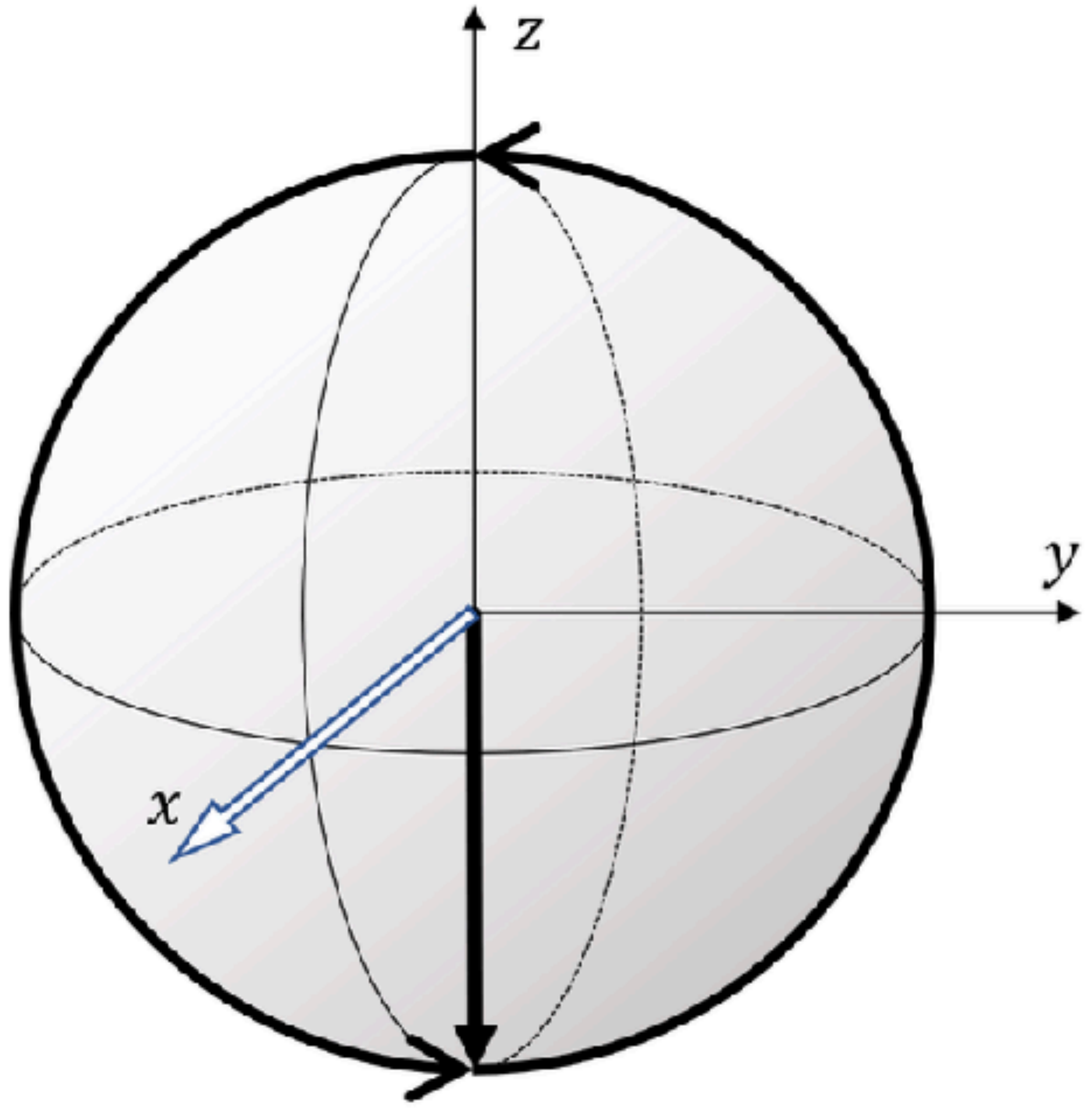
Rabi Oscillation

- EOM $\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\frac{i}{\hbar}[H,\rho]$ of Rabi oscillation is

$$\frac{\mathrm{d}s_x}{\mathrm{d}t} = 0, \quad \frac{\mathrm{d}s_y}{\mathrm{d}t} = -\Omega s_z, \quad \frac{\mathrm{d}s_z}{\mathrm{d}t} = \Omega s_y$$

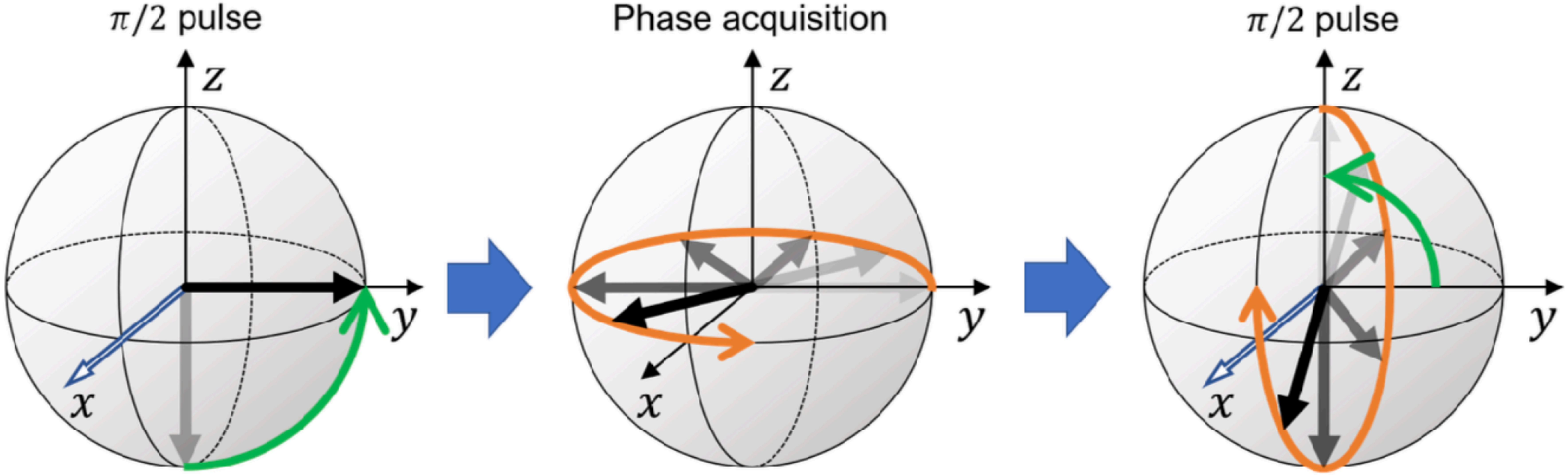
$$\rho = \frac{1}{2}(I + s_x\sigma_x + s_y\sigma_y + s_z\sigma_z)$$

- Rabi oscillation appears for resonant system



https://www.sqei.c.u-tokyo.ac.jp/qed/QEd_textbook.pdf

Ramsey Interference



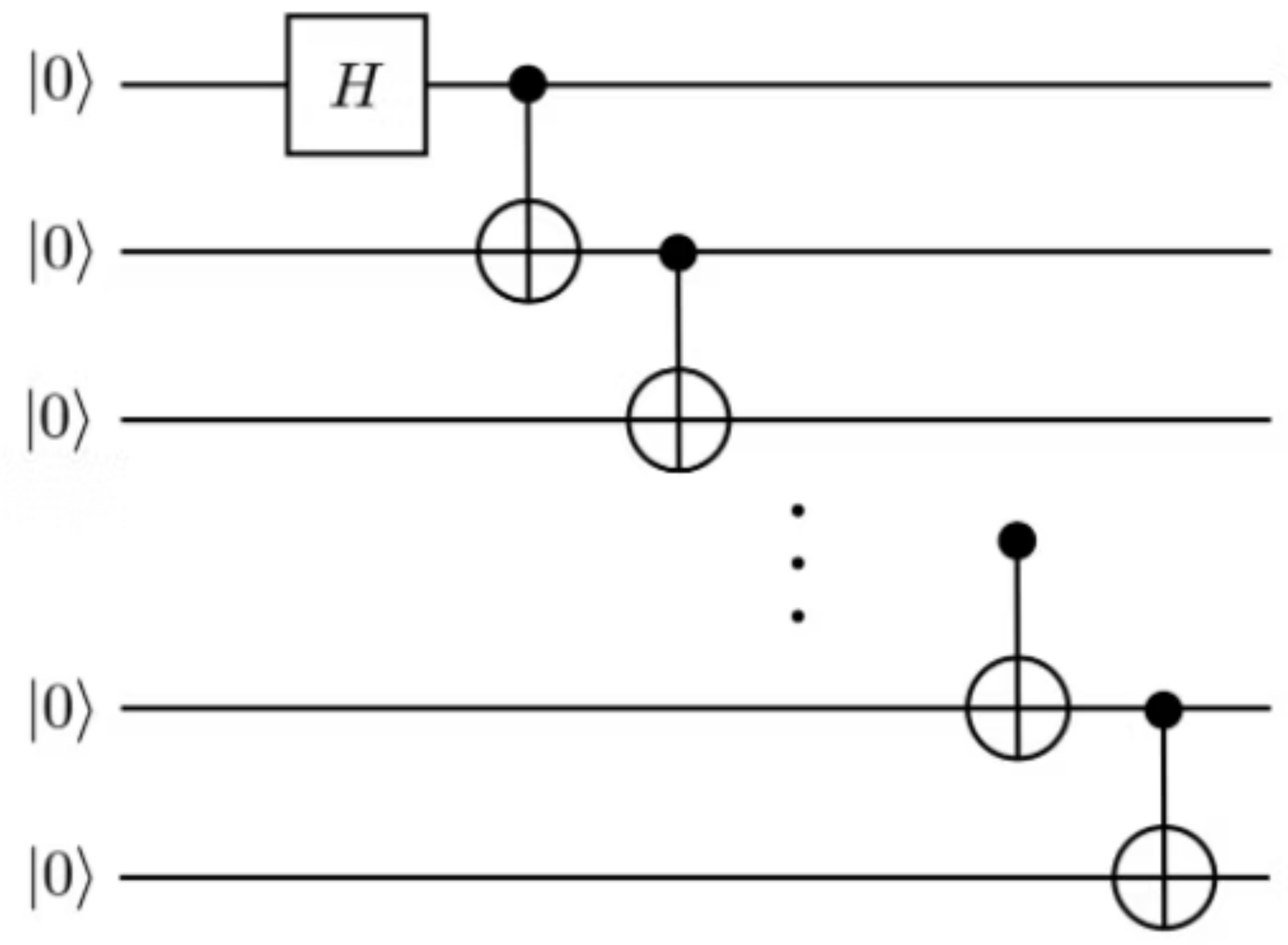
- Noises act as phase acquisition and decrease the excitation probability
- The probability becomes $P = \cos^2\phi$ with angle ϕ from z-axis

https://www.sqei.c.u-tokyo.ac.jp/qed/QEd_textbook.pdf

GHZ State

- GHZ state is often used for quantum sensing

$$\frac{|000\ldots00\rangle + |111\ldots11\rangle}{\sqrt{2}}$$



Made by simple code

https://qiita.com/shnchr/items/696bbfd452955c2b5c2b

Noise Sensing by GHZ State

- Consider electric field sensing

$$H = -\sum_{i}^{N} gE\sigma_z^{(i)}$$

- This time development becomes

$$e^{i\hat{H}t}|GHZ\rangle = e^{iNgEt}\frac{|000\ldots00\rangle}{\sqrt{2}} + e^{-iNgEt}\frac{|111\ldots11\rangle}{\sqrt{2}}$$

c.f. $$e^{i\hat{H}t}\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)^{\otimes N} = \left(e^{igEt}\frac{|0\rangle}{\sqrt{2}} + e^{-igEt}\frac{|1\rangle}{\sqrt{2}}\right)^{\otimes N}$$

Noise Sensing by GHZ State

- **Observable** $\hat{\mathcal{O}} = \sigma_{x,1}\sigma_{x,2}\cdots\sigma_{x,N}$

$$\langle \hat{\mathcal{O}} \rangle = \cos(NgEt)$$

$$\Delta\hat{\mathcal{O}} = \langle \hat{\mathcal{O}}^2 \rangle - \langle \hat{\mathcal{O}} \rangle^2 = \sin^2(NgEt)$$

case of $\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)^{\otimes N}$

$$\langle \hat{\mathcal{O}} \rangle = \cos^N(gEt)$$

$$\Delta\hat{\mathcal{O}} = 1 - \cos^{2N}(gEt)$$

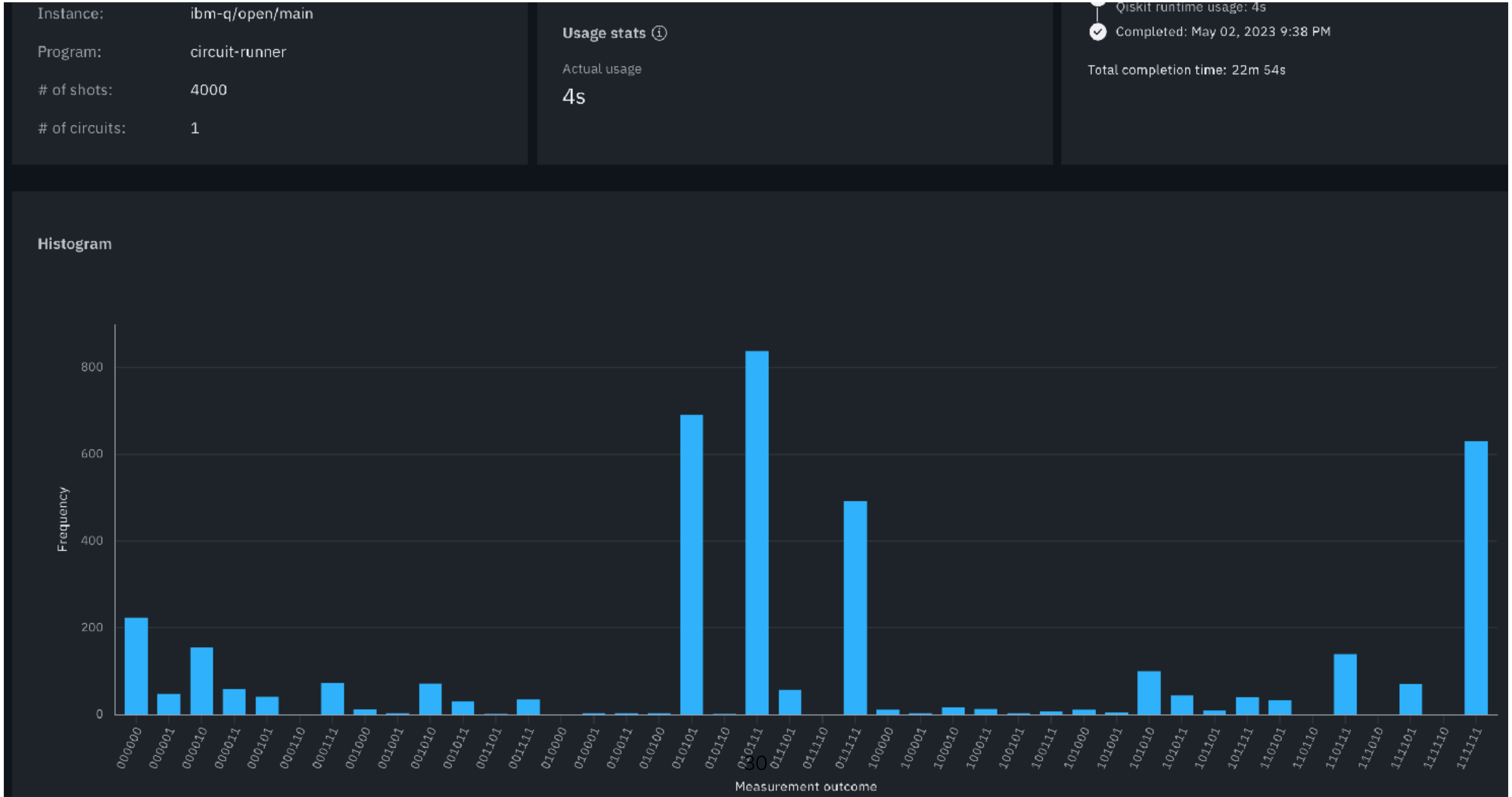
- **Sensitivity** $\delta\phi = \langle \hat{\mathcal{O}} \rangle / \sqrt{\Delta\hat{\mathcal{O}}}$

$$\delta\phi \simeq \frac{1}{NgEt} \propto \underline{\frac{1}{N}}$$

$$\delta\phi \simeq \frac{1}{\sqrt{2N}gEt} \propto \underline{\frac{1}{\sqrt{N}}}$$

Result is Histogram

- Histogram of 4000 trials



Displacement Operator

$$\hat{D}(\beta) = \exp\left(\beta\hat{a}^\dagger - \beta^*\hat{a}\right)$$

- $\hat{D}(\beta)|0\rangle$ is coherent state
- For real β, $\beta(\hat{a}^\dagger - \hat{a}) \propto \hat{p}$ means displacement of momenta

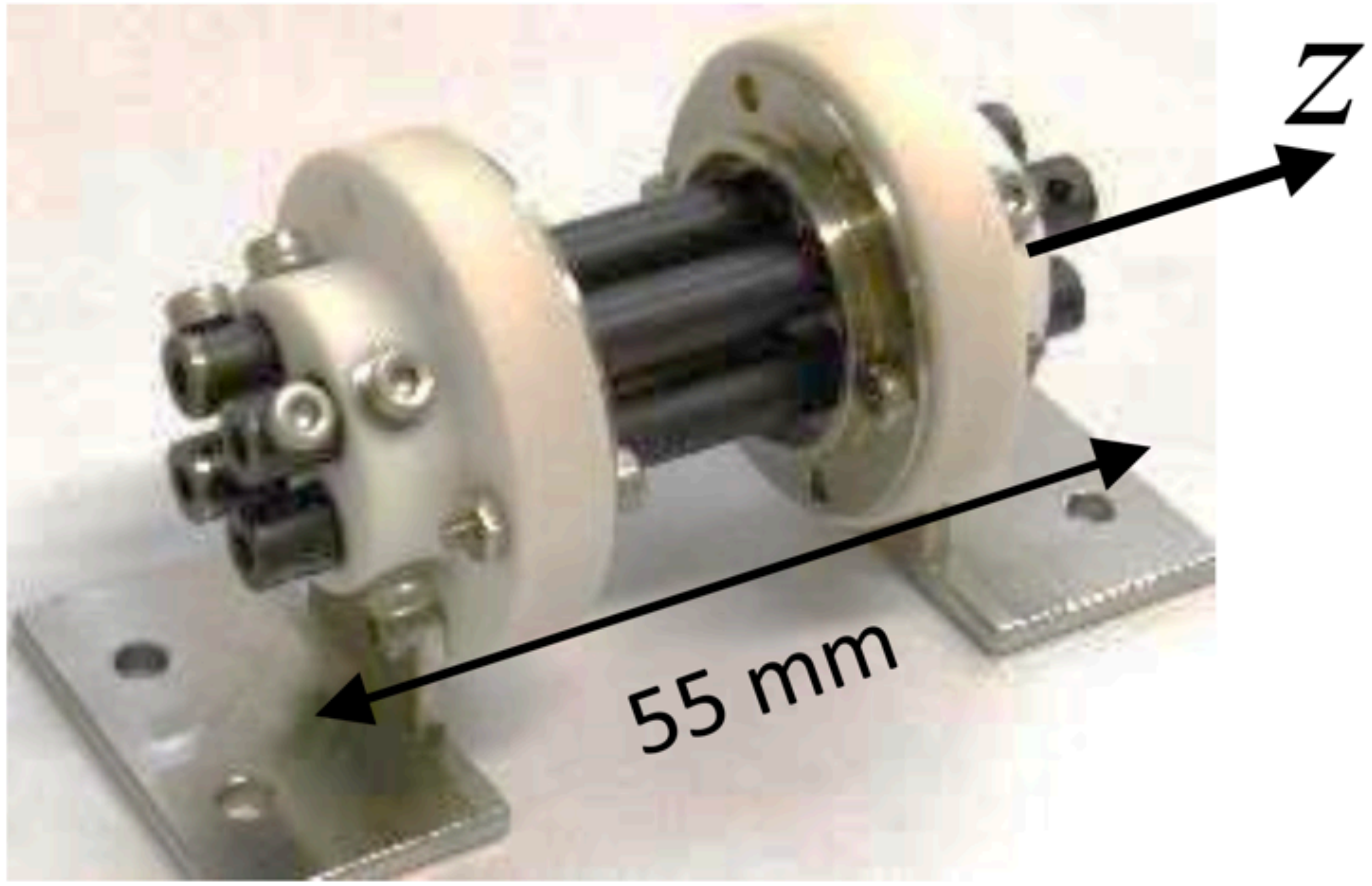
  and Imaginary β, $\beta(\hat{a}^\dagger + \hat{a}) \propto \hat{x}$ means displacement of position
- Unitary $\hat{D}^\dagger(\beta) = \hat{D}^{-1}(\beta) = \hat{D}(-\beta)$,

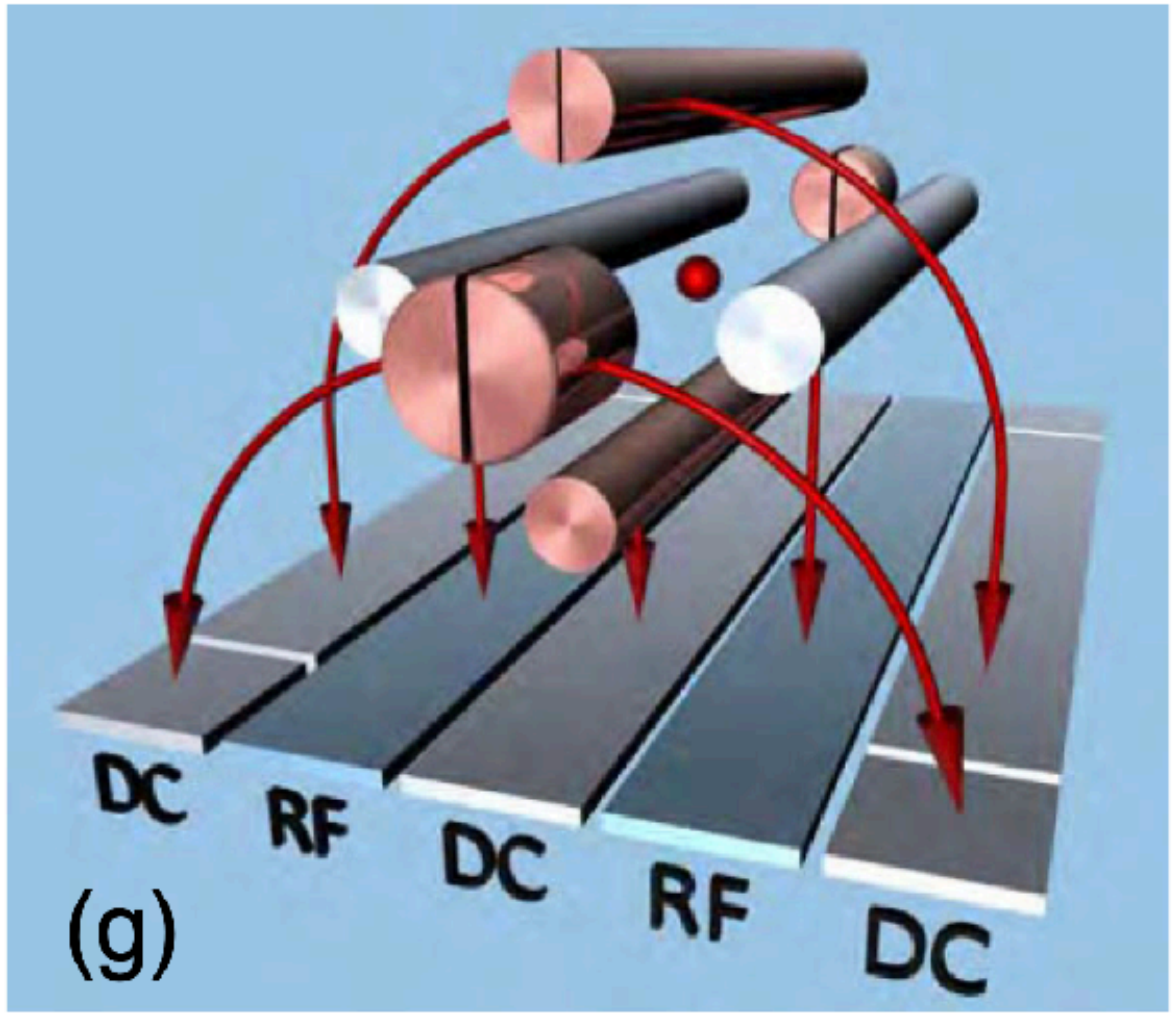
  composition rule $\hat{D}(\alpha)\hat{D}(\beta) = e^{\frac{1}{2}(\alpha\beta^* - \alpha^*\beta)}\hat{D}(\alpha+\beta)$,

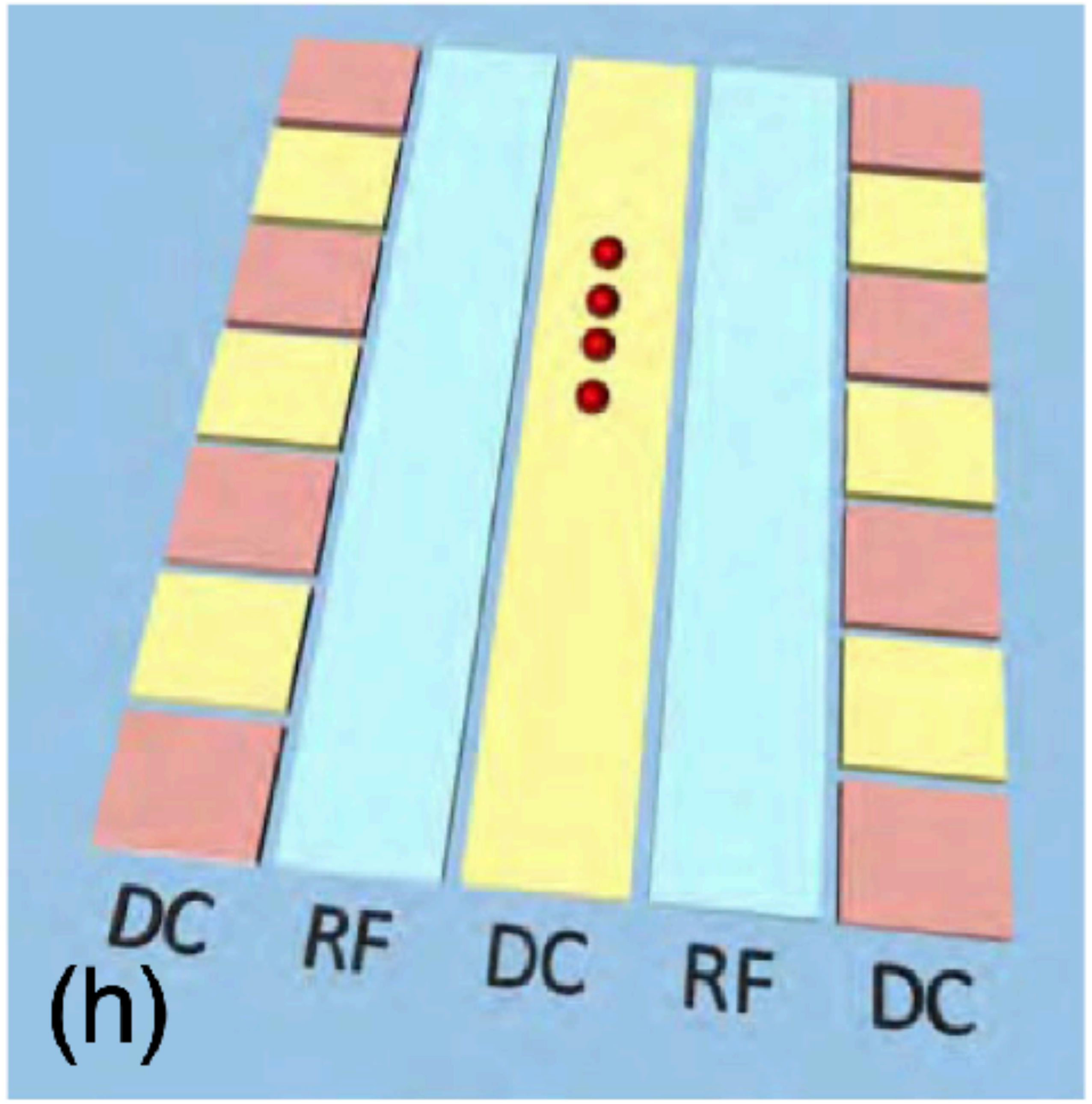
  and transformation $\hat{D}^\dagger(\beta)\hat{a}\hat{D}(\beta) = \hat{a} + \beta$.

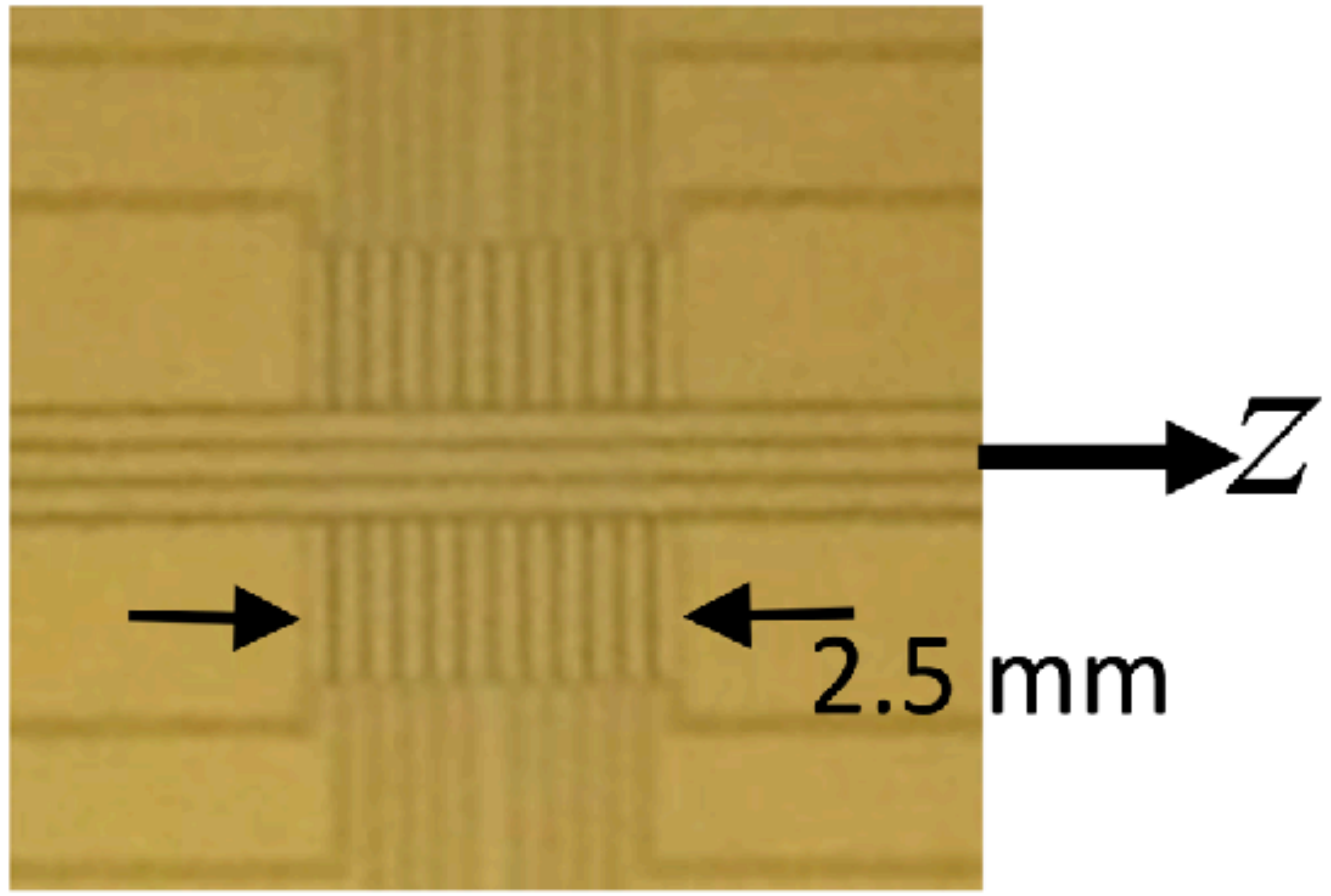
https://kds.kek.jp/event/48400/contributions/261318/attachments/177783/236136/20240329_金茶会_公開用.pdf

Brownnutt et. al., Rev. Mod. Phys. (2015)









Linear Ion Trap

- Scalar potential $\varphi = (U + V\cos\omega t)\,\dfrac{(x^2 - y^2)}{r_o^2}$

- EOM is Mathieu equation

$$\begin{cases} m\dfrac{d^2x}{dt^2} + 2e(U + V\cos\omega t)\,\dfrac{x}{r_o^2} = 0 \\ m\dfrac{d^2y}{dt^2} - 2e(U + V\cos\omega t)\,\dfrac{y}{r_o^2} = 0 \end{cases}$$

- Ion is trapped in stable parameter region

Linear Ion Trap

- Rewrite EOM as

$$\begin{cases} \dfrac{d^2x}{d\xi^2} + (a + 2q\cos 2\xi)x = 0 \\ \dfrac{d^2y}{d\xi^2} - (a + 2q\cos 2\xi)y = 0 \end{cases}$$

$$\omega t = 2\xi \;, \qquad a = \frac{8eU}{mr_o^2\omega^2} \;, \qquad q = \frac{4eV}{mr_o^2\omega^2}$$

$\omega \sim \mathrm{GHz}$

- Ansatz $x(t) = X(t)(1 + q/2\cos(2\xi)) + \mathcal{O}(q^2)$ and time average reduce EOM as

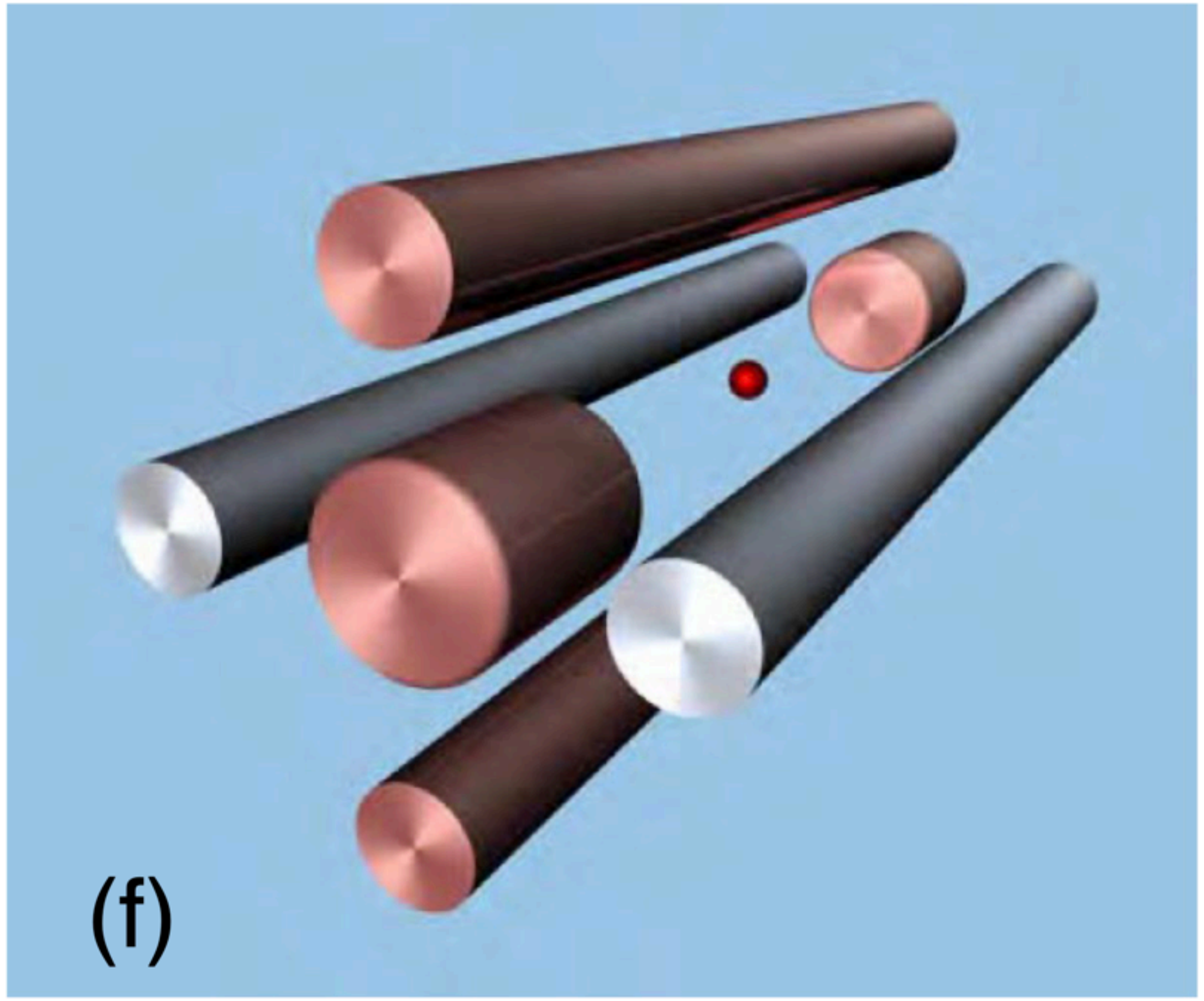
$$\ddot{X}(t) + \omega_x^2 X(t) = 0$$

i.e., motion of harmonic oscillator

Linear Ion Trap

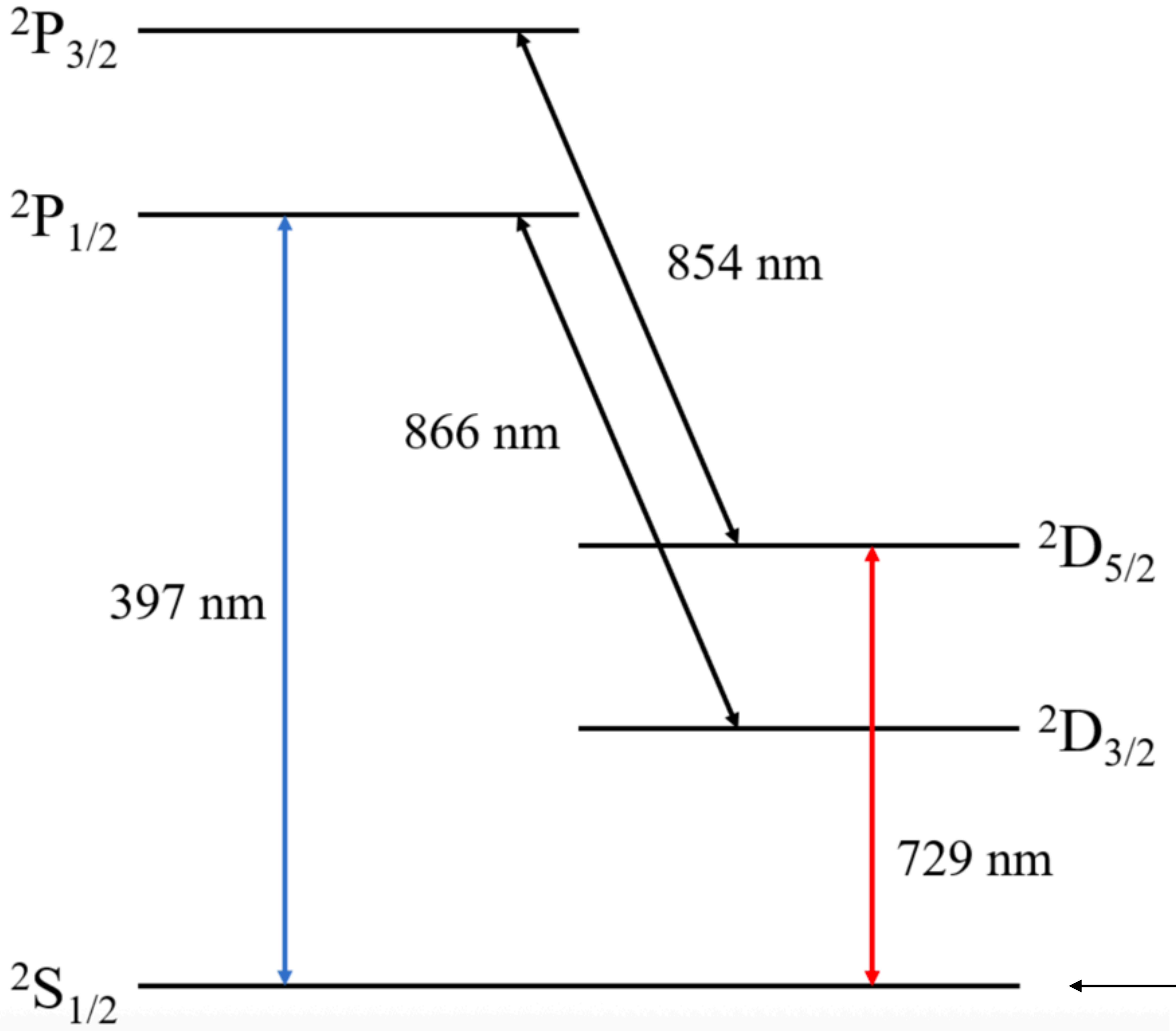
- The Hamiltonian is written by using pseudo potential as

$$H = \sum_{i=1}^{N} \frac{M}{2}\left(\omega_x^2 x_i^2 + \omega_y^2 y_i^2 + \omega_z^2 z_i^2 + \frac{|\vec{p}_i|^2}{M^2}\right) + \sum_{i=1}^{N}\sum_{j>i} \frac{e^2}{4\pi\epsilon_0 |\vec{r}_i - \vec{r}_j|}$$



Brownnutt et. al., Rev. Mod. Phys. (2015)

Energy Structure of $^{40}Ca^{+}$



$|e\rangle$ for $m = -3/2$ $\quad \Gamma = 1.1\ \mathrm{s}^{-1}$

Kreuter et al., 2004, Barton et al., 2000, Staanum et al., 2003

$|g\rangle$ for $m = -1/2$

Jarlaud, et al, 2007.04149

Doppler Cooling

- Tune energy level by Doppler effect of laser

- Ions are cooled to $k_B T \approx \Gamma/2$

Γ : radiative width of transition

- $k_B T \ll \omega_z$ can be satisfied

Regard Ions as Qubits

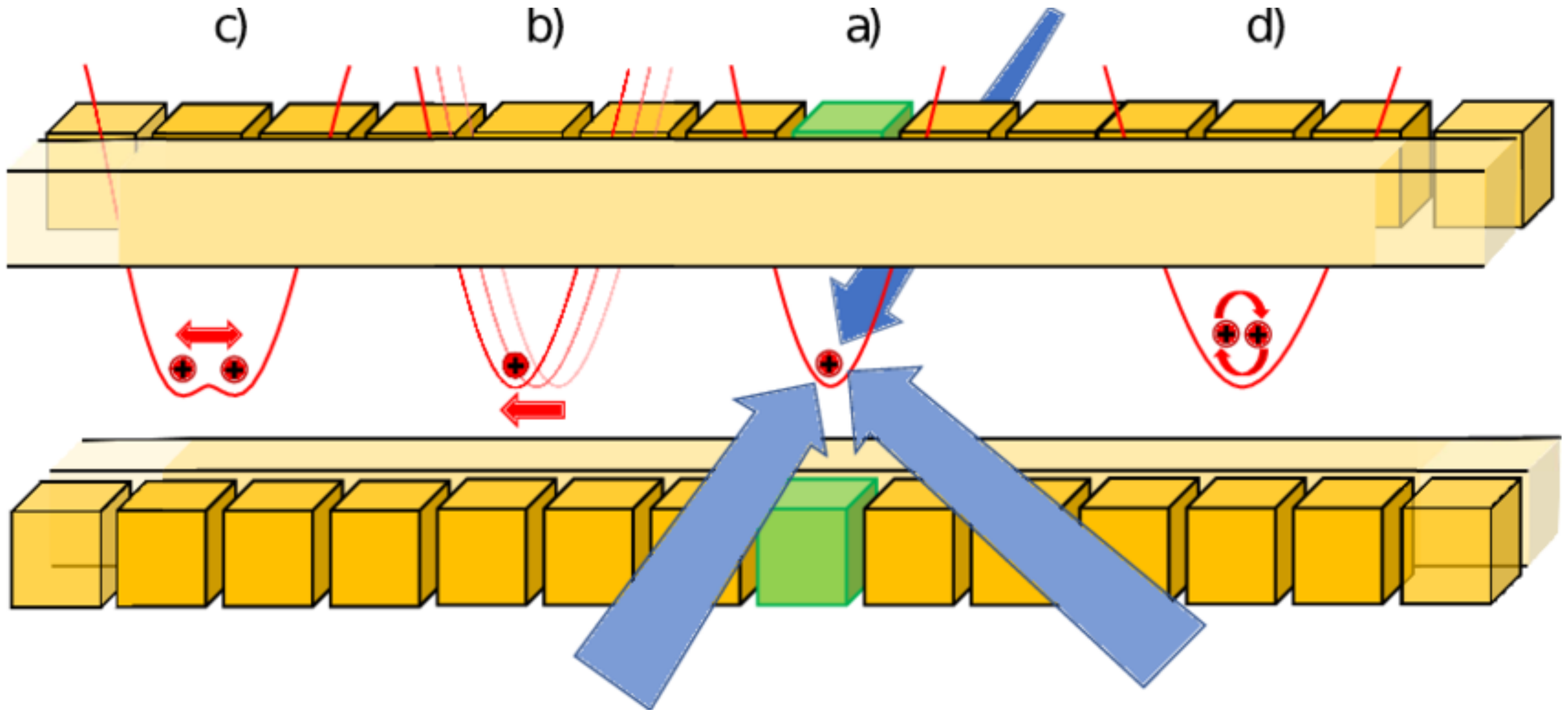
$$\psi_{\text{ion}}(\vec{r}, t) \quad \xleftrightarrow{\text{Connect}} \quad |g\rangle, |e\rangle$$

How do we treat $\vec{r}, t$ dependence?

Dependence of $\vec{r}$

- Quantum operations are safely done if $\lambda_{\text{laser}} \approx \underbrace{\dfrac{1}{\sqrt{2Nm\omega_z}}}_{\text{Typical distance between ions}}$

- Transport and swap of ions are also needed for entanglement by laser



https://www.researchgate.net/figure/Architecture-of-the-linear-segmented-ion-trap-Each-of-the-linearly-arranged-dc-segments_fig1_375504288

Dependence of t

- Time evolution $|\psi\rangle = e^{-iE_g t}\left(|g\rangle + e^{-i\Delta E t}|e\rangle\right)$ cannot be removed

- Resonance laser can tune this frequency and can vanish this effect

- This phase shift can be calculated and we can subtract

Lifetime T_1

- $|e\rangle$ state decays because it is excited state

- The lifetime is labeled as T_1

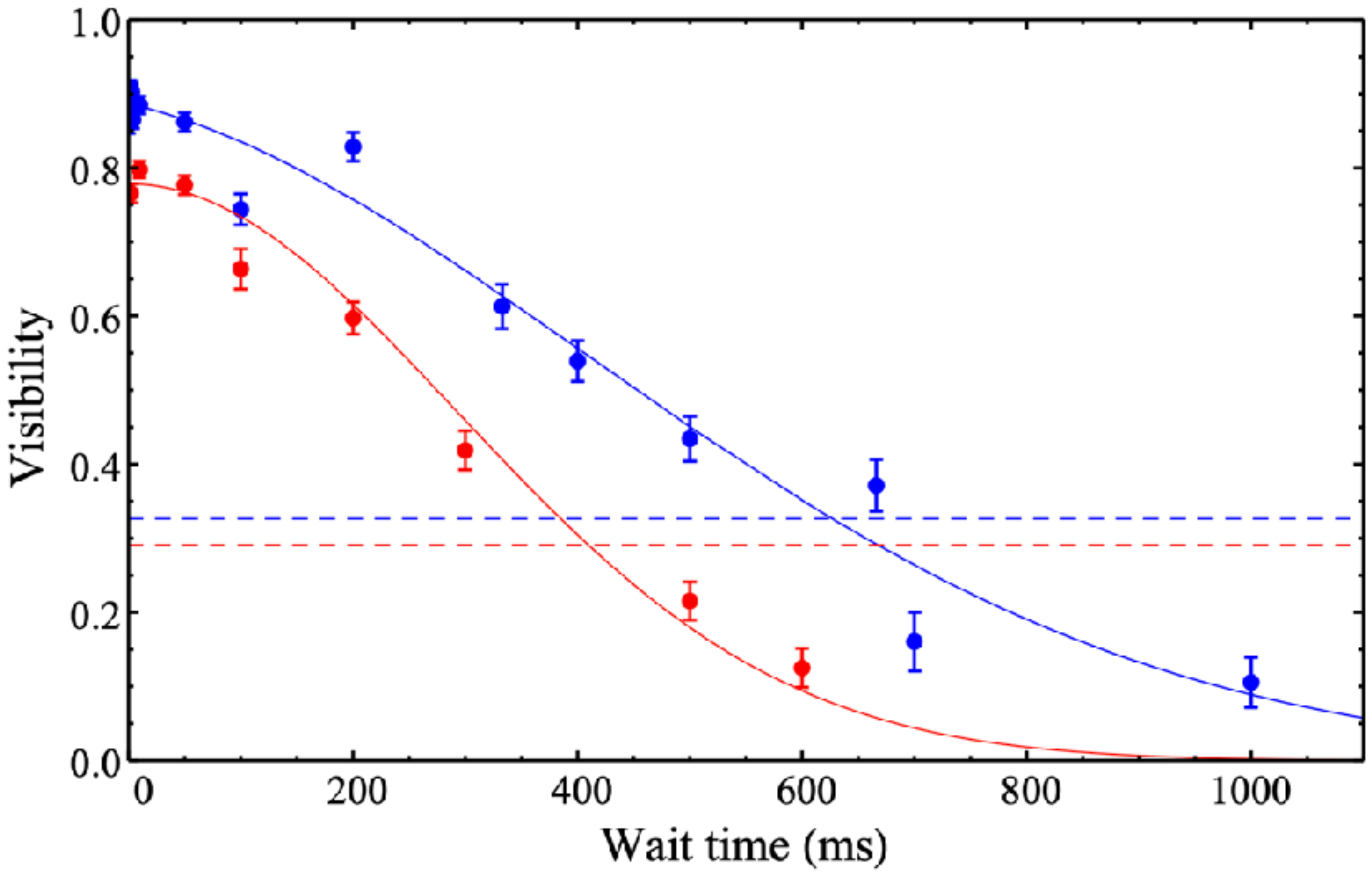
- $T_1(\mathrm{Ca}) \approx 1.1\ \mathrm{s}^{-1}$

Kreuter et al., 2004, Barton et al., 2000, Staanum et al., 2003

Coherence Time T_2

- Decay of relative phase by noises or fluctuation of magnetic field

- $T_2 \sim 0.6\ \mathrm{s}^{-1}$ for $|g,0\rangle + |g,1\rangle$

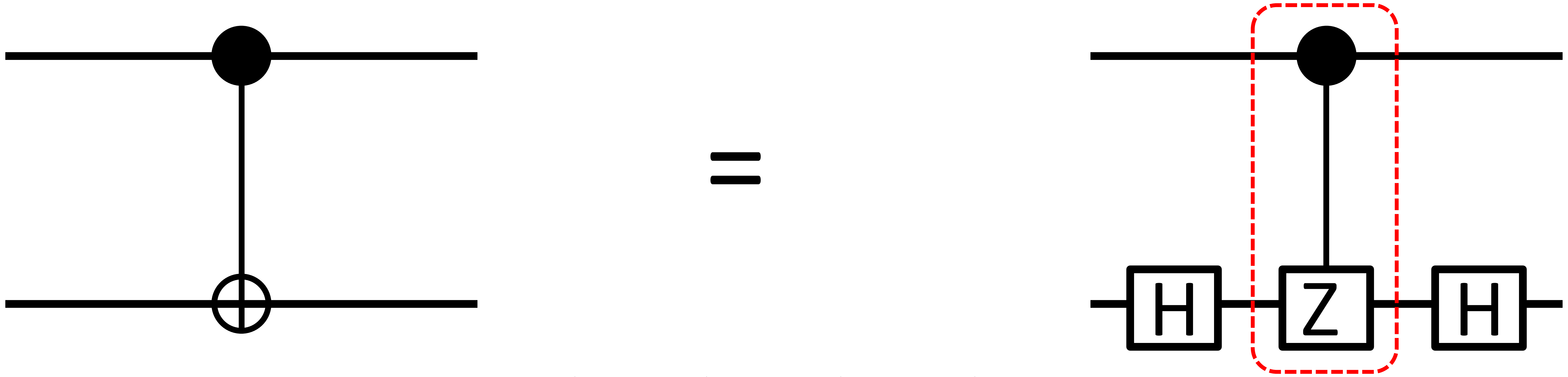


Jarlaud, et al, 2007.04149

６． 量子ビット間相互作用

- Cirac-Zoller ゲートの実装：その１

制御ノットゲート

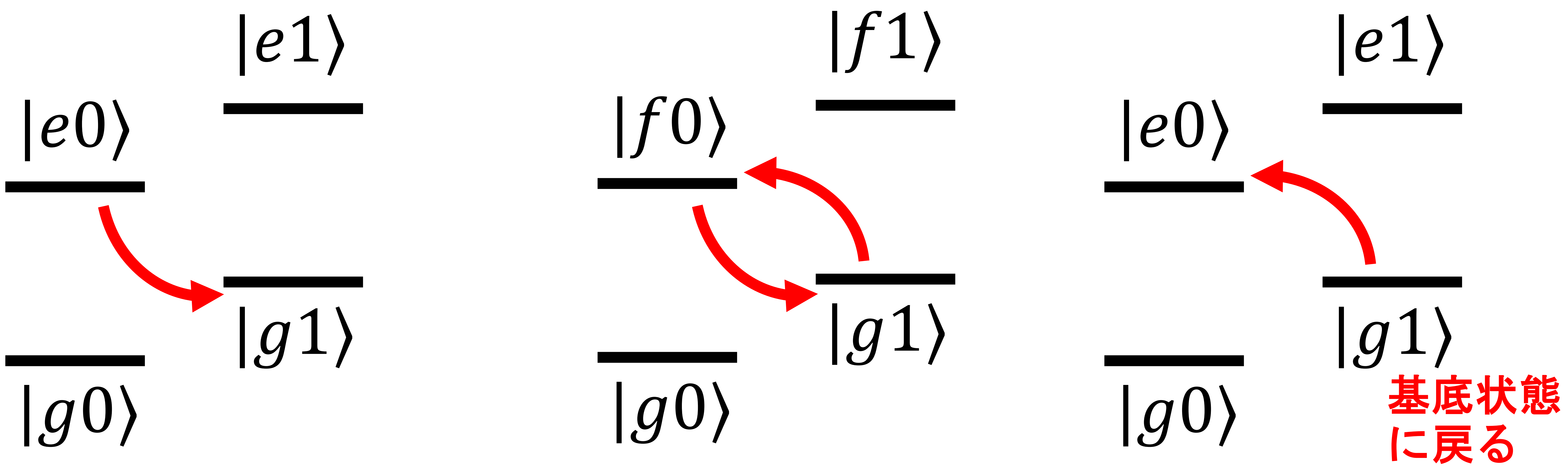


$$
\text{制御Zゲート}: C_z = \begin{array}{c} \begin{array}{cccc} |gg\rangle & |ge\rangle & |eg\rangle & |ee\rangle \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \end{array} \begin{array}{c} \\ |gg\rangle \\ |ge\rangle \\ |eg\rangle \\ |ee\rangle \end{array}
$$

σ_z

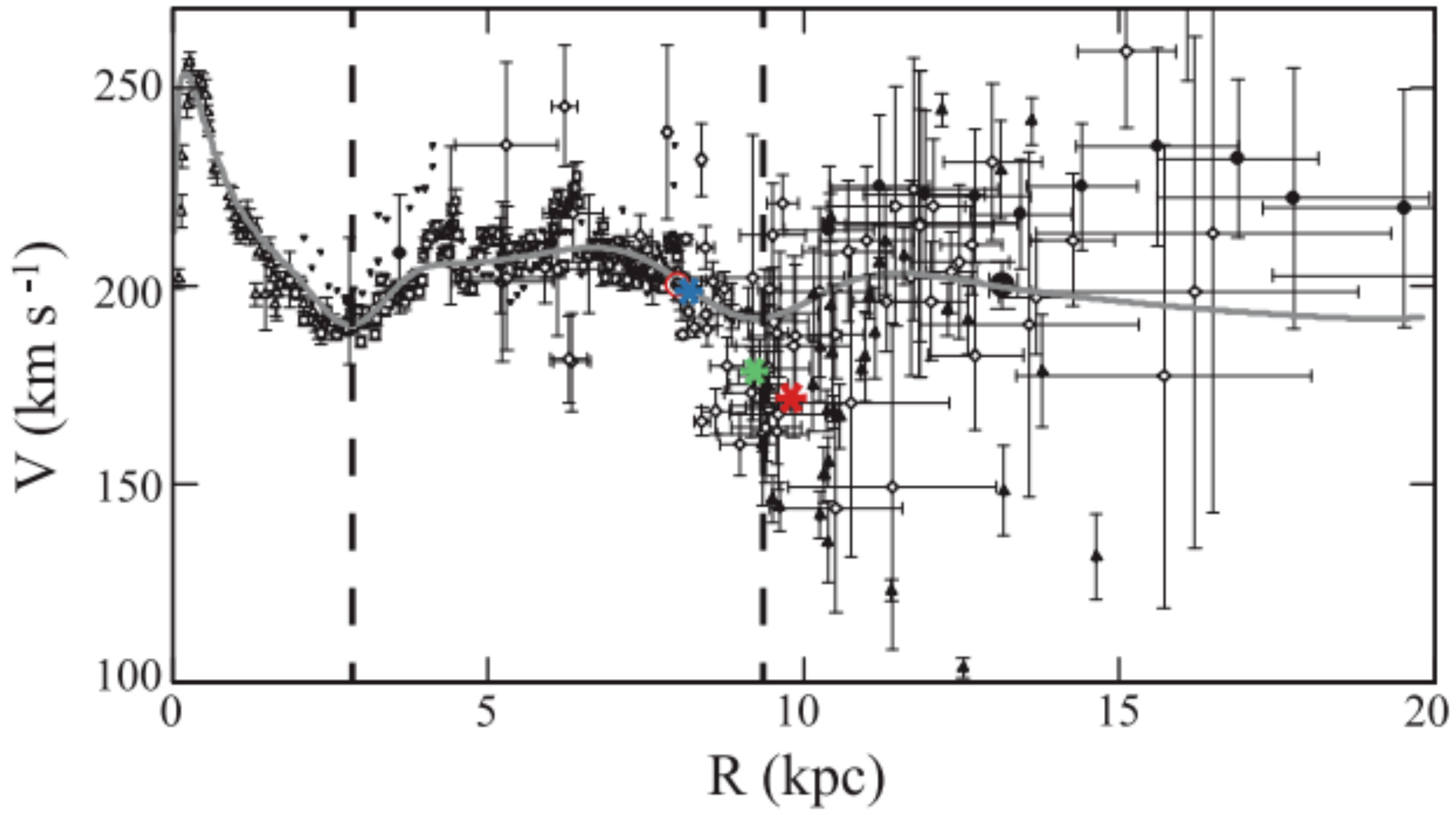
6． 量子ビット間相互作用

- Cirac-Zoller ゲートの実装：その５



$\vert gg0\rangle$	イオン1 RSB半回転	$\vert gg0\rangle$	イオン2 RSB全回転	$\vert gg0\rangle$	イオン1 RSB半回転	$\vert gg0\rangle$
$\vert ge0\rangle$	$\Rightarrow$	$\vert ge0\rangle$	$\Rightarrow$	$\vert ge0\rangle$	$\Rightarrow$	$\vert ge0\rangle$
$\vert eg0\rangle$		$i\vert gg1\rangle$		$-i\vert gg1\rangle$		$\vert eg0\rangle$
$\vert ee0\rangle$		$i\vert ge1\rangle$		$i\vert ge1\rangle$		$-\vert ee0\rangle$

http://veraserver.mtk.nao.ac.jp/hilight/2015_chibueze.html

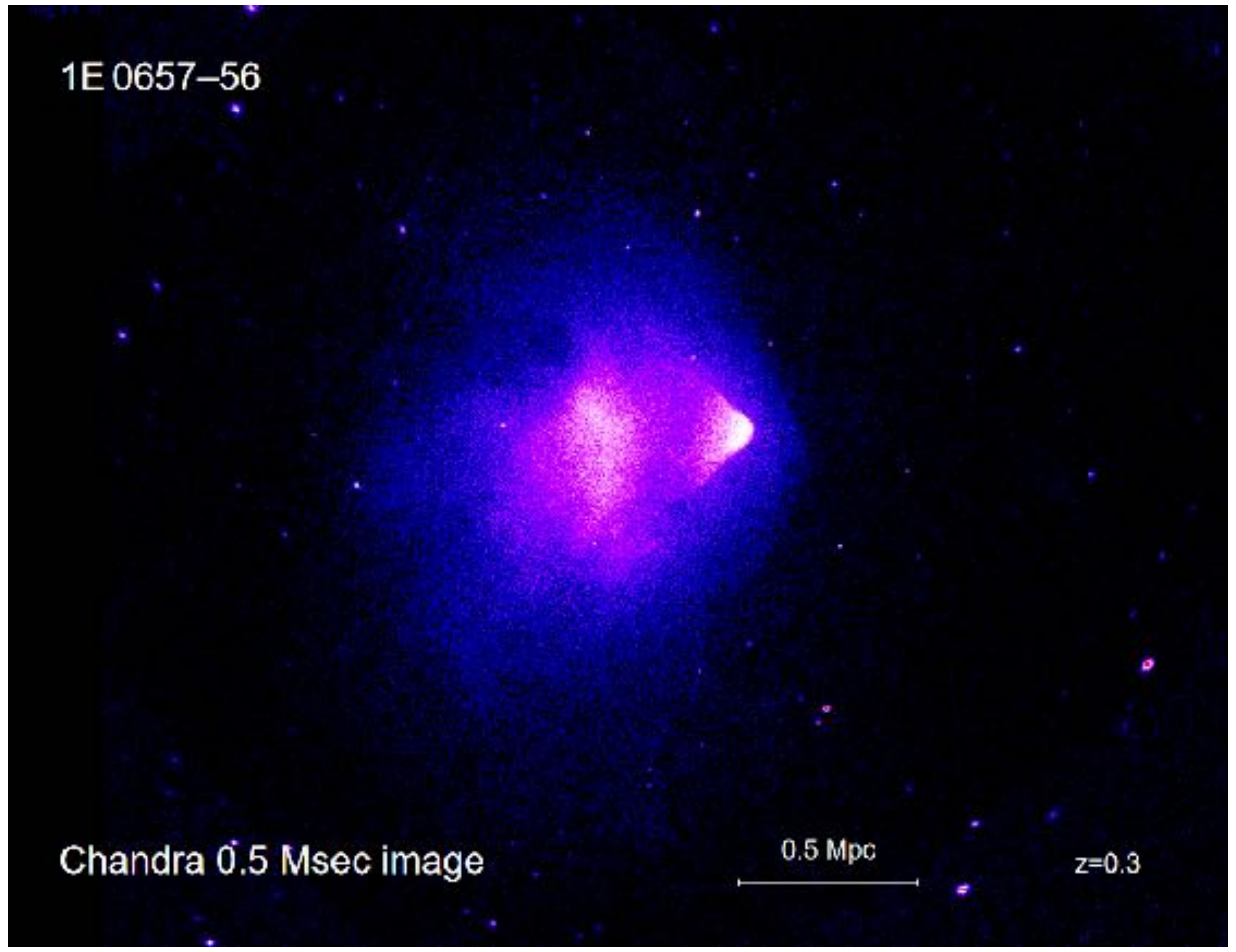


Dark Matter

- Originally motivated by rotational velocity

- Local DM density around the earth $\rho_{DM} \sim 0.45\ \mathrm{GeV\ cm}^{-3}$

- Bullet cluster also suggest the existence of DM



http://cxc.harvard.edu/symposium_2005/proceedings/files/markevitch_maxim.pdf

Axion-Like Particle

- Light ALP DM will oscillate coherently

$$a(t) = a_0 \cos(m_a t - \phi_a)$$

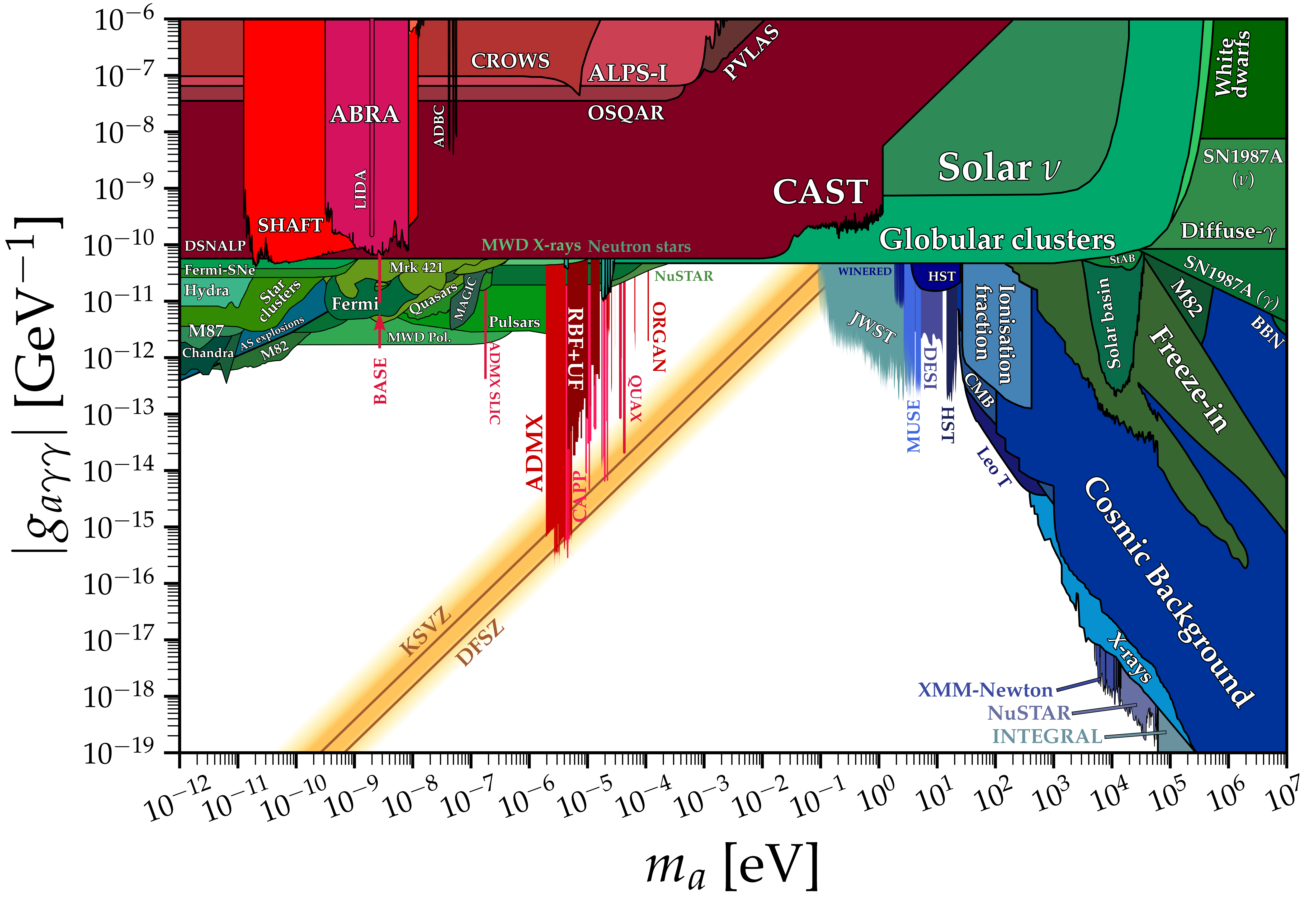
$$\rho_{\rm DM} = m_a^2 a_0^2/2$$

- ALP couples to photon as

$$-\frac{1}{4} g_{a\gamma} a F^{\mu\nu} \tilde{F}_{\mu\nu}$$

- Coherence time

$$T_a = 2\pi/m_a v_{\rm DM}^2$$



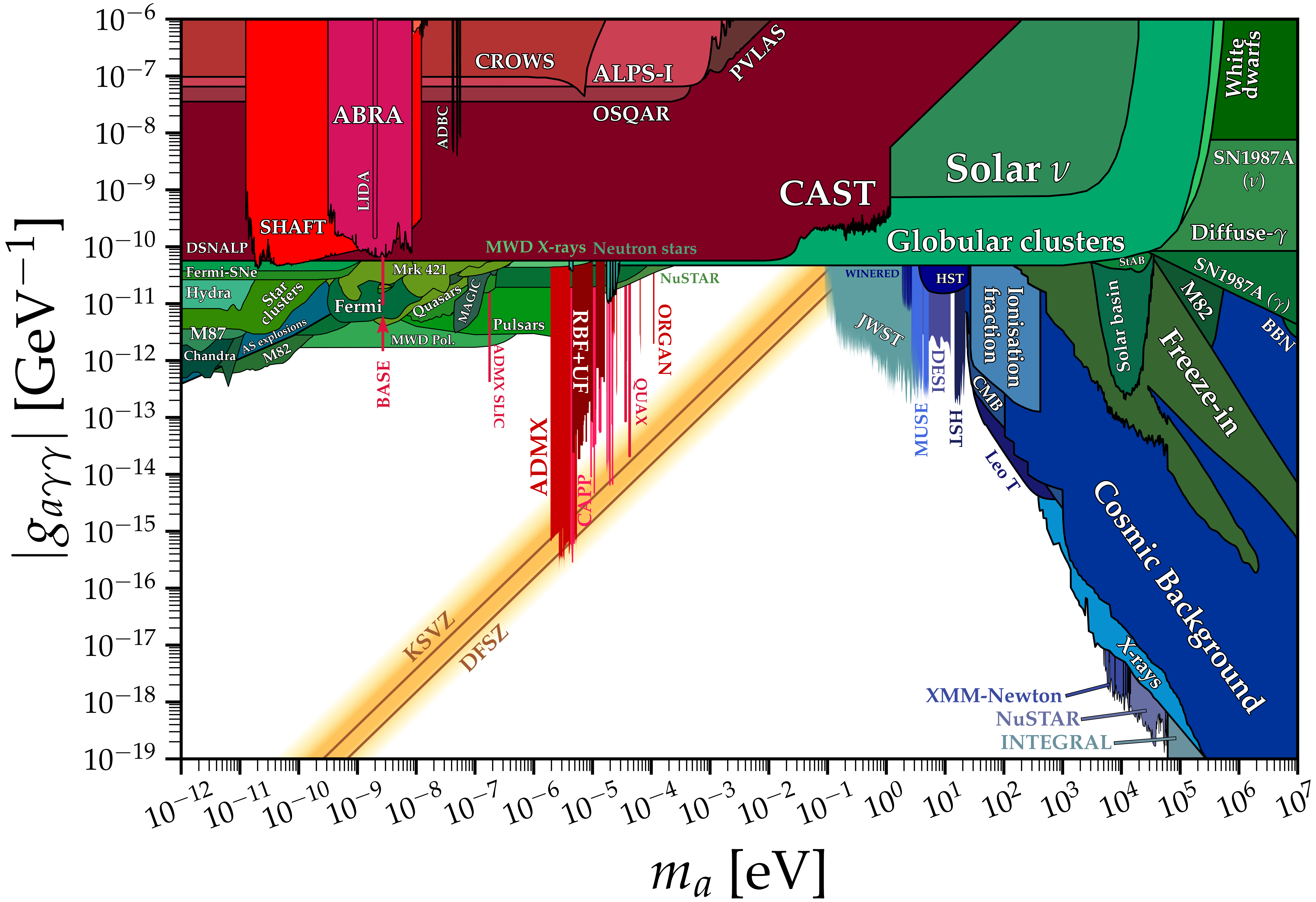
AxionLimits

Axion-Like Particle

- ALP converts to electric field in cavity as Ouellet, Bogorad, (2018)

$$E_{a,z} = \epsilon_a \sqrt{2\rho_{\mathrm{DM}}} \sin\left(m_a t - \phi_a\right)$$

$$\epsilon_a = \frac{B_z}{2}\frac{g_{a\gamma}}{m_a}(m_a R)^2 \left[\left(\log\frac{m_a R}{2} + \gamma - \frac{1}{2}\right)^2 + \left(\frac{\pi}{2}\right)^2\right]^{1/2}$$



Dark Photon

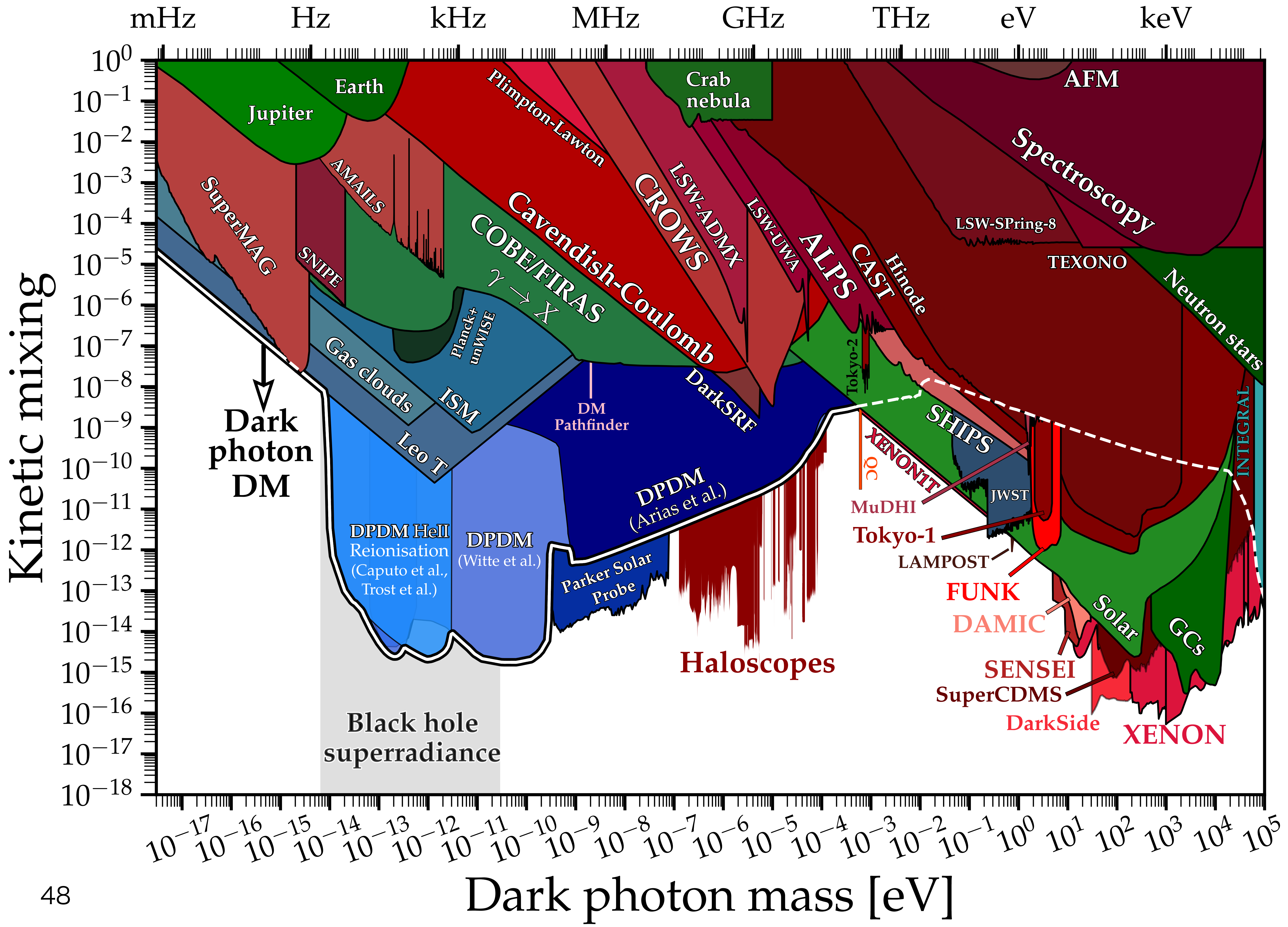
- Kinetic mixing term with photon

$$\frac{\epsilon}{2} F^{\mu\nu} F'_{\mu\nu}$$

$$\vec{E}'(t) = \vec{E}'_0 \sin\left(m_{\mathrm{DP}} t - \phi_{\mathrm{DP}}\right)$$

- Convert to electric field

$$E_{\mathrm{DP},z} = \epsilon_{\mathrm{DP}} \sqrt{2\rho_{\mathrm{DM}}} \sin\left(m_{\mathrm{DP}} t - \phi_{\mathrm{DP}}\right)$$



Detection by Single Ion

$$H_X = eE_{X,z}z$$

$$= e\epsilon_X \sin(m_X t - \phi_X)\sqrt{\frac{\rho_{\mathrm{DM}}}{m_{\mathrm{ION}}\omega_z}}(a^\dagger e^{i\omega_z t} + ae^{-i\omega_z t})$$

- When $\omega_z = m_X$, $E_{X,z}$ excites the oscillator level coherently
- This coherent evolution is expressed by

$$\hat{D}(\beta) = \exp\left(\beta\hat{a}^\dagger - \beta^*\hat{a}\right) \qquad \beta = \alpha_X T, \quad \alpha_X = \frac{e\epsilon_X e^{i\phi_X}}{2}\sqrt{\frac{\rho_{\mathrm{DM}}}{m_{\mathrm{ION}}m_X}}$$

Detection by Single Ion

- For small $\beta = \alpha_X T$,

$$\hat{D}(\beta)\,|\,g,0\rangle \simeq |\,g,0\rangle + \alpha_X T\,|\,g,1\rangle$$

- Therefore, the excitation probability is

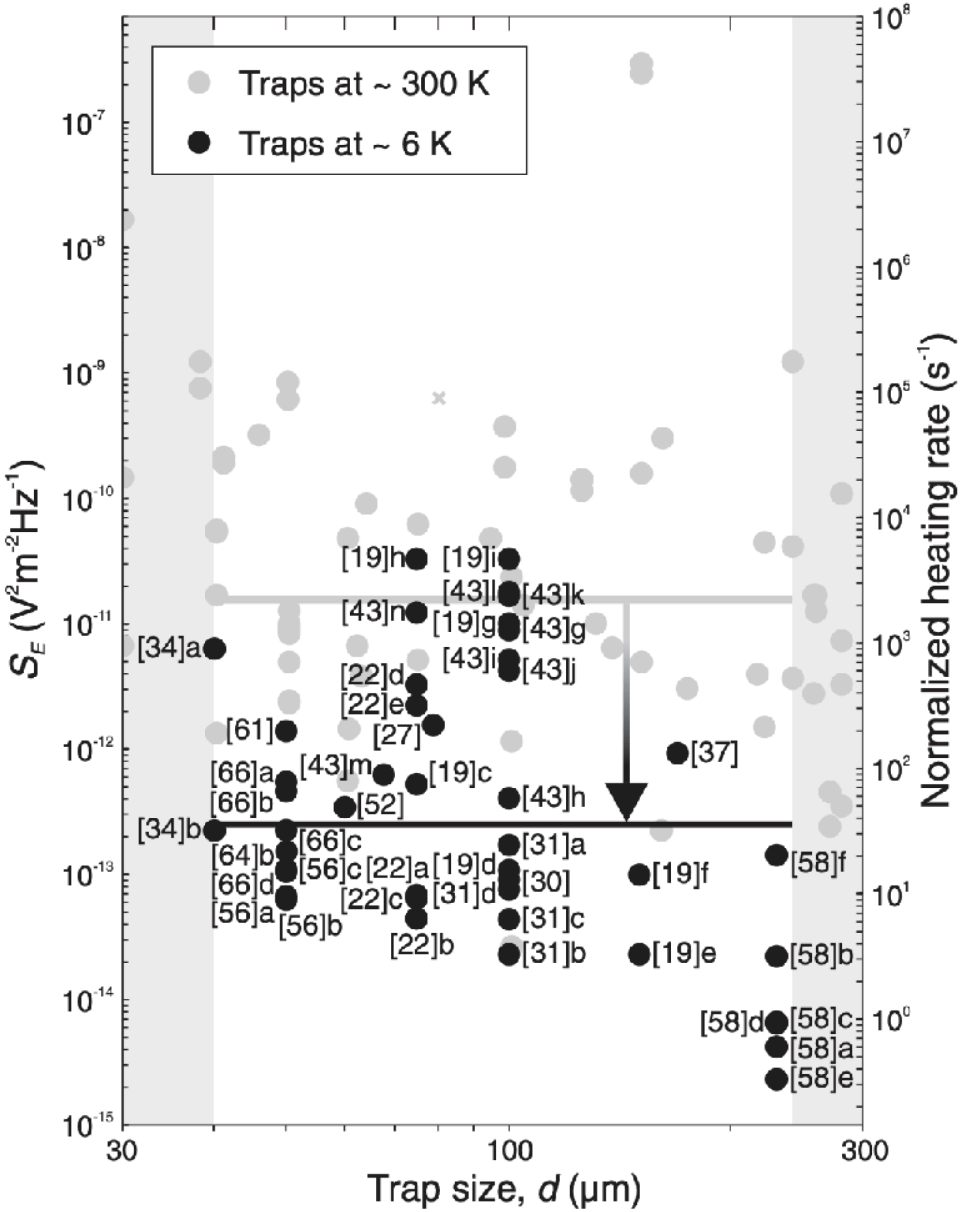
$$P_1(T) \approx \underbrace{\dot{\bar{n}}T}_{\text{Heating noise}} + |\alpha_X|^2 T^2 \qquad T = \frac{2\pi}{m_{\rm DM} v_{\rm DM}^2} \sim 0.4\,{\rm s} \times \left(\frac{10\,{\rm neV}}{m_{\rm DM}}\right)$$

Heating Noises

- Noise comes from electrodes depending on trap size, frequency and, distance from electrodes etc.

- The benchmark value is taken as

$$\dot{\bar{n}} = 0.1 \ \mathrm{s}^{-1}$$



Brownnutt et. al., Rev. Mod. Phys. (2015)

More Serious Calculation

- Same result is obtained by multi-level system

$$H_{\text{total}} = \omega_z a^\dagger a + \sum_j \omega(k_j) b_j^\dagger b_j + H_X + H_{\text{heat}}$$

$$H_{\text{heat}} = \sum_j g_j \left(a^\dagger b_j e^{-i(\omega_z - \omega(k_j))t} + a b_j^\dagger e^{i(\omega_z - \omega(k_j))t} \right)$$

- EOM

$$\begin{cases} \frac{d}{dt} \langle a^\dagger a \rangle = -\gamma \langle a^\dagger a \rangle - i|\alpha_X| e^{i\Delta t} \langle a^\dagger \rangle + i|\alpha_X| e^{-i\Delta t} \langle a \rangle + \gamma \bar{N}, \\ \frac{d}{dt} \langle a \rangle = -i|\alpha_X| e^{i\Delta t} - \frac{\gamma}{2} \langle a \rangle , \\ \frac{d}{dt} \langle a^\dagger \rangle = i|\alpha_X| e^{-i\Delta t} - \frac{\gamma}{2} \langle a^\dagger \rangle , \end{cases}$$

$$\boxed{\langle a^\dagger a \rangle \simeq \gamma \bar{N} t + |\alpha_X|^2 t^2}$$

Detection by Single Ion

- Detection is $|e,0\rangle$ state transited from $|g,1\rangle$ by laser

- 95% C.L. is obtained by ordering

$$S/\sqrt{B} \ > \ 1.645$$

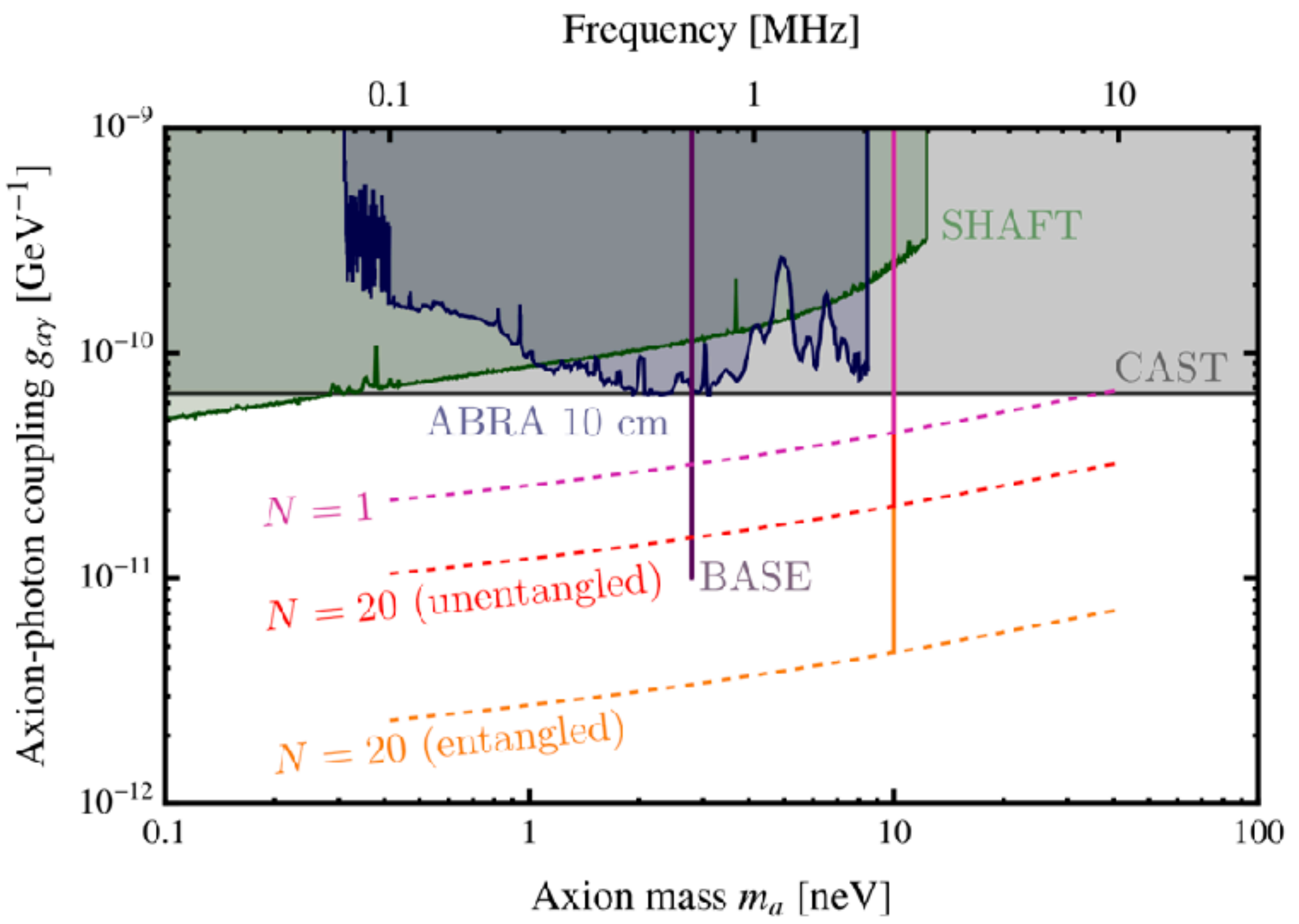
$$S = N_{\mathrm{shot}}|\alpha_X|^2 T^2$$

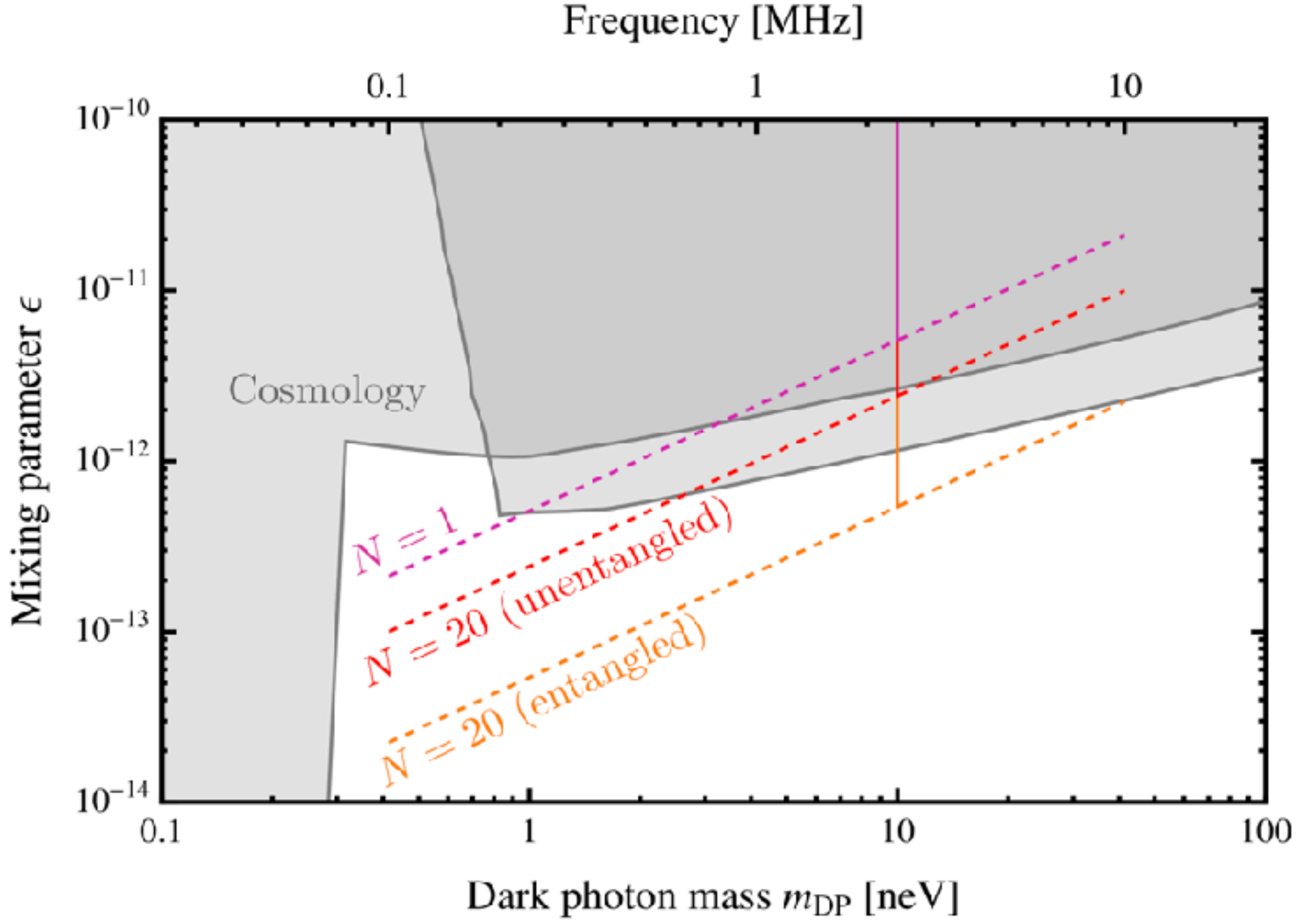
$$B \ = \ N_{\mathrm{shot}}\dot{\bar{n}}T.$$

Sensitivity

$$g_{a\gamma} = 4.4\times10^{-11}\ \mathrm{GeV}^{-1}\times\left(\frac{\dot{\bar{n}}}{0.1\ \mathrm{s}^{-1}}\right)^{1/4}\left(\frac{T_{\mathrm{total}}}{1\ \mathrm{day}}\right)^{-1/4}\left(\frac{T}{0.4\ \mathrm{s}}\right)^{-1/2}\left(\frac{R}{3\ \mathrm{m}}\right)^{-2}$$
$$\times\left(\frac{B_z}{100\ \mathrm{mT}}\right)^{-1}\left(\frac{m_{\mathrm{ION}}}{37\ \mathrm{GeV}}\right)^{1/2}\left(\frac{m_a}{10\ \mathrm{neV}}\right)^{-1/2}\left(\frac{\rho_{\mathrm{DM}}}{0.45\ \mathrm{GeV\ cm}^{-3}}\right)^{-1/2}$$

$$\epsilon = 6.4\times10^{-12}\times\left(\frac{\dot{\bar{n}}}{0.1\ \mathrm{s}^{-1}}\right)^{1/4}\left(\frac{T_{\mathrm{total}}}{1\ \mathrm{day}}\right)^{-1/4}\left(\frac{T}{0.4\ \mathrm{s}}\right)^{-1/2}$$
$$\times\left(\frac{m_{\mathrm{ION}}}{37\ \mathrm{GeV}}\right)^{1/2}\left(\frac{m_{\mathrm{DP}}}{10\ \mathrm{neV}}\right)^{1/2}\left(\frac{\rho_{\mathrm{DM}}}{0.45\ \mathrm{GeV\ cm}^{-3}}\right)^{-1/2}$$

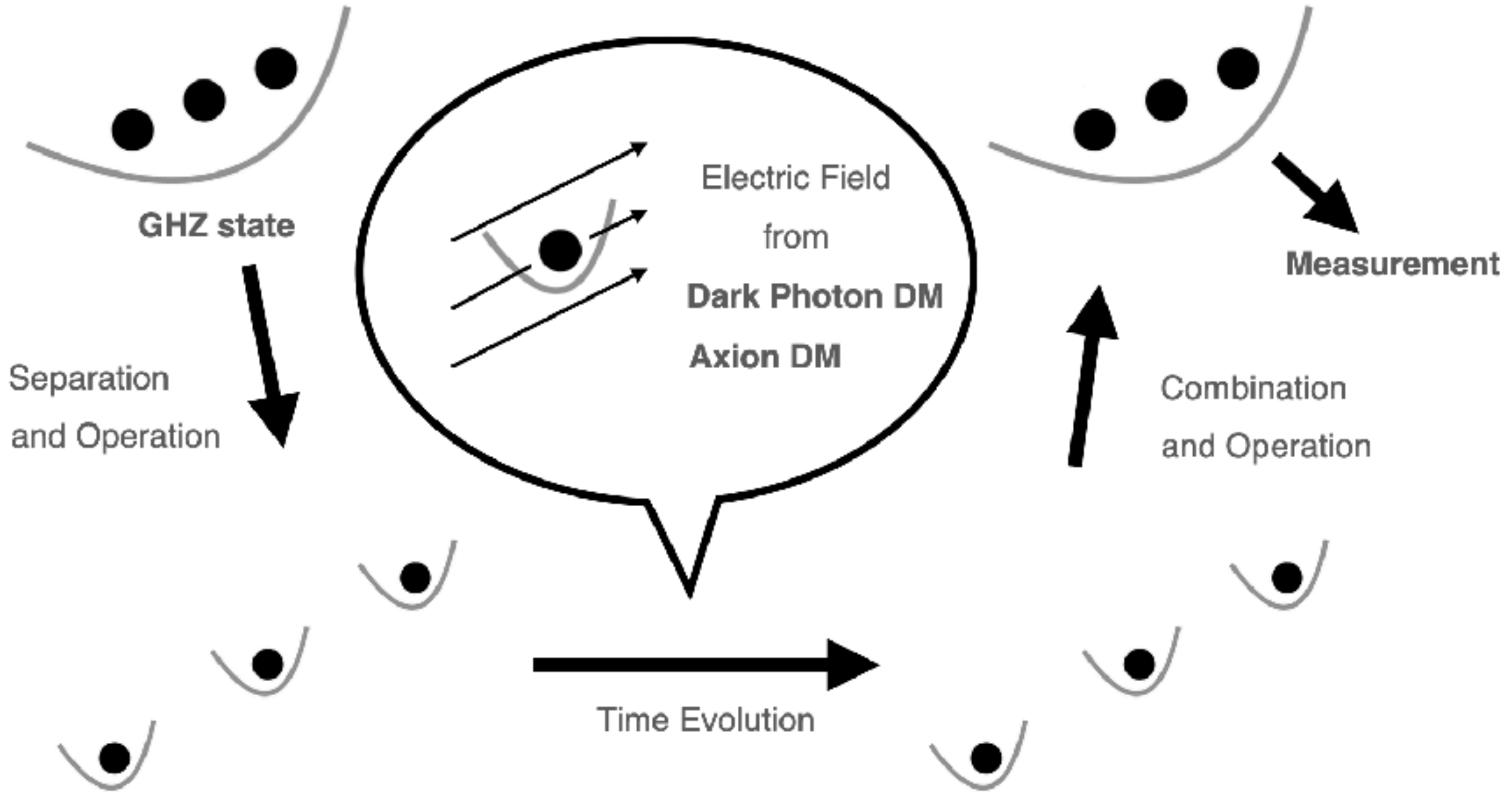




- Corresponding electric field

$$E_z = 3.6\ \mathrm{nV/m}\times\left(\frac{\dot{\bar{n}}}{0.1\ \mathrm{s}^{-1}}\right)^{1/4}\left(\frac{T_{\mathrm{total}}}{1\ \mathrm{day}}\right)^{-1/4}\left(\frac{T}{0.4\ \mathrm{s}}\right)^{-1/2}\left(\frac{m_{\mathrm{ION}}}{37\ \mathrm{GeV}}\right)^{1/2}\left(\frac{\omega_z}{10\ \mathrm{neV}}\right)^{1/2}$$

Operations



- GHZ state as initial state

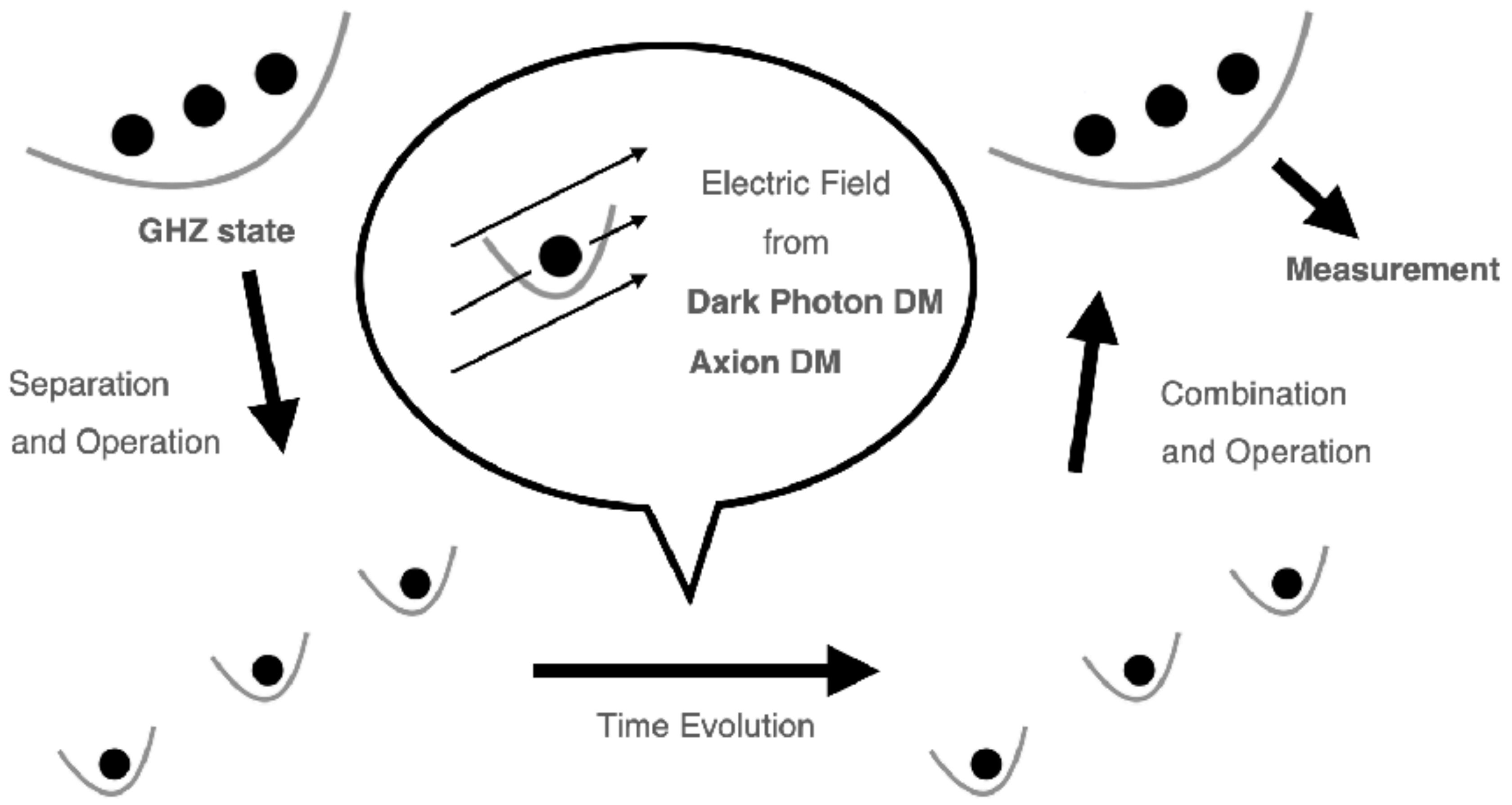
$$|\Psi_{\mathrm{GHZ}}\rangle = \frac{1}{\sqrt{2}}\left(|g, g, \ldots, g, 0\rangle + |e, e, \ldots, e, 0\rangle\right)$$

- Operate H for each qubit, separate and go to osc. mode

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}\left[\left(\frac{|g,0\rangle + |g,1\rangle}{\sqrt{2}}\right)^{\otimes N} + \left(\frac{|g,0\rangle - |g,1\rangle}{\sqrt{2}}\right)^{\otimes N}\right]$$

- Both states are eigenstates of DM operator

Operations



- DM operator act as

$$|\Psi_3\rangle = \frac{1}{\sqrt{2}}\left[e^{iN\beta_{\rm i}}\left(\frac{|g,0\rangle + |g,1\rangle}{\sqrt{2}} + \beta_{\rm r}\frac{|g,0\rangle - |g,1\rangle}{\sqrt{2}}\right)^{\otimes N}\right.$$
$$\left. + e^{-iN\beta_{\rm i}}\left(\frac{|g,0\rangle - |g,1\rangle}{\sqrt{2}} + \beta_{\rm r}\frac{|g,0\rangle + |g,1\rangle}{\sqrt{2}}\right)^{\otimes N}\right]$$

$$\beta = \beta_{\rm r} + i\beta_{\rm i}$$

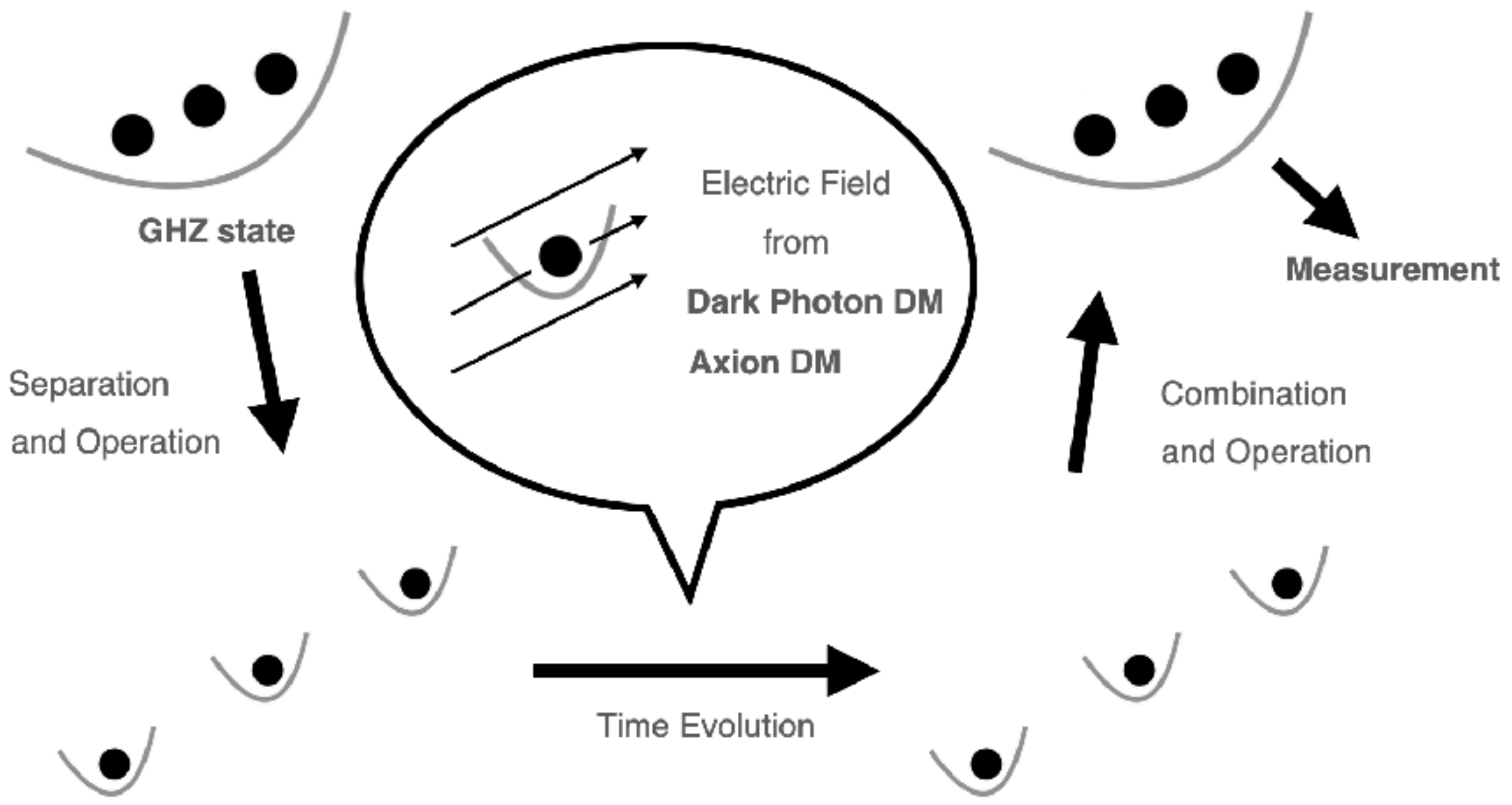
$$\beta = \alpha_X T, \quad \alpha_X = \frac{e\epsilon_X e^{i\phi_X}}{2}\sqrt{\frac{\rho_{\rm DM}}{m_{\rm ION} m_X}}$$

$$D(\beta) = \left(1 - i\beta_{\rm r}\sigma^2_{\rm vib}\right) e^{i\beta_{\rm i}\sigma^1_{\rm vib}} = \left(1 + i\beta_{\rm i}\sigma^1_{\rm vib}\right) e^{-i\beta_{\rm r}\sigma^2_{\rm vib}}$$

- Go back to spin states, $H^{\otimes N}$ gates act as

$$|\Psi_4\rangle = \frac{1}{\sqrt{2}}\left[e^{iN\beta_{\rm i}}\big(\,|g,0\rangle + \beta_{\rm r}\,|e,0\rangle\,\big)^{\otimes N} + e^{-iN\beta_{\rm i}}\big(\,|e,0\rangle + \beta_{\rm r}\,|g,0\rangle\,\big)^{\otimes N}\right]$$

Operations



$$|\Psi_4\rangle = \frac{1}{\sqrt{2}}\left[e^{iN\beta_\mathrm{i}}\left(|g,0\rangle + \beta_\mathrm{r}\cancel{|e,0\rangle}\right)^{\otimes N} + e^{-iN\beta_\mathrm{i}}\left(|e,0\rangle + \beta_\mathrm{r}\cancel{|g,0\rangle}\right)^{\otimes N}\right]$$

- $|e,e,\ldots,e,0\rangle$ to $|e,g,\ldots,g,0\rangle$ by

  $\mathrm{CNOT}_{1,j}\ (j = 2,\ldots,N)$ **and** $H \otimes I^{\otimes N-1}$ **gives**

$$|\Psi_5\rangle \approx |g,g,g,\ldots,g,0\rangle + iN\beta_\mathrm{i}\ |e,g,g,\ldots,g,0\rangle + \cdots$$

- **The excitation probability is**

$$|\langle e,g,g,\ldots,g,0|\Psi_5\rangle|^2 = \underline{N^2}\beta_\mathrm{i}^2$$

https://kds.kek.jp/event/48400/contributions/261318/attachments/177783/236136/20240329_金茶会_公開用.pdf

4. 冷却イオンと量子もつれ状態

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Anton Zeilinger
Prize share: 1/3

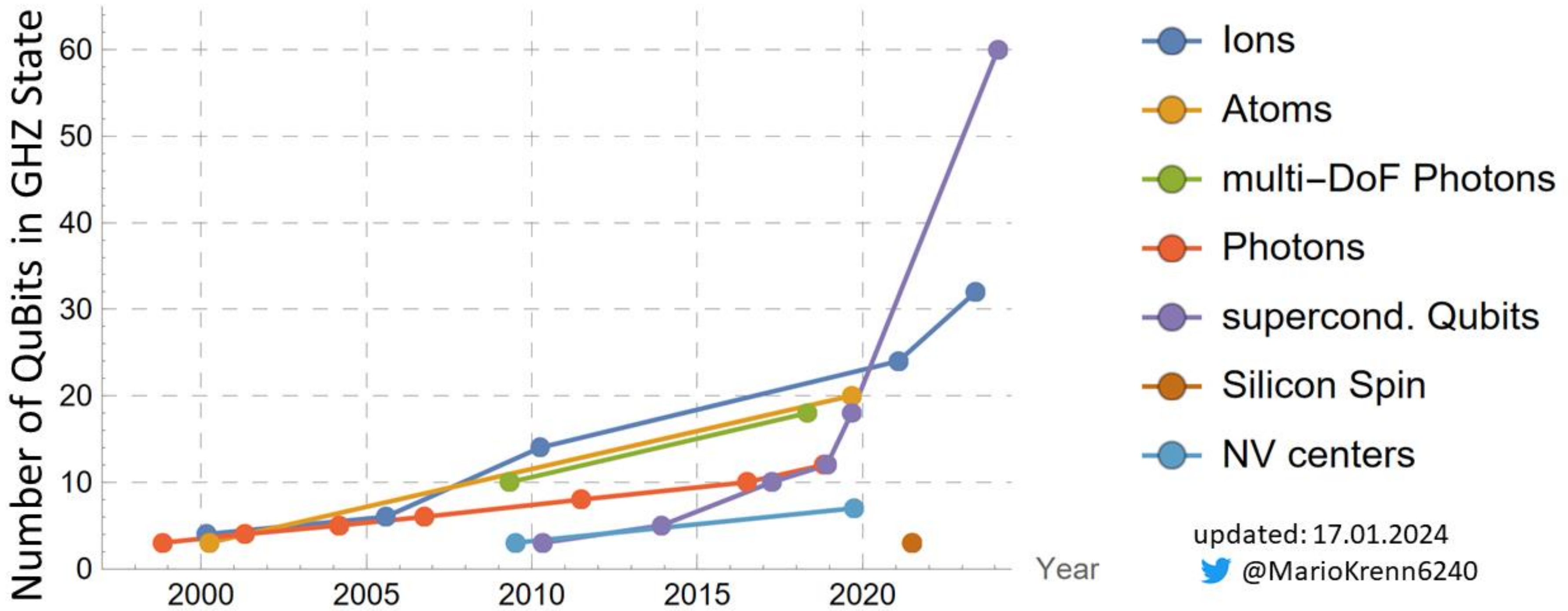
量子もつれ状態の例

$|\psi\rangle = \alpha|e_1\, e_2\rangle + \beta|e_1\, g_2\rangle + \gamma|g_1 e_2\rangle + \delta|g_1 g_2\rangle$

Greenberger-Horne-Zeilinger 状態 （GHZ状態）

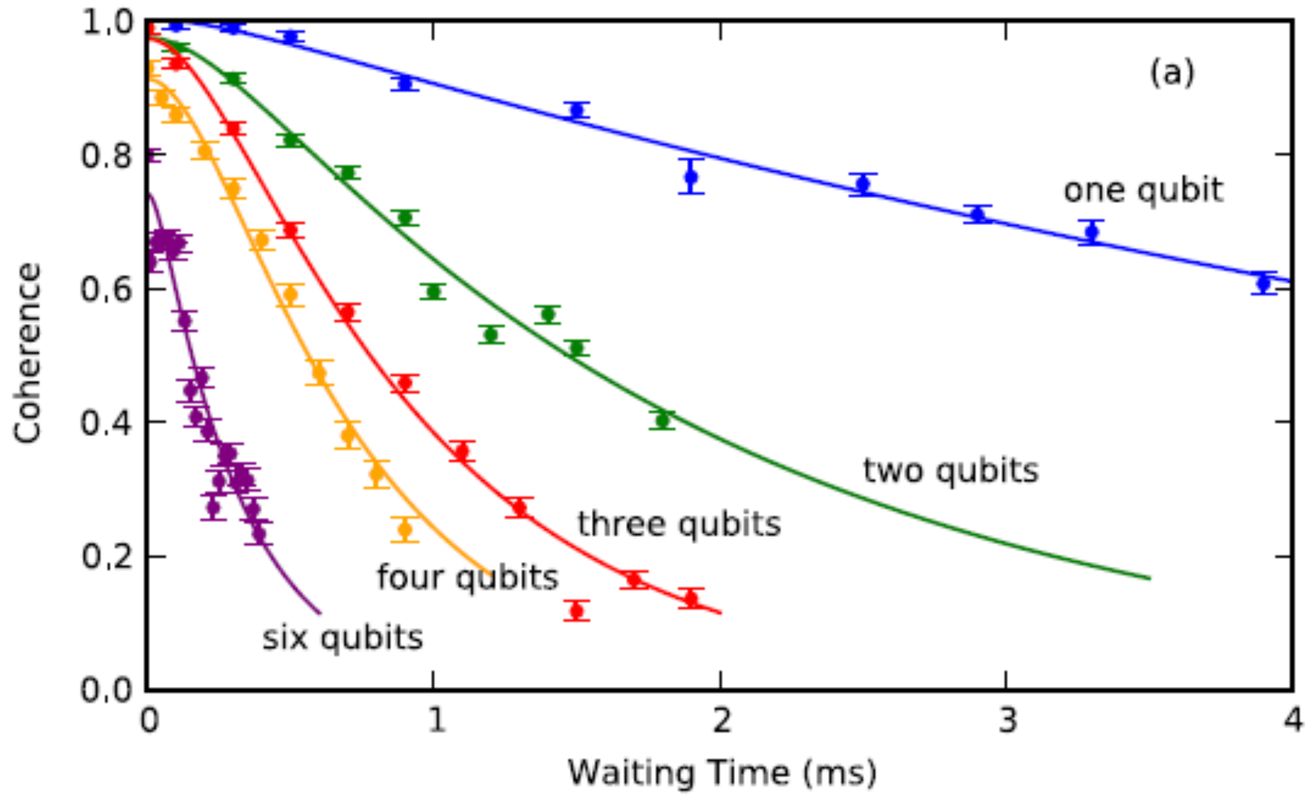
$\dfrac{|ee\cdots e\rangle + |gg\cdots g\rangle}{\sqrt{2}}$ N=2なら $\alpha = \delta = \dfrac{1}{\sqrt{2}}$ $\beta = \gamma = 0$

生成の難しさ



Mario Krenn Xより

状態の維持の難しさ デコヒーレンス



T. Monz *et al*., PRL **106**, 130506(2011)