Exploring chirality structure in nucleon decay

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with

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THE UNIVERSITY OF TOKYO

Take-home: Nucleon decay → Chirality structure

Decay channel: $p \to \pi^0 e^+$ and $p \to \eta e^+$

$$\mathcal{L}_{\text{eff}} = C^{\ell}_{RL}\left[\epsilon_{abc}(u^a_R d^b_R)(u^c_L \ell_L)\right] + C^{\ell}_{LR}\left[\epsilon_{abc}(u^a_L d^b_L)(u^c_R \ell_R)\right] + C^{\ell}_{LL}\left[\epsilon_{abc}(u^a_L d^b_L)(u^c_L \ell_L)\right] + C^{\ell}_{RR}\left[\epsilon_{abc}(u^a_R d^b_R)(u^c_R \ell_R)\right]$$

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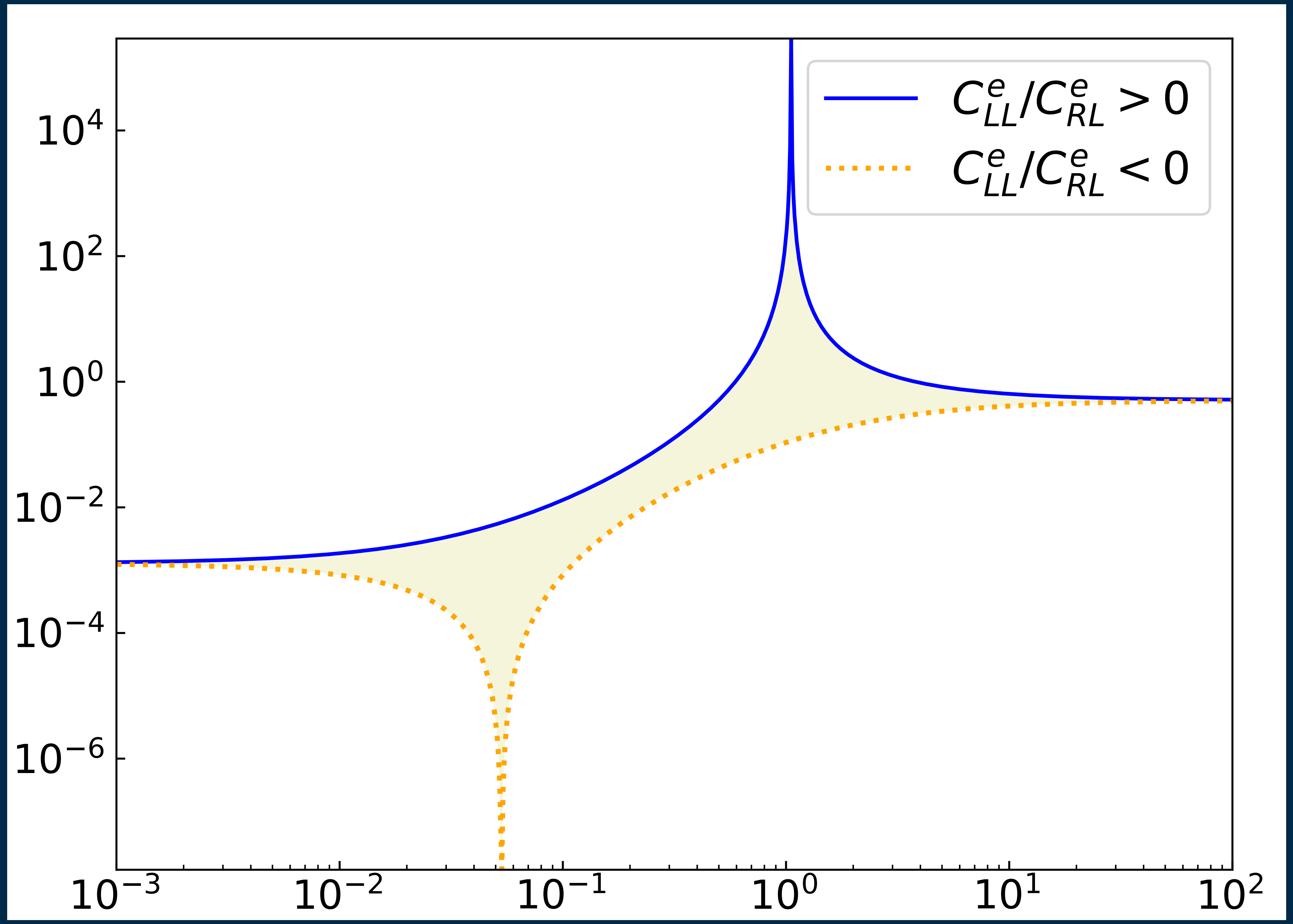
$$\frac{\Gamma(p \to \eta e^+)}{\Gamma(p \to \pi^0 e^+)}$$



$$\left|\frac{C^e_{LL}}{C^e_{RL}}\right|$$

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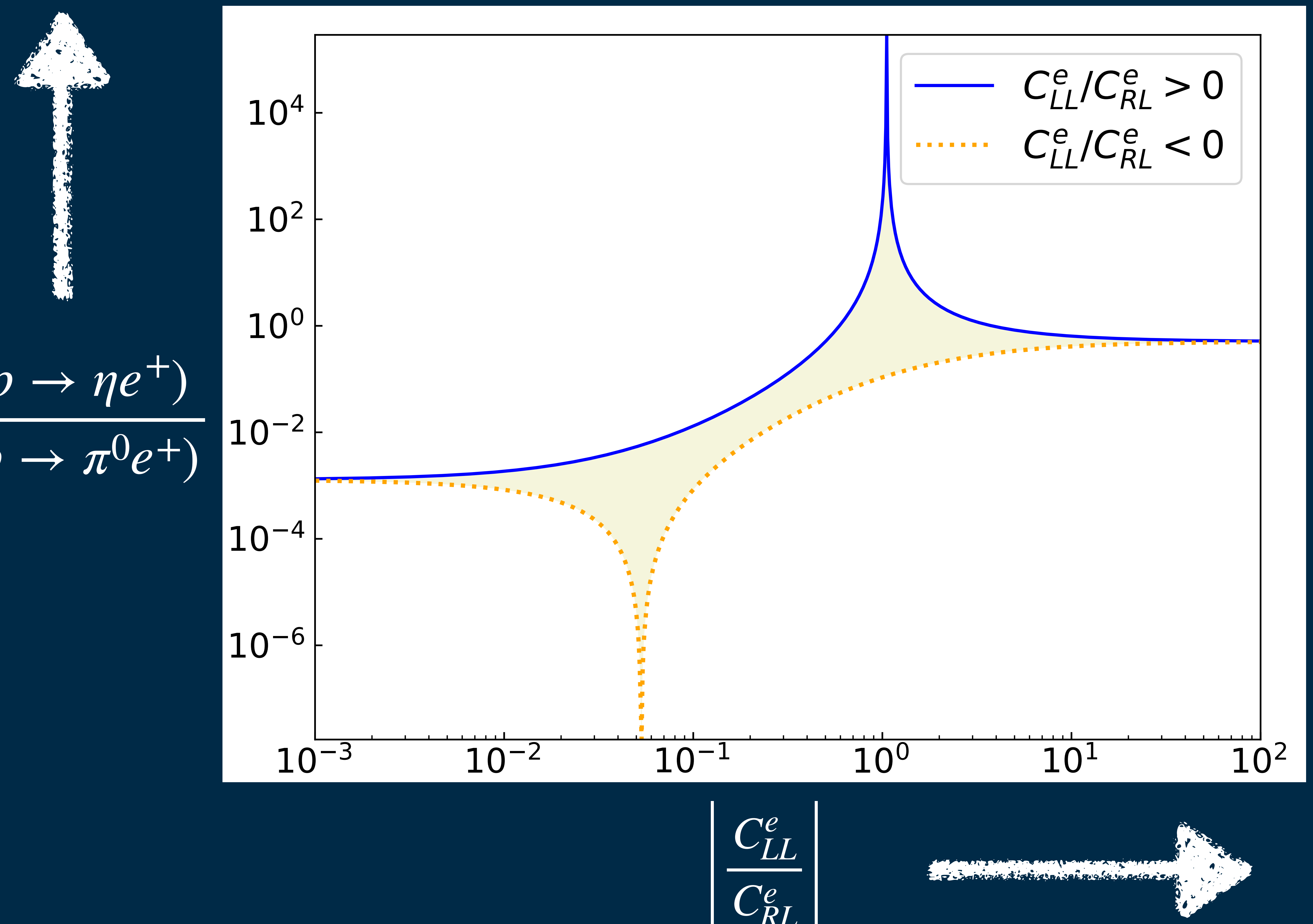

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Application to SUSY SU(5):

- $\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+) \to M_X$ vs $m_{\tilde{g}}$
- $\Gamma(p \to \eta \mu^+)/\Gamma(p \to \pi^0 \mu^+) \to M_X$ vs $m_{\tilde{w}}$
- $\Gamma(n \to \eta \bar{\nu})/\Gamma(n \to \pi^0 \bar{\nu}) \to m_{\tilde{h}}$ vs $m_{\tilde{w}}$

Baryon number violation

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  is nicely incorporated in **Grand Unified Theory (GUT)**

H. Georgi and S. L. Glashow (1974)

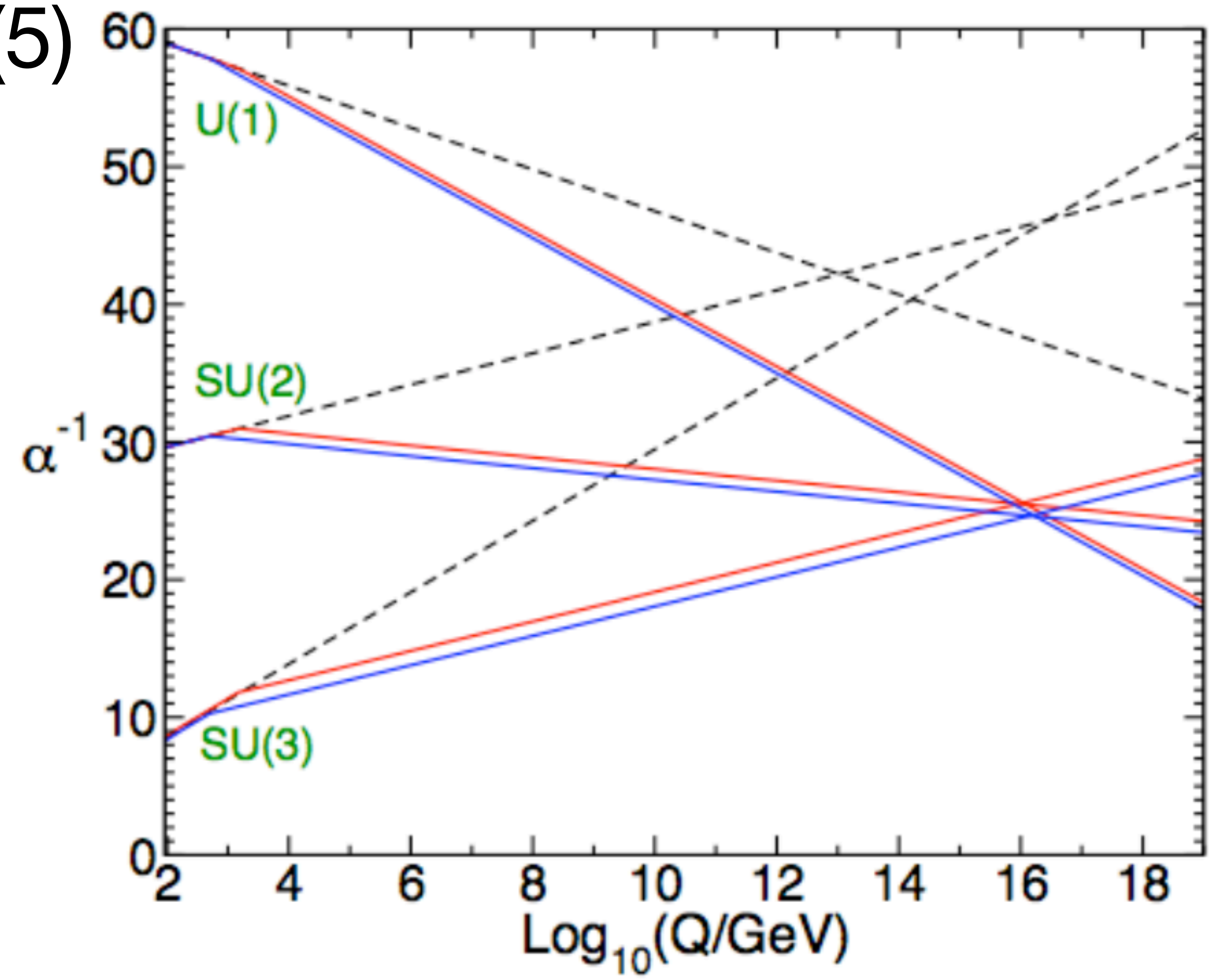

Fig. from P. Langacker (1981)

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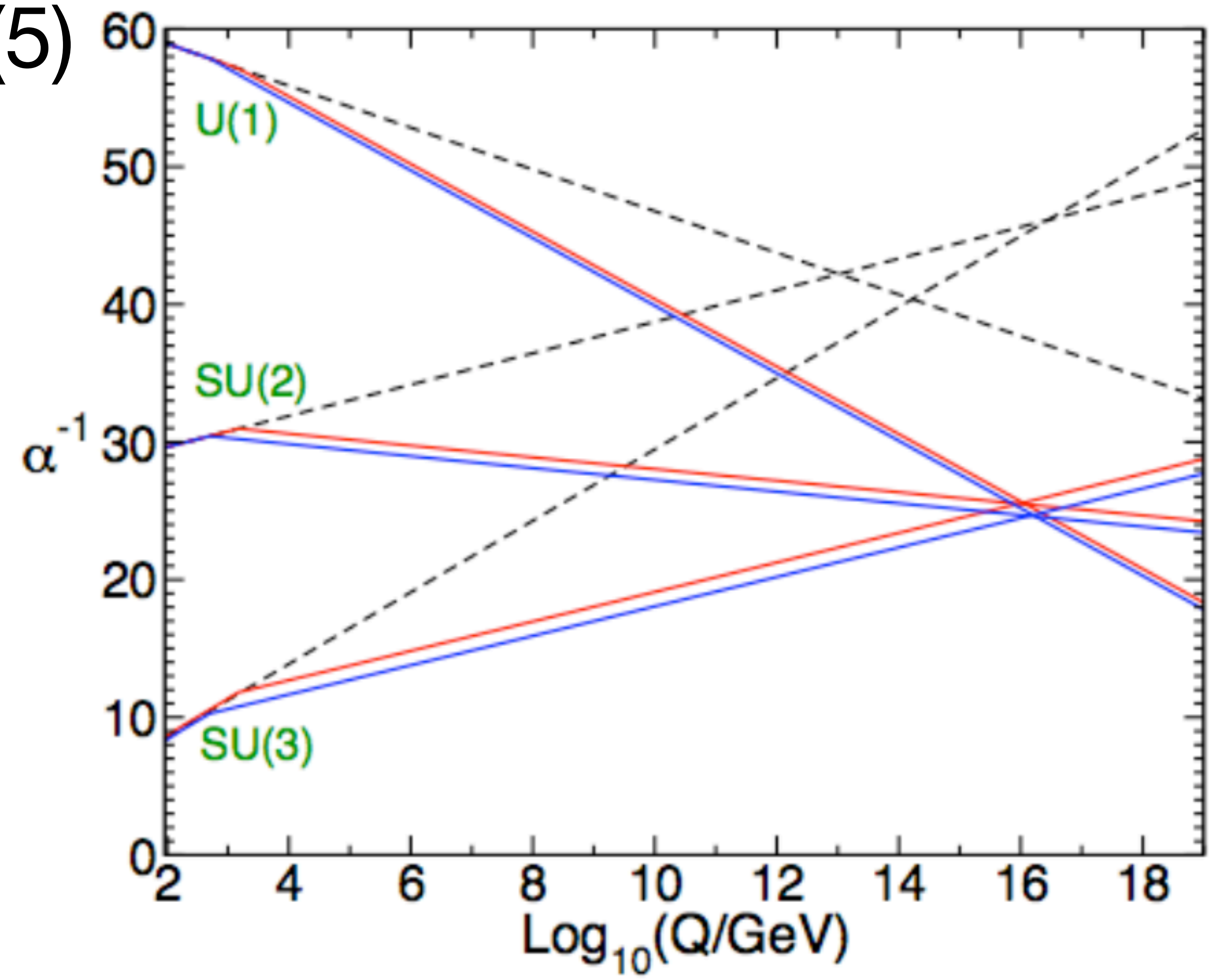
  - Quark-lepton unification

$$\mathbf{10}_i = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & u^\dagger_{Ri3} & -u^\dagger_{Ri2} & u^1_{Li} & d^1_{Li} \\ -u^\dagger_{Ri3} & 0 & u^\dagger_{Ri1} & u^2_{Li} & d^2_{Li} \\ u^\dagger_{Ri2} & -u^\dagger_{Ri1} & 0 & u^3_{Li} & d^3_{Li} \\ -u^1_{Li} & -u^2_{Li} & -u^3_{Li} & 0 & e^\dagger_{Ri} \\ -d^1_{Li} & -d^2_{Li} & -d^3_{Li} & -e^\dagger_{Ri} & 0 \end{pmatrix}, \quad \overline{\mathbf{5}}_i = \begin{pmatrix} d^\dagger_{Ri1} \\ d^\dagger_{Ri2} \\ d^\dagger_{Ri3} \\ e_{Li} \\ -\nu_{Li} \end{pmatrix}$$



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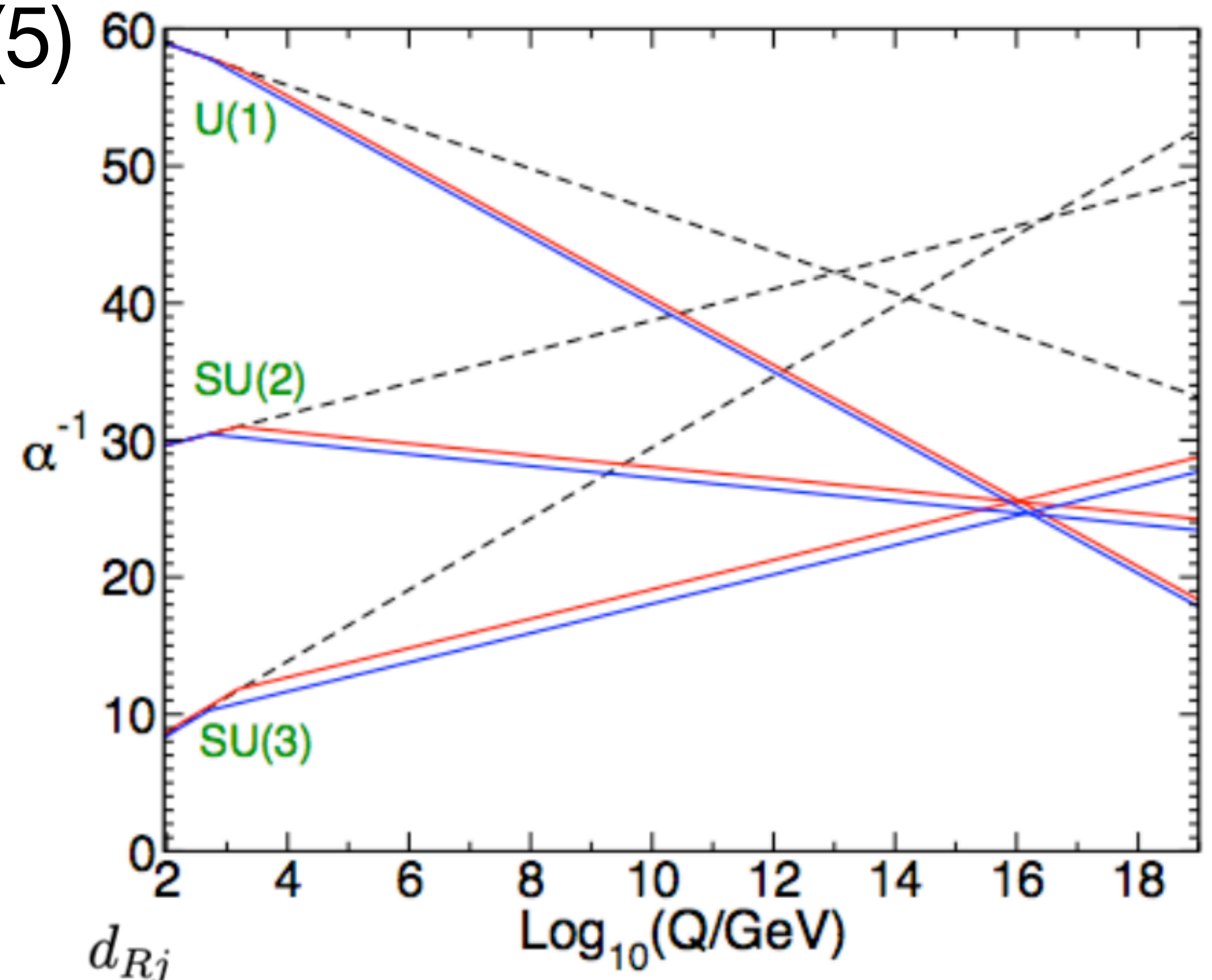

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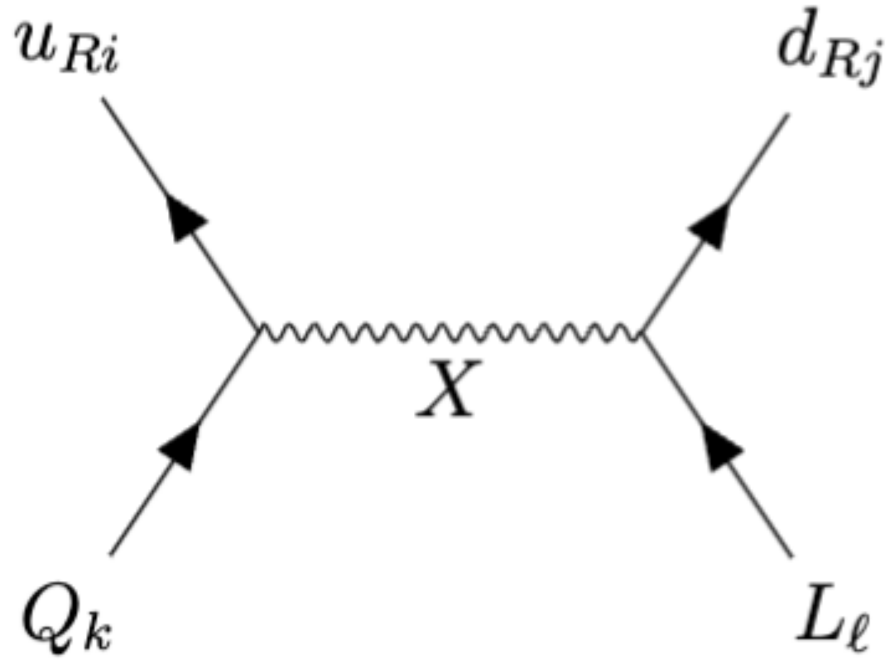

Nucleon decay

Nucleon decay has been the main probe of baryon-number violation.

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Next-generation nucleon decay experiments:

- Hyper-Kamiokande (HK)
- JUNO
- DUNE

Decay Mode	Current [years]	HK sensitivity [years]
$p \to \pi^0 e^+$	2.4×10^{34} [25]	7.8×10^{34} [11]
$p \to \pi^0 \mu^+$	1.6×10^{34} [25]	7.7×10^{34} [11]
$p \to \eta e^+$	1.0×10^{34} [26]	4.3×10^{34} [11]
$p \to \eta \mu^+$	4.7×10^{33} [26]	4.9×10^{34} [11]
$p \to \pi^+ \bar{\nu}$	3.9×10^{32} [27]	
$n \to \pi^- e^+$	5.3×10^{33} [26]	2.0×10^{34} [11]
$n \to \pi^- \mu^+$	3.5×10^{33} [26]	1.8×10^{34} [11]
$n \to \pi^0 \bar{\nu}$	1.1×10^{33} [27]	
$n \to \eta \bar{\nu}$	1.6×10^{32} [28]	

- 90% CL
- 1.9 Megaton-year exposure assumed for the prospect.

Effective interactions for nucleon decay

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F. Wilczek and A. Zee (1979)
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$$\mathcal{L}_{\mathrm{SM,eff}} = \sum_{I,ijkl} C_{(I)}^{ijkl} \mathcal{O}_{ijkl}^{(I)} + \mathrm{h.c.}$$

i, j, k, l: flavor

$\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$

$$\mathcal{O}_{ijkl}^{(1)} = \epsilon_{abc}\epsilon^{\alpha\beta}(u_{Ri}^a d_{Rj}^b)(Q_{Lk\alpha}^c L_{Ll\beta})$$

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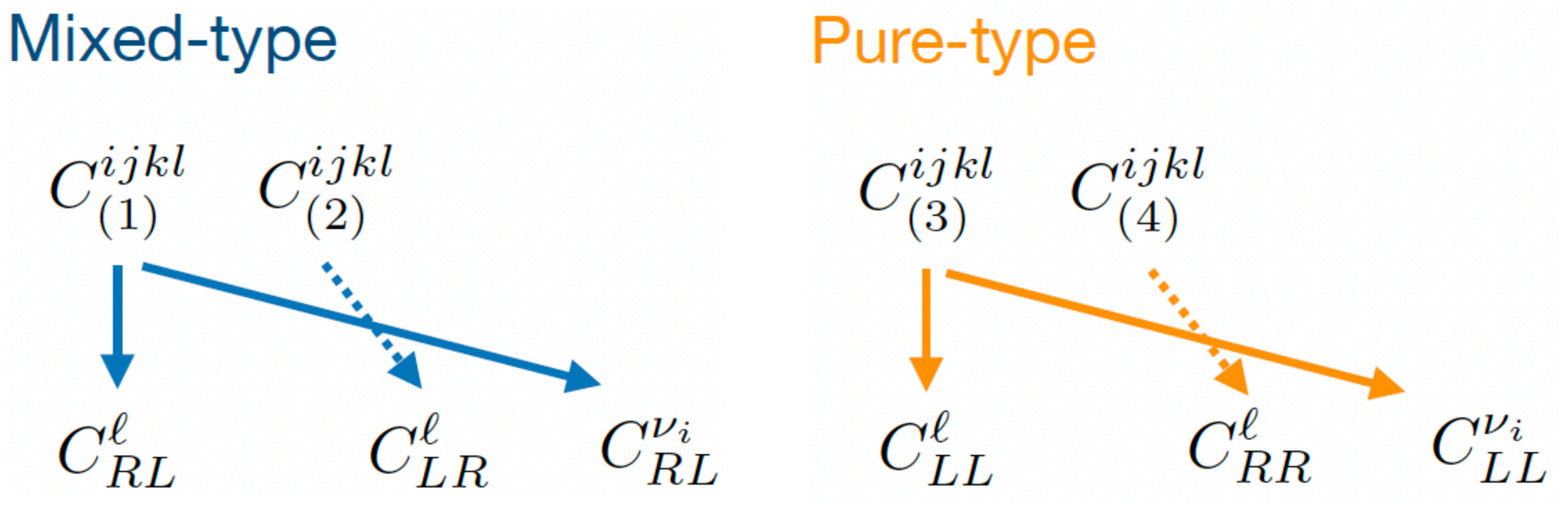
Pure-type

Matching

$$\boxed{\mathrm{SU(3)}_C \otimes \mathrm{SU(2)}_L \otimes \mathrm{U(1)}_Y} \longrightarrow \boxed{\mathrm{SU(3)}_C \otimes \mathrm{U(1)}}$$



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Matching $\Delta S = 0$

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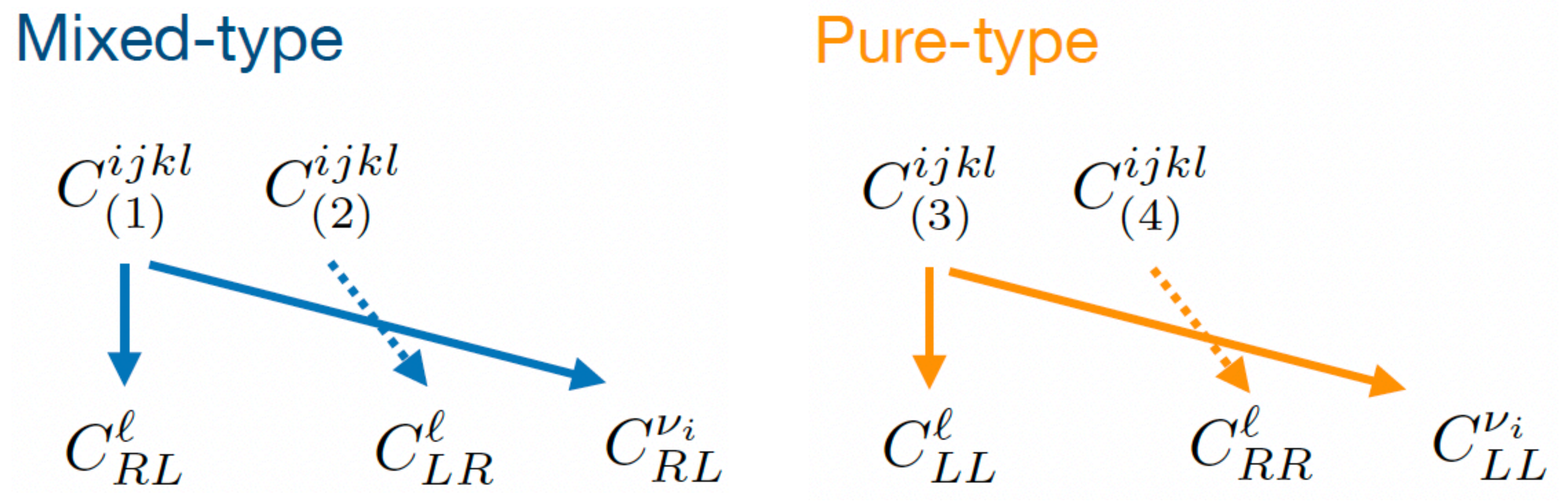

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$n \to \pi^- \ell^+$

$p \to \eta \ell^+$

$\ell = e, \mu$

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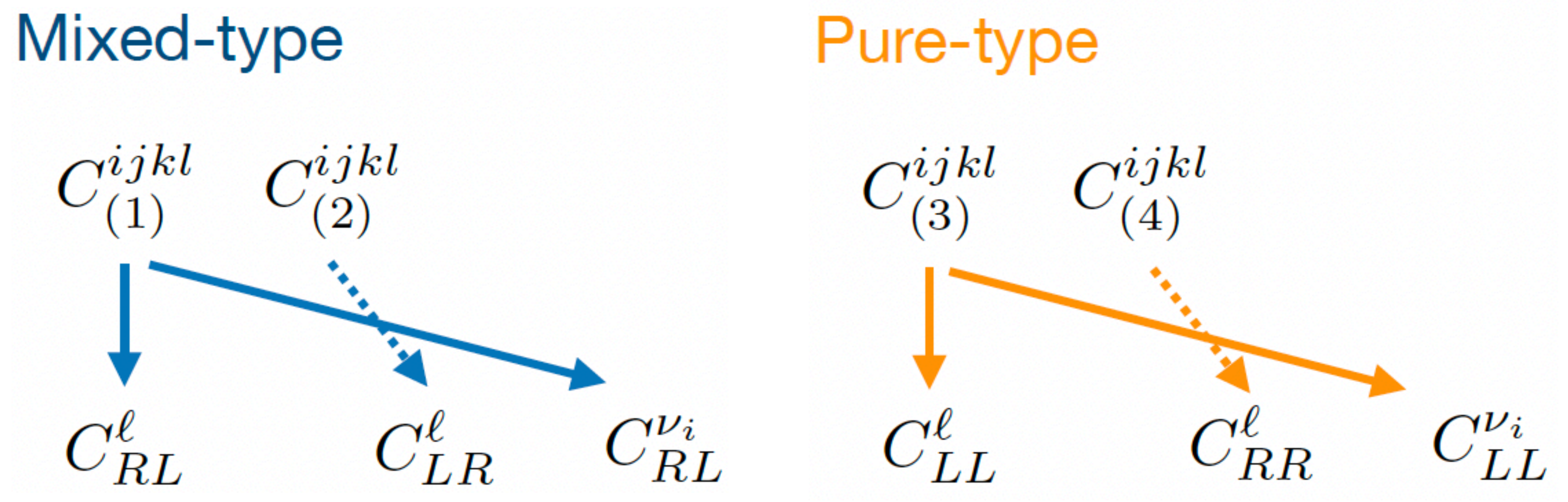

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$\ell = e, \mu$

$p \to \pi^+ \bar{\nu}_i$ $\quad n \to \eta \bar{\nu}_i$
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Chirality structure and UV mechanisms

H. Georgi and S. L. Glashow (1974)

Gauge boson exchange

↓

Dimension-six Kähler

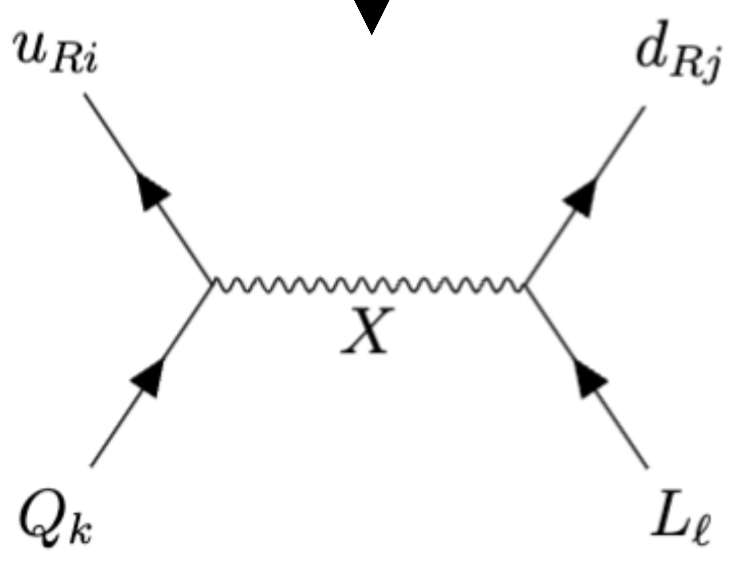
↓

$C^{ijkl}_{(1)}$ $C^{ijkl}_{(2)}$

Mixed-type

Dominant in
non-SUSY GUTs

$X_\mu \psi_{R\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \chi_{L\alpha}$



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Color-triplet Higgs exchange

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Dimension-five superpotential

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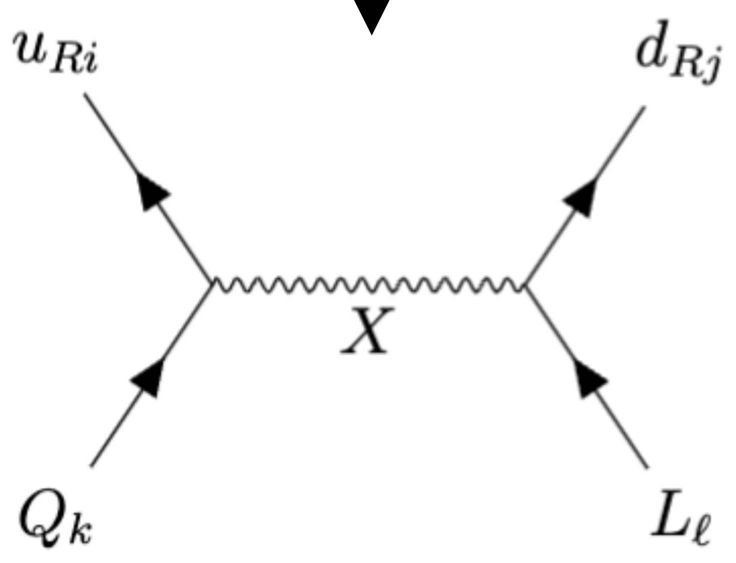


$C^{ijkl}_{(1)}$ $C^{ijkl}_{(2)}$

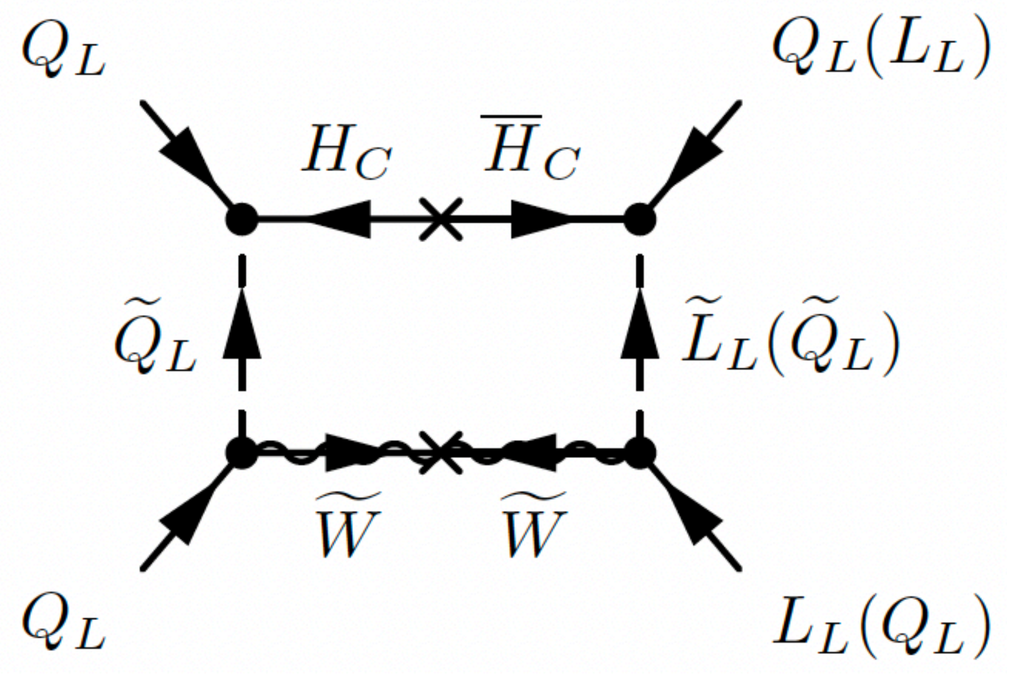
Mixed-type

Wino

Gluino/bino
(flavor violation)

$C^{ijkl}_{(3)}$ $C^{ijkl}_{(4)}$

Pure-type



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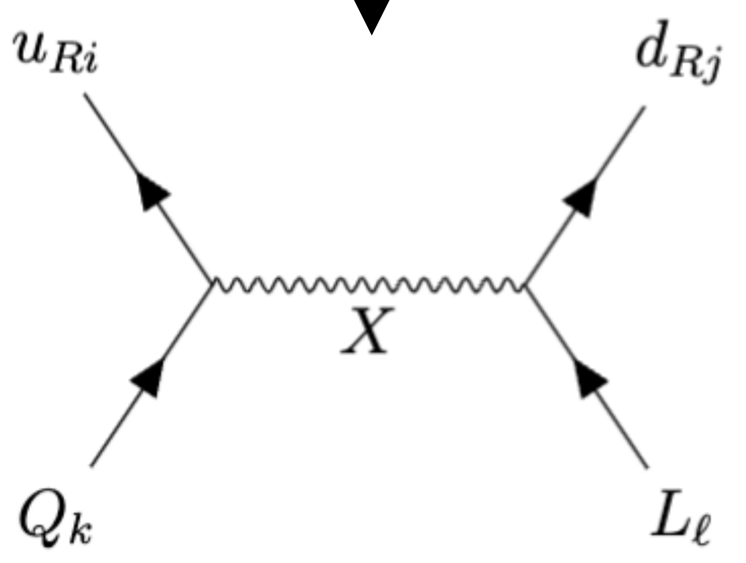

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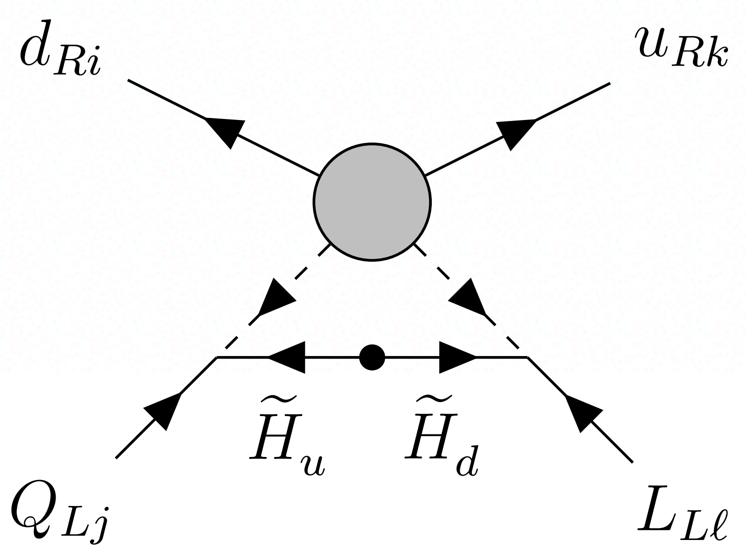

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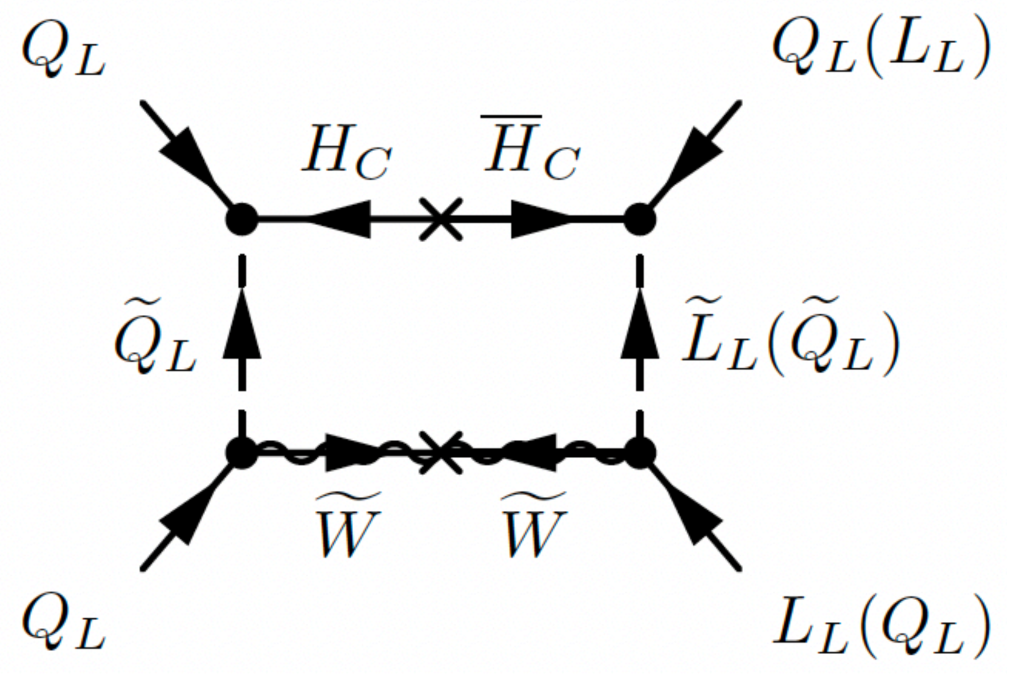

Chirality structure is sensitive to UV physics.

Chirality structure and branching fractions

- **Case 1:** mixed-type only case, $C^{\ell}_{LL} = C^{\ell}_{RR} = 0$, with $m_{\ell} = 0$.

$$\Gamma(N \to \Pi \ell^{+}) = \frac{m_N}{32\pi}\left(1 - \frac{m_{\Pi}^2}{m_N^2}\right)^2 |W^{LR}_{N\Pi\ell,0}|^2 \left[|C^{\ell}_{RL}|^2 + |C^{\ell}_{LR}|^2\right]$$

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Hadron matrix elements

$$\langle \Pi(\boldsymbol{p})|\mathcal{O}_{\chi\chi'}|N(\boldsymbol{k})\rangle = P_{\chi'}\left[\underline{W_0^{\mathcal{O}}(q^2)} + \frac{\not{q}}{m_N} W_1^{\mathcal{O}}(q^2)\right] u_N(\boldsymbol{k}) \qquad \mathcal{O}_{\chi\chi'} = \epsilon_{abc}(q^a_\chi q'^b_\chi) q''^c_{\chi'}$$

$$q^\mu = k^\mu - p^\mu$$

Chirality structure and branching fractions

- **Case 1:** mixed-type only case, $C^{\ell}_{LL} = C^{\ell}_{RR} = 0$, with $m_{\ell} = 0$.

$$\Gamma(N \to \Pi \ell^+) = \frac{m_N}{32\pi}\left(1 - \frac{m_\Pi^2}{m_N^2}\right)^2 \underline{|W^{LR}_{N\Pi\ell,0}|^2} \left[|C^{\ell}_{RL}|^2 + |C^{\ell}_{LR}|^2\right]$$

Hadron matrix elements

$$\langle \Pi(\boldsymbol{p}) | \mathcal{O}_{\chi\chi'} | N(\boldsymbol{k}) \rangle = P_{\chi'} \left[\underline{W_0^{\mathcal{O}}(q^2)} + \frac{\not{q}}{m_N} W_1^{\mathcal{O}}(q^2) \right] u_N(\boldsymbol{k}) \qquad \mathcal{O}_{\chi\chi'} = \epsilon_{abc}(q^a_\chi q'^b_\chi) q''^c_{\chi'}$$

We use the form factors obtained by lattice simulations

Y. Aoki et al. (2017)
J. S. Yoo, et al. (2022)

$\longrightarrow$ Uncertainty: $\mathcal{O}(10)\,\%$

Chirality structure and branching fractions

- **Case 1:** mixed-type only case, $C^{\ell}_{LL} = C^{\ell}_{RR} = 0$, with $m_\ell = 0$.

$$\Gamma(N \to \Pi \ell^+) = \frac{m_N}{32\pi}\left(1 - \frac{m_\Pi^2}{m_N^2}\right)^2 |W^{LR}_{N\Pi\ell,0}|^2 \left[\underline{|C^{\ell}_{RL}|^2 + |C^{\ell}_{LR}|^2}\right]$$

  - Wilson coefficients which reflect UV physics.
  - **Does not depend on whether** $p \to \pi^0 e^+$ **or** $p \to \eta e^+$

Chirality structure and branching fractions

- **Case 1:** mixed-type only case, $C^{\ell}_{LL} = C^{\ell}_{RR} = 0$, with $m_\ell = 0$.

$$\Gamma(N \to \Pi\ell^+) = \frac{m_N}{32\pi}\left(1 - \frac{m_\Pi^2}{m_N^2}\right)^2 |W^{LR}_{N\Pi\ell,0}|^2 \left[|C^{\ell}_{RL}|^2 + |C^{\ell}_{LR}|^2\right]$$

- The ratio $\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$ are **solely determined by low-energy quantities.**

$$\frac{\Gamma(p \to \eta e^+)}{\Gamma(p \to \pi^0 e^+)} = \frac{(1 - m_\eta^2/m_p^2)^2}{(1 - m_\pi^2/m_p^2)^2} \cdot \frac{|W^{XX'}_{p\eta\ell}|^2}{|W^{XX'}_{p\pi^0\ell}|^2} \simeq \begin{cases} 0.0013 & \text{(Mixed only)} \\ 0.51 & \text{(Pure only)} \end{cases}$$

Chirality structure and branching fractions

- **Case 1:** mixed-type only case, $C^{\ell}_{LL} = C^{\ell}_{RR} = 0$, with $m_\ell = 0$.

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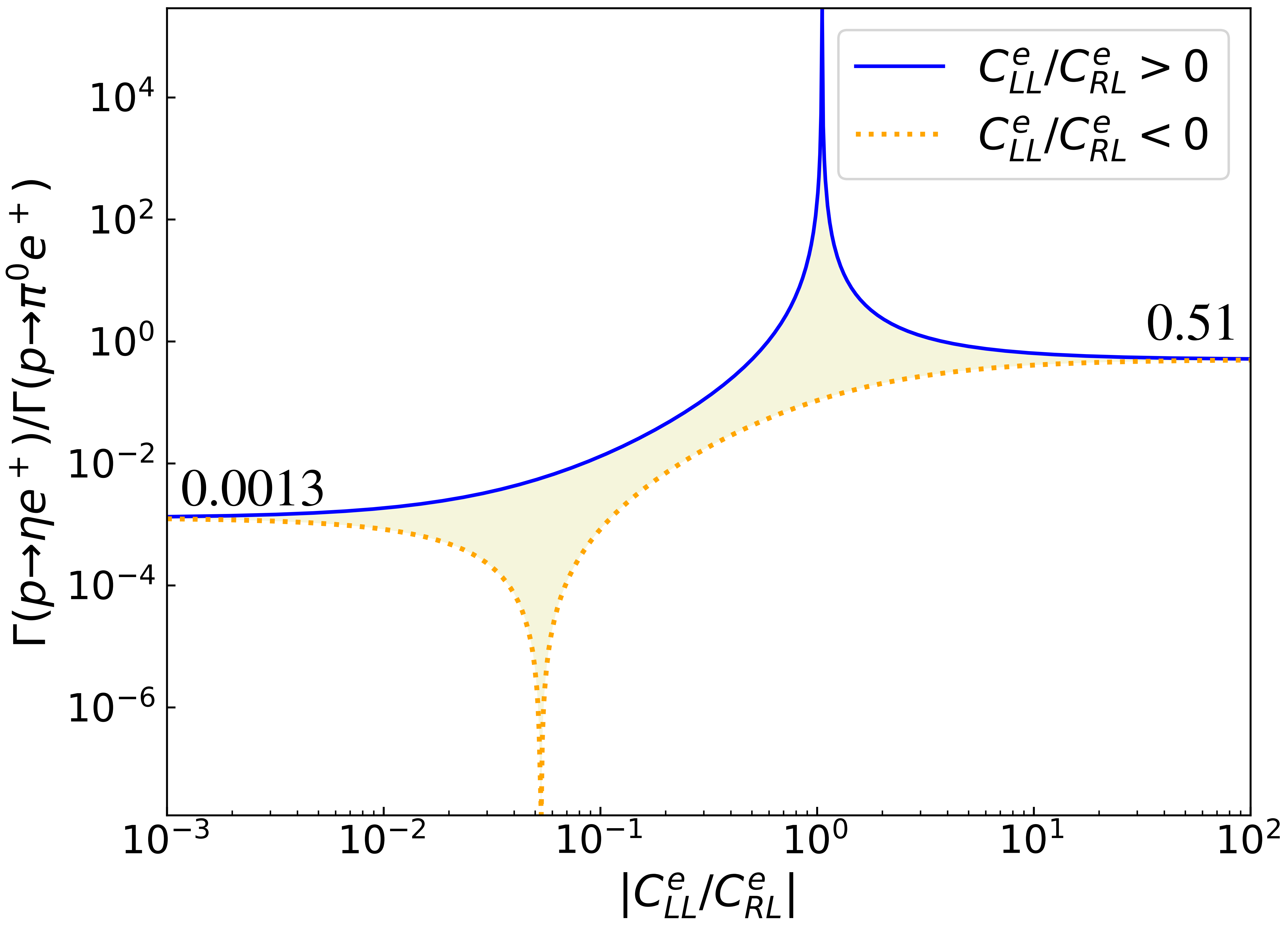
→ Parity invariance of QCD plays the role:

$$\boxed{W^{RL}_{N\Pi\ell,I} = W^{LR}_{N\Pi\ell,I}\,, \qquad W^{LL}_{N\Pi\ell,I} = W^{RR}_{N\Pi\ell,I}}$$

$$\langle \Pi(\boldsymbol{p})|\mathcal{O}_{\chi\chi'}|N(\boldsymbol{k})\rangle = P_{\chi'}\left[W_0^{\mathcal{O}}(q^2) + \frac{\not{q}}{m_N}W_1^{\mathcal{O}}(q^2)\right]u_N(\boldsymbol{k})$$

Chirality structure and branching fractions

- **Case 2:** both-type case, $C^e_{RR} = C^e_{LR} = 0$ with $m_e = 0$.



- $|C^e_{LL}/C^e_{RL}| \ll 1$ and $\gg 1$ approach **orders of magnitude different values.**
- Peaks at
  - $|C^e_{LL}/C^e_{RL}| \simeq 1$ for $C^e_{LL}/C^e_{RL} > 0$
  - $|C^e_{LL}/C^e_{RL}| \simeq 0.1$ for $C^e_{LL}/C^e_{RL} < 0$

$\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$ **is sensitive to chirality structure.**

Chirality structure and UV mechanisms

H. Georgi and S. L. Glashow (1974)

S. Weinberg (1982)
N. Sakai and T. Yanagida (1982)

Gauge boson exchange

Color-triplet Higgs exchange

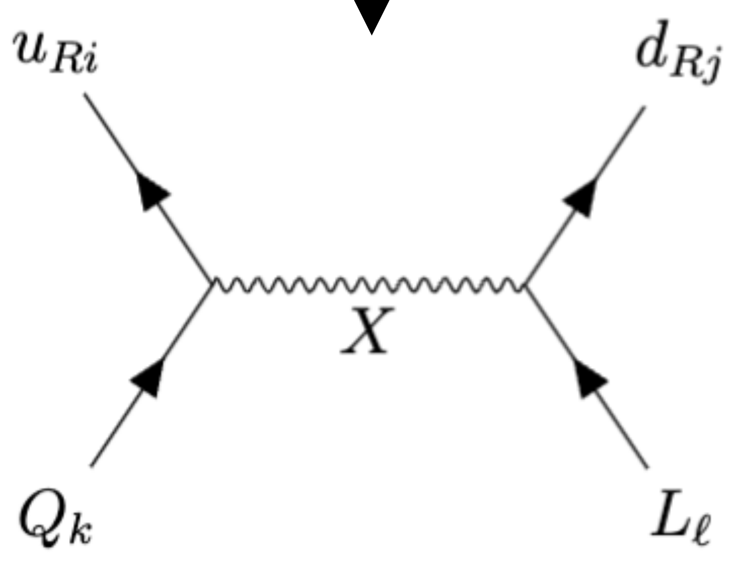
Dominant in non-SUSY GUTs

Dominant in SUSY GUTs

Dimension-six Kähler

Dimension-five superpotential

$X_\mu \psi_{R\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \chi_{L\alpha}$



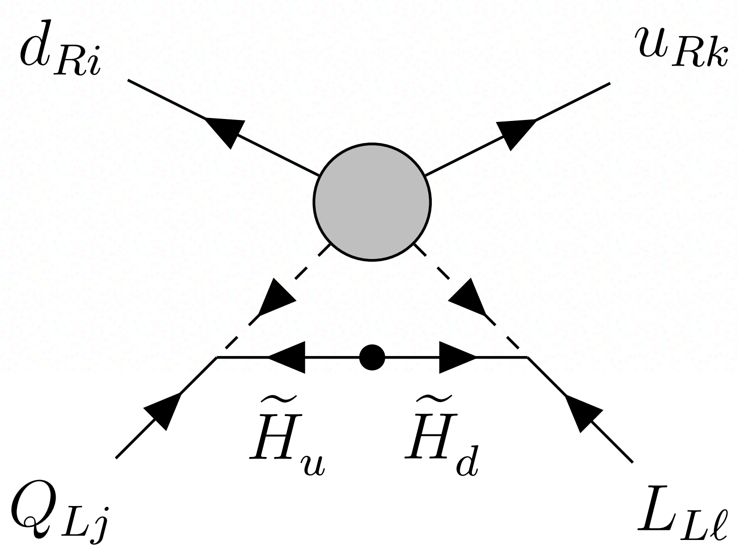
$\propto \frac{1}{M_X^2}$

Higgsino

Wino

$C_{(1)}^{ijkl}$ $C_{(2)}^{ijkl}$

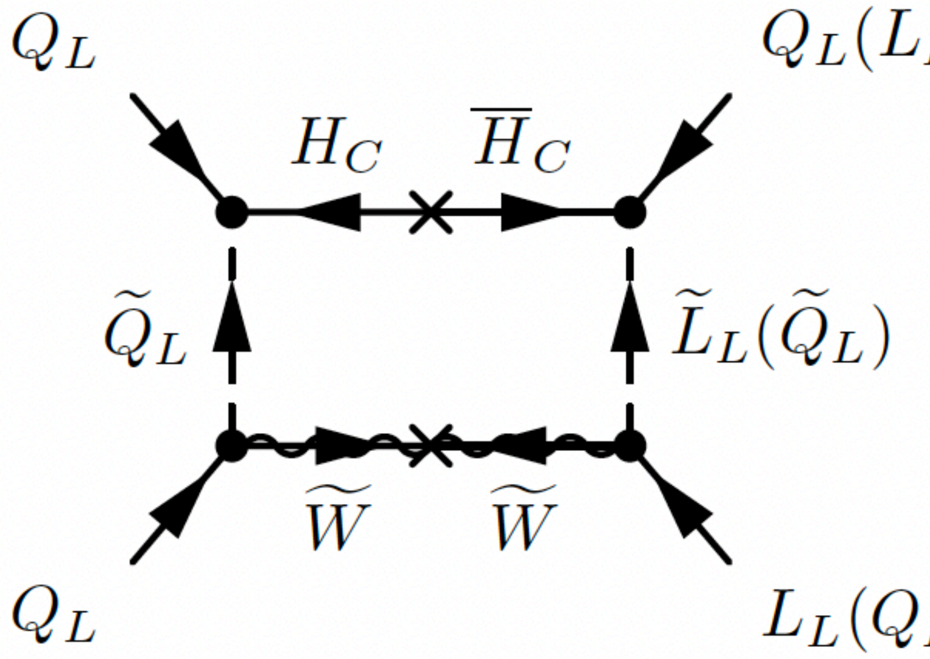
Mixed-type



$\propto \frac{\mu_H}{M_{H_C} M_{\rm SUSY}^2}$

$C_{(3)}^{ijkl}$ $C_{(4)}^{ijkl}$

Pure-type

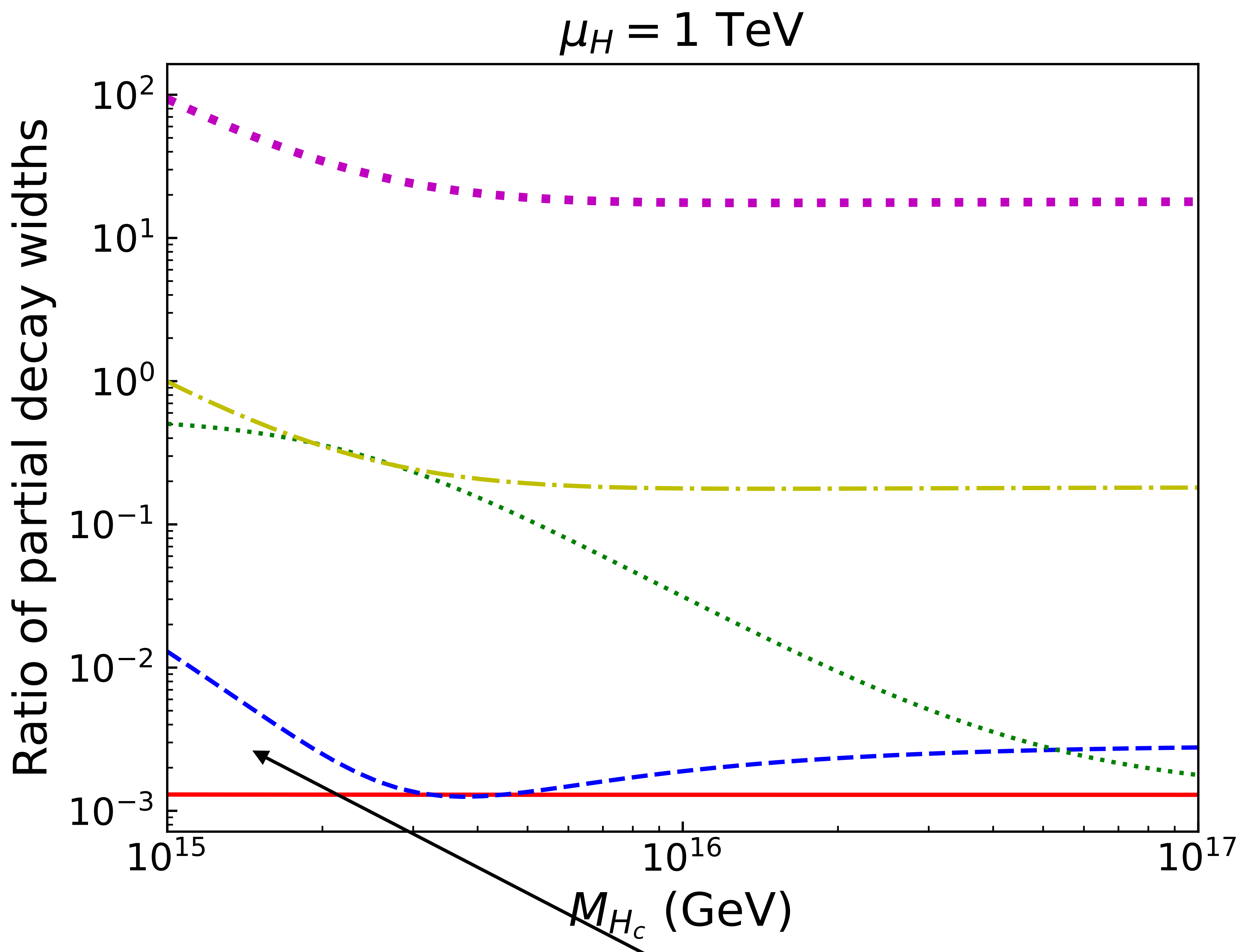


$\propto \frac{m_{\tilde{g}}}{M_{H_C} M_{\rm SUSY}^2}$

- $M_{\rm SUSY}$: sfermion mass
- $m_{\tilde{g}} \leq M_{\rm SUSY}$

The minimal SU(5) with high-scale SUSY

No sfermion flavor violation

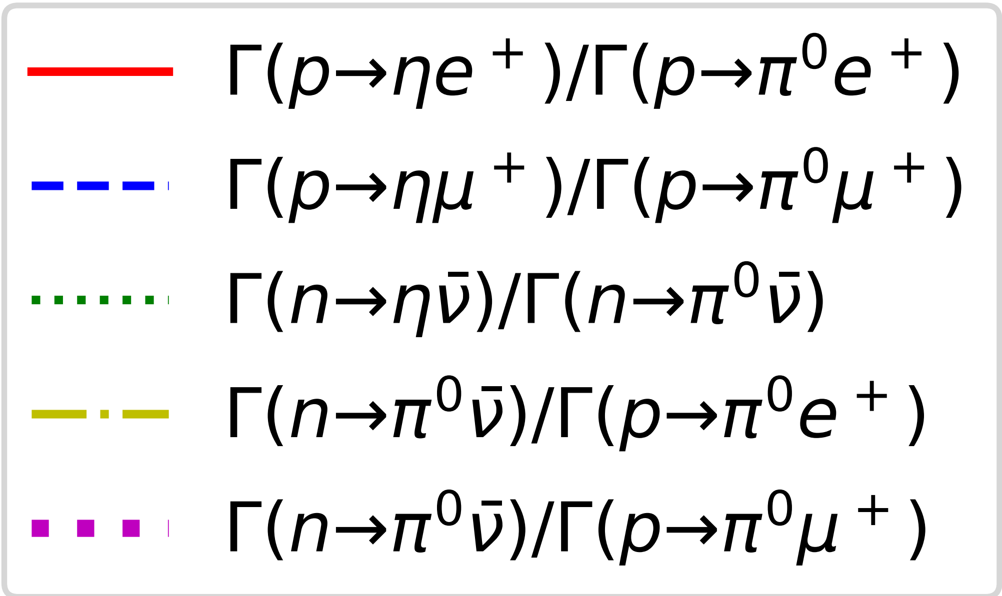


- $M_X = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$

Mini-split type mass spectrum

L. J. Hall, Y. Nomura and S. Shirai (2012)
M. Ibe, S. Matsumoto and T. T. Yanagida (2012)

- $\Gamma(p \to \eta\mu^+)/\Gamma(p \to \pi^0\mu^+)$: wino-exchange contributes (pure-type) as M_{H_C} decreases.



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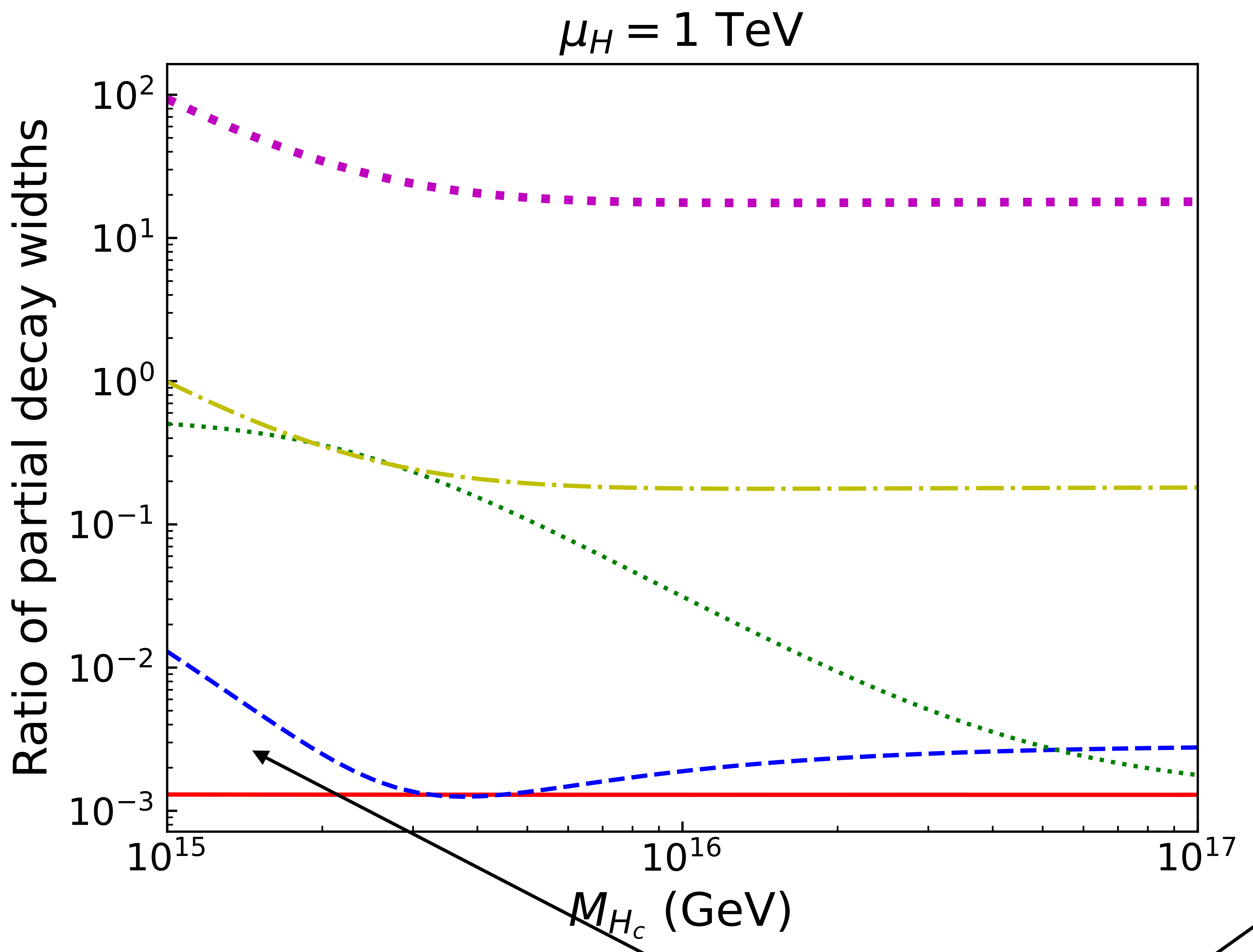

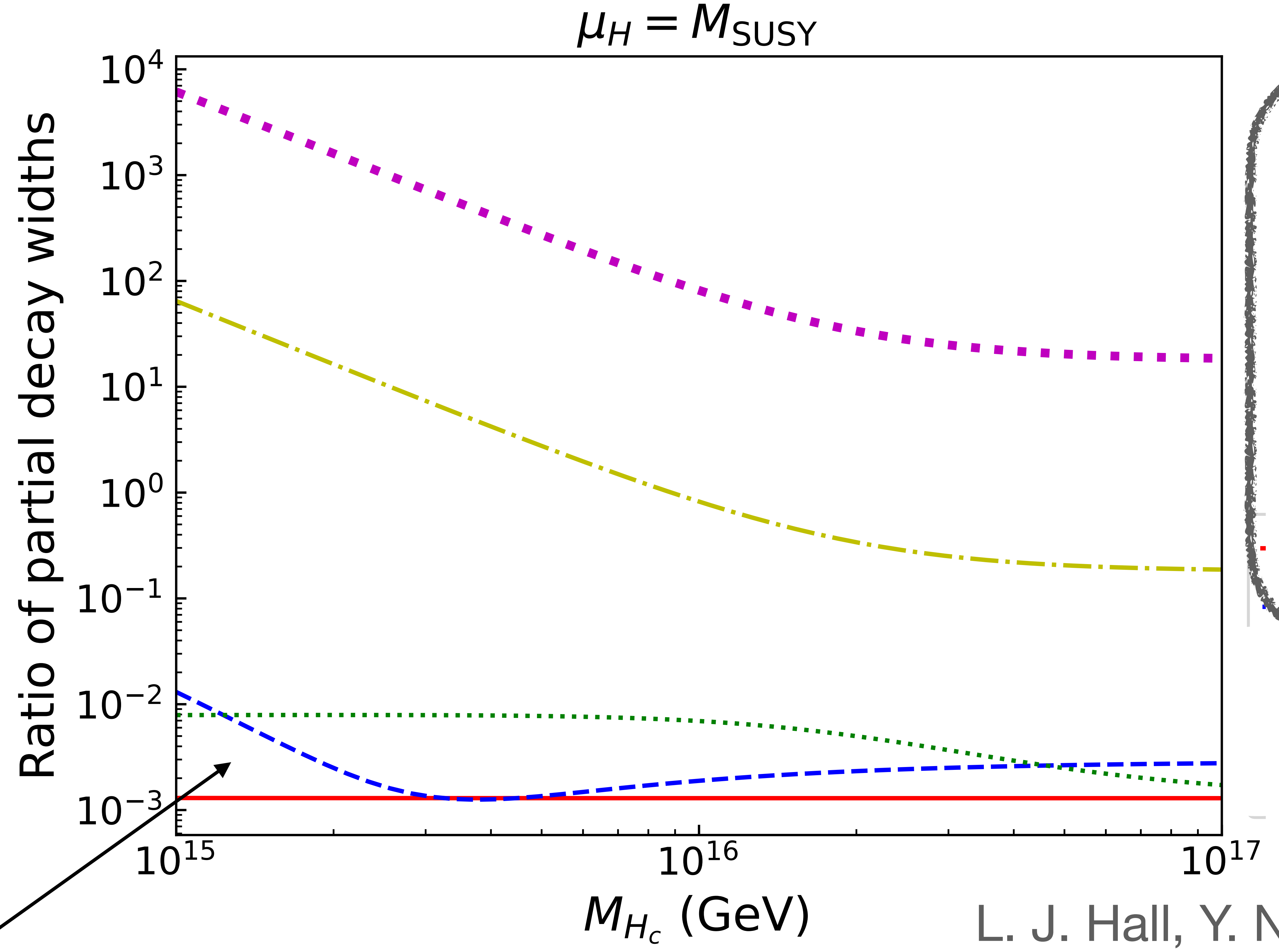

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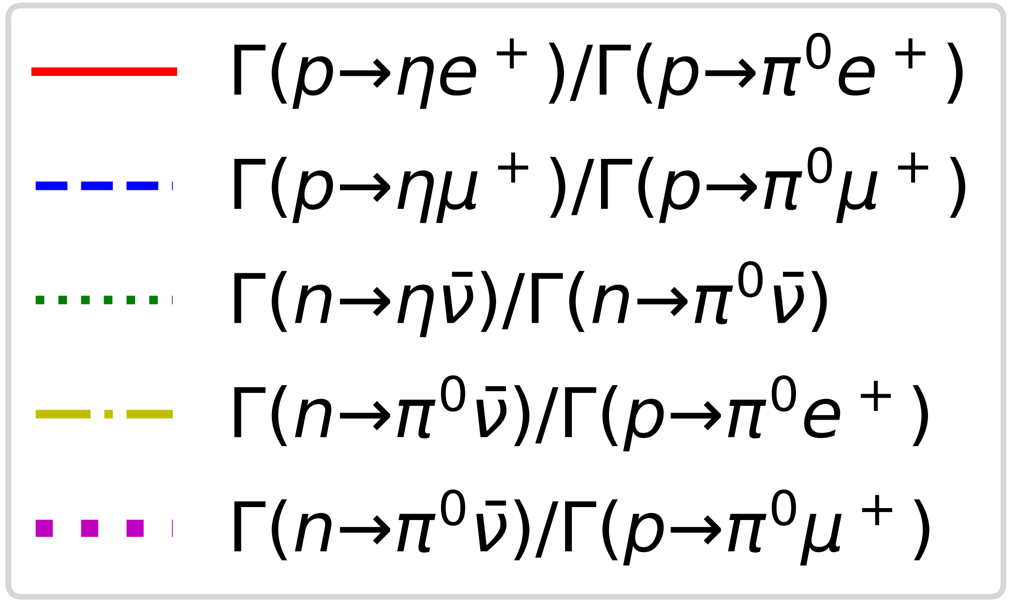
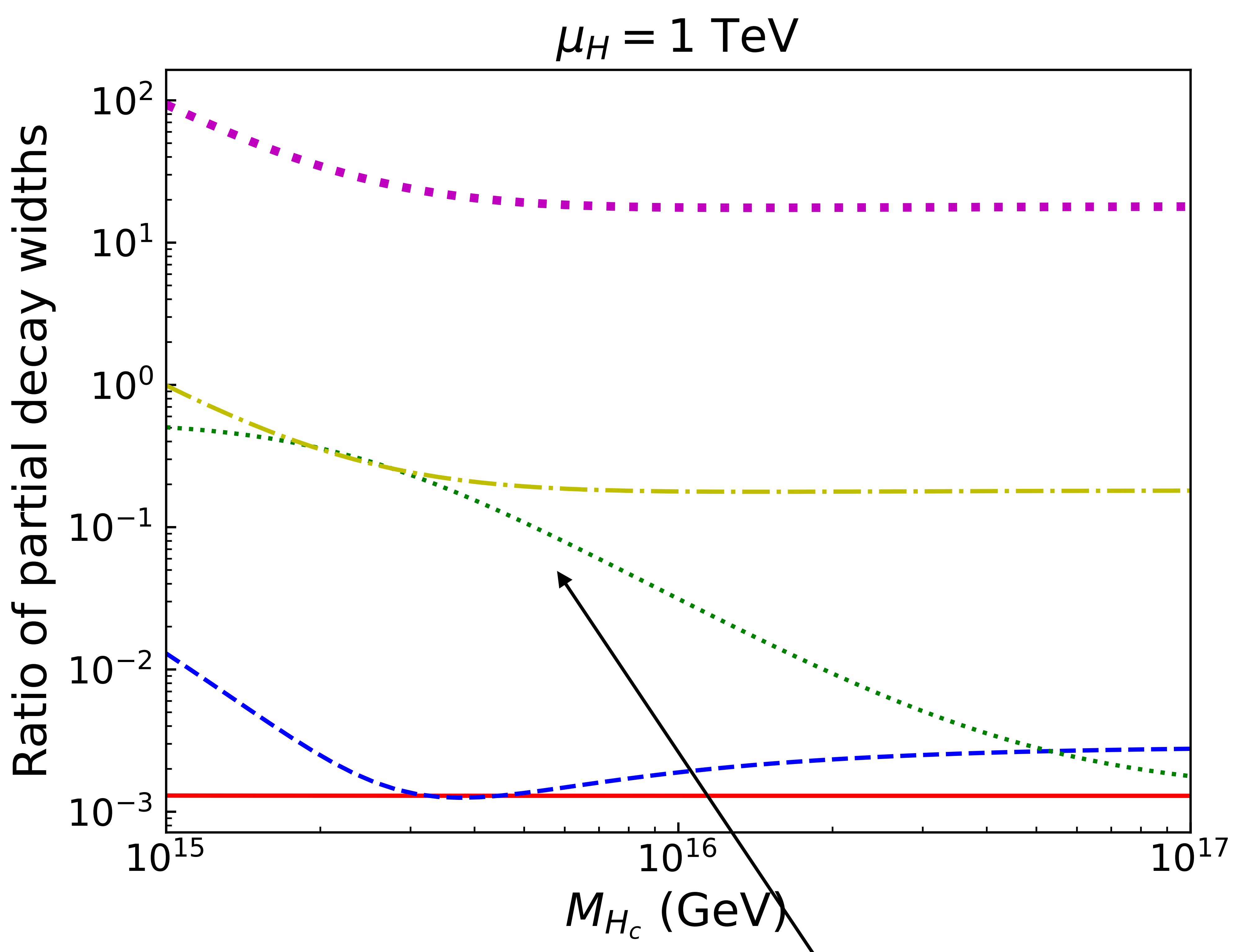
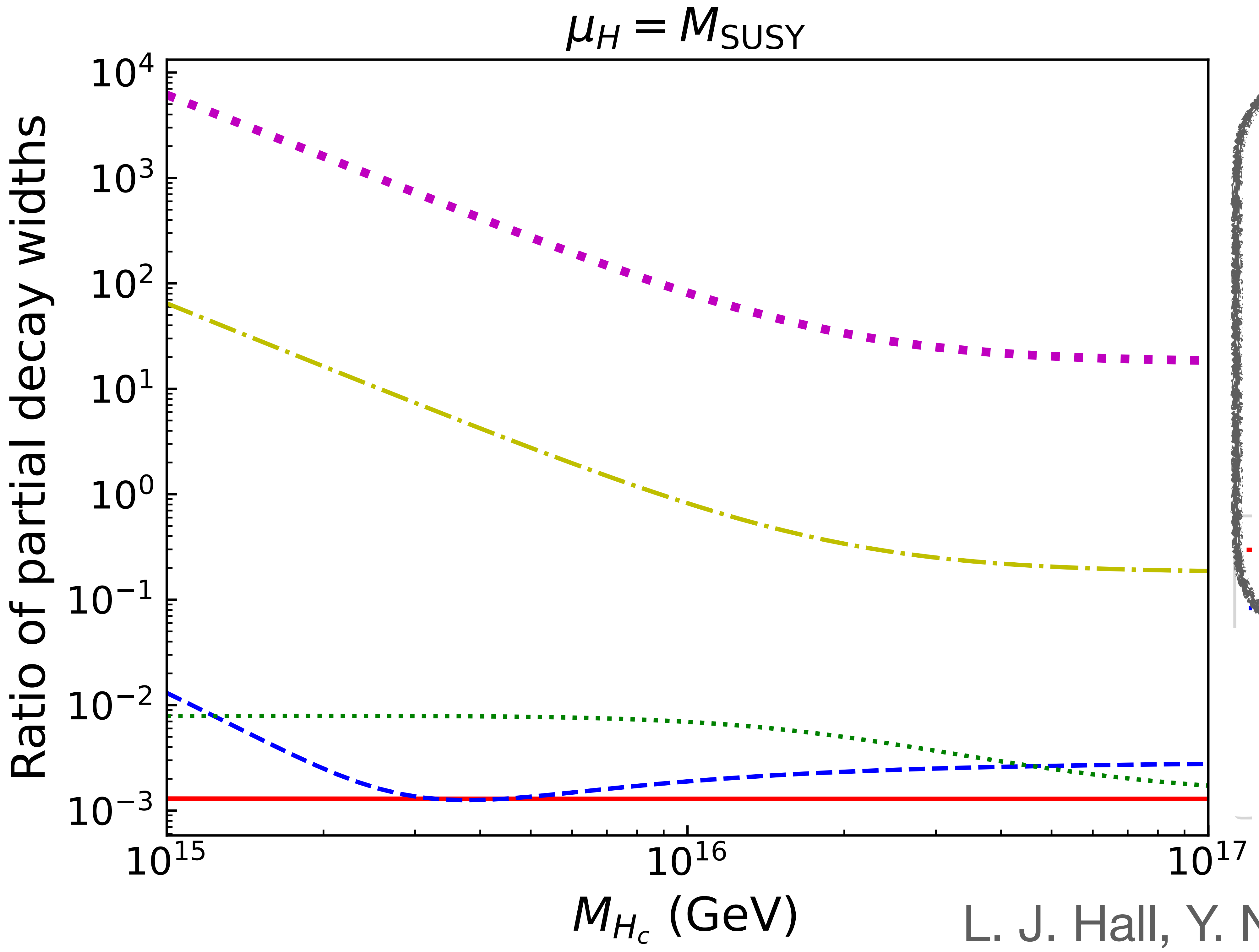


The minimal SU(5) with high-scale SUSY

No sfermion flavor violation





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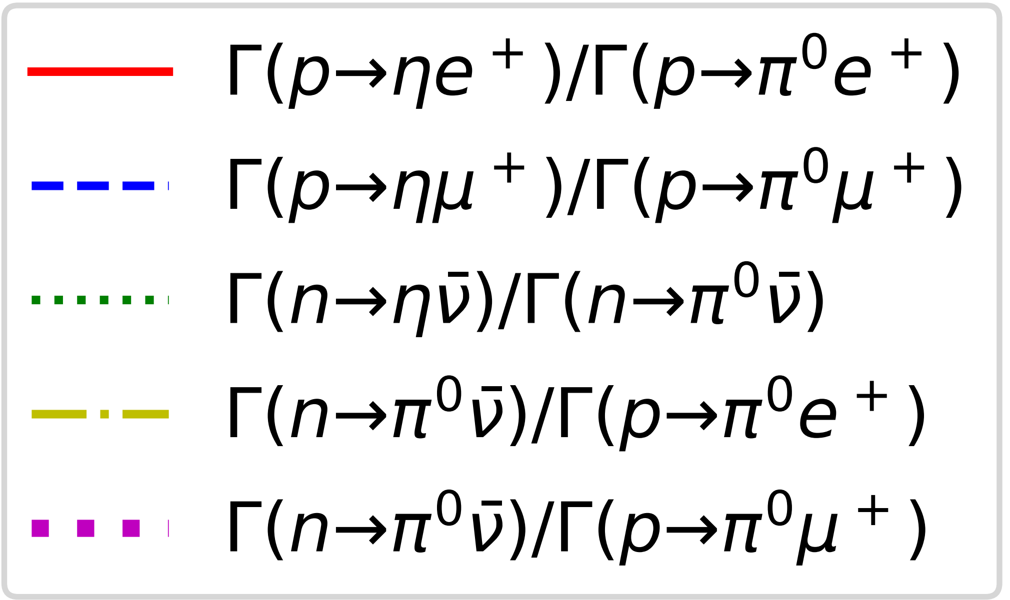
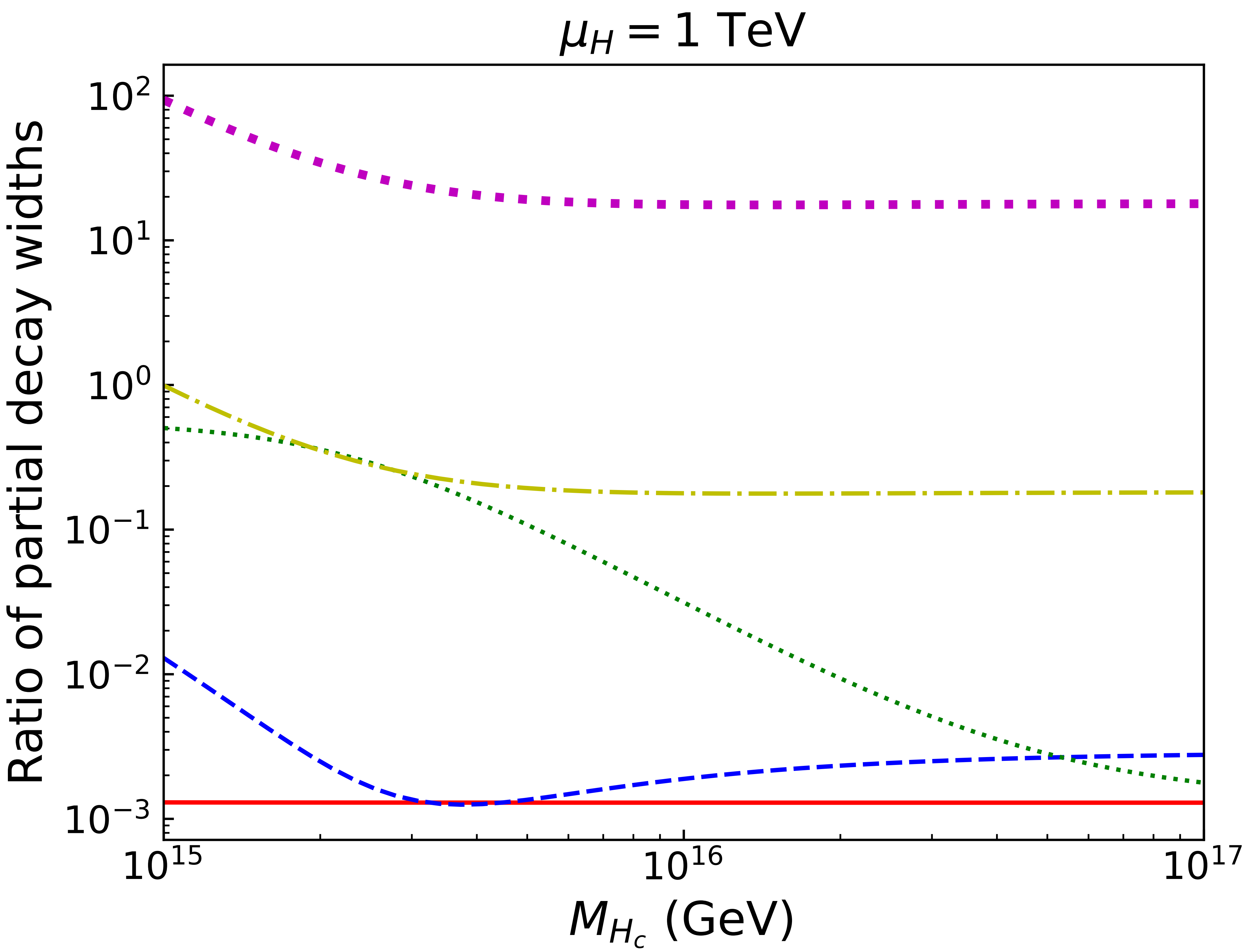


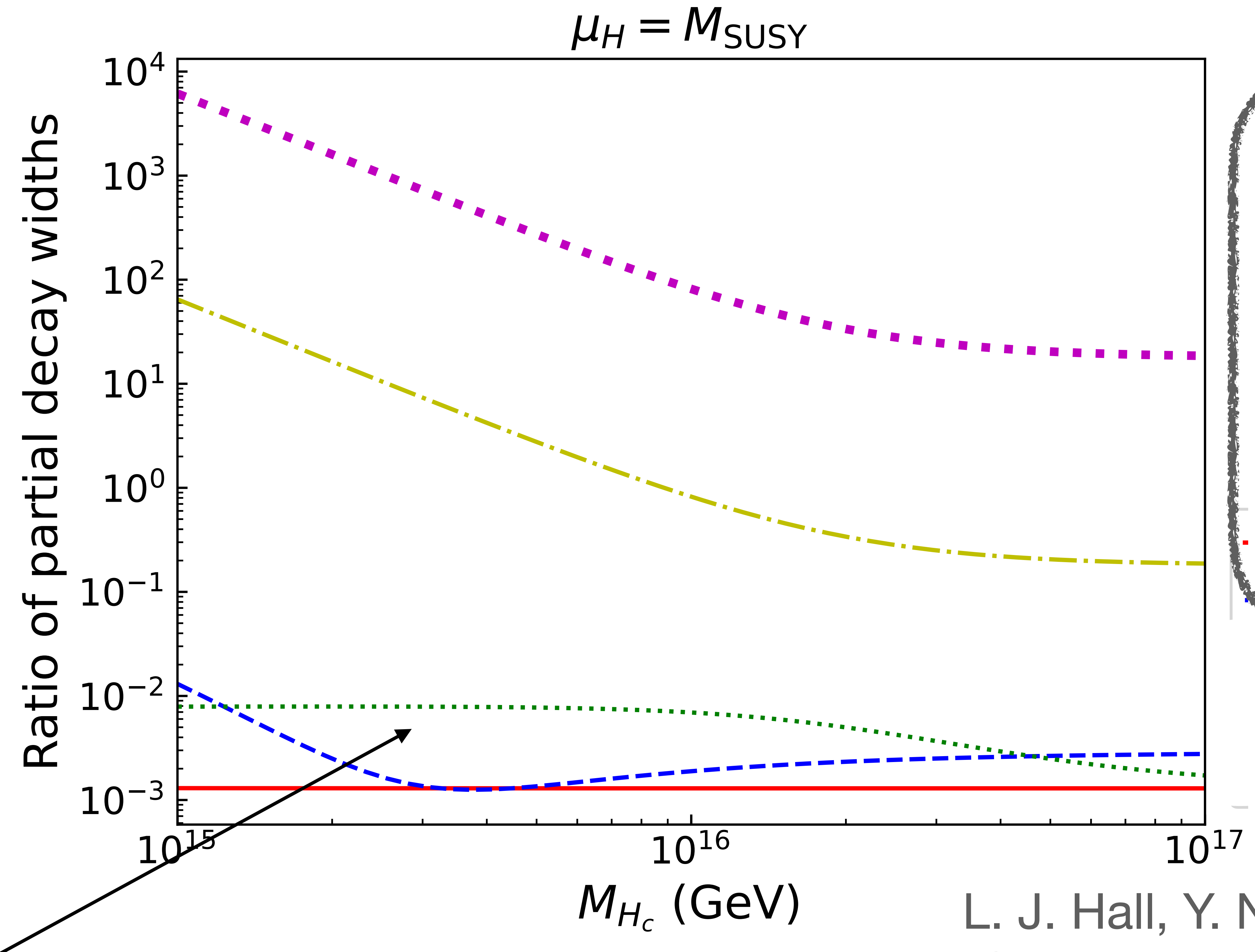
$\Gamma(n \to \eta\bar{\nu})/\Gamma(n \to \pi^0\bar{\nu})$:

- Left: larger contribution of the wino-exchanging process (pure-type).

The minimal SU(5) with high-scale SUSY

No sfermion flavor violation





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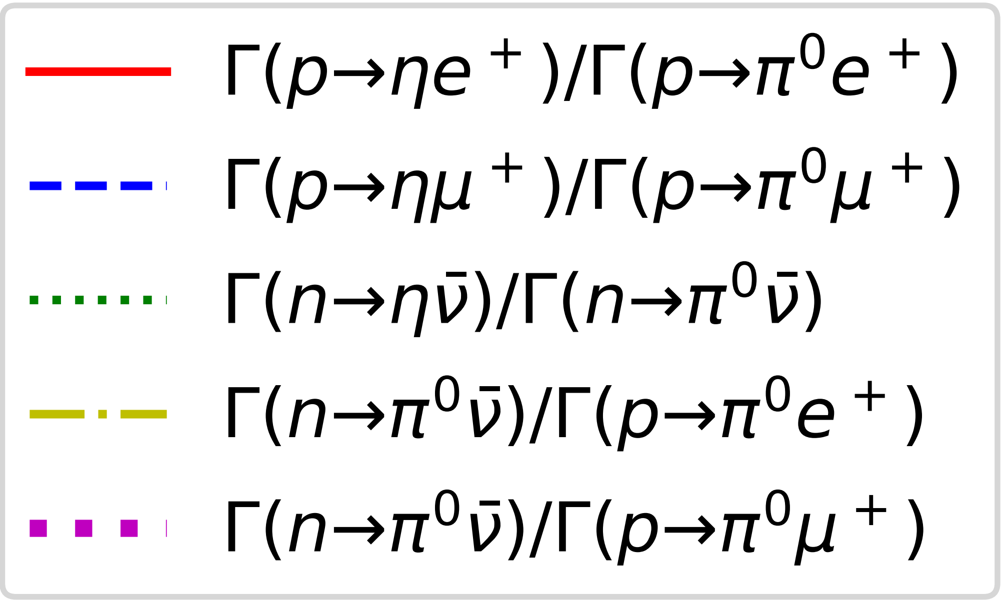


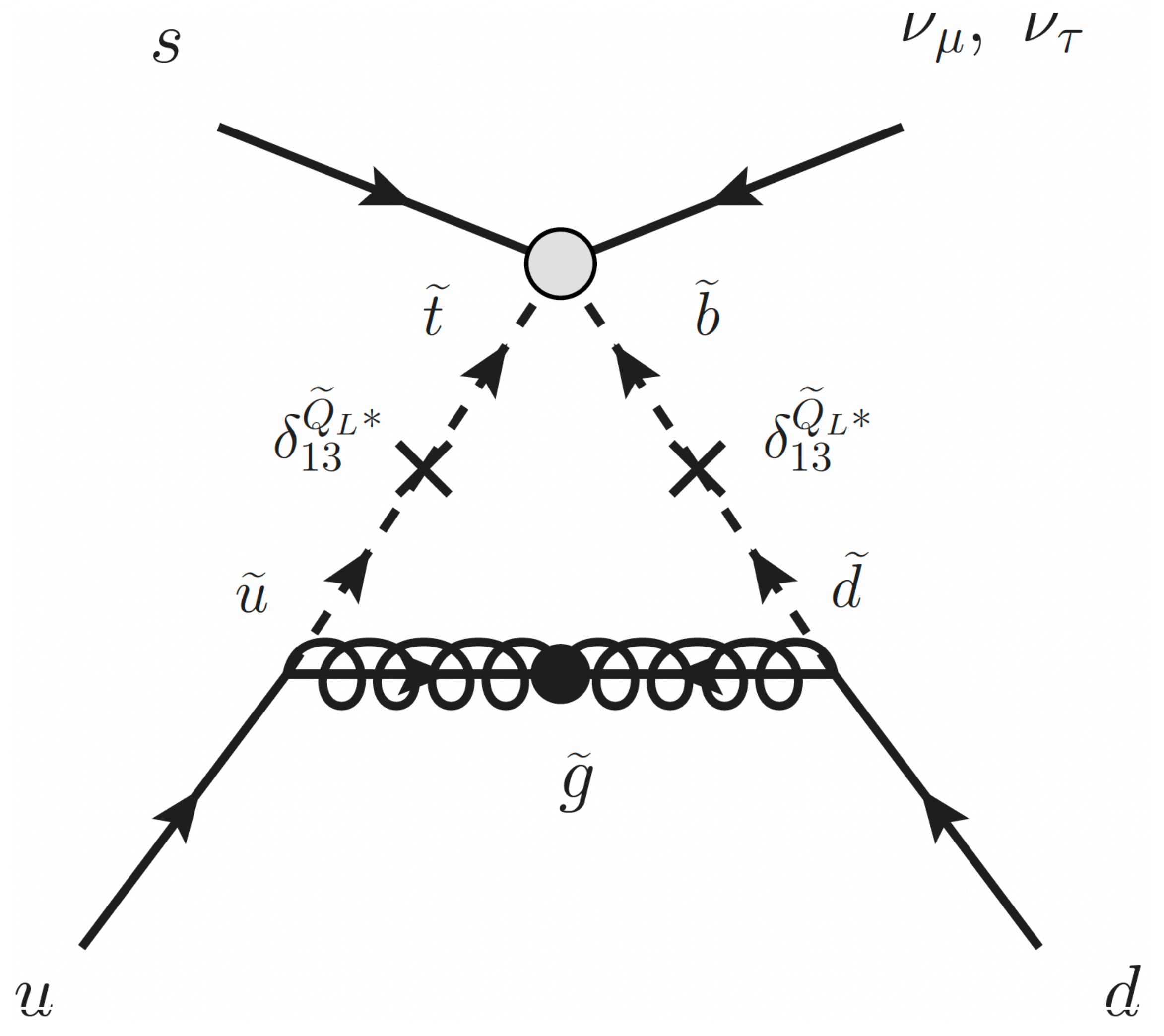
$\Gamma(n \to \eta\bar{\nu})/\Gamma(n \to \pi^0\bar{\nu})$:

- Left: larger contribution of the wino-exchanging process (pure-type).
- Right: higgsino exchange (mixed-type) dominates in smaller M_{H_C}.

Sfermion flavor violation and ratios

- $M_X = 10^{17}$ GeV
- $M_{H_C} = 10^{16}$ GeV
- $M_1 = 5$ TeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\text{SUSY}} = 100$ TeV
- $\tan\beta = 3$
- $\mu_H = 200$ GeV

$$\widetilde{m}^2_{\tilde{f}} = M^2_{\text{SUSY}} \begin{pmatrix} 1 & \delta^{\tilde{f}}_{12} & \delta^{\tilde{f}}_{13} \\ \delta^{\tilde{f}*}_{12} & 1 & \delta^{\tilde{f}}_{23} \\ \delta^{\tilde{f}*}_{13} & \delta^{\tilde{f}*}_{23} & 1 \end{pmatrix}$$



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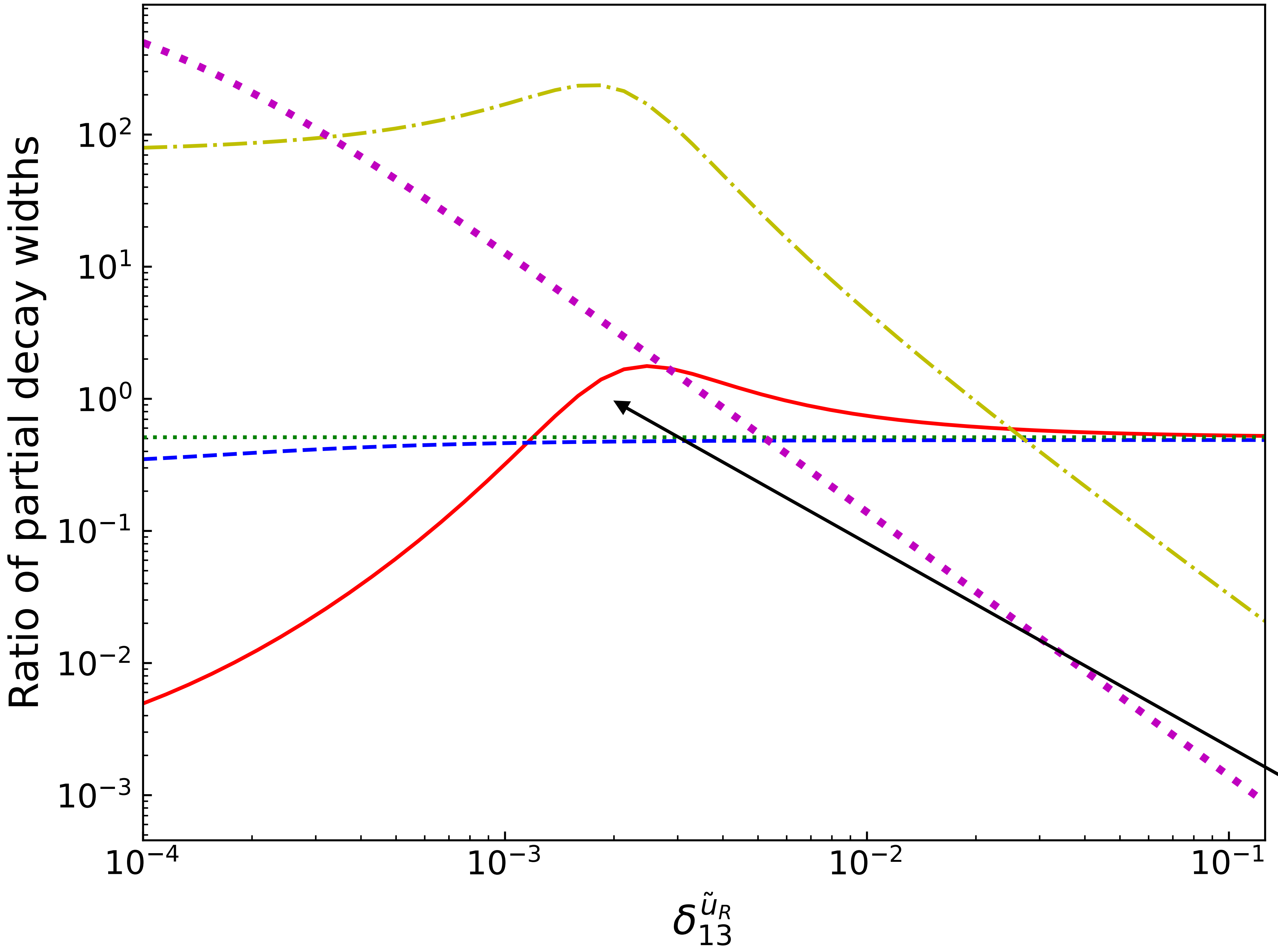

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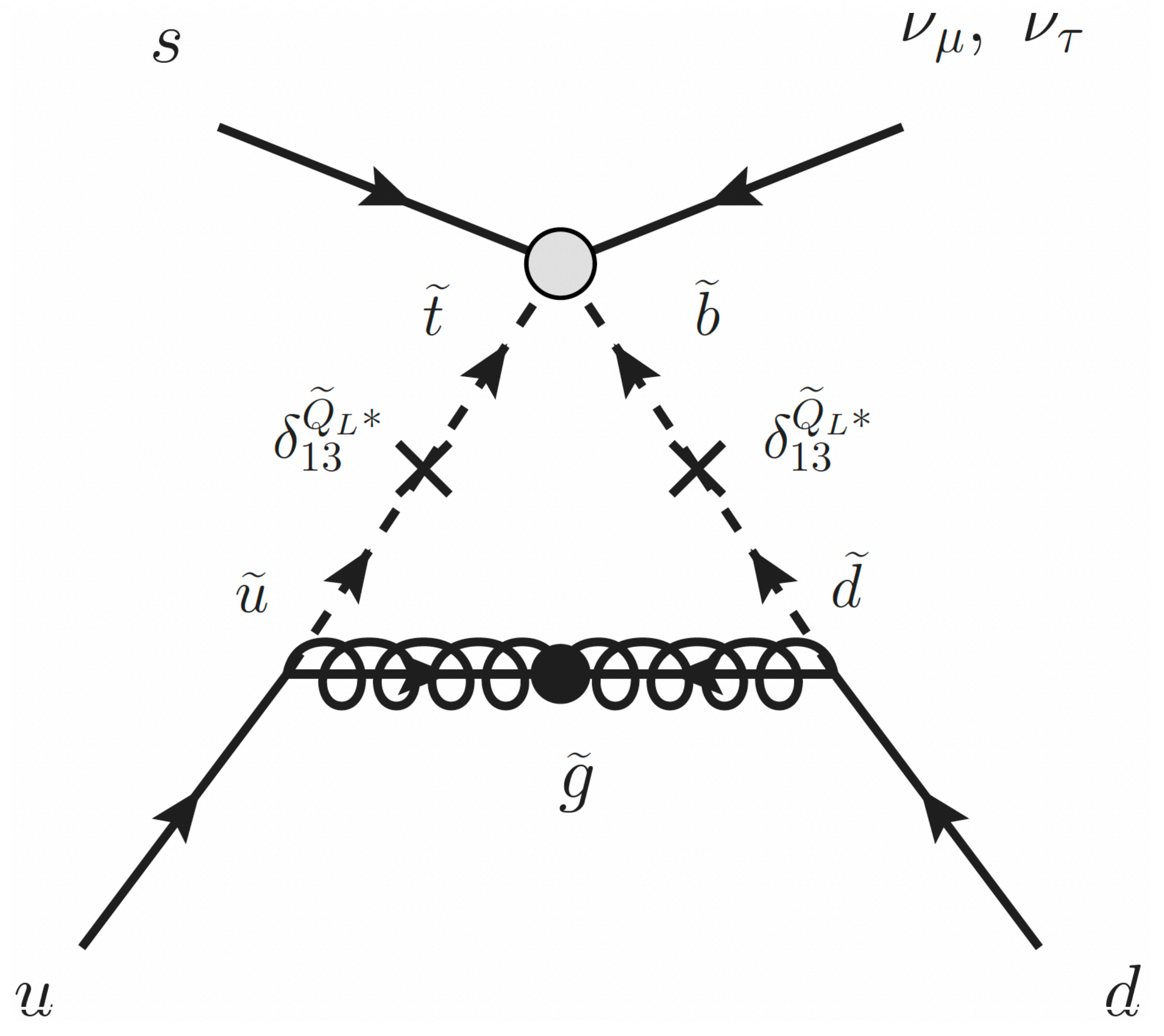


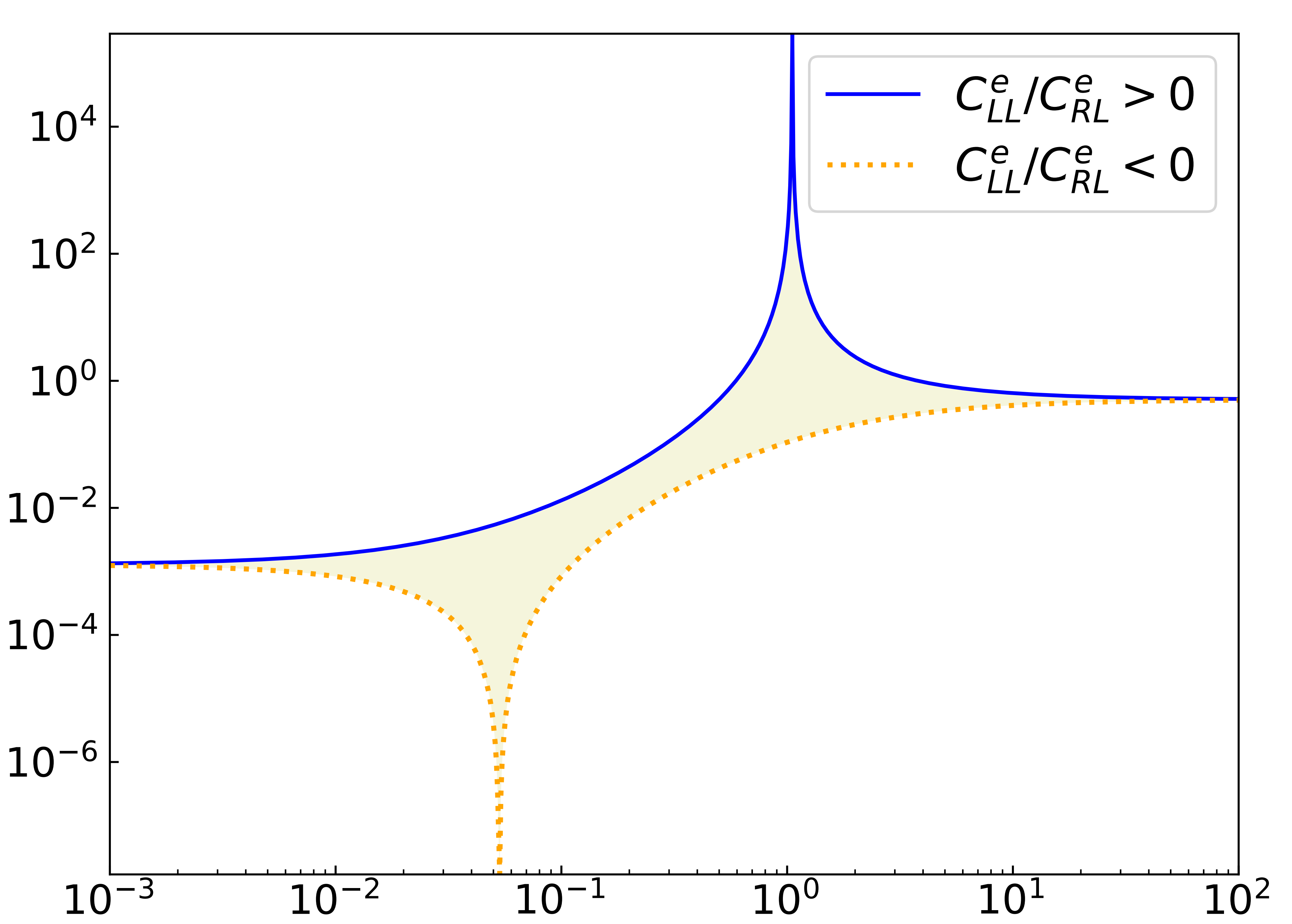
- The gluino exchange dominates as $\delta_{13}^{\tilde{u}_R}$ increases, even in $\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$.

Take-home: Nucleon decay → Chirality structure

Decay channel: $p \to \pi^0 e^+$ and $p \to \eta e^+$

$$\mathcal{L}_{\text{eff}} = C^{\ell}_{RL}\left[\epsilon_{abc}(u^a_R d^b_R)(u^c_L \ell_L)\right] + C^{\ell}_{LR}\left[\epsilon_{abc}(u^a_L d^b_L)(u^c_R \ell_R)\right] + C^{\ell}_{LL}\left[\epsilon_{abc}(u^a_L d^b_L)(u^c_L \ell_L)\right] + C^{\ell}_{RR}\left[\epsilon_{abc}(u^a_R d^b_R)(u^c_R \ell_R)\right]$$

$$\frac{\Gamma(p \to \eta e^+)}{\Gamma(p \to \pi^0 e^+)}$$



$$\left|\frac{C^e_{LL}}{C^e_{RL}}\right|$$

Application to SUSY SU(5):

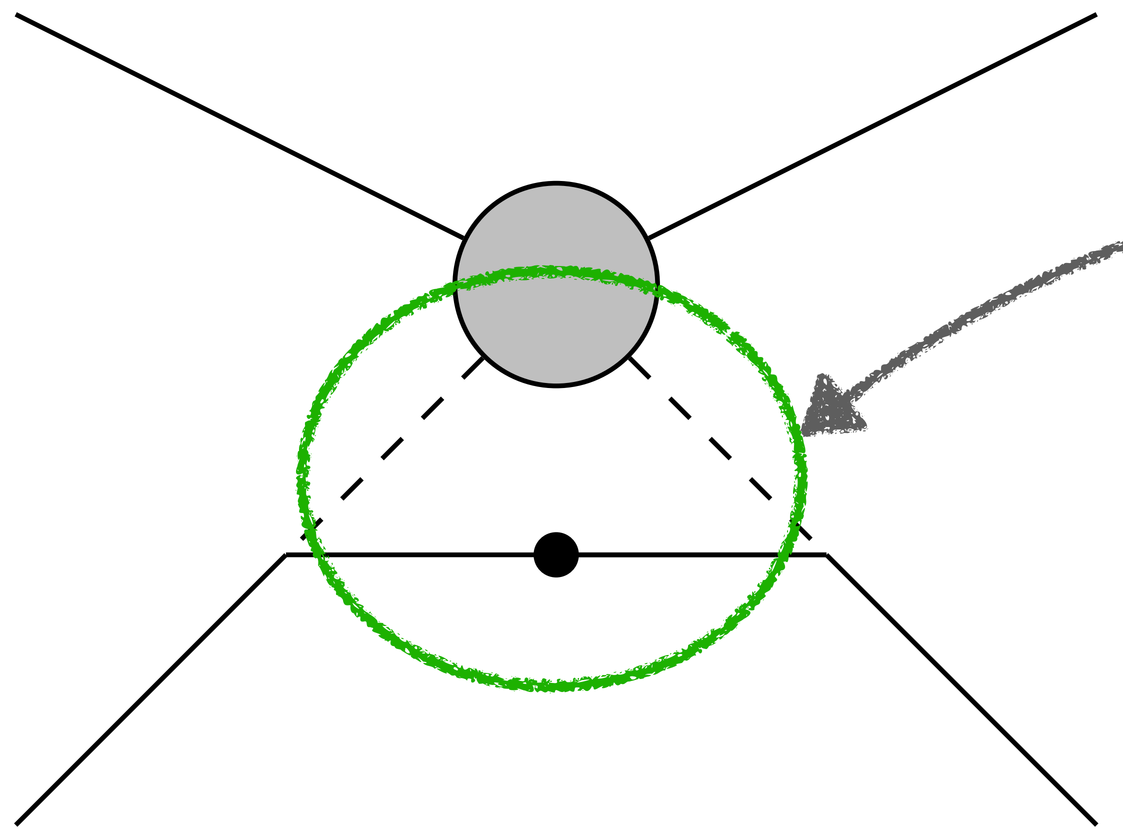
- $\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+) \to M_X$ vs $m_{\tilde{g}}$
- $\Gamma(p \to \eta \mu^+)/\Gamma(p \to \pi^0 \mu^+) \to M_X$ vs $m_{\tilde{w}}$
- $\Gamma(n \to \eta \bar{\nu})/\Gamma(n \to \pi^0 \bar{\nu}) \to m_{\tilde{h}}$ vs $m_{\tilde{w}}$

Backup

SUSY dimension-five nucleon decay operators

P. Nath and R. L. Arnowitt (1988)
J Hisano, H. Murayama and T. Yanagida (1993)

- Higgsino/gaugino-dependence



Loop function:

$$F(M, m_1, m_2) \equiv \int \frac{d^4p}{\pi^2} \frac{i}{(\not{p} - M)(p^2 - m_1^2)(p^2 - m_2^2)}$$

$$= \frac{M}{m_1^2 - m_2^2}\left(\frac{m_1^2}{m_1^2 - M^2}\ln\frac{m_1^2}{M^2} - \frac{m_2^2}{m_2^2 - M^2}\ln\frac{m_2^2}{M^2}\right)$$

$$\to \frac{M}{m_1^2},$$

When $M \ll m_1 \sim m_2$

The dimension-five contribution is proportional to $m_{\tilde{g}}/(M_{H_C} M^2_{\mathrm{SUSY}})$

Current bounds and future prospects

Decay Mode	Current [years]	HK sensitivity [years]
$p \to \pi^0 e^+$	2.4×10^{34} [25]	7.8×10^{34} [11]
$p \to \pi^0 \mu^+$	1.6×10^{34} [25]	7.7×10^{34} [11]
$p \to \eta e^+$	1.0×10^{34} [26]	4.3×10^{34} [11]
$p \to \eta \mu^+$	4.7×10^{33} [26]	4.9×10^{34} [11]
$p \to \pi^+ \bar{\nu}$	3.9×10^{32} [27]	
$n \to \pi^- e^+$	5.3×10^{33} [26]	2.0×10^{34} [11]
$n \to \pi^- \mu^+$	3.5×10^{33} [26]	1.8×10^{34} [11]
$n \to \pi^0 \bar{\nu}$	1.1×10^{33} [27]	
$n \to \eta \bar{\nu}$	1.6×10^{32} [28]	

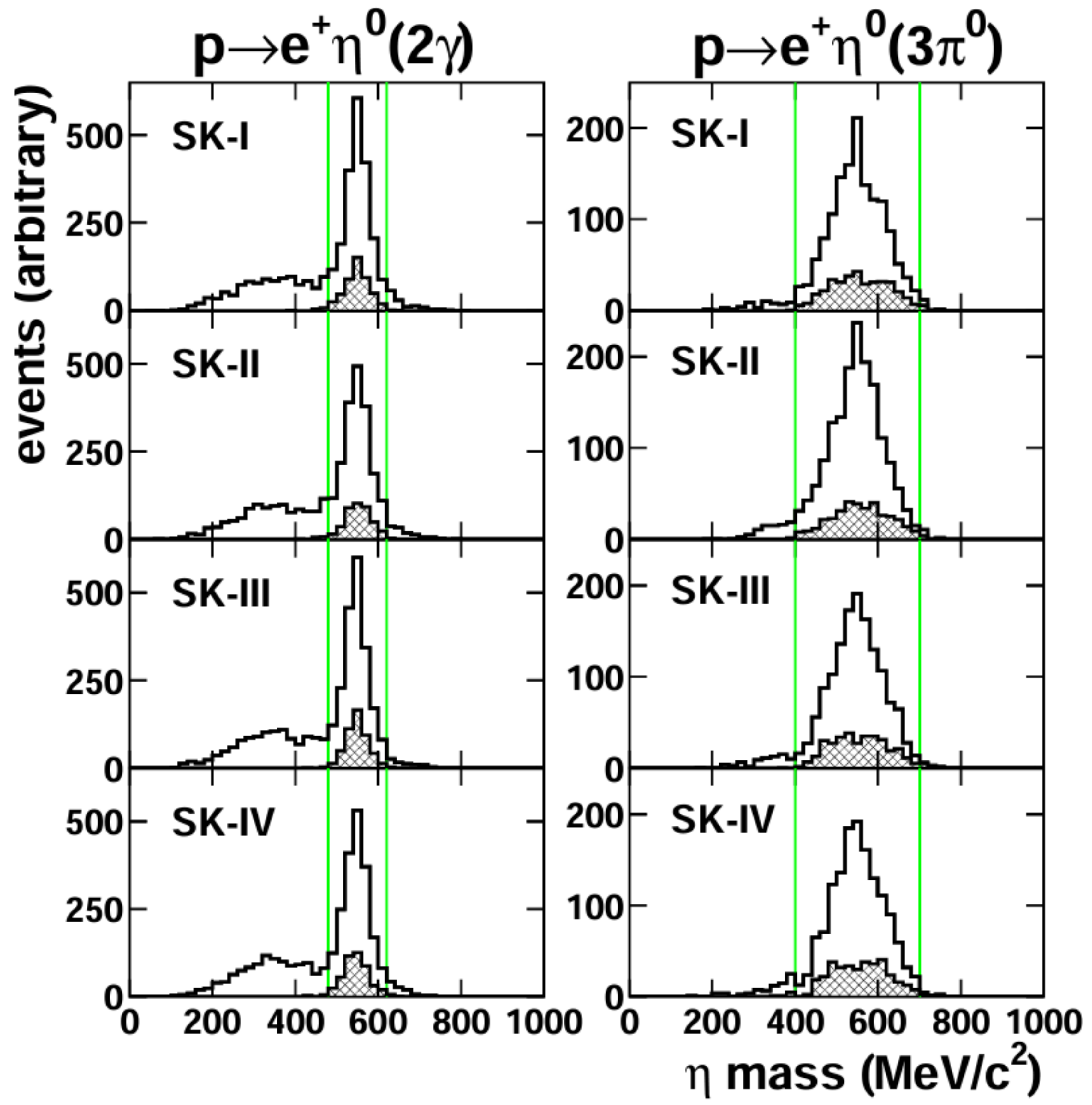
[11]: Hyper-Kamiokande Collaboration [arXiv: 1805.04163]
[25]: Super-Kamiokande Collaboration [arXiv: 2010.16098]
[26]: Super-Kamiokande Collaboration [arXiv: 1705.07221]
[27]: Super-Kamiokande Collaboration [arXiv: 1305.4391]
[28]: C. McGrew et al. (1999)

- Units: 10^{33} years
- 90% CL
- 1.9 Megaton-year exposure is assumed for the prospect.

Event Reconstruction

Super-Kamiokande Collaboration [arXiv: 1705.07221]

- Eta mass reconstruction at SK



- Left: $\eta \to 2\gamma$, branching ratio = 39%
- Right: $\eta \to 3\pi^0$, branching ratio = 33%
- Open histogram: Monte-Carlo events
- Hatched histogram:

  - left: true $\eta \to 2\gamma$
  - right: true $\eta \to 3\pi^0$

Effective interactions for nucleon decay

S. Weinberg (1979)
D. V. Nanopoulos and S. Weinberg (1979)

- Assume the tree-level exchange of a scalar or vector boson
  - Vector boson V_μ can induce $\mathcal{O}^{(1)}_{ijk\ell}$, $\mathcal{O}^{(2)}_{ijk\ell}$

    Renormalizable interaction: $V_\mu \psi_{R\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \chi_{L\alpha}$

    ψ_R include the conjugate of ψ_L
  - Scalar boson S can induce $\mathcal{O}^{(1)}_{ijk\ell}$, $\mathcal{O}^{(2)}_{ijk\ell}$, $\mathcal{O}^{(3)}_{ijk\ell}$, $\mathcal{O}^{(4)}_{ijk\ell}$.

    Renormalizable interactions: $S(\psi_L \chi_L)$, $S(\psi_R \chi_R)$

$$\mathcal{O}^{(1)}_{ijk\ell} \equiv \epsilon_{abc}\epsilon_{\alpha\beta}\left(u^a_{Ri} d^b_{Rj}\right)\left(Q^{c\alpha}_k L^\beta_\ell\right),$$
$$\mathcal{O}^{(2)}_{ijk\ell} \equiv \epsilon_{abc}\epsilon_{\alpha\beta}\left(Q^{a\alpha}_i Q^{b\beta}_j\right)\left(u^c_{Rk} e_{R\ell}\right),$$
$$\mathcal{O}^{(3)}_{ijk\ell} \equiv \epsilon_{abc}\epsilon_{\alpha\beta}\epsilon_{\gamma\delta}\left(Q^{a\alpha}_i Q^{b\gamma}_j\right)\left(Q^{c\delta}_k L^\beta_\ell\right),$$
$$\mathcal{O}^{(4)}_{ijk\ell} \equiv \epsilon_{abc}\left(u^a_{Ri} d^b_{Rj}\right)\left(u^c_{Rk} e_{R\ell}\right),$$

- In non-SUSY GUTs, the gauge boson exchange typically dominates: mixed-type
- In SUSY-GUT, the one-loop contribution can be significant: both can contribute

The direct method

Y. Aoki et al. (2017)
J. S. Yoo, et al. (2022)

- We use hadron matrix elements evaluated by lattice simulations.

I	0		1	
$\chi\chi'$	LR	LL	LR	LL
$W_{p\pi^+\nu}$, $\sqrt{2}W_{p\pi^0\ell}$, $\sqrt{2}W_{n\pi^0\nu}$, $W_{n\pi^-\ell}$	-0.159	0.151	0.169	-0.134
$W_{p\eta\ell}$, $W_{n\eta\nu}$	0.006	0.113	0.044	-0.044

- $W^{RR} = W^{LL}$, $W^{RL} = W^{LR}$
- In units of GeV^2

$$\langle \Pi(\boldsymbol{p}) | \mathcal{O}_{\chi\chi'} | N(\boldsymbol{k}) \rangle = P_{\chi'} \left[W_0^{\mathcal{O}}(q^2) + \frac{\not{q}}{m_N} W_1^{\mathcal{O}}(q^2) \right] u_N(\boldsymbol{k})$$

$$\mathcal{O}_{\chi\chi'} = \epsilon_{abc}(q_\chi^a q_\chi'^b) q_{\chi'}''^c$$

- $\mathcal{O}(m_\ell/m_p)$ → only relevant for anti-muon

Positron channels: $\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$

- To understand the behavior, let us use the expressions calculated from chiral lagrangian (i.e., the so-called indirect method M. Claudson, M. B. Wise, and L. J. Hall (1982))

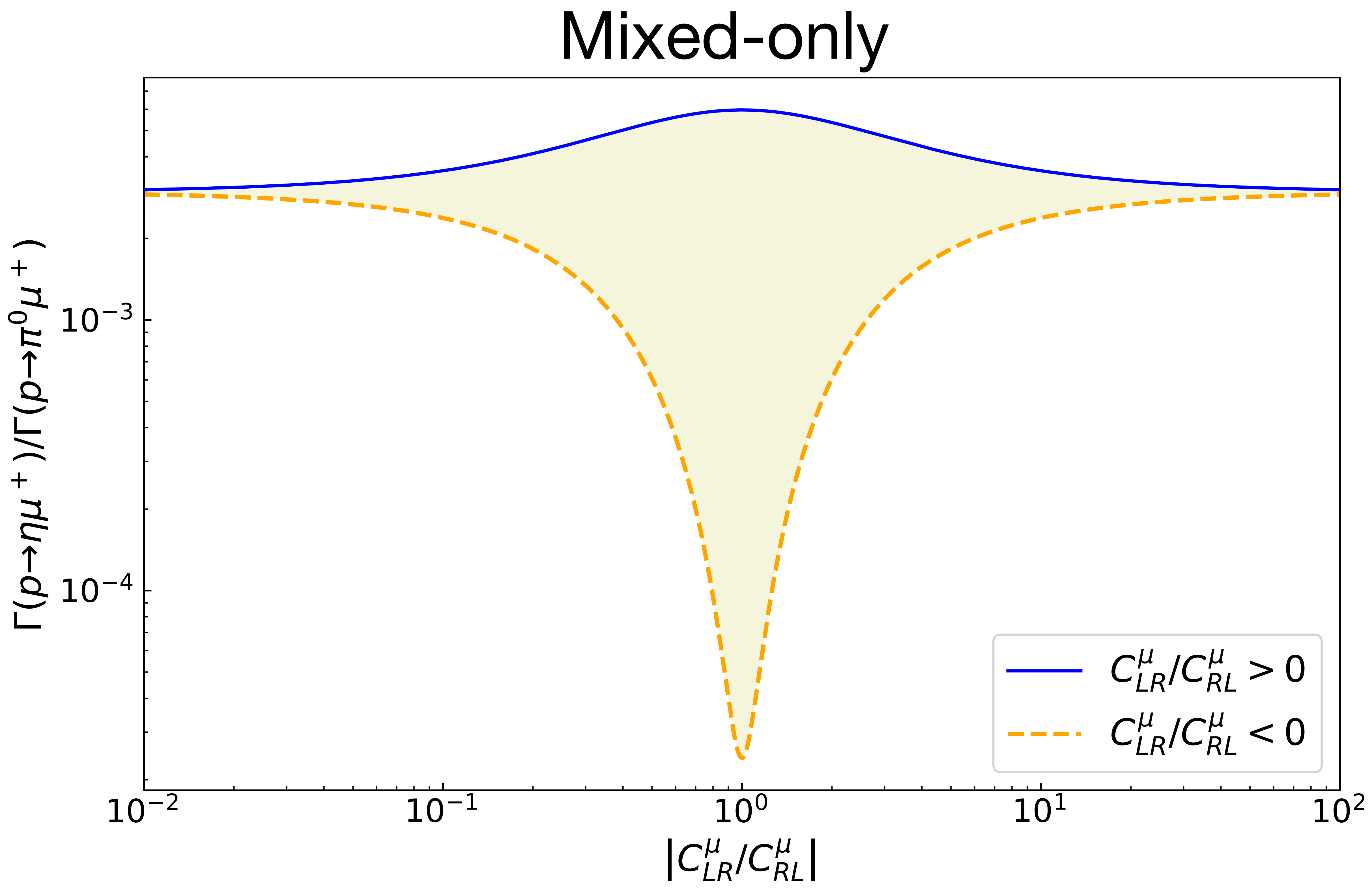
$$\Gamma(p \to \pi^0 e^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 \frac{(1+D+F)^2\alpha^2}{2f^2}\left[|C^e_{RL} - C^e_{LL}|^2 + |C^e_{LR} - C^e_{RR}|^2\right]$$

$$\Gamma(p \to \eta e^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_\eta^2}{m_p^2}\right)^2 \frac{\alpha^2}{6f^2}\Big[|(1+D-3F)C^e_{RL} + (3-D+3F)C^e_{LL}|^2 + |(1+D-3F)C^e_{LR} + (3-D+3F)C^e_{RR}|^2\Big] ,$$

- $p \to \pi^0 e^+$ is suppressed when $C^e_{LL}/C^e_{RL} = C^e_{RR}/C^e_{LR} = 1$
- $p \to \eta e^+$ is suppressed when $C^e_{LL}/C^e_{RL} = C^e_{RR}/C^e_{LR} = -(1+D-3F)/(3-D+3F) \simeq -0.1$

Chirality structure and branching fractions

- **Case 3:** mixed- or pure-only case, but with nonzero m_μ.



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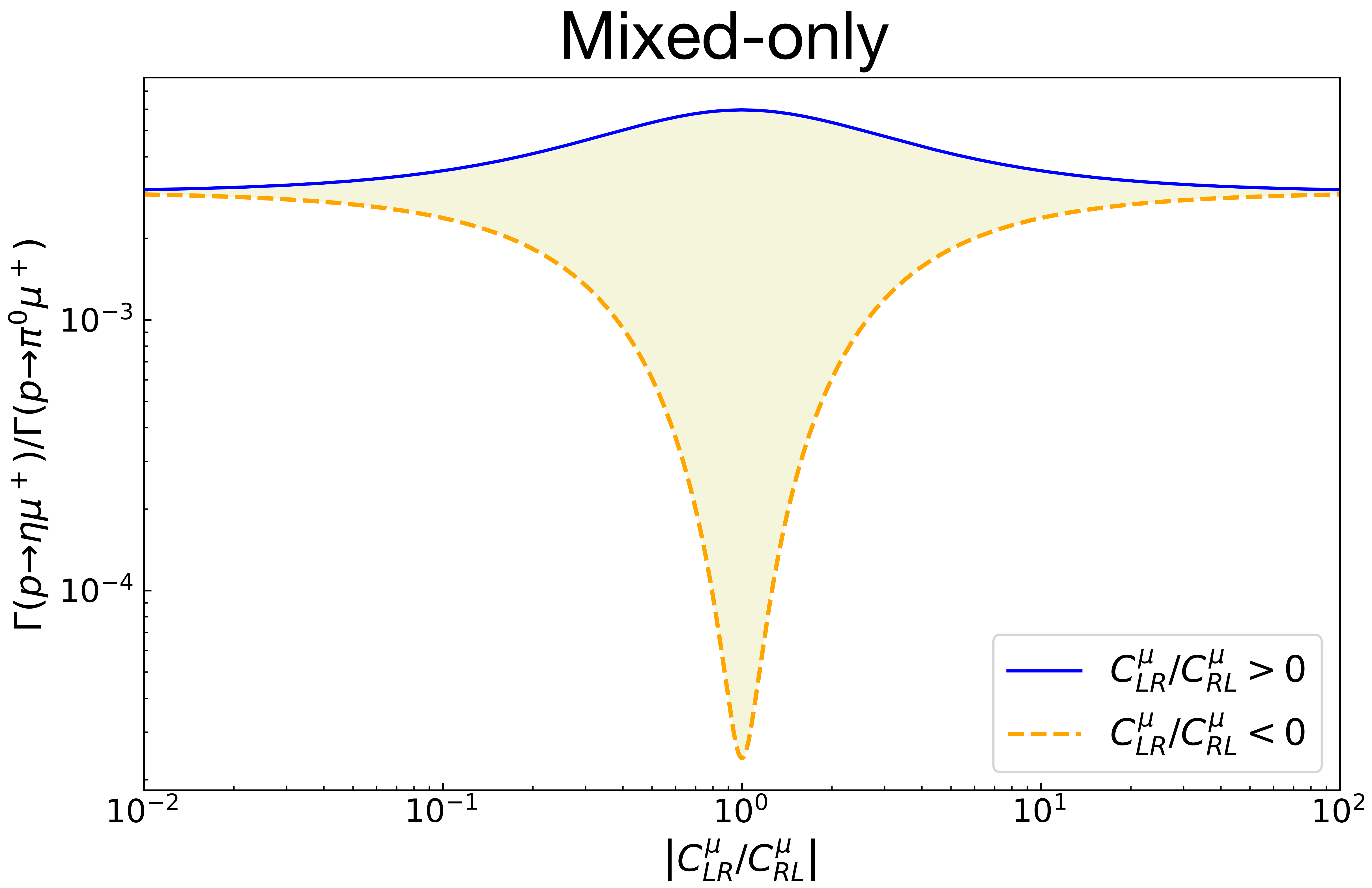
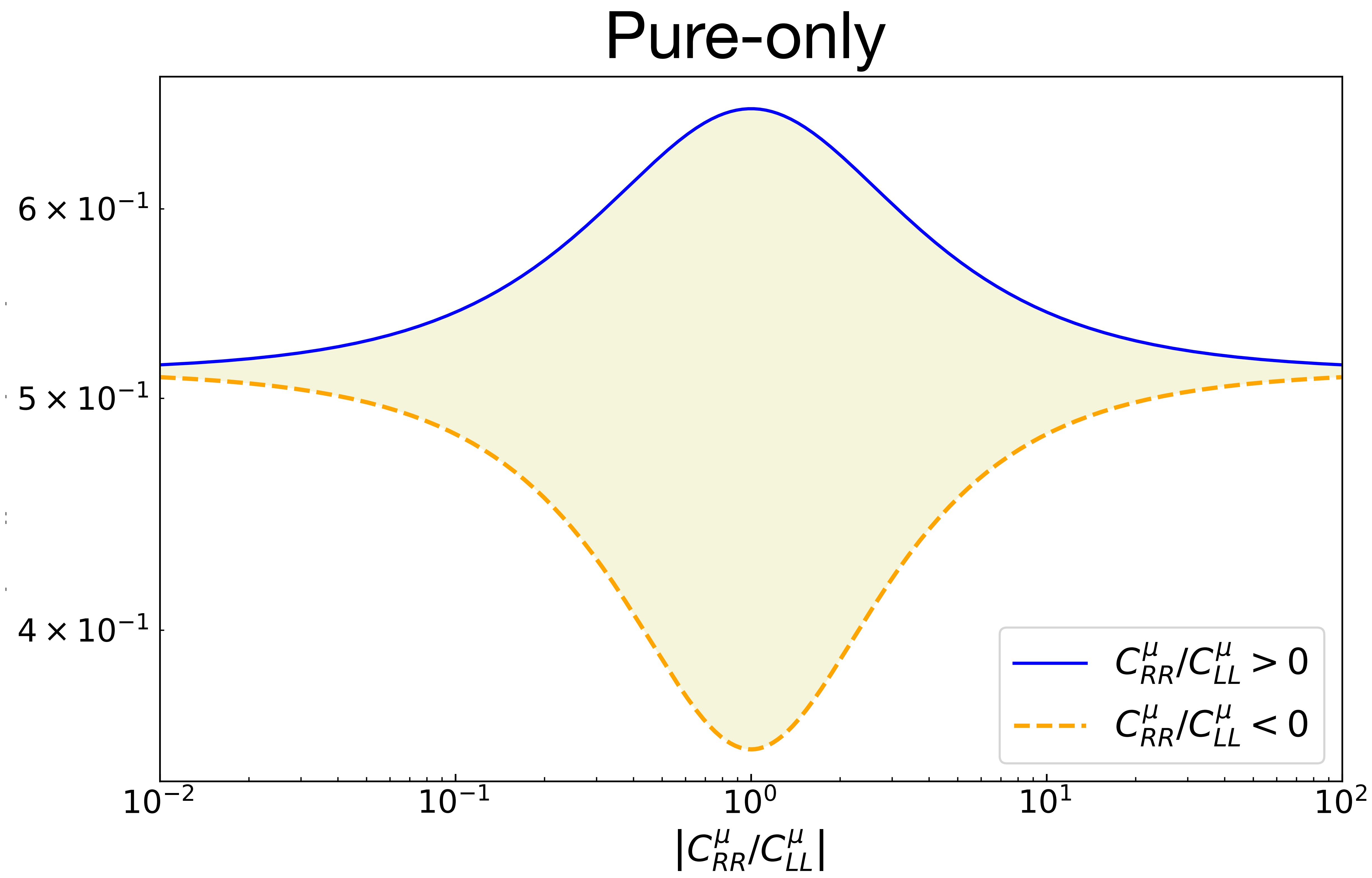




$\Gamma(p \to \eta\mu^+)/\Gamma(p \to \pi^0\mu^+)$ **can distinguish between different mixed/pure operators.**

Anti-muon channels: $\Gamma(p \to \eta\mu^+)/\Gamma(p \to \pi^0\mu^+)$

- For the mixed-only case, up to $\mathcal{O}(m_\mu/m_p)$,

$$\Gamma(p \to \Pi\mu^+) = \frac{m_p}{32\pi}\left(1-\frac{m_\Pi^2}{m_p^2}\right)^2 \left(W^{LR}_{p\Pi\ell,0}\right)^2 \left[|C^\mu_{RL}|^2 + |C^\mu_{LR}|^2\right] ,$$
$$\times \left[1 + \underline{\frac{4m_\mu}{m_p}\left\{\frac{W^{LR}_{p\Pi\ell,1}}{W^{LR}_{p\Pi\ell,0}} + \left(1-\frac{m_\Pi^2}{m_p^2}\right)^{-1}\right\}\frac{\mathrm{Re}(C^\mu_{RL}C^{\mu*}_{LR})}{|C^\mu_{RL}|^2 + |C^\mu_{LR}|^2}}\right]$$

  - The source of the dependence on $|C^e_{LR}/C^e_{RL}|$
  - This effect is enhanced when $|W^{LR}_{p\Pi\mu,1}| \gg |W^{LR}_{p\Pi\mu,0}|$

I	0		1	
$\chi\chi'$	LR	LL	LR	LL
$W_{p\pi^+\nu}$, $\sqrt{2}W_{p\pi^0\ell}$, $\sqrt{2}W_{n\pi^0\nu}$, $W_{n\pi^-\ell}$	−0.159	0.151	0.169	−0.134
$W_{p\eta\ell}$, $W_{n\eta\nu}$	0.006	0.113	0.044	−0.044

Form factors obtained by lattice QCD in units of GeV^2

Y. Aoki et al. (2017)
J. S. Yoo, et al. (2022)

Other channels

- The ratio of neutrino channels is related to that of charged leptons in either mixed- or pure-only cases.

$$\Gamma(N \to \Pi\bar{\nu}) = \frac{m_N}{32\pi}\left(1 - \frac{m_\Pi^2}{m_N^2}\right)^2 \left|W_{N\Pi\nu,0}^{RL}\right|^2 \sum_i |C_{RL}^{\nu_i}|^2$$

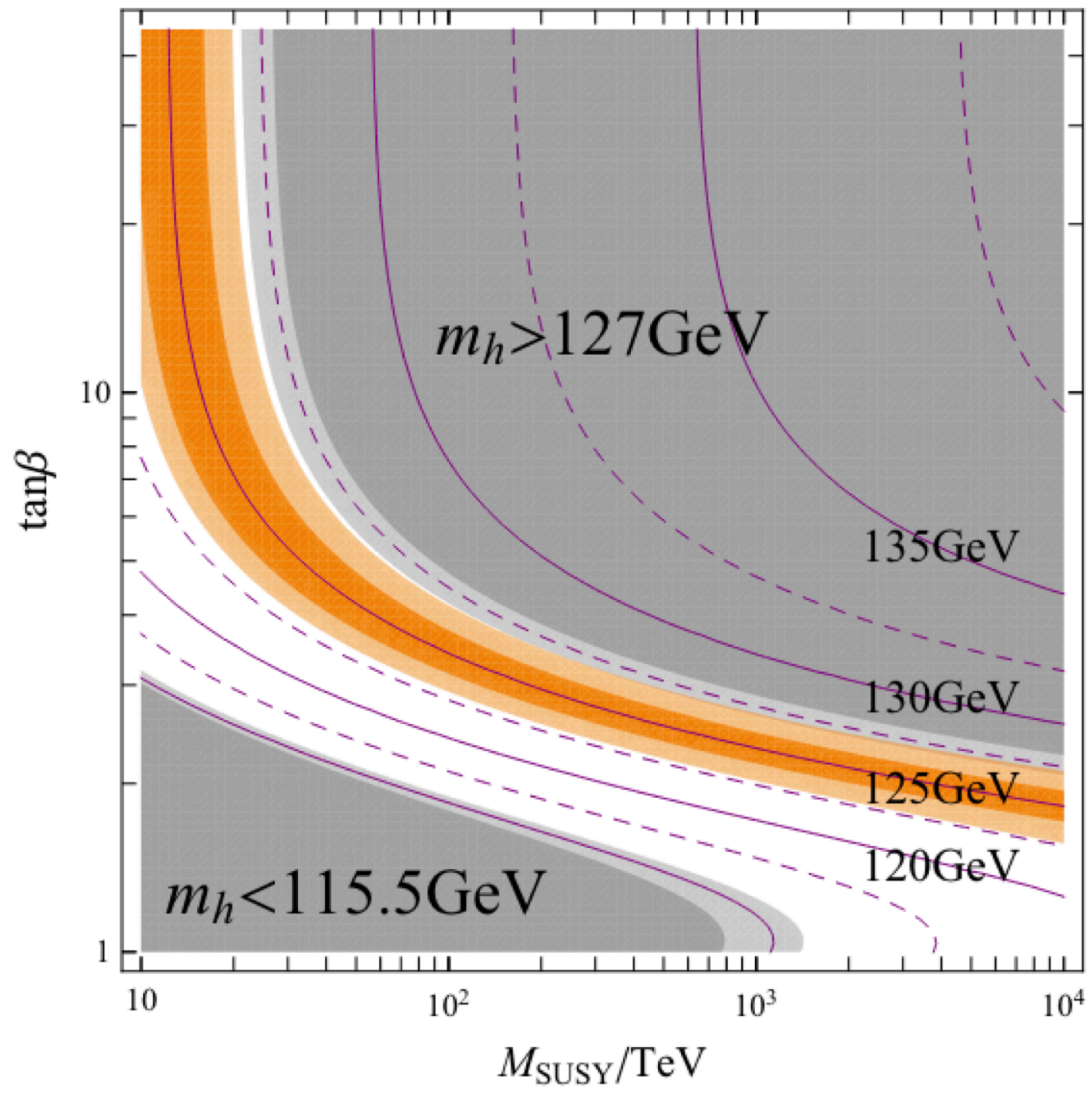
$$\frac{\Gamma(n \to \eta\bar{\nu})}{\Gamma(n \to \pi^0\bar{\nu})} = \frac{(1 - m_\eta^2/m_n^2)^2}{(1 - m_\pi^2/m_n^2)^2} \cdot \frac{|W_{n\eta\nu}^{\chi\chi'}|^2}{|W_{n\pi^0\nu}^{\chi\chi'}|^2} \simeq \frac{\Gamma(p \to \eta e^+)}{\Gamma(p \to \pi^0 e^+)}$$

- Due to the presence of more Wilson coefficients (=six for the neutrino ratio), it is difficult to extract information about the Wilson coefficients on generic grounds. The same applies to $\Gamma(n \to \pi^0\bar{\nu})/\Gamma(p \to \pi^0\ell^+)$.
- Those ratios can still become powerful probes when considering specific UV models.

High-scale SUSY

M. Ibe, S. Matsumoto, T. T. Yanagida (2012)

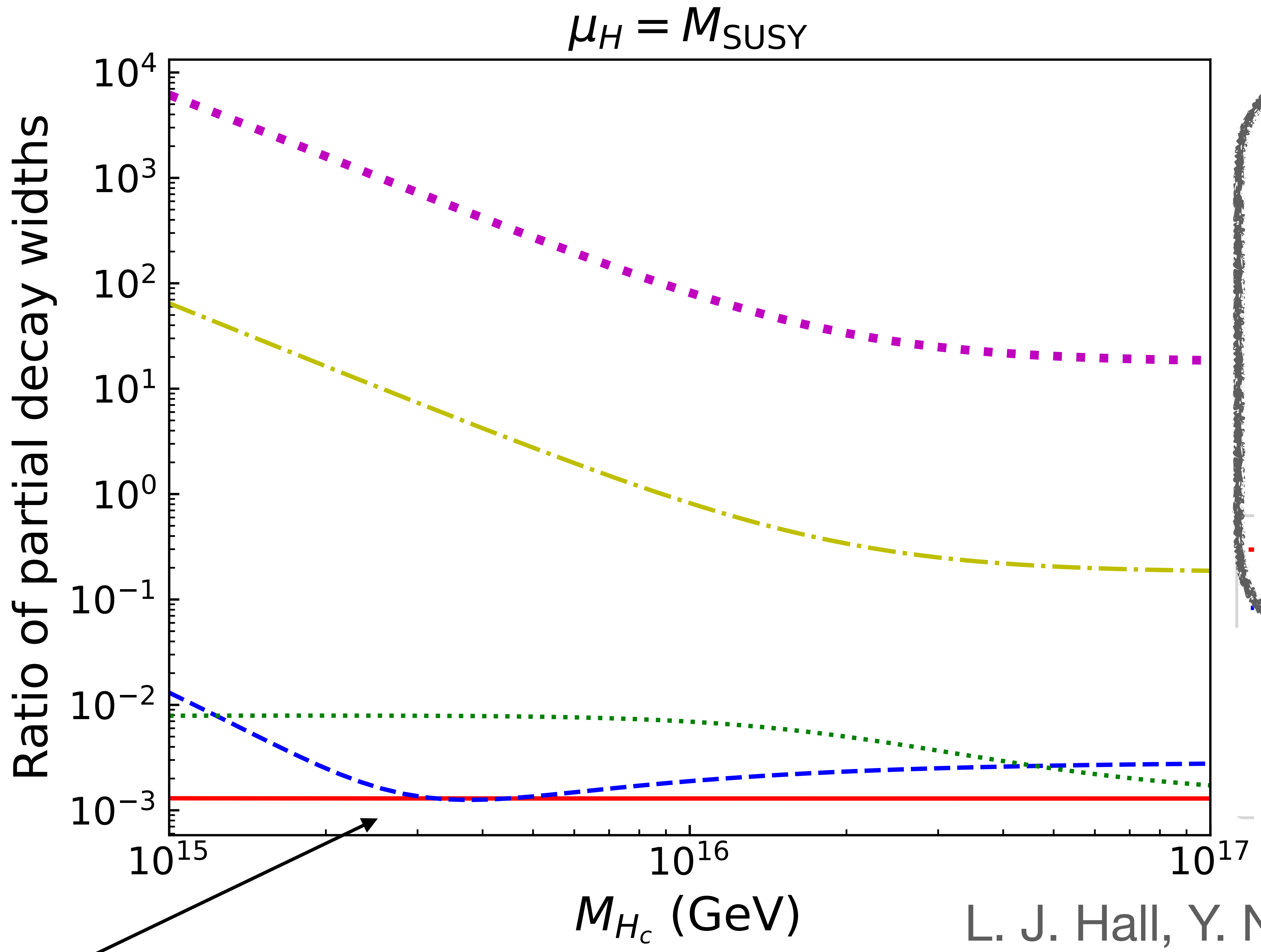
- Relation between SUSY-breaking scale and $\tan\beta$



For $M_{\mathrm{SUSY}} = 10^2$ TeV,

$\tan\beta \sim 3$ is needed to reproduce the correct Higgs mass.

The minimal SU(5) with high-scale SUSY

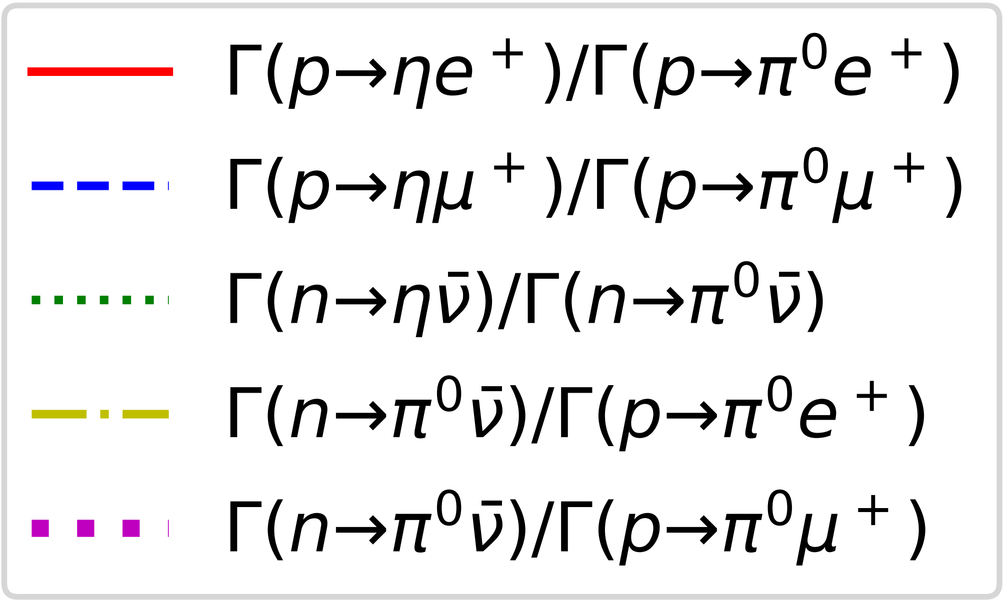


- $M_X = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$

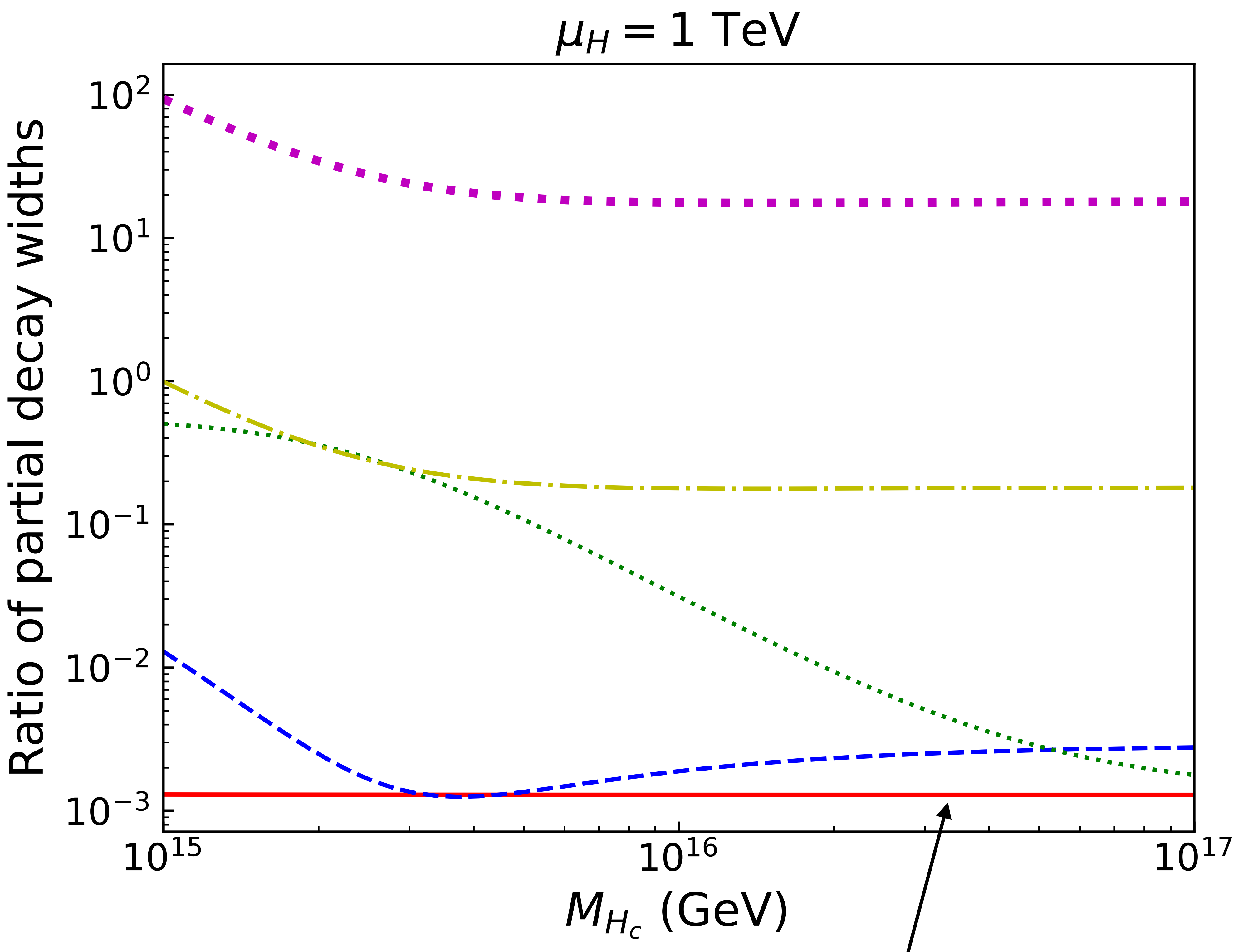
Mini-split type mass spectrum

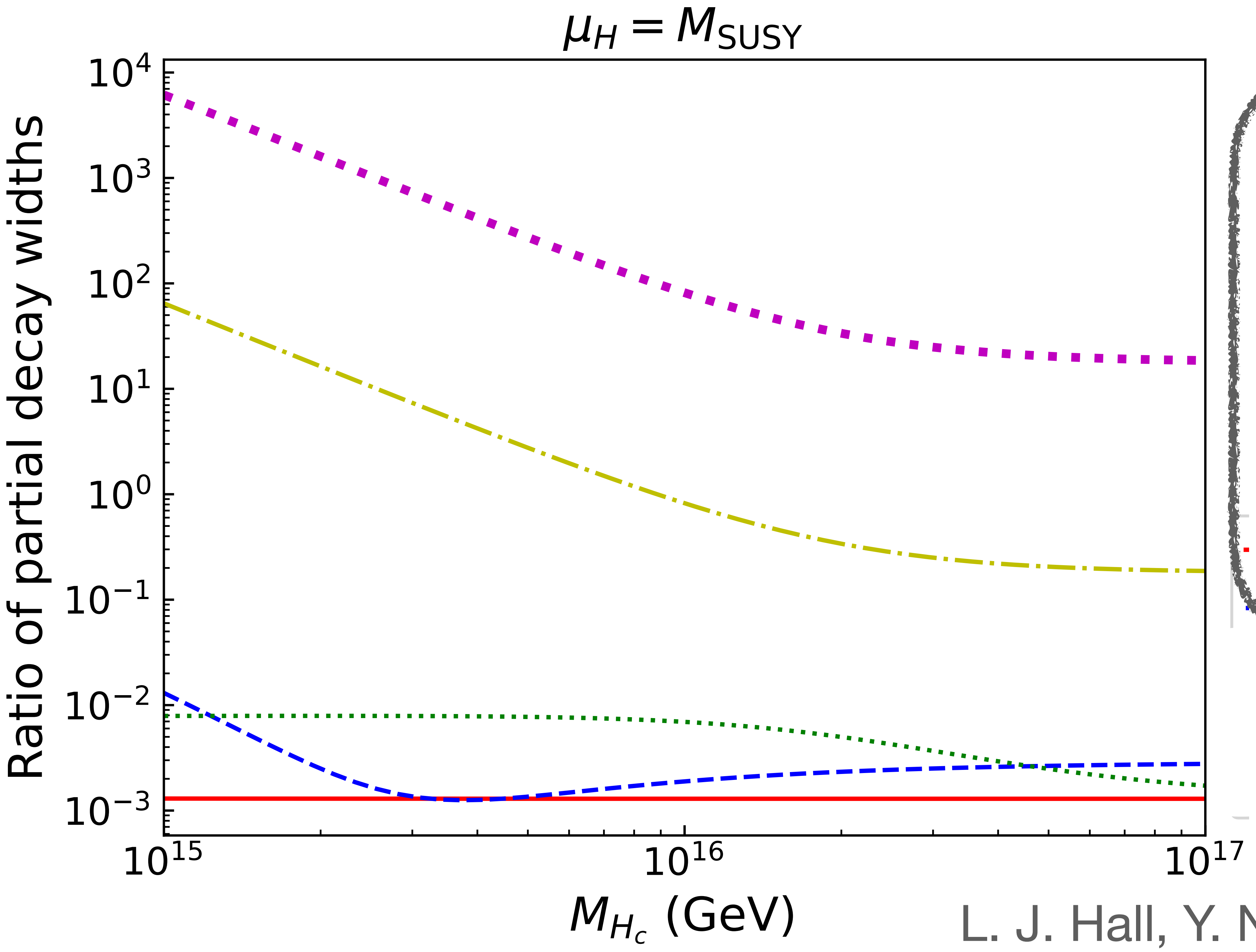
L. J. Hall, Y. Nomura and S. Shirai (2012)
M. Ibe, S. Matsumoto and T. T. Yanagida (2012)

$\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$: gauge-boson/higgsino exchange (mixed-type) dominates.



The minimal SU(5) with high-scale SUSY





- $M_X = 10^{16}$ GeV
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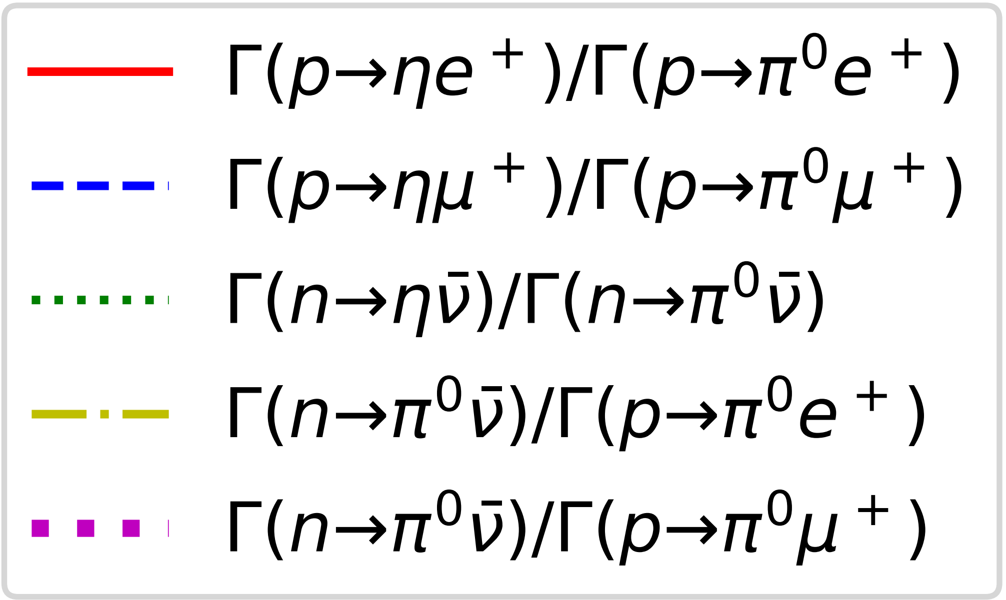
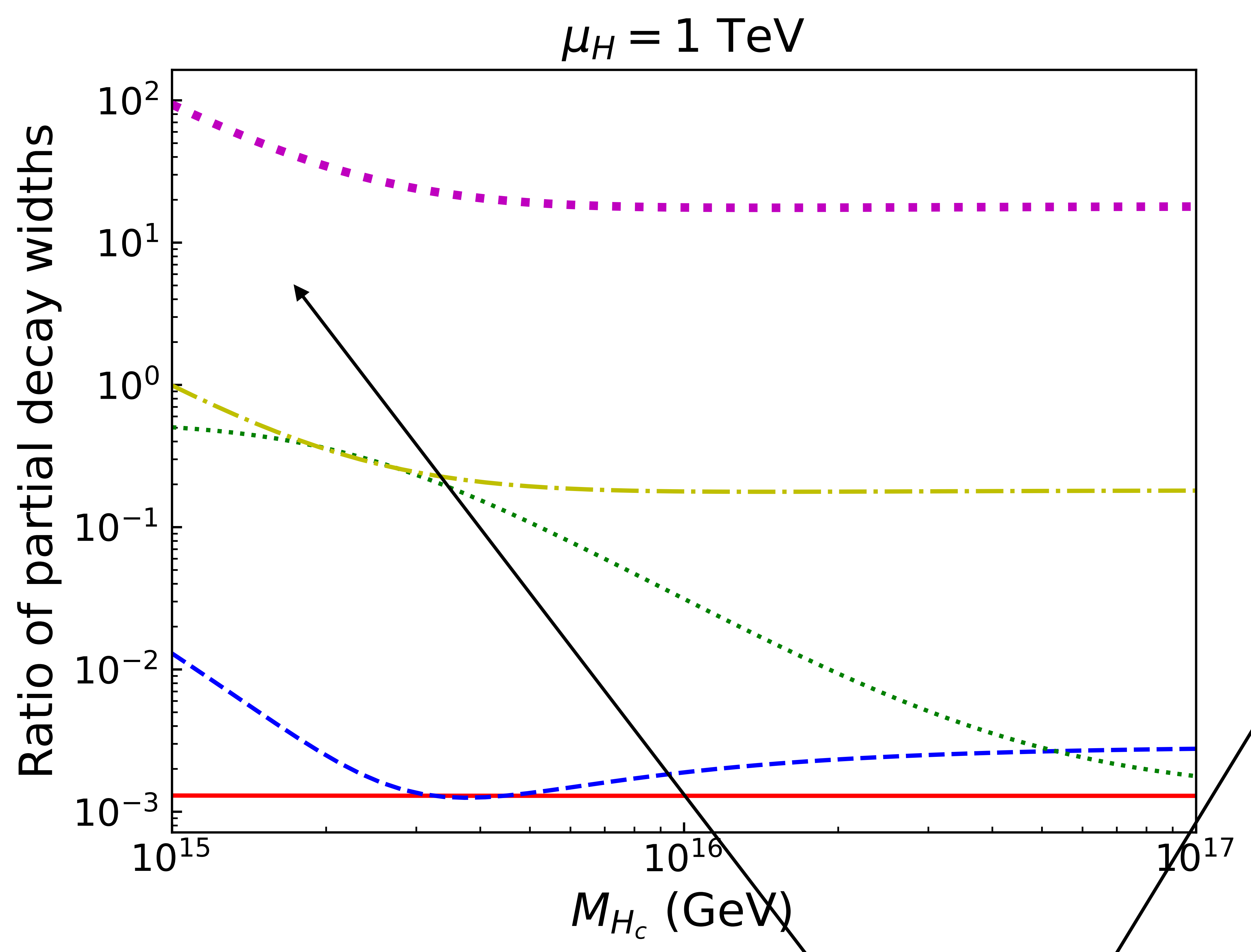
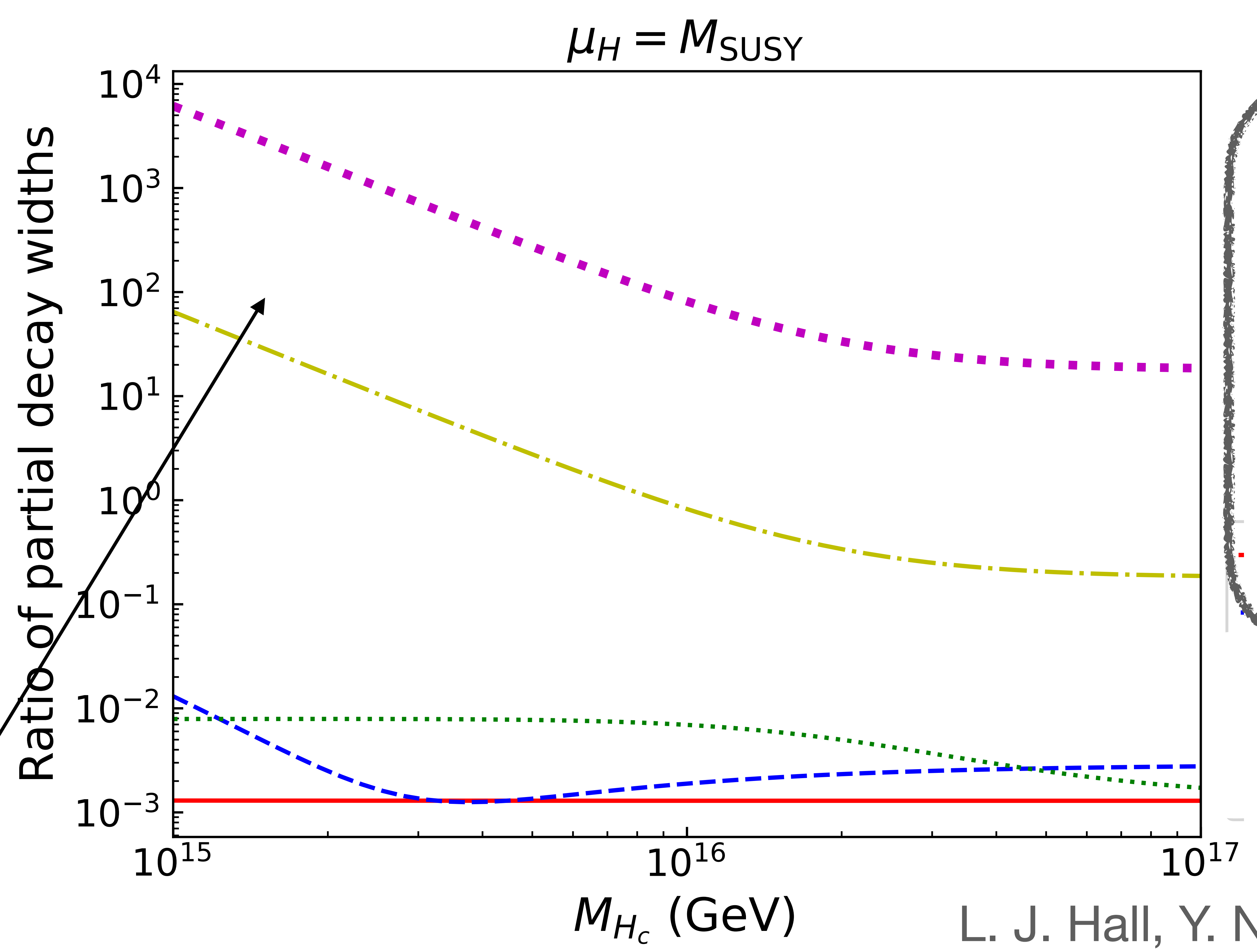


$\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$: gauge-boson exchange (mixed-type) dominates.

The minimal SU(5) with high-scale SUSY





- $M_X = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$

Mini-split type mass spectrum

L. J. Hall, Y. Nomura and S. Shirai (2012)
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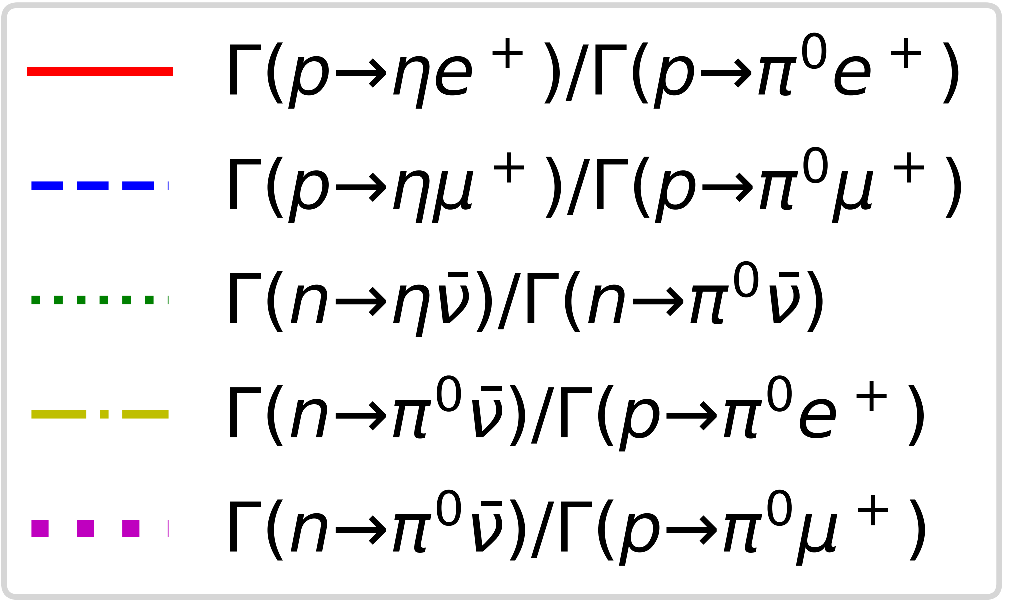
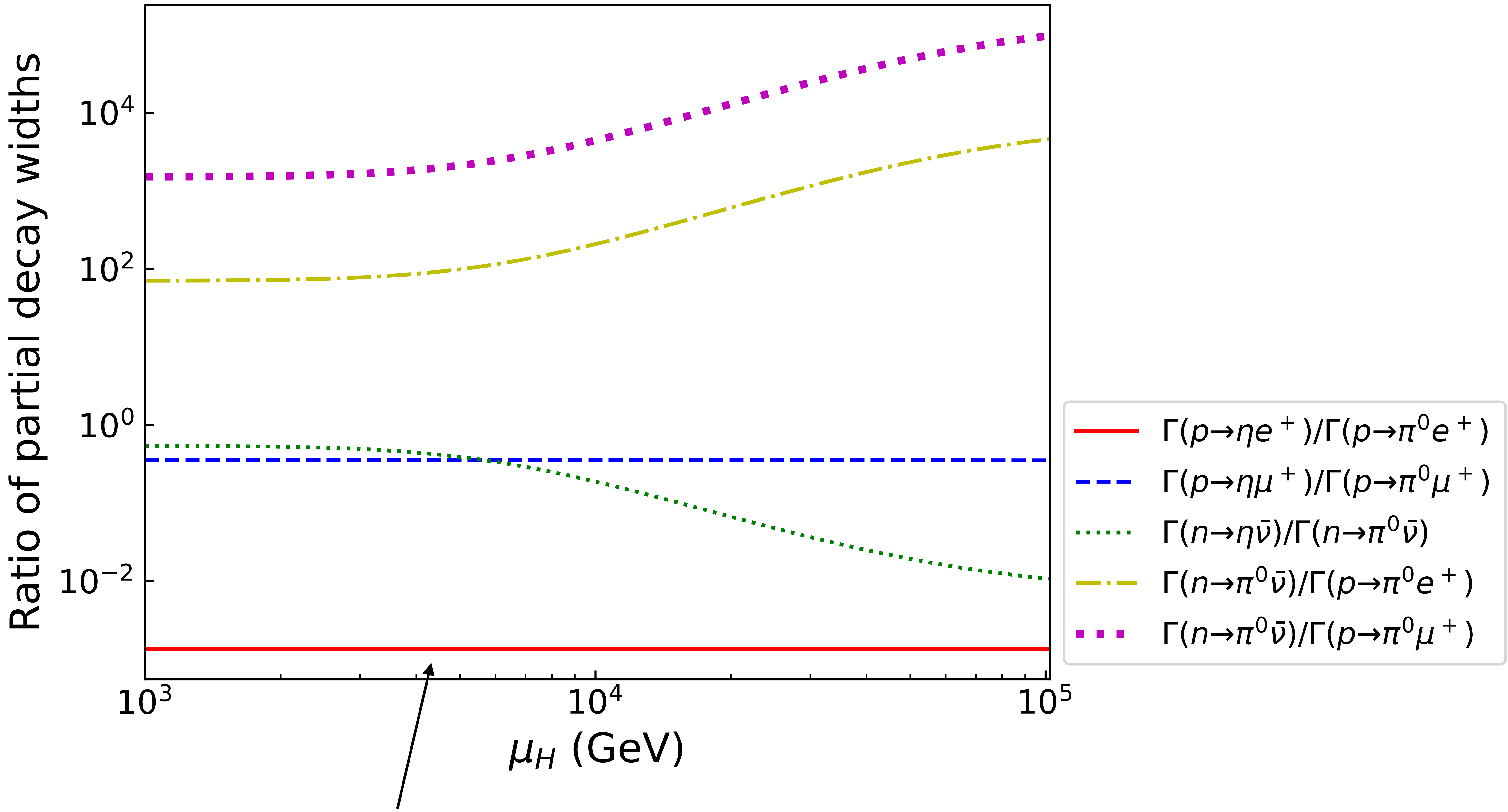


- $\Gamma(p \to \eta\mu^+)/\Gamma(p \to \pi^0\mu^+)$: wino-exchange contributes (pure-type) as M_{H_C} decreases.
- $\Gamma(n \to \pi^0\bar{\nu})/\Gamma(p \to \pi^0 e^+)$ and $\Gamma(n \to \pi^0\bar{\nu})/\Gamma(p \to \pi^0\mu^+)$ show the same tendency.

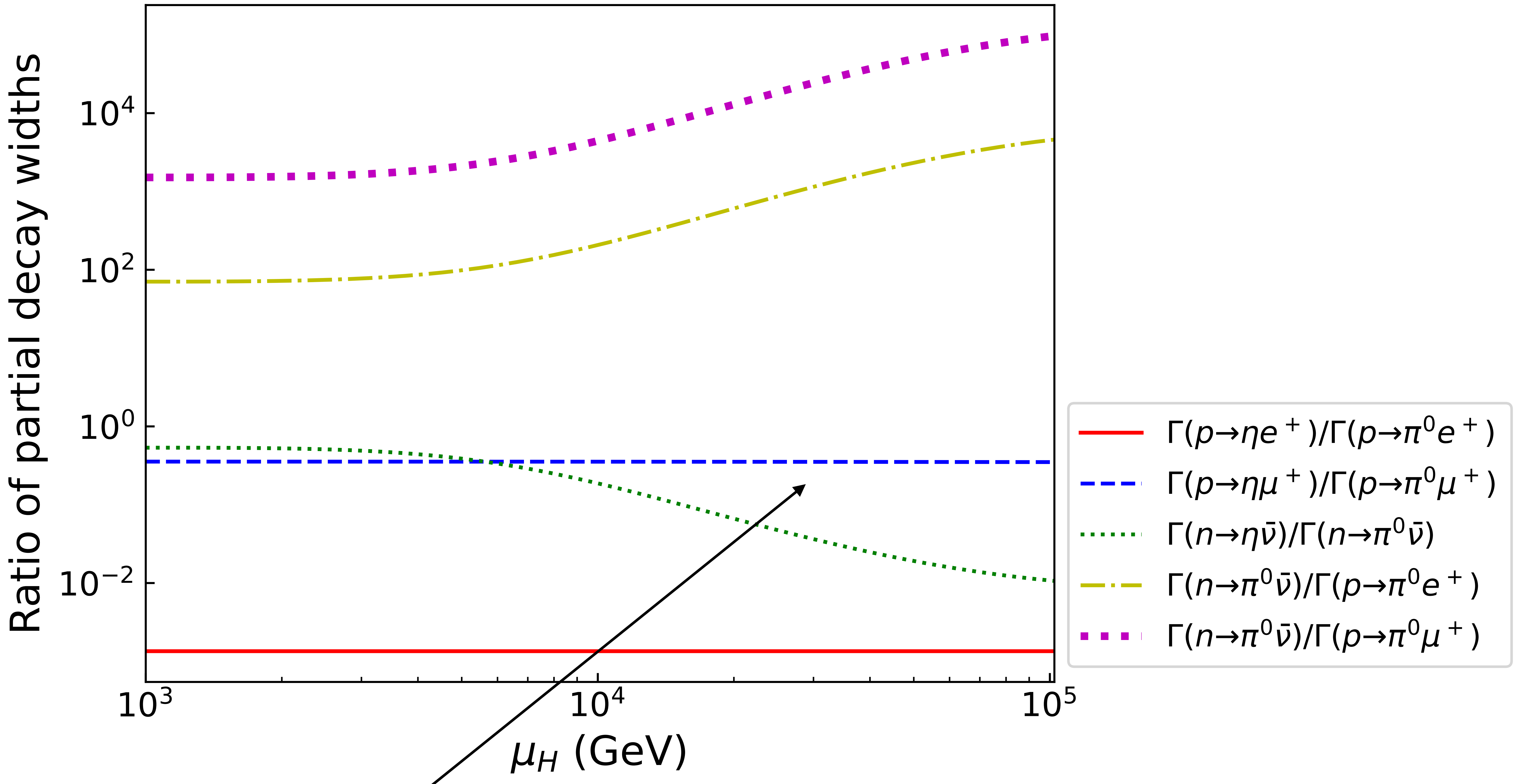
The minimal SU(5) with high-scale SUSY



- $M_X = 10^{17}$ GeV
- $M_{H_C} = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$

$\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$: gauge-boson exchange (mixed-type) dominates even when the GUT gauge boson is heavy.

The minimal SU(5) with high-scale SUSY

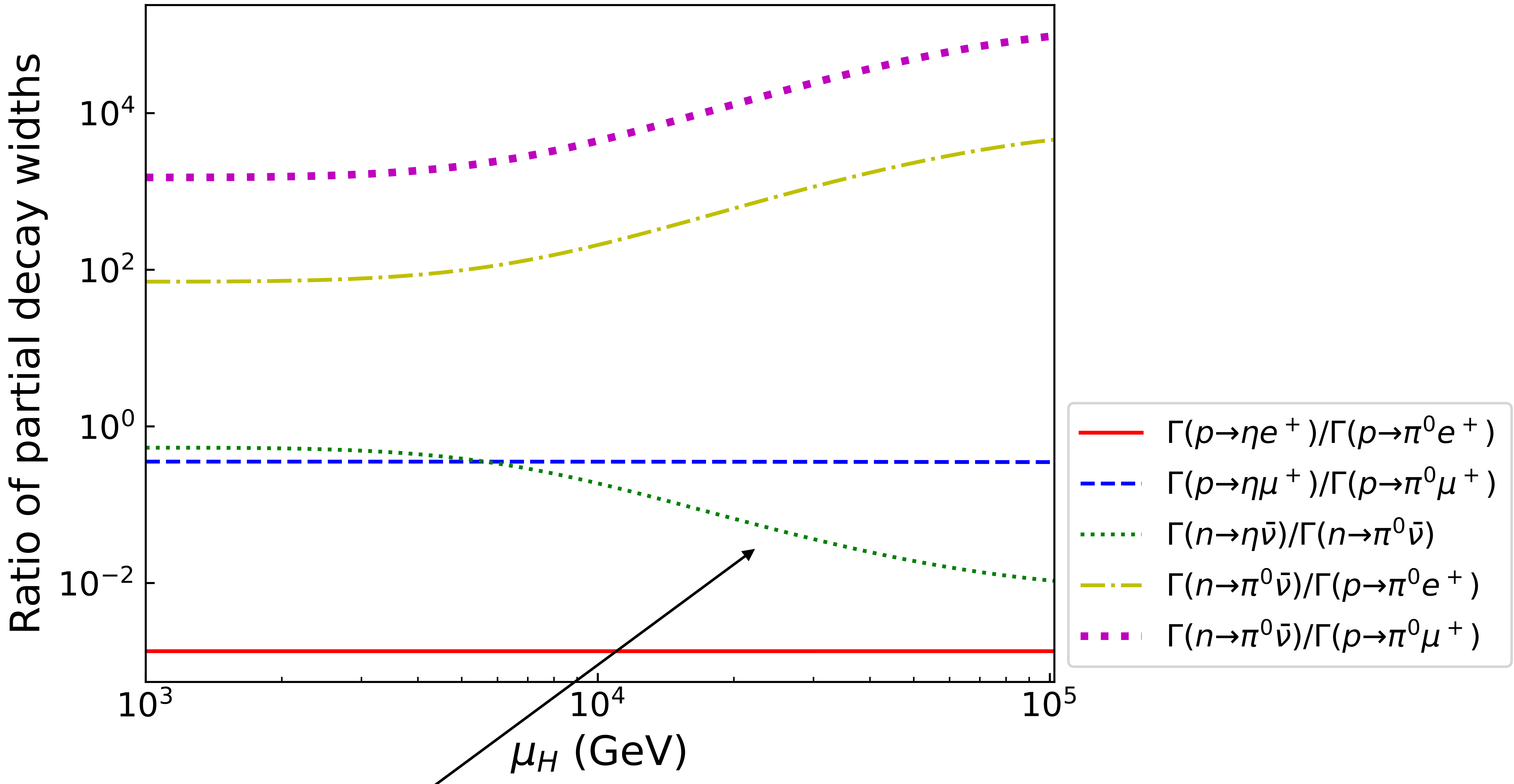


- $M_X = 10^{17}$ GeV
- $M_{H_C} = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\text{SUSY}} = 100$ TeV
- $\tan\beta = 3$

$\Gamma(p \to \eta\mu^+)/\Gamma(p \to \pi^0\mu^+)$: wino contribution (mixed-type) dominates regardless of the higgsino mass.

→ this ratio is sensitive to dim-5 wino v.s. the GUT gauge boson competition.

The minimal SU(5) with high-scale SUSY

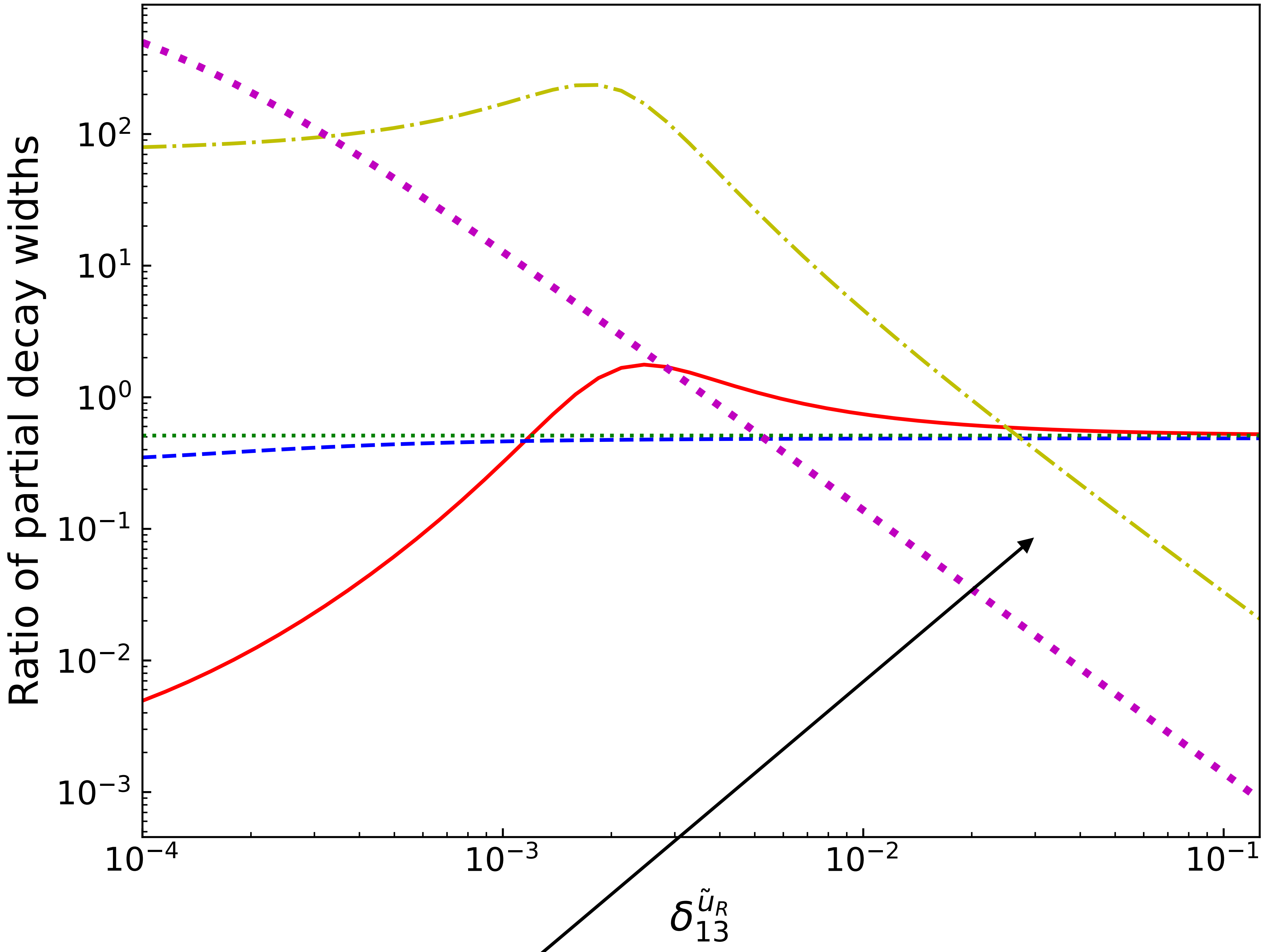


- $M_X = 10^{17}$ GeV
- $M_{H_C} = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$

$\Gamma(n \to \eta\bar{\nu})/\Gamma(n \to \pi^0\bar{\nu})$: higgsino contribution (mixed-type) becomes larger as its mass increases.

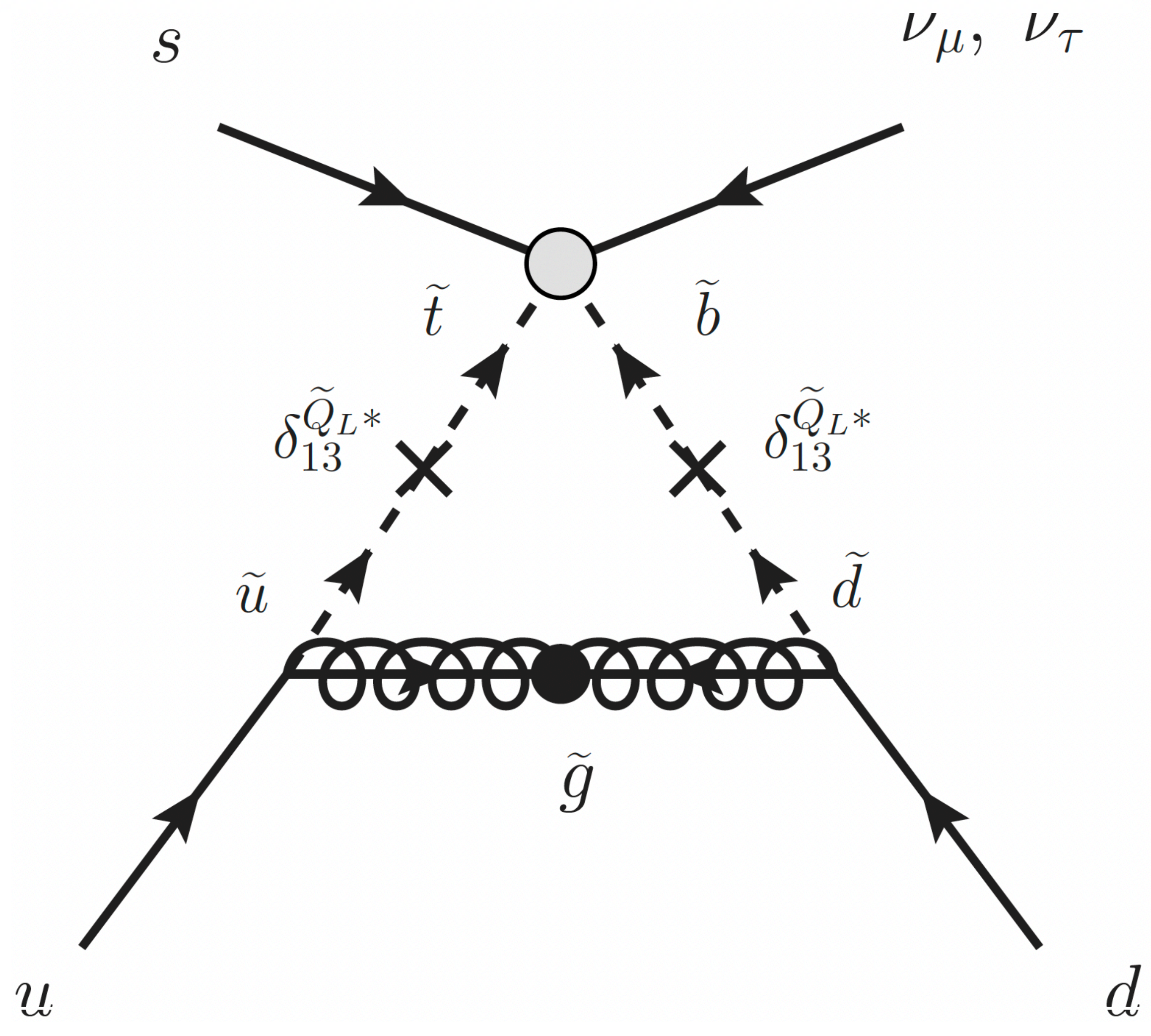
→ **this ratio is most useful to discriminate the higgsino contribution from that of wino.**

Sfermion flavor violation and ratios



- $M_X = 10^{17}$ GeV
- $M_{H_C} = 10^{16}$ GeV
- $M_1 = 5$ TeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\mathrm{SUSY}} = 100$ TeV
- $\tan\beta = 3$
- $\mu_H = 200$ GeV

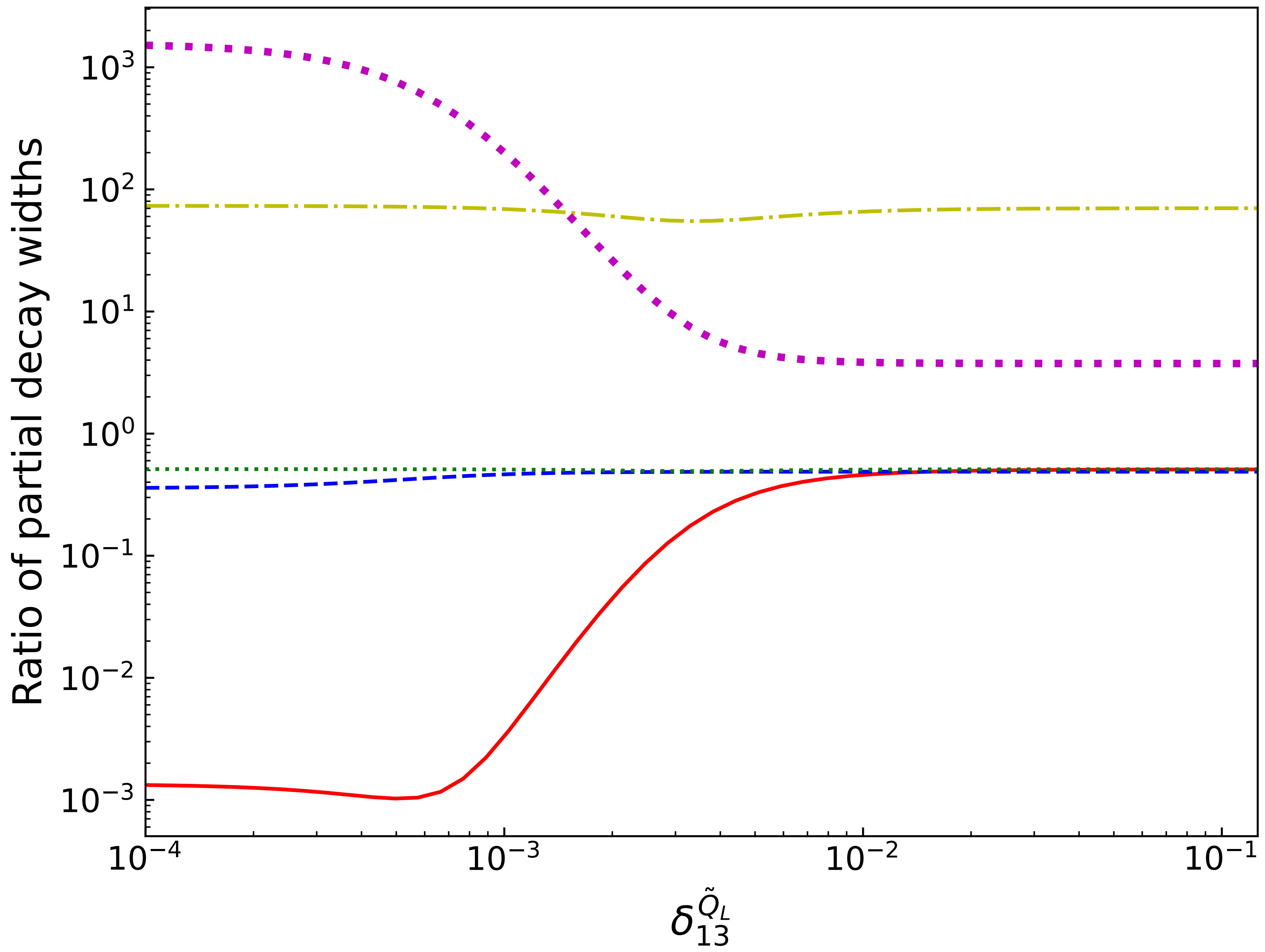
$$\widetilde{m}^2_{\tilde{f}} = M^2_{\mathrm{SUSY}} \begin{pmatrix} 1 & \delta_{12}^{\tilde{f}} & \delta_{13}^{\tilde{f}} \\ \delta_{12}^{\tilde{f}*} & 1 & \delta_{23}^{\tilde{f}} \\ \delta_{13}^{\tilde{f}*} & \delta_{23}^{\tilde{f}*} & 1 \end{pmatrix}$$



- The gluino exchange dominates as $\delta_{13}^{\tilde{u}_R}$ increases, even in $\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$.
- $\Gamma(n \to \pi^0 \bar{\nu})/\Gamma(p \to \pi^0 \ell^+)$ is a useful probe of this scenario.

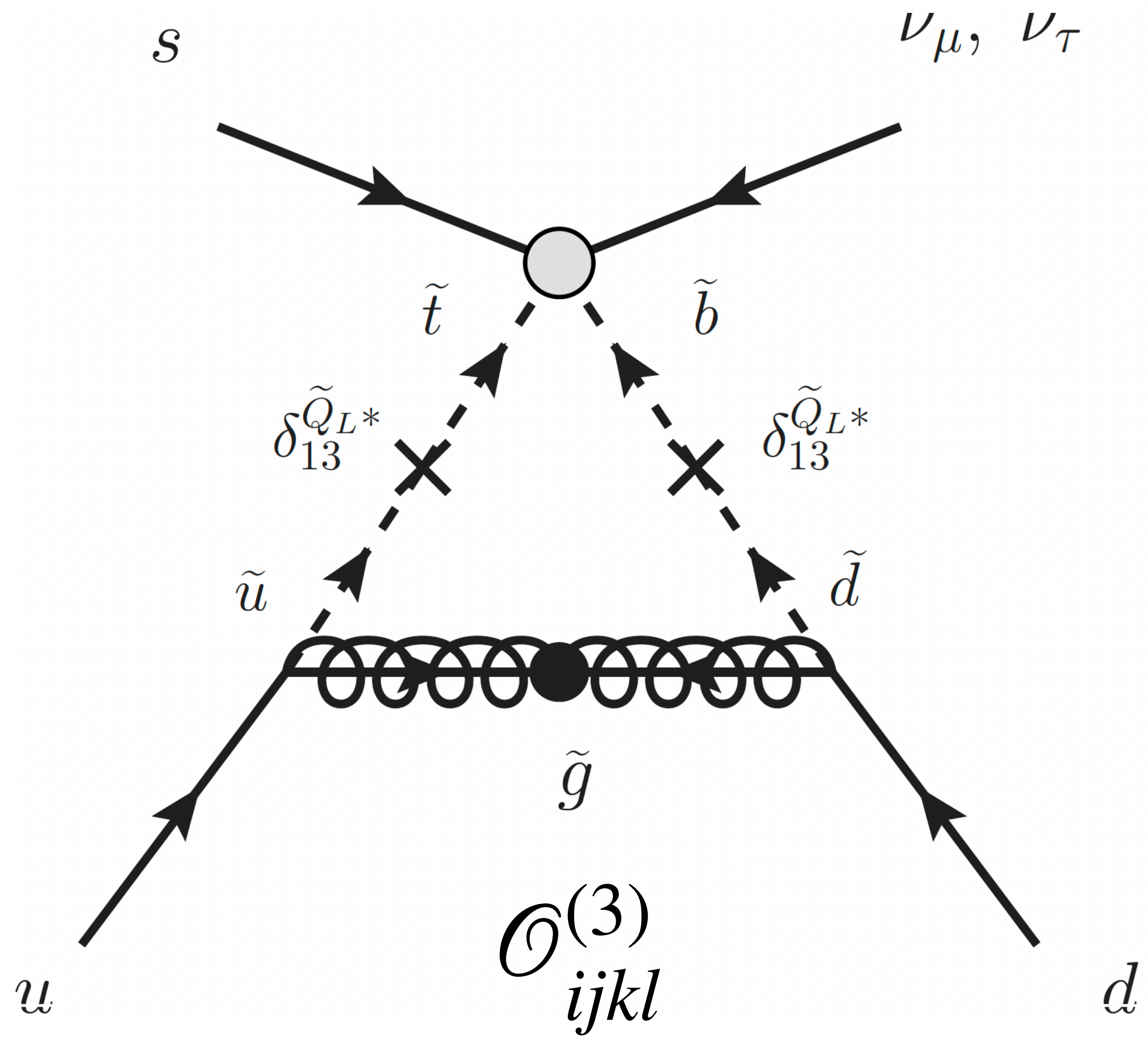
$\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$
$\Gamma(p \to \eta \mu^+)/\Gamma(p \to \pi^0 \mu^+)$
$\Gamma(n \to \eta \bar{\nu})/\Gamma(n \to \pi^0 \bar{\nu})$
$\Gamma(n \to \pi^0 \bar{\nu})/\Gamma(p \to \pi^0 e^+)$
$\Gamma(n \to \pi^0 \bar{\nu})/\Gamma(p \to \pi^0 \mu^+)$

Sfermion flavor violation and ratios



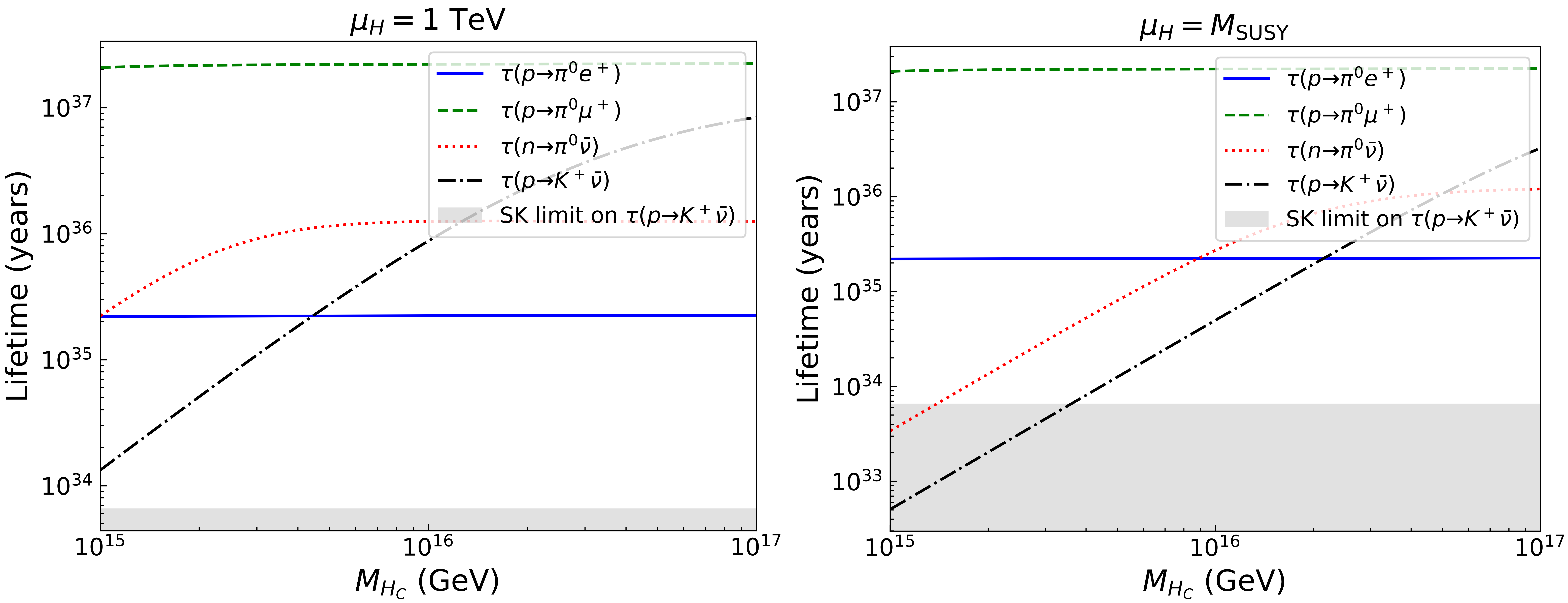
- $M_X = 10^{17}$ GeV
- $M_{H_C} = 10^{16}$ GeV
- $M_1 = 5$ TeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\text{SUSY}} = 100$ TeV
- $\tan\beta = 3$
- $\mu_H = 200$ GeV

$$\widetilde{m}^2_{\tilde{f}} = M^2_{\text{SUSY}} \begin{pmatrix} 1 & \delta^{\tilde{f}}_{12} & \delta^{\tilde{f}}_{13} \\ \delta^{\tilde{f}*}_{12} & 1 & \delta^{\tilde{f}}_{23} \\ \delta^{\tilde{f}*}_{13} & \delta^{\tilde{f}*}_{23} & 1 \end{pmatrix}$$



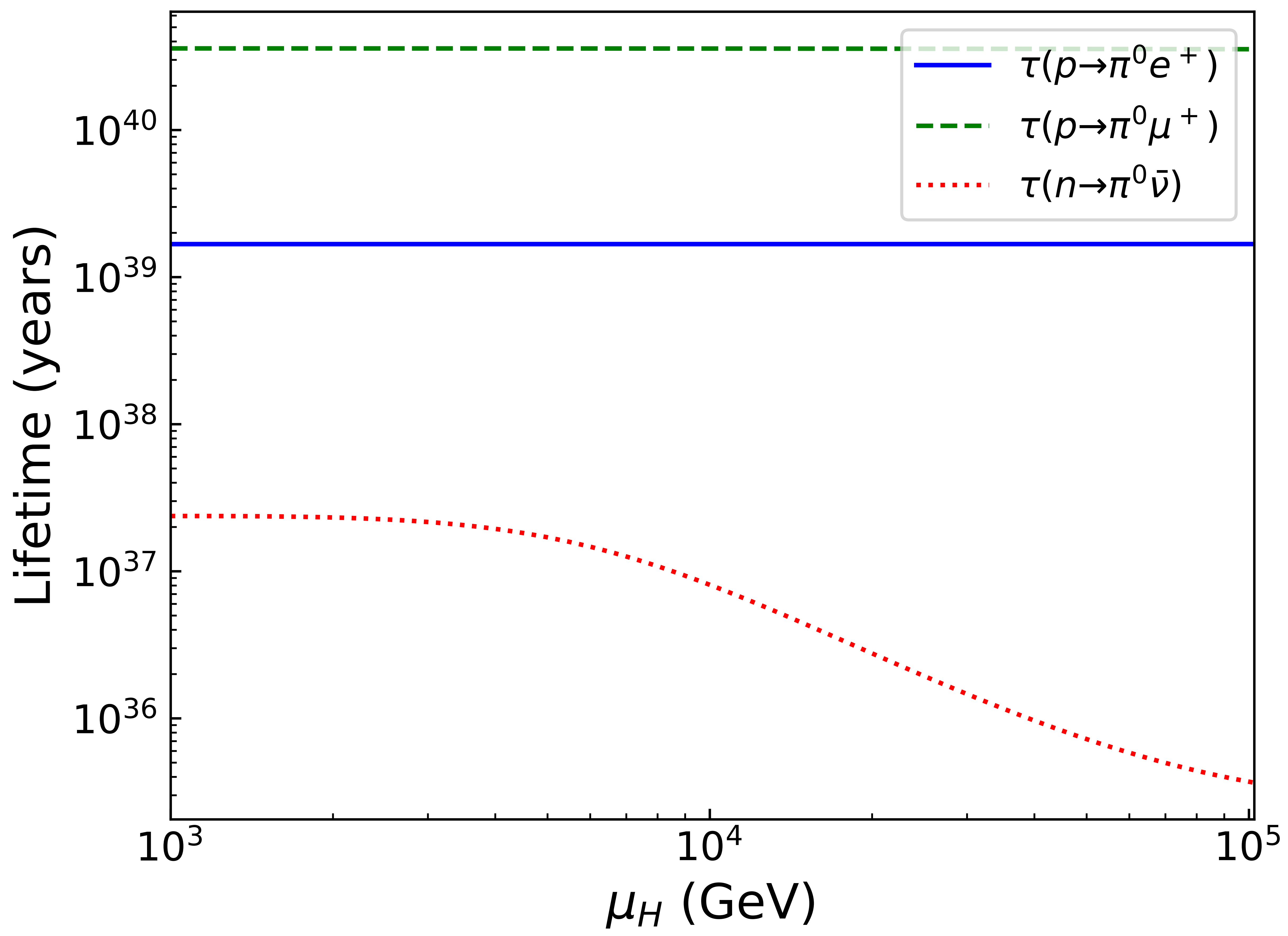
- The gluino exchange dominates as $\delta_{13}^{\tilde{Q}_L}$ increases, even in $\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$.
- $\Gamma(n \to \pi^0\bar{\nu})/\Gamma(p \to \pi^0\mu^+)$ is a useful probe of this scenario.

The minimal SU(5) with high-scale SUSY



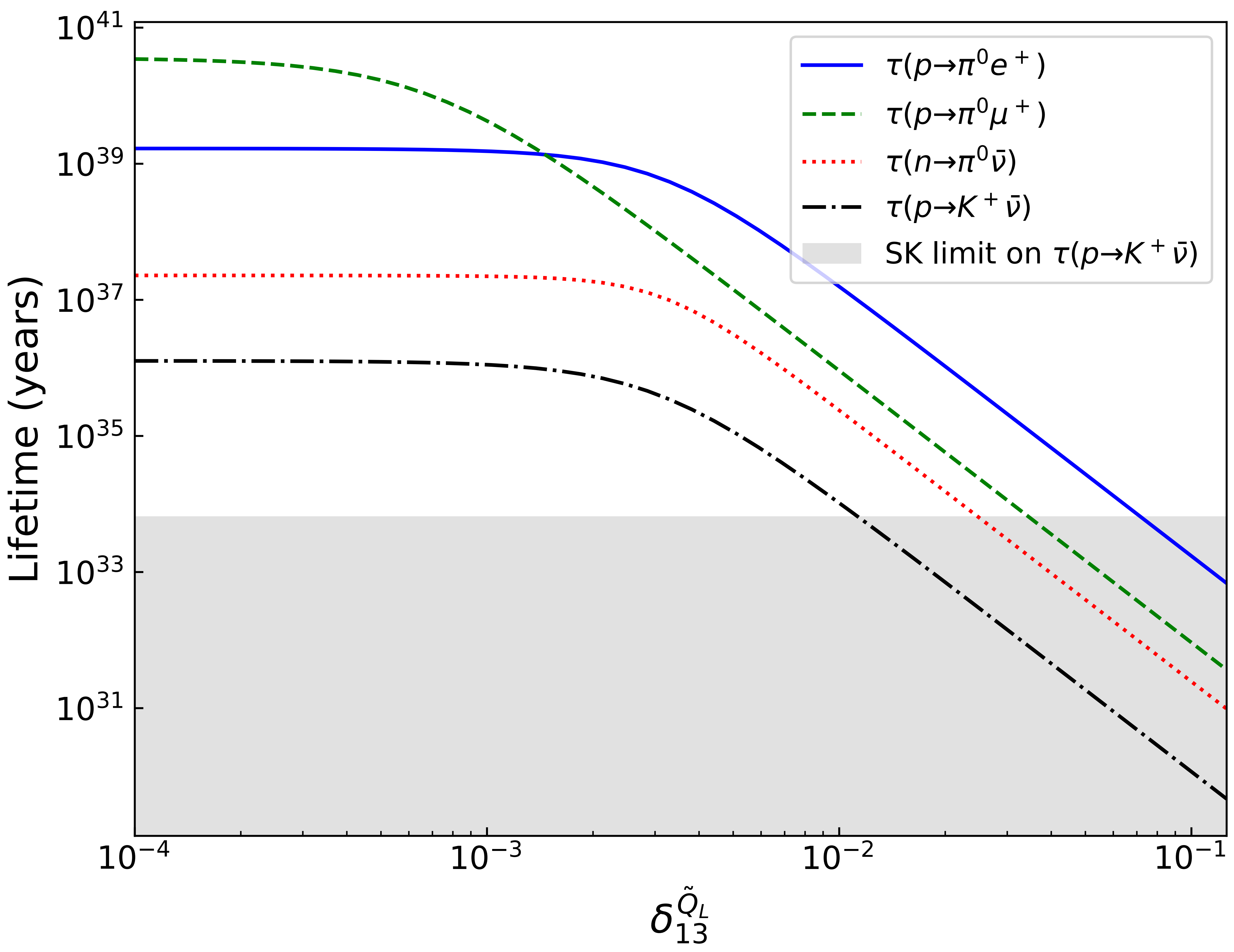
- $M_X = 10^{16}$ GeV, $M_2 = 1$ TeV, $M_3 = 10$ TeV, $M_{\mathrm{SUSY}} = 100$ TeV, $\tan\beta = 3$

The minimal SU(5) with high-scale SUSY



- $M_X = 10^{17}$ GeV, $M_{H_C} = 10^{16}$ GeV, $M_2 = 1$ TeV, $M_3 = 10$ TeV, $M_{\rm SUSY} = 100$ TeV, $\tan\beta = 3$

Sfermion flavor violation



In the presence of flavor violation,

- various channels could be accessible by upcoming experiments, or
- even ruled out by current experimental limits for large $\delta_{13}^{\tilde{f}}$

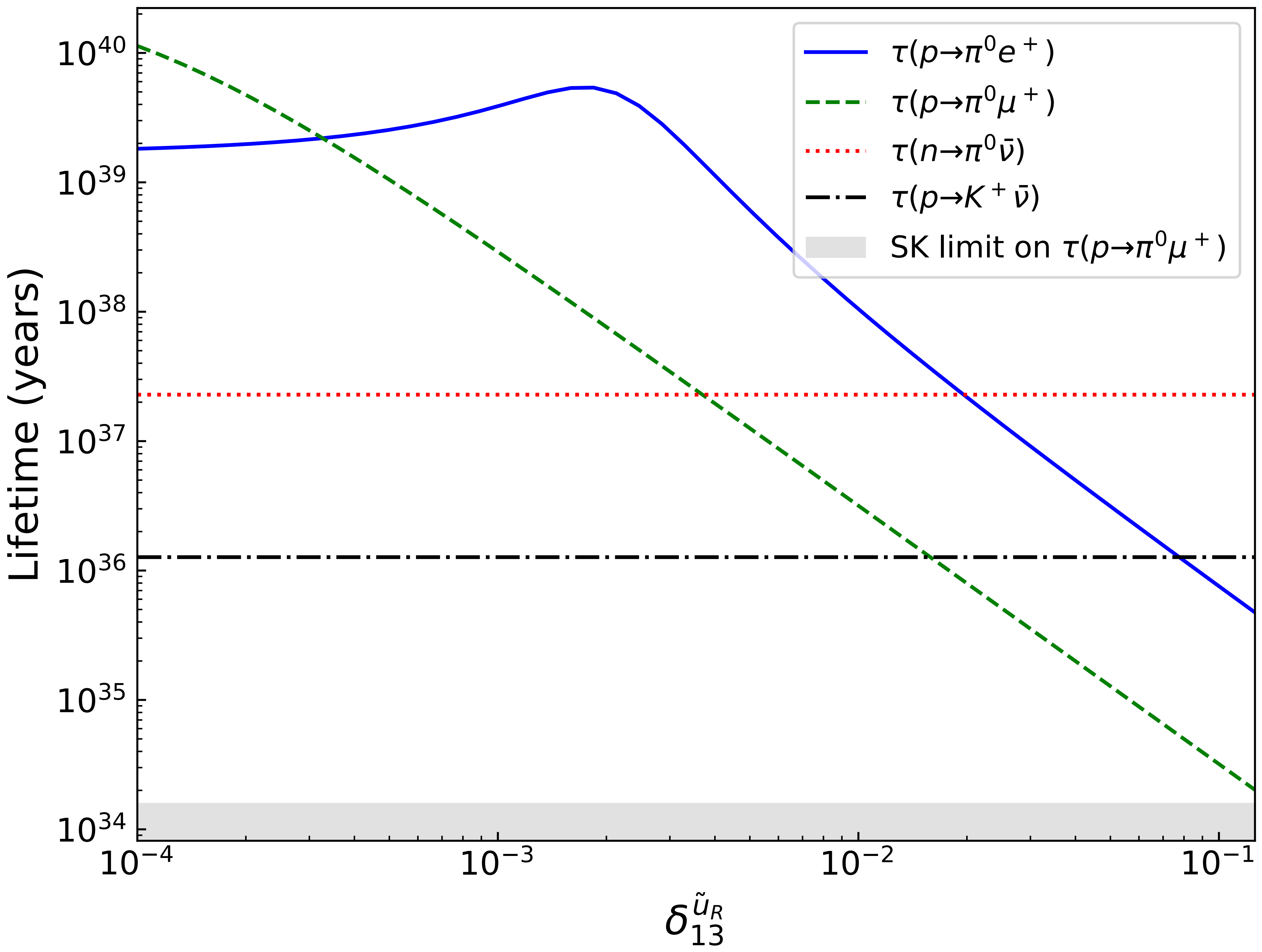
→ can readily be avoided if M_{SUSY} or M_{H_C} is larger

$M_X = 10^{17}$ GeV, $M_{H_C} = 10^{16}$ GeV, $\mu_H = 200$ GeV,

$M_1 = 5$ TeV, $M_2 = 1$ TeV, $M_3 = 10$ TeV, $M_{\text{SUSY}} = 100$ TeV, $\tan\beta = 3$

$$\widetilde{m}^2_{\tilde{f}} = M^2_{\text{SUSY}} \begin{pmatrix} 1 & \delta_{12}^{\tilde{f}} & \delta_{13}^{\tilde{f}} \\ \delta_{12}^{\tilde{f}*} & 1 & \delta_{23}^{\tilde{f}} \\ \delta_{13}^{\tilde{f}*} & \delta_{23}^{\tilde{f}*} & 1 \end{pmatrix}$$

Sfermion flavor violation



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$M_1 = 5$ TeV, $M_2 = 1$ TeV, $M_3 = 10$ TeV, $M_{\rm SUSY} = 100$ TeV, $\tan\beta = 3$

Uncertainties in hadron matrix elements

- As we have seen in Aoki san's talk, the uncertainties in the nucleon decay matrix elements obtained by the lattice QCD calculation are currently $\sim 10\,\%$.

- **We expect the uncertainties to be significantly reduced if, instead of the matrix elements themselves, the ratios of them are estimated directly by the calculation.** This is because each matrix element is not independent, but correlated with each other.

- At leading order in the chiral perturbation,

$$W^{LR}_{p\pi^+\nu,0} = \frac{1+D+F}{f}\alpha\ , \qquad W^{LR}_{p\pi^+\nu,1} = -\frac{2(D+F)}{f}\alpha\ ,$$

$$W^{LL}_{p\pi^+\nu,0} = \frac{1+D+F}{f}\beta\ , \qquad W^{LL}_{p\pi^+\nu,1} = -\frac{2(D+F)}{f}\beta\ ,$$

$$W^{LR}_{p\eta\ell,0} = -\frac{1+D-3F}{\sqrt{6}f}\alpha\ , \qquad W^{LR}_{p\eta\ell,1} = \frac{2(D-3F)}{\sqrt{6}f}\alpha\ ,$$

$$W^{LL}_{p\eta\ell,0} = \frac{3-D+3F}{\sqrt{6}f}\beta\ , \qquad W^{LL}_{p\eta\ell,1} = \frac{2(D-3F)}{\sqrt{6}f}\beta\ ,$$

$$\langle 0|\epsilon_{abc}(u^a_R d^b_R)u^c_L|p\rangle \equiv \alpha P_L u_p\ ,$$

$$\langle 0|\epsilon_{abc}(u^a_L d^b_L)u^c_L|p\rangle \equiv \beta P_L u_p\ ,$$

$$\alpha = -0.01257(111)\ \mathrm{GeV}^3\ ,$$

$$\beta = 0.01269(107)\ \mathrm{GeV}^3\ .$$

$\alpha \simeq -\beta$ to $\sim 1\,\%$

If we take their ratios, f, α, β will cancel out.

Comments on $W^{\chi\chi'}_{p\eta\ell,1}$

- In previous lattice simulations, the values of $W^{\chi\chi'}_{p\eta\ell,1}(0)$ were not estimated. However, the following combination is evaluated in Y. Aoki et al. (2017).

$$W^{\chi\chi'}_{p\eta\ell,\mu} \equiv W^{\chi\chi'}_{p\eta\ell,0} + \frac{m_\mu}{m_N} W^{\chi\chi'}_{p\eta\ell,1}$$

- We extract $W^{\chi\chi'}_{p\eta\ell,1}(0)$ from $W^{\chi\chi'}_{p\eta\ell,\mu}(m_\mu^2)$ by using $W^{\chi\chi'}_{p\eta\ell,0}(0)$, neglecting m_μ^2 dependence.