中山大學天琴中心
TIANQIN CENTER FOR GRAVITATIONAL PHYSICS, SYSU

Effects of electroweak phase transition on dark matter relic density

↓

One-loop analysis of dark matter constraints in a complex scalar extension of the Standard Model

arXiv: 2506.23199

collaborator : Koichi Funakubo (Saga Univ.)

Sun Yat-sen Univ. (Guangdong, China)
Chikako Idegawa

21st Oct. 2025 Summer Institute 2025

Introduction: Dark matter

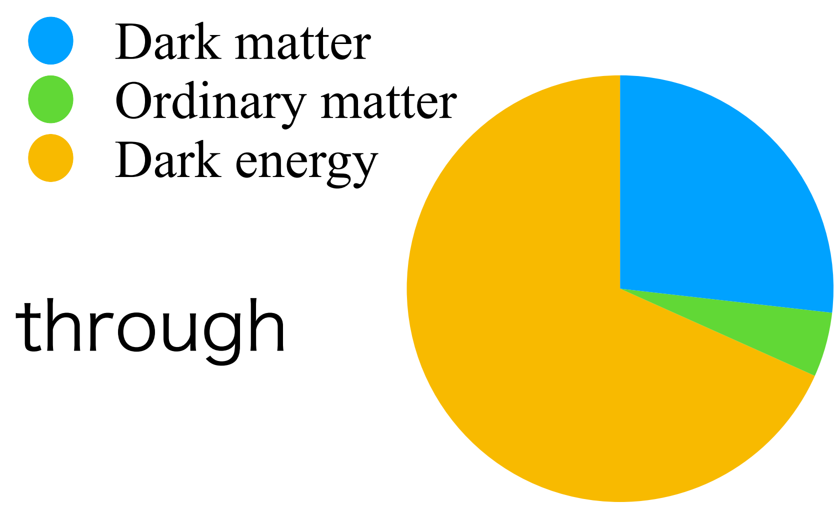
Dark matter (DM) has been suggested to exist by cosmological observations, but is impossible to observe optically

DM relic density

Planck 2018 results

$$\boxed{\Omega_{\mathrm{DM}} h^2 = 0.1200 \pm 0.0012}$$

One candidate for dark matter:
Weekly interacting massive particle (**WIMP**)



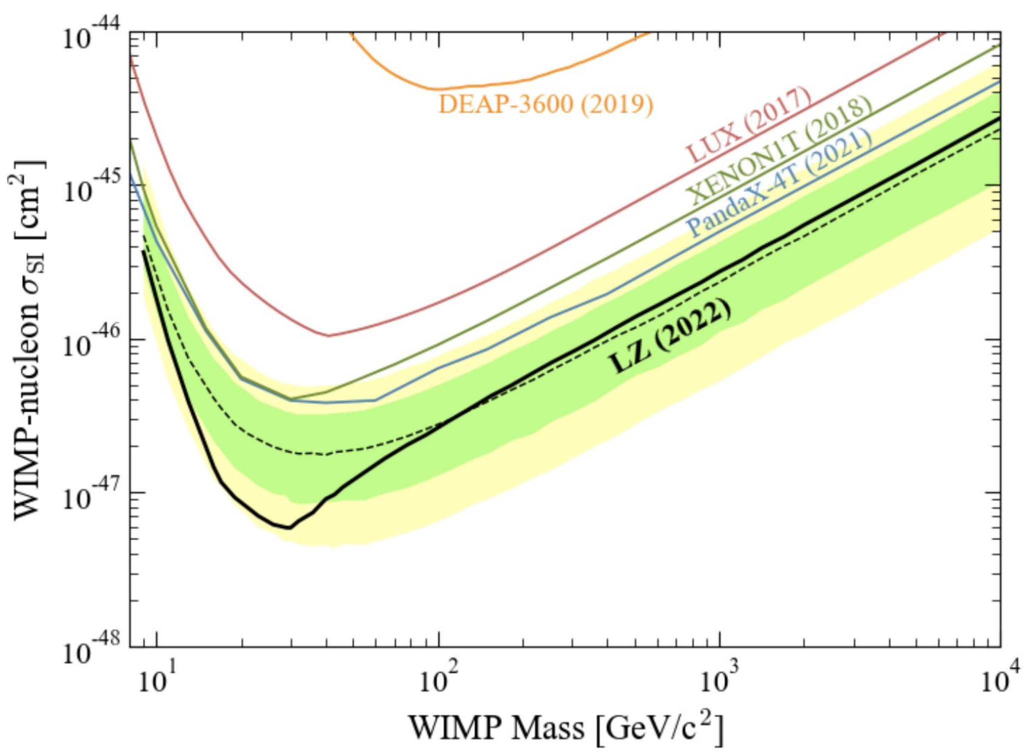
WIMP can explain the observed DM relic density through a thermal production process.

WIMP mass range : $O(1)\mathrm{GeV} - O(10)\mathrm{TeV}$

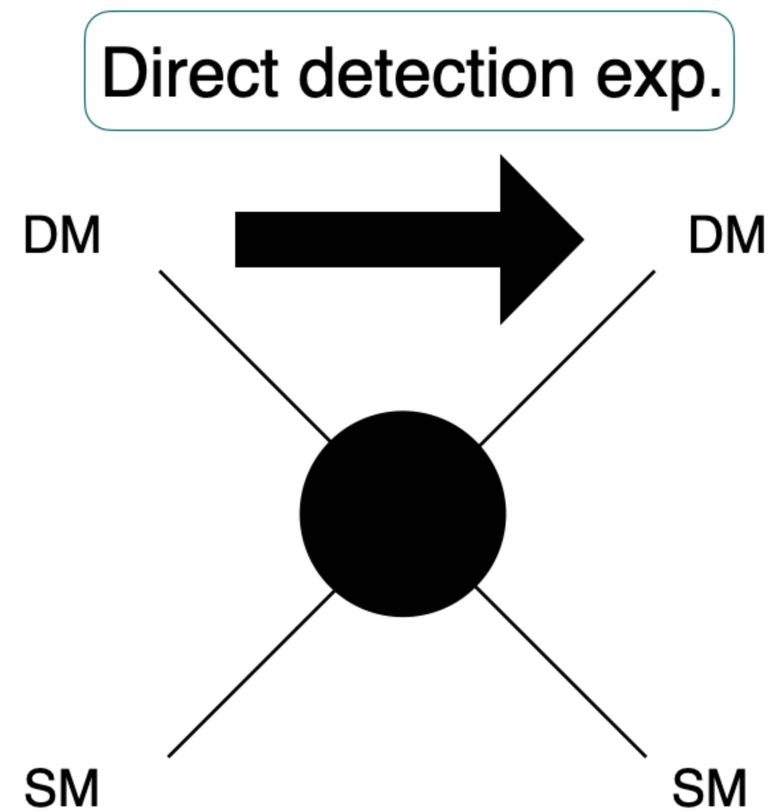
DM suggests physics **beyond the Standard Model** (SM)

Introduction: Dark matter

Direct detection experiment :
DM from space collides with liquid xenon stored in an underground laboratory, and the signal emitted when the nucleon recoils is detected.



LZ Collaboration (Jul 8, 2022)

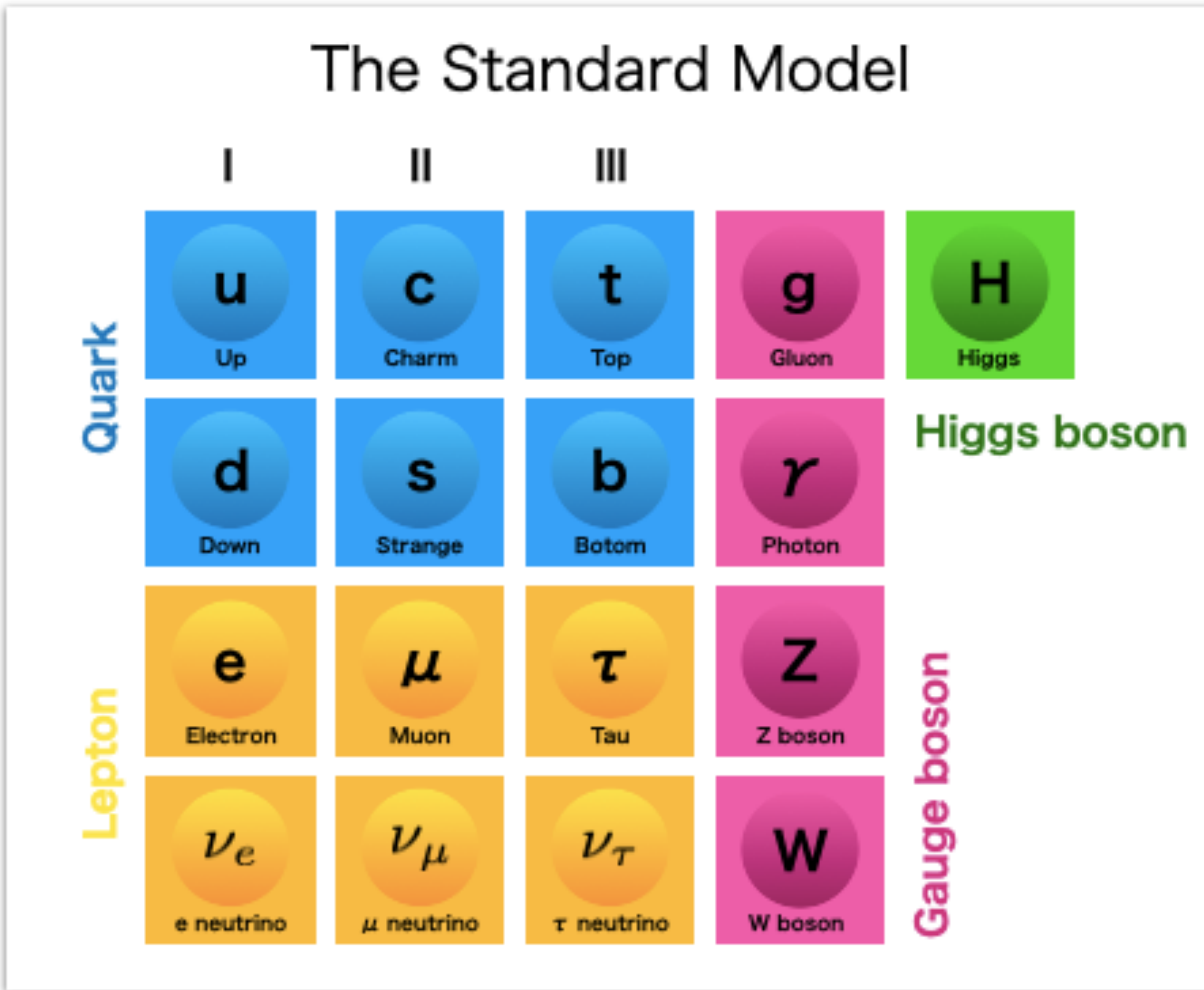


The DM-nucleon scattering cross section is very suppressed.

Constructing a model that includes WIMP DM without violating this bound is a major challenge.

The Model

Complex scalar extension of the SM (CxSM)



One complex scalar

$+$ S

The Model : CxSM

The scalar potential of the SM Higgs H and the complex scalar S

$$V_0 = \frac{m^2}{2}|H|^2 + \frac{\lambda}{4}|H|^4 + \frac{\delta_2}{2}|H|^2|S|^2 + \frac{b_2}{2}|S|^2 + \frac{d_2}{4}|S|^4$$
$$+ \left(a_1 S + \frac{b_1}{4} S^2 + \text{ c.c. } \right)$$

all coefficients including a_1, b_1 are real

At $T = 0$,

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \quad S = (v_S + s + \underline{i\chi})/\sqrt{2}$$

DM particle

Z_2 symmetry $\chi \to -\chi$ guarantees the stability of the DM

Condition for the bounded from below

$$\lambda > 0, \quad d_2 > 0, \quad \left([\delta_2 \geq 0], \quad \text{or} \quad \left[\delta_2 < 0 \text{ and } \lambda d_2 \geq \delta_2^2 \right] \right)$$

Tree level

We denote the classical background fields of the Higgs doublet and singlet as

$$\langle H\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\varphi \end{pmatrix}, \ \langle S\rangle = \frac{1}{\sqrt{2}}\left(\varphi_S + i\varphi_\chi\right),$$

Mass eigenstates

▷ h, s $\quad \begin{pmatrix} h \\ s \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$

$$m^2_{h_1,h_2} = \frac{1}{2}\left(\frac{\lambda}{2}v^2 + \Lambda^2 \mp \sqrt{\left(\frac{\lambda}{2}v^2 - \Lambda^2\right)^2 + 4\left(\frac{\delta_2}{2}vv_S\right)^2}\right) \qquad \Lambda^2 = \frac{d_2 v_S^2}{2} - \frac{\sqrt{2}a_1}{v_S}$$

h_1 is the currently observed 125 GeV Higgs boson

▷ χ (DM) $\quad m^2_\chi = -b_1 - \sqrt{2}\frac{a_1}{v_S}$

One-loop level

Zero-temperature effective potential

$$V_{\text{eff}}\left(\varphi, \varphi_S, \varphi_\chi\right) = V_0\left(\varphi, \varphi_S, \varphi_\chi\right) + V_{\text{CW}}\left(\varphi, \varphi_S, \varphi_\chi\right)$$

$$V_0 = \sqrt{2}\left(a_1 + a_1^{(1)}\right)\varphi_s + \frac{m^2 + m^{(1)2}}{4}\varphi^2 + \frac{b_2 + b_1 + b_2^{(1)} + b_1^{(1)}}{4}\varphi_s^2 + \frac{b_2 - b_1 + b_2^{(1)} - b_1^{(1)}}{4}\varphi_\chi^2$$
$$+ \frac{\lambda + \lambda^{(1)}}{16}\varphi^4 + \frac{d_2 + d_2^{(1)}}{16}(\varphi_S^2 + \varphi_\chi^2)^2 + \frac{\delta_2 + \delta_2^{(1)}}{8}\varphi^2(\varphi_S^2 + \varphi_\chi^2),$$

$$V_{\text{CW}}\left(\varphi, \varphi_s, \varphi_\chi\right) = \sum_i \frac{n_i \bar{m}_i^4}{64\pi^2}\left(\log\frac{\bar{m}_i^2}{\bar{\mu}^2} - \gamma_i\right).$$

Quantities labeled with the superscript (1): coefficients of the finite counterterms

$$\delta_2^{(1)} = -\frac{2}{v v_S}\frac{\partial^2 V_{\text{CW}}}{\partial\varphi\partial\varphi_S},$$

$$\lambda^{(1)} = -\frac{2}{v^2}\left(\left\langle\frac{\partial^2 V_{\text{CW}}}{\partial\varphi^2}\right\rangle - \frac{1}{v}\left\langle\frac{\partial V_{\text{CW}}}{\partial\varphi}\right\rangle\right),$$

$$m^{2(1)} = -\frac{\lambda^{(1)}}{2}v^2 - \frac{\delta_2^{(1)}}{2}v_S^2 - \frac{2}{v}\left\langle\frac{\partial V_{\text{CW}}}{\partial\varphi}\right\rangle$$

$$d_2^{(1)} = -\frac{2}{v_S^2}\left(\frac{\partial^2 V_{\text{CW}}}{\partial\varphi_S^2} - \frac{1}{v_S}\frac{\partial V_{\text{CW}}}{\partial\varphi_s}\right),$$

$$b_2^{(1)} + b_1^{(1)} = -\frac{\delta_2^{(1)}}{2}v^2 - \frac{d_2^{(1)}}{2}v_S^2 - \frac{2}{v_S}\frac{\partial V_{\text{CW}}}{\partial\varphi_S},$$

$$b_2^{(1)} - b_1^{(1)} = -\frac{d_2^{(1)}}{2}v_S^2 - \frac{\delta_2^{(1)}}{2}v^2 - 2\frac{\partial^2 V_{\text{CW}}}{\partial\varphi_\chi^2}.$$

Vacuum stability in the φ_χ-direction

From the viewpoint of the DM stability, no spontaneous VEV should develop along the φ_χ-direction.

$$(\varphi, \varphi_S, \varphi_\chi) = (0, v'_S, 0) \to (\varphi, \varphi_S, \varphi_\chi) = (v, v_S, 0)$$

Tree

The curvature of V_0 at the origin corresponds to

$$\frac{\partial^2 V_0}{\partial \varphi_\chi^2} = \bar{m}_\chi^2(\varphi, \varphi_S) = \frac{b_2}{2} - \frac{b_1}{2} + \frac{\delta_2}{4}\varphi^2 + \frac{d_2}{4}\varphi_S^2$$

For $\delta_2 > 0$, $d_2 > 0$, the condition $b_2 - b_1 > 0$ ensures convexity of V_0 along $\varphi_\chi = 0$.

Most stringent bound comes from the origin.

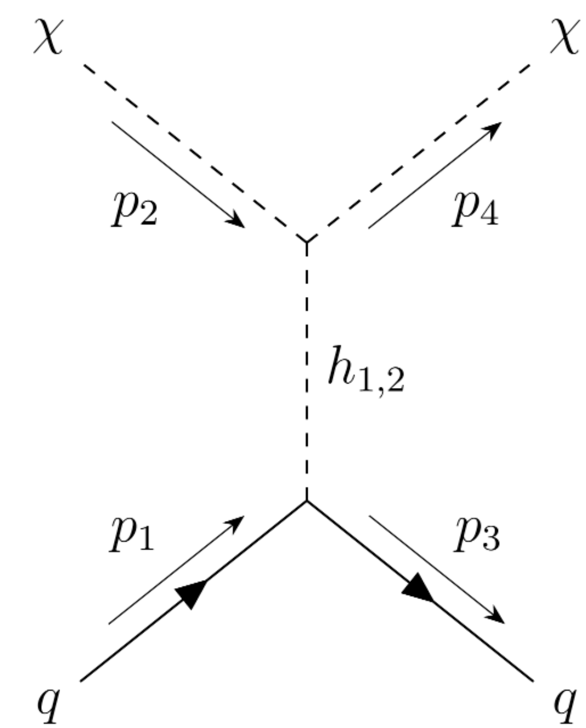
$$\left.\frac{\partial^2 V_0}{\partial \varphi_\chi^2}\right|_{\varphi=\varphi_S=\varphi_\chi=0} = \frac{b_2 - b_1}{2} = m_\chi^2 - \frac{d_2}{4}v_S^2 - \frac{\delta_2}{4}v^2 > 0.$$

One-loop

$$\left.\frac{\partial^2 V_{\rm eff}}{\partial \varphi_\chi^2}\right|_{\varphi=\varphi_S=\varphi_\chi=0} = \frac{b_2 - b_1 + b_2^{(1)} - b_1^{(2)}}{2} + \left.\frac{\partial^2 V_{\rm CW}}{\partial \varphi_\chi^2}\right|_{\varphi=\varphi_S=\varphi_\chi=0} > 0.$$

DM-quark scattering

The scattering of DM χ and quarks q in the CxSM occurs with h_1 and h_2 as mediator particles.



The scattering amplitudes mediated by h_1 or h_2

$$i\mathcal{M}_{h_1} = -i\frac{m_f}{vv_S}\frac{m_{h_1}^2 + \dfrac{\sqrt{2}a_1}{v_S}}{t - m_{h_1}^2}\sin\alpha\cos\alpha\ \bar{u}(p_3)u(p_1),$$

$$i\mathcal{M}_{h_2} = +i\frac{m_f}{vv_S}\frac{m_{h_2}^2 + \dfrac{\sqrt{2}a_1}{v_S}}{t - m_{h_2}^2}\sin\alpha\cos\alpha\ \bar{u}(p_3)u(p_1),$$

$$\begin{aligned} i\left(\mathcal{M}_1 + \mathcal{M}_2\right) &= i\frac{m_f}{vv_S}\bar{u}\left(p_3\right)u\left(p_1\right)\sin\alpha\cos\alpha \\ &\times\left\{\left(-\frac{m_{h_1}^2}{t - m_{h_1}^2} + \frac{m_{h_2}^2}{t - m_{h_2}^2}\right)\right. \\ &\left.+\frac{\sqrt{2}a_1}{v_S}\left(-\frac{1}{t - m_{h_1}^2} + \frac{1}{t - m_{h_2}^2}\right)\right\} \end{aligned} \simeq 0$$

@ $\alpha \to$ small

or

@ $m_{h_1} \simeq m_{h_2}$

Abe, Cho, Mawatari (2021)

DM abundance

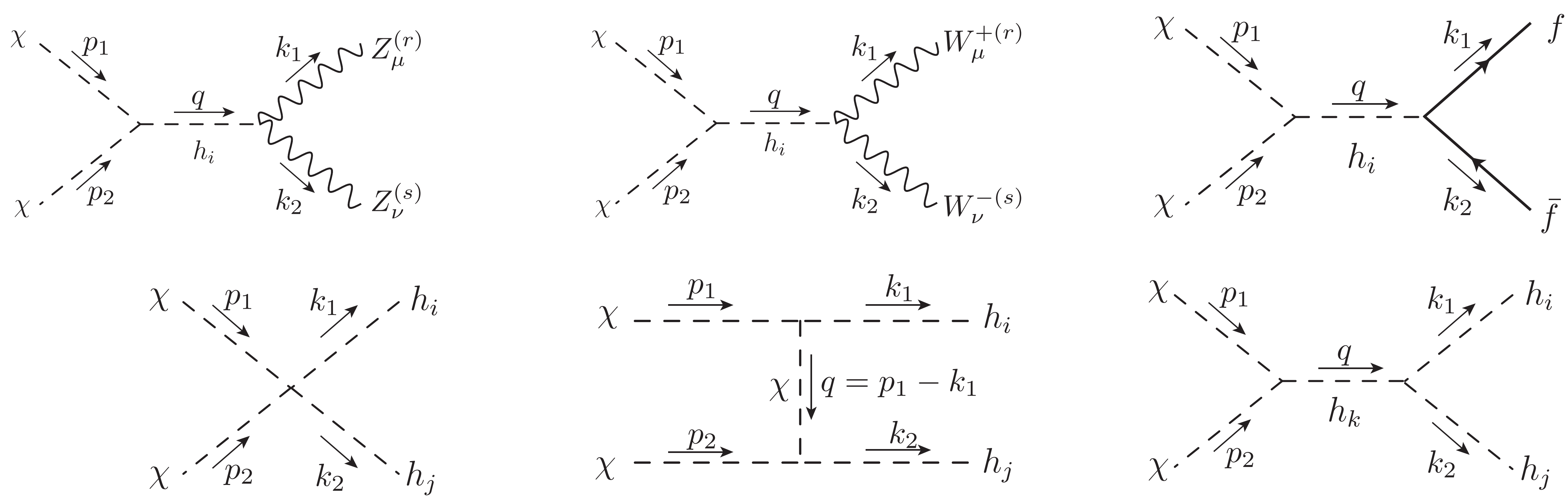
The Boltzmann equation of dimensionless quantities $Y(z) = \dfrac{n_\chi}{s}$

$$\frac{dY_\chi(z)}{dz} = -\lambda(z)\left(Y_\chi(z)^2 - Y_{\rm eq}(z)^2\right)$$

The final DM abundance $\Omega_\chi h^2 = 2.7506 \times 10^8 \cdot m_\chi \text{ GeV} \cdot Y_\chi(\infty)$

can be compared with the observed value $\Omega_{\rm DM} h^2 = 0.1200 \pm 0.0012$

Tree-level DM annihilation process of the CxSM



Numerical results

Explore the parameter space that satisfies three key conditions:

1) Scalar potential is convex downward in the φ_χ-direction along $\varphi_\chi = 0$

2) Consistency with the latest bounds of LZ experiment

3) Reproduction of the observed DM relic abundance

Tree v.s. One-loop

	Tree	**One-loop**
Scalar potential	V_0	$V_{\rm eff}$
DM Annihilation / DD amplitudes	tree diagrams only	tree + one-loop effective χ-vertices
Counterterms	–	included

Parameter scan

$$10 \leq v_s \ \mathrm{GeV} \leq 1000$$

$$-500^3 \leq a_1/\mathrm{GeV}^3 \leq 0$$

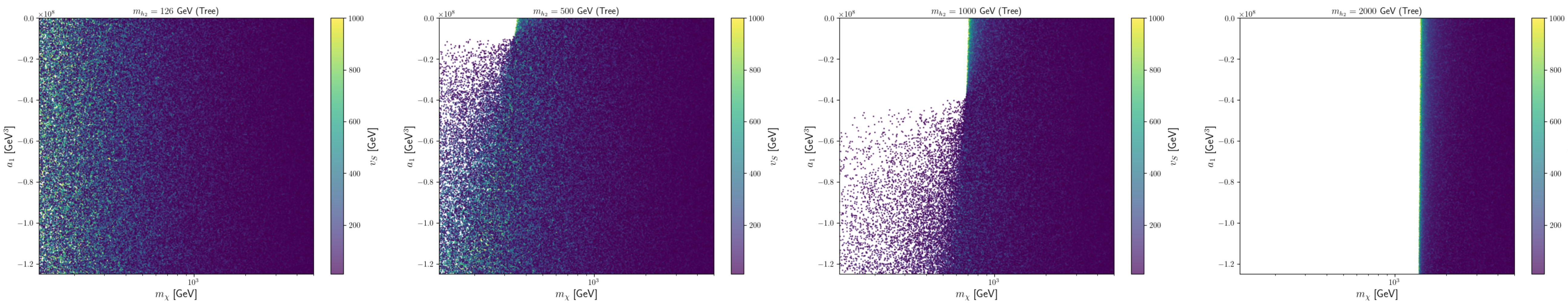
$$125 \leq m_\chi \ \mathrm{GeV} \leq 5000.$$

$$\alpha = -\frac{\pi}{14}$$

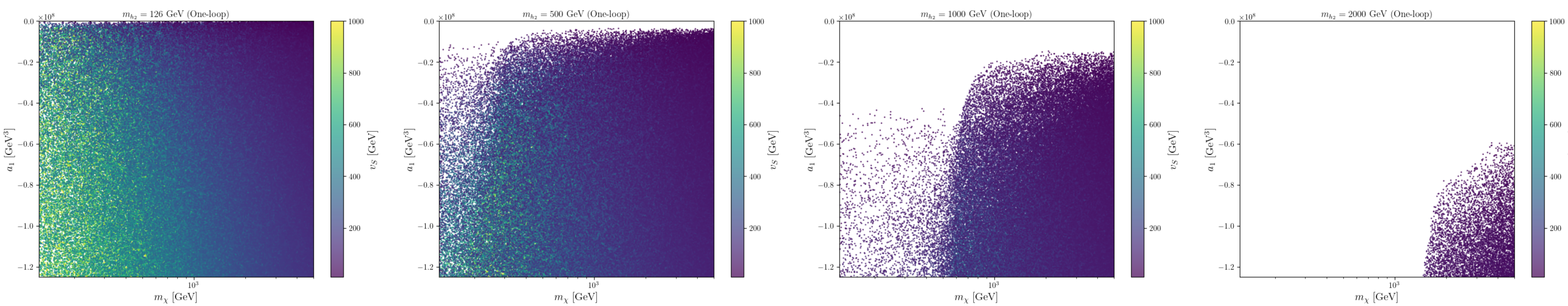
Numerical results

1) Vacuum stability at φ_χ-direction

Tree $\left.\dfrac{\partial^2 V_0}{\partial\varphi_\chi^2}\right|_{\varphi=\varphi_S=\varphi_\chi=0} = \dfrac{b_2 - b_1}{2} = m_\chi^2 - \dfrac{d_2}{4}v_S^2 - \dfrac{\delta_2}{4}v^2 > 0.$



One-loop $\left.\dfrac{\partial^2 V_{\rm eff}}{\partial\varphi_\chi^2}\right|_{\varphi=\varphi_S=\varphi_\chi=0} = \dfrac{b_2 - b_1 + b_2^{(1)} - b_1^{(2)}}{2} + \left.\dfrac{\partial^2 V_{\rm CW}}{\partial\varphi_\chi^2}\right|_{\varphi=\varphi_S=\varphi_\chi=0} > 0.$

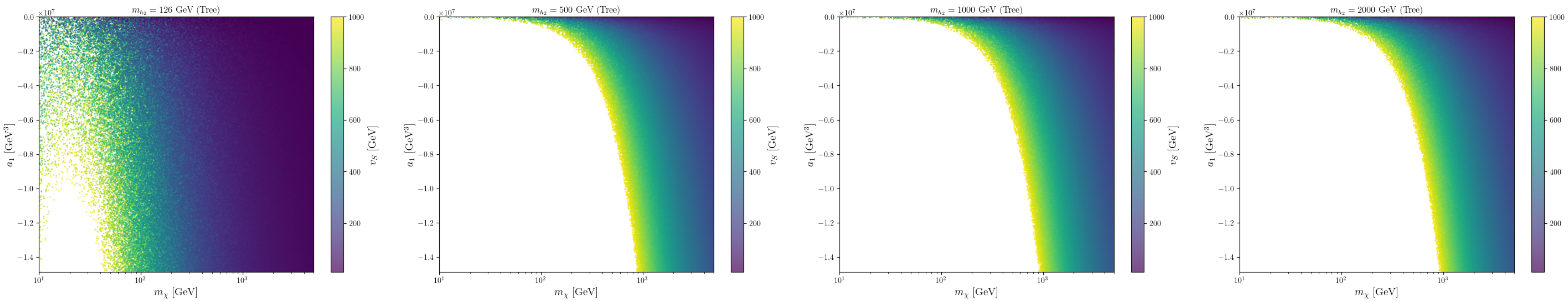


$$b_2^{(1)} - b_1^{(1)} = -\frac{d_2^{(1)}}{2}v_S^2 - \frac{\delta_2^{(1)}}{2}v^2 - 2\frac{\partial^2 V_{\rm CW}}{\partial\varphi_\chi^2} \qquad \delta_2^{(1)} = -\frac{2}{v v_S}\frac{\partial^2 V_{\rm CW}}{\partial\varphi\partial\varphi_S}, \qquad d_2^{(1)} = -\frac{2}{v_S^2}\left(\frac{\partial^2 V_{\rm CW}}{\partial\varphi_S^2} - \frac{1}{v_S}\frac{\partial V_{\rm CW}}{\partial\varphi_s}\right),$$

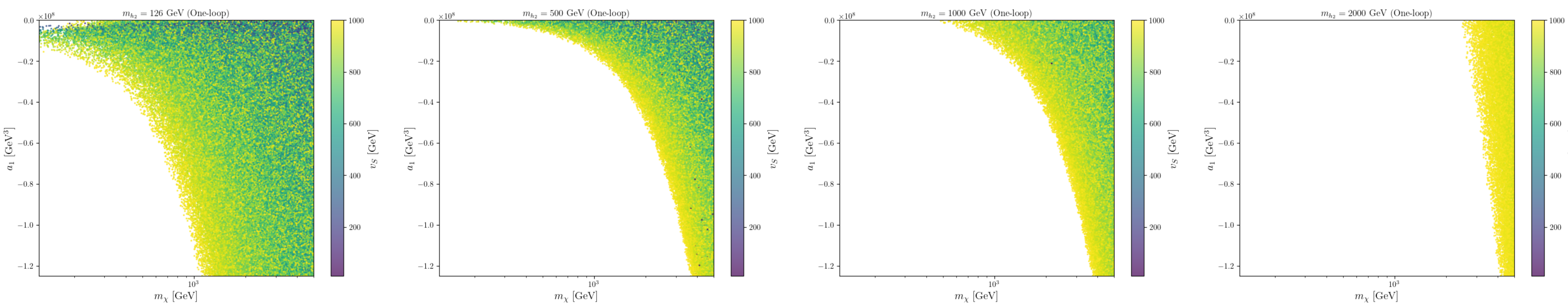
Numerical results

2) DM direct detection bounds

Tree



One-loop



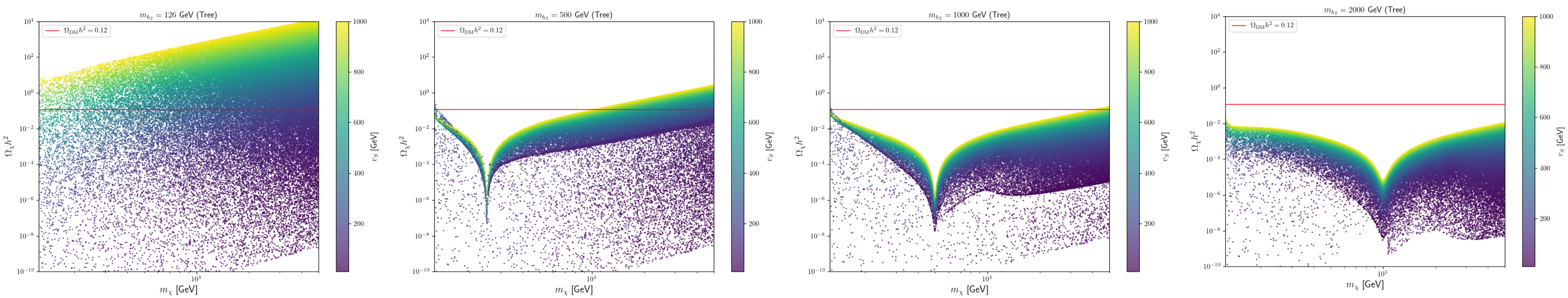
As m_{h_2} increases, effective couplings become stronger.
These strong couplings enhance the scattering processes, making it more likely for the points to be excluded by the LZ bound.

Numerical results

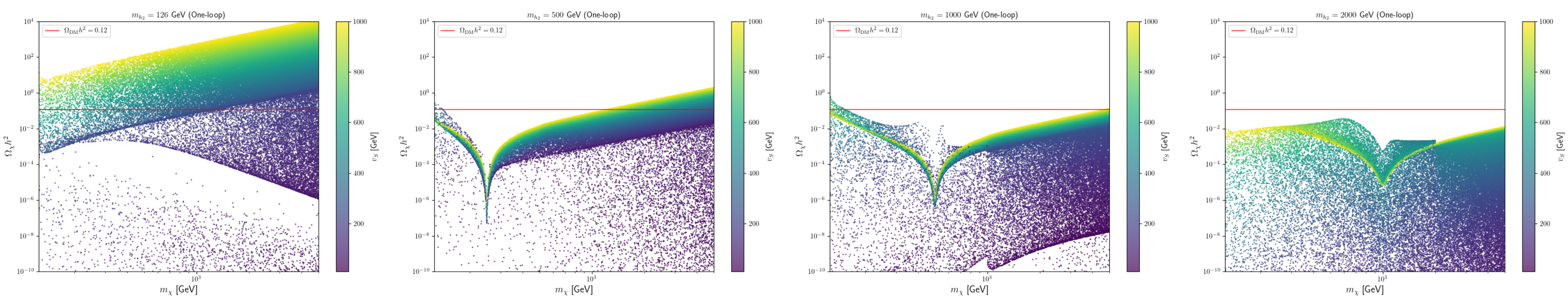
3) DM relic abundance

Tree

$$a_1 = -246^3\ \mathrm{GeV}^3$$



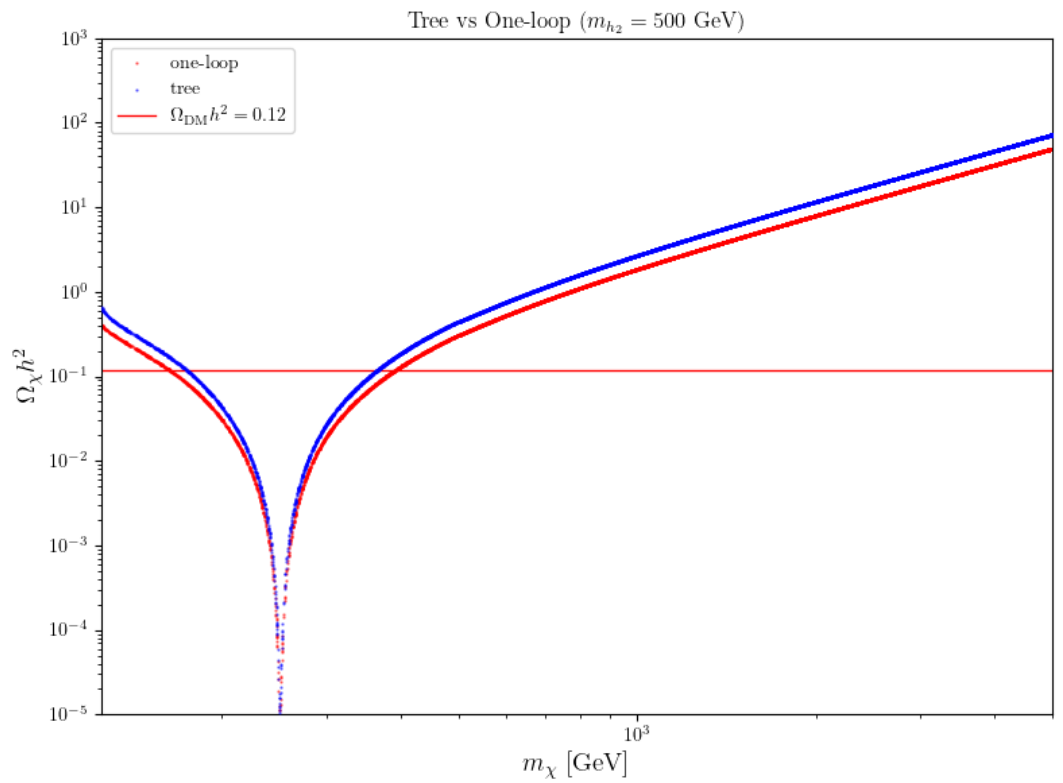
One-loop

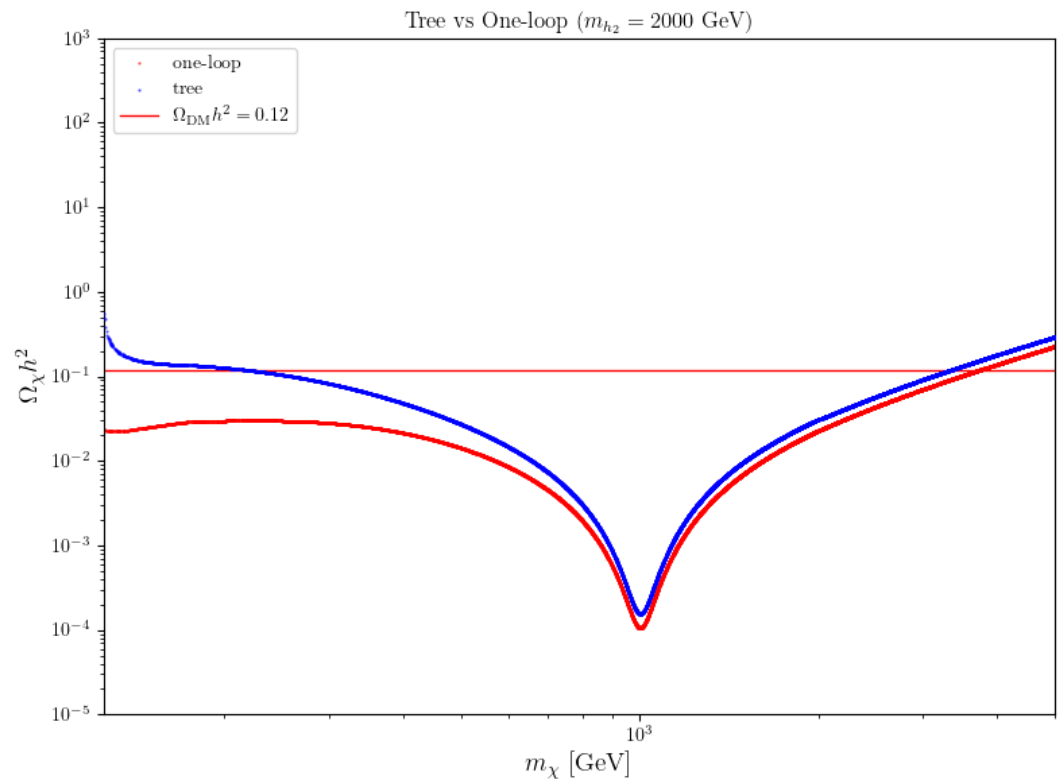


Numerical results

3) DM relic abundance

$$v_S = 5000 \text{ GeV} \quad a_1 = -246^3 \text{ GeV}^3$$



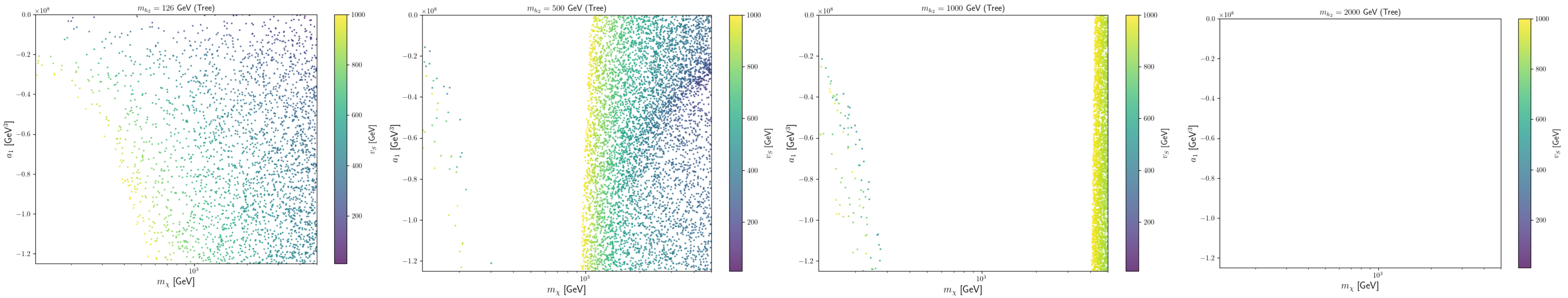


When h_2 is heavy, effective couplings, especially at the one-loop level, becomes stronger, and DM annihilation often occurs, causing a decrease in abundance.

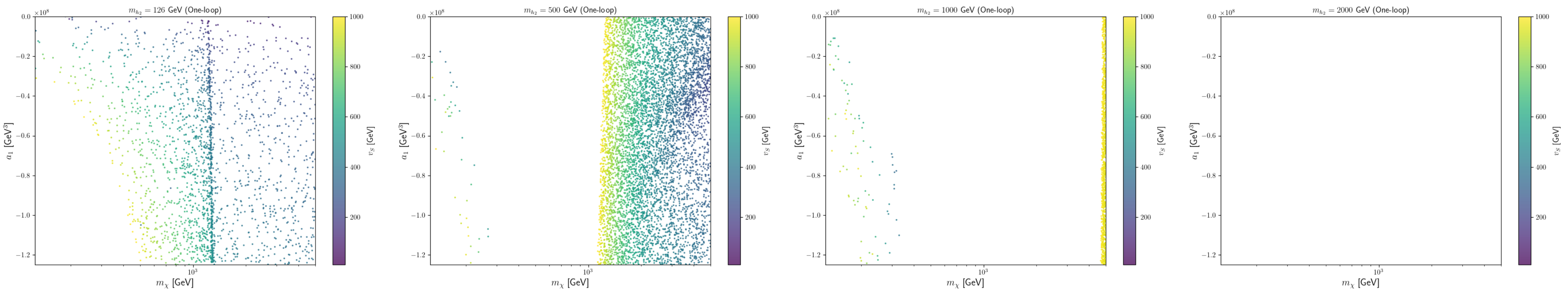
Numerical results

3) DM relic abundance

Tree



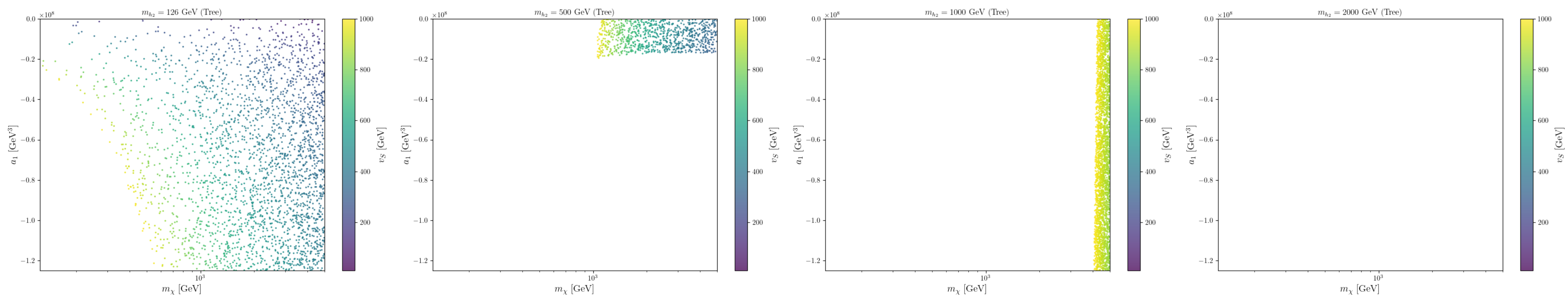
One-loop



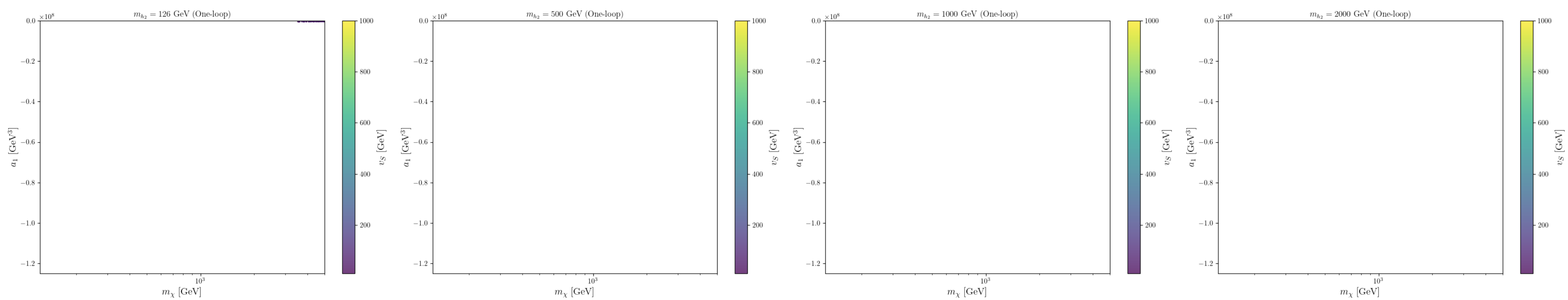
Numerical results

Combining

Tree



One-loop



Summary and future work

We studied the parameter space of the **CxSM** which includes a **WIMP DM** candidate.

Three key constraints were examined:

1) Effective potential is convex downward in the φ_χ-direction along $\varphi_\chi = 0$
2) Consistency with the latest bounds of LZ experiment
3) Reproduction of the observed DM relic abundance

Considering up to one-loop level, it is difficult to find parameters that satisfy all of these conditions (but there is a small allowed region when $m_{h_2} = 126$ GeV).

To solve this, one possible solution is to introduce a scalar cubic term.

$$c_1 H^\dagger H(S + \text{ h.c.})\,, c_2|S|^2(S + \text{ h.c.})\,, c_3(S + \text{ h.c.})^3$$

$$\Updownarrow$$

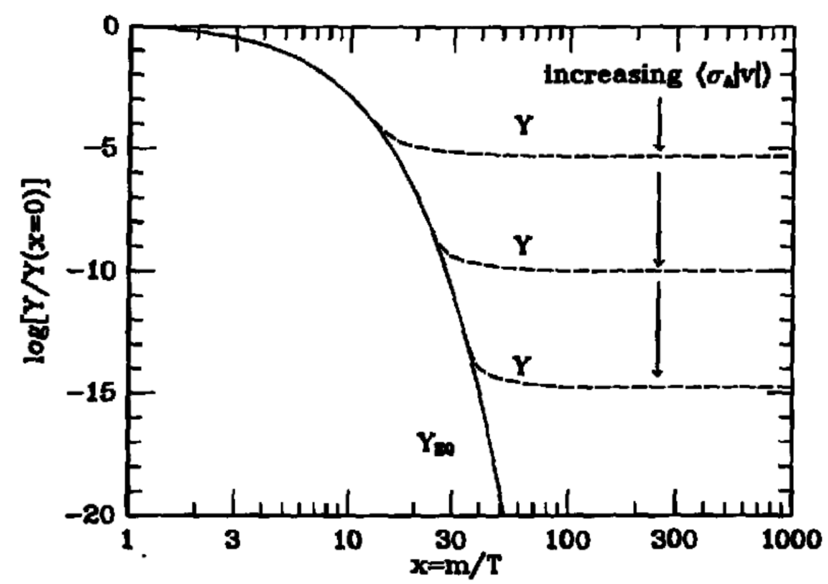
$\tilde{b} \equiv b_2 - b_1$, Effective vertices of $\chi - \chi - h_i$, D_i $(i = 1,2)$

Summary and future work

Boltzman equation:
the fundamental equation describing the time evolution of the particle number density

Dimensionless quantities $Y(z) = \frac{n_\chi}{s}$ are often used

Kolb and Turner “The Early Universe”



$$\frac{dY(z)}{dz} = -\underbrace{\lambda(z)}_{\propto \langle\sigma v\rangle} \left(Y(z)^2 - Y_{\rm eq}(z)^2\right)$$

$$z = \frac{m_{\rm DM}}{T}$$

Conventional calculation

Particle masses at $T = 0$ are used to solve the Boltzman eq.

Due to **phase transitions in the early universe**, particle masses change with temperature through the varying vacuum expectation value.

-> Taking into account temperature effects, we estimate DM abundance.

Back up

Parameters

Tree

$$m^2 = -\frac{\lambda}{2}v^2 - \frac{\delta_2}{2}v_S^2$$

$$b_2 = -\frac{\delta_2}{2}v^2 - \frac{d_2}{2}v_S^2 - \sqrt{2}\frac{a_1}{v_S} - b_1.$$

$$\lambda = \frac{2}{v^2}\left(m_{h_1}^2 \cos^2\alpha + m_{h_2}^2 \sin^2\alpha\right),$$

$$\delta_2 = \frac{1}{v v_S}\left(m_{h_1}^2 - m_{h_2}^2\right)\sin 2\alpha,$$

$$d_2 = 2\left(\frac{m_{h_1}}{v_S}\right)^2 \sin^2\alpha + 2\left(\frac{m_{h_2}}{v_S}\right)^2 \cos^2\alpha + 2\sqrt{2}\frac{a_1}{v_S^3},$$

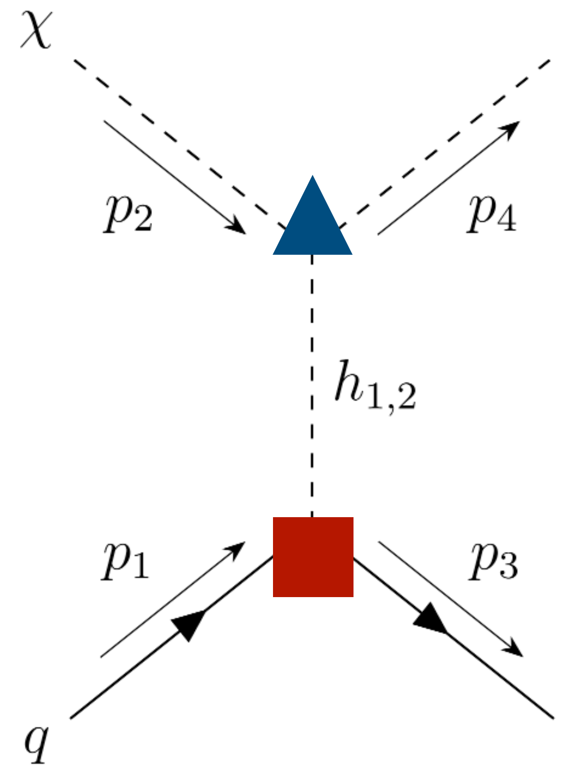
$$b_1 = -m_\chi^2 - \frac{\sqrt{2}}{v_S}a_1.$$

One-loop

$$m_\chi^2 = \frac{b_2 + b_2^{(1)} - b_1^{(1)} - b_1^{(1)}}{2} + \frac{d_2 + d_2^{(1)}}{4}\left(\varphi_S^2 + 3\varphi_\chi^2\right) + \frac{\delta_2 + \delta_2^{(1)}}{4}\varphi^2 + \frac{\partial^2 V_{\mathrm{CW}}}{\partial \varphi_\chi^2}$$

DM-quark scattering

The scattering of DM χ and quarks q in the CxSM occurs with h_1 and h_2 as mediator particles.



■ Yukawa interaction

$$\mathcal{L}_Y = \frac{m_q}{v}\bar{q}q(h_1\cos\alpha + h_2\sin\alpha)$$

▲ Scalar 3-point interaction

$$\mathcal{L}_S = g_{h_1\chi\chi}h_1\chi^2 + g_{h_2\chi\chi}h_2\chi^2$$

$$g_{h_1\chi\chi} \equiv \frac{m_{h_1}^2 + \frac{\sqrt{2}a_1}{v_S}}{2v_S}\sin\alpha$$

$$g_{h_2\chi\chi} \equiv -\frac{m_{h_2}^2 + \frac{\sqrt{2}a_1}{v_S}}{2v_S}\cos\alpha$$

$$\begin{pmatrix} h \\ s \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

Higgs search with degenerate scalars

gauge eigenstates

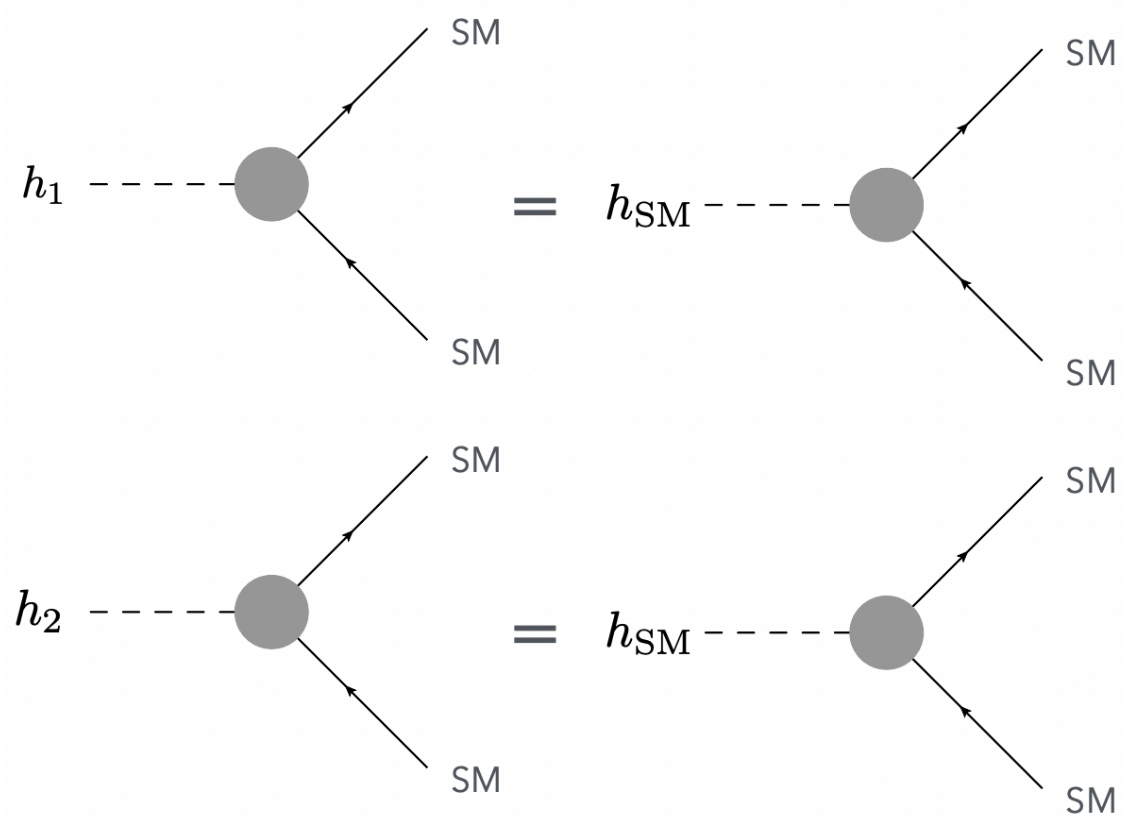
$$(h, s)$$

2×2 mixing matrix

$$\xrightarrow{O(\alpha)}$$

mass eigenstates

$$(h_1, h_2)$$



$\times \cos\alpha$

$\times \sin\alpha$

$$h_1 = h\cos\alpha + s\sin\alpha$$

$$h_2 = -h\sin\alpha + s\cos\alpha$$

$$\Gamma\left(h_1 \to \text{SM}\right) = \Gamma\left(h_{\text{SM}} \to \text{SM}\right)\left(m_{h_1}\right) \times \cos^2\alpha$$

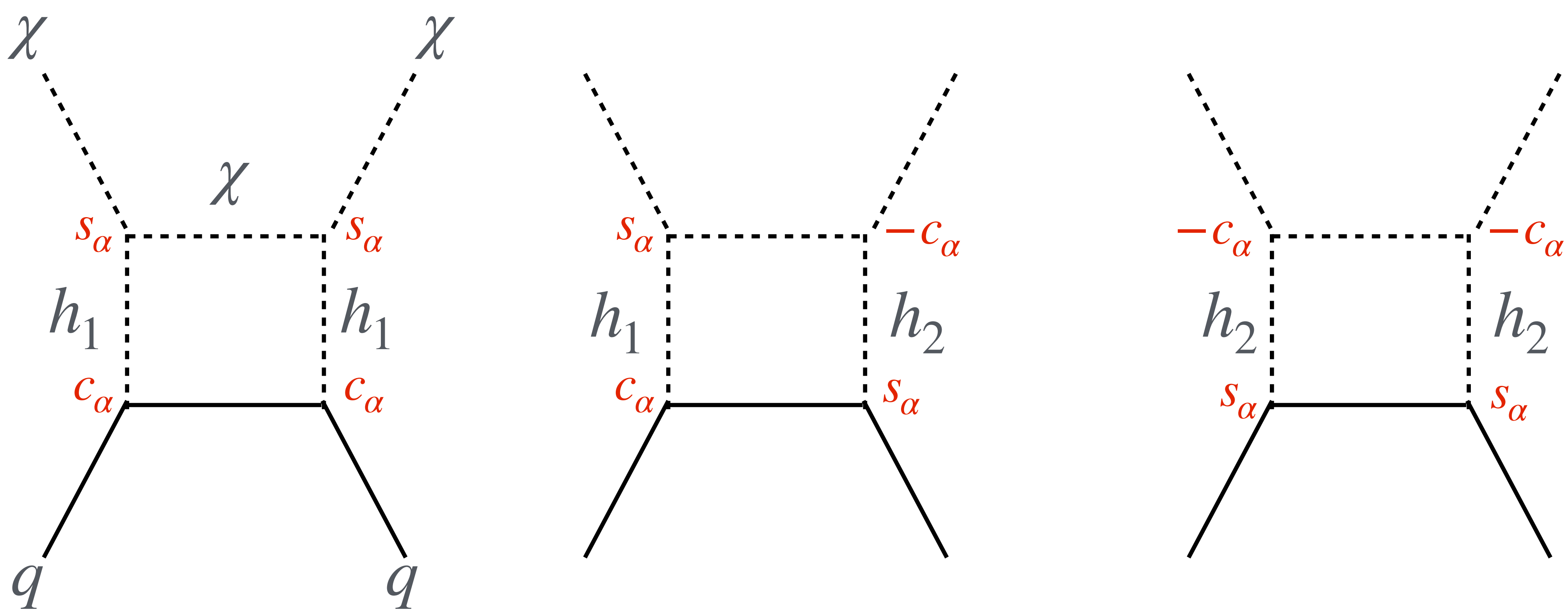
$$\Gamma\left(h_2 \to \text{SM}\right) = \Gamma\left(h_{\text{SM}} \to \text{SM}\right)\left(m_{h_2}\right) \times \sin^2\alpha$$

$$\boxed{\Gamma\left(h_1 \to \text{SM}\right) + \Gamma\left(h_2 \to \text{SM}\right) \simeq \Gamma\left(h_{\text{SM}} \to \text{SM}\right) \ \text{ for } m_{h_1} \simeq m_{h_2}}$$

Degenerate scalar scenario

Degenerate scalar scenario@ one-loop   Azevedo etal., 1801.06105

$$\sigma_{\chi N}^{\rm NLO} = \sin 2\alpha \left(\frac{\mu_{\chi N} f_N m_N}{m_{h_1} m_{h_2}} \right)^2 \frac{\boxed{m_{h_1}^2 - m_{h_2}^2}}{v^3 v_S^3} \times \text{loop func.} \qquad \propto m_{h_1}^2 - m_{h_2}^2$$

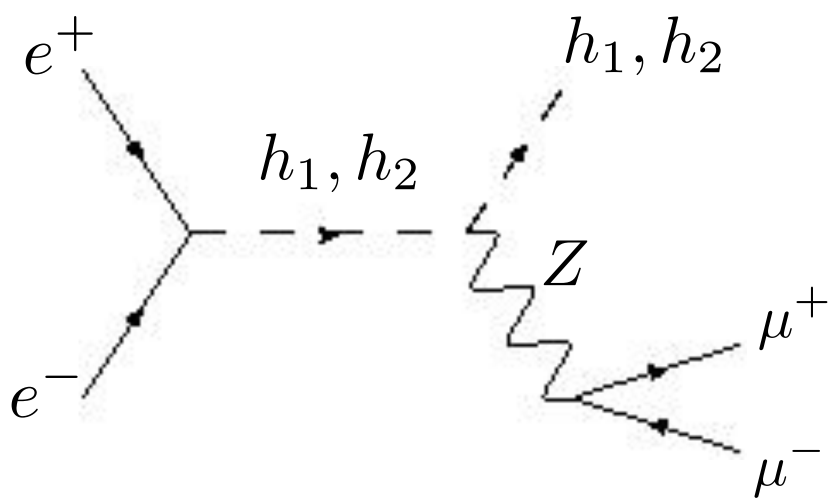


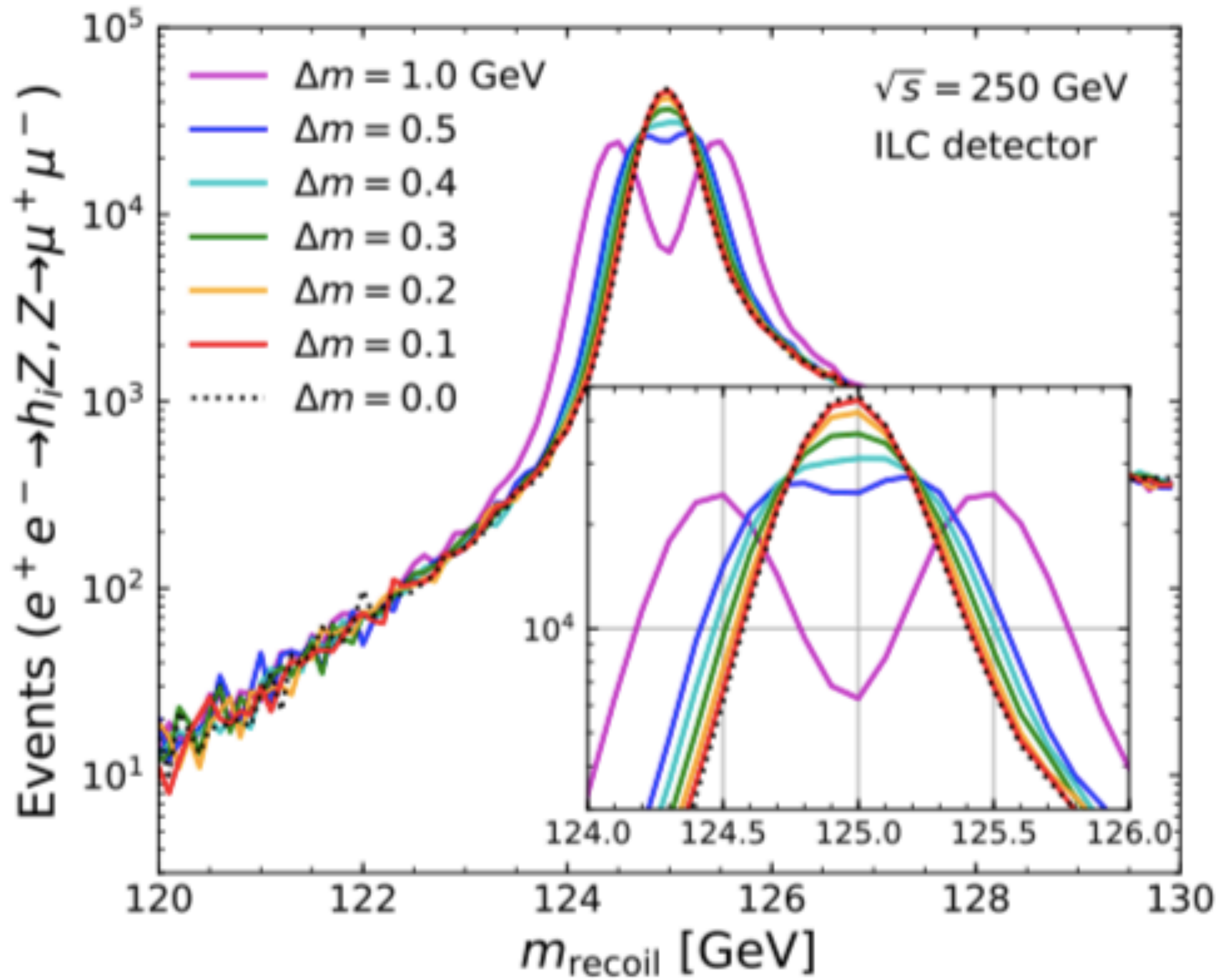
$(s_\alpha c_\alpha)^2 f(m_{h_1}, m_{h_1}) \qquad -2(s_\alpha c_\alpha)^2 f(m_{h_1}, m_{h_2}) \qquad (-s_\alpha c_\alpha)^2 f(m_{h_2}, m_{h_2})$

Sum $= (s_\alpha c_\alpha)^2 (f(1,1) - f(1,2)) + (s_\alpha c_\alpha)^2 (f(2,2) - f(2,1))$ –> 0 for $m_{h_1} \sim m_{h_2}$

Degenerate scalar scenario

S.Abe, G.C.Cho, K.Mawatari,
Phys.Rev.D 104 (2021) 3, 035023





Boltzman eq. approximation

The Boltzmann equation of dimensionless quantities $Y(z) = \frac{n_\chi}{s}$

$$\frac{dY_\chi(z)}{dz} = -\lambda(z)\left(Y_\chi(z)^2 - Y_{\rm eq}(z)^2\right)$$

$$\lambda(z) = \frac{1}{8}\sqrt{\frac{\pi g_*(T)}{45}}\frac{m_{\rm Pl}}{m_\chi}\frac{\int_{4z^2}^\infty dx \mathcal{F}\left(T^2 x\right)\sqrt{x-4z^2}K_1(z)}{\left[z^2 K_2\left(z\right)\right]^2}, \quad \langle \sigma v\rangle = \frac{1}{2s\sqrt{1-4m_\chi^2/s}}\mathcal{F}(s)$$

Approximation

$$Y_\chi(\infty) \simeq \frac{Y_\chi\left(z_d\right)}{1+Y_\chi\left(z_d\right)A_d} = \frac{\left(1+\delta_d\right)Y_\chi^{\rm eq}\left(z_d\right)}{1+\left(1+\delta_d\right)Y_\chi^{\rm eq}\left(z_d\right)A_d}$$

conventional expression

$$z_d \simeq \log\left[\frac{\delta_d\left(2+\delta_d\right)}{1+\delta_d}\frac{\lambda(z)\left(\hat{Y}_\chi^{\rm eq}(z)\right)^2}{\hat{Y}_\chi^{\rm eq}(z) - \frac{d\hat{Y}_\chi^{\rm eq}(z)}{dz}}\right]_{z=z_d}$$

$$A_d = \int_{z_d}^\infty dz \lambda(z)$$

our expression

$$z_d \simeq \log\left[\frac{\delta_d\left(2+\delta_d\right)}{1+\delta_d}\frac{1}{8\pi^3}\sqrt{\frac{45}{2g_{\rm eff}(T)}}\frac{m_{\rm Pl}}{m_\chi}\mathcal{F}\left(4m_\chi^2\right)\right] - \frac{1}{2}\log\log\left[\frac{\delta_d\left(2+\delta_d\right)}{1+\delta_d}\frac{1}{8\pi^3}\sqrt{\frac{45}{2g_{\rm eff}(T)}}\frac{m_{\rm Pl}}{m_\chi}\mathcal{F}\left(4m_\chi^2\right)\right],$$

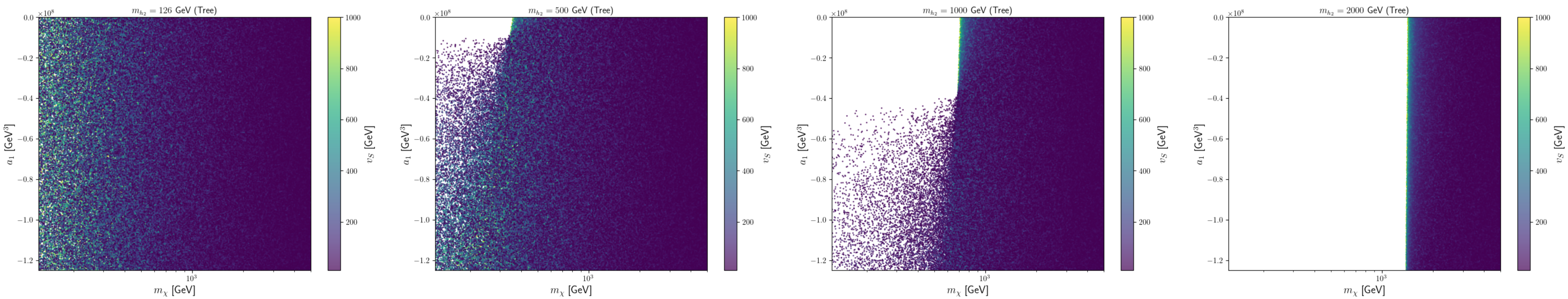
$$A_d \simeq \frac{1}{2}\sqrt{\frac{\pi g_*(T)}{45}}\frac{m_{\rm Pl}}{m_\chi}\cdot\mathcal{F}\left(4m_\chi^2\right)\frac{1}{z_d}\left(1-\frac{27}{16z_d}+\frac{319}{128z_d^2}+\cdots\right)$$

Numerical results

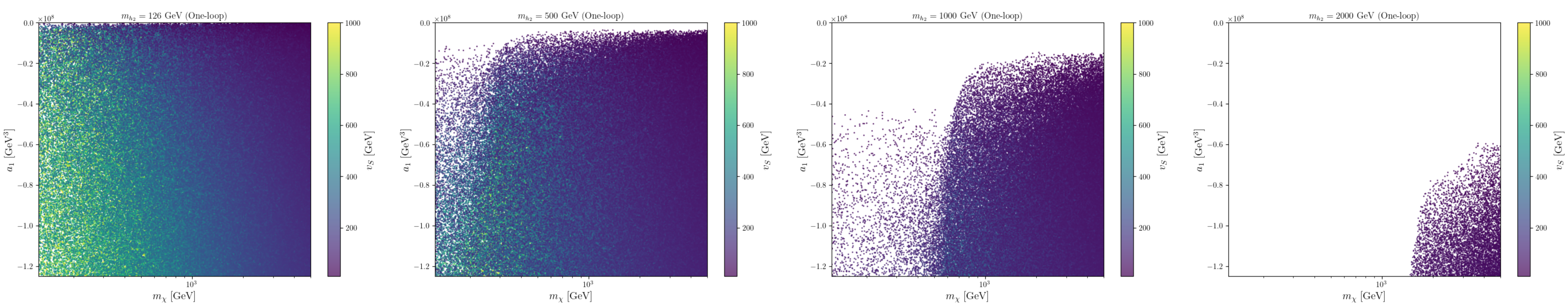
1) Vacuum stability at φ_χ-direction

Tree

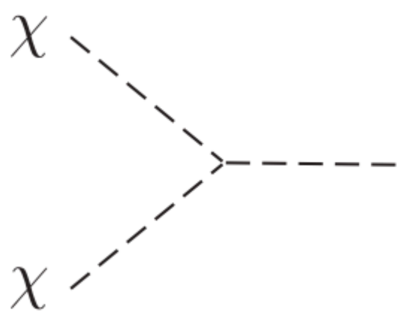
$$m_\chi^2 > \frac{1}{2}\left\{m_{h_1}^2 \sin^2\alpha + m_{h_2}^2 \cos^2\alpha + \sqrt{2}\frac{a_1}{v_S} + \frac{v}{v_S}\left(m_{h_1}^2 - m_{h_2}^2\right)\right\}$$



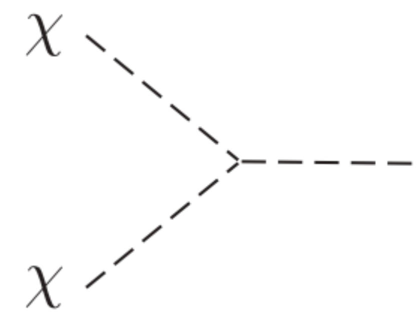
One-loop



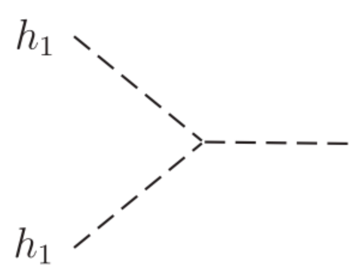
Feynman rules



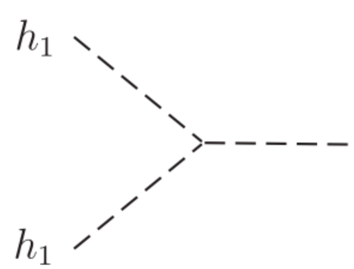
$$h_1 = -i\frac{\bar{D}_1 v}{2}$$



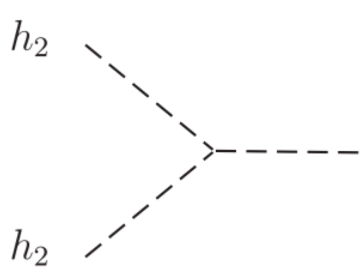
$$h_2 = -i\frac{\bar{D}_2 v}{2}$$



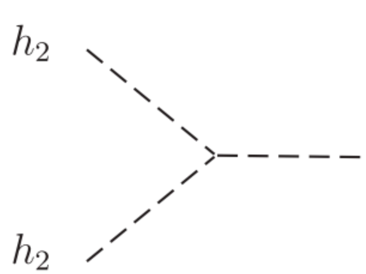
$$h_1 = -\frac{3i}{2}\left(\bar{L}_1 c_\alpha^2 + \bar{D}_1 s_\alpha^2 + \frac{4c_3}{3v} s_\alpha^3\right) v \equiv -\frac{3i}{2}\lambda_{111} v$$



$$h_2 = -\frac{i}{2}\left[\bar{L}_2 c_\alpha^2 + \bar{D}_2 s_\alpha^2 - 2(\bar{L}_1 - \bar{D}_1) c_\alpha s_\alpha + \frac{4c_3}{v} c_\alpha s_\alpha^2\right] v \equiv -\frac{i}{2}\lambda_{112} v$$

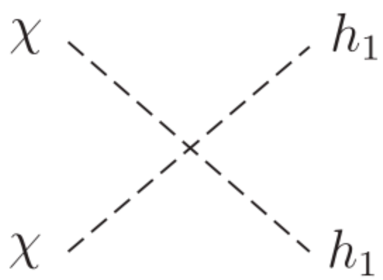


$$h_1 = -\frac{i}{2}\left[\bar{L}_1 c_\alpha^2 + \bar{D}_1 s_\alpha^2 - 2(\bar{L}_2 - \bar{D}_2) c_\alpha s_\alpha + \frac{4c_3}{v} c_\alpha^2 s_\alpha\right] v \equiv -\frac{i}{2}\lambda_{122} v$$

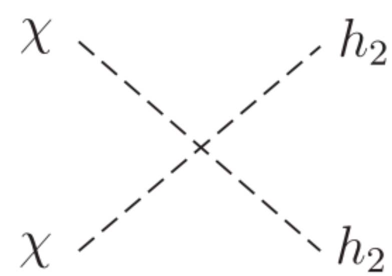


$$h_2 = -\frac{3i}{2}\left(\bar{L}_2 c_\alpha^2 + \bar{D}_2 s_\alpha^2 + \frac{4c_3}{3v} c_\alpha^3\right) v \equiv -\frac{3i}{2}\lambda_{222} v$$

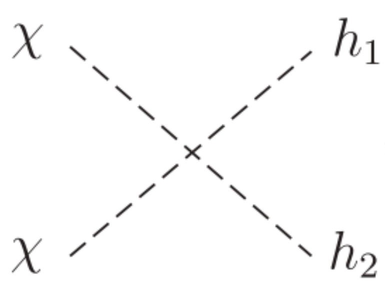
Feynman rules



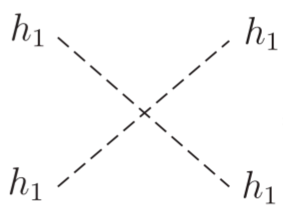
$$= -i\frac{\delta_2 \cos^2\alpha + d_2 \sin^2\alpha}{2}$$



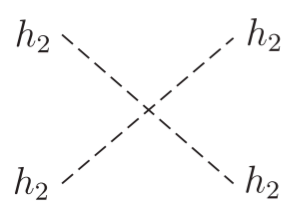
$$= -i\frac{\delta_2 \sin^2\alpha + d_2 \cos^2\alpha}{2}$$



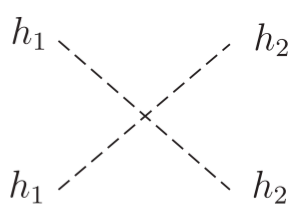
$$= i\frac{\delta_2 - d_2}{2}\sin\alpha\cos\alpha$$



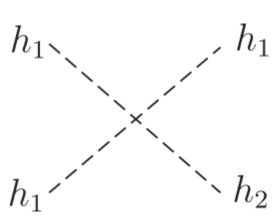
$$= -\frac{3i}{2}\left(\lambda c_\alpha^4 + d_2 s_\alpha^4 + 2\delta_2 c_\alpha^2 s_\alpha^2\right)$$



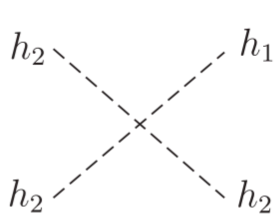
$$= -\frac{3i}{2}\left(\lambda s_\alpha^4 + d_2 c_\alpha^4 + 2\delta_2 c_\alpha^2 s_\alpha^2\right)$$



$$= -\frac{i}{2}\left[3(\lambda + d_2)c_\alpha^2 s_\alpha^2 + \delta_2(1 - 6c_\alpha^2 s_\alpha^2)\right]$$



$$= -\frac{3i}{2}\left[(\delta_2 - \lambda)c_\alpha^2 + (d_2 - \delta_2)s_\alpha^2\right] c_\alpha s_\alpha$$



$$= -\frac{3i}{2}\left[(\delta_2 - \lambda)s_\alpha^2 + (d_2 - \delta_2)c_\alpha^2\right] c_\alpha s_\alpha$$

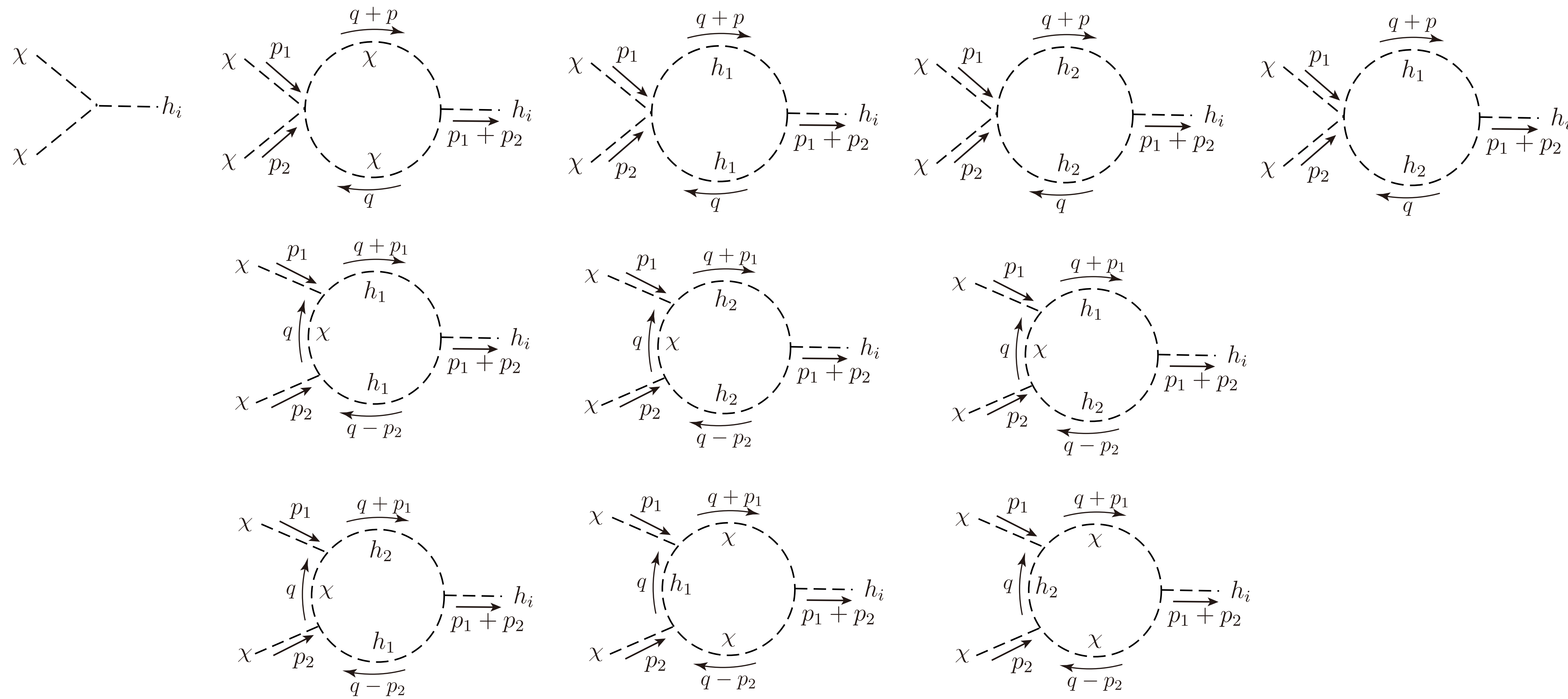
Finite counterterms

$$
\begin{aligned}
\delta_2^{(1)} &= -\frac{2}{v v_S}\frac{\partial^2 V_{\rm CW}}{\partial\varphi\partial\varphi_S} \\
&= -\frac{d_2\delta_2}{64\pi^2}\log\frac{m_\chi^2}{\bar\mu^2} \\
&\quad -\frac{2}{v v_S}\left[\left\langle\frac{\partial^2\bar m_{h_2}^2}{\partial\varphi\partial\varphi_S}\right\rangle\frac{m_{h_2}^2}{32\pi^2}\left(\log\frac{m_{h_2}^2}{\bar\mu^2}-1\right)+\left\langle\frac{\partial\bar m_{h_2}^2}{\partial\varphi}\right\rangle\left\langle\frac{\partial\bar m_{h_2}^2}{\partial\varphi_S}\right\rangle\frac{1}{32\pi^2}\log\frac{m_{h_2}^2}{\bar\mu^2}\right], \\
d_2^{(1)} &= -\frac{2}{v_S^2}\left(\frac{\partial^2 V_{\rm CW}}{\partial\varphi_S^2}-\frac{1}{v_S}\frac{\partial V_{\rm CW}}{\partial\varphi_s}\right), \\
&= -\frac{d_2^2}{64\pi^2}\log\frac{m_\chi^2}{\bar\mu^2} \\
&\quad -\frac{2}{v_S^2}\left[\left(\left\langle\frac{\partial^2\bar m_{h_2}^2}{\partial\varphi_S^2}\right\rangle-\frac{1}{v_S}\left\langle\frac{\partial\bar m_{h_2}^2}{\partial\varphi_S}\right\rangle\right)\frac{m_{h_2}^2}{32\pi^2}\left(\log\frac{m_{h_2}^2}{\bar\mu^2}-1\right)+\left\langle\frac{\partial\bar m_{h_2}^2}{\partial\varphi_S}\right\rangle^2\frac{1}{32\pi^2}\log\frac{m_{h_2}^2}{\bar\mu^2}\right].
\end{aligned}
$$

$$
\begin{aligned}
D_1^{(1)} &= D_1\left(1-\frac{d_2}{64\pi^2}\log\frac{m_\chi^2}{\bar\mu^2}\right) \\
&\quad -\frac{2}{v v_S}\left[\left(\left\langle\frac{\partial^2\bar m_{h_2}^2}{\partial\varphi\partial\varphi_S}\right\rangle c_\alpha+\left\langle\frac{\partial^2\bar m_{h_2}^2}{\partial\varphi_S^2}\right\rangle s_\alpha-\frac{s_\alpha}{v_S}\left\langle\frac{\partial\bar m_{h_2}^2}{\partial\varphi_S}\right\rangle\right)\frac{m_{h_2}^2}{32\pi^2}\left(\log\frac{m_{h_2}^2}{\bar\mu^2}-1\right)\right. \\
&\quad \left.+\left(\left\langle\frac{\partial\bar m_{h_2}^2}{\partial\varphi}\right\rangle c_\alpha+\left\langle\frac{\partial\bar m_{h_2}^2}{\partial\varphi_S}\right\rangle s_\alpha\right)\left\langle\frac{\partial\bar m_{h_2}^2}{\partial\varphi_S}\right\rangle\frac{1}{32\pi^2}\log\frac{m_{h_2}^2}{\bar\mu^2}\right], \\
D_2^{(1)} &= D_2\left(1-\frac{d_2}{64\pi^2}\log\frac{m_\chi^2}{\bar\mu^2}\right) \\
&\quad -\frac{2}{v v_S}\left[\left(-\left\langle\frac{\partial^2\bar m_{h_2}^2}{\partial\varphi\partial\varphi_S}\right\rangle s_\alpha+\left\langle\frac{\partial^2\bar m_{h_2}^2}{\partial\varphi_S^2}\right\rangle c_\alpha-\frac{c_\alpha}{v_S}\left\langle\frac{\partial\bar m_{h_2}^2}{\partial\varphi_S}\right\rangle\right)\frac{m_{h_2}^2}{32\pi^2}\left(\log\frac{m_{h_2}^2}{\bar\mu^2}-1\right)\right. \\
&\quad \left.+\left(-\left\langle\frac{\partial\bar m_{h_2}^2}{\partial\varphi}\right\rangle s_\alpha+\left\langle\frac{\partial\bar m_{h_2}^2}{\partial\varphi_S}\right\rangle c_\alpha\right)\left\langle\frac{\partial\bar m_{h_2}^2}{\partial\varphi_S}\right\rangle\frac{1}{32\pi^2}\log\frac{m_{h_2}^2}{\bar\mu^2}\right]
\end{aligned}
$$

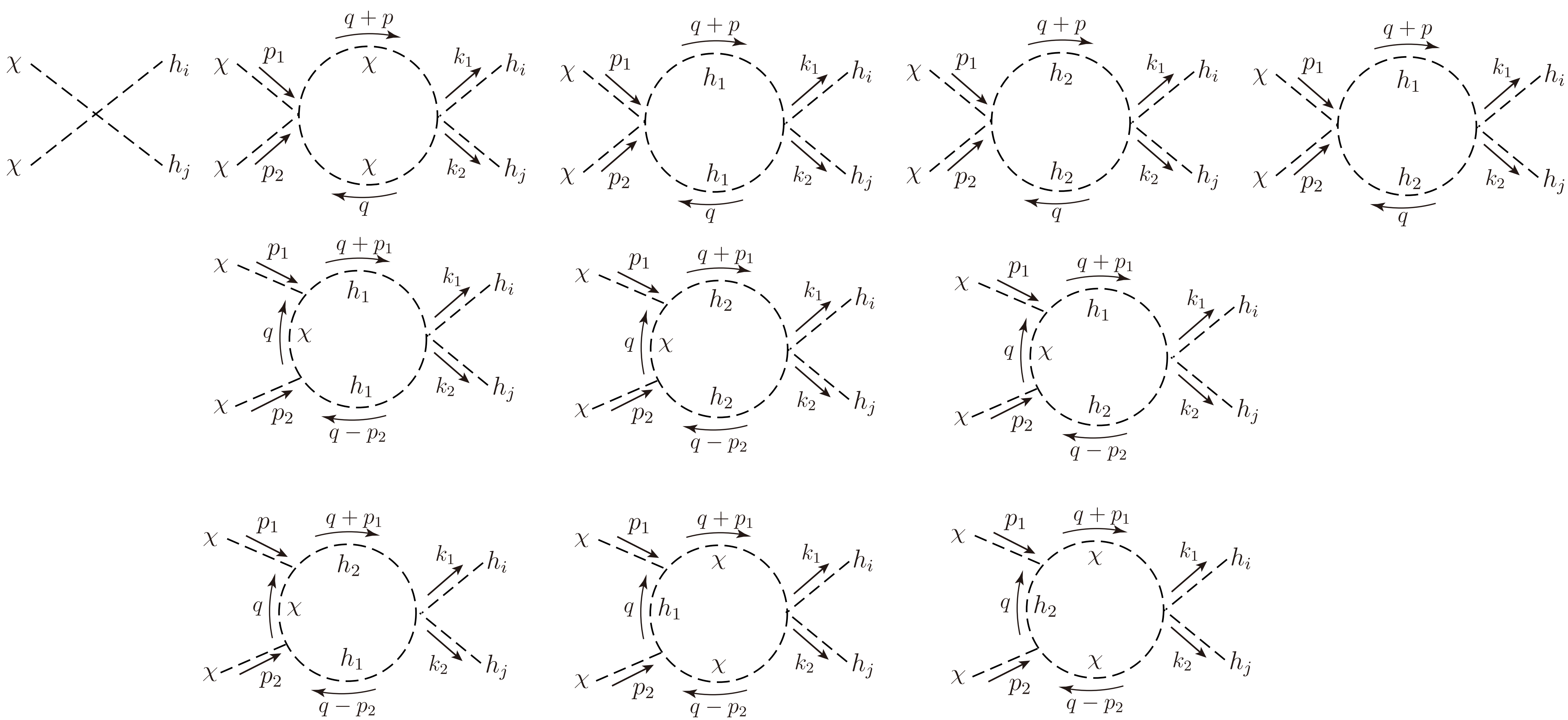
One-loop corrections to the vertices including χ-particle

$\chi\chi h_i$-vertex



One-loop corrections to the vertices including χ-particle

$\chi\chi h_i h_j$-vertex



One-loop corrections to the vertices including χ-particle

For both the DM direct detection and the relic abundance, the relevant amplitudes are dominated by contributions with vanishing incoming momenta. In addition, we omit the contribution from the diagrams containing the h_1-propagator here.

Accordingly, the one-loop level vertex functions are given as follows.

The three-point functions

$$
\begin{aligned}
&\Gamma_{\chi\chi h_1}(0) = \Gamma^{\text{basic}}_{\chi\chi h_1}(0) + \Gamma^{\text{higher}}_{\chi\chi h_1}(0) \\
&= -\frac{\varphi}{2}\left\{ D_1 + \delta^{(1)} c_\alpha^2 + d_2^{(1)} s_\alpha^2 - \frac{3D_1 d_2}{64\pi^2}\log\frac{m_\chi^2}{\bar{\mu}^2} - \frac{\left(\delta_2 s_\alpha^2 + d_2 c_\alpha^2\right)\lambda_{122}}{64\pi^2}\log\frac{m_{h_2}^2}{\bar{\mu}^2} \right. \\
&\left. -\frac{D_2\varphi^2}{64\pi^2}\left[\frac{D_2\lambda_{122}m_\chi^2 - D_1^2 m_{h_2}^2}{\left(m_{h_2}^2 - m_\chi^2\right)^2}\log\frac{m_{h_2}^2}{m_\chi^2} - \frac{D_2\lambda_{122} - D_1^2}{m_{h_2}^2 - m_\chi^2}\right]\right\},
\end{aligned}
$$

$$
\begin{aligned}
&\Gamma_{\chi\chi h_2}(0) = \Gamma^{\text{basic}}_{\chi\chi h_2}(0) + \Gamma^{\text{higher}}_{\chi\chi h_2}(0) \\
&= -\frac{\varphi}{2}\left\{ D_2 + \delta^{(1)} s_\alpha^2 + d_2^{(1)} c_\alpha^2 - \frac{3D_2 d_2}{64\pi^2}\log\frac{m_\chi^2}{\bar{\mu}^2} - \frac{3\left(\delta_2 s_\alpha^2 + d_2 c_\alpha^2\right)\lambda_{222}}{64\pi^2}\log\frac{m_{h_2}^2}{\bar{\mu}^2} \right. \\
&\left. -\frac{D_1^2\varphi^2}{64\pi^2}\left[\frac{3\lambda_{222}m_\chi^2 - D_2 m_{h_2}^2}{\left(m_{h_2}^2 - m_\chi^2\right)^2}\log\frac{m_{h_2}^2}{m_\chi^2} - \frac{3\lambda_{222} - D_2}{m_{h_2}^2 - m_\chi^2}\right]\right\}.
\end{aligned}
$$

One-loop corrections to the vertices including χ-particle

The four-point functions

$$\begin{aligned}
&\Gamma_{\chi\chi h_1 h_1}(0) = \Gamma^{\text{basic}}_{\chi\chi h_1 h_1}(0) + \Gamma^{\text{higher}}_{\chi\chi h_1 h_1}(0) \\
&= -\frac{\left(\delta_2 + \delta_2^{(1)}\right) c_\alpha^2 + \left(d_2 + d_2^{(1)}\right) s_\alpha^2}{2} + \frac{3d_2\left(\delta_2 c_\alpha^2 + d_2 s_\alpha^2\right)}{128\pi^2} \log\frac{m_\chi^2}{\bar{\mu}^2} \\
&\quad + \frac{\delta_2 s_\alpha^2 + d_2 c_\alpha^2}{128\pi^2}\left[3\left(\lambda + d_2 - 2\delta_2\right) c_\alpha^2 s_\alpha^2 + \delta_2\right] \log\frac{m_{h_2}^2}{\bar{\mu}^2} \\
&\quad + \frac{D_2^2 \varphi^2}{128\pi^2}\left[\frac{\left(3\left(\lambda + d_2 - 2\delta_2\right) c_\alpha^2 s_\alpha^2 + \delta_2\right) m_\chi^2 - \left(\delta_2 c_\alpha^2 + d_2 s_\alpha^2\right) m_{h_2}^2}{\left(m_{h_2}^2 - m_\chi^2\right)^2} \log\frac{m_{h_2}^2}{m_\chi^2} \right.\\
&\quad \left. - \frac{3\left(\lambda + d_2 - 2\delta_2\right) c_\alpha^2 s_\alpha^2 + \left(\delta_2 - d_2\right) s_\alpha^2}{m_{h_2}^2 - m_\chi^2}\right],
\end{aligned}$$

$$\begin{aligned}
&\Gamma_{\chi\chi h_2 h_2}(0) = \Gamma^{\text{basic}}_{\chi\chi h_2 h_2}(0) + \Gamma^{\text{higher}}_{\chi\chi h_2 h_2}(0) \\
&= -\frac{\left(\delta_2 + \delta_2^{(1)}\right) s_\alpha^2 + \left(d_2 + d_2^{(1)}\right) c_\alpha^2}{2} + \frac{3d_2\left(\delta_2 s_\alpha^2 + d_2 c_\alpha^2\right)}{128\pi^2} \log\frac{m_\chi^2}{\bar{\mu}^2} \\
&\quad + \frac{3\left(\delta_2 s_\alpha^2 + d_2 c_\alpha^2\right)}{128\pi^2}\left(\lambda s_\alpha^4 + d_2 c_\alpha^4 + 2\delta_2 s_\alpha^2 c_\alpha^2\right) \log\frac{m_{h_2}^2}{\bar{\mu}^2} \\
&\quad + \frac{D_1^2 \varphi^2}{128\pi^2}\left[\frac{3\left(\lambda s_\alpha^4 + d_2 c_\alpha^4 + 2\delta_2 c_\alpha^2 s_\alpha^2\right) m_\chi^2 - \left(\delta_2 s_\alpha^2 + d_2 c_\alpha^2\right) m_{h_2}^2}{\left(m_{h_2}^2 - m_\chi^2\right)^2} \log\frac{m_{h_2}^2}{m_\chi^2} \right.\\
&\quad \left. - \frac{3\left(\lambda s_\alpha^4 + d_2 c_\alpha^4 + 2\delta_2 c_\alpha^2 s_\alpha^2\right) - \left(\delta_2 s_\alpha^2 + d_2 c_\alpha^2\right)}{m_{h_2}^2 - m_\chi^2}\right],
\end{aligned}$$

$$\begin{aligned}
&\Gamma_{\chi\chi h_1 h_2}(0) = \Gamma^{\text{basic}}_{\chi\chi h_1 h_2}(0) + \Gamma^{\text{higher}}_{\chi\chi h_1 h_2}(0) \\
&= c_\alpha s_\alpha \left\{ \frac{\delta_2 + \delta_2^{(1)} - d_2 - d_2^{(1)}}{2} - \frac{3d_2\left(\delta_2 - d_2\right)}{128\pi^2} \log\frac{m_\chi^2}{\bar{\mu}^2} \right.\\
&\quad + \frac{3\left(\delta_2 s_\alpha^2 + d_2 c_\alpha^2\right)}{128\pi^2}\left[\left(\delta_2 - \lambda\right) s_\alpha^2 + \left(d_2 - \delta_2\right) c_\alpha^2\right] \log\frac{m_{h_2}^2}{\bar{\mu}^2} \\
&\quad + \frac{D_2^2 v^2}{128\pi^2}\left[\frac{3\left(\left(\delta_2 - \lambda\right) s_\alpha^2 + \left(d_2 - \delta_2\right) c_\alpha^2\right) m_\chi^2 - \left(\delta_2 - d_2\right) m_{h_2}^2}{\left(m_\chi^2 - m_{h_2}^2\right)^2} \log\frac{m_{h_2}^2}{m_\chi^2} \right.\\
&\quad \left.\left. - \frac{3\left(\left(\delta_2 - \lambda\right) s_\alpha^2 + \left(d_2 - \delta_2\right) c_\alpha^2\right) - \left(\delta_2 - d_2\right)}{m_{h_2}^2 - m_\chi^2}\right]\right\}.
\end{aligned}$$