Composite Dark Matter with Forbidden Annihilation

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[JHEP **09** (2024) 064 (arXiv:2404.03963)], work in progress

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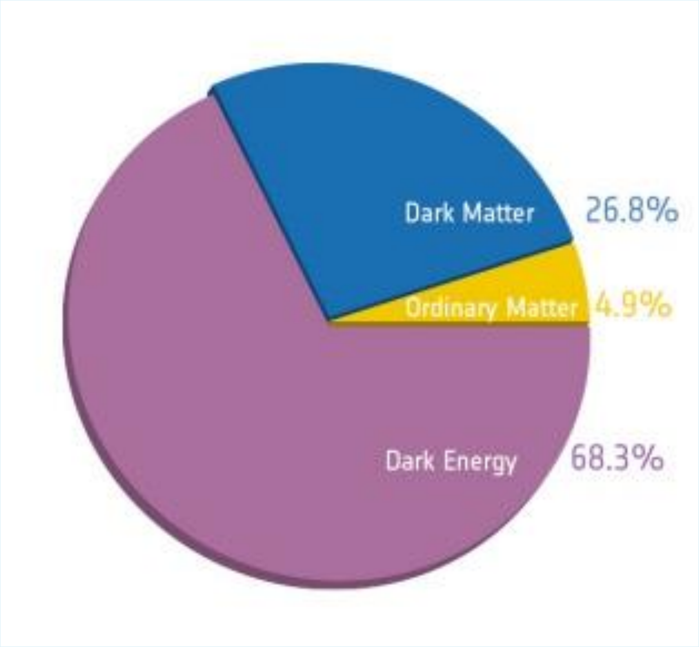
Introduction

Dark Matter (DM)

energy density: $\Omega_{\mathrm{DM}}h^2 \simeq 0.12$

Planck Collaboration (2018)

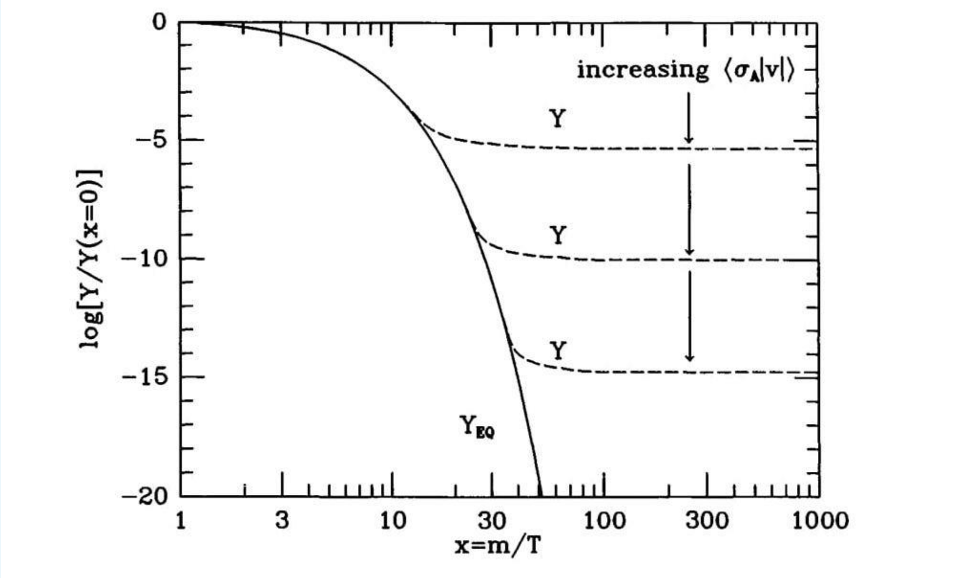


https://sci.esa.int/web/planck/-/51557-planck-new-cosmic-recipe

Freeze-out mechanism (e.g. WIMP DM)

Decoupling DM from thermal bath in the early universe

$$\Omega_{\mathrm{DM}}h^2 \sim 0.12 \frac{2\times 10^{-26}\ \mathrm{cm}^3/\mathrm{s}}{\langle \sigma v\rangle_{\mathrm{ann}}}$$



Kolb, Turner *"The Early Universe"*

Motivation

Indirect detection

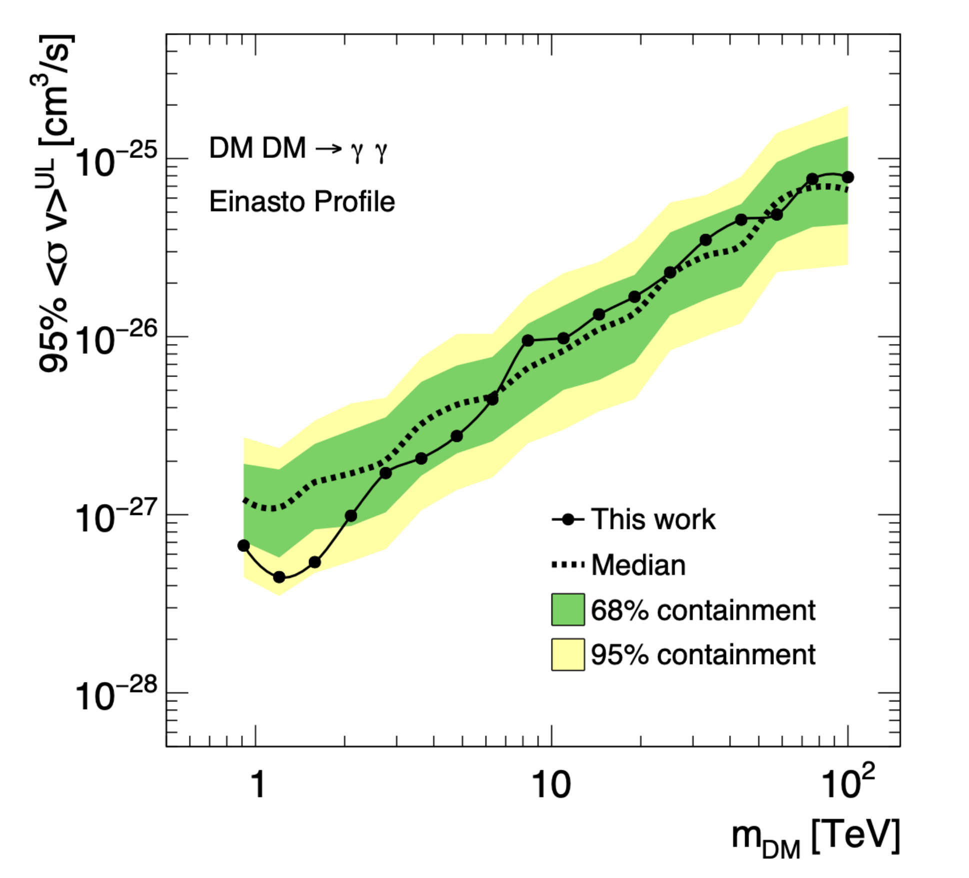
Sensitivity to DM with $O(1-100)$ TeV mass

Heavy WIMP as DM candidate

$$\langle \sigma v \rangle_{\text{ann}} \sim \frac{\alpha_{\text{DM}}^2}{m_{\text{DM}}^2}$$

$\alpha_{\text{DM}} \sim \mathcal{O}(1)$ for $m_{\text{DM}} \sim \mathcal{O}(10)$ TeV

QCD-like DM model



MAGIC Collaboration [PRL **130** (2023)]

Composite DM Model

$SU(N)_d$ gauge symmetry
& $SU(2)_W$ triplet dark quark (3-flavor)

Bai and Hill [PRD **82** (2010)]

	$SU(N)_d$	$SU(3)_c$	$SU(2)_W$	$U(1)_Y$
ψ	N	$\mathbf{1}$	$\mathbf{3}$	0
$\bar{\psi}$	$\bar{N}$	$\mathbf{1}$	$\mathbf{3}$	0

dark quark mass $m_q < \Lambda_d$

Chiral symmetry breaking: $SU(3)_L \times SU(3)_R \to SU(3)_V \supset SU(2)_W$ by $\langle \bar{\psi}\psi \rangle \sim v_d^3$

Dark pion $SU(2)_W$ 3-plet χ $\oplus$ $SU(2)_W$ 5-plet π

Chiral Lagrangian

$$\mathcal{L} \supset \frac{f_d^2}{4}\mathrm{tr}[D_\mu U D^\mu U^\dagger] + v_d^3 \mathrm{tr}[MU + M^\dagger U^\dagger] + \mathcal{L}_{\mathrm{WZW}}$$

$$U = \exp\left(\frac{\sqrt{2}i}{f_d}(\Pi_3 + \Pi_5)\right)$$

$$M = \mathrm{diag}(m_q, m_q, m_q)$$

Parameters

dark pion decay const. f_d

dark quark mass m_q

Accidental Symmetry & Stability of DM

G-Parity [Lee & Yang Nuovo. Cim. **10** (1956)]

	$SU(N)_d$	$SU(3)_c$	$SU(2)_W$	$U(1)_Y$
ψ	N	$\mathbf{1}$	$\mathbf{3}$	0
$\bar{\psi}$	$\bar{N}$	$\mathbf{1}$	$\mathbf{3}$	0

$$U = \exp\left(\frac{\sqrt{2}i}{f_d}(\Pi_3 + \Pi_5)\right) \to U^T$$

$SU(2)_W$ 3-plet (anti-symmetric) $SU(2)_W$ 5-plet (symmetric)

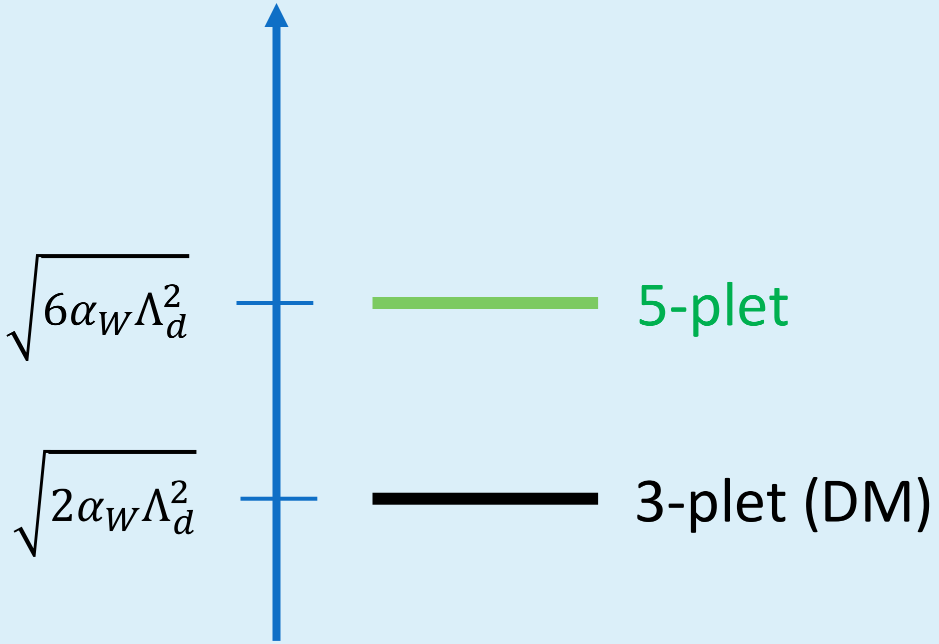
$$\Pi_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i\chi^0 & \frac{\chi^- - \chi^+}{\sqrt{2}} \\ i\chi^0 & 0 & -i\frac{\chi^- + \chi^+}{\sqrt{2}} \\ -\frac{\chi^- - \chi^+}{\sqrt{2}} & i\frac{\chi^- + \chi^+}{\sqrt{2}} & 0 \end{pmatrix}, \quad \Pi_5 = \begin{pmatrix} \frac{\pi^0}{\sqrt{6}} - \frac{\pi^{++} + \pi^{--}}{2} & -i\frac{\pi^{++} - \pi^{--}}{2} & \frac{\pi^+ + \pi^-}{2} \\ -i\frac{\pi^{++} - \pi^{--}}{2} & \frac{\pi^0}{\sqrt{6}} + \frac{\pi^{++} + \pi^{--}}{2} & i\frac{\pi^+ - \pi^-}{2} \\ \frac{\pi^+ + \pi^-}{2} & i\frac{\pi^+ - \pi^-}{2} & -\sqrt{\frac{2}{3}}\pi^0 \end{pmatrix}$$

G-parity $\Pi_3 \to -\Pi_3$ (odd) $\Pi_5 \to \Pi_5$ (even)

DM candidate: χ^0

decay into EW gauge bosons

Mass Spectrum & Annihilation Processes

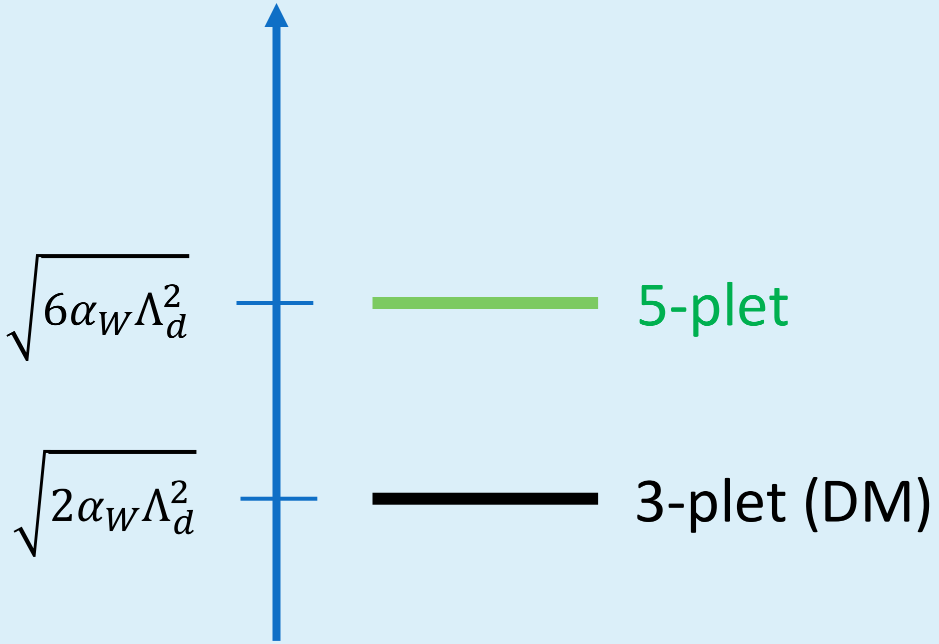


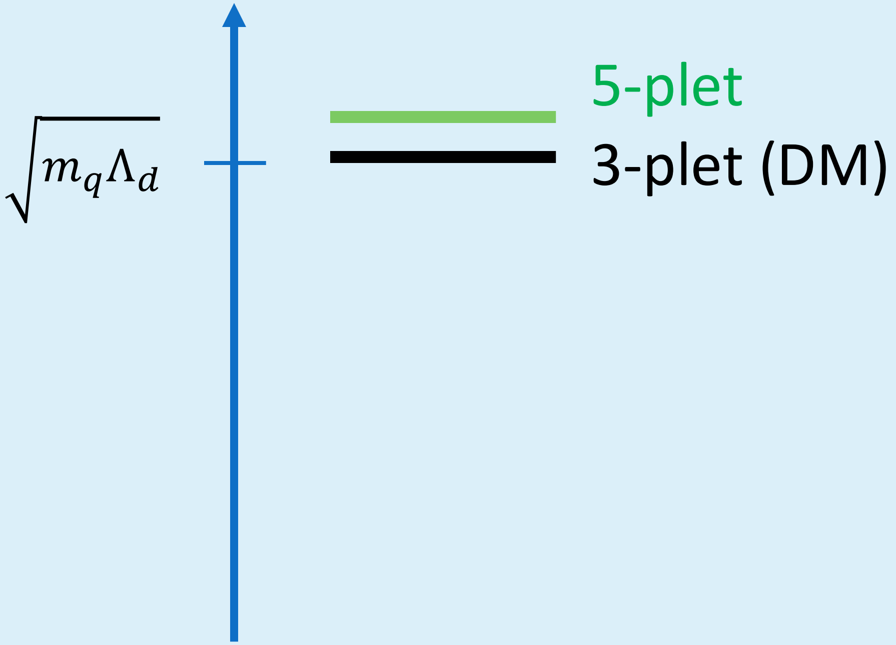
①large mass splitting : DM+DM→EW gauge bosons

$$\langle \sigma v \rangle_{WW} \simeq \frac{4\pi \alpha_W^2}{m_\chi^2}$$

$m_\chi \sim 1.8\ \mathrm{TeV}$ (leading order)

Mass Spectrum & Annihilation Processes





①large mass splitting : DM+DM→EW gauge bosons

$$\langle\sigma v\rangle_{WW} \simeq \frac{4\pi\alpha_W^2}{m_\chi^2}$$

$m_\chi \sim 1.8\ \mathrm{TeV}$ (leading order)

②small mass splitting : DM+DM→ 5-plet

Forbidden channel Abe, Sato, **TY** [JHEP **09** (2024)]

$$\langle\sigma v\rangle_{\pi\pi} \propto \frac{m_\chi^2}{f_d^4}\exp\left(-\frac{m_\pi - m_\chi}{T}\right)$$

$m_\chi \sim \mathcal{O}(1-10)\ \mathrm{TeV}$ (leading order)

Leading Order Calculation

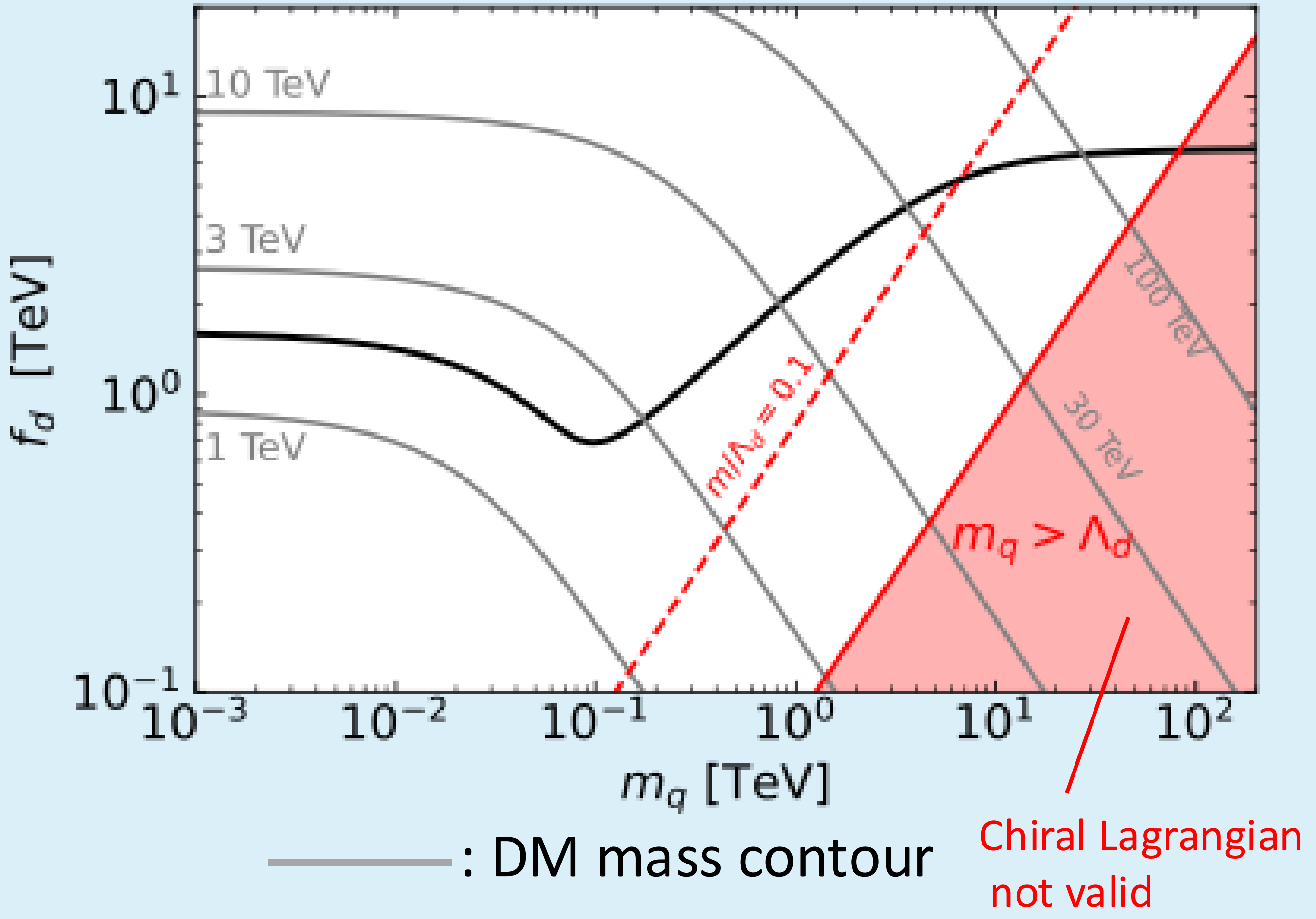
Abe, Sato, TY [JHEP **09** (2024)]

small $m_q,\ m_\chi \sim 1.8\ \mathrm{TeV}$

large $m_q,\ m_\chi \sim O(1-10)\ \mathrm{TeV}$

→**Forbidden channels contribute to relic abundance**

$N=3$



Leading Order Calculation

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small $m_q,\ m_\chi \sim 1.8\ \mathrm{TeV}$

large $m_q,\ m_\chi \sim O(1-10)\ \mathrm{TeV}$

→**Forbidden channels contribute to relic abundance**

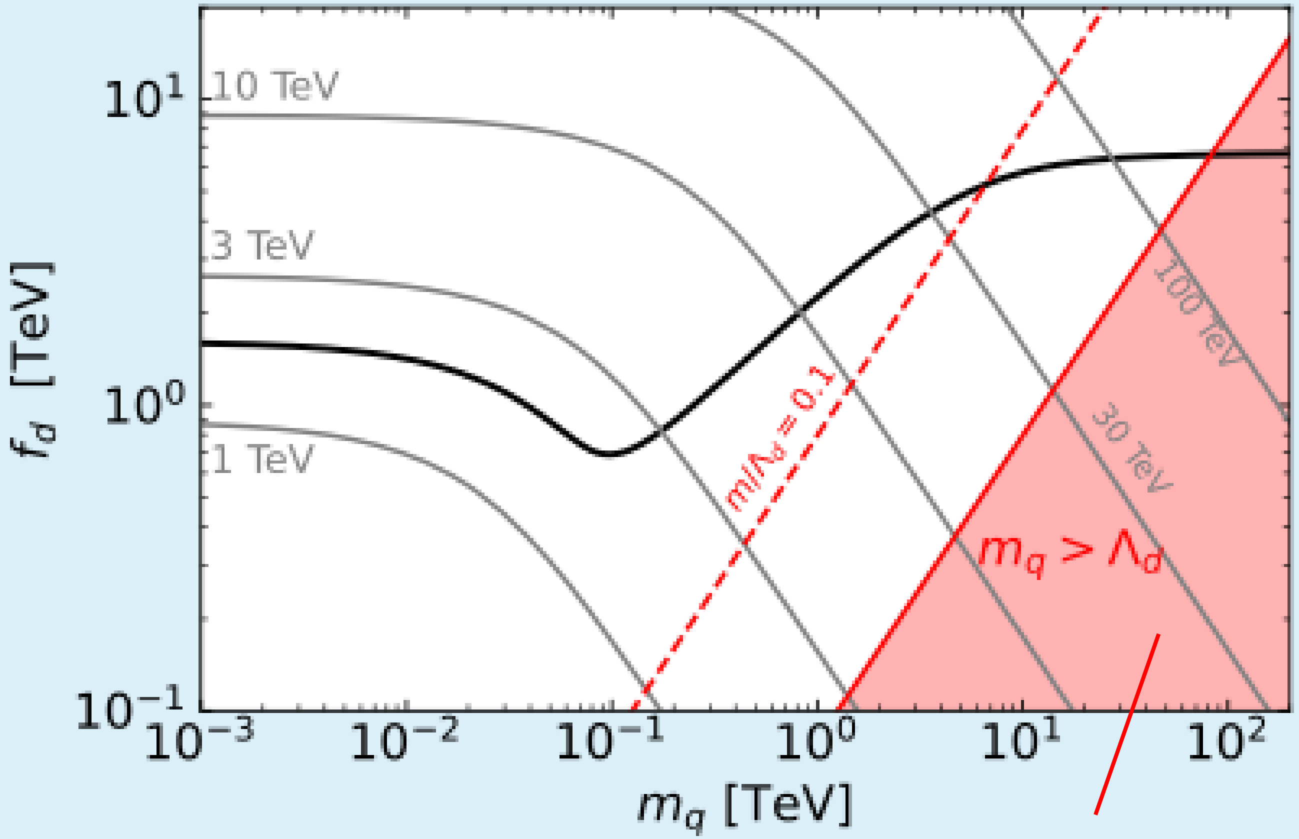
Next step…

Sommerfeld Effect (SE)

EW interaction of DM affects annihilation cross sections.

[Hisano, Matsumoto, Nojiri, Saito PRD **71** (2005)]

$N = 3$



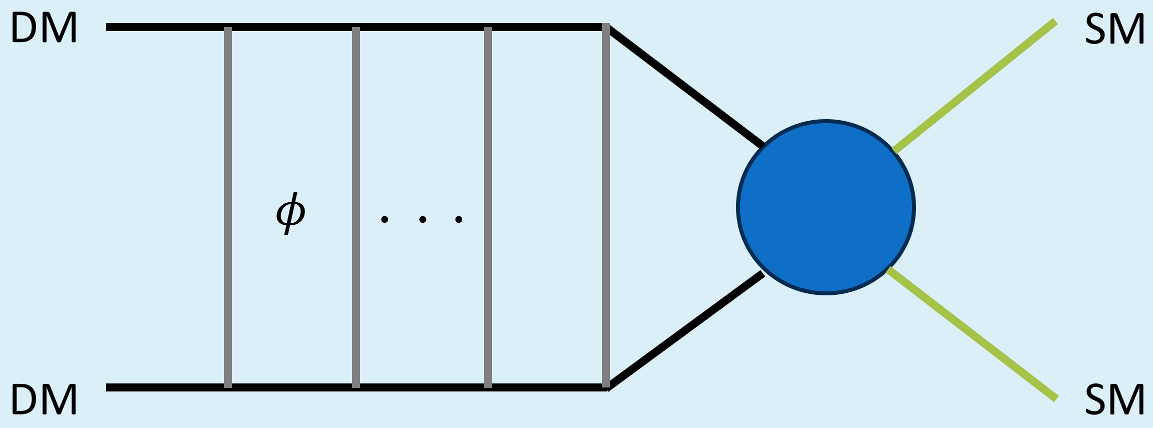
: DM mass contour

Chiral Lagrangian not valid

Sommerfeld Effect (SE)

Leading order $\sigma_{\rm LO}$:
Only short-range effect is included.

If DM has long-range interactions,
Resummation is needed!



Including SE, $\sigma_{\rm SE} \simeq |\psi(\mathbf{0})|^2 \sigma_{\rm LO}$

[Arkani-Hamed, Finkbeiner, Slatyer, Weiner, PRD **79** (2009)]

↑ obtained by solving Schroedinger eq.

SE affects cross section when $m_{\rm DM} \gg m_\phi$!

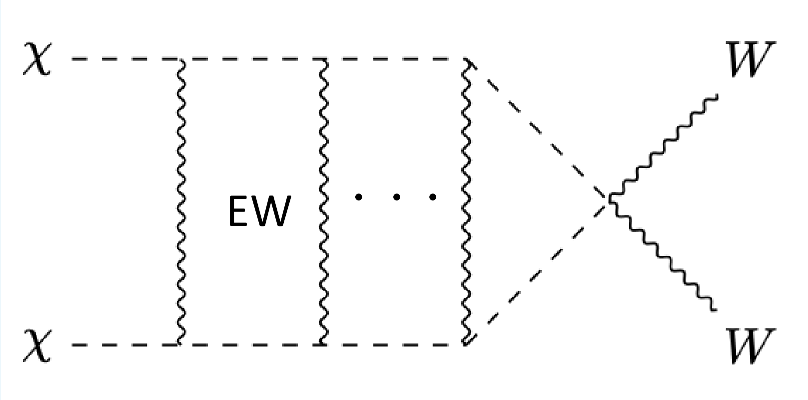
Sommerfeld Effect of Composite DM

$m_\chi \gg m_{W,Z}$ → EW interaction behaves as long-range force

[1] $m_\pi \gg m_\chi$,

$m_\chi \simeq 1.8\ \mathrm{TeV}$ (w/o SE) ⇒ $m_\chi \simeq 2.5\ \mathrm{TeV}$ (w/ SE)

Cirelli, Strumia, Tamburini [NPB **787** (2007)]
Katayose, Matsumoto, Shirai, Watanabe [JHEP**09** (2021)]

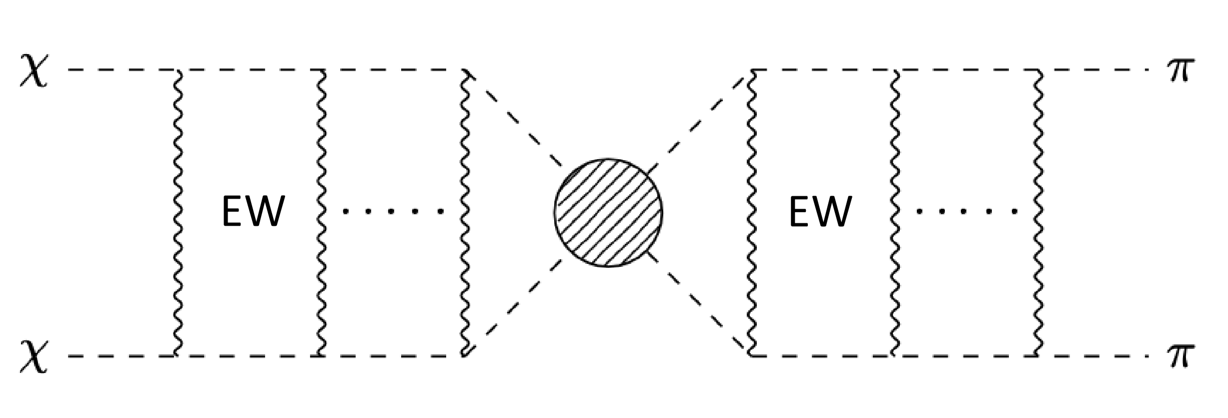


[2] $m_\pi \simeq m_\chi \gg m_{W,Z}$

Forbidden channel $\chi\chi \to$ 5-plet

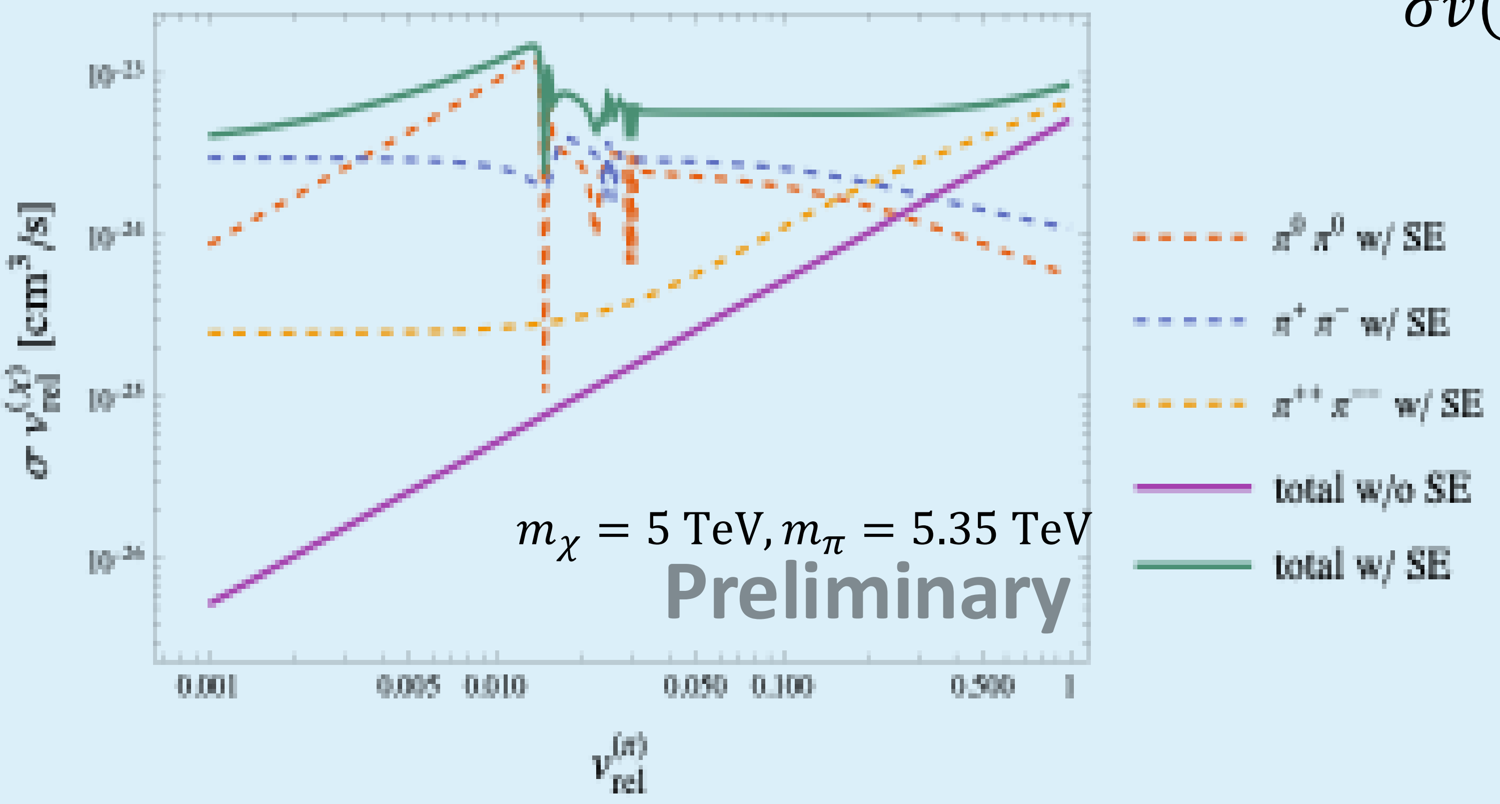
SE in final states ?

Cui, Luo [JHEP **01** (2021)]



Enhanced Cross Section

$$\mathcal{L} \supset \frac{f_d^2}{4}\mathrm{tr}\left[\partial_\mu U \partial^\mu U^\dagger\right] + v_d^3 \mathrm{tr}[MU + \mathrm{h.c.}]$$



w/o SE,

$$\sigma v(\chi^0\chi^0 \to \pi\pi) \simeq \frac{|\boldsymbol{p}_\pi|}{32\pi m_\pi^3}|\mathcal{M}|^2 \propto v_{\rm rel}^{(\pi)}$$

w/ SE, $\quad \mathcal{M} \to \psi_\chi(0)\mathcal{M}\psi_\pi(0)$

At Low $v_{\rm rel}^{(\pi)}$, SE $\propto 1/v_{\rm rel}^{(\pi)}$
(Coulomb force of final two-body)

Enhanced Cross Section

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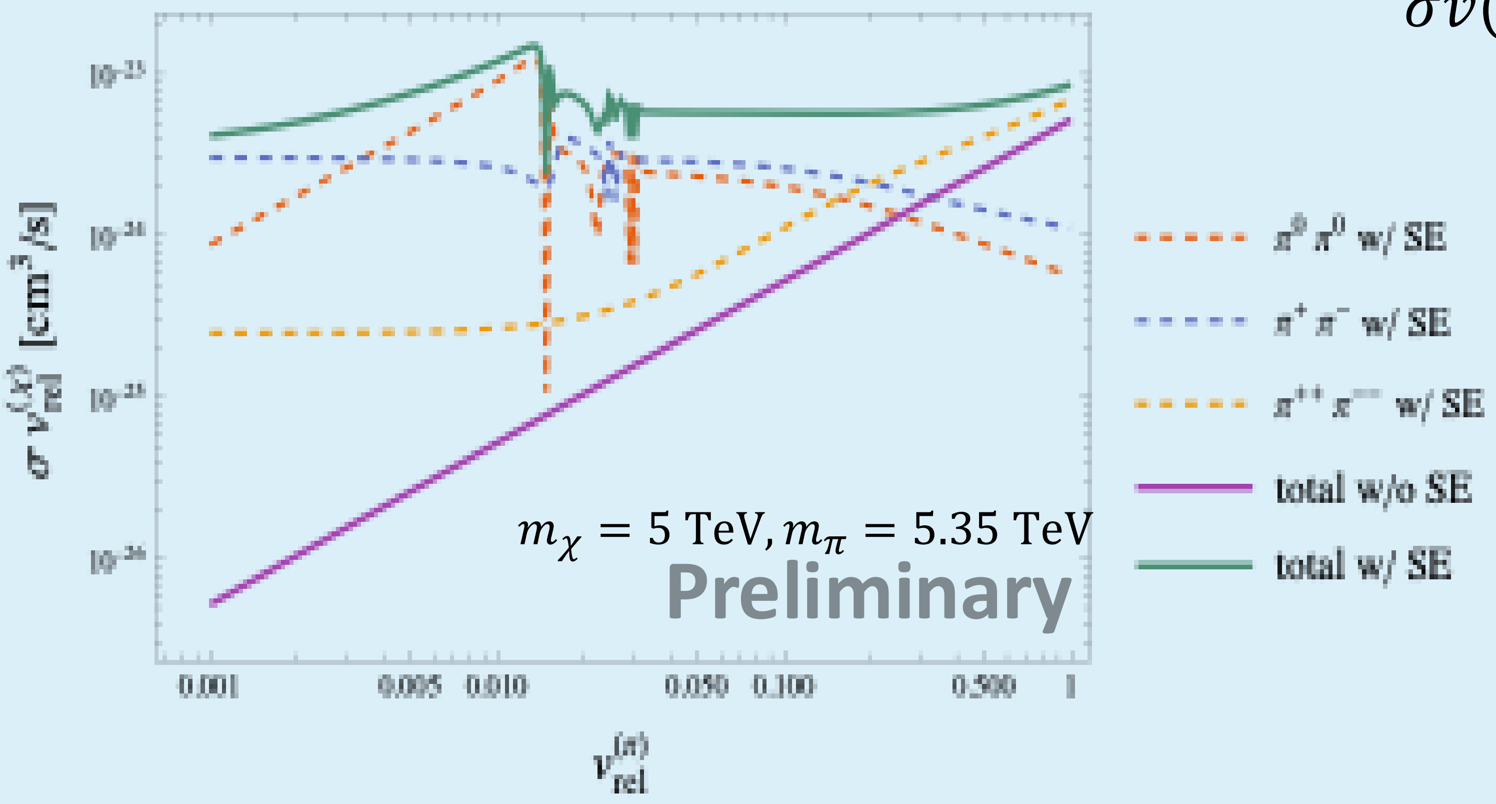

w/o SE,

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w/ SE, $\quad \mathcal{M} \to \psi_\chi(0)\mathcal{M}\psi_\pi(0)$

At Low $v_{\mathrm{rel}}^{(\pi)}$, SE $\propto 1/v_{\mathrm{rel}}^{(\pi)}$
(Coulomb force of final two-body)

What happens if π decays?
(work in progress…)

Summary & Future Prospect

Summary

- Composite DM → DM w/ $O(1-10)$ TeV mass
- DM candidate : $SU(2)_W$ triplet dark pion χ

Forbidden channel $\chi\chi \to \pi\pi$ contributes to the relic abundance

- EW interacting heavy DM → **Sommerfeld Effect (SE)**

SE in final states significantly affect annihilation cross sections!

Future prospect

Decay width of particles in final states?

Comparison w/ current constraints from indirect detection experiments

Back Up

Toy Model

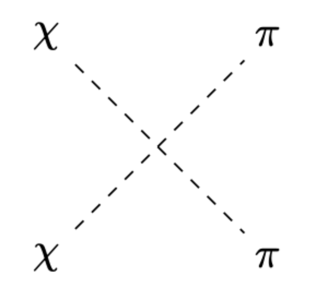
$\chi\chi \to \pi\pi$

$$\mathcal{L} \supset v_d^3 \mathrm{tr}[MU + \mathrm{h.c.}] \supset \frac{4mv_d^3}{3f_d^4}\left(\frac{1}{2}(\chi^0)^2 + \chi^+\chi^-\right)\left(\frac{1}{2}(\pi^0)^2 + \pi^+\pi^- + \pi^{++}\pi^{--}\right).$$

$\psi_{i,f}$: wave func. of initial/ final two-body

amplitude $$\mathcal{M} \propto \psi_i(0)\psi_f(0)\mathcal{M}_{\mathrm{LO}}$$

$$\mathcal{M}_{\mathrm{LO}} =$$



SE factor $$S \sim |\psi_i(0)|^2 |\psi_f(0)|^2$$

initial $$\left[-\frac{1}{2\mu_\chi}\nabla^2 + V_\chi(\boldsymbol{r}) - \frac{p^2}{2\mu_\chi}\right]\Psi_\chi(\boldsymbol{r}) = 0$$

$$\Psi_\chi(r) = \begin{pmatrix} \psi_{\chi^0\chi^0}(r) \\ \psi_{\chi^+\chi^-}(r) \end{pmatrix}$$

V_χ: 2×2 matrix

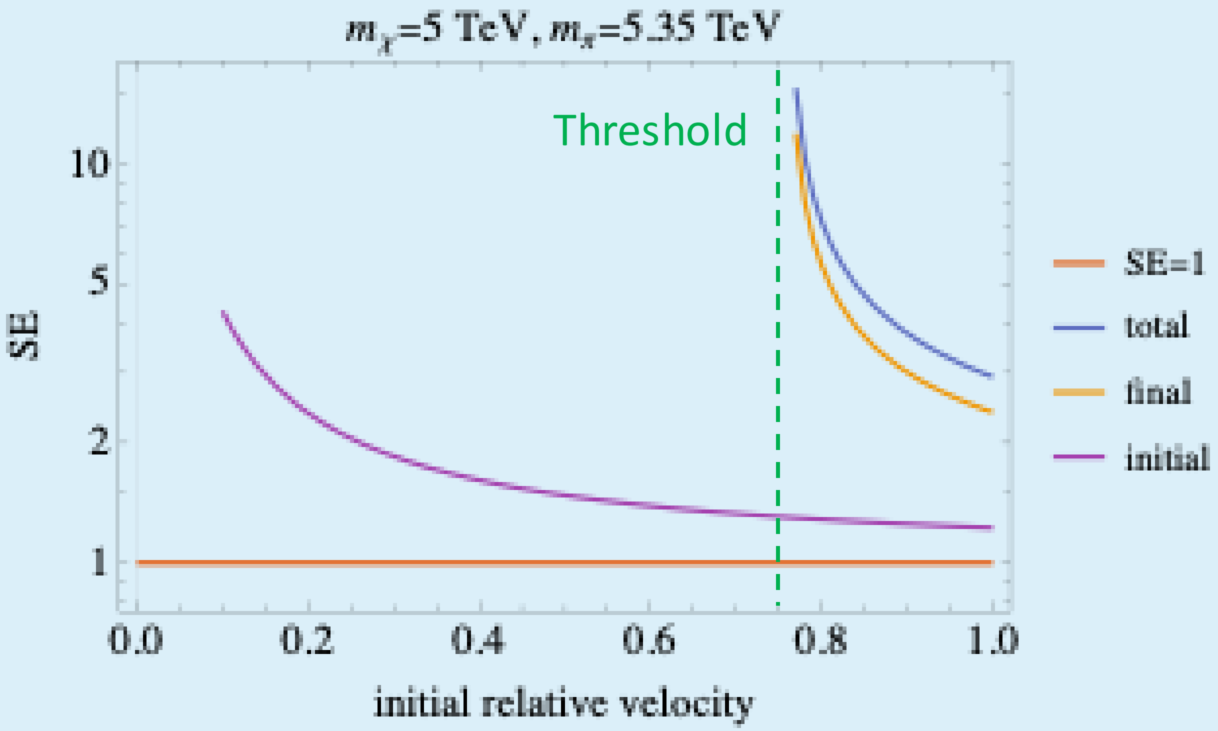
final $$\left[-\frac{1}{2\mu_\pi}\nabla^2 + V_\pi(\boldsymbol{r}) - \frac{p^2}{2\mu_\pi}\right]\Psi_\pi(\boldsymbol{r}) = 0$$

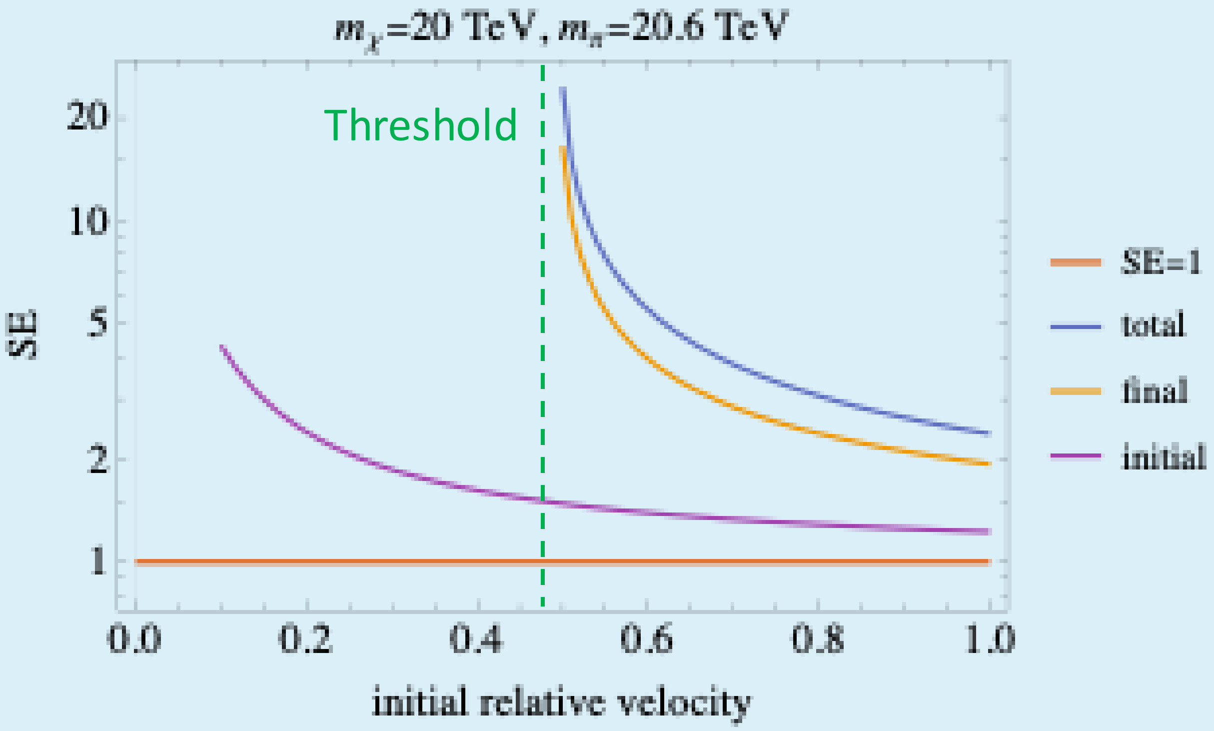
$$\Psi_\pi(r) = \begin{pmatrix} \psi_{\pi^0\pi^0}(r) \\ \psi_{\pi^+\pi^-}(r) \\ \psi_{\pi^{++}\pi^{--}}(r) \end{pmatrix}$$

V_π : 3×3 matrix

SE factors

threshold velocity of initial two-body : $v_{rel} \geq 2\sqrt{\left(\frac{m_\pi}{m_\chi}\right)^2 - 1}$





SE in final states mainly contribute to σv!

Interaction terms

$$\mathcal{L} \supset \frac{f_d^2}{4}\mathrm{tr}\left[\partial_\mu U \partial^\mu U^\dagger\right] + v_d^3\mathrm{tr}[MU + \mathrm{h.c.}]$$

$$\mathcal{L}_m \supset \lambda_0^{(1)} m_\Pi^2 \left(\chi^0\chi^0/2 \quad -\chi^+\chi^-\right) M_{0,0} \begin{pmatrix} \pi^{++\dagger}\pi^{--\dagger} \\ -\pi^{+\dagger}\pi^{-\dagger} \\ \pi^0\pi^0/2 \end{pmatrix}$$

$$\begin{aligned}
\mathcal{L}_{\chi'\chi\pi'\pi} = \frac{1}{4f_d^2}\Big[& \partial_\mu\chi^+\chi^+\left(\frac{1}{\sqrt{6}}\partial^\mu\pi^{--}\pi^0 - \frac{1}{2}\partial^\mu\pi^-\pi^- + \frac{1}{\sqrt{6}}\partial^\mu\pi^0\pi^{--}\right) \\
& + \partial_\mu\chi^-\chi^-\left(\frac{1}{\sqrt{6}}\partial^\mu\pi^{++}\pi^0 - \frac{1}{2}\partial^\mu\pi^+\pi^+ + \frac{1}{\sqrt{6}}\partial^\mu\pi^0\pi^{++}\right) \\
& + \partial_\mu\chi^+\chi^-\left(\frac{1}{2}\partial^\mu\pi^0\pi^0 + \frac{1}{2}\partial^\mu\pi^+\pi^- + \frac{1}{3}\partial^\mu\pi^{++}\pi^{--} + \frac{1}{3}\partial^\mu\pi^{--}\pi^{++}\right) \\
& + \partial_\mu\chi^-\chi^+\left(\frac{1}{2}\partial^\mu\pi^0\pi^0 + \frac{1}{2}\partial^\mu\pi^-\pi^+ + \frac{1}{3}\partial^\mu\pi^{--}\pi^{++} + \frac{1}{3}\partial^\mu\pi^{++}\pi^{--}\right) \\
& + \partial_\mu\chi^0\chi^0\left(\frac{1}{3}\partial^\mu\pi^{++}\pi^{--} + \frac{1}{6}\partial^\mu\pi^+\pi^- + \frac{1}{6}\partial^\mu\pi^-\pi^+\frac{1}{3}\partial^\mu\pi^{--}\pi^{++}\right) \\
& + \partial_\mu\chi^+\chi^0\left(\frac{1}{2\sqrt{3}}\partial^\mu\pi^-\pi^0 + \frac{1}{3\sqrt{2}}\partial^\mu\pi^+\pi^{--} + \frac{2}{3\sqrt{2}}\partial^\mu\pi^{--}\pi^+\right) \\
& + \partial_\mu\chi^-\chi^0\left(\frac{1}{2\sqrt{3}}\partial^\mu\pi^+\pi^0 + \frac{1}{3\sqrt{2}}\partial^\mu\pi^-\pi^{++} + \frac{2}{3\sqrt{2}}\partial^\mu\pi^{++}\pi^-\right) \\
& + \partial_\mu\chi^0\chi^+\left(\frac{1}{2\sqrt{3}}\partial^\mu\pi^0\pi^- + \frac{1}{3\sqrt{2}}\partial^\mu\pi^+\pi^{--} + \frac{2}{3\sqrt{2}}\partial^\mu\pi^{--}\pi^+\right) \\
& + \partial_\mu\chi^0\chi^-\left(\frac{1}{2\sqrt{3}}\partial^\mu\pi^0\pi^+ + \frac{1}{3\sqrt{2}}\partial^\mu\pi^-\pi^{++} + \frac{2}{3\sqrt{2}}\partial^\mu\pi^{++}\pi^-\right)\Big]
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{\chi'\chi'\pi\pi} = \frac{1}{4f_d^2}\Big[& \partial_\mu\chi^+\partial^\mu\chi^+\left(\frac{1}{2}\pi^-\pi^- - \frac{1}{\sqrt{6}}\pi^{--}\pi^0\right) \\
& + \partial_\mu\chi^-\partial^\mu\chi^-\left(\frac{1}{2}\pi^+\pi^+ - \frac{1}{\sqrt{6}}\pi^{++}\pi^0\right) \\
& + \partial_\mu\chi^0\partial^\mu\chi^0\left(-\frac{1}{3}\pi^+\pi^- - \frac{4}{3}\pi^{++}\pi^{--}\right) \\
& + \partial_\mu\chi^+\partial^\mu\chi^-\left(\pi^0\pi^0 + \frac{5}{3}\pi^+\pi^- + \frac{2}{3}\pi^{++}\pi^{--}\right) \\
& + \partial_\mu\chi^0\partial^\mu\chi^+\left(-\frac{1}{\sqrt{3}}\pi^0\pi^- - \sqrt{2}\pi^{--}\pi^+\right) \\
& \partial_\mu\chi^0\partial^\mu\chi^-\left(-\frac{1}{\sqrt{3}}\pi^0\pi^+ - \sqrt{2}\pi^{++}\pi^-\right)\Big]
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{\chi\chi\pi'\pi'} = \frac{1}{4f_d^2}\Big[& \chi^+\chi^+\left(\frac{1}{2}\partial_\mu\pi^-\partial^\mu\pi^- - \frac{1}{\sqrt{6}}\partial_\mu\pi^{--}\partial^\mu\pi^0\right) \\
& + \chi^-\chi^-\left(\frac{1}{2}\partial_\mu\pi^+\partial^\mu\pi^+ - \frac{1}{\sqrt{6}}\partial_\mu\pi^{++}\partial^\mu\pi^0\right) \\
& + \chi^0\chi^0\left(-\frac{1}{3}\partial_\mu\pi^+\partial^\mu\pi^- - \frac{4}{3}\partial_\mu\pi^{++}\partial^\mu\pi^{--}\right) \\
& + \chi^+\chi^-\left(\partial_\mu\pi^0\partial^\mu\pi^0 + \frac{5}{3}\partial_\mu\pi^+\partial^\mu\pi^- + \frac{2}{3}\partial_\mu\pi^{++}\partial^\mu\pi^{--}\right) \\
& + \chi^0\chi^+\left(-\frac{1}{\sqrt{3}}\partial_\mu\pi^0\partial^\mu\pi^- - \sqrt{2}\partial_\mu\pi^{--}\partial^\mu\pi^{++}\right) \\
& \chi^0\chi^-\left(-\frac{1}{\sqrt{3}}\partial_\mu\pi^0\partial^\mu\pi^+ - \sqrt{2}\partial_\mu\pi^{++}\partial^\mu\pi^{--}\right)\Big] \qquad (1.14)
\end{aligned}$$

SU(N) Composite DM Model

Bai, Hill [Phys. Rev. D **82** (2010)], Antipin, Redi, Strumia, Vigiani [JHEP **07**(2015)]

$SU(N)_d$ gauge symmetry

$$\Psi_i \equiv \begin{pmatrix} \psi_i \\ \bar{\psi}_i^\dagger \end{pmatrix}$$

$\psi, \bar{\psi}$: dark quark, anti-dark quark (3-flavor)

Renormalizable Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \bar{\Psi}_i(i\gamma^\mu D_\mu - m)\Psi_i - \frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} + \frac{g_d^2\theta}{32\pi^2}G^A_{\mu\nu}\tilde{G}^{A\mu\nu}$$

Dark quark confines at the scale Λ_d→ dark baryons, dark pions

Accidental symmetries

U(1) global symmetry: $\Psi_i \to e^{i\alpha}\Psi_i$ Stability of dark baryons

G-parity: $\Psi_i \to \exp(i\pi I_2)\Psi_i^c$ Stability of dark pions

Dark Pion Matrices

$SU(2)_W$ triplet χ

$$\Pi_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i\chi^0 & \frac{\chi^- - \chi^+}{\sqrt{2}} \\ i\chi^0 & 0 & -i\frac{\chi^- + \chi^+}{\sqrt{2}} \\ -\frac{\chi^- - \chi^+}{\sqrt{2}} & i\frac{\chi^- + \chi^+}{\sqrt{2}} & 0 \end{pmatrix}$$

$SU(2)_W$ quintuplet π

$$\Pi_5 = \begin{pmatrix} \frac{\pi^0}{\sqrt{6}} - \frac{\pi^{++} + \pi^{--}}{2} & -i\frac{\pi^{++} - \pi^{--}}{2} & \frac{\pi^+ + \pi^-}{2} \\ -i\frac{\pi^{++} - \pi^{--}}{2} & \frac{\pi^0}{\sqrt{6}} + \frac{\pi^{++} + \pi^{--}}{2} & i\frac{\pi^+ - \pi^-}{2} \\ \frac{\pi^+ + \pi^-}{2} & i\frac{\pi^+ - \pi^-}{2} & -\sqrt{\frac{2}{3}}\pi^0 \end{pmatrix}$$

Dark pion fields $$U = \exp\left(\frac{\sqrt{2}i}{f_d}(\Pi_3 + \Pi_5)\right)$$

$U \in SU(3)_V$

G-parity

$$W_{\mu\nu} \to W_{\mu\nu}, \quad U \to U^T$$

$$\Pi_3 \to -\Pi_3, \quad \Pi_5 \to \Pi_5$$

π decay process $$\mathcal{L}_{\rm WZW} \supset -\frac{g^2 N}{16\sqrt{2}\pi^2 f_d}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}[\Pi_5 W_{\mu\nu} W_{\rho\sigma}]$$

G-parity

$$\Psi \to \exp(i\pi I_2)\Psi^c$$

Ψ: Dirac Spinor

Charge conjugation of dark quarks

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \to \begin{pmatrix} \overline{\psi_1} \\ -\bar{\psi}_2 \\ \bar{\psi}_3 \end{pmatrix} \qquad \begin{pmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \\ \bar{\psi}_3 \end{pmatrix} \to \begin{pmatrix} \psi_1 \\ -\psi_2 \\ \psi_3 \end{pmatrix}$$

Multiplying by $\exp(i\pi I_2) = \mathrm{diag}(-1,1,-1)$,

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \to \begin{pmatrix} -\overline{\psi_1} \\ -\bar{\psi}_2 \\ -\bar{\psi}_3 \end{pmatrix} \qquad \begin{pmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \\ \bar{\psi}_3 \end{pmatrix} \to \begin{pmatrix} -\psi_1 \\ -\psi_2 \\ -\psi_3 \end{pmatrix}$$

$$U_{ij} \sim \bar{\psi}_i \psi_j \to \psi_i \bar{\psi}_j \sim U_{ji}$$

Dark Pion Mass

Bai, Hill [Phys. Rev. D **82** (2010)], Antipin, Redi, Strumia, Vigiani [JHEP **07**(2015)]

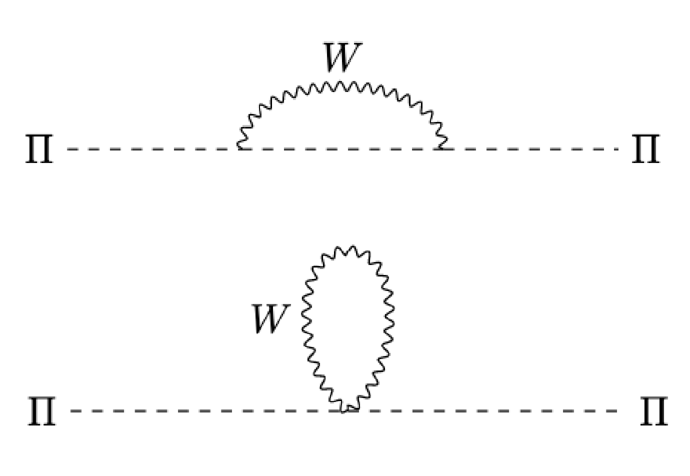
Explicit breaking of chiral symmetry→ Dark pion mass

① Dark quark mass term $$m_\chi^2 = m_\pi^2 = \frac{4mv_d^3}{f_d^2}$$

② $SU(2)_W$ radiative corrections

$$\delta m^2 \sim C^2(R)\alpha_2(\Lambda_d)\Lambda_d^2$$

$C^2(R)$: Casimir op. of rep. R $C^2(\mathbf{3}) = 2$, $C^2(\mathbf{5}) = 6$



mass splitting among multiplet

Cirelli, Fornengo, Strumia [Nucl. Phys. B **753** (2005)]

$$m_Q - m_0 \simeq \alpha_2 Q^2 m_W \sin^2\frac{\theta_W}{2} \quad (\text{for} \quad m \gg m_W)$$

$$\simeq 166 \times Q^2 \text{ MeV}$$

Dark baryon

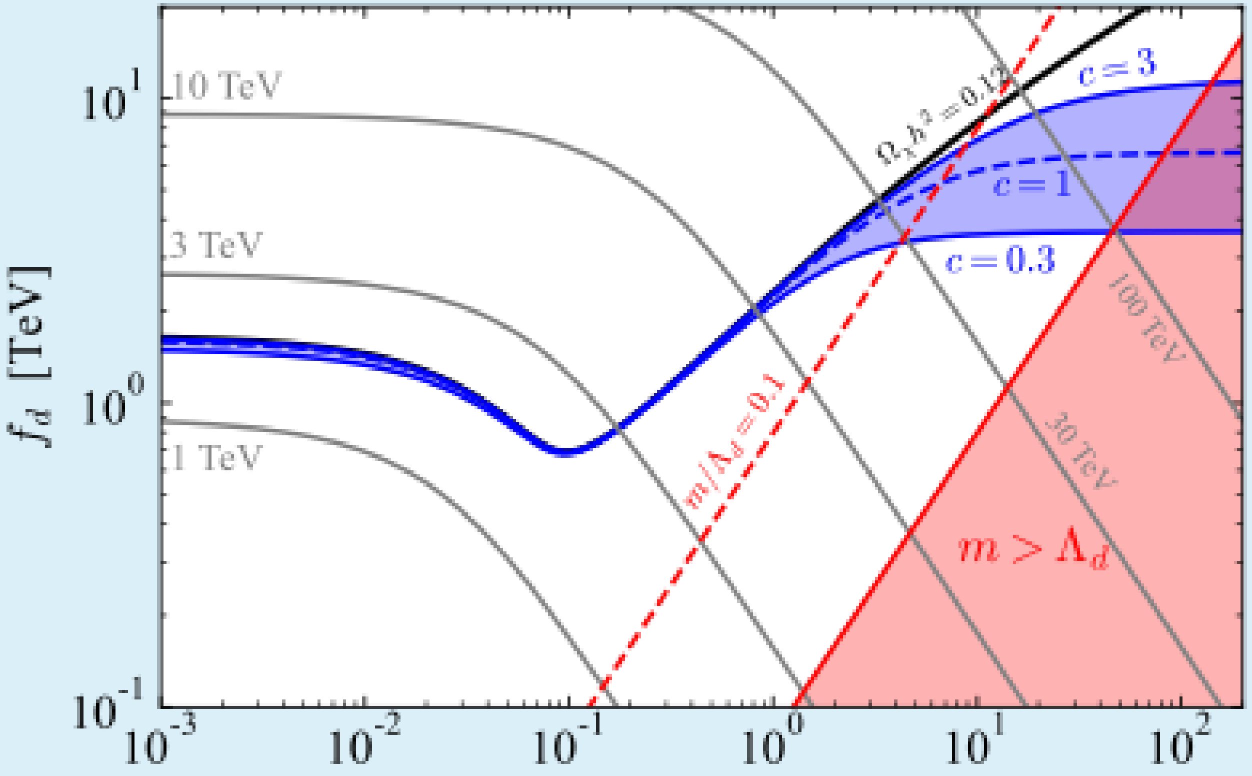
DM components $\quad \Omega_{\rm DM} h^2 = \Omega_\chi h^2 + \Omega_B h^2$

energy density of dark baryon

annihilation cross section (s-wave)

$$\langle \sigma_B v \rangle \simeq c \frac{4\pi}{m_B^2}, \quad c \sim O(1) \quad m_B \sim \Lambda_d \sim 4\pi f_d$$

$$\Omega_B h^2 \simeq 0.12 \left(\frac{f_d}{6.66\,\mathrm{TeV}} \right)^2 \left(\frac{1.0}{c} \right)$$



Dark quark mass [TeV]

CP phase

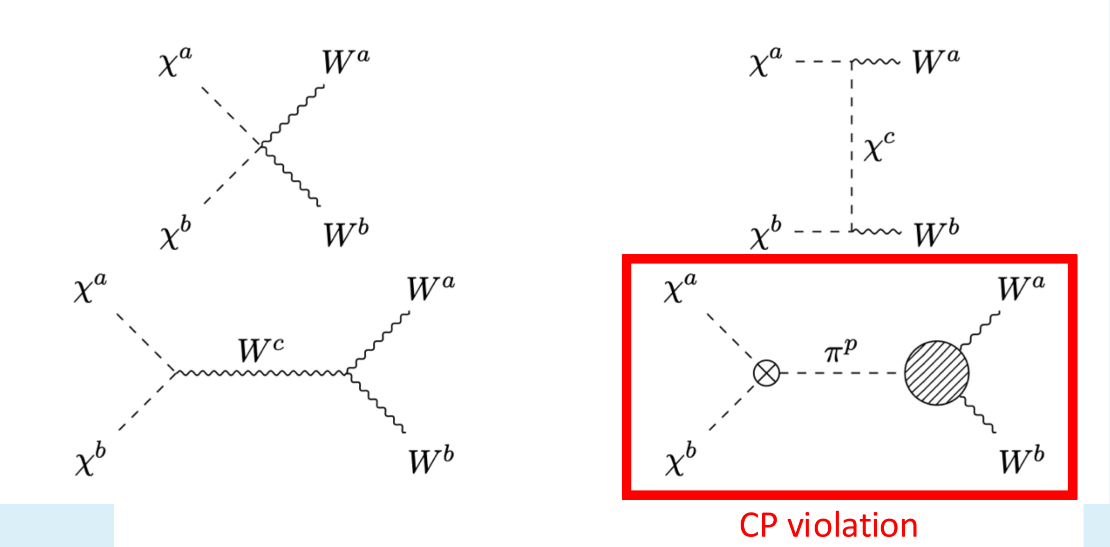
$\boldsymbol{\theta \neq 0}$

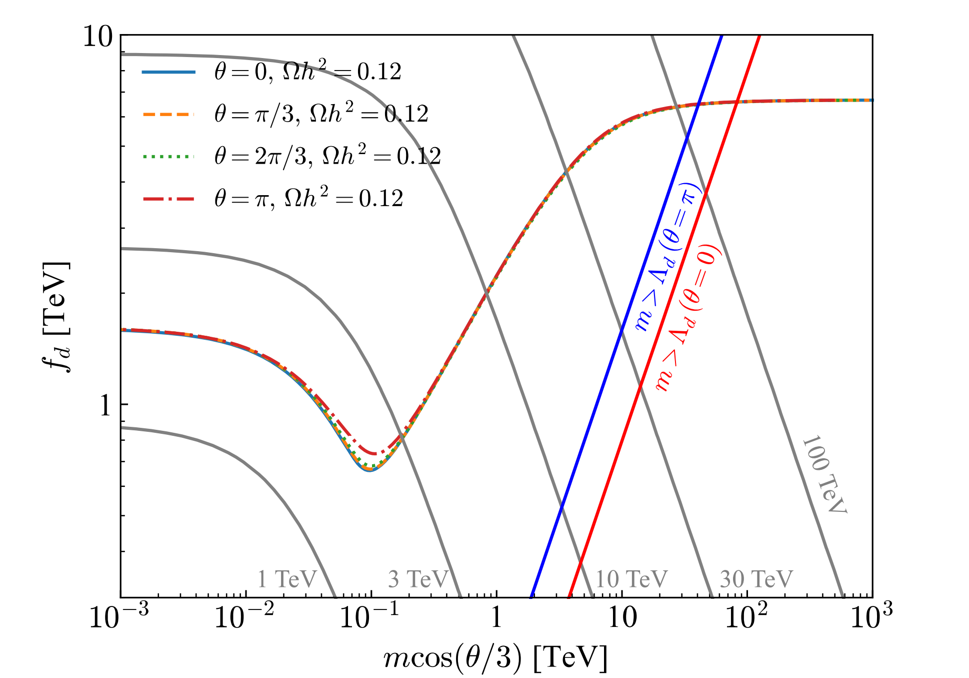
$$\chi\chi \to WW$$

annihilation cross section

$$\langle \sigma v \rangle_{WW} = \langle \sigma v \rangle_{WW}^{\mathrm{MDM}} + \langle \sigma v \rangle_{WW}^{\mathrm{WZW}}$$

$$\langle \sigma v \rangle_{WW}^{\mathrm{MDM}} \simeq \frac{4\pi\alpha^2}{m_\chi^2} \qquad \langle \sigma v \rangle_{WW}^{\mathrm{WZW}} \propto \frac{g^4}{m_\chi^2}\left(\frac{m_q \sin\frac{\theta}{3}}{m_\chi}\right)^2 \frac{m_\chi^4}{m_\pi^4}$$

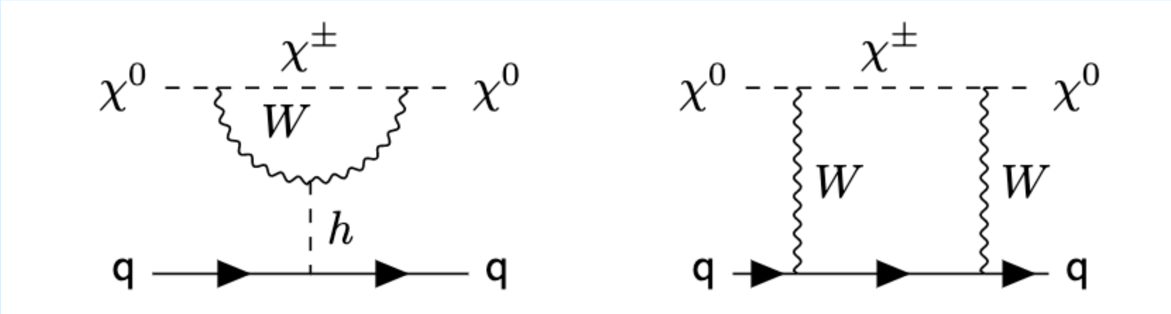


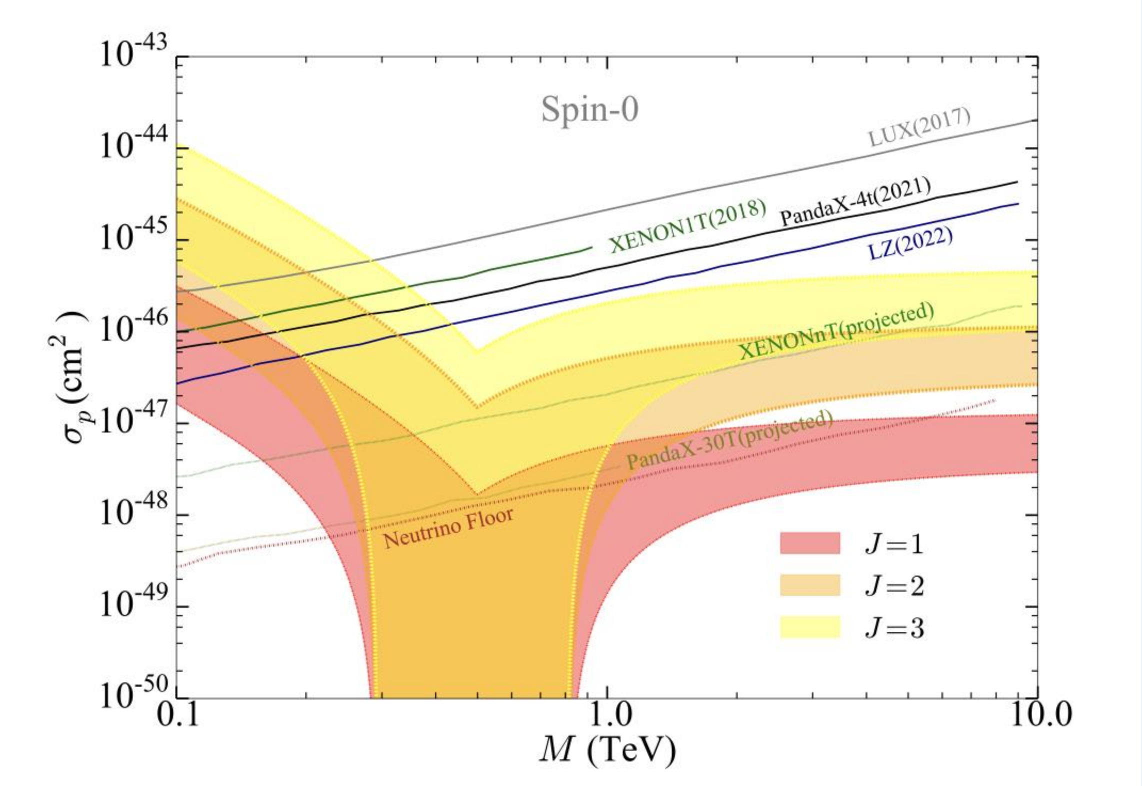


Direct Detection

Elastic scattering suppressed at one-loop level

Cirelli, Fornengo, Strumia [Nucl. Phys. B **753** (2005)]





cross section: $\sigma_{\mathrm{SI}} \sim O(10^{-47})\ \mathrm{cm}^2$

Hisano, Ishiwata, Nagata [JHEP **06** (2015)]

Cheng, Ding, Hill, [Phys. Rev. D **108** (2023)]

Above the neutrino fog if $m_\chi \lesssim 4\ \mathrm{TeV}$

Leading Order analysis

Cheng, Ding, Hill, [Phys. Rev. D **108** (2023)]

Collider Experiments

①production of χ : $pp \to W^{\pm *} \to \chi^{\pm}\chi^{0}$ Cirelli, Fornengo, Strumia [Nucl. Phys. B **753** (2005)]

How to detect Charged component decay $\chi^{\pm} \to \chi^{0}\pi^{\pm}_{\mathrm{QCD}}$ $\pi^{\pm}_{\mathrm{QCD}}$: soft

Disappearing tracks of $\chi^{\pm}$: $c\tau \sim 6\ \mathrm{cm}$

Might be tested by future 100 TeV pp-collider? [Chiang, Cottin, Du, Fuyuto, Ramsey-Musolf [JHEP **01** (2021) 198]

②production of π

$$pp \to W^{*} \to \pi W$$
$$pp \to \rho_d^{*} \to \pi\pi$$

Kilic, Okui, Sundrum [JHEP **02** (2010)], Draper, Kozaczuk, Yu [Phys. Rev. D **98** (2018)]

π decays into EW gauge bosons → clean signals

Quintuplet decay

Decay into EW gauge bosons

$$\mathcal{L}_{\rm WZW} \supset -\frac{g^2 N}{16\sqrt{2}\pi^2 f_d}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}[\Pi_5 W_{\mu\nu} W_{\rho\sigma}]$$

Decay Width $\Gamma_\pi \sim \left(\frac{\alpha_W}{4\pi}\right)^2 \frac{m_\pi^3}{f_d^2}$

Lifetime $\tau_\pi = \frac{1}{\Gamma_\pi}$

Range of weak interaction $r_W \sim \frac{1}{m_W}$

$r_W \ll c\tau_\pi$

two-body π affected by SE before its decay

Cross Section Calculation [Blum, Sato, Slatyer (2016), Parikh, Sato, Slatyer (2024)]

DM is non-relativistic → Analysis w/ Schroedinger eq. is effective!

Schroedinger eq. of DM two-body state

$$\left[-\frac{1}{2\mu}\nabla^2 + \underbrace{V(\vec{r})}_{\text{long-range}} + \underbrace{u\delta^3(\vec{r})}_{\text{short-range (annihilation)}}\right]\psi(\vec{r}) = E\psi(\vec{r})$$

Boundary condition

$$\psi(\vec{r}) \to \underbrace{e^{ikz}}_{\text{incoming plain wave}} + \underbrace{f(\theta)\frac{e^{ikr}}{r}}_{\text{outgoing scattered wave}} \quad (r \to \infty)$$

Flux of probability

$$\vec{j}(r) \equiv \frac{1}{\mu}\mathrm{Im}[\psi^* \nabla \psi]$$

divergence of the flux

$$\nabla \cdot \vec{j} = 2\mathrm{Im}u\, \delta^3(\vec{r})|\psi(\vec{r})|^2$$

Cross Section Calculation

[Blum, Sato, Slatyer (2016), Parikh, Sato, Slatyer (2024)]

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$$\psi(\vec{r}) \to \underbrace{e^{ikz}}_{\text{incoming plain wave}} + \underbrace{f(\theta)\frac{e^{ikr}}{r}}_{\text{outgoing scattered wave}} \quad (r \to \infty)$$

incoming flux

$$j_{in} = \frac{k}{\mu} = v$$

divergence of the flux

$$\nabla \cdot \vec{j} = 2\mathrm{Im}u\ \delta^3(\vec{r})|\psi(\vec{r})|^2$$

annihilation cross section

$$\sigma \times j_{in} = \int d^3r\ \nabla \cdot \vec{j}_{out}(r)$$

$$\therefore \sigma v = 2\mathrm{Im}\ u\left|\psi(\vec{0})\right|^2$$

$$\Rightarrow \sigma v \simeq \left|\psi_0(\vec{0})\right|^2 \times (\sigma v)_{\mathrm{LO}} \text{ (s-wave)}$$

[Hisano, Matsumoto, Nojiri, Saito (2002)]

where $\left[-\frac{1}{2\mu}\nabla^2 + V(\vec{r})\right]\psi_0(\vec{r}) = E\psi_0(\vec{r})$

Schroedinger Equations

Cirelli, Fornengo, Strumia [Nucl. Phys. B **753** (2007)]

$$\left[-\frac{1}{2\mu_\chi}\nabla^2 + V_\chi(\boldsymbol{r}) - \frac{p^2}{2\mu_\chi}\right]\Psi_\chi(\boldsymbol{r}) = 0 \qquad \left[-\frac{1}{2\mu_\pi}\nabla^2 + V_\pi(\boldsymbol{r}) - \frac{p^2}{2\mu_\pi}\right]\Psi_\pi(\boldsymbol{r}) = 0$$

$$V_\chi(r) \equiv \begin{pmatrix} 0 & -\sqrt{2}B \\ -\sqrt{2}B & -A+2\Delta \end{pmatrix} \qquad V_\pi(r) \equiv \begin{pmatrix} 0 & -3\sqrt{2}B & 0 \\ -3\sqrt{2}B & -A+2\Delta & -2B \\ 0 & -2B & -4A+8\Delta \end{pmatrix}$$

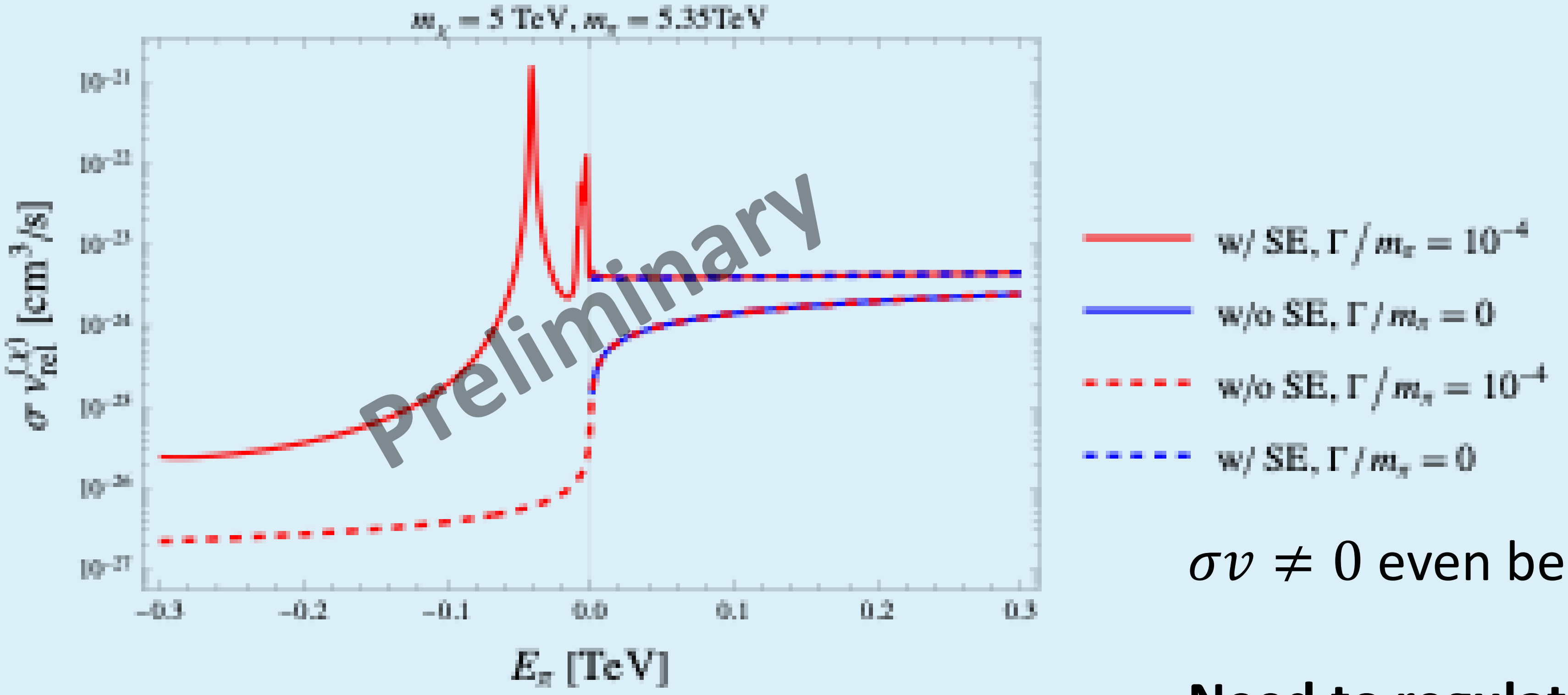
$$A \equiv \frac{\alpha}{r} + \frac{\alpha_W c_W^2}{r} e^{-m_Z r} \qquad B \equiv \frac{\alpha_W}{r} e^{-m_W r} \qquad \Delta \equiv 166\ \mathrm{MeV}$$

Boundary conditions   Plain wave outside the potential

$$\frac{\psi_\chi^{i\prime}(r)}{\psi_\chi^i(r)} = i\sqrt{p^2 - m_\chi V_\chi(\infty)_{ii}} \qquad \frac{\psi_\pi^{i\prime}(r)}{\psi_\pi^i(r)} = i\sqrt{p^2 - m_\pi V_\pi(\infty)_{ii}}$$

Including Decay Rate

$E_\pi \to E_\pi + i\Gamma$ in the Schroedinger eqs. of ψ_π



$\sigma v \neq 0$ even below threshold

Need to regulate the resonance

[Parikh, Sato, Slatyer (2024)]